Solving Non-Linear Equations Using *Excel*

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Abstract

Nonlinear equations arise naturally in mathematics, engineering, and the sciences, yet solving them analytically is often impractical. This paper demonstrates how Microsoft *Excel* can be effectively used as a computational tool to solve nonlinear equations and systems, making the process accessible to students and educators without requiring advanced programming knowledge. Through a series of examples utilizing *Excel's Goal Seek*, *Solver*, and iterative capabilities, we illustrate Excel's strengths as an intuitive platform for nonlinear problem-solving.

1. Introduction

Nonlinear equations are those where the relationship between variables is not linear; they cannot be expressed in the form $ax + b = 0$. Nonlinear equations may involve exponential, logarithmic, trigonometric, or other complex functions. Examples include:

- $\log_2(3x - 1) + 2x$ (Logarithmic Equation)
- $e^x - 3x = 0$ (Exponential Equation)
- $\sin(x) - \frac{x}{2} = 0$ (Trigonometric Equation)

Nonlinear equations model a wide range of real-world phenomena:

- Mathematics: Chaos theory, dynamical systems, and optimization problems.
- Engineering: Nonlinear behavior in electrical circuits, structural stress analysis.
- Applied Sciences: Quantum mechanics, biological growth models, economic equilibrium.

Traditional methods for solving nonlinear equations include:

- Analytical Methods: Suitable for simple cases (e.g., quadratic formula).
- Numerical Methods: Bisection, Newton-Raphson, Secant, and Fixed-Point Iteration.

Challenges with traditional approaches include programming requirements, divergence of iterative methods, and limited accessibility for non-specialists. Microsoft Excel offers a practical alternative, providing user-friendly tools that eliminate the need for advanced programming.

2. Why *Excel*?

Excel's widespread availability, intuitive interface, and lack of requirement for coding skills make it ideal for teaching and learning mathematical problem-solving.

- Visual learning: Built-in graphing tools illustrate solutions dynamically.
- Interactive experimentation: Immediate feedback encourages exploration and understanding.

Excel provides several features to support solving nonlinear equations:

- *Goal Seek*: Solves single-variable equations easily.
- *Solver*: Tackles complex systems and constrained optimization problems.
- Iterative Calculation: Supports custom numerical methods step-by-step.

Excel bridges the gap between theoretical concepts and real-world applications, making nonlinear problem-solving approachable even to those without a programming background.

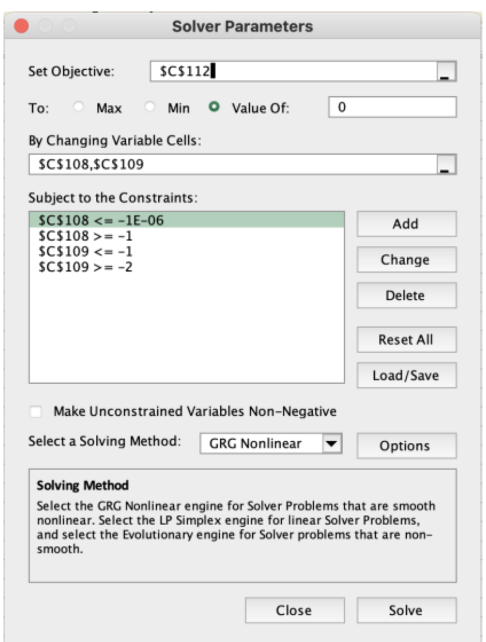
3. Examples

A variety of problems were solved using *Excel's Solver* and *Goal Seek* tools:

Example 1: Solve the following equation for the constant C_F using solver in *Excel*.

- $\frac{0.242}{\sqrt{C_F}} = \log(R_N \cdot C_N)$; where $R_N = 12000$.

  Solution by *Solver* is;



Example 2: Solve the following equation for the constant *e* using *Solver* in *Excel*

$$(4e^4 + 14e^4 + 9)\sinh^{-1}(e) = (9e + 8e^3)\,(1 + e^2)^{1/2}$$

Solution of the equation by *Solver*:

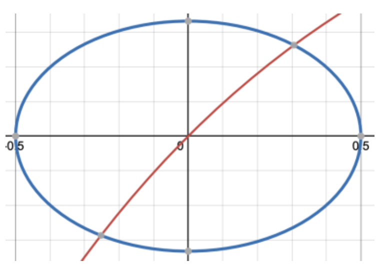
Example 2: Solution By Solver			
		e	1.676554261
		Equation	-1.98813E-07

Example 3: Solve the nonlinear system of equations by *Solver*

$$9y^2 + 4x^2 = 1$$

$$y = 1 - e^{-x}$$

The graph suggests there are two solutions. We get those solutions by *Solver* given below.



It provides us the same solutions.

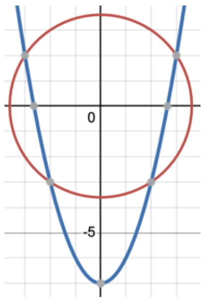
Example 3 Solution by Solver		
	initial gues of x	0.30600456
	y =Sqrt(1-4x^2)/3	0.2636169
	y=1-e^(-x)	0.26361674
	Difference	1.6332E-07
	initial gues of x	-0.2527785
	y =-Sqrt(1-4x^2)/3	-0.2875977
	y=1-e^(-x)	-0.287598
	Difference	2.908E-07

Example 4**:** Solve the system of nonlinear equations by *Solver.*

$$\boldsymbol{x^2 + y^2 = 13}$$

$$\boldsymbol{x^2 - y = 7}$$

The graph suggests that there are four solutions.

