CONCRETE FADING WITH TAPE DIAGRAMS AND ITS EFFECTS ON STUDENTS IN HIGH SCHOOL MATH

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ABSTRACT

Mathematics in high school can be a challenge for some students, while for others, it will just 'click'. Algebra I, in particular, may act as the basis for building the mathematical foundation. For students who struggle with mathematics, concrete fading is an instructional technique that offers them support.

The presentation consisted of two parts. The first part involved describing a variation of concrete fading defined in this particular study that analyzed the impact the instructional technique had on students in an Algebra I class solving linear equations and word problems. The variation of concrete fading involved using a tape diagram that enabled students to help visualize the quantitative relationships embedded in the story problem (Chen, 2022).

The second part allowed participants to work on various problems while using the tape diagram to help solve the problems. The objectives of this session were for participants to be able to: define concrete fading, define a variation of concrete fading with a tape diagram, envision how a tape diagram can be used to solve word problems that involve linear equations and understand how a tape diagram can be used to help visualize the quantitative relationship embedded in the word problem.

Throughout the presentation, participants experienced how concrete fading with tape diagrams could be used in a high school Algebra I class. Participants took part in a discussion as a whole group and then within smaller groups, where they were given practice problems involving the tape diagram. Participants learned how to use the tape diagram along with its benefits and challenges.

INTRODUCTION TO CONCRETE FADING

Concrete fading is based on Bruner's (1966) modes of representations that follows a three-stage progression, in which Fyfe and Nathan (2018) argued as an instructional technique (Chen, 2022). The three stage progression included three stages: enactive, iconic and symbolic (Bruner, 1966; Fyfe & Nathan, 2018). All of the stages relate to each other as they all represent the same underlying concept as shown in an example below:

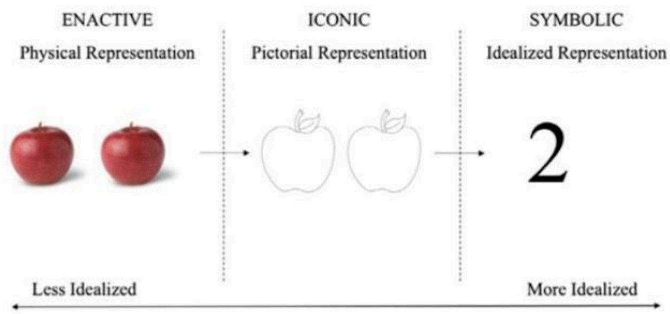


Figure 1
Three-stage Progression (Fyfe & Nathan, 2018, p. 2)

The first stage is the enactive stage, where students have physical objects that can allow them to physically feel and see the concept (Chen, 2022). Next, students progress to the second stage, the iconic stage, where the physical objects are removed and students see a picture of the concept. The final stage, the symbolic stage, is where students no longer have physical objects or see pictures; rather, they see symbols. In relation to Figure 1, the concept is the numeral 2. In the first stage, students are working with two physical apples to visualize the concept of numeral 2 (Chen, 2022). In the second stage, students phase out the physical objects and see the picture of two apples. Finally, in the third stage, the physical objects and pictures are phased out, so the students only see the numeral 2 in symbols. Throughout all of the three stages, the underlying concept was the same for every stage, which is the numeral 2 (Chen, 2022).

VARIATION OF CONCRETE FADING

The presentation applied a variation of concrete fading that was applied in my dissertation manuscript, “Concrete Fading and its Effect on Students’ Algebraic Problem Solving and Computational Skills” (Chen, 2022). The first stage applied the Theory of Realistic Mathematics Education, RME (Heuvel-Panhuizen & Drijvers, 2014), by beginning with a real-world context, such as a word problem that encouraged students to relate to the context and real-world application (Gravemeijer, 1994; Barnes, 2005). The second stage is the ‘fading’ stage that includes, in this variation, a tape diagram. The tape diagram will help students create the final stage, which is the symbolic, mathematical equation that is embedded in the word problem. The tape diagram was proven beneficial for students in a study of elementary school students solving word problems conducted by Ng and Lee (2009). Students who used this particular model were much more detailed in their answer (Ng & Lee, 2009; Chen, 2022). The final stage is where students create an equation that is based on the word problem, which is supported by the tape diagram. The comparison of models is shown in figure 2 below.

Study	Stage 1: Concrete	Stage 2: Semi-concrete	Stage 3: Abstract

Fyfe and Nathan, 2018	Real object	Sketch	Number
Ng & Lee, 2009	Text	Pictorial	Symbolic
Current study	Real-world context supported by RME (Heuvel-Panhuizen & Drijvers, 2014)	Tape diagram	Equation

Figure 2
Comparison of Models (Chen, 2022, p. 14)

