Answers

Exercise 1

1 $\frac{2}{3}$ **2** $\frac{1}{3}$ **3** $\frac{1}{2}$
4 $\frac{3}{4}$, 75% **5** $\frac{1}{4}$, 0.25 **6** 0.15, 15%
7 $\frac{7}{20}$, 35% **8** $2\frac{2}{3}$ **9** $3\frac{2}{5}$
10 $\frac{7}{3}$ **11** $\frac{11}{6}$

Exercise 1*

1 $\frac{2}{7}$ **2** $\frac{1}{6}$ **3** $\frac{1}{2}$ **4** $\frac{3}{40}$
5 $\frac{5}{6}$ **6** $\frac{2}{3}$ **7** $\frac{4}{9}$ **8** $\frac{7}{40}$
9 680 **10** 0.765 **11** 39 **12** 24
13 26.25 **14** 41.86

Exercise 2

1 5 **2** 13 **3** 13 **4** −3 **5** −12
6 −6 **7** −2 **8** 3 **9** 10

Exercise 2*

1 −2 **2** 23 **3** 18 **4** 18 **5** 12
6 12 **7** 36 **8** 49 **9** 144

Exercise 3

1 1.8 **2** 5.52 **3** 0.684 **4** $31.80
5 $45.50 **6** 15% **7** 10% **8** 5%
9 Percentage increase in value $= \frac{468}{7800} \times 100 = 6\%$
10 1980−2000 ⇒ 2.86% increase, 2009−2010 ⇒ 4.17% increase, increase per year probably tailing off

Exercise 3*

1 0.75 **2** $171 **3** 52.5
4 68.005 km **5** $80.04 **6** 13%
7 Profit = €4.25; percentage profit $= \frac{4.25}{34} \times 100 = 12.5\%$
8 0.0402%
9 11.1%; 12.5%
10 1975−2005 100 m men improved by 1.8% women by 5.2%
1500 m men improved by 2.9% women by 4.5%
Women showed more improvement than men (plus students' own explanations).

Exercise 4

1 10^5 **2** 10^{10} **3** 10^3
4 10^{13} **5** 10^2 **6** 10^3
7 10^3 **8** 10^1 **9** 4.56×10^2
10 1.2345×10^2 **11** 5.68×10^2 **12** 7.0605×10^2
13 4000 **14** 4 090 000 **15** 560
16 7 970 000 **17** 10 000 **18** 8.4×10^9

Exercise 4*

1 4.5089×10^4 **2** 2.983×10^7 **3** 10^3
4 10^5 **5** 10^{21} **6** 10^0 or 1
7 6.16×10^6 **8** 40 **9** 9.1125×10^{16}
10 2.5×10^4 **11** 9.653×10^8 **12** 10^{10}
13 Saturn 10 cm, Andromeda Galaxy 10^{11} cm (1 million km)

Exercise 5

1 800 **2** 3780 **3** 0.44 **4** 0.506
5 34.78 **6** 3.0 **7** 1×10^5 **8** 1×10^5

Exercise 5*

1 10 **2** 45.7 **3** 0.069 **4** 0.0495
5 9.00 **6** 7.0 **7** 1.06×10^5 **8** 9.88×10^4

Exercise 6 (Revision)

1 $\frac{1}{3}$ **2** $\frac{2}{7}$ **3** $\frac{1}{5}$ **4** $\frac{1}{8}$
5 $\frac{1}{2}$ **6** $\frac{1}{4}$ **7** $\frac{1}{5}$ **8** $\frac{3}{4}$
9 $\frac{1}{10}$ **10** $\frac{3}{10}$ **11** $\frac{2}{5}$ **12** $\frac{3}{4}$
13 50 **14** 60 **15** 300 **16** 8
17 −16 **18** −48 **19** $-\frac{1}{3}$ **20** 48
21 $\frac{1}{4}$ **22** $\frac{1}{10}$ **23** $\frac{3}{4}$ **24** $\frac{3}{5}$
25 $\frac{7}{20}$ **26** 150 m **27** $360 **28** 1650 m
29 £2040 **30** $90 **31** €897 **32** $56.25
33 6×10^{10} **34** 4×10^2 **35** 5.6×10^{16} **36** 1230
37 1240 **38** 1240 **39** 54 300 **40** 54 400
41 1.234 **42** 1.235 **43** 1.231 **44** 1.204
45 1.201

Exercise 6* (Revision)

1 $\frac{1}{11}$ **2** $\frac{3}{20}$ **3** $\frac{9}{10}$ **4** $\frac{1}{8}$
5 42 g **6** 37.5% **7** 12%
8 **a** 56.7 s **b** 19%
9 **a** 1.54 m **b** 23.2%
10 **a** €118 800 **b** €120 000 $\left(1 - \frac{x^2}{10^4}\right)$
11 7450 **12** 0.0745 **13** 74 500 **14** 74 500 000
15 5.3×10^6 **16** 8.8×10^{14} **17** 5.0×10^1 **18** 3.7×10^8
19 3.5×10^8 **20** 6.3×10^{10} **21** 0.201 **22** 0.00201
23 3080 **24** 47 600 **25** 0.079 **26** 0.072
27 0.063 **28** 0.111

Exercise 7

1 $5a$ **2** $2ab$ **3** $2a + 3b$ **4** $7xy$
5 $4ab$ **6** $7xy$ **7** $-3pq$ **8** $y - xy$
9 $-6x + 2$ **10** $2cd$ **11** $-4xy$ **12** $2ab + 5bc$
13 0 **14** $2gh - 5jk + 7$

Exercise 7*

1 $5ab$ **2** $4xy - 2xz$ **3** $3x + 3$ **4** $3y + 2z$
5 $-xy$ **6** $4ab - b$ **7** $6ab$ **8** 0
9 $3ab + 3bc$ **10** $3q^2$ **11** $x + 1$ **12** $a^3 + 2a^2 + a$
13 $h^3 + h^2 + 3h + 4$ **14** $7a^2b - 3ab$

Exercise 8

1 $6a$ **2** $2x^2$ **3** $3x^3$ **4** $15a^5$ **5** $6st$
6 $4rs^2$ **7** $2a^2b^2$ **8** $4y^3$ **9** $12x^3$ **10** $20a^3$

Exercise 8*

1 $10xy$ **2** $12x^5$ **3** $8a^3$ **4** $24x^3$ **5** $15x^4y^2$
6 $6a^7$ **7** $18y^3$ **8** $36x^5y^3$ **9** $30a^3b^3c^5$ **10** $56xy^4$

Exercise 9

1 $4x + 4y$ **2** $3a - 3b$ **3** $10 + 15a$
4 $2b - 8c$ **5** $-6a - 24$ **6** $4x - 12$
7 $-a + 2b$ **8** $5a + 4b$ **9** $3t - 18$
10 $6x + y$

Exercise 9*

1 $5x + 10y$ **2** $6x - 15y$ **3** $12m - 8$
4 $2x - 2y + 2z$ **5** $15a + 5b - 20c$ **6** $2x - 3y + 4$
7 $3y - x$ **8** $-4x - 4y + 4z$ **9** $-1.4x - 2.2$
10 $-6x - 3y$

Exercise 10

1 4 **2** 15 **3** 25 **4** 100 **5** 12
6 15 **7** 2.4 **8** 13.5 **9** 26.6 **10** 1.4
11 0.985 **12** 6.8

Exercise 10*

1 99.9 **2** 5.13 **3** 40.7
4 580 **5** 8.49 **6** 38.8

Exercise 11

1 $x = 3$ **2** $x = -1$ **3** $x = -2$ **4** $x = 2$
5 $x = 8$ **6** $x = 1$ **7** $x = -6$ **8** $x = 1$
9 $x = -2$ **10** $x = \frac{2}{3}$ **11** $x = \frac{5}{9}$ **12** $x = \frac{4}{3}$
13 $x = -1$ **14** 238, 239 **15** $x = 10$; 40, 80, 60
16 $x = 25$; 75, 50, 55 **17** -15

Exercise 11*

1 $x = 4$ **2** $x = 11$ **3** $x = -2$ **4** $x = -5$
5 $x = -4$ **6** $x = 5$ **7** $x = 0$ **8** 72, 74, 76
9 11, 44, 67 kg **10** 14 **11** $x = 2.5$ **12** 45 m (2 s.f.)
13 4.20 cm **14** $1\frac{2}{3}$ km

Exercise 12

1 $x = 1$ **2** $x = 2$ **3** $x = 4$ **4** $x = -3$
5 $x = 1$ **6** $x = -1$ **7** $x = 0$ **8** $x = -\frac{1}{2}$
9 $x = 2, 38$ **10** 9

Exercise 12*

1 $x = 4$ **2** $x = -2$ **3** $x = 1\frac{1}{2}$ **4** $x = \frac{4}{5}$
5 $x = \frac{7}{9}$ **6** $x = 3$ **7** $x = 5$ **8** $x = -9$
9 $x = 0.576$ (3 s.f.) **10** $x = 30$ **11** 42 years

Exercise 13

1 $x = 13$ **2** $x = 3$ **3** $x = 2$
4 $x = 4$ **5** $x = \frac{5}{2}$

Exercise 13*

1 $x = 8$ **2** $x = 5$ **3** $x = 4$ **4** $x = 9$
5 $x = \frac{3}{4}$ **6** 6 hits **7** 15 **8** 4 nights

Exercise 14 (Revision)

1 $3x - 2$ **2** ab **3** $6a$ **4** $2a^2$
5 a^3 **6** $2a^4$ **7** $4a^4$ **8** $-4ab - 5a$
9 $x + 7y$ **10** $x = 7$ **11** $x = 4.8$ **12** $x = 2$
13 145, 146, 147
14 **a** $4x + 12 = 54$ **b** $x = 10.5, 10.5, 16.5$

Exercise 14* (Revision)

1 $4xy^2 - 3x^2y$ **2** $2x^3y^3$ **3** 1 **4** $2x^3y + xy^3 + x^4$
5 $x = 20$ **6** $x = \frac{5}{4}$ **7** $x = -6$ **8** $x = 2$
9 $x = 4$ **10** 72 m^2 **11** 11 years old **12** 6 m/s
13 \$12 800

Exercise 15

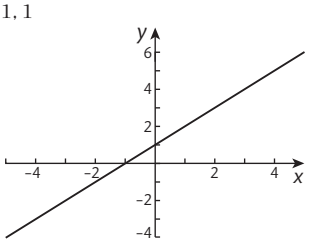
1 1 **2** 0.5 **3** 3 **4** $\frac{1}{4}$ **5** $-\frac{1}{4}$
6 10m **7** 1.5 m **8** 2.325 m **9** 2 m
10 **a** 14m **b** $\frac{1}{30}$ **11** **a** 2 **b** 159 m

Exercise 15*

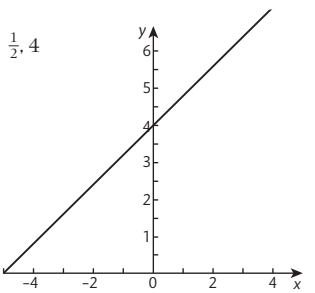
1 $\frac{3}{8}$ **2** $-\frac{3}{4}$ **3** 52 **4** No **5** Yes
6 **a** $\frac{1}{6}$ cm **b** 0.1 cm **7** 26m **8** $p = -2$

Exercise 16

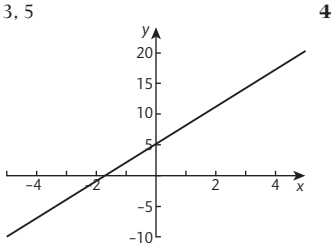
1 1, 1



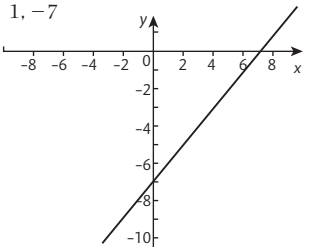
2 $\frac{1}{2}$, 4



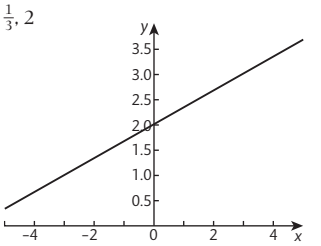
3 3, 5



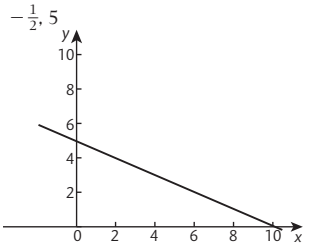
4 1, −7



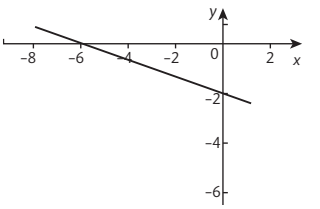
5 $\frac{1}{3}$, 2



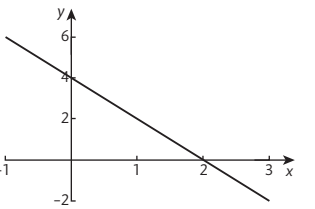
6 $-\frac{1}{2}$, 5



7 $-\frac{1}{3}$, −2



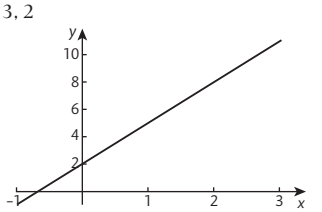
8 −2, 4



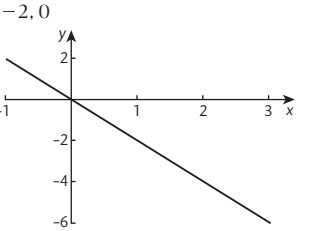
9 $y = 2x + 1$
10 $y = -x + 2$ **11** $y = 2x + 4$ **12** $y = -5x - 1$
13 For example:
a $y = x - 2$ **b** $y = 5 - x$ **c** $y = 2$

Exercise 16*

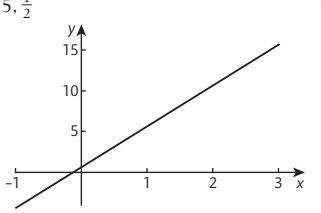
1 3, 2



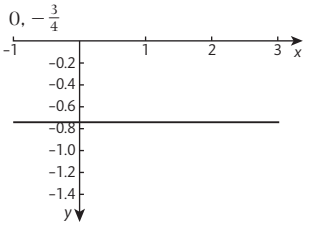
2 −2, 0



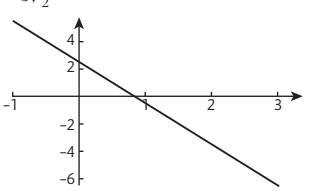
3 5, $\frac{1}{2}$



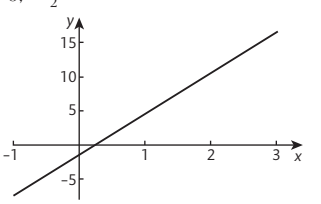
4 0, $-\frac{3}{4}$



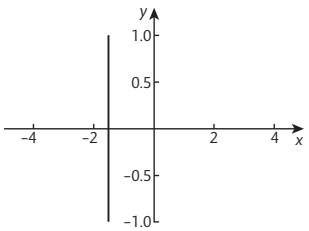
5 −3, $\frac{5}{2}$



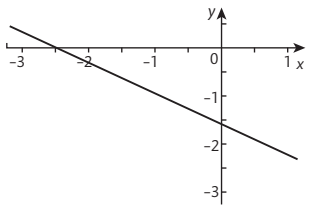
6 6, $-\frac{3}{2}$



7 Infinity (vertical line)



8 $-\frac{2}{3}, -\frac{5}{3}$



9 $y = 2.5x - 2.3$ **10** $y = \frac{x}{4} + 1$

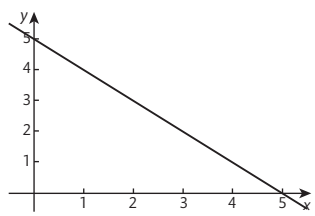
11 $2y = 5x - 7$ **12** $7x + 6y = 84$

13 For example:

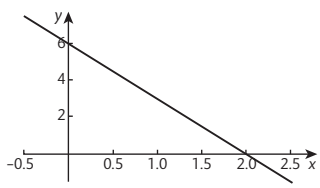
a $x = 3$ **b** $y = 3 - x$ **c** $y = 4x - 2$

Exercise 17

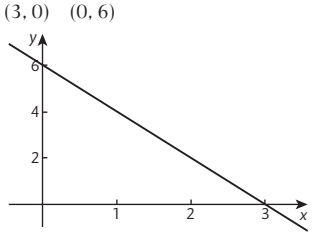
1 (5, 0) (0, 5)



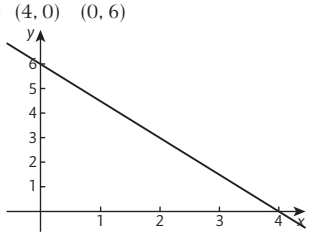
2 (2, 0) (0, 6)



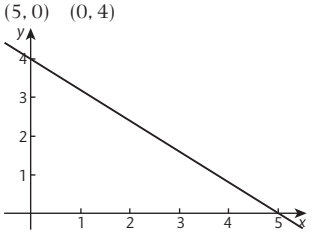
3 (3, 0) (0, 6)



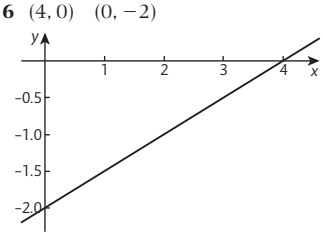
4 (4, 0) (0, 6)



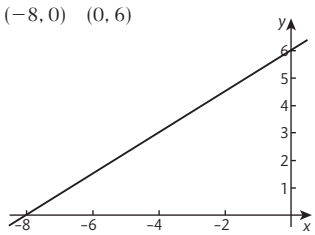
5 (5, 0) (0, 4)



6 (4, 0) (0, −2)

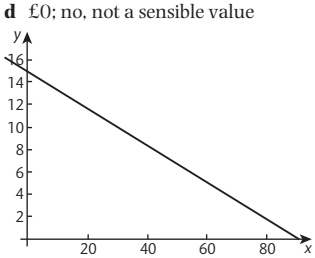


7 (−8, 0) (0, 6)



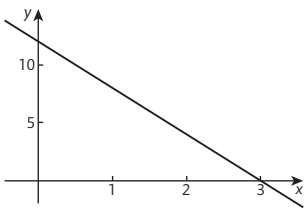
8 **b** £10 **c** 42 000

d £0; no, not a sensible value

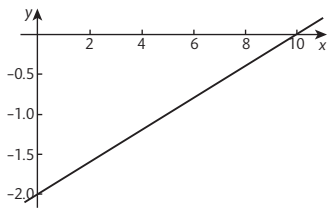


Exercise 17*

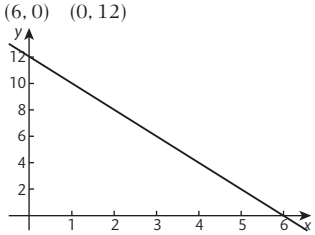
1 (3, 0) (0, 12)



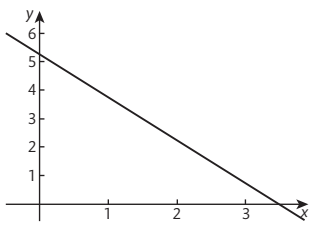
2 (10, 0) (0, −2)



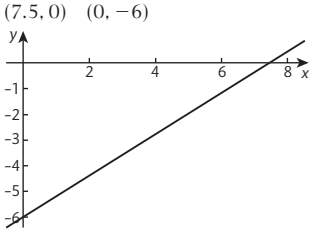
3 (6, 0) (0, 12)



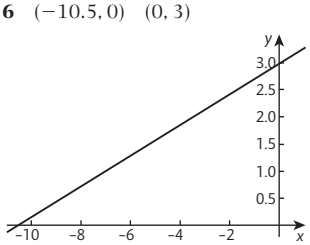
4 (3.5, 0) (0, 5.25)



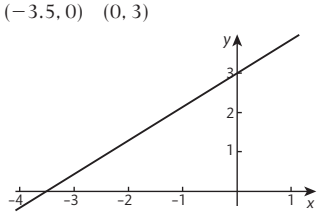
5 (7.5, 0) (0, −6)



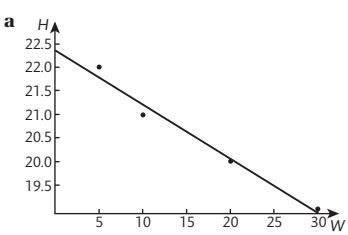
6 (−10.5, 0) (0, 3)



7 (−3.5, 0) (0, 3)



8 **a**



b 23 **c** -9, 22; $9H + W = 200$

d 92 weeks; no, longer, unlikely to continue linear

Exercise 18 (Revision)

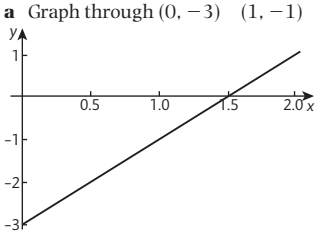
1 **a** 2 **b** -1

2 4.5 m

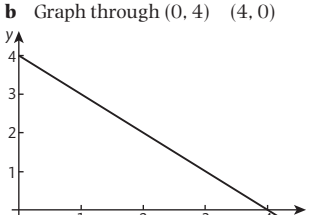
3 **a** 3, -2 **b** -2, 5

4 **a** $y = 2x - 1$ **b** $y = -3x + 2$

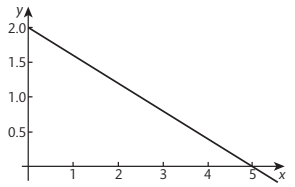
5 **a** Graph through (0, −3) (1, −1)



b Graph through (0, 4) (4, 0)



c Graph through (5, 0) (0, 2)



6 $y = 2x + 4$; $4x = 2y + 7$
$x - 3y = 1$; $9y = 3x + 4$
$4x - 3y = 12$; $3y = 4x - 1$
$3x - 4y = 12$; $4y = 3x + 7$

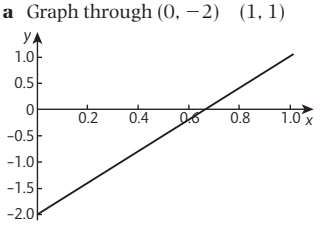
7 **a** $\frac{2}{3}$ **b** $-\frac{1}{2}$

Exercise 18* (Revision)

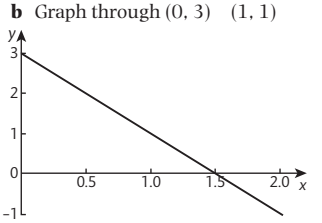
1 **a** $\frac{1}{2}$ **b** -2.5

2 5 m

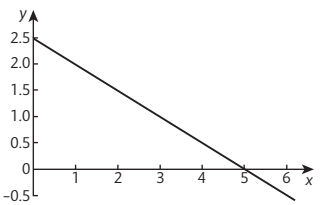
3 **a** Graph through (0, −2) (1, 1)



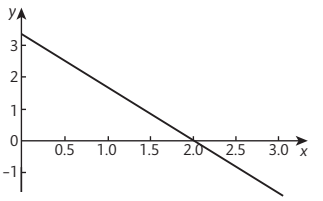
b Graph through (0, 3) (1, 1)



c Graph through (0, 2.5) (1, 2)

