

Instructor – Hello.

Let's calculate beta for a two-tailed test.

Our example will be example 9-11 from the textbook.

In that example, what we find is that the ideal diameter for an item is 2.25.

And so the company wants to test to see if there is a difference from this ideal value.

So they're interested in knowing if it's too high or too low compared to that 2.25.

So they want to conduct a two-tailed test.

Before they take a sample, and their sample size will be 20 items, they know that the population standard deviation is 0.005.

But the question that the manager has is, what is the likelihood that they would commit a type II error? And the likelihood, remember, of a type II error is denoted by beta, which is the probability of committing that type II error and not rejecting the null hypothesis when, in fact, the null hypothesis is false.

So while the ideal measurement value is 2.25, they would like to know the probability of committing a type II error if the true mean is 2.255.

So how likely is it that they would conclude that they are at 2.25 whenever the true mean is at 2.255?

So we need to calculate beta, the probability of committing a type II error in order to be able to answer that question.

Now because this is a two-tailed test, we will go to the template labeled two-tailed test, and we will use that test.

So let's take a look here.

I'm going to clear out the inputs, and we will go through our problem.

And everything that is in a cell that is shaded gray requires an input from the data analyst.

Now the company has decided to take a sample of 20 items.

Their hypothesized mean, what they believe is that it's at the ideal diameter of 2.25.

The population standard deviation is known, and it's 0.005.

So it's small.

Let's suppose they use an alpha value of 0.05.

Now we can take a look here and see what has been calculated for us.

What we see is the standard error of the mean is calculated using the formula for the standard error, the definition of a standard error for the mean, which is the standard deviation divided by the square root of the sample size.

Now this is a two-tailed test.

So we will have a rejection region in the lower tail and a rejection region in the upper tail.

And the values of z, such that we would reject the null hypothesis will

be any z-value that is less than or equal to negative 1.96,
or any z-value that is greater than
or equal to a positive 1.96.

Now we could also state these rejection regions
in terms of values for the sample mean.

So when we take our sample and we calculate the sample mean,
 \bar{x} , then we would reject the null hypothesis.

If we have a sample mean that is less than or equal to 2.2478
or if the sample mean is greater than or equal to 2.2522.

Now the question that the manager has is,
what is the probability that he would fail to reject the null
hypothesis when, in fact, it's false
if the true mean is 2.255.

So if that's the true mean, how likely
is it that he's going to say I have not
rejected the null hypothesis, and I'm
concluding that the hypothesized mean value is the 2.25?
well we see that what happens is we calculate z-values
for the true population mean based off of these values
that we've computed down here for the values of the sample
mean for which the null hypothesis is rejected.

We then use those values to calculate
the probability of committing a type II error and the power
of the test.

We use Excel functions to do that.

These calculations are made for you automatically
once you provided the inputs.

And we can see in this case that the probability of committing
a type II error is very low, 0.006.

This test has real strong power, a high power at 0.994,
meaning that we would correctly reject the null hypothesis when
it is false with a probability of 0.9940.

So we can see in this case that because the standard deviation
for the population is so very small,
we have a very small standard error of the mean.

So we're going to have a very small margin of error,
and we see that the probability of committing a type II error
is very small.

Now also recall that one of the things that
will affect the probability of committing this type II error
is the population standard deviation.

And because the population standard deviation
in this example is so small, we have a very small margin
of error.

So we have high power of the test, low probability of committing a type II error.