FLIPPING THEORY OF INTEREST

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This article's purpose is to describe the journey of two experienced college mathematics professors who decided to work together to flip almost all of an introductory actuarial science class they each taught at their respective institutions. The article will provide a brief overview of what is meant by flipping a class and then walk the reader through the multi-year process and the results of this experience.

Active Learning and Flipped Classrooms

The last 25 years has seen a great deal of pedagogical attention focused on measuring the value of active learning in the STEM classroom versus that of the traditional model of lecturing. Active learning can be described as any learning activity in which the student participates or interacts with the learning process as opposed to passively receiving information. A lot of this work began in the discipline of Physics with the work of E. Mazur (Mazur, 1997) and R.R. Hake (Hake, 1998). Their results showed demonstrably superior learning gains made by students whose classrooms featured significant amounts of active learning activities versus traditional, passive, lecture-dominated classrooms. These results have been replicated in mathematics classrooms, as demonstrated by several large studies involving Calculus classrooms (Epstein, 2013).

Active learning can take on many guises, from the full-bore approach exemplified by the Moore Method to a more modest approach in which the instructor allows students time in class to work through examples on their own or with the assistance of a peer. The relatively recent ability to post and share videos online easily has led to another active learning approach described as *flipping the classroom*. In this approach, the instructor creates video lectures of the material to be covered and requires their students to watch these videos prior to class. Students then spend the class time working through problems related to this material in small groups with assistance, as needed, from the instructor. It is of interest to note that many people point to Khan Academy as one reason for the increasing popularity of this teaching methodology (Thompson, 2011) and (Sparks, 2011)

In fact, it is not unusual to see some instructors use actual Khan Academy video lectures to provide the outside-of-class content.

The recent nature of flipping as a pedagogical approach means that there is not yet a consensus on the effectiveness of this active learning approach; however, a recent meta-analysis of studies done on flipped classrooms did show modest improvements in both student learning outcomes and student satisfaction in flipped classrooms in comparison to more traditional lecture-style classes (Lag, 2019).

Theory of Interest

The two authors have been aware of the flipped classroom technique for several years and had been looking for an ideal opportunity to implement it. That opportunity arose in 2019 when both Dr. Axtell and Dr. Stickles were assigned to teach a Theory of Interest class at their respective institutions during the fall semester. This course is typically taken by second-year actuarial science students (or mathematics students hoping to become actuaries). The list of topics can be tersely described as a thorough exploration of the time-value of money and how this plays out in annuities, bonds, loans, asset-liability matching, immunization, and a study of interest rates. For reference, the text used was Ruckman (2005).

For students hoping to become credentialed actuaries, this material is the focus of the Exam FM-Financial Mathematics of the Society of Actuaries (called Exam 2 by the Casualty Actuarial Society) that many actuarial-focused students try to pass while undergrads. The authors felt that this material might be ideal for exploring in a flipped environment due to its (relative) mathematical approachability. Much of the content of the course revolves around various applications of a small number of mathematical techniques involved in moving piles of money through time, and as such can be compared to the fast-food chain, Taco Bell, which appears to take the same 4 or 5 ingredients and come out with new products on a regular basis.

In Theory of Interest, the mathematical tools in question are exponentiation and geometric series (both finite and infinite). As such, the authors felt that a series of short videos viewed before class could adequately provide the necessary mathematical background for many of the topics in the course, freeing up class time for students to work together in further mastering the techniques and ideas through a series of carefully selected exercises. This use of class time aligns well with the need students hoping to pass Exam FM/2; namely, they need to work a lot of problems to develop both an understanding of the material and the necessary computational speed to pass the professional exam. The co-authors also decided that videos would not feature derivations of the various formulas used in the course. Such derivations would be done in class. Instead, videos would focus on introducing ideas and presenting preliminary examples.

The authors both taught a partially flipped version of Theory of Interest in the fall of 2019. In the years that have followed, we have substantially modified the scope of what

we flip in the course and what we do not. The remainder of this article discusses the processes and factors that influenced the decisions we made.

