# TECH-ENHANCED MATHEMATICS FOR EC-6 PRESERVICE TEACHERS – BEYOND THE CALCULATOR!


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## Abstract

Many people have a limited view of technology for the mathematics classroom. As mathematics educators, we would like to expand preservice teachers' views of effective classroom technology beyond calculator usage by introducing a variety of technologies that bring multi-modal approaches into their lessons. The purpose of these other technological tools is to support and facilitate conceptual understanding of critical ideas in mathematics. In this paper, we address how we broaden their views by implementing non-calculator-based technology into the mathematical preparation of preservice teachers pursuing certification for grades EC-6.


## Introduction

For grades EC – 6, one of the goals of the curriculum is to help students build conceptual understanding and numerical fluency as they explore and develop strategies and processes for operations with whole numbers, rational numbers, and integers. In addition, the curriculum seeks to develop conceptual understanding of geometry, measurement, probability, statistics, proportionality, and the foundation for algebraic thinking (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Teachers who provide students with multi-modal approaches create a classroom atmosphere that fosters robust conceptual understanding and opportunities for rich discourse (Lemke, 2002; Smith et al., 2023; Wilkerson et al., 2022).

Many people have a limited view of technology for the mathematics classroom, and think of only the calculator. As mathematics educators, we would like to expand preservice teachers' views of effective classroom technology beyond calculator usage by introducing them to a variety of technologies that bring multi-modal approaches into their lessons. The purpose of these other technological tools is not to de-emphasize the computational skills that are being developed or to replace the interaction with hands-on manipulatives. Instead, the purpose is to support and facilitate the conceptual understanding of critical ideas in mathematics. In this paper, we address how we broaden their views by implementing non-calculator-based technology into preservice teacher courses at the university.

## Overview – Mathematics Component of EC-6 Preservice Teacher Preparation

At our institution, students pursuing teacher certification for grades EC-6 take four 3-hour mathematics courses. These four courses are listed below:

- MATH 1314 (College Algebra)
- MATH 3303 (Concepts of Elementary Mathematics I)
- MATH 3305 (Concepts of Elementary Mathematics II)
- MATH 4305 (Concepts of Elementary Mathematics III)

First, each preservice teacher must make a C or better in MATH 1314 (College Algebra). Once College Algebra is complete with the desired grade, along with at least 45 credit hours overall, then the preservice teacher can start the Concepts series.

In MATH 3303 (Concepts of Elementary Mathematics I), students address the content of problem solving, set theory, and place value, including place value in other bases. Another critical content piece in this course is operations on whole numbers, integers, and rational numbers. Next, students move to MATH 3305 (Concepts of Elementary Mathematics II). This second course in the series focuses on the content areas of geometry, measurement, probability, and data collection and statistics. The Concepts series concludes with MATH 4305 (Concepts of Elementary Mathematics III). This course contains units on number theory, proportionality, and algebraic reasoning. Preservice teachers must make a C or better in each course before moving on to the next course in the series.

## Importance of Visualization Models

The Concepts courses are taught from an active-learning perspective with an emphasis on multi-modal communication. Students are immersed in discovery activities using manipulatives to develop conceptual understanding (Piaget, 1972; Hoong et al., 2015) while working in groups to articulate their thinking and develop and refine multi-modal communication skills (Lemke, 2002; Lemke, 2004).

Instruction is grounded in the work of learning theorists such as Piaget and Bruner as we address content. Piagetian thinking is comprised of four stages of cognitive development classified as sensorimotor intelligence, preoperational thinking, concrete operational thinking, and formal operational thinking (Piaget, 1972). Most school-aged children fall in the latter three stages of development described by Piaget; therefore, it is important for preservice teachers to experience ways in which children should learn mathematics.

Furthermore, we develop content with the Concrete-Pictorial-Abstract approach, which is based on Bruner's enactive, iconic, and symbolic modes of representation (Hoong et al., 2015). Throughout the courses, we engage the preservice teachers in hands-on modeling with physical manipulatives (concrete), then we have them create graphs, sketches, and diagrams to represent their thinking (pictorial). These two stages are followed by intentional connections between the concrete and pictorial models to discover and develop an understanding of the algorithms (abstract). Technology plays an important role in this sequence of concept development, particularly at the pictorial stage, as preservice teachers expand their understanding of multi-modal communication.

Multi-modal communication is comprised of four modes: natural language, visual representations, mathematical symbols, and manual-technical tools (Lemke, 2002; Lemke, 2004). These modes are accomplished by students explaining their understandings through oral and written communication while supporting their explanations with visual representations, mathematical symbols, and/or manipulatives or other mathematical tools.