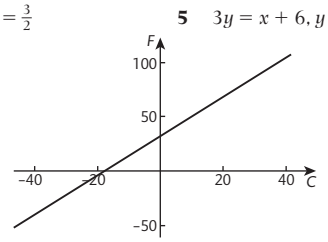


d Graph through (2, 0) $\left(0, 3\frac{1}{3}\right)$



4 $b = \frac{3}{2}$ **5** $3y = x + 6$, $y = 22 - 3x$

6 **a**



b 80°F ≈ 27°C **c** −40°
−22°F ≈ −30°C
25°C ≈ 77°F

7 $y = x + 2$, $y = x - 2$, $y = -x + 2$, $y = -x - 2$

8 $y = 3$, $y = -3$, $x = 3$, $x = -3$, $y = x$, $y = -x$

Exercise 19

1 $a = 102°$, $b = 78°$ **2** $a = 65°$
3 $a = 73°$, $b = 34°$ **4** $a = 57°$, $b = 123°$
5 $a = 31°$, $b = 31°$ **6** $a = 124°$, $b = 56°$
7 $a = 58°$, $b = 32°$
8 **a** 45° **b** 135° **c** 1080°
9 9 sides **10** $x = 74°$; 74° and 148°

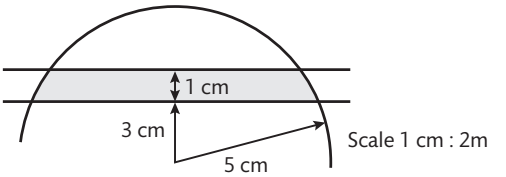
Exercise 19*

1 $a = 137°$, $b = 43°$ **2** $a = 36°$
3 34°, 85° **4** $180 - 2x$
5 $x = 50°$ **6** $a = 56°$, $b = 34°$
7 $a = 40°$, $b = 113°$ **8** $a = 73°$
9 $a = 56°$, $b = 38°$

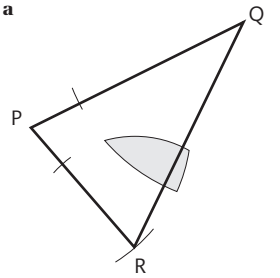
Exercise 20

1 5.9 cm

2

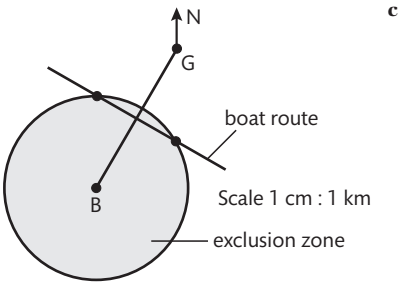


3 **a**



b 275 km

4 **a** and **b**



c 3.87 km

5 152 m

6 161 m

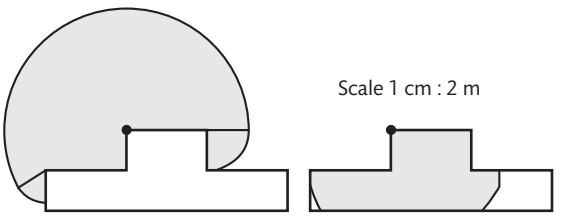
Exercise 20*

1 5.4 cm

2 6.8 cm

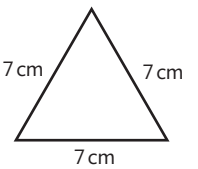
3, 5 Student's own diagrams

4

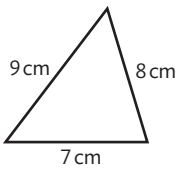


Exercise 21 (Revision)

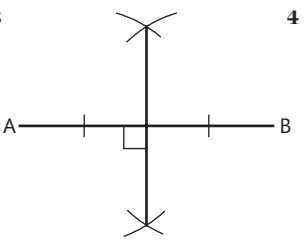
1



2

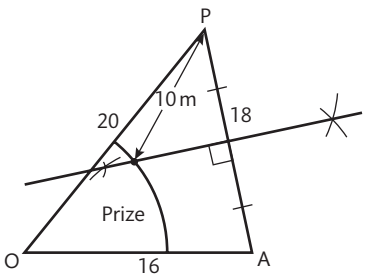


3



4 Measure and check arcs are included.

5



6 $x = 143°, y = 37°$ **7** $x = 30°, y = 60°$
8 $x = 69°, y = 42°$ **9** $x = 65°, y = 115°$
10 $x = 40°, y = 140°$ **11** $x = 77°, y = 103°$
12 $x = 60°, y = 120°$ **13** $x = 25°, y = 115°$
14 $x = 100°, y = 75°, z = 135°$

Exercise 21* (Revision)

1 **a** 36 **b** 144° **c** 1440°
2 **a** 22 sides **b** $16\frac{4}{11}°$
3 **b** 30.7 m **4** **c** 13.5 m
5 $x = 46\frac{2}{3}°, y = 133\frac{1}{3}°$ **6** $x = 66°$
7 $x = 160°$ **8** $x = 36°, y = 106°, z = 38°$
9 $x = 70°, y = 60°$ **10** $x = 60°, y = 30°$
11 $x = 30°, y = 120°$ **12** $x = 36°$
13 $x = 20°$ **14** $x = y = 33°, z = 83°$
15 $x = 150°$ **16** $x = 36°$

Exercise 22

1 **a** Any two vegetables **b** Any two colours
c Any two letters **d** Any two odd numbers
2 **a** {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}
b {1, 4, 9, 16, 25, 36, 49, 64, 81, 100}
c For example, {Mathematics, Science, English, ...}
d {2, 3, 5, 7, 11, 13, 17, 19}
3 **a** {the first four letters of the alphabet}
b {days of the week beginning with T}
c {first four square numbers} **d** {even numbers}
4 **a** False **b** False
c False **d** True
5 b and c

Exercise 22*

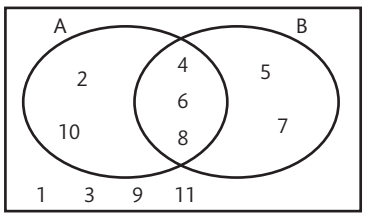
1 **a** Any two planets **b** Any two polygons
c Any two chemical elements **d** Any two square numbers
2 **a** {2, 3, 4}
b {1, 4, 6}
c {1, 5, 7, 35}
d {1, 10, 100, 1000, 10 000, 100 000}
3 **a** {seasons of the year} **b** {conic sections}
c {first five powers of 2} **d** {Pythagorean triples}
4 **a** True **b** False
c True **d** False
5 a, c and d

Exercise 23

1 **a** 16 **b** $n(T) = 14$; 14 pupils like toffee
c $n(C \cap T) = 12$; 12 pupils like both chocolate and toffee
d 21
2 **a** Pink Rolls-Royce cars
b There are no pink Rolls-Royce cars in the world.
3 **a** 35 **b** 3 **c** 20 **d** 2 **e** 64
4 **a** 39 **b** 22 **c** 8 **d** 12 **e** 70

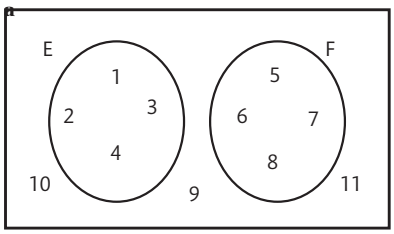
Exercise 23*

1 **a**



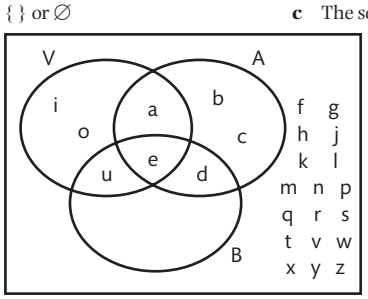
b {4, 6, 8}, 3 **c** Yes
d {1, 2, 3, 5, 7, 9, 10, 11} **e** Yes

2 **a**



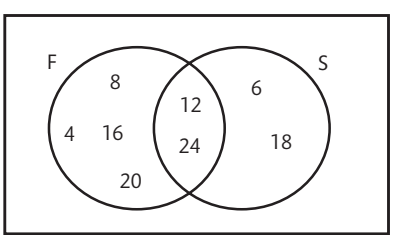
b { } or ∅ **c** The sets don't overlap (*disjoint*).

3 **a**



b {a, e}, {a, i, o}, {u} **c** {e}

4 **a**

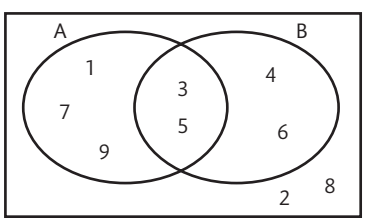


b {12, 24} **c** 12
d LCM is 12. It is the smallest element of F ∩ S.
e **(i)** 24 **(ii)** 40

5 {a, b, c} {a, b} {a, c} {b, c} {a} {b} {c}, ∅, 2^n

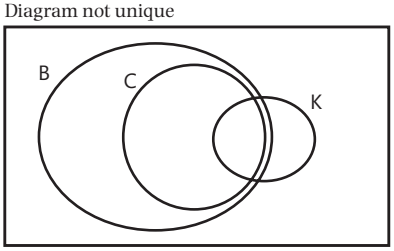
Exercise 24

1 **a**



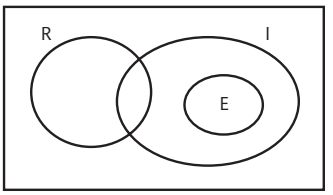
b {1, 3, 4, 5, 6, 7, 9}, 7 **c** Yes
d {2, 8} **e** No

2 **a** Diagram not unique



b All cards that are black or a king or both
c All cards that are black or a king or both
d All cards that are red or king or both

3 **a**



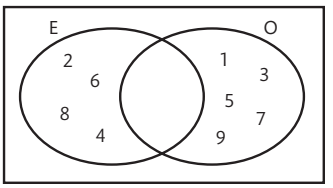
b An isosceles right-angled triangle.

c Isosceles triangles, triangles that are isosceles or right−angled or both.

d Equilateral triangles, ∅

Exercise 24*

1 **a**



b {1, 2, 3, 4, 5, 6, 7, 8, 9} **c** $E \cap O = \varnothing$ **d** $E \cap O = \mathscr{E}$

2 **a** Pizzas containing ham or cheese or both.

b Pizzas containing both ham and cheese.

c All the pizzas contain ham or cheese or both.

3 $B \subset A$ or $B = \varnothing$

Exercise 25 (Revision)

1 **a** any 2 spices **b** any 2 pets

c any 2 fruits **d** any 2 colours

2 **a** {4, 9, 16, 25} **b** {1, 2, 3, 4, 6, 8, 12, 24}

c {a, e, i} **d** {April, June, Sept, Nov}

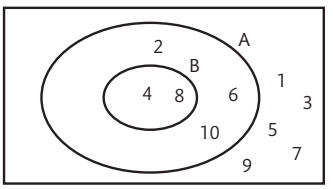
3 **a** {prime numbers less than 10}

b {even numbers between 31 and 39}

c {days in weekend} **d** {vowels}

4 **a** F **b** F **c** T **d** T

5 **a**

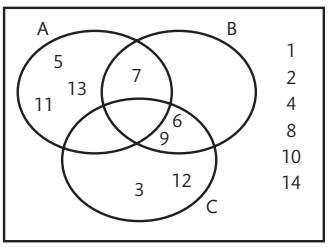


b A′ = {1, 3, 5, 7, 9} odd numbers

c $n(B') = 8$ **d** Yes

6 **a** C = {3, 6, 9, 12}

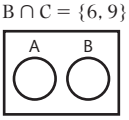
b



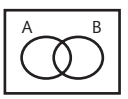
c $A \cup B = \{5, 6, 7, 9, 11, 13\}$

d $B \cap C = \{6, 9\}$ **e** $A \cap C = \varnothing$

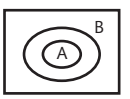
7 **a**



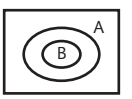
b



c



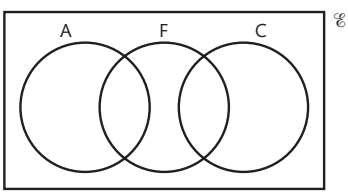
d



8 **a** {females born in Africa}

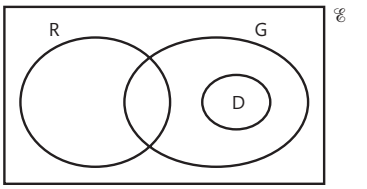
b ∅

c She was born in Africa *or* China

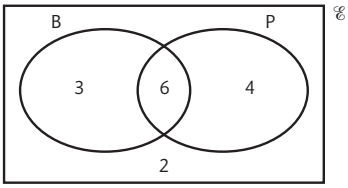


9 **a** There are no red dresses

b All dresses are green



10 **a**



b 15

Exercise 25* (Revision)

1 **a** {4, 8, 12, 16}

b {Red, Orange, Yellow, Green, Blue, Indigo, Violet}

c {CAT, CTA, ATC, ACT, TAC, TCA}

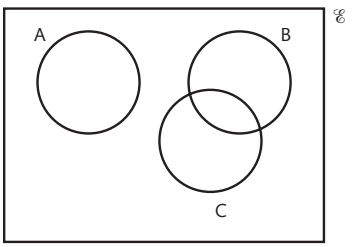
d {2, 3, 6}

2 **a** {factors of 12} **b** {1st five Fibonacci numbers}

c {suits of playing cards} **d** {3D shapes}

3 **a** 9

b

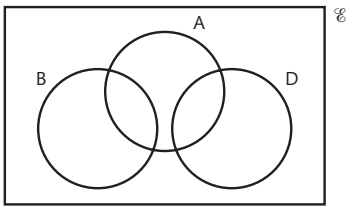


4 **a** {ace of diamonds}

b ∅

c {all diamonds plus 3 other aces}

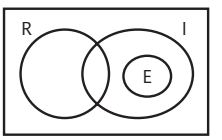
d 2



5 **a** {right-angled isosceles triangles} **b** equilateral triangle

c ∅

d

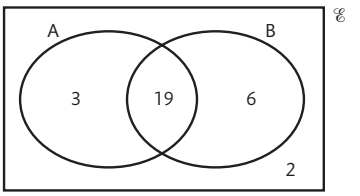


6 **a** {20} **b** {12, 24} **c** Yes

7 {2, 4, 6, 8, 10, 12, 14}

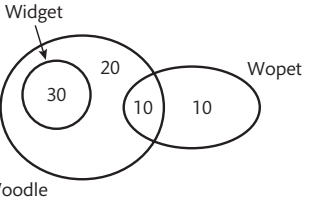
8 **a** 8 **b** 2

9 **a**



b 19

10 **a**



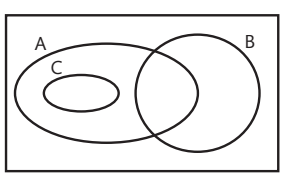
b 20

Multiple choice 1

1 C **2** C **3** B **4** D **5** A
6 D **7** D **8** A **9** B **10** D

Self-assessment 1

1 **a** 3.85×10^5 **b** 3250
2 **a** 36.58 **b** 37
3 **a** 4.00×10^2 **b** 4.81×10^5
4 **a** 1.20×10^6 **b** 2.60×10^3
5 **a** £4.62 **b** £42.77
6 **a** $8ab$ **b** $3xy + 3x$ **c** $3a^3b$
7 **a** $x = 3$ **b** $a = 3.5$ **c** $x = 3$
8 **a** $x + 1$ **b** $x + x + 1 + x + 2 = 525$ **c** $x = 174$
9 $x - 3y = 12$ and $6y - 2x = 7$
10 **a** 2 **b** -4
11 **a** 400 m **b** $q = 7$
12 **b** AC = 8.6 cm
13 **c** RS = 7.9 cm, area = 31.5 cm^2
14 $p = 80$
15 **d** $x = 0.6$, $y = 3.71$
16 A gradient $= \frac{1}{2}$ intercept $= 4$ equation is $y = \frac{1}{2}x + 4$
B gradient $= -2$ intercept $= 8$ equation is $y = -2x + 8$
17 **a** $A \cap C = \varnothing$ **b** $C \cup D = C$ **c** $A \cap B \neq \varnothing$
18 **a** 13 **b** 7
19 **a** and **c**



b 90°, 45°, 45°
20 **a** {6, 12, 18} **b** {multiples of 6 less than 19}
c {odd numbers less than 19}
d {3, 9, 15} **e** {odd multiples of 3 less than 19}

Exercise 26

1 10^{-1} **2** 10^{-3} **3** 10^{-3}
4 10^1 **5** 0.001 **6** 0.001 2
7 0.000 001 **8** 0.046 7 **9** 5.43×10^{-1}
10 7×10^{-3} **11** 6.7×10^{-1} **12** 1×10^2
13 100 **14** 10 000 **15** 128
16 30 **17** 0.018

Exercise 26*

1 10 **2** 0.011 **3** 0.01
4 0.0011 **5** 1000 **6** 10^1
7 10^4 **8** 10^2
9 5000 viruses **10** 66 000 viruses
11 **a** 10^{27}, 27 zeros
c 10^7
d 2×10^{23}, 2×10^{16} cm, $(2 \times 10^{16}) \div (4 \times 10^9) \approx 5 \times 10^6$ times!

Exercise 27

1 $\frac{6}{7}$ **2** $\frac{2}{5}$ **3** $1\frac{2}{9}$ **4** $\frac{1}{2}$
5 $\frac{23}{24}$ **6** $4\frac{5}{12}$ **7** $\frac{5}{18}$ **8** $\frac{6}{7}$
9 $\frac{3}{5}$ **10** $\frac{3}{25}$ **11** 3 **12** $1\frac{1}{2}$
13 $4\frac{7}{12}$ **14** $2\frac{3}{4}$ **15** $4\frac{4}{5}$ **16** $\frac{3}{10}$

Exercise 27*

1 $\frac{3}{4}$ **2** $\frac{3}{5}$ **3** $\frac{19}{20}$ **4** $\frac{3}{4}$
5 $\frac{1}{5}$ **6** $\frac{7}{10}$ **7** $\frac{2}{9}$ **8** $\frac{3}{8}$
9 $7\frac{2}{3}$ **10** $6\frac{1}{2}$ **11** $6\frac{7}{9}$ **12** $1\frac{7}{15}$
13 $2\frac{2}{3}$ **14** $\frac{5}{6}$ **15** $4\frac{1}{2}$ **16** $1\frac{7}{8}$

Exercise 28

1 \$168, \$224 **2** 94 kg, 658 kg
3 574, 410 **4** 8.1, 5.4

Exercise 28*

1 \$45 : \$75 **2** 111 ml **3** \$32 **4** 1 mg

Exercise 29

1 $2^4 = 16$ **2** $2^5 = 32$ **3** $2^2 = 4$
4 $5^3 = 125$ **5** $3^6 = 729$ **6** $2^{10} = 1024$
7 $(0.1)^3 = 0.001$ **8** $2.1^2 = 4.41$ **9** $4^4 = 256$
10 $20^4 = 160\,000$

Exercise 29*

1 $8^3 = 512$ **2** $7^3 = 343$ **3** $5^3 = 125$
4 $6^1 = 6$ **5** $5^3 = 125$ **6** $10^6 = 1\,000\,000$
7 $2^0 = 1$ **8** $5^3 = 125$ **9** 2 097 152, 524 288
10

No. times done	No. of sheets in pile	Height of pile
3	8	0.8 mm
5	32	3.2 mm
10	1024	0.102 m
50	1.13×10^{15}	1.13×10^8 km

42 times

Exercise 30

1 Yen 180 **2** Aus\$2.38 **3** NZ\$2.75
4 **a** £8.40 **b** £75.60
5 **a** 105 mm/h **b** 1.75 mm/min
6 **a** 0.9 s **b** 66.7

Exercise 30*

1 **a** HK \$15 **b** ¥2667
2 ≈ 12 Miles **3** ≈ 50
4 **a** 82.5 s **b** 12.1 litres/s
5 **a** 94.4 m/s **b** 0.001 s

Exercise 31

	km	m	cm	mm
1	5	5000	5×10^5	5×10^6
2	2	2000	2×10^5	2×10^6
3	0.05	50	5000	5×10^4
4	1	10^3	10^5	10^6

Exercise 31*

	km	m	cm	mm
1	2.5×10^4	2.5×10^7	2.5×10^9	2.5×10^{10}
2	5×10^3	5×10^6	5×10^8	5×10^9
3	5×10^{-4}	0.5	50	500
4	9×10^3	9×10^6	9×10^8	9×10^9

5 **a** 2×10^{14} **b** 1×10^6

Exercise 32

	km^2	m^2	cm^2	mm^2
1	2	2×10^6	2×10^{10}	2×10^{12}
2	5×10^{-5}	50	5×10^5	5×10^7
3	6×10^{-4}	6×10^2	6×10^6	6×10^8
4	10	10^7	10^{11}	10^{13}

Exercise 32*

	km^2	m^2	cm^2	mm^2
1	6×10^{-3}	6000	6×10^7	6×10^9
2	6	6×10^6	6×10^{10}	6×10^{12}
3	2×10^9	2×10^{15}	2×10^{19}	2×10^{21}
4	7×10^{-2}	7×10^4	7×10^8	7×10^{10}

Exercise 33

	km²	m²	cm²	mm²
1	1	10^9	10^{15}	10^{18}
2	8×10^{-9}	8	8×10^6	8×10^9
3	4×10^{-12}	4×10^{-3}	4×10^3	4×10^6
4	10^{-3}	10^6	10^{12}	10^{15}

5 1000 **6** 10

Exercise 33*

	km²	m²	cm²	mm²
1	6×10^{-7}	600	6×10^8	6×10^{11}
2	3×10^{-7}	3×10^2	3×10^8	3×10^{11}
3	5×10^7	5×10^{16}	5×10^{22}	5×10^{25}
4	4×10^{-6}	4×10^3	4×10^9	4×10^{12}

5 5.12×10^5 **6** 10^3 **7** 10^{45}

Exercise 34 (Revision)

1 $\frac{29}{35}$ **2** $\frac{1}{35}$ **3** $\frac{6}{35}$
4 $1\frac{1}{14}$ **5** $3\frac{26}{35}$ **6** $1\frac{16}{35}$
7 $2\frac{34}{35}$ **8** $2\frac{11}{40}$ **9** 12 m : 24 m
10 45 kg : 60 kg **11** \$160 : \$240
12 160 min : 200 min **13** £19 : £38 : £76
14 64 km : 96 km : 192 km **15** 1.23×10^{-2}
16 1.24×10^{-2} **17** 1.60×10^{-4}
18 8.89×10^{-3} **19** 4.31×10^3
20 1.02×10^8 **21** 2.50×10^{-12}
22 2.93×10^{-8} **23** 3.62×10^{-8}
24 5.61×10^{-17} **25** 2.10×10^7
26 2.46×10^5
27 **a** \$150 **b** \$1650 **c** \$75
28 **a** \$1.20 **b** \$6 **c** \$14.40
29 **a** 162 **b** 180 **c** 12 960
30 **a** **(i)** 46.7 km **(ii)** 17 056 km
b **(i)** 0.214 days **(ii)** 0.0749 days
c 5.41×10^2 mm/s

Exercise 34* (Revision)

1 $\frac{3}{44}$ **2** $\frac{27}{35}$ **3** $\frac{3}{5}$
4 $\frac{25}{63}$ **5** 4 **6** $2\frac{89}{100}$
7 10 **8** $\frac{2m}{5}$
9 $X : Y : Z = £3000 : £6000 : £2000$
10 **a** 500 mm **b** 20 m²
11 1.85×10^{-6} **12** 2.66×10^{-22}
13 2.42×10^{-6} **14** 1.62×10^{-3}
15 **a** 4.08×10^{-7} **b** 1.76×10^{-9}
c 3.87×10^{-11} **d** 4.83×10^{-4}
16 2.90×10^{-5} km
17 **a** 3.02×10^{30} mm² **b** 8.28%
18 6.32×10^{-13} km/s