Class & Institutional Information

Dr. Mike Axtell works at the University of St. Thomas (UST) in Minnesota. UST is a mid-size comprehensive university with an undergraduate enrollment of approximately 6500. UST has been a Center of Actuarial Excellence since 2011 and graduates about 30 actuarial science majors each year. Class sizes in the actuarial program run between 15 and 25. As such, the institution has a significant number of students who choose to attend due to their desire to become credentialed actuaries. The Theory of Interest class is typically taken during a student's second year, though approximately 30% of students will take it in the first year. The sole prerequisite is two semesters of Calculus. The school offers one or two sections of the class each semester, and each course meets either 3 times a week for 65 minutes each or twice a week for 100 minutes each. The flipped approach was conducted in the fall of 2019 with one section of 19 students. The class met in the 3x65 minute format. Axtell had previously taught this course numerous times.

Dr. Joe Stickles teaches at Millikin University, a private, independent university in Illinois that has an enrollment of around 2000. The actuarial science program is recognized as an Advanced Curriculum program by the Society of Actuaries. Enrollment in courses specifically designed for actuarial science students typically run around 10 students, and we graduate between 3 and 5 actuarial students each year. Like UST, Millikin's Theory of Interest class has a prerequisite of two semesters of Calculus; however, with the program being small, this course is offered on an every-other-year basis. This means students may take the course during their second or their third year. During fall of 2019, we offered one section of the course that met for 50 minutes three times per week.

Stage I – Summer&Fall 2019

The summer of 2019 saw Stickles and Axtell work together to decide which course topics they wished to cover in a flipped manner, and which they would cover via a more standard approach of lecturing and small group activities. The decision-making process was influenced as much by pedagogy as by how much time we could devote to creating video content during this particular summer.

The authors decided they wished the students to have an early taste of flipping during the second day of class when covering simple interest and discount, followed by an extended block later in the semester in which flipping was used for about four weeks straight. This extended block covered annuities and loans, topics that the instructors perceived to be most approachable via a series of short videos introducing the ideas and working some examples. The table below provides a breakdown of how topics were covered.

Covered via Flipping	**Covered via in-class lecturing**

Simple Interest and Discount Level Annuities Arithmetic Progression Annuities Continuously Payable Varying Annuities Geometric Progression Annuities Loans	Compound Interest and Discount Constant Force of Interest Varying Interest Rates NPV/IRR/Other Rates of Return Bonds Duration and Convexity Asset-Liability Matching Term Structure of Interest Rates Interest Rate Swaps

All told, approximately one-third of the instructional days utilized a flipped approach.

As mentioned above, one significant reason for only doing one-third of the course in a flipped format was the significant time it took to develop the videos for the topics listed above. Axtell and Stickles spent the summer storyboarding and then shooting 54 videos to support the topics in the left column above. The videos vary between just over one minute to just under seven minutes in length, with the majority in the three-to-five minutes range. Videos were created at UST's educational technology center on a light board set-up. The actual taping of the videos took approximately four full days. In addition to making the videos, the co-authors created a series of exercises for students to work through in small groups during class time. The time required to create these videos and exercises was the major factor in deciding how much of the course to flip. A representative video of from the summer of 2019 can be found through the link below:
Continuously Varying Payments Video

The upfront time demands on flipping a course should not be underestimated. That being said, subsequent offerings of the course require far less preparatory time in that it is more a question of updating and tweaking the material as opposed to wholesale creation. An important part of maintaining such a course is monitoring and adapting the exercises students work on in small groups during class. These exercises must cement key ideas and challenge students to apply these ideas in a variety of settings.

The Semester Experience

We now give an example of a typical flipped class 'day'.