We use the work of these learning theorists as a backdrop for our Concepts classes so that our preservice teachers can experience both content and pedagogy that is grounded in evidenced-based practices.

While technology should not replace the process of students working with manipulatives and then building graphs, sketches, and diagrams, it can (and should) be used appropriately to enhance and reinforce conceptual understanding. We strive to embed the appropriate use of technology into our Concepts courses so that our preservice teachers can experience a variety of tools. Furthermore, we believe that preservice teachers will be more likely to use technology in their future classrooms if they have positive technology experiences during the Concepts courses.

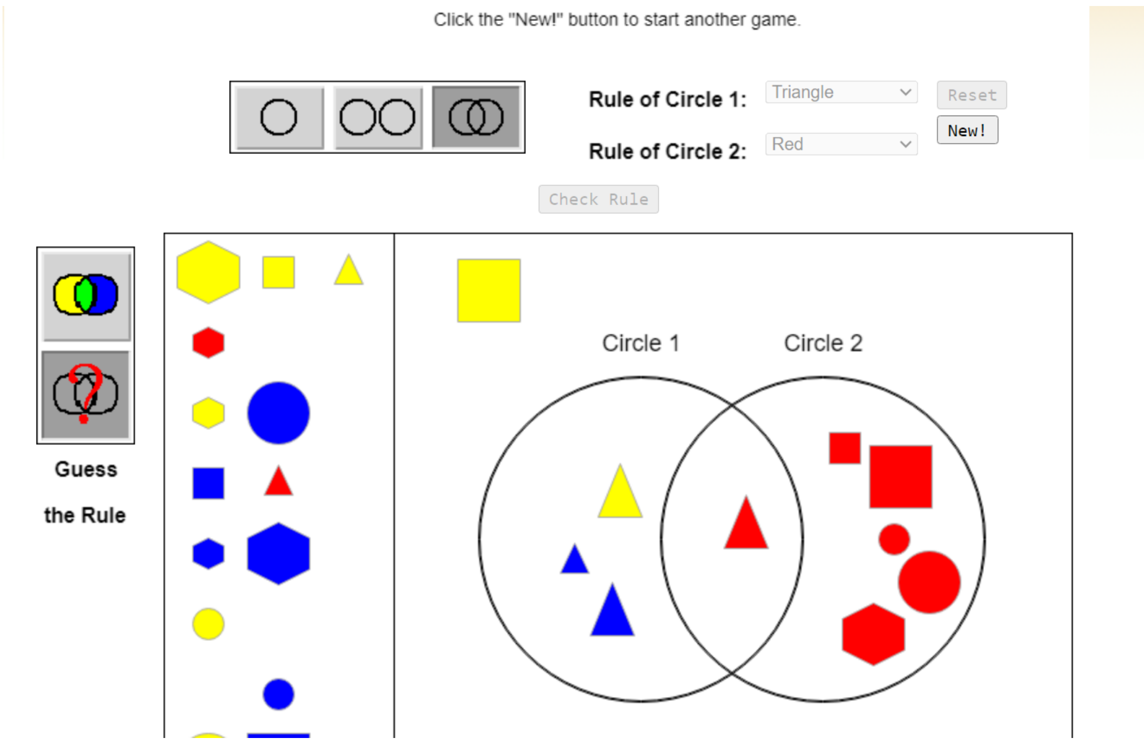
## Implementation in MATH 3303 – Concepts of Elementary Mathematics I

The mathematical content of MATH 3303 (Concepts of Elementary Mathematics I) includes problem solving, set theory, place value and other bases, along with operations on whole numbers, integers, and rational numbers. Next, we will describe some technology tools that we use in this course to enhance instruction, after the preservice teachers have interacted with the content at the concrete and pictorial levels.

First, the Venn Diagram Shape Sorter (Shodor, n.d.) has been an effective tool for the set theory unit in Concepts of Elementary Mathematics I. Students can choose the Venn Diagram layout and the rule for each set, then drag the attribute blocks to the appropriate region of the Venn Diagram. Additionally, there is an option where students can choose "Guess the Rule" and then determine the rule through trial and error. The Shape Sorter is self-checking so students receive immediate feedback as they interact with the technology. Figure 1 shows a screen capture from the tool.