Exercise 35

1 4 **2** $3y$ **3** 2 **4** $\frac{3a}{b}$
5 $\frac{b}{2}$ **6** $\frac{3a}{b}$ **7** $4c$ **8** $\frac{a}{2}$
9 $\frac{4}{x}$ **10** $2b$ **11** $\frac{1}{5b^2}$ **12** $\frac{a}{4}$

Exercise 35*

1 $\frac{1}{2}$ **2** $\frac{2}{b}$ **3** $\frac{x}{4}$ **4** $\frac{a}{2}$
5 $\frac{2}{b}$ **6** $\frac{6}{b^2}$ **7** $3y$ **8** $\frac{a}{2b}$
9 $\frac{3}{abc}$ **10** $\frac{a}{(4b^2)}$ **11** $\frac{1}{a^2b^2}$ **12** $\frac{3a}{8b}$

Exercise 36

1 $\frac{5x^2}{4}$ **2** $\frac{x^3z}{y}$ **3** 1 **4** $\frac{4c^2}{5}$ **5** 6
6 $\frac{ab}{2}$ **7** $\frac{b}{6}$ **8** 1 **9** $\frac{2}{y}$ **10** $\frac{b}{2c}$

Exercise 36*

1 $\frac{x^2}{6}$ **2** $2a^3$ **3** $\frac{3x}{z}$ **4** $\frac{9pq^2}{10}$ **5** $\frac{1}{2x}$
6 $\frac{5xy^3}{z^2}$ **7** y **8** $\frac{x^2}{5z}$ **9** $\frac{3x^4}{8y}$ **10** $\frac{1}{2a^2b^2}$

Exercise 37

1 $\frac{3x}{4}$ **2** $\frac{7x}{12}$ **3** $\frac{a}{12}$
4 $\frac{4a+3b}{12}$ **5** $\frac{5x}{12}$ **6** $\frac{a}{2}$
7 $\frac{3a+4b}{12}$ **8** $\frac{5x}{12}$ **9** $\frac{a}{6}$
10 $\frac{3a+8b}{12}$

Exercise 37*

1 $\frac{7x}{18}$ **2** $\frac{5a}{21}$ **3** $\frac{14x+20y}{35}$
4 $\frac{a}{4}$ **5** $\frac{17}{6b}$ **6** $\frac{2d+3}{d^2}$
7 $\frac{7-3x}{10}$ **8** $\frac{y-2}{30}$ **9** $\frac{3x+5}{12}$
10 $\frac{2a-1}{a(a-1)}$

Exercise 38

1 $x = \pm 3$ **2** $x = \pm 6$ **3** $x = \pm 4$
4 $x = \pm 8$ **5** $x = \pm 3$ **6** $x = \pm 1$
7 $x = 13$ **8** $x = \pm 4$ **9** $x = 16$
10 $x = 81$

Exercise 38*

1 $x = \pm 5$ **2** $x = \pm 5$ **3** $x = \pm 7$
4 $x = \pm 9$ **5** $x = 81$ **6** $x = 7$
7 $x = \pm 4$ **8** $x = \pm 5$ **9** $x = 10$ or -16
10 $x = \pm 3$

Exercise 39

1 4.5 cm **2** 44 cm; 154 cm²
3 $h = 8$ cm **4** $YZ = 4$ cm
5 **a** 9.42×10^8 km **b** 108 000 km/h
6 $h = 4.7$ cm **7** 1.5×10^8 km
8 0.1 second

Exercise 39*

1 14 cm **2** AC = 12 cm; 4.2 cm
3 $h = 5.5$ cm **4** $\sqrt{9^2 + 12^2} > 14$; no
5 5.30 cm² **6** 10.5 cm² for each
7 15.9 km **8** 220 m
9 8.37 cm

Exercise 40

1 $2^{10} = 1024$ **2** $4^7 = 16\,384$ **3** $2^6 = 64$
4 $7^3 = 343$ **5** $2^{12} = 4096$ **6** $6^8 = 1\,679\,616$
7 a^5 **8** c^4 **9** e^6
10 a^9 **11** c^5 **12** $12 \times a^6$
13 $6a^5$ **14** $2e^2$

Exercise 40*

1 $6^{12} = 2.18 \times 10^9$ **2** $7^6 = 1.18 \times 10^5$
3 $8^{12} = 6.87 \times 10^{10}$ **4** $4^{17} = 1.72 \times 10^{10}$
5 a^{12} **6** $3c^6$ **7** $2e^8$
8 $8g^{12}$ **9** $48j^{12}$ **10** $24m^7$
11 $27a^6$ **12** 2 **13** $8b^4$
14 6

Exercise 41

1 $<$ **2** $<$ **3** $>$
4 $<$ **5** $x \leqslant 0, x > 2$ **6** $-3 < x \leqslant 3$
7 $x > 5$ **8** $x \leqslant 4$ **9** $x < 3$
10 $x \geqslant 3$ **11** $x \geqslant 9$ **12** $x < 4$
13 $x < 0$ **14** $x > 3$ **15** $\{5, 6\}$
16 $\{3, 4\}$ **17** $\{0, 1\}$ **18** $1 \leqslant x < 5$ $\{1, 2, 3, 4\}$
19 $1\frac{1}{2} < x \leqslant 3$ $\{2, 3\}$

Exercise 41*

1 $x \leqslant 0$ or $x > 3$; $0 \geqslant x > 3 \rightarrow 0 \geqslant 3$ **2** $x \leqslant 2.5$
3 $x < 5\frac{1}{3}$ **4** $x < 1.5$ **5** $x < -3\frac{1}{5}$
6 $x \leqslant 2$ **7** $-1 < x \leqslant 3$ **8** $x \leqslant 7$ $\{7, 6, 5, 4\}$
9 23 **10** $\{1, 2, 3\}$

Exercise 42 (Revision)

1 3 **2** x **3** $3x$
4 4 **5** a **6** $6x$
7 $\frac{9y}{20}$ **8** $\frac{2x}{15}$ **9** $\frac{4a + b}{10}$
10 $x = \pm 4$ **11** $x = \pm 6$ **12** $x = 20$
13 a^{10} **14** b^2 **15** c^{12}
16 $>$ **17** $<$ **18** $<$
19 $=$ **20** $-3 < x \leqslant 2$ **21** $x > 5$
22 $x \leqslant 4.5$ **23** $x \geqslant 2$ **24** $x \geqslant 1$
25 $\{3, 4\}$ **26** 3.24 cm **27** 11.34 km

Exercise 42* (Revision)

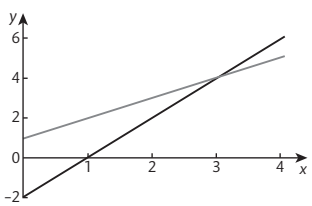
1 $\frac{4a}{b}$ **2** $\frac{5x}{y}$ **3** $\frac{b}{4a}$
4 $\frac{b}{2}$ **5** $\frac{5}{xy}$ **6** $\frac{18b}{a}$
7 $\frac{8a}{5}$ **8** $\frac{7}{12b}$ **9** $\frac{2x + 6}{21}$
10 $x = \pm 3$ **11** $x = 2$ **12** $x = \pm 4$
13 a^4 **14** $4b^6$ **15** $81c^7$
16 $-3 < x \leqslant 0, -2$ **17** $x < -4.4$ **18** $x > -4$
19 $x \leqslant 4.5$ **20** 37 **21** $\{-3, -2, -1, 0\}$
22 50.1 cm **23** 0.39 s
24 1130 km/s = 4.07×10^6 km/h

Exercise 43

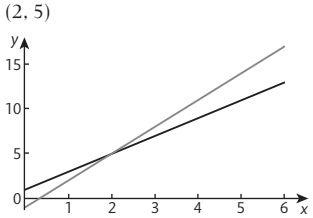
1

x	0	2	4
$y = x + 1$	1	3	5
$y = 2x - 2$	−2	2	6

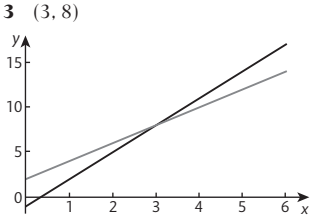
Intersection point is (3, 4).



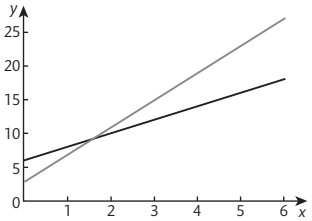
2 (2, 5)



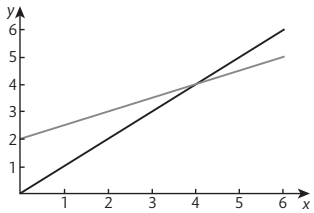
3 (3, 8)



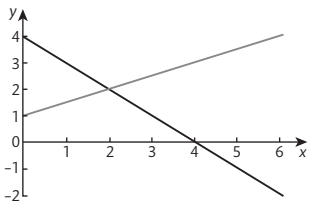
4 (1.5, 9)



5 (4, 4)

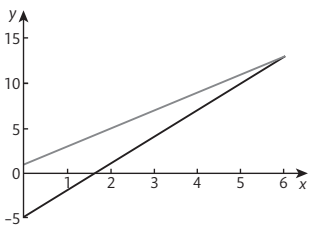


6 (2, 2)

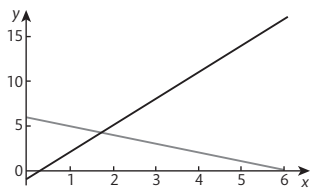


Exercise 43*

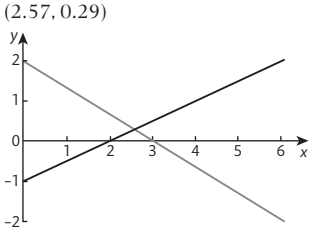
1 (6, 13)



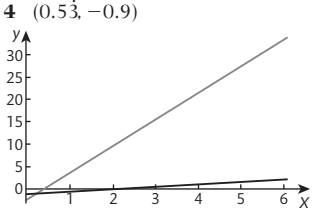
2 (1.75, 4.25)



3 (2.57, 0.29)

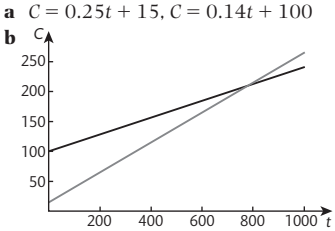


4 (0.5$\dot{3}$, −0.9)



5 **a** $C = 0.25t + 15$, $C = 0.14t + 100$

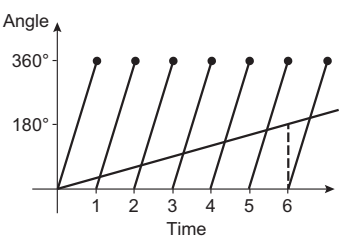
b



c 773 min

6 Angles: 0°, 90°, 180°, 270°, 0°, 90°, 180°, 270°, 0°, 90°, 180°

a and **b**

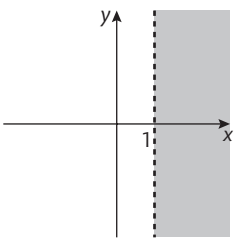


c 1 h 5.45 min; 2 h 10.9 min; 3 h 16.4 min; 4 h 21.8 min; 5 h 27.3 min

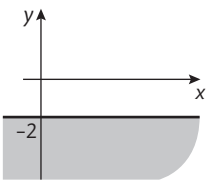
Exercise 44

1 $x \leqslant 2$ **2** $y > 4$
3 $y \leqslant -2$ **4** $x + y \geqslant 6$

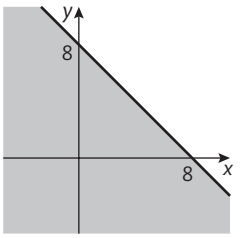
5



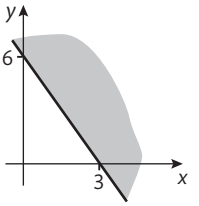
6



7



8



Exercise 44*

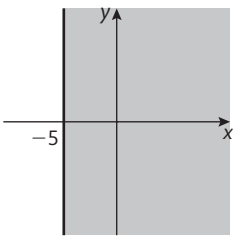
1 $y > -2$

2 $2x + y \geqslant 6$

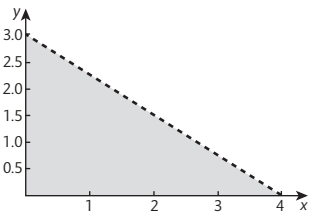
3 $y - x < 4$

4 $2y + x \leqslant 4$

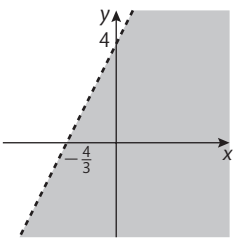
5



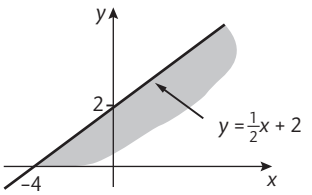
6



7



8



Exercise 45

1 $2 < x < 5$

2 $-2 < y \leqslant 3$

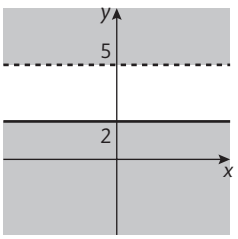
3 $x \leqslant -3, x \geqslant 4$

4 $y \geqslant 9$ or $y < 3$

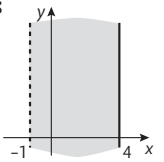
5 $x + y > 3$ and $x - y \geqslant 2$

6 $y < x + 3, 2y + x \leqslant 6$ and $y \geqslant 0$

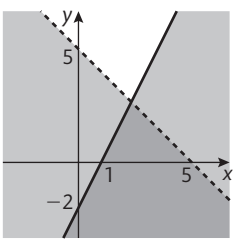
7



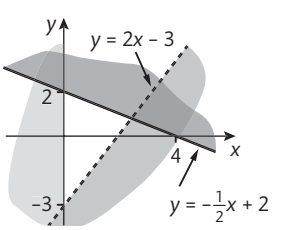
8



9



10



Exercise 45*

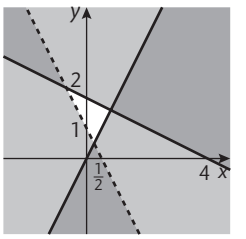
1 $-3 \leqslant x < 4$

2 $2y + x \geqslant 10$ or $2y + x \leqslant 4$

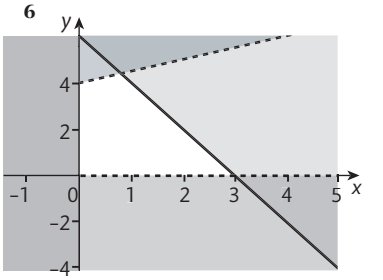
3 $y \geqslant 0, y < 2x + 4, 4x + 3y \leqslant 12$

4 $x \geqslant 0, y \geqslant 0, y < -\frac{3x}{2} + 9$ and $y \leqslant -\frac{2x}{3} + 6$

5

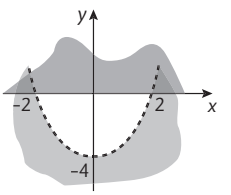


6



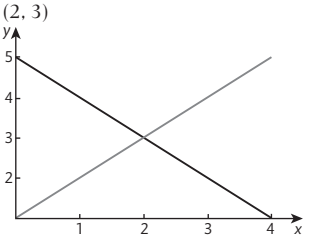
7 **b** $y < x + 2, y < 2 - 2x, 2y > -x - 2$ **c** 0

8

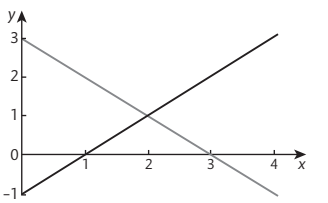


Exercise 46 (Revision)

1 (2, 3)



2 (2, 1)



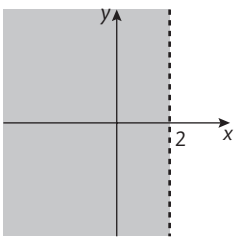
3 (2, 1) **4** (4, 3)

5 $(-1, 3)$ **6** $(-2, -2)$ **7** (2, 2) **8** $\left(\frac{6}{5}, \frac{2}{5}\right)$

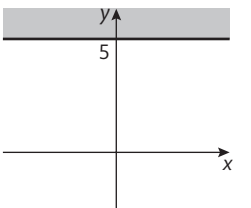
9 $\left(\frac{14}{3}, \frac{4}{3}\right)$ **10** $\left(\frac{7}{3}, -\frac{5}{3}\right)$ **11** $x \geqslant 1$ **12** $x < 3$

13 $y \geqslant 2$ **14** $y < 1$ **15** $x + y \leqslant 4$ **16** $y > x + 1$

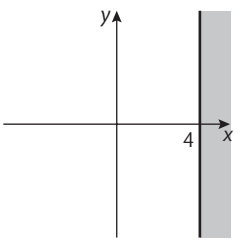
17



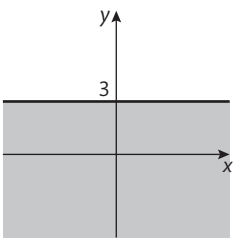
18



19



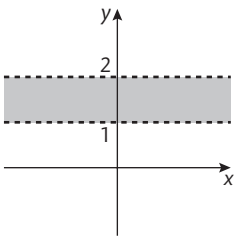
20



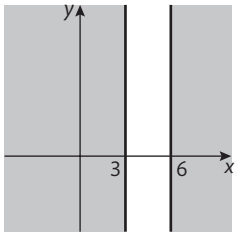
21 $x < 1, x \geqslant 3$

22 $-1 \leqslant y \leqslant 2$

23

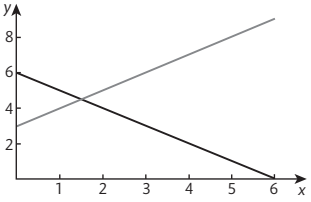


24

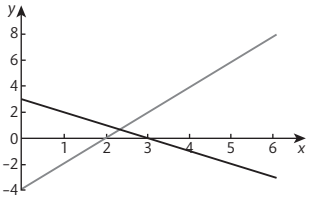


Exercise 46* (Revision)

1 $\left(\frac{3}{2}, \frac{9}{2}\right)$



2 $\left(\frac{7}{3}, \frac{2}{3}\right)$



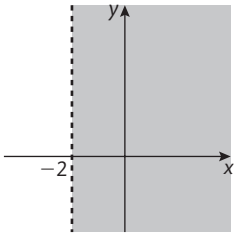
3 (2, 1) **4** (4, 2)

5 $\left(\frac{16}{3}, \frac{10}{3}\right)$ **6** $\left(\frac{16}{5}, -\frac{2}{5}\right)$ **7** $\left(\frac{16}{7}, \frac{15}{7}\right)$ **8** $\left(\frac{30}{13}, \frac{42}{13}\right)$

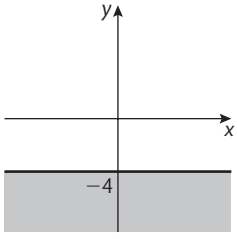
9 $\left(-\frac{8}{5}, \frac{6}{5}\right)$ **10** $\left(-\frac{7}{5}, -\frac{4}{5}\right)$ **11** $x \leqslant -1$ **12** $y < 3$

13 $x > -2$ **14** $y \leqslant -1$ **15** $x + 2y > 6$ **16** $y \geqslant 2x - 1$

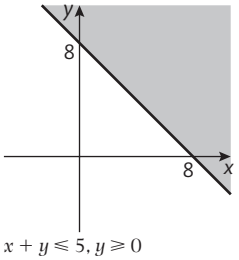
17



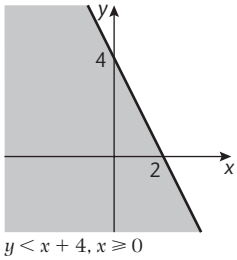
18



19



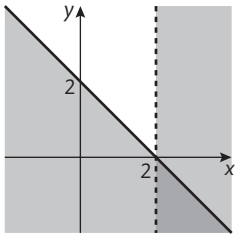
20



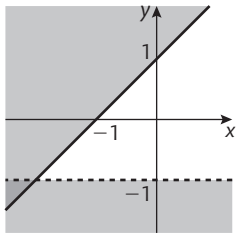
21 $x + y \leqslant 5, y \geqslant 0$

22 $y < x + 4, x \geqslant 0$

23



24



Exercise 47

1 x: hyp, y: opp, z: adj **2** x: opp, y: adj, z: hyp

3 $\frac{4}{3}$ **4** 5.8

5 87 **6** 100 **7** 6.66 cm **8** 8.20 cm

9 11.3 cm **10** 87.5 m **11** 100 m^2

Exercise 47*

1 14.4 cm **2** $x = 200$ **3** 8.45 m

4 100 m **5** 22.4 m **6** $x = 8.4$ cm, $y = 4.85$ cm

7 $x = 27.5$ cm, $y = 9.24$ cm

8 **a** 25.4 m **b** 18.3 km/h

Exercise 48

1 45° **2** 15° **3** 70.0° **4** 45° **5** 75°

6 37.9° **7** 28.2° **8** 23.4° **9** 15°

Exercise 48*

1 **a** 69° **b** 139°

2 60°

3 160°

4 **a** 080.5° **b** 260.5°
c 108.4° **d** 236.3°

5 36.4 m

6 173.2 cm^2

7 13.9°

Exercise 49 (Revision)

1 7.00 **2** 6.71 **3** 6.99

4 11.0 **5** 8.57 **6** 6.93

7 59.0° **8** 32.5 **9** 58.0°

10 5.20 cm^2 **11** 30° **12** $\theta = 46.3°$

13 $\theta = 5.20°$ **14** $\theta = 59.6°$ **15** $\theta = 16.1°$

Exercise 49* (Revision)

1 $x = 6.53$ cm, $y = 1.55$ cm **2** $x = 34.6$ cm, $y = 29.1$ cm

3 $x = 8.39$ m, $y = 3.53$ m **4** $x = 12.1$ cm, $y = 5.12$ cm

5 549 m

6 **a** 063.4° **b** 243°

7 **a** 1.01 m **b** Undesirable to have too large a blind distance.

8 18.4° **9** 71.6°

10 56.3° **11** 144 cm

12 23.3 m **13** 50.4 m

14 correct proof **15** correct proof

Exercise 50

1

Score		Frequency
1	𝍸 ||||	9
2	𝍸 𝍸	10
3	𝍸 ||	7
4	𝍸	5
5	||||	4
6	𝍸	5
		Total = 40

2

Weight in kg		Frequency
$1.0 \leqslant w < 1.5$	||	2
$1.5 \leqslant w < 2.0$	𝍸	5
$2.0 \leqslant w < 2.5$	𝍸 ||||	9
$2.5 \leqslant w < 3.0$	𝍸 𝍸 ||	12
$3.0 \leqslant w < 3.5$	𝍸 ||||	9
$3.5 \leqslant w < 4.0$	𝍸 |||	8
$4.0 \leqslant w < 4.5$	||||	4
$4.5 \leqslant w < 5.0$	|	1
		Total = 50

3 Mean = 10, Median = 12, Mode = 14

4 Mean = 15, Median = 16, Mode = 4

Exercise 50*

1 Mean = 48.9 s, median = 45 s.

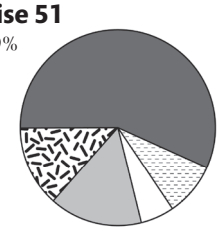
2 Mean 92, median 91, mode 91; therefore either median or mode.