In this class period, we covered arithmetically increasing annuities and perpetuities. The students were assigned to watch and take notes on five videos totaling just over twenty minutes of run-time. The first video provided time-value diagrams for the base cases of an increasing annuity-immediate and annuity-due as well as their present and accumulated value formulas. The next two videos provided a basic example and a more involved example of these. Video four provided time-value diagrams for a base case of increasing perpetuities immediate and due as well as their present value formulas. Finally, video five provided an involved example of determining the present value of an

increasing perpetuity. The videos were made available via YouTube, but they could have been posted to the learning system of the institution as well.

These notes were checked for completeness at the start of class. After this quick check, students formed groups of 3 or 4 and worked through four or five involved exercises on the topics mentioned above. The instructor circulated around the room, checking on work and answering questions that students had on the exercises. For some of the more challenging class exercises, the instructor invited a group with a good solution to write it up on whiteboards up front and take the entire class through their solution. In this way, the instructor made sure that all groups will have solutions to all of the in-class exercises by the end of the period.

The class period described above also points to an unexpected benefit of the flipped classroom. Students appeared more engaged in the derivation of certain key formulas. In the day described above, the instructor spent the last 15-20 minutes of class deriving the present value formulas for both the base case of the arithmetically increasing annuity-immediate and the arithmetically increasing perpetuity-due. Having watched videos, worked problems, and either presented or seen their peers present solutions to some problems on this topic, students were well-positioned to follow the derivation of one or two of the formulas they had been working with. In addition, sitting back and watching the instructor step through a proof of the formula provided an appreciated change of pace in the students' otherwise self-directed study of the material.

Following the class, students would have several more exercises on the topic to work as homework. In addition, students would be required to begin viewing the videos for the next class period.

Phase 1 Results

An article focusing on the experiential aspects of a small roll-out of flipping cannot hope to provide convincing evidence of the superiority of one pedagogical method over another. Rather, we will present our views and the views of our students along with some very limited data comparing learning outcomes of students from the flipped semester versus students in previous semesters. We offer our perspective along with exam score data, as well as student engagement, evaluation scores, and feedback.

We used final exam scores as a preliminary look into student learning. In particular, we compared final exam scores between sections of the course in which the final exam had not changed in recent years. These scores provided assurances that students in our flipped classrooms are doing no worse than in the past, and possibly somewhat better. So, the endeavor at least appears to pass the Hypocratic muster. At UST, the final exam given at the end of the partially flipped class was the same as the one given to the courses Axtell had Axtell taught in a more traditional manner the previous two semesters and was graded using the same rubric. The class average for the exam during the partially flipped semester was an 86.3% (n = 20), while the previous two courses taught by Axtell saw

averages of 82.1% (n = 20) and 85% (n = 18), respectively. (Please note that while final exam scores of Millikin University students also increased from fall 2017 to fall 2019, we only include non-anecdotal data from UST since the sample sizes of n = 7 and n = 9, respectively, at Millikin were considerably smaller.)

Another measure of the learning experience is student engagement as witnessed by the professors. Both Axtell and Stickles observed a far higher level of student engagement during the flipped semester with both the material and with each other. This is not a surprise given the far more active nature of the classroom activities. However, this level of engagement was maintained during the more traditional lecture-based class periods that followed the block of flipped lessons with students far more engaged even with far fewer in-class problems worked in small groups.

A final measure of success is how students perceived the experience. This was measured in two ways – the official student evaluation of the course at UST (IDEA is the student evaluation system used at UST) and a specific questionnaire developed by the co-authors that students anonymously filled in (see Appendix A). (Again, due to small enrollment numbers and response rates at Millikin University, we do not include evaluation data from these students.)

We start with the very blunt instrument of student assessments of the course and instructor. We again acknowledge the incredibly small sample size that we are dealing with and claim nothing other than that the presence of a flipped component to the course did not appear to lower student views of the course or instructor. In fact, it **may** have helped raise both. This lack of harm is an important consideration to untenured colleagues who may face (or perceive) significant pressure to maintain strong evaluation scores.

Axtell IDEA Scores

Semester	Sp'16	F'16	Sp'17	F'17	F'19 (flipped)
Summary IDEA Score	60/4.8	56/4.6	58/4.6	59/4.7	63/4.9

The second, more focused, instrument we have for student views of the experience is a brief questionnaire that students completed anonymously at the end of the fall 2019 semester (see Appendix A). Some specific responses are recorded below, and the overall result of the data is shown in the following table.

