EXPLORING SOLIDS WITH TECHNOLOGY

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Spatial reasoning, or being able to think and reason in three dimensions, includes spatial visualization, orientation, and relations (Rich & Brendefur, 2018). Being able to think spatially in the early years of school is associated with later academic success (Cheng & Mix, 2013; Clements & Sarama, 2007), thus making a focus on spatial reasoning skills for every child a matter of equitable education (Temple et al., 2020). The study of solids in geometry is a primary content area in which to develop these skills. Attributes of three-dimensional solids are often considered in kindergarten, composition of solids in first grade, and surface area and volume of solids in sixth grade (Common Core State Standards Initiative, 2021).

Although elementary teachers are expected to help their students develop spatial reasoning, the spatial visualization skills of students majoring in elementary education are lower than those of students majoring in mathematics or science education, likely due to differences in their previous experience (Erkek et al., 2017; Marchis, 2017; Metzger, 2019). For this reason, mathematics teacher educators must provide quality experiences to support the growth of spatial awareness and spatial reasoning skills in pre-service elementary teachers (Blanco et al., 2019).

Three-dimensional models are integral to the understanding of solids, and advancements in technology have provided new ways to design such models. For example, 3D printing technology allows the creation of models that would be difficult to build with available materials. In addition, virtual models are becoming an important addition to traditional physical models since they allow greater flexibility in design and manipulation (Moral-Sanchez & Siller, 2022) and allow embodiment within a learning environment (Thomsen et al., 2023). Virtual reality has been shown to have a positive effect on learning geometric concepts and on student engagement while studying geometry (Song & Lee, 2002; Su et al., 2022). The project described in this paper uses technology, including virtual reality, to support preservice elementary teachers' understanding of the attributes of three-dimensional geometric figures.

Project Description

This project involved writing and implementing activities that used technology to help develop preservice elementary teachers' understanding of attributes of three-dimensional figures. The implementation took place within a Mathematics for Elementary Teachers course at Indiana University Indianapolis that is designed to develop deep conceptual understanding of geometric concepts addressed in preschool through eighth grade. Funding for this project came from a $1000 Extended Reality Initiative Faculty Fellows Grant from Indiana University Indianapolis. These funds were used to purchase a Meta Quest 2 headset and software, the Shapes 3D app for iPad, and a 3D printer and filament.

The project consisted of four activities designed to be completed during class. The activities were implemented in one summer section of the course and in two fall sections of the course. Approximately 35 students were enrolled in the three sections, although not all students were present during each session in which the activities were implemented. Two different instructors implemented the activities. Students were guided through Activities 1, 2, and 4 using written instructions, which are included in Appendix A.

Activity 1 focused on the attributes of pyramids, prisms, and Platonic solids. Before class instruction on the topic, students manipulated virtual models found on the free website **https://polyhedra.tessera.li/**. Their instructions guided them to examine faces, bases, and other attributes of the three-dimensional figures, to look for patterns, and to summarize their findings.

The focus of Activity 2 was the various heights that are referenced in surface area and volume formulas. Students used the Shapes 3D iOS app that was installed on several iPads and iPhones to draw altitudes and slant heights of pyramids and heights of prisms. They then discussed the relationship of the various measures of height to calculating surface area and volume. Class instruction explaining height followed Activity 2.

For Activity 3, students used the Surface Area lesson in Prisms Math, which was installed on a Meta Quest 2 virtual reality (VR) headset. The view of the student operating the headset was cast to a screen in the classroom so that a group of students could work together through the problem posed: design a container for a 3D printer that uses less material to construct than a cube. This activity took place after nets had been discussed but before surface area instruction.

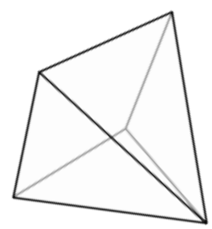
Activity 4, with a focus on volume, made use of a 3D printer and consisted of three parts, each of which was completed on a different day. On Day 1, students determined the dimensions of a specified solid with a given volume. On Day 2, students used free software at www.tinkercad.com to create files for a 3D printer. After the models were printed, the students measured the models on Day 3, calculating their volume and comparing the result to the volume they were given on Day 1.

Analysis and Results

In order to determine the effectiveness of the project, students completed a reflection after each activity. In addition, for Activities 1 and 2, students took a pretest before the activity and then took the same test as a posttest after the activity. Two questions from the Pretest/Posttest for Activity 1 are shown in Figure 1, and three questions from the Pretest/Posttest for Activity 2 are shown in Figure 2.