3 10.5 years

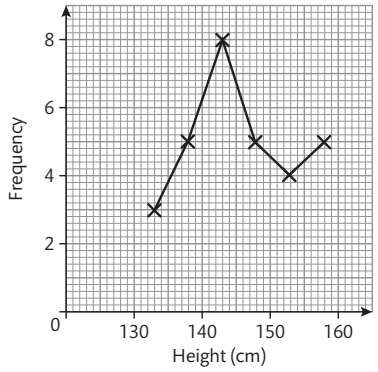
4 For example: 1, 2, 3, 4, 5, 16; mean = 5.2

Exercise 51

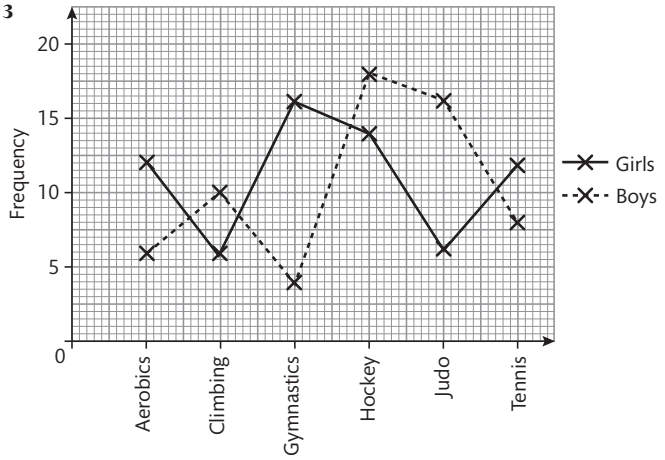
1 29%



2



3



4 7; 6.1

Exercise 51*

1 **a** 2.6; 2 **2** 720 pupils

3 14 min 41 s

4 Let 0 ⩽ random number ⩽ 0.5 be a head, 0.5 < random number ⩽ 1 be a tail. Generate random numbers to simulate the experiment.

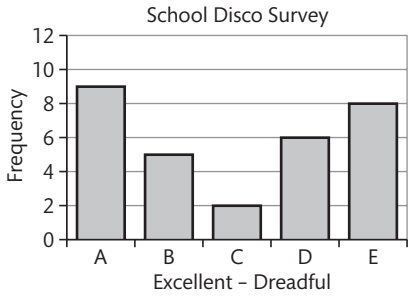
Exercise 52 (Revision)

1 mean = 21, median = 21, mode = 20

2 **a** mean = 1.69, median = 1.72 **b** new mean 1.76

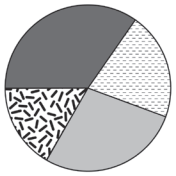
3 **a**

A	9
B	5
C	2
D	6
E	8



b Most felt strongly one way or the other, not so many thought it was average.

4 Strawberries 125°, bananas 75°, yoghurt 100°, iced water 60°.



5 **a**

Score	Frequency
1	7
2	10
3	5
4	10
5	6
6	12

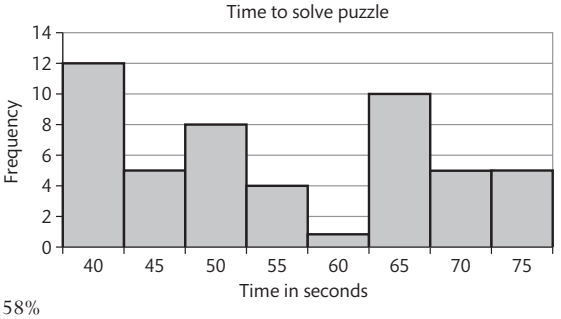
b

c No clear bias. (Sample is too small to draw conclusions from.)

6 **a**

Times	Frequency
40–45	12
45–50	5
50–55	8
55–60	4
60–65	1
65–70	10
70–75	5
75–80	5

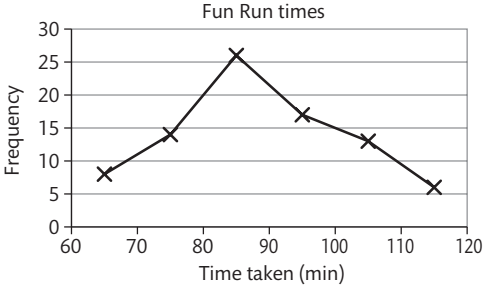
b



c 58%

7 **a** 85

b



c 23.5%

8 **a** 14.2 m **b** 1.40 m

9 **a** mean = 0.8, median = 0.5, mode = 0 **b** 0.909

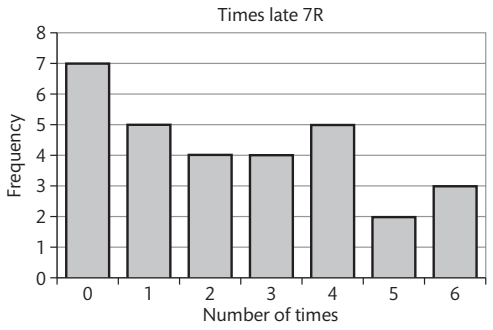
Exercise 52* (Revision)

1 **a** mean = 7.45, median = 7.45 **b** new mean = 8.2

2 **a** Mean = 2.43, median = 2, mode = 0

b The mode as it is the most flattering figure.

c

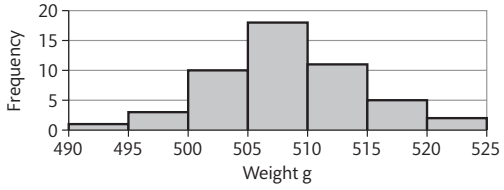


3 540

4 **a**

Weight g	Frequency
$490 \leqslant w < 495$	1
$495 \leqslant w < 500$	3
$500 \leqslant w < 505$	10
$505 \leqslant w < 510$	18
$510 \leqslant w < 515$	11
$515 \leqslant w < 520$	5
$520 \leqslant w < 525$	2

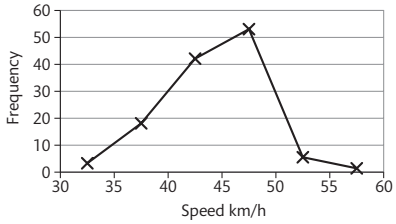
b



c Evidence suggests the mean is between 505 and 510 so 500 is probably minimum weight.

5 **a** 126

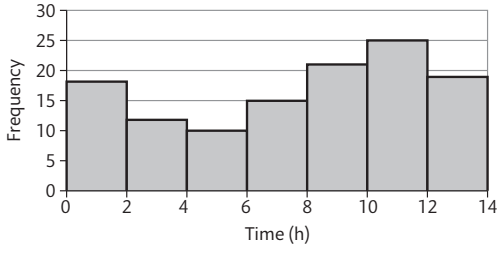
b



c Speed limit is probably 50 km/h as there is a sharp cut off at that speed.

6 **a** 15

b



c 38.3%

7 1.74 m

8 **a** 83 cm **b** 9.37 cm

Multiple choice 2

1	A	**2**	D	**3**	A	**4**	D	**5**	D
6	C	**7**	B	**8**	B	**9**	C	**10**	A

Self-assessment 2

1 **a** 0.0504 **b** 0.00002

2 **a** $\frac{\cancel{3}^{1}}{\cancel{5}_{1}} \times \frac{\cancel{15}^{3}}{\cancel{21}_{7}} = \frac{3}{7}$ **b** $\frac{39}{12} + \frac{28}{12} = \frac{67}{12} = 5\frac{7}{12}$

c $\frac{\cancel{14}^{2}}{\cancel{15}_{3}} \times \frac{\cancel{25}^{5}}{\cancel{7}_{1}} = \frac{10}{3} = 3\frac{1}{3}$

3 **a** 295 kg : 472 kg **b** \$0.64 : \$2.24 : \$1.60

4 68

5 78.4 km/h

6 27.8 seconds

7 **a** $3a^2$ **b** $\frac{1}{2x}$ **c** $3b^2$

8 $\frac{7a + 2b}{14}$ **b** $\frac{y}{10}$ **c** $\frac{35x - 12y - 81}{20}$

9 **a** ± 5 **b** 16 **c** ± 6

10 **a** 68 °F **b** 37.8 °C **c** $-40°$

11 **a** y^{13} **b** b^9 **c** $16a^2b^8$

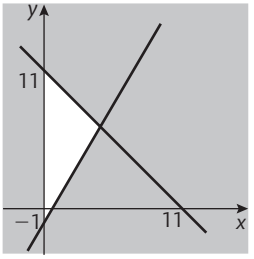
12 **a** b^9 **b** $49a^8$ **c** p^8

13 **a** $\frac{7d}{16b}$ **b** $\frac{7a^2c^3}{b}$ **c** 3

14 **a** $x > 12$ **b** $x \leqslant 7$

15 $-5, -4, -3, -2, -1, 0, 1, 2$

16 **a, b, d**



c (4, 7)

17 **a** $x = 4.37$ **b** $y = 2.33$ **c** $z = 67.4°$

18 125°

19 **a** mean = 23.4, median = 23, mode = 27 **b** 23

20 $p = 80$, $q = 48$, $r = 56$, $s = 64$, $t = 50$

Exercise 53

1 **a** 1.1 **b** 1.2 **c** 1.30
d 1.01 **e** 1.15 **f** 1.25

2 '... by 1.03×1.04'

3 660 pupils

4 '... by $(1.04)^3$'

5 \$515.21

6 \$98 400

Exercise 53*

1 **a** 1.125 **b** 1.04 **c** 1.05

2 90 km

3 \$436.80

4 Result is a multiplying factor of $1.2 \times 0.8 = 0.96$. That is a 4% decrease. You do not get back to the original quantity because the 80% reduction is applied to the increased value.

5 \$692.84

6 \$10 100, \$20 200, \$40 700

7 \$8870, \$3940, \$1750

8 \$4.39 million

Exercise 54

1 7, 14, 21, 28, 35 **2** 6, 12, 18, 24, 30

3 1, 2, 3, 4, 6, 12 **4** 1, 2, 3, 5, 6, 10, 15, 30

5 $2 \times 2 \times 7$ **6** $2 \times 2 \times 3 \times 5$

7 No **8** $2 \times 3 \times 5 \times 7$

9 $2 \times 2 \times 2 \times 11$ **10** Divisible by 7

Exercise 54*

1 No
2 3, 7, 19
3 3, 7, 11
4 11
5 1, 3, 5, 15, 25, 75
6 1, 2, 3, 6, 9, 18, 27, 54
7 $3 \times 5 \times 11$
8 $3 \times 7 \times 19$
9 59, 61
10 $2^3 \times 3^2 \times 7$
11 $2^4 \times 3^2 \times 7$

Exercise 55

1 2 **2** 5 **3** 22 **4** 6
5 30 **6** 30 **7** $2x$ **8** $3a$
9 $4y^2$ **10** xy **11** $6ab$ **12** $12xy$
13 $\frac{3}{4}$ **14** $\frac{4}{7}$ **15** $\frac{1}{2}$ **16** $\frac{3}{16}$
17 $\frac{1}{5}$ **18** $\frac{7}{24}$ **19** $\frac{17}{140}$ **20** $\frac{11}{30}$
21 $\frac{7}{12}$

Exercise 55*

1 HCF = 6, LCM = 36
2 HCF = 15, LCM = 210
3 HCF = y, LCM = $6xyzc$
4 HCF = $2xy$, LCM = $12xy$
5 HCF = xy, LCM = x^2yz
6 HCF = xy, LCM = x^3y^4
7 HCF = $3xyz$, LCM = $18x^2y^2z^2$
8 $\frac{13}{24}$
9 $\frac{13}{36}$
10 $\frac{93}{140}$
11 $\frac{29}{72}$

Exercise 56

1 6.96 **2** 3.35 **3** 6.96 **4** 3.35
5 134 **6** 0.384 **7** 12.9 **8** 16.1
9 2.58 **10** 14.6 **11** 2.69 **12** 7.16
13 11.3 **14** 1.22 **15** 625 **16** 191
17 191 **18** 245 000 **19** 1.75×10^{10}

Exercise 56*

1 3.43 **2** 0.005 80 **3** −1.01 **4** 12.4
5 −0.956 **6** 15.2 **7** 0.103 **8** 0.0454
9 3.60 **10** No, because $a^2 + b^2 \neq (a + b)^2$

Exercise 57 (Revision)

1 1.12, 1.25, 75%, 99%
2 0.88, 0.75, 75%, 99%
3 **a** \$535 **b** \$572.45 **c** \$612.52
4 **a** £104 **b** £108.16 **c** £121.67
5 **a** €26 250 **b** €27 562.50 **c** €31 907.04
6 **a** £41 400 **b** £38 088 **c** £29 658.67
7 **a** 2 **b** 5 **c** 8 **d** 2
8 **a** 12 **b** 42 **c** 120 **d** 630
9 2.64 **10** 4.37 **11** 0.245 **12** 6.75
13 5.94 **14** 0.314 **15** 27.3 **16** 2 400 000
17 26 200 **18** 15.6 **19** 755 000 **20** 25.5

Exercise 57* (Revision)

1 **a** €3345.56 **b** 12 years
2 **a** \$134 391.64 **b** 14 years
3 £7366.96
4 **a** \$425 570.14 **b** \$597 895.41
5 €652.70
6 **a** £16 769.97 **b** £13 887.21
7 **a** HCF = 10, LCM = 420
b HCF = 28, LCM = 420
c HCF = 14 , LCM = 210
8 **a** HCF = $2xyz$, LCM = $12x^2y^2z^2$
b HCF = $5pq$, LCM = $140pq$
c HCF = $6a^2b^3c^2$, LCM = $36a^4b^3c^4$
9 862 000 **10** 2.79 **11** 2.31
12 5.68 **13** 14.7 **14** 0.104

Exercise 58

1 $x(x + 3)$ **2** $x(x - 4)$ **3** 5(a − 2b)
4 $x(y - z)$ **5** $2x(x + 2)$ **6** $3x(x - 6)$
7 $ax(x - a)$ **8** $3xy(2x - 7)$ **9** $3pq(3p + 2)$
10 $a(p + q - r)$

Exercise 58*

1 $5x^3(1 + 3x)$ **2** $3x^2(x - 6)$ **3** $3x^2y^2(3x - 4y^2)$
4 $x(x^2 - 3x - 3)$ **5** $ab(c^2 - b + ac)$ **6** $4pq(pqr^2 - 3r + 4q)$
7 $3x(10x^2 + 4y - 7z)$ **8** $0.1h(2h + g - 3g^2h)$
9 $\frac{xy}{16}(2x^2 - 4y + xy)$ **10** $\pi r(r + 2h)$
11 $4pqr(4pr^2 - 7 - 5p^2q)$ **12** $(a + b)(x + y)$
13 $(x - y)^2(x - y + 1)$

Exercise 59

1 $x + 1$ **2** $\frac{(x + y)}{z}$ **3** 2
4 $\frac{(a - b)}{b}$ **5** $\frac{t}{r}$ **6** $\frac{x}{z}$

Exercise 59*

1 $x + y$ **2** $\frac{1}{(z + 1)}$ **3** $2 + 3x^2$
4 $\frac{2}{3}(x - 3y^2)$ **5** y **6** $\frac{2x}{z}$
7 1 **8** $\frac{b}{a}$ **9** 5
10 $-x$

Exercise 60

1 $x = 8$ **2** $x = -10$ **3** $x = 2$
4 $x = 0$ **5** $x = -6$ **6** $x = 5$
7 $x = -4$ **8** $x = 6$ **9** $x = 14$
10 $x = 3$ **11** $x = 0$ **12** 6 km
13 90 cm

Exercise 60*

1 $x = 9$ **2** $x = \frac{3}{5}$ **3** $x = 9$
4 $x = -6$ **5** $x = 0$ **6** $x = \frac{1}{9}$
7 $x = 3$ **8** $x = -1$ **9** $x = 7$
10 84 years **11** 60 km, 3 h

Exercise 61

1 $x = 2$ **2** $x = -3$ **3** $x = \frac{3}{5}$
4 $x = -8$ **5** $x = 10$ **6** $x = -2.4$
7 $x = 50$ **8** $x = -25$ **9** $x = \frac{5}{3}$
10 $x = \pm 3$

Exercise 61*

1 $x = 4$ **2** $x = -8$ **3** $x = \frac{1}{6}$
4 $x = -64$ **5** $x = 4$ **6** $x = \pm 8$
7 $x = \pm 2$ **8** $x = 0.32$ **9** $x = \frac{5}{6}$
10 $x = \frac{(a + b)}{ab}$

Exercise 62

1 (5, 3) **2** $x = 3, y = 2$ **3** (1, 2)
4 $x = 1, y = 1$ **5** (1, −1)

Exercise 62*

1 (8, 3) **2** $x = 4, y = 5$ **3** (1, 5)
4 $x = 0, y = -2$ **5** (−1, 5)

Exercise 63

1 (2, 5) **2** $x = 5, y = 1$ **3** (1, 3)
4 $x = 5, y = -1$ **5** (2, 1)

Exercise 63*

1 (3, −1) **2** $x = 1, y = 2$ **3** (−0.4, 2.6)

4 $x = 7, y = 3$ **5** (0.5, 0.75) **6** $x = 4, y = 6$
7 $(-0.6, -0.8)$ **8** $x = 0.4, y = 0.5$

Exercise 64

1 $x = 3, y = 1$ **2** $x = 1, y = 4$ **3** $x = 1, y = 6$
4 $x = -1, y = 2$ **5** $x = 3, y = -1$

Exercise 64*

1 $x = 1, y = 2$ **2** $x = 4, y = 1$ **3** $x = 2, y = 1$
4 $x = 1, y = -2$ **5** $x = -3, y = \frac{1}{2}$

Exercise 65

1 29, 83 **2** 12, 16 **3** 9, 4
4 $x = 2, y = 3$, area = 180 **5** Burger 99p, cola 49p
6 27 at 20p, 12 at 50p **7** 11

Exercise 65*

1 (2, 3) **2** $\frac{12}{17}$ **3** 1.5 m s^{-1} **4** 16
5 7.5 km **6** 37 **7** 50 m

Exercise 66 (Revision)

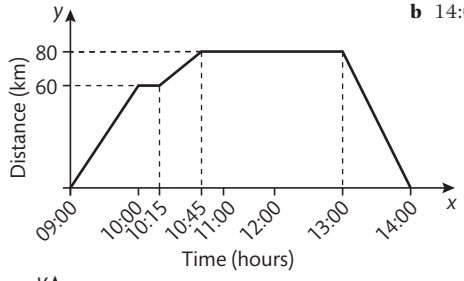
1 $x(x - 8)$ **2** $3x(x + 4)$ **3** $6xy(y - 5x)$
4 $3x(4x^2 + 3x - 5)$ **5** $x - 1$ **6** $\frac{(x + y)}{(x - y)}$
7 $x = 4$ **8** 6 **9** -4
10 2 **11** 24 **12** $(-1, 3)$
13 (0, 3) **14** (2, 2) **15** (1, 3)
16 CD $7.50, Tape $3.50 **17** 19 at 10c, 11 at 20c

Exercise 66* (Revision)

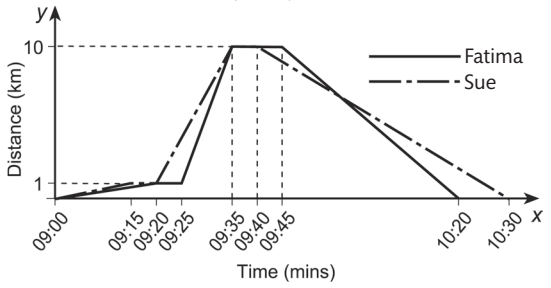
1 $3x^3(x - 4)$ **2** $\frac{2}{3}\pi r^2(2r + 1)$
3 $6x^2y(4xy - 3)$ **4** $3a^2b^2c^2(5b - 3a + 7c)$
5 $\frac{x}{y}$ **6** x **7** $x = \frac{1}{3}$
8 $x = -4$ **9** $x = 6$ **10** $x = \frac{1}{2}$
11 70 years **12** (2, 3) **13** (4, 1)
14 (4, 1.5) **15** $\left(3\frac{1}{3}, 2\right)$ **16** $a = \frac{3}{11}, b = \frac{2}{11}$
17 Azim is 38, Chen is 14

Exercise 67

1 **a** 65 km/h **b** 50 km/h **c** 12:00
d 72.5 km **e** 11:08 approx
2 **a** 09:30 for half an hour **b** 09:00 and 10:54
c 20 miles **d** 80 mph yes! **e** 53.3 mph
f 53.3 mph
3 **a**



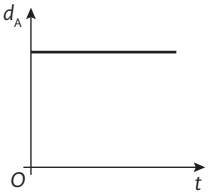
b 14:00
4 **a**



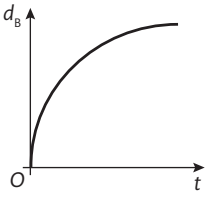
b Fatima at 10:20, Sue at 10:30
c 09:20, 09:35–09:40, 09:57
d Fatima: 18.5 km/h, Sue: 15 km/h

Exercise 67*

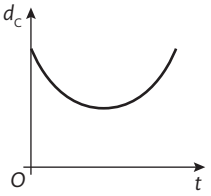
1 **a**



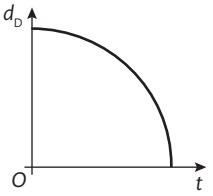
b



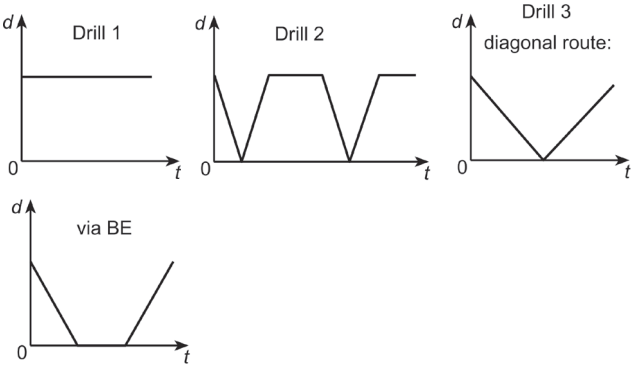
c



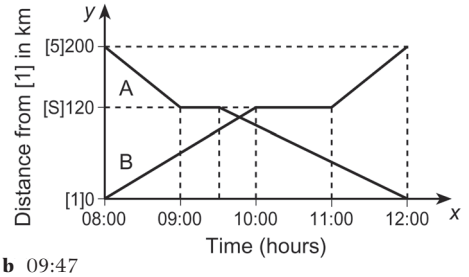
d



2



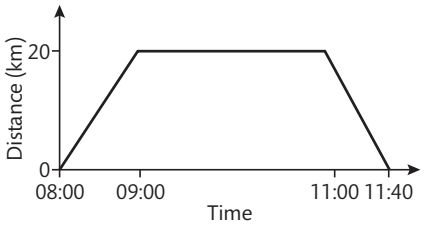
3 **a** **(i)** B & C joint 1st, A 3rd **(ii)** C 1st, B 2nd, A 3rd
(iii) A 1st, B 2nd, C 3rd
b 28.5 s **c** B **d** **(i)** A **(ii)** C
4



b 09:47
c 48 km/h, 80 km/h
d A: 57 km/h B: 67 km/h

Exercise 68

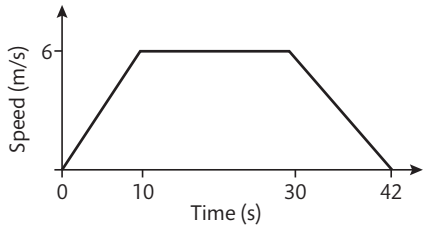
1 **a** 2 m/s^2 **b** -4 m/s^2 **c** 150 m **d** 10 m/s
2 **a** 3.5 km/h^2 **b** -7 km/h^2 **c** 10.5 km **d** 3.5 km/h
3 **a** 2 m/s^2 **b** -1 m/s^2 **c** 8000 m **d** 50 m/s
4 **a** 15 m/s **b** 4s **c** 9 m/s
5 **a** **b** 11:40



6 5 m/s^2

Exercise 68*

1 a



b $0.6\ \text{m/s}^2$

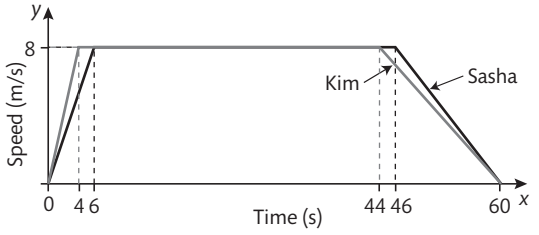
c $-0.5\ \text{m/s}^2$ **d** 4.43 m/s (3 s.f.)