**Figure 1**

*Screen Capture – Venn Diagram Shape Sorter from Shodor*

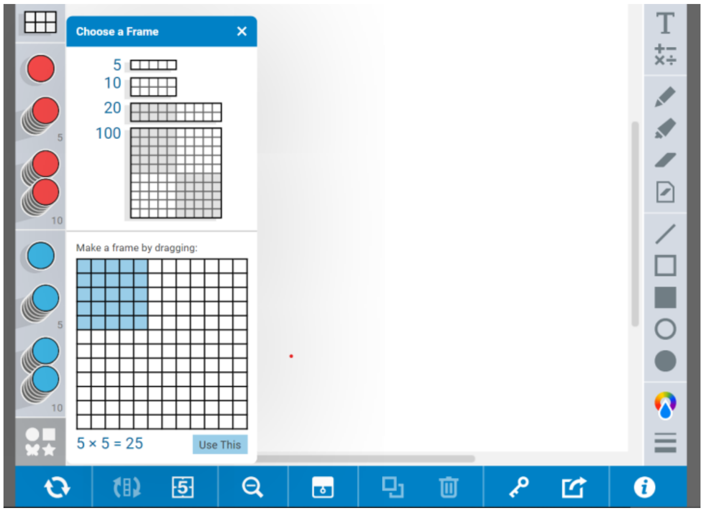



A second technology tool that we have found effective is the Number Frame tool from the Math Learning Center (n.d.). We use this tool to enhance the numerical fluency unit. Within the application, students can choose a 5-frame, 10-frame, 20-frame, or 100-frame. Then, they can drag colored counters onto the frame in units, bundles of 5, or bundles of 10. This tool can be used to reinforce many foundational concepts including cardinality of set, subitizing, unitizing, one more than, one less than, or equal to. Furthermore, operations with whole numbers can be illustrated effectively in this virtual environment, allowing students to reinforce the conceptual development that they experienced with the hands-on manipulatives.

The tool can be used across the grade bands K-6 by expanding its use to introduce operations on integers since the counters can be customized to different colors to model two-color counters. The site contains several annotation tools as well, facilitating the bridge from the pictorial model to the abstract. A screen capture from the Number Frame tool illustrating the area model of multiplication is shown in Figure 2 below.

**Figure 2**

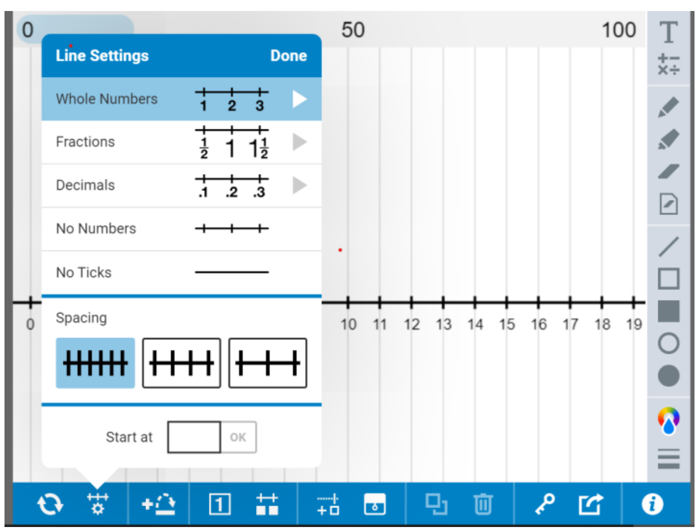
*Screen Capture – Number Frames from the Math Learning Center*




Third, the Number Line tool (Math Learning Center, n.d.) has many options for building measurement pictorial models to enhance conceptual understanding for operations on whole numbers, integers, and rational numbers. Students can customize the line settings, the spacing, the starting value, and the step value for the number line. Operations can be illustrated using the “jump” tool. Figure 3 contains a screen capture from this technology tool.