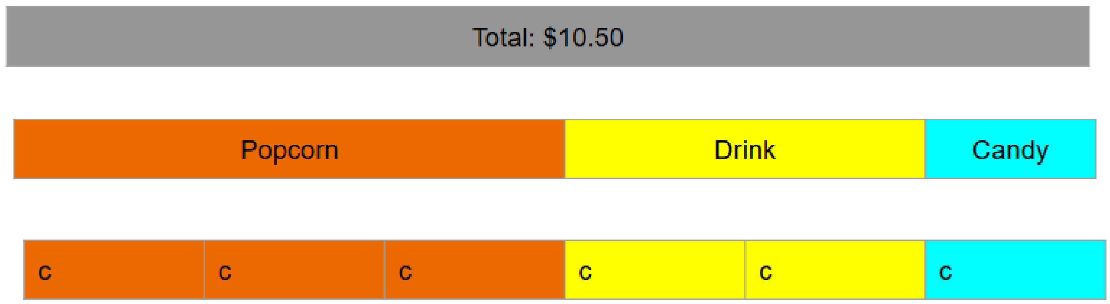
PRACTICING WITH CONCRETE FADING

Participants were introduced to the concept of concrete fading as well as the variation that I used as discussed above. Next, they had the opportunity to work on word problems involving linear equations while applying the concrete fading method with tape diagrams. The first stage was a word problem that applied RME (Heuvel-Panhuizen & Drijvers, 2014), by having the word problem consist of something they can relate to and envision. Second, they represent that word problem as a tape diagram. Finally, they create an equation from the tape diagram and solve for the variable.

I showed a few examples to the participants before distributing the handout. Not many of them were familiar with the concept of concrete fading. However, some participants were familiar with the second stage, which involved using the tape diagram.

The first example of the session was from Brack (2023): *Eva bought popcorn, candy and a drink at the movies. The popcorn was three times as expensive as the candy and the drink was twice as expensive as the candy. Eva spent a total of $10.50. How much did the drink cost?* Participants were given one minute to try to solve using a tape diagram until the solution was shown, which is shown below in figure 3.

Since the price of the popcorn was three times as much as the candy, there are three c's under the popcorn to represent the cost. Similarly, since the drink was twice as much as the candy, there are two c's to represent that cost. Since everything was in terms of the candy, there was just one c to represent the cost of the candy. The tape diagram showed that now that everything was in terms of 'c' and the total of everything was $10.50, all of those c's added up to $10.50. Thus, the equation generated by the word problem and tape diagram was: 6c-10.50. To solve how much the candy cost, participants simply divided by 6 to show that one candy cost $1.75. Not all tape diagrams will look the same as some of the participants had slightly different tape diagrams, but all the tape diagrams showed the same underlying concept.



$$6c=10.50$$

$$c=1.75$$

Figure 3
Solution to popcorn problem (Brack, 2023)

The session then concluded with participants completing the handout that I created. The handout consisted of 10 problems. Some of the problems required more steps to construct a tape diagram to solve while others did not require as many steps. I monitored participants as they worked in case anyone had any questions. It did turn out that most of the participants struggled trying to use the tape diagram to solve the following problem: *A club has 600 members. 60% of them were males. When 200 new members joined the club, the percentage of male members was reduced to 50%. How many of the* ***new*** *members were males?* (*Tape diagrams* 2014)

This problem presented a challenge because participants solved it without using tape diagrams, but used multiple equations instead. Thus, using a tape diagram was a little bit more difficult for this particular problem. Since the problem used percentages it was easier to split the tape diagram in a multiple of 10 so that each block could present 1% of 10%. For simplicity, I split it in blocks of 10 as shown below in figure 4. The tape diagram can help students better understand because it can help students visualize what 60% of 600 is and also 50% of the new total when 200 more members were added. Since 50% is half and the number of males was reduced to 50% of the new total, then students could count half the number of tiles and add up each number in that box of the tape diagram. In this case, each box was 80, so they could count 80 five times to get 400. The equations helped by dividing to see what each box would be and then subtracting the new total of 400 from the old total of 360 to get an answer of 40 males.

Since we're looking at percentages, portion the tape diagram into 10 or 100.

Splitting into $\frac{600\ members}{10\ spaces} = 60\ members\ per\ one\ space$

|--600 members—---------------------------------|

60	60	60	60	60	60				

60+60+60+60+60+60=360

When adding 200 more members, 50% of 600+200=800 is 400.

Since we're looking at percentages, portion the tape diagram into 10 or 100.

Splitting into $\frac{800\ members}{10\ spaces} = 80\ members\ per\ one\ space$

|--800 members—-------------------------------------|

80	80	80	80	80					

80+80+80+80+80=400 males

400-360=40

Total # of males - the 'old' male members= 'new' male members

40 of the **new** members were male.

Figure 4

Solution to percentage problem (*Tape diagrams* 2014)

CONCLUSION

The variation of concrete fading that was defined in this presentation began with a real-world context that students were able to relate to, then phased into constructing a tape diagram that represented the quantitative relationship embedded in that particular word problem (Chen, 2022) and then concluded with students generating an equation from that tape diagram. Concrete fading with tape diagrams can be beneficial for students, especially with lower-level students using the tape diagram as discussed earlier (Ng & Lee, 2009). It is important to note, however, that solving the word problems in this particular format is not the 'only' way, rather, it is a method that can be proven to be more beneficial for students and has the potential to help lower-level students.

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