2 a $S = 120$ m/s **b** 9600 m **c** 80 m/s

3 a $t = 10$ s, so distance = 1900 m

b $-3\ \text{m/s}^2$ **c** 47.5 m/s

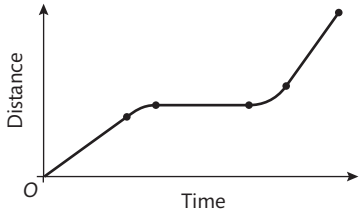
4



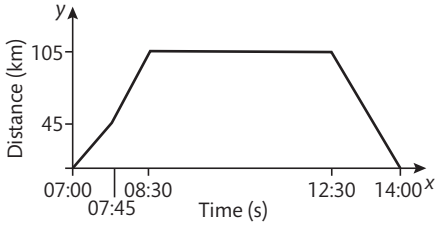
b Dead heat **c** 6.67 m/s (to 3 s.f.) for both runners

d 400 m **e (i)** Kim **(ii)** Kim

5



6



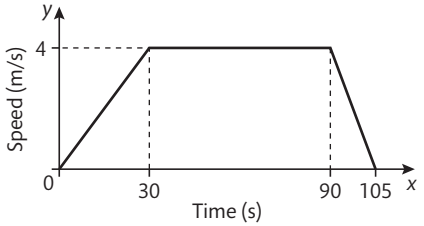
She is just in time to meet her husband.

Exercise 69 (Revision)

1 a 20 min **b** 10:00 **c** 10 km/h **d** $3\frac{1}{3}$ km

2 a 0.4 m/s **b** 10 min **c** 0.2 m/s

3



a $\frac{2}{15}\ \text{m/s}^2$ **b** $0\ \text{m/s}^2$ **c** $-\frac{4}{15}\ \text{m/s}^2$ **d** $3\frac{1}{7}$ m/s

4 a 400 m **b** 1050 m **c** 10.5 m/s **d** $0.33\ \text{m/s}^2$

5 a $3\ \text{m/s}^2$ **b** $0\ \text{m/s}^2$ **c** $-2\ \text{m/s}^2$ **d** 43.3 m/s

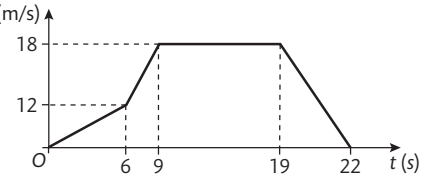
Exercise 69* (Revision)

1 a 50 m/s **b** 0.5 s at 30 m/s approx

2 a 32 m **b** $-\frac{2}{3}\ \text{m/s}^2$ **c** 3.2 m/s

3 a $x = 6$

b V (m/s)



c (i) $2\ \text{m/s}^2$ **(ii)** 13.1 m/s

4 a (i) 6.4 m/s **(ii)** 23.0 km/h

b 7.08 m/s **c** $0.590\ \text{m/s}^2$

5 a False; it is constant at $\frac{2}{3}\ \text{m/s}^2$ **b** True

c True **d** False; it is 72 km/h

Exercise 70

1 2.46 **2** 5.07 **3** 8.09 **4** 9.44 cm

5 8.76 **6** 1.61 m **7** 67.6 m

Exercise 70*

1 6.57 m

2 a 2.5 m **b** 4.33 m **c** $20.6\ \text{m}^2$ **d** 31.7°

3 a 107 m **b** 79.7 m

4 155 m

5 a 2.25 km **b** 3.90 km **c** 4.5 km

6 452 m

Exercise 71

1 48.6° **2** 46.1° **3** 78.7° **4** 1.72°

5 70.5°

Exercise 71*

1 37.8° **2** 58.6° **3** 57.3°

4 014.9°; 194.9° **5** 72.7°

Exercise 72

1 5 **2** $x = 8.66$ **3** 5.18 **4** $a = 30°$

5 60° **6** 9.24 **7** 32.2° **8** 49.7°

9 1.38 cm **10** 41.6° **11** 62.3° **12** 250 m

13 10.0°

Exercise 72*

1 18 **2** $x = 20$

3 7.96 **4** $x = 17.3$ $y = 34.6$

5 16.8° **6** 50.2°

7 a 4.66 km north **b** 17.4 km west

8 a 195 m **b** 442 m

9 22.2 m **10** $d = 611$ m

11 7.99 km **12** 1.5 m

Exercise 73 (Revision)

1 $x = 14.1$ cm, $d = 70.5°$ **2** $x = 7.87$ m, $d = 10.2°$

3 $x = 16.7$ km, $d = 39.9°$ **4** $x = 11.7$ cm, $d = 31.2°$

5 $x = 2.38$ m, $d = 4.62°$ **6** $x = 14.3$ km, $d = 79°$

7 $43.3\ \text{cm}^2$ **8** 33.7°

9 a 0.5 **b** $f = 30°$

10 a 20.5 m **b** 19.1 m **c** 20.7 m

Exercise 73* (Revision)

1 Ascends in 3 min 52 s, so reaches surface with 8 seconds to spare.

2 a 17.2 km, 284° **b** 18 : 11 : 10

3 3.56 m

4 a 16.2 m **b** 16.2 s **c** 432 m

5 a 13.9 km, 280° **b** He arrives at 17:55

6 3.13 m/s **7** 2.5 km

8 1.79 m **9** $p = 25$

10 $q = 5$

Exercise 74

1 a

Score	1	2	3	4	5	6
f	7	8	5	4	3	3

b 2.9 **c** 2.5

2 a 50 **b** 1.44 **c** 1

3 a 80 **b** 2.4 **c** 40%

4 26.1

Exercise 74*

1 197 cm **2** 16 min **3** 25.0 s

Exercise 75 (Revision)

1 a 20 **b** 1 **c** 0 **d** 1.4

2 a 40 **b** 2 **c** 1 **d** 2.05

3 a 50 **b** 16−20 **c** 15.3

4 a 24 **b** $4 < w \leqslant 6$ **c** 4.75

5 a 25 **b** $17.5 < t \leqslant 20.5$ **c** 17.92 min

6 a 32 **b** $14 \leqslant a < 15$ **c** 14.3

d Decrease as below the mean

Exercise 75* (Revision)

1 a 8 **b** 73 **c** 74 **d** 73.2

2 a 7 **b** 2 **c** 26.7%

3 a 28 days ⇒ February **b** 16−20 **c** 15.3

4 a 1000 **b** 601−800 **c** 770.5

5 a 540 **b** $3000 < h \leqslant 3300$ **c** 3360

6 a $22 - x$ **b** 9

Multiple choice 3

1 D **2** B **3** C **4** B **5** C

6 B **7** B **8** D **9** C **10** C

Self-assessment 3

1 a £466.19 **b** £473.90

2 £10 675

3 9.45×10^{12} km = a light year.

4 a 0.09215784314 **b** 9.22×10^{-2}

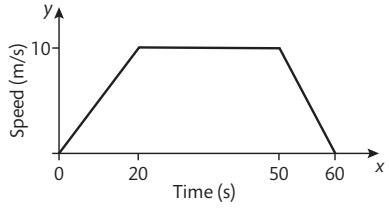
5 HCF = 6, LCM = 60

6 a $\frac{5}{xy}$ **b** $\frac{2(x+3)}{21}$

7 $x = \frac{1}{9}$

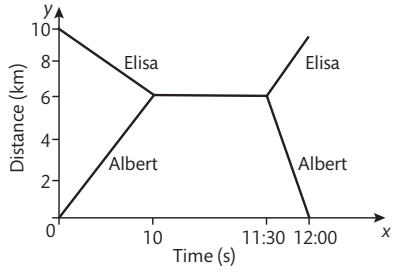
8 $x = -0.4$, $y = -9.2$

9 a



b (i) 0.5 m/s² **(ii)** −1 m/s² **(iii)** 7.5 m/s

10 a



b Eliza home 12:00, Albert home 12:00

c Eliza 1.48 m/s, Albert 2.22 m/s

11 a 6.43 cm **b** 7.66 cm **c** 3.66 cm

d 60.3° **e** 120° **f** 12.9 cm²

12 a 5 m **b** 30.0° **c** 14.1 m

13 $x = 25.8°$, $y = 10.3$ cm

14 2.6 goals/game

15 a $\Sigma fx = 72$ goals **b** 1.44 goals/game

Exercise 76

1 \$442 **2** 100% **3** \$40 **4** \$60

5 13.3% **6** \$76.20 **7** \$74.10

Exercise 76*

1 \$44 **2** \$180 000 **3** \$60 **4** 11.1%

5 \$73 000 **6** 34.4% **7** \$450

Exercise 77

1 150 **2** 3 **3** 300 **4** 8

5 200 **6** 6×10^7 **7** 4×10^3 **8** 2.3×10^4

9 8.7×10^4 **10** 5×10^2 **11** 2×10^6 **12** 1×10^2

13 7×10^6 **14** 4×10^6 **15** 8×10^2

Exercise 77*

1 4 **2** 8 **3** 100 cm² **4** 68 cm²

5 1.2×10^9 **6** 2×10^3 **7** 7.06×10^8 **8** 50 000

9 0.2 **10** \$6 000 000 **11** 10 000 **12** 2×10^6

13 3×10^{-1} **14** 2×10^7 **15** 1×10^{-3}

Exercise 78 (Revision)

1 \$120 **2** \$20 **3** \$2500 **4** 80 seconds

5 1×10^8 **6** 4×10^{13} **7** 6×10^{15} **8** 1×10^{11}

9 2×10^2 **10** 2×10^3 **11** 5×10^7 **12** 5×10^8

13 4×10^{14} **14** 3×10^{15} **15** 1×10^3 **16** 2×10^{10}

Exercise 78* (Revision)

1 \$4329 **2** \$1811.59 **3** \$409.36 **4** \$2573.53

5 \$1283.76 **6** \$888 889 **7** \$6863.56 **8** \$2326.24

9 2×10^6 **10** 4×10^{-3} **11** 2×10^{-12}

12 5×10^6 **13** 2×10 **14** 8×10^{-3}

Exercise 79

1 $x = a - 2$ **2** $x = 5 + p$ **3** $x = c - a$ **4** $x = \frac{b}{5}$

5 $x = \frac{(b-a)}{3}$ **6** $x = \frac{(t-s)}{2}$ **7** $x = \frac{(4-b)}{a}$ **8** $x = \frac{(f+g)}{e}$

9 $x = \frac{c}{(a+b)}$ **10** $x = \frac{(d-8b)}{c}$ **11** $x = \frac{(a-3b)}{3}$ **12** $x = \frac{(c-ab)}{a}$

13 $x = ab$ **14** $x = \frac{qr}{p}$ **15** $x = r(p+q)$ **16** $x = cd + b$

Exercise 79*

1 $x = \frac{(c-b)}{a}$ **2** $x = cd + b$ **3** $x = \frac{cd}{b}$

4 $x = \frac{e-ac}{a}$ **5** $x = \frac{P-b^2}{\pi}$ **6** $x = \frac{Td^2}{b}$

7 $x = \pi - b$ **8** $x = \frac{(ab-c)}{d}$ **9** $x = \frac{a}{b}$

10 $x = \frac{(a+b)}{c}$ **11** $x = \frac{s}{(p-q)}$ **12** $r = \frac{A}{2\pi}$

13 $h = \frac{3V}{\pi r^2}$ **14** $x = \frac{(y-c)}{m}$ **15** $s = \frac{(v^2-u^2)}{2a}$

16 $a = 2m - b$ **17** $a = \frac{S(1-r)}{(1-r^n)}$ **18** $a = \frac{s}{n} - \frac{(n-1)d}{2}$

Exercise 80

1 $x = \sqrt{\left(\frac{b}{a}\right)}$ **2** $x = \sqrt{(ab)}$ **3** $x = \sqrt{(2D - C)}$

4 $x = \sqrt{(a(c-b))}$ **5** $x = \sqrt{\left(\frac{(c-2b)}{a}\right)}$ **6** $x = \frac{t}{a+d}$

7 $x = \frac{ab}{a-1}$ **8** $x = \frac{2b-a}{a-b}$ **9** $r = \sqrt{\left(\frac{A}{4\pi}\right)}$

10 $v = \sqrt{(ar)}$ **11** $r = \sqrt[3]{\frac{3V}{4\pi}}$ **12** $l = \left(\frac{T}{2\pi}\right)^2$

Exercise 80*

1 $x = \sqrt{\left(\frac{S}{R}\right)}$ **2** $x = \sqrt{\frac{g-a}{c}}$ **3** $x = \frac{c}{b-a}$

4 $x = \frac{c-f}{e+d}$ **5** $x = \frac{\tan b + ac}{(1-a)}$ **6** $x = t(p^2 - s)$

7 $x = \sqrt{(Ab - Da)}$ **8** $r = \sqrt{\left(\frac{3V}{\pi h}\right)}$ **9** $v = \sqrt{(2gh)}$

10 $x = \sqrt{\frac{1}{y} - a^2}$ **11** $a = b - \sqrt{12s}$ **12** $Q = \frac{1}{p}\left(\frac{S}{r}\right)^2$

13 $d = \left(\frac{k}{F}\right)^3$ **14** $\sqrt{1 - \frac{1}{y^2}}$ **15** $x = \frac{p(y+1)}{y-1}$

Exercise 81

1 **a** 155 min **b** 2 kg

2 **a** 18 cm² **b** 10 cm

3 **a** 0.15 cm **b** 200°C

4 **a** \$2350 **b** \$25 750

5 $A = \frac{\pi r^2}{4}$, $P = r\left(2 + \frac{\pi}{2}\right)$

a $A = 19.6$ cm², $P = 17.9$ cm

b 11.28 cm **c** 14.0 cm **d** 4.55

Exercise 81*

1 **a** \$22 **b** 400

2 **a** 53.8 m **b** 79.1 km/h

3 **a** 209 **b** 3 **c** 8

4 **a** 27.3 **b** 12 **c** 6.63

5 $A = r^2\left(1 + \frac{\pi}{4}\right)$, $P = r\left(4 + \frac{\pi}{2}\right)$

a $A = 28.6$ cm², $P = 22.3$ cm **b** 4.10 cm

c 12.6 cm **d** 3.12

Exercise 82 (Revision)

1 $x = \frac{b}{a}$ **2** $x = ac$ **3** $y = \frac{a-c}{b}$ **4** $y = \sqrt{\frac{d}{b}}$

5 $y = \frac{b^2}{a}$ **6** $y = \frac{d}{a-c}$ **7** $y = \frac{bc}{c-1}$

8 **a** 45 m/s **b** $t = \frac{v-20}{5}$ **c** 2 s

9 **a** \$26 **b** 100 **c** \$1800

10 **a** $A = 4n + 2$ **b** 402 cm²

c $n = \frac{A-2}{4}$ **d** 53

Exercise 82* (Revision)

1 $x = \frac{c-b}{a}$ **2** $x = \frac{b}{a-d}$ **3** $x = \frac{ab - \tan c}{a}$

4 $y = \sqrt{\frac{a}{b-c}}$ **5** $y = \frac{ac-d}{a-b}$ **6** $y = b - d(c-a)^2$

7 **a** 20 **b** $A = \frac{N-2}{0.4}$ **c** 70 m²

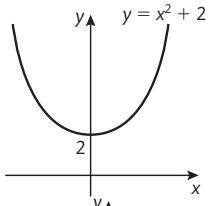
8 **a** 2.81 s **b** $\ell = 10\left(\frac{t}{2\pi}\right)^2$ **c** 1.58 m

9 **a** \$14 **b** $n = \frac{200}{C-10}$ **c** 160

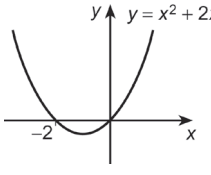
10 **a** 291 cm² **b** $h = \frac{A}{2\pi r} - r$ **c** 15.9 cm

Exercise 83

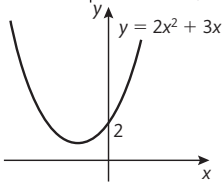
1



2

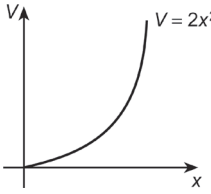


3



4 **b**

x	0	0.4	0.8	1.2	1.6	2.0
V	0	0.32	1.28	2.88	5.12	8

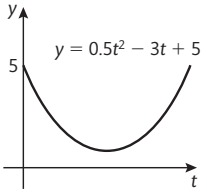


c 1.41 m x 1.41 m **d** 0.72 m³

e $1.22 \text{ m} \leqslant x \leqslant 1.8 \text{ m}$

5 **a**

t	0	1	2	3	4	5	6
y	5	2.5	1	0.5	1	2.5	5

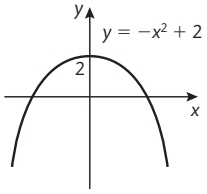


b 5 m **c** 0.5 m, 3 s

d 0 m **e** $0 \leqslant t \leqslant 0.76$, $5.24 \leqslant t \leqslant 6$

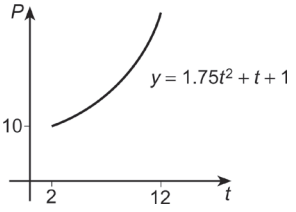
Exercise 83*

1



2 **a** $k = 1.75 \rightarrow P = 1.75t^2 + t + 1$

t	2	4	6	8	10	12
P	10	33	70	121	186	265

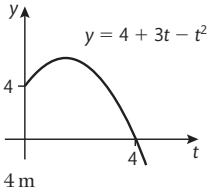


c Accurate answer is 49 750 000. Only approximate answers will be available from the graph.

d 7.2 days approx.

3 **a**

t	0	1	1.5	2	3	4
y	4	6	6.25	6	4	0

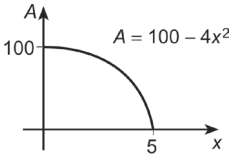


b 4 m **c** 4 p.m.

d 6.25 m, 1.30 p.m. **e** Between 12.23 p.m. and 2.37 p.m.

4 **a** $A = (10 - 2x)^2 + 4x(10 - 2x) = 100 - 4x^2$

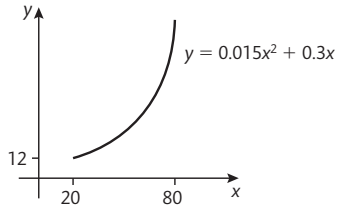
b



c $2.5 \leqslant x \leqslant 3.5$.

5 **a**

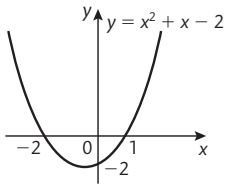
x	20	30	40	50	60	70	80
y	12	22.5	36	52.5	72	94.5	120



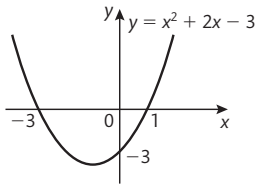
b 61.9 m **c** 48.6 mph **d** 0.7 s

Exercise 84 (Revision)

1

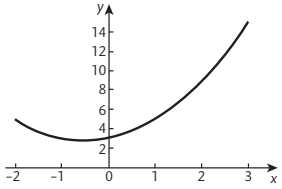


2



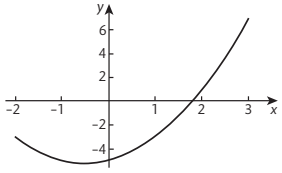
3

x	−2	−1	0	1	2	3
y	5	3	3	5	9	15



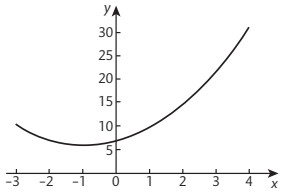
4

x	−2	−1	0	1	2	3
y	−3	−5	−5	−3	1	7



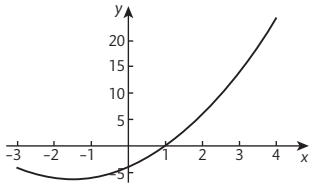
5

x	−3	−2	−1	0	1	2	3	4
y	10	7	6	7	10	15	22	31



6

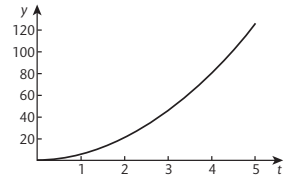
x	−3	−2	−1	0	1	2	3	4
y	−4	−6	−6	−4	0	6	14	24



7 **a** 4.9 cm² **b** 4.5 cm **c** 12.9 cm

8 **a**

t	0	1	2	3	4	5
y	0	5	20	45	80	125

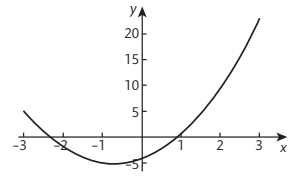


b **(i)** 61 m **(ii)** 2.2 s

Exercise 84* (Revision)

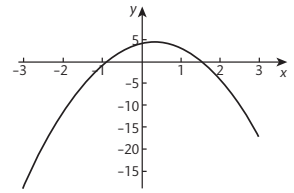
1

x	−3	−2	−1	0	1	2	3
y	5	−2	−5	−4	1	10	23



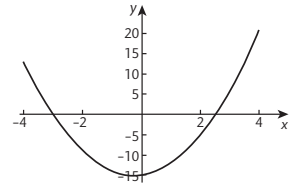
2

x	−3	−2	−1	0	1	2	3
y	−29	−12	−1	4	3	−4	−17



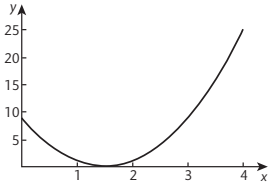
3

x	−4	−3	−2	−1	0	1	2	3	4
y	13	0	−9	−14	−15	−12	−5	6	21

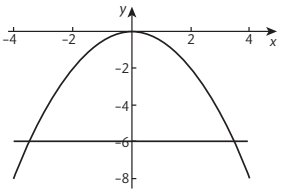


4

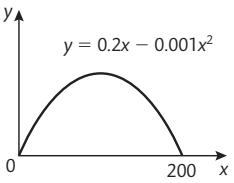
x	0	1	2	3	4
y	9	1	1	9	25



5 **a**



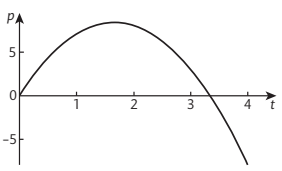
b 28.3 m

6 **a** 

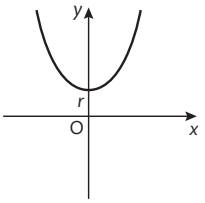
b 10 m **c** $29 \leqslant x \leqslant 170$