Figure 1
Pretest Posttest Questions Activity 1

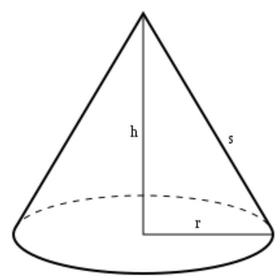
1. How many bases does a prism have?
2. Identify this figure.



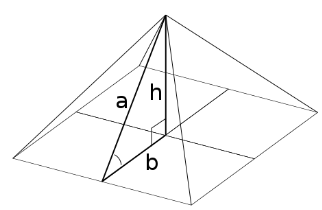
Triangular Pyramid
Triangular Prism
Square Pyramid
Square Prism

Figure 2
Pretest Posttest Questions Activity 2

1. Identify the altitude of the cone.



2. Identify the slant height of the pyramid.



3. Identify the height of the prism.

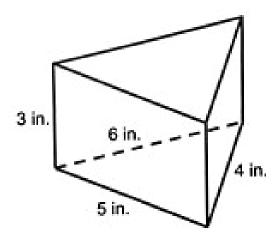


Table 1 summarizes the results from the questions contained in Figures 1 and 2. Five students completed both the pretest and posttest for Activity 1. None of the five students were able to give the number of bases of a prism before the activity, and three of the five students answered this question correctly after the activity. One of the five students was able to correctly identify a square pyramid before the activity, and this increased to four out of five after the activity. In Activity 2, more than half of the 16 students who completed both the pretest and posttest were able to correctly identify an altitude of a cone and a slant height of a pyramid before the activity. This increased to nearly all of the 16 students after the activity. Only one of the 16 students could identify the height of a triangular prism before the activity, and only five could identify this height after the activity.

Table 1
Pretest and Posttest Results

	Number of Students	Number Correct Pretest	Number Correct Posttest
Activity 1			
Question 1	5	0	3
Question 2	5	1	4
Activity 2			
Question 1	16	10	15
Question 2	16	14	16
Question 3	16	1	5

Responses to reflection questions indicated that, in general, students enjoyed and were engaged with the activities. Students enjoyed being able to manipulate the shapes in Activities 1 and 2 and actively building a box in Activity 3, they enjoyed the collaborative aspect of Activities 2 and 3, and they felt that their understanding was deepened. Their reflections also indicated that they still had questions about the concepts after the activities; for example, one student wondered whether a shape could be both a prism and a pyramid after completing Activity 1, and another wondered whether height location would change with different shapes. The virtual reality headset, used in Activity 3, generated the most comments, with some students mentioning how fun it was to use virtual reality and others questioning its applicability for students with disabilities or for students who often experienced motion sickness.

Discussion

All four of the activities appeared to support student learning and increase student engagement. As the pretest-posttest results indicate, more students understood concepts addressed in Activities 1 and 2 after completing the activities than before. Both instructors noted that further instruction on attributes of solids was not necessary after students completed Activity 1 and that students performed better overall on assessments of those concepts than in previous semesters. Students also evidenced through class discussion a stronger understanding of surface area and volume than in previous semesters after Activities 3 and 4. Student reflections indicated engagement in the first three activities, and a common request was for more class time to explore the virtual models. Students were delighted by the outcome of their printed models, and all students wanted to keep the models they had designed.

As part of the activity reflections, students were asked for suggestions for change, and these suggestions were incorporated as much as possible after each implementation. Although the VR headset in Activity 3 provided an immersive experience and intrigued some students, many were reluctant to try it after seeing the steep learning curve their classmates experienced as they became accustomed to the controls. The VR headset was the most expensive and least effective technology used in this project. Besides the time it took to learn to navigate the environment and the limitations of having only one headset, students were disappointed that there was only one example to complete.

This project reinforced research findings that technology can positively affect student learning and engagement in the study of geometry (Song & Lee, 2002; Su et al., 2022). Students understood attributes of solids through exploration of virtual objects and retained that knowledge through the end of the course. They exercised ownership of their learning by designing and printing solids, and they stayed actively engaged throughout all activities. The limitations of the VR headset were primarily those of software, since at the time of this project, little appropriate software was available. Next steps include designing a VR lesson for a university computer science class to develop. This could lead to a series of open source lessons for use with VR headsets in K-12 and college classrooms.

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Appendix A

Activity 1: Properties

VR website: https://polyhedra.tessera.li/

1. **Prisms**

   - Explore the Prisms (not the Antiprisms!) in the right column at the top of the site.
   - What patterns do you see in the number of faces in each prism?
   - What patterns do you see in the shapes of the faces in each prism?
   - The BASES of each prism are two parallel, congruent polygons. In this exploration, they are a different color from the other faces (except for one example). How are these bases related to the name of the prism? Why are the bases not a different color in the square prism?
   - What is the shape of all the faces of each prism that are NOT bases?