**Figure 3**

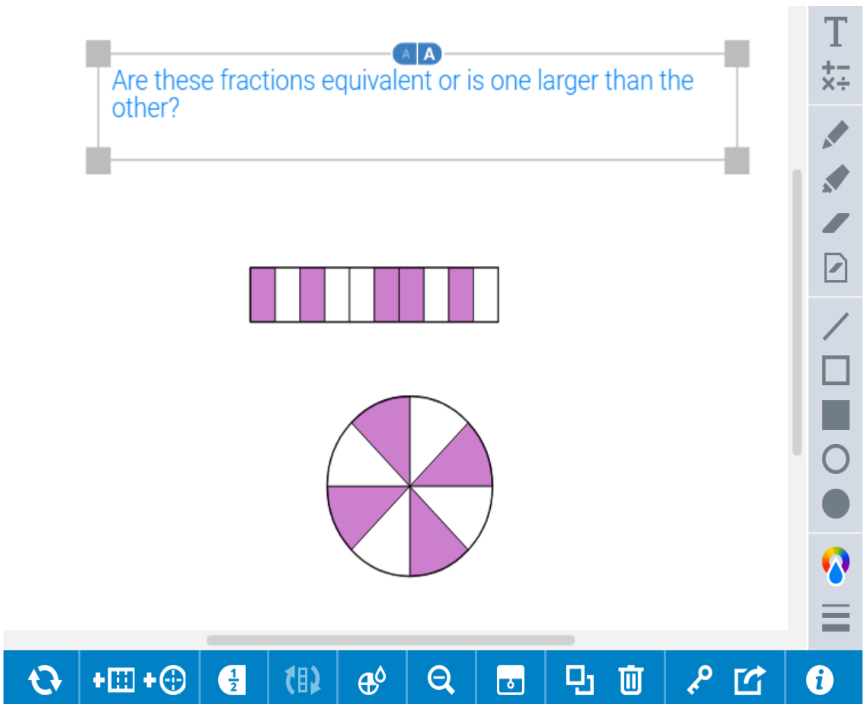
*Screen Capture – Number Line from the Math Learning Center*



Next, the Math Learning Center (n.d.) has a Fraction tool that creates representations for fraction bars (measurement model) or fraction circles (area model). Students can choose the number of partitions to create as they explore fraction equivalencies and operations with fractions. Figure 4 illustrate two types of fraction models in the context of exploring equivalent fractions. It is important that students experience a variety of representations so that they become more flexible in moving among the measurement, area, and set models for fractions.

**Figure 4**

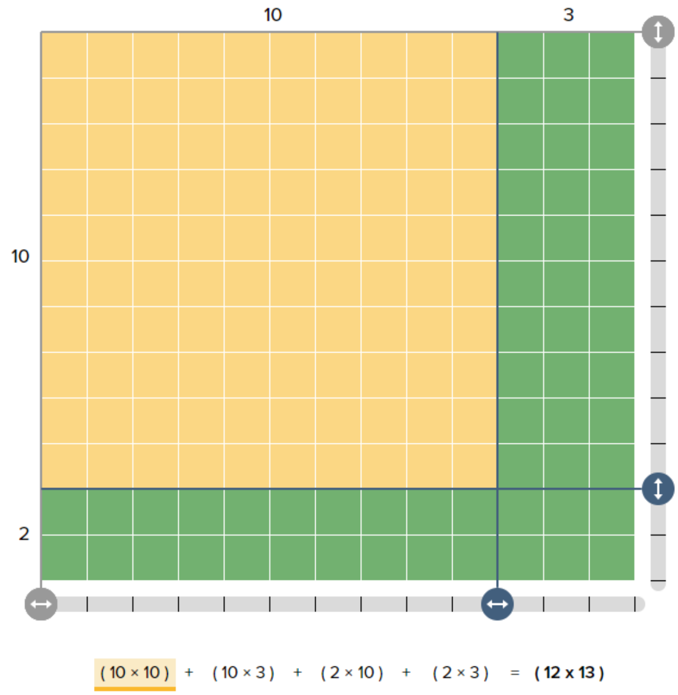
*Screen Capture – Fractions from the Math Learning Center*




The last technology tool for MATH 3303 that we wish to share in this paper is the Partial Product Finder (Math Learning Center, n.d.). This tool allows the student to build models for multiplication like they would build with base-ten blocks. Various regions within the model change color as you hover over them with the mouse, facilitating the connections between the pictorial model and the development of the traditional algorithm for multiplication. Furthermore, this area model allows for students to see how expanded notation, the distributive property of multiplication, and the associative properties of addition and multiplication can be utilized to develop the traditional algorithm for multiplication. A model for $12 \times 13$ is shown in Figure 5. Note that the $10 \times 10$ region in the array is highlighted below in the expression for the sum of the partial products.

**Figure 5**

*Screen Capture – Partial Product Finder from the Math Learning Center*




There are many more wonderful technology tools beyond those selected for this paper that fit well with the content in MATH 3303. We have used these tools during class in groups, as well as posted links to them in Canvas for students to use on their own after the completion of the lesson. In addition, we have incorporated the tools into assessments which will be discussed later.