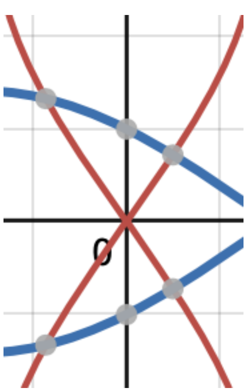


Example 4 Solution by Solver				
initial gues of x	-3.0000001		**initial gues of x**	3
y=Sqrt(13-x^2)	1.9999998		**y=Sqrt(13-x^2)**	2
y=x^2-7	2.00000081		**y=x^2-7**	2.000001
Difference	-1.016E-06		Difference	-1E-06
initial gues of x	-1.9999993		**initial gues of x**	1.999999
y=-Sqrt(13-x^2)	-3.0000004		**y=-Sqrt(13-x^2)**	-3
y=x^2-7	-3.0000026		**y=x^2-7**	-3
Difference	2.1785E-06		Difference	2.18E-06

Example 5:

Solve the system of nonlinear equations by *Solver*.

$$\sin x + y^2 = 1$$
$$\cos y + x^2 = 1$$



The graph suggests that there are four solutions. We obtain those solutions by *Solver*. This part of the solution is the first case when x and y are both positive.

Example 5 Solution by Solver		
	initial gues of x	0.49942183
	initial gues of y	0.7218599
	Equation(1)	-2.107E-07
	Equation(2)	2.0769E-07
	Objective	8.7547E-14

This is the case where *x* is positive but *y* is negative;

initial gues of x	0.49942175
initial gues of y	-0.7218601
Equation(1)	1.0231E-09
Equation(2)	-4.681E-09
Objective	2.2956E-17

This is the case where *x* is negative, but *y* is positive;

initial gues of x	-0.8719609
initial gues of y	1.32875578
Equation(1)	1.6325E-08
Equation(2)	1.024E-08
Objective	3.7137E-16

This is the case where *x* is negative, and *y* is also negative;

initial gues of x	-0.8719609
initial gues of y	-1.3287558
Equation(1)	1.6325E-08
Equation(2)	1.024E-08
Objective	3.7137E-16

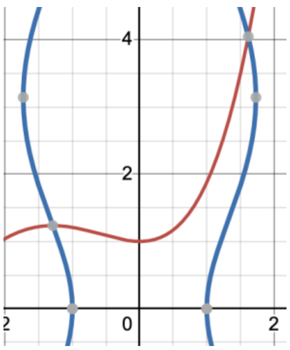
Example 6:

Solve the system of nonlinear equations by *Solver*.

$$\sin x + y \quad = e^x$$

$$\cos y + x^2 = 2$$

The graph suggests that there are two solutions. We obtain those solutions by *Solver*. This of the solution is the case where *x* and *y* are both positive.



This is the case where *x* and *y* are both non-negative.

Example 6 Solution by Solver		
	initial gues of x	1.61823688
	initial gues of y	4.04531405
	Equation(1)	7.7057E-09
	Equation(2)	-1.216E-08
	Objective	2.0718E-16

This is the case where *x* is non-positive and *y* is non-negative

initial gues of x	-1.2928748
initial gues of y	1.23610809
Equation(1)	-2.413E-07
Equation(2)	1.438E-08
Objective	5.8423E-14

Example 7**:** Use *Solver* to determine the best-fit line between the number of planets (x, independent variable) and their respective mean distance from the Sun (*y*, dependent variable). Use the predicted values of *y* from Bode's Law and the observed values from the given table. Case *x*=10 planets.

Solution by *Solver*

Example 3: Solution By Solver

Planet	B: Actual Y - Actual Distance	C: Predicted Distance - independent Variable	D: Fitted Y - Regression Line	E: Error (B - D)	Error Squared E^2		Initail Slope (m) Estimate	Initail y-intercept (b) Estimate
Mercury	0.387	0.4	2.65770658	-2.2707	5.156108		0.543744548	2.44020876
Venus	0.723	0.7	2.82082995	-2.0978	4.40089			
Earth	1	1	2.98395331	-1.984	3.936071			
Mars	1.524	1.6	3.31020004	-1.7862	3.190511		y = 0.544 x + 2.44	
Astroids	2.8	2.8	3.9626935	-1.1627	1.351856			
Jupiter	5.201	5.2	5.26768041	-0.0667	0.004446			
Saturn	9.538	10	7.87765424	1.66035	2.756748			
Uranus	19.18	19.6	13.0976019	6.0824	36.99557			
Neptune	30.06	38.8	23.5374972	6.5225	42.54304			
Plutus	39.52	77.2	44.4172879	-4.8973	23.98343			
				SUM	124.3187			

4. Conclusions

Excel's *Solver* and *Goal Seek* tools offer a powerful and accessible approaches to solving nonlinear equations without requiring sophisticated programming. Throughout a number of examples, it was demonstrated that *Excel* can:

- Solve both single nonlinear equations and complex systems;
- Allow visual understanding of solution behavior;
- Provide optimization for real-world applications, such as minimizing the error in empirical laws like Bode's Law;
- Serve as a practical bridge between academic mathematics and professional problem-solving.

Excel is not merely a simple spreadsheet program; it can serve as a robust numerical engine for educators, students, and professionals seeking efficient solutions to nonlinear problems.

References

1. J. Biesiada, 2015. *Numerical Methods and Optimization in Excel*. Springer.
2. D. M. Bourg, 2006. *Excel Scientific & Engineering Cookbook.* Sebastopol, CA: O'Reilly Media.

3. C. E. Froberg, 1966. *Introduction to Numerical Analysis.* Reading, MASS: Addison-Wesley Publishers.
4. F. D. Stacey & P. M. Davis, 2008. *Physics of the Earth.* Cambridge University Press.
5. M. Sullivan, 2020. *Algebra & Trigonometry.* Hoboken, NJ: Pearson Publications.