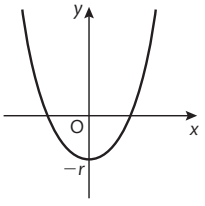
7 **a** $k = 3$

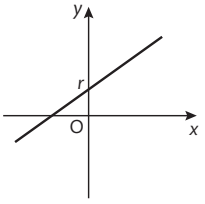
t	0	1	2	3	4
p	0	7	8	3	−8

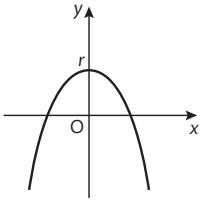
b 

c **(i)** £8333 at $t = 1.7$ months **(ii)** $t > 3.3$ months

8 **a** 

b 

c 

d 

Exercise 85

1 $a = 230°, b = 25°$ **2** $a = 280°, b = 50°$
3 $a = 100°, b = 260°$ **4** $a = 40°, b = 320°$
5 $a = 110°, b = 35°$ **6** $a = 60°, b = 60°$
7 $a = 108°, b = 72°, c = 54°$ **8** $a = 62°, b = 62°, c = 28°$
9 $a = 55°$ **10** $a = 45°$
11 $a = 90°, b = 32°, c = 58°$ **12** $a = 50°$

Exercise 85*

1 **a** 37° **b** 53°
2 **a** x **b** $2x$
3 **a** 22.5° **b** 45°
4 OUT $= x$; TOU $= 180° - 2x$; TUV $= 90° - x$
5 OBA $= 90° - x$; AOB $= 2x$; COB $= 180° - 2x$
6 **a** $\sqrt{(r^2 - x^2)}$ **b** $\sqrt{(r^2 - x^2)}$
7 **a** 12 cm **b** 54 cm^2
c 7.2 cm
8 **a** 9 cm **b** 108 cm^2

Exercise 86

1 $a = 124°$ **2** $a = 56°$
3 $a = 102°$ **4** $a = 78°$
5 $a = 56°$ **6** $a = 36°, b = 36°$
7 $a = 50°, b = 130°$ **8** $a = 67°, b = 113°$
9 $a = 100°, b = 160°$ **10** $a = 40°, b = 140°$
11 $a = 67°, b = 85°$ **12** $a = 105°$

Exercise 86*

1 Correct proof **2** Angle HOI $= 100° \Rightarrow x = 50°$
3 126°
4 Angle GOI $= 2 \times 100 = 200 \Rightarrow x = 160°$
5 $x = 116°, y = 64°$ **6** $x = 27°$
7 Correct proof **8** Correct proof

Exercise 87

1 T_1, T_2 **2** $x = 6$
3 $x = 5\ y = 8$ **4** **a** 3 cm **b** 14 cm
5 **a** 7.5 cm **b** 10.5 cm
6 1.5 m

Exercise 87*

1 E(4, 2), F(4, 4)
2 **b** 16 cm **c** 15 cm
3 **a** proof **b** DE = 11.25 cm **c** BD = 3.75
4 PQS, RPS, RPQ
a PS $= \frac{20}{3}$, RS $= \frac{16}{3}$ **b** $\frac{50}{3}$
5 18 m **6** 15 m
7 239 litres **8** 135°

Exercise 88

1 10.3 cm **2** 8.94 cm **3** 11.8 cm
4 29.2 km **5** 3.16 m
6 **a** $\sqrt{(y^2 - r^2)}$ **b** $\sqrt{(y^2 - r^2)}$

Exercise 88*

1 12.4 cm **2** 8.77 m **3** 17 : 28
4 11.6 cm **5** 27.5 m
6 **a** 48 m **b** 24 m

Exercise 89

1 Yes SAS **2** No **3** No
4 Yes ASA **5** Yes RHS
6 No

Exercise 89*

1 Correct proof **2** Correct proof
3 Correct proof **4** Correct proof
5 Correct proof **6** Correct proof

Exercise 90 (Revision)

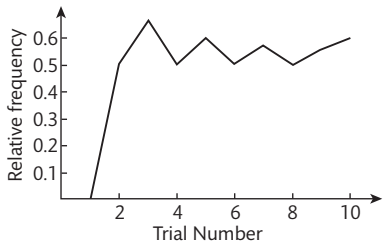
1 $a = 50°, b = 280°$ **2** $a = 90°, b = 30°$
3 $a = 70°, b = 20°$ **4** $a = 55°, b = 70°$
5 $a = 60°$ **6** $a = 140°$
7 $a = 50°$ **8** $a = 140°$
9 $a = 40°, b = 20°$ **10** $a = 120°, b = 30°$
11 $a = 65°, b = 115°$ **12** $a = 50°, b = 130°$
13 $a = 18, b = 14$ **14** $a = 10.5$
15 $a = 12, b = 12$ **16** $a = 16, b = 3$
17 $a = 6.40$ **18** $b = 4.47$
19 $c = 15.0$ **20** AC = 36.6 cm
21 **a** No **b** Yes SSS **22** SAS

Exercise 90* (Revision)

1 $a = 60°, b = 300°$ **2** $a = 90°, b = 45°$
3 $2a = 36°, 3a = 54°$ **4** $a = 55°, b = 35°$
5 $a = 100°$ **6** $a = 80°$
7 $a = 290°$ **8** $a = 102°$
9 $a = 40°, b = 60°$ **10** $a = 35°, b = 25°$
11 $a = 110°, b = 70°$ **12** $a = 60°, b = 60°$
13 $a = 4.5$ **14** $a = 6, b = 4.5$
15 $a = 4.5, b = 2.5$
16 **a** The angles of both triangles are the same: 55°, 60°, 65°
b $a = 3.61, b = 2.87$
17 $a = 5.39$ **18** $a = 5.20$
19 **a** $r = 11.7$ **b** $a = 18.7$ **20** **a** XC = 2 **b** AC = 4.47
21 Correct proof **22** Correct proof

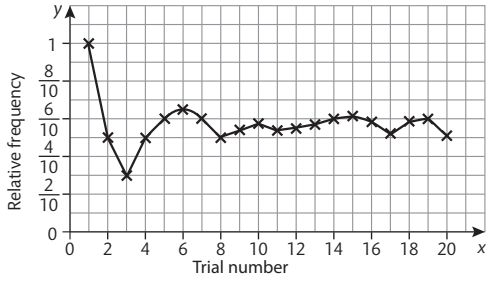
Exercise 91

1 **a** p(odd) $= \frac{6}{10}$



b Inconclusive. More trials would improve the experiment.

2 **a** p(a) $= \frac{3}{5}$



b More likely to rain than not in first 20 days of October.

3 p(vowel) $= \frac{9}{20}$

Exercise 91*

1 **a** p(L success) $= \frac{8}{12}$ p(R success) $= \frac{4}{12}$

b Learning curve, so warm up before playing. Practise more from RHS.

2 Work in pairs. p(A) $\rightarrow \frac{2}{\pi}$ as the number of trials increases.

3 **a** p(W) $= \frac{12}{20} = \frac{3}{5}$; p(P) $= \frac{8}{20} = \frac{2}{5}$

b No. of $W = \frac{3}{5} \times 100 = 60 \rightarrow$ No. of $P = 40$

Exercise 92

1 **a** p(g) $= \frac{4}{10} = \frac{2}{5}$ **b** p(a) $= \frac{3}{10}$

c p(t) $= 0$ **d** p(S) $= \frac{9}{10}$

2 **a** p(O) $= \frac{4}{14} = \frac{2}{7}$ **b** p(T) $= \frac{1}{14}$

c p(vowel) $= \frac{6}{14} = \frac{3}{7}$ **d** p(N or non-vowel) $= \frac{8}{14} = \frac{4}{7}$

3 **a** p(R) $= \frac{1}{2}$ **b** p(K) $= \frac{1}{13}$

c p(mult of 3) $= \frac{3}{13}$ **d** p($AJQK$) $= \frac{4}{13}$

4 **a** $\frac{3}{10}$ **b** $\frac{2}{5}$ **c** $\frac{3}{10}$ **d** $\frac{9}{10}$

5 **a** $\frac{1}{10}$ **b** $\frac{1}{2}$ **c** $\frac{3}{10}$ **d** $\frac{2}{5}$

6 **a** $\frac{3}{50}$ **b** $\frac{43}{50}$ **c** $\frac{2}{25}$ **d** $\frac{19}{20}$

7 p(S) $= \frac{3}{5}$

8 0.8

9 **a** p(P) $= \frac{1}{2}$ **b** p(R') $= \frac{5}{6}$ **c** p(WRP) $= 1$ **d** p(Y) $= 0$

10 **a** $\frac{3}{29}$ **b** $\frac{5}{29}$ **c** $\frac{17}{29}$ **d** impossible

Exercise 92*

1 **a** **(i)** $\frac{5}{36}$ **(ii)** $\frac{1}{12}$ **(iii)** $\frac{1}{12}$ **(iv)** $\frac{5}{12}$

Black \ White	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b 7

2 **a**

Green \ Red	2	3	5	7
11	9	8	6	4
13	11	10	8	6
17	15	14	12	10
19	17	16	14	12

(i) $\frac{1}{8}$ **(ii)** $\frac{5}{16}$ **(iii)** $\frac{7}{16}$ **(iv)** $\frac{3}{16}$

b 4, 9, 11, 15, 16, 17; all with probability $\frac{1}{16}$

3 **a** $\frac{9}{25}$ **b** $\frac{14}{25}$ **c** $\frac{6}{25}$ **d** $\frac{9}{25}$

Second spin \ First spin	1	2	3	4	5
1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	12	15
4	4	8	12	16	20
5	5	10	15	20	25

4 **a** $\frac{10}{110} = \frac{1}{11}$ **b** $\frac{113}{130}$ **c** 850

5 **a** $\frac{1}{2}$ **b** $\frac{3}{18} = \frac{1}{6}$ **c** $\frac{13}{18}$ **d** $\frac{5}{18}$

6 **a** $\frac{1}{5}$ **b** $\frac{1}{5}$ **c** $\frac{2}{15}$ **d** $\frac{1}{15}$

7 **a**

Die 2 \ Die 1	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

b **(i)** $\frac{1}{36}$ **(ii)** 0 **(iii)** $\frac{11}{36}$ **(iv)** $\frac{8}{36}$

8 **a**

White box \ Black box	10	30	50
9	10	30	50
16	16	30	50
25	25	30	50
36	36	36	50

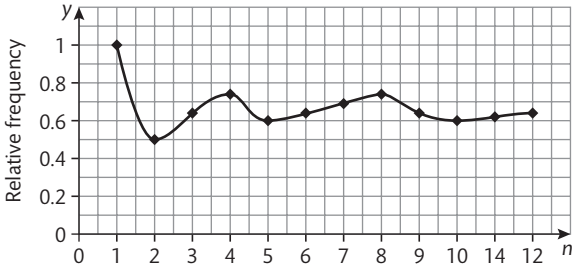
b **(i)** $\frac{1}{12}$ **(ii)** 0 **(iii)** $\frac{11}{12}$ **(iv)** $\frac{4}{12} = \frac{1}{3}$

9 $f = 5$

10 **a** $\frac{\pi}{3\sqrt{3}}$ **b** Approx. 40 darts.

Exercise 93 (Revision)

1 $\frac{2}{3}$; More trials for a better estimate



2 $\frac{13}{15}$

3 **a** $\frac{7}{51}$ **b** $\frac{1}{17}$ **c** $\frac{1}{3}$ **d** 0

4 HH, HT, TH, TT **a** $\frac{1}{4}$ **b** $\frac{1}{2}$

5 **a** $\frac{1}{6}$ **b** $\frac{1}{6}$ **c** 0 **d** 1

6 $\frac{10}{494} = \frac{5}{247}$

7 **a** 0 **b** $\frac{9}{25}$ **c** $\frac{16}{25}$ **d** $\frac{9}{25}$

8 **a** $\frac{1}{13}$ **b** $\frac{2}{13}$ **c** $\frac{3}{4}$ **d** $\frac{3}{26}$

9 **a** $\frac{1}{8}$ **b** $\frac{1}{2}$ **c** $\frac{3}{4}$ **d** $\frac{5}{8}$

10 **a** RG GR GG **b** $\frac{1}{2}$

Exercise 93* (Revision)

1 **a** 2002, $\frac{7}{10}$; 2003, $\frac{3}{5}$; 2004, $\frac{2}{5}$

b Decrease in numbers from 2002 is suggested by the data

2 **a** $\frac{1}{12}$ **b** $\frac{3}{4}$ **c** $\frac{11}{36}$

3 **a** **(i)** $\frac{1}{11}$ **(ii)** $\frac{2}{11}$ **(iii)** 0

b Z or *U*, $\frac{2}{11}$

4 **a** $\frac{6}{25}$ **b** $\frac{19}{25}$ **c** $\frac{3}{25}$ **d** $\frac{9}{25}$

5 \$45

6 HHH, HHT, HTH, HTT, THH, TTH, TTT, THT

a $\frac{1}{8}$ **b** $\frac{3}{8}$ **c** $\frac{1}{2}$

7 **a** $\frac{25}{28}$ **b** $\frac{1}{6}$ **c** 102

8 **a**

Blue Box	Pink Box: △	○	☆
□	□△	□○	□☆
▭	▭△	▭○	▭☆
☆	☆△	☆○	☆☆

b **(i)** $\frac{5}{9}$ **(ii)** $\frac{4}{9}$

9 10

10 $\frac{8}{245}$

Multiple choice 4

1 D **2** A **3** B **4** C **5** A

6 D **7** A **8** A **9** C **10** D

Self-assessment 4

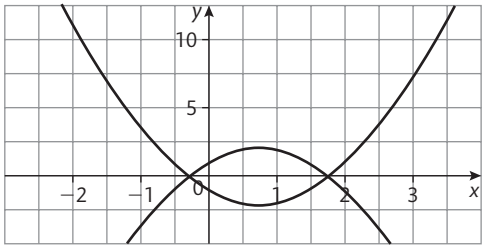
1 \$14 286 **2** \$22 727

3 **a** 500 000 **b** ≈ 500

4 $x = ab - Vg$ **5** $x = \frac{cd + ab}{a + c}$

6 $x = (PR)^2 - Q$

7 **a**, **b** $x = -2.8$ or 1.8 for both equations.



8 **a** $x = 13$ **b** $y = 13.2$

9 12.0 units **10** $x = 6\frac{2}{3}, y = 22.5$

11 $x = 15, y = 9$ **12** $x = 130°, y = 25°, z = 65°$

13 $x = 70°, y = 55°, z = 35°$ **14** $x = 130°, y = 65°, z = 115°$

15 $x = 124°, y = 34°, z = 62°$

16 **a**

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b **(i)** $\frac{1}{12}$ **(ii)** $\frac{7}{36}$ **(iii)** $\frac{5}{6}$ **(iv)** 0

17 **a** $\frac{3}{26}$ **b** $\frac{3}{26}$ **c** $\frac{11}{26}$ **d** $\frac{8}{13}$

18 **a** HHH HHT HTH THH TTH THT HTT TTT

b **(i)** $\frac{1}{8}$ **(ii)** $\frac{3}{8}$ **(iii)** $\frac{7}{8}$

19 **a** $e = \frac{\left(t\sqrt{\frac{5}{h}} - 1\right)}{\left(t\sqrt{\frac{5}{h}} + 1\right)}$ **b** $\frac{2}{3}$

20 BC common to both △s so SSS

Exercise 94

1 **a** 4 days **b** 2 days **c** $2\frac{2}{3}$ days

2 **a** 8 people **b** 16 people **c** 32 people

3 **a** 60 years **b** 15 years **c** 1200 years

4 **a** 12 hours **b** 72 km/h

5 **a** 32 km/litre **b** 20 litres

Exercise 94*

1 **a**

Number of light bulbs (*N*)	Power of each bulb (*P*)
6	500
5	600
2	1500
30	100

b $NP = 3000$

2 **a** 12 g/cm^3 **b** 1.5 cm^3

3

Number of men	Number of tunnels	Time in years
100 000	4	4
100 000	2	2
20 000	8	40
400 000	2	0.5

4 **a** 5 **b** 1000 **c** 2.26×10^8 tonnes

5 **a** 16 000 kg

b **(i)** 4 min **(ii)** 0.6 min **(iii)** 1 min **(iv)** 8 sec

Exercise 95

1 0.375 **2** 0.08 **3** 0.28125 **4** $0.\dot{2}$

5 $0.1\dot{8}$ **6** $0.2\dot{6}$ **7** $0.3\dot{8}$ **8** $\frac{9}{12}$

9 $\frac{3}{20}, \frac{5}{64}$ **10** $\frac{1}{3}$ **11** $\frac{5}{9}$ **12** $\frac{7}{9}$

13 $\frac{7}{90}$ **14** $\frac{1}{30}$ **15** $\frac{5}{90} = \frac{1}{18}$

Exercise 95*

1 $0.4\dot{6}$ **2** $0.04\dot{6}$ **3** $2.\dot{3}\dot{0}$ **4** $0.3\dot{0}\dot{1}$

5 $\frac{11}{16}, \frac{7}{40}$ **6** $\frac{19}{20}, \frac{3}{25}, \frac{5}{64}$ **7** $\frac{24}{99} = \frac{8}{33}$ **8** $\frac{10}{33}$

9 $9\frac{19}{990}$ **10** $\frac{3}{110}$ **11** $\frac{412}{999}$ **12** $\frac{128}{333}$

13 $\frac{11}{90}$ **14** $\frac{28}{495}$ **15** $0.\dot{0}3\dot{7}$

Exercise 96 (Revision)

1 **a** 2 days **b** 1 day **c** $\frac{1}{2}$ day

2 **a** $2\frac{1}{2}$ days **b** $1\frac{1}{4}$ days **c** $\frac{1}{2}$ day

3

Number of years, *n*	Number of men, *m*
1	12 000
2	6000
3	4000
4	3000
6	2000
12 000 ÷ *m*	*m*
n	12 000 ÷ *n*

4 **a** **(i)** $2\frac{1}{2}$ days **(ii)** $1\frac{1}{4}$ days **(iii)** 1 day **(iv)** $\frac{5}{p}$ days

b **(i)** 10 people **(ii)** 5 people **(iii)** 4 people **(iv)** $\frac{20}{d}$ people

5 **a** 9 hours **b** 90 km/hr

6 **a** $0.\dot{3}$ **b** $0.\dot{1}$ **c** $0.08\dot{3}$ **d** $0.0\dot{6}$

7 **a** $\frac{4}{9}$ **b** $\frac{7}{9}$ **c** $\frac{53}{99}$ **d** $\frac{82}{99}$

8 **a** $\frac{301}{999}$ **b** $\frac{707}{999}$ **c** $\frac{7}{198}$ **d** $\frac{409}{99\,900}$

Exercise 96* (Revision)

1

Number of women, w	Length of dry-stone wall, x m	Time of construction, t days
1	8	8
3	18	6
4	24	6
6	12	2
$2\frac{2}{3}$	32	12
w	x	$x \div w$
w	wt	t
$x \div t$	x	t

2

Number of bees, b	Length of bee's journey, x km	Mass of honey, m g
10	150	10
20	375	50
100	750	500
500	300	1000
10^6	1000	6.67×10^6
b	x	$bx \div 150$
b	$150m \div b$	m
$150m \div x$	x	m

3

Number of grass-cutters, n	Number of rugby pitches, r	Time, t hours
2	1	0.5
$1\frac{1}{3}$	2	1.5
4	4	1
$r \div t$	r	t
n	tn	t
n	r	$r \div n$

4

Number of houses, h	Mass of waste, w kg	Time, t weeks
1	20	1
100	20 000	10
962	10^6	52
h	w	$w \div 20h$
$w \div 20t$	w	t
h	$20ht$	t

5 **a** $\frac{107}{333}$ **b** $\frac{34}{45}$ **c** $\frac{25}{66}$ **d** $\frac{203}{198}$

6 $\frac{254}{713}$

Exercise 97

1 $x^2 + 5x + 4$ **2** $x^2 - 4x - 21$ **3** $x^2 - 4x - 12$

4 $x^2 - 8x + 15$ **5** $x^2 + 6x + 9$ **6** $x^2 - 8x + 16$

7 $x^2 - 25$ **8** $x^2 - 6x - 16$ **9** $15x^2 - 7x - 2$

10 $x^3 + 2x^2 - 5x - 10$

Exercise 97*

1 $x^2 + 4x - 21$ **2** $x^2 - 9$

3 $x^2 + 24x + 144$ **4** $12x^2 - 25x + 12$

5 $x^2 + x(b - a) - ab$ **6** $16x^2 - 40x + 25$

7 $15x^3 + 21x^2 + 5x + 7$ **8** $8x + 8 = 8(x + 1)$

9 $\frac{a^2}{4} - \frac{ab}{5} + \frac{b^2}{25}$ **10** $10x^5 + 11x^4 + 3x^3$

11 4 **12** $x = -\frac{5}{3}$

13 $a = 3, b = 1$

Exercise 98

1 **a** $x^2 + 3x + 2$ **b** $3x + 2$ **c** $x = 3$

2 **a** $\pi(x^2 + 8x + 16)$ **b** $\pi(8x + 16)$ **c** $x = 2$

3 **a** $5x^2 + 25x + 30$ **b** $2x^2 + 30x + 62$

4 **a** $7x^2 + 35x$ **b** $2x^2 + 66x + 140$

5 $x = 6$

6 **a** $(x + 2)^2 = (x + 1)^2 + 49$ **b** $x = 23$

Exercise 98*

1 **a** $\pi(x^2 + 12x + 36)$ **b** $x = 0.75$

2 **a** $4x^2 + 37x + 40$ **b** $x = 1.5$

3 $x = 6$

4 $x = 2$

5 **a** $2n$ is divisible by 2 and so is even; $2n + 1$ is then odd.

b $(2n + 1)(2m + 1) = 4mn + 2n + 2m + 1$; $4mn$, $2n$ and $2m$ are even so this is odd.