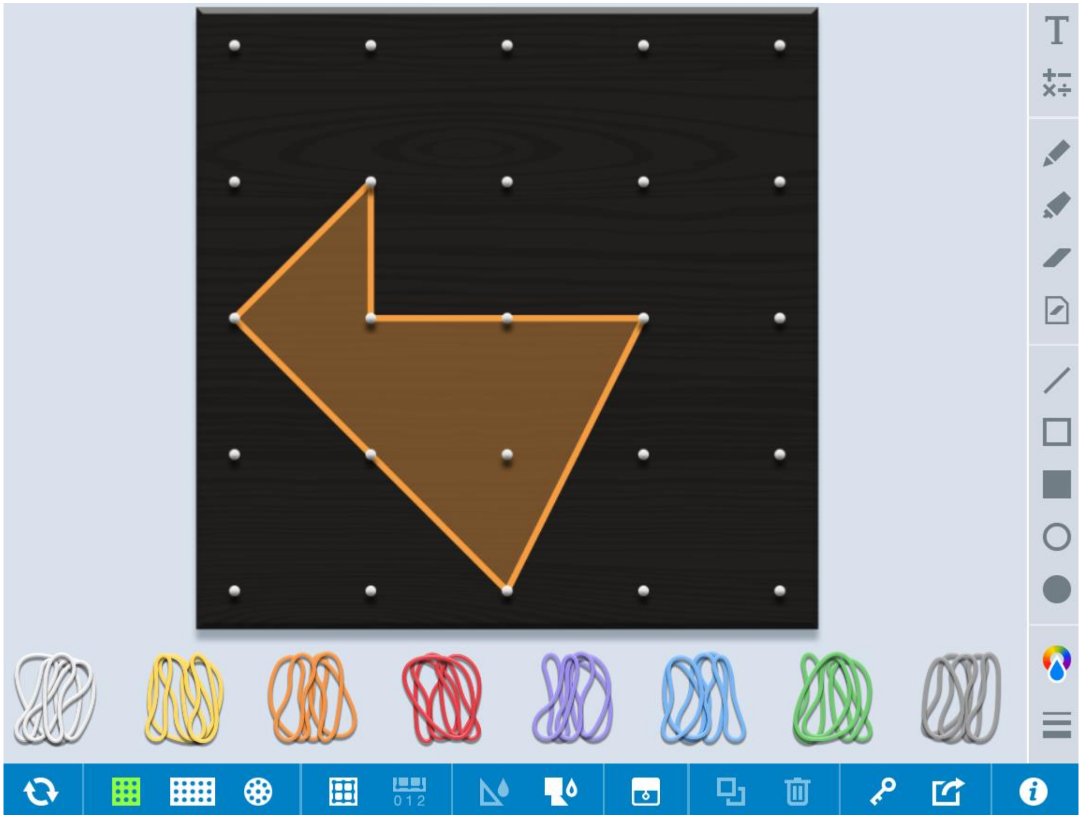
**Implementation in MATH 3305 – Concepts of Elementary Mathematics II**

Moving on to the second course in our Concepts series, MATH 3305 includes the content of geometry, measurement, probability, and data collection and statistics. Next, we will share a selection of our favorite technology tools that we utilize in this course.

First, the virtual geoboard (Math Learning Center, n.d.) is a tool that we use for a variety of topics in both geometry and measurement. We use it to reinforce polygon vocabulary and attributes, as well as measurement concepts such as area and perimeter. Problems involving the Pythagorean Theorem work well with the virtual geoboard too. The annotation tools that are included allow students to record their problem-solving processes, and the share-image features allow preservice teachers to create professional-looking images to import into other documents as needed. Figure 6 contains a screen capture from this technology tool. Students could use this image to classify the polygon, explore the concepts of concave and convex, calculate the shaded area, or calculate the perimeter of the polygon. In addition, the image could be used as students practice classifying angles as acute, obtuse, right, or reflex.

**Figure 6**

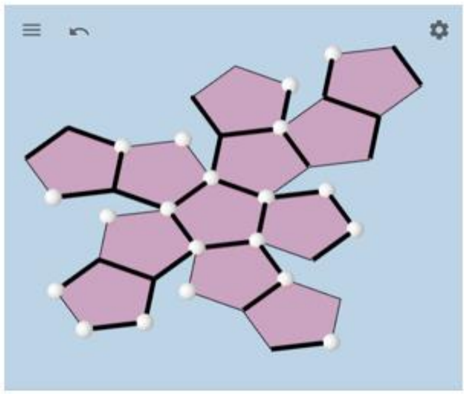
*Screen Capture – Virtual Geoboard from the Math Learning Center*