Combined Student Responses

Student Response Type	Did Not Like Flipped Experience	Neutral or Mixed Views	Liked Flipped Experience for the Selected Topics	Wished All Topics Delivered in a Flipped Format
Number of Student Responses (out of 26)	2	3	17	4

These responses indicate that the vast majority of the students found benefit to the flipped sections of the course and, in general, felt that some topics are best covered in a more traditional manner. This result is roughly speaking in line with a far larger study of student views on flipped classrooms described in Nouri (2016).

Student quotes:

- I liked the videos because if I wanted to I could refer to them whenever.
- I liked both methods (flipped and lecture), and I think there is a place for both classroom styles. I think that some, more challenging ideas should be taught in-class while less challenging material is better online. I loved working with groups in class.
- I was much better in the lecture setting than the flipped classroom setting. The videos were hard for me to pay attention to.
- I preferred learning via flipped classroom. I found I was able to better focus and learn outside of the classroom setting. It also helped to review the videos again before tests.
- I think the video notes prepared me better for homework/exams because it gave me time to fully understand the topic prior to working problems/discussing in class.
- The videos helped a lot with exposing me to the material before we learned it. I felt like it allowed us to move quicker in class if everyone had watched the videos because then we wouldn't have to go over basic explanations. However, I don't think the videos were a complete substitution for the material. Even after watching them, we still had to go over it in class to fully understand it.
- I didn't have any real preference to either method. I enjoyed using the flipped method just because it allowed me to see the material ahead of class without having to reference a textbook which I thought was very beneficial. Because of this, I believe there were some topics that were more difficult that the flipped format would have been helpful with.

Phase 2: 2020-present

The first time a teacher tries something new in the classroom leads to lots of ideas for improvement. This experience was no different. In the years since 2019, we have modified our flipped approach towards this course in two main ways. First, we have increased the number of topics covered via the flipped format to the point where now approximately 70% of the course material is delivered via a flipped format. We believe the remaining 30% of the material is better covered via in-class lectures accompanied by practice problems due to the more complex nature of these topics. The table below illustrates the current list of course topics and how we cover them.

Covered via Flipping	**Covered via in-class lecturing**

Simple Interest and Discount Compound Interest and Discount Constant Force of Interest Varying Interest Rates Level Annuities Arithmetic Progression Annuities Continuously Payable Varying Annuities Geometric Progression Annuities Loans Term Structure of Interest Rates	Bonds NPV/IRR/Other Rates of Return Duration and Convexity Asset-Liability Matching

The topics above were chosen with the same criteria in mind that we used during Phase 1. Namely, we flipped the topics whose level of complexity is amenable to an 8-minute (or so) explanation. Topics that are more complex or nuanced continue to be taught via lectures. We also continue to walk students through the derivations of key formulas and ideas via in-class mini-lectures at the end of class. Student attitudes toward the flipped format continue to be strongly positive. We have not yet tried to flip a topic and realize via student performance and feedback that it was a mistake. (We believe this is due as much to luck as any brilliant teacher insight!)

The second change involves the use of now-common technologies to actually create the videos. As mentioned earlier, in 2019 we utilized an actual studio to create the videos. This involved reserving the studio, getting trained in on the technology, shooting the videos, transferring the videos onto a hard drive, and then uploading the videos to YouTube. The Covid-19 pandemic changed all of this by accelerating the development and emergence of low-cost, lower-effort methods of creating and sharing videos. The two authors have each found different styles of video production that best fit them.

Axtell has adopted a very low-tech approach. He wrote the content of the video up on his office whiteboard (approx. 6'x4'), opened up a Zoom room, and then recorded himself narrating the material at the whiteboard with his laptop. The Zoom recording was then posted to YouTube. An example can be found at the link: Compound Interest Video

Stickles preferred to type the video content up into a Beamer document (a LaTeX file used to create presentation slides), and then recorded a voice over narration of these slides. Recordings were again uploaded to YouTube. An example can be found here: Callable Bonds

In thinking through the video-creation method that best suited each of us, we kept a few key ideas in mind.