c $(2n - 1)(2n + 1) + 1 = 4n^2 = (2n)^2$

6 **a** $10 - x$ **b** $5 - x$ **c** $x = 3.9$

Exercise 99

1 $x(x - 3)$ **2** $x(x + 2)$ **3** $x(x - 31)$

4 $x(x + 42)$ **5** $(x - 4)(x + 4)$ **6** $(x - 7)(x + 7)$

Exercise 99*

1 $x(x - 312)$ **2** $x(x + 51)$ **3** $(x - 8)(x + 8)$

4 $(x - 11)(x + 11)$ **5** $(x - 15)(x + 15)$ **6** $4((x - 2)(x + 2))$

Exercise 100

1 $a = 1$ **2** $a = 4$ **3** $a = -1$

4 $a = -2$ **5** $a = 2$ **6** $a = -1$

Exercise 100*

1 $a = 3$ **2** $a = 4$ **3** $a = -7$

4 $a = -3$ **5** $a = -8$ **6** $a = \frac{1}{2}$

Exercise 101

1 $(x - 2)(x - 1)$ **2** $(x - 1)(x - 3)$ **3** $(x - 4)(x - 3)$

4 $(x + 4)^2$ **5** $(x - 1)(x - 8)$ **6** $(x - 1)^2$

Exercise 101*

1 $(x + 7)(x + 3)$ **2** $(x - 2)(x - 6)$ **3** $(x - 8)^2$

4 $(x - 6)(x - 12)$ **5** $(x + 9)(x + 5)$ **6** $(x + 12)^2$

Exercise 102

1 $(x + 3)(x - 2)$ **2** $(x + 2)(x - 5)$ **3** $(x + 2)(x - 6)$

4 $(x + 1)(x - 10)$ **5** $(x + 7)(x - 2)$ **6** $(x + 8)(x - 1)$

Exercise 102*

1 $(x + 6)(x - 5)$ **2** $(x + 4)(x - 6)$ **3** $(x + 12)(x - 5)$

4 $(x + 5)(x - 14)$ **5** $(x + 8)(x - 15)$ **6** $(x - 5)(x + 15)$

Exercise 103

1 $(x - 1)(x - 2)$ **2** $(x + 3)(x - 1)$ **3** $(x + 1)(x + 12)$

4 $(x - 2)(x - 6)$ **5** $(x - 4)^2$ **6** $(x - 4)(x + 5)$

Exercise 103*

1 $(x + 10)(x - 2)$ **2** $(x + 2)(x - 9)$ **3** $(x + 9)(x + 4)$

4 $(x - 4)(x - 8)$ **5** $(x + 12)(x - 4)$ **6** $(3 - x)(x + 1)$

Exercise 104

1 $x = -1$ or $x = -2$ **2** $x = -4$ or $x = 1$ **3** $x = 7$ or $x = 2$

4 $x = -8$ **5** $x = 0$ or $x = 10$

Exercise 104*

1 $x = -8$ or $x = 4$ **2** $x = -21$ or $x = 5$ **3** $x = 0$ or $x = 8$

4 $x = -\frac{3}{2}$ or $x = \frac{3}{4}$ **5** $x = -1$ or $x = 1$ or $x = -\frac{5}{2}$

Exercise 105

1 $x = 1$ or $x = 2$ **2** $x = -2$ or $x = 1$ **3** $x = -2$ or $x = -4$
4 $x = 4$ or $x = -3$ **5** $x = 5$ or $x = 3$ **6** $x = -4$

Exercise 105*

1 $x = 4$ or $x = 5$ **2** $x = -3$ or $x = 8$ **3** $x = -9$ or $x = -12$
4 $x = 14$ or $x = 4$ **5** $x = -16$ or $x = -6$ **6** $x = 3$ or $x = -1$
7 $x = -15$ or $x = 8$

Exercise 106

1 $x = 0$ or $x = 2$ **2** $x = 0$ or $x = -7$ **3** $x = 0$ or $x = 25$
4 $x = 0$ or $x = -23$ **5** $x = -2$ or $x = 2$ **6** $x = -5$ or $x = 5$

Exercise 106*

1 $x = 0$ or $x = 125$ **2** $x = 0$ or $x = -231$ **3** $x = -8$ or $x = 8$
4 $x = -13$ or $x = 13$ **5** $x = \pm\sqrt{7}$

Exercise 107

1 3, −4 **2** 5, −6 **3** −4, 5
4 −6, 7 **5** 5, −7 **6** −4, 6
7 **a** $x^2 + 3x$ **b** $x = 3$
8 **a** $x^2 + x$ **b** $x = 6$
9 10 cm by 4 cm **10** 8 cm by 5 cm
11 $x = 3$ **12** $x = 2$

Exercise 107*

1 11, 13 or −13, −11 **2** 8, 12 or −12, −8 **3** 30 cm by 40 cm
4 20 m by 100 m **5** 1 s and 2 s **6** 2 s
7 8, 9 or −9, −8 **8** 7, 9 or −9, −7 **9** 20
10 18 **11** 4 **12** 20 cans

Exercise 108 (Revision)

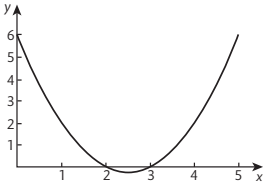
1 $x^2 - 10x + 21$ **2** $x^2 + 4x + 4$ **3** $2x^2 - 7x - 15$
4 **a** $x^2 + 5x + 6$ **b** $5x + 6$ **c** $x = 4$
5 $(x - 6)(x + 6)$ **6** $(x + 3)(x + 1)$ **7** $(x + 4)(x - 2)$
8 $x = 6$ or $x = -2$ **9** $x = 0$ or $x = 5$ **10** $x = -6$ or $x = 6$
11 $x = -4$ or $x = 5$ **12** 20 cm by 30 cm

Exercise 108* (Revision)

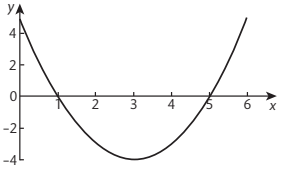
1 $x^2 - 3x - 108$ **2** $4x^2 - 12x + 9$ **3** $6x^2 + 7x - 3$
4 4.25 m **5** 27 cm **7** $x = -11$ or $x = 11$
7 $x = 0$ or $x = 7$ **8** $x = -7$ or $x = 8$ **9** $x = 9$ or $x = 6$
10 4, 10 **11** **b** $x = 30$ **12** 400 cm^2

Exercise 109

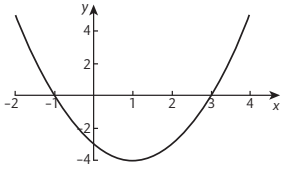
1 $x = 3$ or $x = 2$



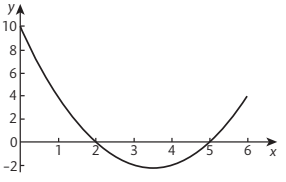
2 $x = 1$ or $x = 5$



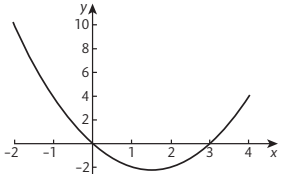
3 $x = -1$ or $x = 3$



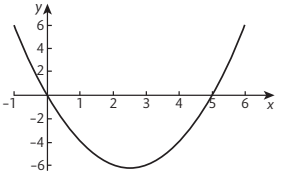
4 $x = 2$ or $x = 5$



5 $x = 0$ or $x = 3$

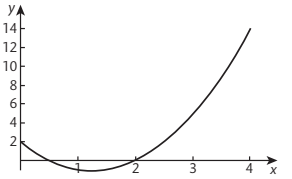


6 $x = 0$ or $x = 5$

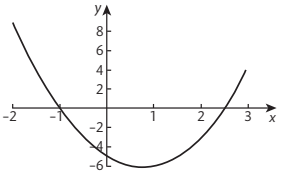


Exercise 109*

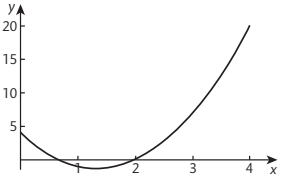
1 $x = \frac{1}{2}$ or $x = 2$



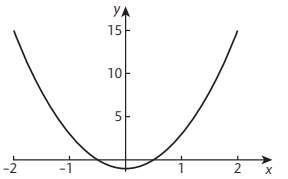
2 $x = -1$ or $x = 2\frac{1}{2}$



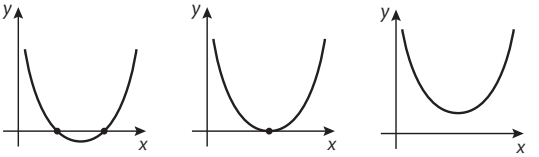
3 $x = \frac{2}{3}$ or $x = 2$



4 $x = -\frac{1}{2}$ or $x = \frac{1}{2}$



5 $y = x^2 - 6x + 5$ **6** $y = x^2 - 7x + 10$
7 $y = x^2 - 6x + 9$ **8** $y = -3x - 4$
9 Two solutions One solution No solutions



Exercise 110

1 $x = -2.2$ or $x = 2.2$ **2** $x = -1.7$ or $x = 1.7$
3 $x = -1$ or $x = 2$ **4** $x = -2.3$ or $x = 1.3$
5 $x = -3.8$ or $x = 1.8$ **6** $x = -1.6$ or $x = 3.6$
7 $x = 0.6$ or $x = 3.4$ **8** $x = -3.7$ or $x = -0.3$
9 $x = -2.9$ or $x = 3.4$ **10** $x = -3.1$ or $x = 2.6$

Exercise 110*

1 $x = -1.3$ or $x = 2.3$ **2** $x = -2.6$ or $x = 1.6$
3 $x = -2.6$ or $x = -0.4$ **4** $x = -0.7$ or $x = 2.7$
5 $x = 2$ **6** $x = -1$
7 $x = -2.7$ or $x = 2.2$ **8** $x = -2$ or $x = 2.5$
9 $x = -2.8$ or $x = 3.2$ **10** $x = -2.8$ or $x = 2.5$

Exercise 111

1 **a** $x = 0$ or $x = 3$ **b** $x = -0.56$ or $x = 3.56$
c $x = 0.38$ or $x = 2.62$ **d** $x = -0.24$ or $x = 4.24$
e $x = -0.79$ or $x = 3.79$ **f** $x = 0.21$ or $x = 4.79$
2 **a** $x = 0$ or $x = 2$ **b** $x = -1.45$ or $x = 3.45$
c $x = 0.29$ or $x = 1.71$ **d** $x = -0.62$ or $x = 1.62$
e $x = -0.73$ or $x = 2.73$ **f** $x = 0.59$ or $x = 3.41$
3 **a** $x = 1$ or $x = 3$ **b** $x = -0.65$ or $x = 4.65$
c $x = 0.70$ or $x = 4.30$ **d** $x = -0.56$ or $x = 3.56$
4 **a** $x = -1$ or $x = 4$ **b** $x = 0.38$ or $x = 2.62$
c $x = -1.24$ or $x = 3.24$ **d** $x = 0.59$ or $x = 3.41$
5 (2.71, 3.5), no
6 (1.1, −2), no

Exercise 111*

1 **a** $x = 5$ or $x = 0$ **b** $x = 4.30$ or $x = 0.70$
c $x = 3.73$ or $x = 0.27$ **d** $x = 0.76$ or $x = 5.24$
2 **a** $x = 0.5$ or $x = 0$ **b** $x = -1.19$ or $x = 1.69$
c $x = -0.82$ or $x = 1.82$ **d** $x = -0.62$ or $x = 1.62$
3 **a** $x = -1.78$ or $x = 0.28$ **b** $x = -2.35$ or $x = 0.85$
c $x = -2.28$ or $x = -0.22$
4 **a** $x = -0.67$ or $x = 1$ **b** $x = 1.33$ or $x = -1$
c $x = -0.26$ or $x = 1.26$
5 **a** $y = x + 2$ **b** $y = -2x - 1$
6 **a** $y = -x - 3$ **b** $y = 3x - 5$

Exercise 112 (Revision)

1 **a** $-2.6, 2.6, y = 7$ **b** $0, 1, y = x$
c $-1.3, 2.3, y = x + 3$ **d** $-2.6, 1.6, y = 4 - x$
e $1, y = 2x - 1$ **f** $-2.4, 0.4, y = 1 - 2x$

2 **a** $-1.6, 0.6, y = 0$ **b** $-1, 0, y = -1$
c $-2, 1, y = 1$ **d** $-2.6, 1.6, y = 3$
e $-3.3, 0.3, y = -2x$ **f** $-1, 1, y = x$

3 **a** $y = 2$ **b** $y = x$
c $y = 1 - x$ **d** $y = 3 + 2x$

4 **a** $y = 0$ **b** $y = -3$
c $y = 3$ **d** $y = x$

5 $x = 0.6$ or $x = 3.4$

6 $x = -1.4$ or $x = 3.4$

Exercise 112* (Revision)

1 **a** $0.4, 2.6, y = 0$ **b** $-0.8, 3.8, y = 4$
c $0.3, 3.7, y = x$ **d** $-0.2, 4.2, y = x + 2$
e $-1.4, 3.4, y = 6 - x$ **f** $-1.3, 2.3, y = 4 - 2x$
g $k = -2.25$

2 **a** $y = -1$ **b** $y = 6$
c $y = x$ **d** $y = 2x + 2$
e $y = -4x + 3$ **f** $y = -2x^2 - 3x$

3 (2, 3)

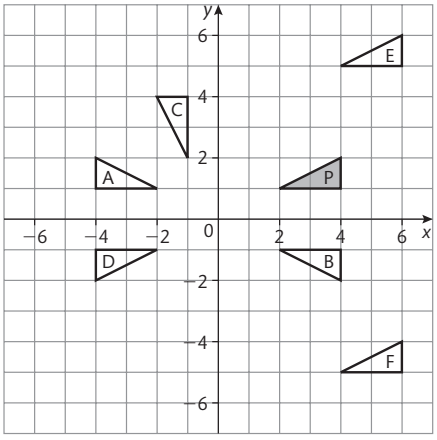
4 (72, 14)

5 $x = 1.14$ or -2.64

6 $x = 1.14$ or -2.64

Exercise 113

1



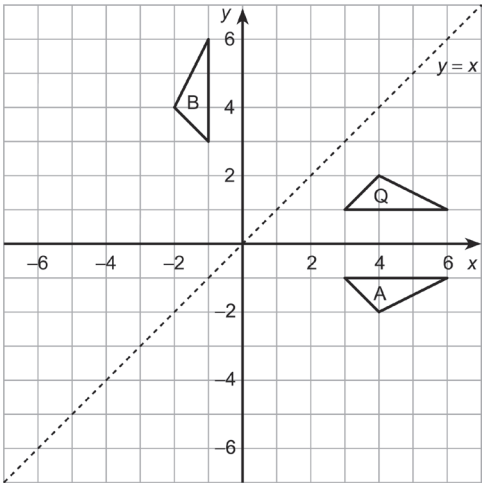
2

Object	Reflection in line	Image
A	$x = 5$	B
F	$x = 5$	G
G	$x = 5$	F
A	$x = 7$	C
D	$x = 12$	B
G	$x = 9$	H
K	$x = 24$	J
I	$x = 12$	G
H	$x = 18$	K
E	$x = 10$	A
J	$x = 15$	G
D	$x = 17$	E

3

Object	Transformation		Image
B	Rotation	Centre (0, 0) Angle 90°	A
A	Translation	Vector $\begin{pmatrix}3\\1\end{pmatrix}$	C
B	Rotation	Centre (1, 2) Angle 90°	C
A	Rotation	Centre (0, −1) Angle 90°	F
F	Translation	Vector $\begin{pmatrix}0\\2\end{pmatrix}$	E
A	Rotation	Centre (−1, 0) Angle 90°	E
A	Rotation	Centre (0, 0) Angle 180°	G
G	Translation	Vector $\begin{pmatrix}-2\\0\end{pmatrix}$	H
A	Rotation	Centre (−1, 0) Angle 180°	H
B	Rotation	Centre (−1, 1) Angle −90°	H
H	Translation	Vector $\begin{pmatrix}2\\0\end{pmatrix}$	G
B	Rotation	Centre (0, 0) Angle −90°	G

4 **a**

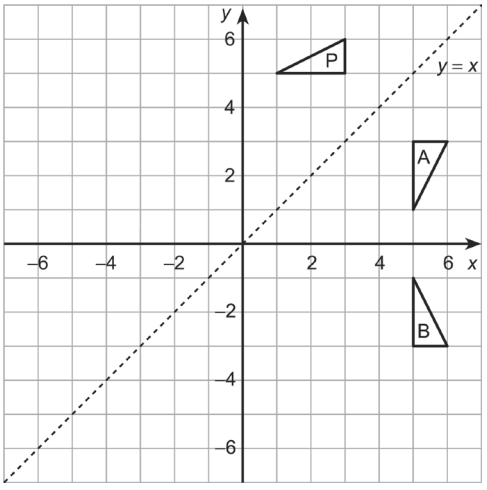


c Rotation 90° about (0, 0)

Exercise 113*

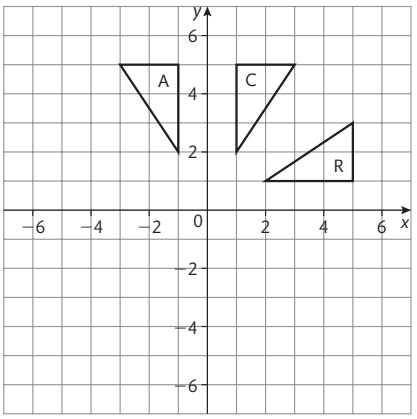
1 **a** (1, −2) **b** (1, 2) **c** (1, 10)
d (1, 18) **e** (4, 1) **f** (5, 2)
g (6, 3) **h** (12, 9)

2 **a**



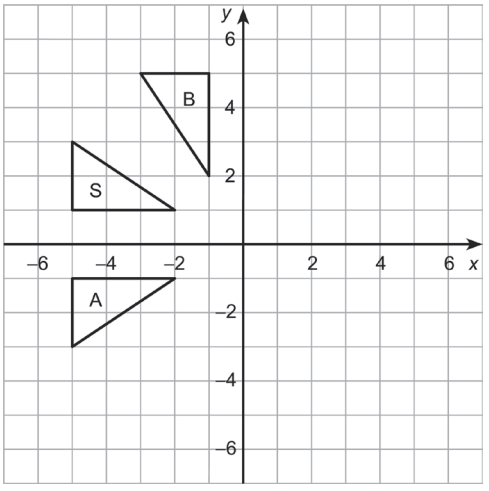
c Rotation −90° about (0, 0)

3 **a**



c Reflection in $y = x$

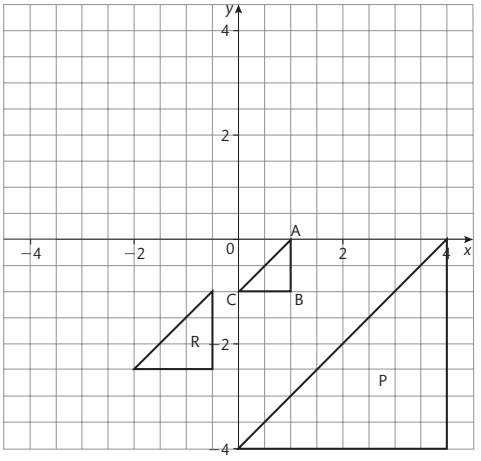
4 **a**



c Reflection in $y = -x$

Exercise 114

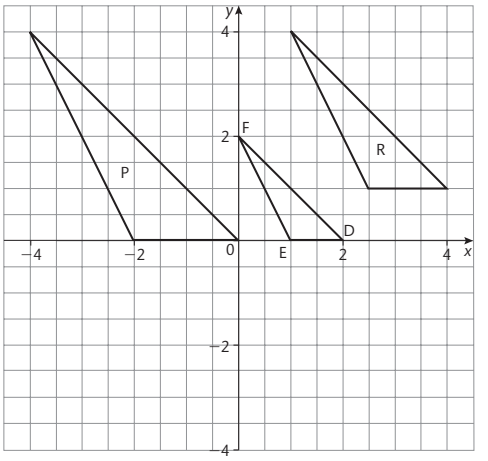
1 and **2**



3 B to A: (10, 8) SF = 2; A to E: (8, 0) SF = $\frac{1}{2}$; D to B: (12, 4) SF = $\frac{3}{2}$; E to D: (14, 12) SF = $\frac{2}{3}$; D to C: (4, 4) SF = 2

Exercise 114*

1 and **2**



3 A to C: (2, 11) SF = 2; A to D: (3, 11) SF = 4; C to D: (5, 11) SF = 2; C to E: (4, 10) SF = 3; B to C: (4, 5) SF = $\frac{1}{2}$; E to D: (1, 7) SF = $\frac{2}{3}$

Exercise 115

1 Reflection in $y = 2$; reflection in x-axis; translation $(0, -4)$
2 Reflection in $x = 4$; reflection in y-axis; translation $(-8, 0)$
3 Reflection in y-axis; reflection in $y = 2$; rotation 180° about (0, 2)
4 Reflection in $x = 4$; reflection in $y = 2$; rotation 180° about (4, 2)
5 Reflection in y-axis; reflection in $x + y = 0$; rotation 90° about (0, 0)
6 Reflection in y-axis; reflection in $x + y = 0$; rotation 90° about (0, 0)
7 Reflection in $x = -4$; reflection in $y = 2$; rotation 180° about $(-4, 2)$
8 Reflection in $y = -2$; reflection in y-axis; rotation 180° about $(0, -2)$

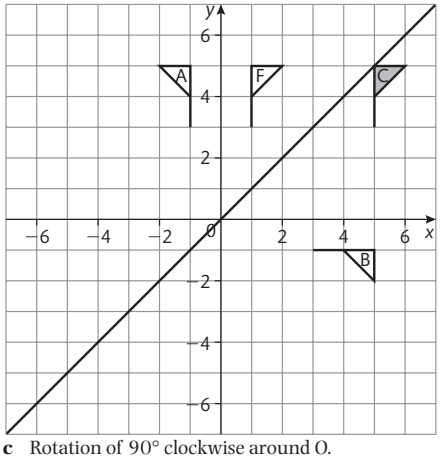
Exercise 115*

1 Reflection in $x + y = 6$; rotation −90° about (2, 4)
2 Rotation 90° about $(-3, -1)$; translation (2, 4)
3 Rotation 143° about $(-2, 2)$; reflection in $y = 2x + 6$
4 Reflection in $x + y = 8$; translation $(-8, -8)$

Exercise 116 (Revision)

1 **a** (3, −4) **b** (−3,4) **c** (4,−3) **d** (10, −2)
2 **a** (−3,−5) **b** (3, 5) **c** (−5,−3) **d** (−8, 8)
3 **a** A′(1, −2), B′(1, −6), C′(8, −2)
b A′(−2, 1), B′(−6, 1), C′(−2, 8)
c A′(−4, 6), B′(−4, 10), C′(3, 6)
d A′(2, 0), B′(2, 8), C′(16, 0)

4



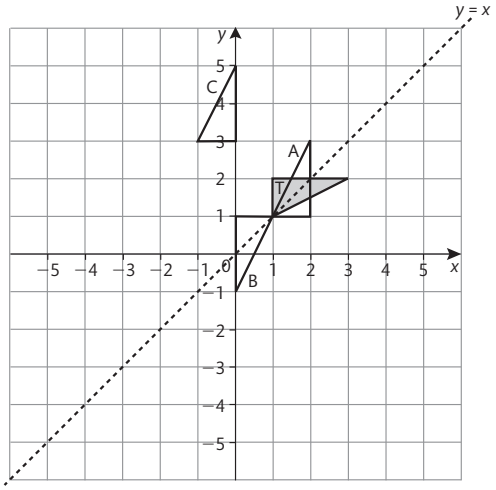
c Rotation of 90° clockwise around O.
e Translation along $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$

5 $x = -11, y = -1$

Exercise 116* (Revision)

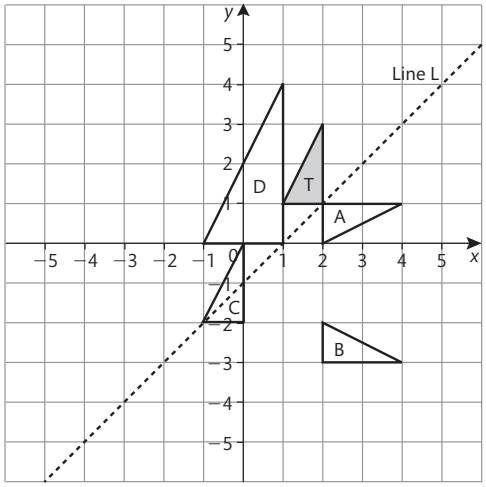
1 A(1, −5), B(−1, −5), C(1, −9)