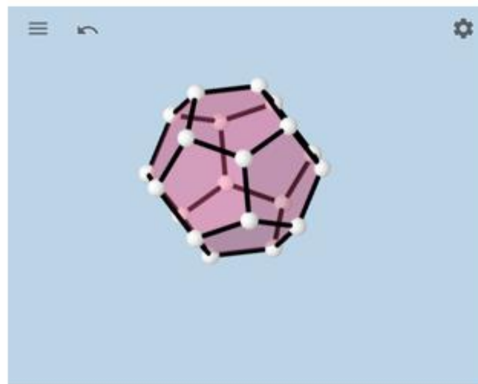


Next, GeoGebra (https://www.geogebra.org/) contains a plethora of resources and animations that pair nicely with the content in MATH 3305 (Concepts of Elementary Mathematics II). Some of the GeoGebra sketches are stand-alone, and others come in a bundle (called a book) where students can interact with a series of sketches that are intentionally scaffolded to build content knowledge. The sketch captured in Figure 7 below is a sample of what we use in MATH 3305 when we explore the Platonic Solids. The sketch animates the net for a dodecahedron as it folds up into the 3-D figure, then unfolds again to the 2-D net (Brzezinski, 2018). Prior to using the animations, the preservice teachers have had experiences with cutting out and folding nets, as well as identifying attributes of regular polyhedra. In addition, the sketch could be used to practice counting faces, vertices, and edges as students verify that the Platonic Solids satisfy the definition of a regular polyhedron. Finally, the sketch could be used to reinforce the relationship among the faces, vertices, and edges as summarized in Euler's Formula.

**Figure 7**

*Screen Capture – Animations in GeoGebra*

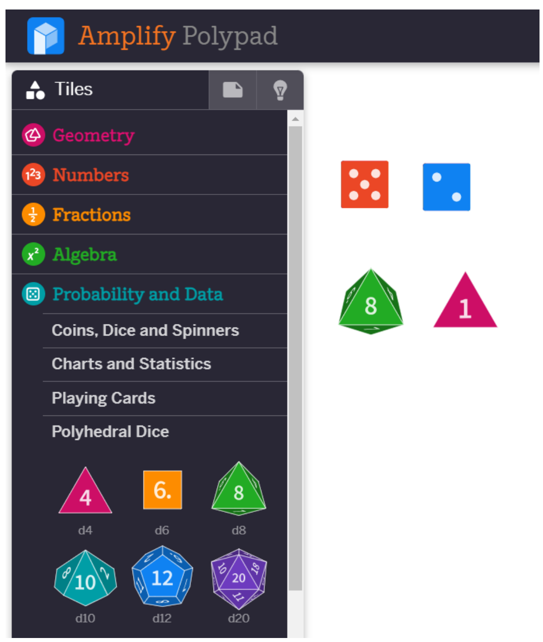




Third, the resources in Polypad (Polypad, n.d.) offer a large variety of virtual manipulatives. One of our favorites is the ability to roll virtual dice during our probability unit in MATH 3305. The students have the option of choosing any of the polyhedral dice, and dice can be rolled individually or in groups. There is a *Tabulate* feature as well that collects the outcomes for several rolls, and the data from the table can then be graphed with the click of a button. The screen capture in Figure 8 below shows a sample of the dice that are available in the technology tool.