2. **Pyramids**

   - Explore the pyramids in the left column on the second screen of the site.
   - What patterns do you see in the number of faces in each pyramid?
   - What patterns do you see in the shapes of the faces in each pyramid?
   - Except for the triangular pyramid, the BASE of each pyramid is a different color and shape from the other faces. How is the base related to the name of the pyramid? Why is the base of the triangular pyramid not a different color from the other faces?
   - What is the shape of all the faces of each pyramid that are NOT the base?

3. Platonic Solids

- Explore the five platonic solids in the top row on the left at the top of the site and fill in the table below.
- Can you tell where the name of each platonic solid comes from?

Name	**What kind of shape is used to make the solid? (What shape are the faces?)**	**How many shapes are used to form the faces of the solid?**
Tetrahedron		
Cube		
Octahedron		
Dodecahedron		
Icosahedron		

Activity 2: All the Heights

Shapes 3D, iOS app

Heights in Pyramids

There are two "heights" in a pyramid (and in a cone).

- The "slant height" is on the surface of the pyramid or cone. It is the length of a segment connecting the vertex and the center of a side of the base. (There is a more general and accurate definition, but this is sufficient for right square pyramids.) Slant height is abbreviated with a lower case *s* or a lower case *l*.

- The "altitude" is often just called the "height" of the pyramid or cone. It is the length of a segment connecting the vertex and the center of the base. Altitude is abbreviated with a lower case *h*.

1. Which is longer: the slant height or the altitude? Why?

2. One of these heights is used to calculate the surface area and one of these is used to calculate the volume. Which do you think makes sense to use for the surface area? Which do you think makes sense to use for the volume?

3. Think of unfolding the pyramid into its net. What is the relation of the slant height to the triangles used in the net?

4. Use the Shapes 3D app to draw the slant height and the altitude of a square pyramid.
    - Launch the Shapes Drawing app.
    - Choose Pyramids.
    - Choose a square pyramid by selecting the square and then the rotating pyramid.
    - Choose the Segments option in the menu at the bottom left.
    - Touch the two endpoints of the line segment you want to draw.
    - When you have drawn both an altitude and a slant height, have a teacher initial _____ before continuing.

Heights in Prisms

The height of a prism is its altitude – that is, the perpendicular distance between the bases. This is abbreviated *h*, like the altitude of a pyramid.

If the prism is triangular, there is another height that is ALSO abbreviated *h*. This is the height of the triangular base, which is part of the familiar formula for the area of a triangle, $A = \frac{1}{2}bh$.

Use the Shapes 3D app to draw both heights in a triangular prism.

- Choose Prisms in the app.
- Choose an equilateral triangular prism by selecting the equilateral triangle and then the rotating prism.
- Choose the Segments option in the menu at the bottom left.
- Touch the two endpoints of the line segment you want to draw.
- When you have drawn both heights drawn, have a teacher initial _____ before continuing.

Activity 4: Volume

CAD Software: https://www.tinkercad.com/

Part 1. Design a solid with a specific volume.

1. Choose one of the following shapes and volumes:
   a) Hemisphere, Volume 25 cm^3
   b) Cylinder, Volume 40 cm^3
   c) Cube, Volume 50 cm^3
   d) Cone, Volume 30 cm^3
   e) Pyramid, Volume 25 cm^3
   f) Triangular Prism, Volume 40 cm^3

2. Design a solid in the given shape with the given volume.
3. You'll probably have to use algebra! Round your answers to the nearest hundredth.

Part 2. Model your solid using CAD software and export your model so that it can be printed.

1. Go to **https://www.tinkercad.com/** and sign up for a personal account.
2. Choose Create and 3D Designs.
3. Drag the shape from the menu on the left to the grid.
4. Stretch or shrink the dimensions to match your design. You can type in the values – it's probably easier. *The dimensions shown are in millimeters, not centimeters. You will have to convert!*
5. When you are ready, choose Export at the top right of the screen.
6. Choose .STL under For 3D Print.
7. OPTIONAL: You can try opening the file. If your computer has 3D software, you will see your model and you can rotate it.
8. Email the .STL file (you should be able to find it under Downloads) to TEACHER EMAIL

Part 3. Check the model!

1. Swap your model with another group.
2. Measure a model from another group and calculate the volume.
3. Check the calculations against the specifications and explain any discrepancies.