- How much effort is required to create an effective video? Or perhaps, is the amount of additional effort needed to create a more polished video justified by an expectation that students will learn better from it?
- How effective is the video in

    - conveying the material, and
    - holding student interest (keep them short!)?
- How adaptable are the videos to future classes?

It is then a matter of each teacher finding the balance that is right for them. Axtell felt that his approach was very low effort and held student interest (by having a human in-frame) while not being adaptable to future classes if the content of the video needed to change. In contrast, Stickles' material is more adaptable to any necessary future change but does require a greater up-front cost to create.

Conclusion

Flipping a classroom is a large undertaking. Teachers should carefully consider whether the intended audience and material make it a good potential map for flipping. We would also recommend adopting an incremental approach of gradually flipping a course over a number of semesters (or even years) in order to spread out the up-front costs as well as to ensure the appropriateness of the material and audience to the flipped methodology. The authors would also freely invite any readers to freely utilize any and all materials posted to the YouTube channel linked to in this article. All videos are posted on YouTube at Theory of Interest YouTube Channel.

References

Epstein, J. (2013). The Calculus Concept Inventory – Measurement of the Effect of Teaching Methodology in Mathemtics. *Notices of the AMS, 60*(8), 1018-1026.

Hake, R. (1998). Interactive Engagement versus traditional methods: A six-thousand student survey of mechanics test data for physics courses. *Amer. J. Physics*, 64-74.

Lag, T., & Saele, R. G. (2019). Does the Flipped Classroom Improve Student Learning and Satisfaction: A Systematic Review and Meta-analysis. *AERA Open, 5*(3), 1-17.

Mazur, E. (1997). *Peer Instruction: A User's Manual.* Prentice Hall.

Nouri, J. (2016). The Flipped Classroom: For Active, Effective and Increased Learning – Especially for Low Achievers. *International Journal of Educational Technology in Higher Education, 13*(33). Retrieved from https://educationaltechnologyjournal.springeropen.com/articles/10.1186/s41239-016-0032-z

Ruckman, C., & Francis, J. (2005). *Financial Mathematics: A Practical Guide for Actuaries and other Business Professionals.* Farmington, CT: BPP Professional Education.

Sparks, S. (2011, September 27). Lectures Are Homework in Schools Following Khan Academy Lead. *Education Week*. Retrieved from https://www.edweek.org/teaching-learning/lectures-are-homework-in-schools-following-khan-academy-lead/2011/09

Thompson, C. (2011, July 15). How Khan Academy is Changing the Rules of Education. *Wired*. Retrieved from https://www.wired.com/2011/07/ff-khan/

Appendix A

Flipped Classroom Feedback Form

ACSC 264 – Fall 2019 – Prof. Axtell

NO NAMES!

I am seeking your feedback in comparing the educational experience you had with Chapter 2, 6-11 (in which you watched videos prior to class and worked problems in class; i.e., **flipped**) versus the remaining chapters of the text which were covered in a more traditional lecture format (with reading questions preceding the **lecture**) with additional problems worked outside of class.

1. How did the two methods compare for you? Did one work better than the other? Please share why one worked better than the other?

2. If the **flipped** format worked well for you, please indicate which of the topics below might also work well with a flipped approach:

Topic	YES – flip it!	NO – stick with reading questions and lectures
Compound Interest/Discount		
Constant Force of Interest		
Varying forces of interest		
NPV/IRR/Time-weighted/dollar-weighted		
Bonds		
Duration and Convexity		
Asset—liability matching		
Immunization		
Spot and forward rates		
Interest rate swaps		

3. If you like the **flipped** approach in general, were there certain topics that you think did NOT work well with the flipped approach? We covered simple interest, annuities and loans via the flipped format.