**Figure 8**

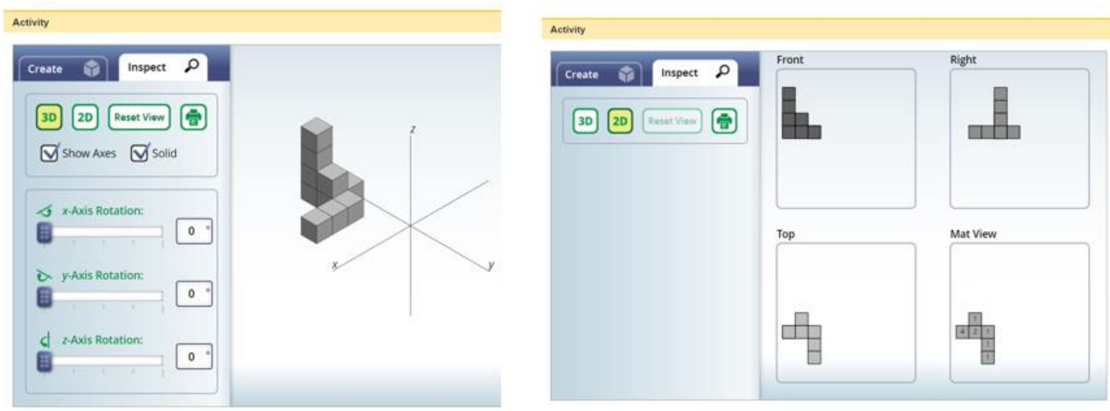
*Screen Capture – Dice in Polypad*



Fourth, Illuminations from the National Council of Teachers of Mathematics has many virtual tools. One that has proven to be particularly helpful in MATH 3305 is the Isometric Drawing Tool (National Council of Teachers of Mathematics, n.d.). This technology tool allows students to build representations of 3-D figures with virtual blocks, then turn the figure to view it from different perspectives. In addition, the students can switch to a 2-D view to look at profiles of the figure from the front, right, and top, along with a mat view that shows the number of blocks that are stacked in each region. Reinforcing spatial reasoning and visualization is an important geometry topic, and this technology tool does a wonderful job with helping the students visualize the figures and hone their conceptual understanding. Figure 9 shows a sample 3-D figure with its corresponding 2-D profile images.

**Figure 9**

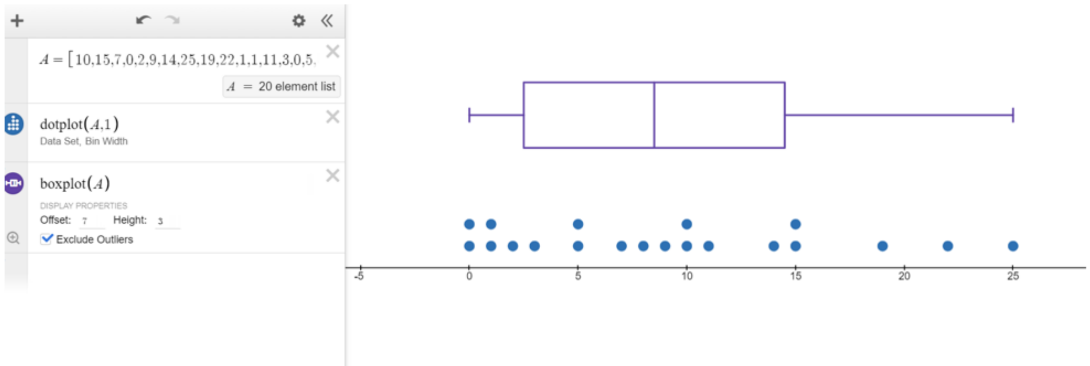
*Screen Capture – Isometric Drawing Tool from NCTM*




Fifth, Desmos (https://www.desmos.com/calculator) has many features that work well with the data collection and statistics unit in MATH 3305. Specifically, the *dotplot* and *boxplot* commands have been effective in helping students grasp the relationship between the two representations. Students can see regions of the dataset that are clustered tightly together results in a "shorter" corresponding region of the boxplot, while regions of the dataset that are more scattered results in a "longer" corresponding region of the boxplot. Students often have the misconception that the longer sections of the boxplot correspond with a greater number of values from the dataset. With both of these graphs on the same axis, students can address this misconception by adjusting the height of the boxplot and lowering it so that it is superimposed on the dotplot. Next, the graphs can help reinforce what students have learned about measures of central tendency and measures of variation. For example, students can explore how the mean and the median are affected if the dataset contains an extreme value (outlier). Figure 10 shows a sample dataset with 20 values, along with a dotplot and boxplot of the data.

**Figure 10**

*Screen Capture – Dotplots and Boxplots from Desmos*




**Implementation in MATH 4305 – Concepts of Elementary Mathematics III**

MATH 4305 (Concepts of Elementary Mathematics III) addresses number theory, proportionality, and algebraic reasoning. We use a variety of technology-enhanced visualization models in this course such as virtual prime factor trees (Math Playground, n.d.), virtual Cuisenaire rods (Strategic Intervention Solutions, n.d.), and virtual algebra tiles (Polypad, n.d.), to name a few. The sources are listed in the References section of this paper.

Instead of describing the sample of technology tools for this course, we would like to describe a culminating technology-enhanced project that we use in MATH 4305 – the Google Classroom project. In short, the preservice teachers are put into groups of 3 to 4 students. Throughout the project, they play the role of teacher and student, as many times as the length of the semester allows.

During the role as teacher, each preservice teacher is assigned a mathematics topic to teach using the medium of the Google Classroom. For the assigned mathematics topic, the preservice teacher must find or create a video to refresh their "students/peers" on the topic, as well as find or create at least 20 minutes of activities for the "students/peers" to complete to enhance their knowledge of the topic. Next, the preservice teacher must create a way to assess the learning of the "students/peers." Finally, the preservice teacher must write a reflection over the experience based on their teacher role.

During the role of student, each preservice teacher will engage with the content that each of their peers created for whatever mathematics topic they were assigned. They watch the videos, complete the activities, and complete the assessments that were created by the "teacher/peer." At the end of the project, each preservice teacher provides peer review feedback for the other preservice teachers in the class, in addition to writing a reflection over the experience based on the student role.

The project meets several objectives. First, the preservice teachers are reviewing the content for both the topic they were assigned (teacher role) and the topics with which they engaged (student role). Secondly, they have opportunity to think like a teacher as they create content and assessments. Finally, they develop and strengthen a variety of technology skills as they learn how to create and post the content in Google Classroom for their peers to see.

**Tech-Enhanced Assessment across Concepts I, II, and III**

The last technology tool that we wish to share in the scope of this paper is a way that we have leveraged these virtual manipulatives and video technology to allow our preservice teachers to deepen their understanding of the content while demonstrating proficiency with some of these virtual tools.

Students in our courses take in-person exams during which they are expected to use physical manipulatives and give multi-modal responses where they draw pictorial models to show their processes, explain in words their strategies, and solve problems using algorithms where appropriate. We have added a video component to these exams in order to have an accountability piece with using some of the virtual tools described in this paper.

The video component deploys through a Canvas quiz, typically the afternoon following the in-person portion of the test. Students receive two to three prompts where they prepare a response to a question, then record a video for their finished product. Many of these prompts require the students to demonstrate proficiency with a virtual manipulative. Students practice with the virtual manipulative as they plan for the video, then they start over and narrate the solution as they share their screen. We use a Canvas rubric to grade the videos.

We have been pleased with the results of incorporating video test prompts into these courses. The preservice teachers are able to demonstrate their conceptual understanding as they prepare to make their videos. Their content knowledge is strengthened, and they have opportunity to practice and refine their multi-modal communication skills as they narrate the video showing how they create the models using the virtual manipulatives. They are able to communicate with mathematical symbols and other tools as they use the annotation features of the program.

Figure 11 shows a sample video prompt test question that requires the students to use a virtual geoboard. The goal of this particular video test prompt is to address the misconception that many students have about the relationship between perimeter and area. This sample question has the student create two non-congruent figures that have the same area, but different perimeters. As they record the video, the preservice teachers must move into “teacher-mode” and offer explanations that they might use with their future students. While not shown as a figure in this paper, the next video prompt question had students address another common misconception by creating two non-congruent figures that have the same perimeter, but different areas.

**Figure 11**

*Screen Capture – Video Test Prompts*

Prepare for your video:

Click here to access the virtual geoboard.

If for some reason that link doesn't take you directly there, here is the website: https://apps.mathlearningcenter.org/geoboard/

Once you are there, change your geoboard to the **larger rectangle** so that you will have plenty of work area.

Then, create **2 different figures** according to the following criteria:

- Your 2 figures must be different shapes from each other - no congruent figures allowed.
- Each of your figures must have an area of 7.5 square units.
- Each of your figures must have a different perimeter.

Practice with the virtual geoboard until you are confident that you have addressed the three bullets above. You can already have them pre-made for when you start recording the video. :-)

Make your video:

Once you have your 2 figures created on the virtual geoboard (and you are sure that they satisfy the criteria in the bulleted list above), then make a brief video (4 minutes max) where you **share your screen** and address the following questions:

1) How would you help your students see that the area of each of your figures is 7.5 square units?

2) What is the perimeter of each of your figures? Be sure to clearly state your answers in the video. As you do the perimeter of each figure, explain how you did your calculations.

*Remember that you can use the tools within the virtual geoboard to assist you with your explanations (for example, additional rubberbands, shading the interior of the figure, the drawing tool, etc...).*

Asking students to narrate their solution while they model with the virtual manipulatives has been a great way to give the preservice teachers practice articulating their solution strategies, conceptual understanding, and their multi-modal communication skills. This assessment technique is used in each of the courses in the Concepts series.

## Concluding Remarks

By implementing tools like those described in this paper, we aim to broaden the view of preservice teachers that technology is more than just a calculator. The non-calculator-based technologies available to visualize and model concepts within the EC-6 curriculum are plentiful, and preservice teachers benefit from experiencing a selection of these tools during their teacher education program. It is essential that the tools chosen are grounded in evidence-based practices and highlight the conceptual foundations of the content. The use of these tools can facilitate the development of strong content knowledge in tandem with multi-modal communication skills. Furthermore, the preservice teachers expand their repertoire of appropriate tools that can be implemented effectively in their future classrooms.

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