2 **a** and **b**



c Translation along vector $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$

3 **a**–**d**



e Rotation of −90° about (0, −1)

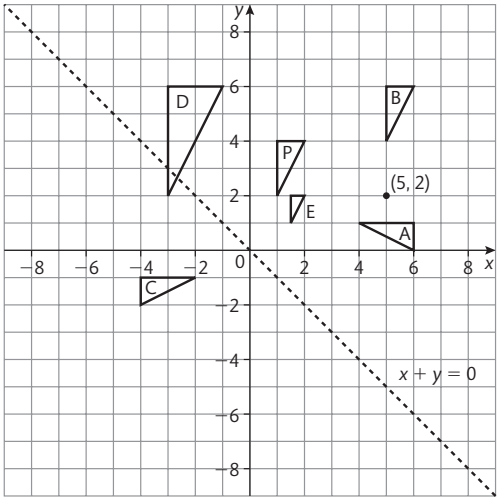
f Enlargement of scale factor +2 centre (−1, −4)

4 **a** Reflection in y-axis **b** Reflection in x-axis

c Reflection in $y = x + 2$ **d** Reflection in $y = x - 2$

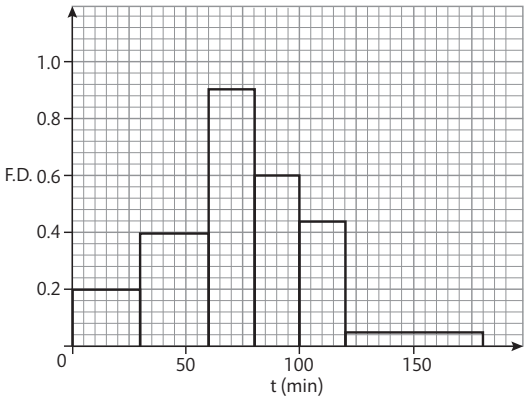
e Rotation 90° about (−2, 0) **f** Rotation 90° about (0, −2)

5

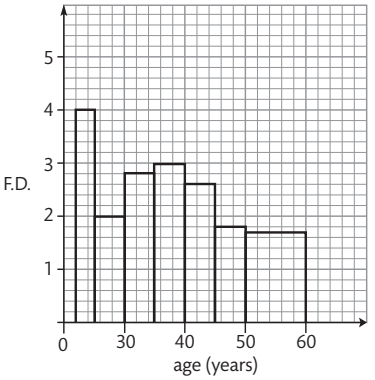


Exercise 117

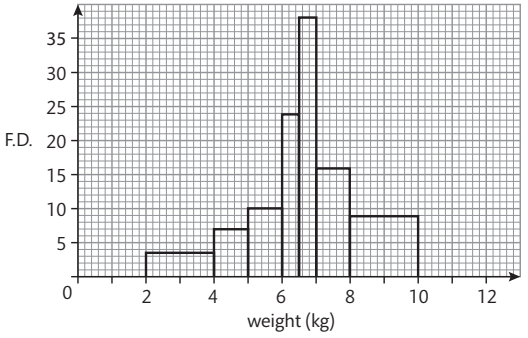
1 f.d.: 0.20, 0.40, 0.90, 0.60, 0.45, 0.05



2 f.d.: 4.0, 2.0, 2.8, 3.0, 2.6, 1.8, 1.7



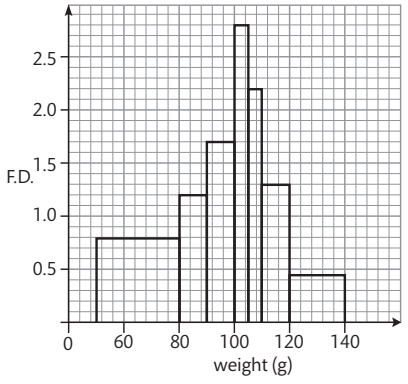
3 **a** f.d.: 3.5, 7, 10, 24, 38, 16, 9



b 6.5–7 kg

c 55%

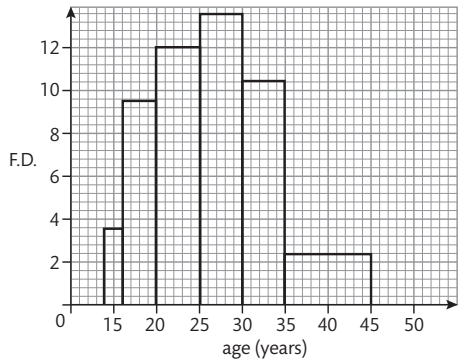
4 **a** f.d.: 0.8, 1.2, 1.7, 2.8, 2.2, 1.3, 0.45



b 100–105 g **c** 33% **d** 98

5 **a** f.d.: 3.5, 9.5, 12, 13.6, 10.4, 2.5

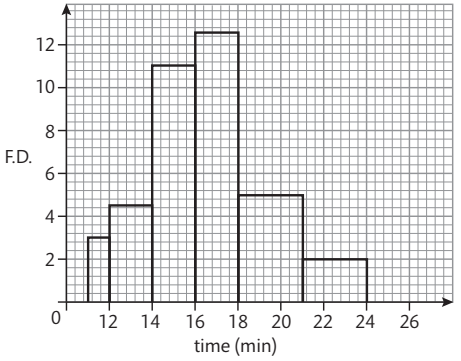
b



c $\bar{x} = 26.8$ years

6 **a** f.d.: 3.0, 4.5, 11, 12.5, 5.0, 2.0

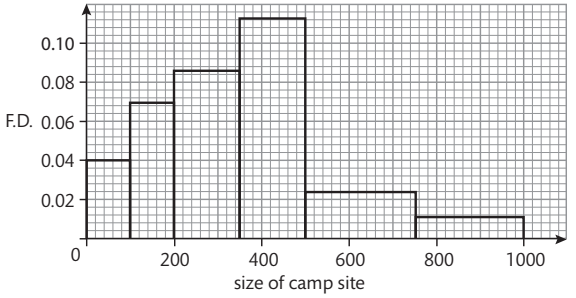
b



c $x = 16$ min 40 s, **d** 16.5 min

Exercise 117*

1 **a** f.d.: 0.04, 0.07, 0.087, 0.113, 0.024, 0.012

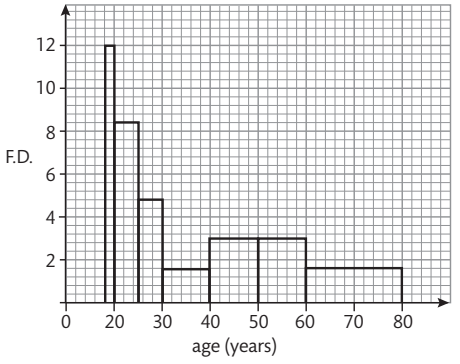


b 51.4%

c $\bar{x} = 368.5$

d 359

2 **a** f.d.: 12, 8.4, 4.8, 1.6, 3.0, 3.0, 1.7

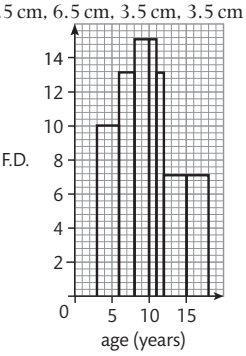


b 26.5% **c** $x = 40.0$ years

3 **a** f.d.: 10, 13, 15, 15, 13, 7, 7

b $\bar{x} = 9.77$ years

c 6.5 cm, 7.5 cm, 7.5 cm, 6.5 cm, 3.5 cm, 3.5 cm

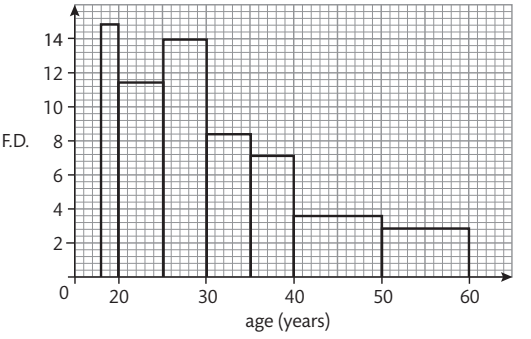


4 **a** f.d.: 15, 11.4, 13.8, 8.4, 7.2, 3.6, 3.0

b $x = 32.45$ years

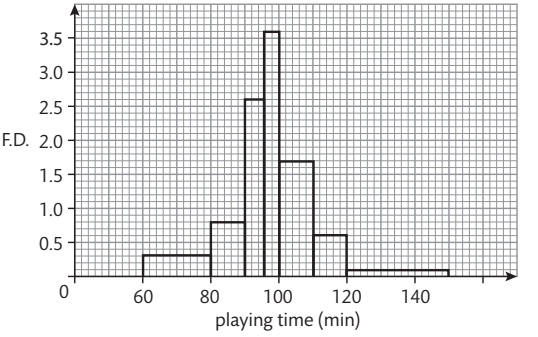
c 3.8 cm, 4.6 cm, 2.8 cm, 2.4 cm, 1.2 cm, 1 cm

d 29.6 years



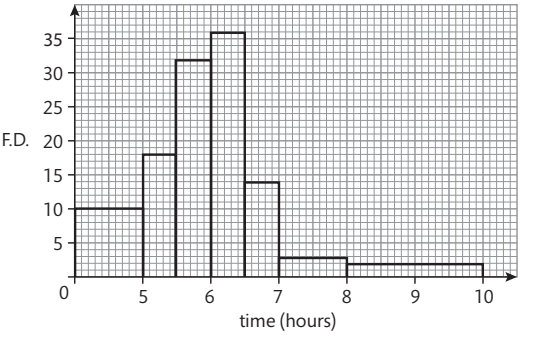
5 **a** 6, 8

b f.d.: 3.6, 1.7, 0.6, 0.1



c $\bar{x} = 97.7$ min

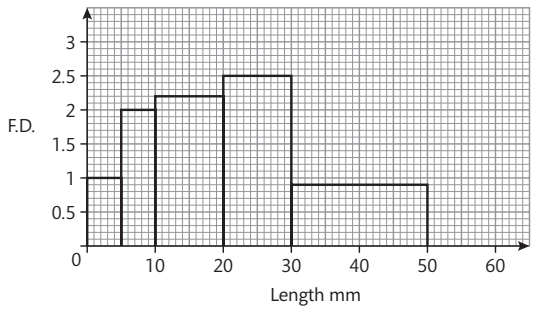
6 **a** 3, 4 **b** f.d.: 10, 18, 32, 36



c $x = 6.01$ h

Exercise 118 (Revision)

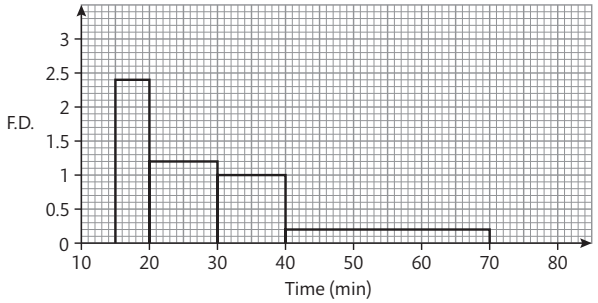
1 a



Length, l mm	F.D.
$0 < l \leqslant 5$	1
$5 < l \leqslant 10$	2
$10 < l \leqslant 20$	2.2
$20 < l \leqslant 30$	2.5
$30 < l \leqslant 50$	0.9

b 7 **c** 22.0 mm

2 a

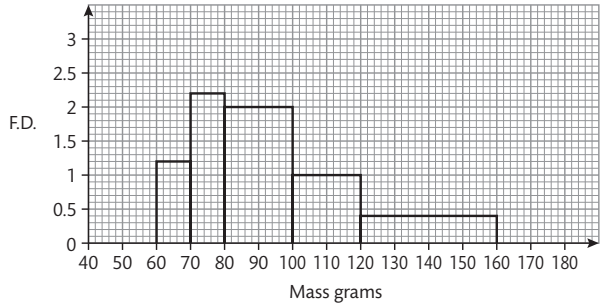


Time, t min	F.D.
$15 < t \leqslant 20$	2.4
$20 < t \leqslant 30$	1.2
$30 < t \leqslant 40$	1
$40 < t \leqslant 70$	0.2

b 2 **c** 17 (16.8) **d** 26.7 min

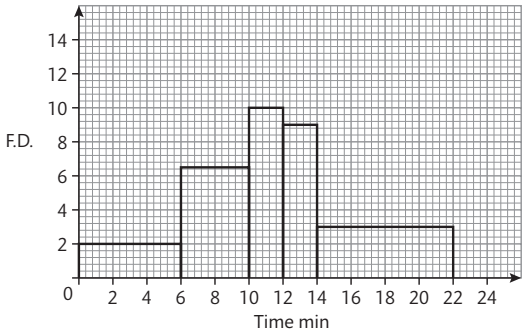
3 a b

Mass, m g	Frequency	F.D.
$60 < m \leqslant 70$	12	1.2
$70 < m \leqslant 80$	22	2.2
$80 < m \leqslant 100$	40	2
$100 < m \leqslant 120$	20	1
$120 < m \leqslant 160$	16	0.4



c 41

4

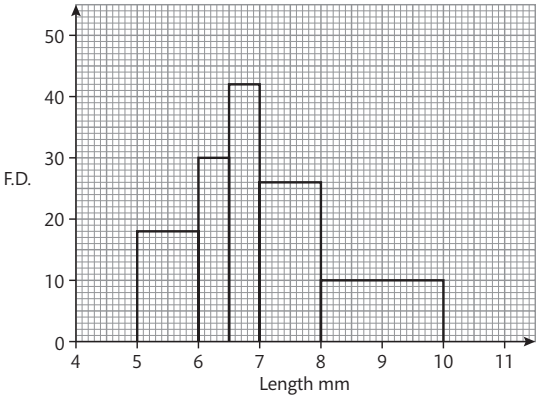


Time, t min	Frequency	F.D.
$0 < t \leqslant 6$	12	2
$6 < t \leqslant 10$	26	6.5
$10 < t \leqslant 12$	20	10
$12 < t \leqslant 14$	18	9
$14 < t \leqslant 22$	24	3

c 57% **d** 11.2 min

Exercise 118* (Revision)

1 a

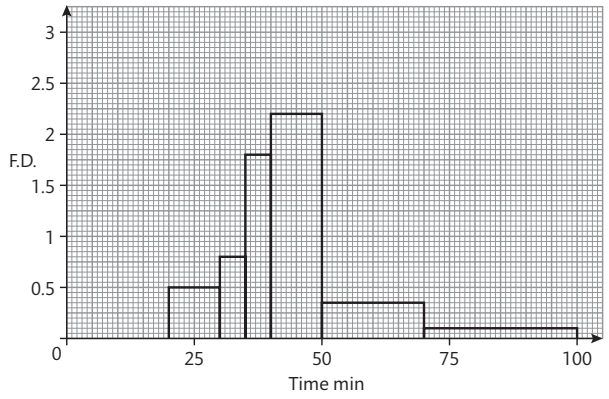


Length, l mm	F.D.
$5 < l \leqslant 6$	18
$6 < l \leqslant 6.5$	30
$6.5 < l \leqslant 7$	42
$7 < l \leqslant 8$	26
$8 < l \leqslant 10$	10

b 76 **c** 7.10 mm

2 a

Time, t min	F.D.
$20 < t \leqslant 30$	0.5
$30 < t \leqslant 35$	0.8
$35 < t \leqslant 40$	1.8
$40 < t \leqslant 50$	2.2
$50 < t \leqslant 70$	0.35
$70 < t \leqslant 100$	0.1

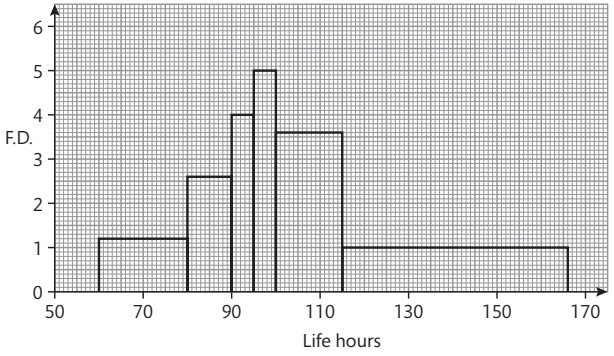


b 71.2% **c** 43.2 min **d** $40 < t \leqslant 50$

3 **a, c**

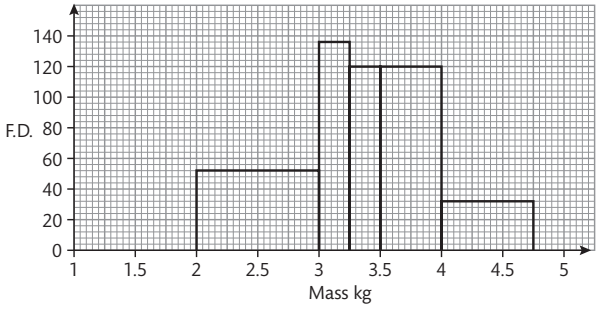
Life, t hours	Frequency	F.D.
$60 < t \leqslant 80$	24	1.2
$80 < t \leqslant 90$	26	2.6
$90 < t \leqslant 95$	20	4
$95 < t \leqslant 100$	25	5
$100 < t \leqslant 115$	54	3.6
$115 < t \leqslant x$	51	1

b $x = 166$



4 **a, b**

Mass, m kg	Frequency	F.D.
$2 < m \leqslant 3$	52	52
$3 < m \leqslant 3.25$	34	136
$3.25 < m \leqslant 3.5$	30	120
$3.5 < m \leqslant 4$	60	120
$4 < m \leqslant 4.75$	24	32



c 0.83 **d** 3.37 kg

Multiple choice 5

1 A **2** D **3** C **4** C **5** D
6 C **7** B **8** D **9** A **10** D

Self-assessment 5

1 **a** 4 girls **b** 40 cars

2

Number of men, m	Number of humous tubs, h	Time, t min
2	120	5
4	480	10
6	1080	15
m	h	$h \div (12m)$
$h \div (12t)$	h	t
m	$12mt$	t

3 **a** $\frac{5}{9}$ **b** $\frac{37}{990}$ **c** $\frac{2908}{4995}$ **d** $\frac{35\,117}{33\,300}$

4 $\frac{263}{329}$

5 **a** $x^2 + 7x + 10$ **b** $y^2 + 2y - 99$ **c** $9 - 6p + p^2$

6 **a** $15x^2 + x - 6$ **b** $a^2 - 6ab + 9b^2$

7 **a** $x(x - 3)$ **b** $(x - 7)(x + 7)$
c $(x + 1)(x + 2)$ **d** $(x - 8)(x + 1)$

8 **a** $(x - 7)(x - 3)$ **b** $3p^2(p + 2)(p - 2)$

9 **a** $x = -3$ or -4 **b** $x = -2$ or 3
c $x = 0$ or 3 **d** $x = 2$ or 10

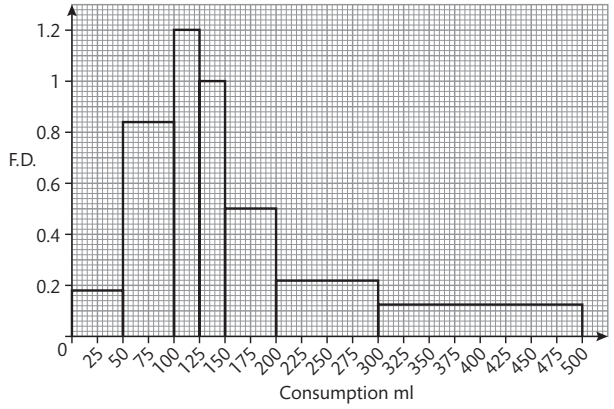
10 **b** $x = 6$

11 **a** $y = 6$ **b** $y = 2x + 1$ **c** $y = 2 - x$

12 **d** Translation $\begin{pmatrix}10\\10\end{pmatrix}$ **e** $x + y = 14$
f 180° **g** (11, 3)
i (15, 15), $\frac{1}{3}$

13

Consumption, m, ml	Frequency
$0 < m \leqslant 50$	9
$50 < m \leqslant 100$	42
$100 < m \leqslant 125$	30
$125 < m \leqslant 150$	25
$150 < m \leqslant 200$	25
$200 < m \leqslant 300$	22
$300 < m \leqslant 500$	27



c 134 ml **d** 50.1% **e** 171.5 ml

Exercise 119

1 \$270 **2** \$285.60 **3** \$234 **4** \$208.80
5 \$319.20 **6** \$338.40 **7** \$18.38 **8** \$14 per hour

Exercise 119*

1 **a** \$549 **b** 8 hours
2 **a** \$609
b 12 hours
3 \$9
4 \$10/hour
5 **a** \$6 240 000 **b** \$11.90

6 **a** \$2307.69 per week **b** \$0.23 per minute
7 B better than A in year 3
8 B better than A by year 4

Exercise 120

1 **a** Aus \$277.50 **b** R\$457.50
c CNY 1707.50 **d** €185
2 **a** JMD \$39 750 **b** MXN 19 290
c £960 **d** €1110
3 **a** \$300 **b** \$900
4 **a** \$80 **b** \$2500
5 \$37 736
6 \$77 761
7 Jamaica: \$500 000, Mexico: \$510 000 so Jamaican house cheaper by \$10 000
8 UK: \$300 000, France: \$290 000 so French house cheaper by \$10 000

Exercise 120*

1 **a** £24.15 **b** €27.92
c Aus \$41.89 **d** R\$69.06
2 **a** JMD\$3879.94 **b** £93.70
c MXN1882.87 **d** €108.35
3 41.4 million
4 R\$1 648 649
5 MXN 1941, R\$414
6 CNY6209.08, €1345.45, £1745.45
7 JMD 4 240 000 = U\$ 160 000
8 \$15 000

Exercise 121 (Revision)

1 \$555 Australian **2** £283.78
3 €27.92 **4** Jamaican \$206.07
5 Jam\$53 000, Pesos 25 720, Reais 10 980
6 £13 663 **7** \$2990
8 Gerard: €865.38/week, Marcelle: €860/week. Gerard by €5.38/week.

Exercise 121* (Revision)

1 2533.33 Ringitts and 6504.76 Yuan
2 4160 Rupees and 1438 Yuan
3 $x = 47\,619$ **4** $y = 10\,841\,270$
5 $y = 20$ **6** $y = 14$
7 \$2.40 **8** €3.69

Exercise 122

1 **a** $y = 5x$ **b** 30 **c** 5
2 **a** d = 4t **b** 60 **c** 45
3 **a** $e = \frac{M}{20}$ **b** 5 m **c** 120 kg
4 1950 sales
5 Yes, as 210 people would turn up to swim

Exercise 122*

1 **a** $v = 9.8t$ **b** 49 m/s **c** 2.5 s
2 **a** $c = \frac{m}{3}$ **b** £2.50 **c** 600 g
3 **a** $d = 150m$ **b** 1500 km **c** 266.7 g
4 **a** $m = 6.5n$ **b** 975 kg **c** 1540 approx.
5 **a** $h = \frac{3y}{2}$ **b** 0.75 m **c** 4 months

Exercise 123

1 **a** $y = 4x^2$ **b** 144 **c** 4
2 **a** $p = 2q^2$ **b** 18 **c** 7
3 **a** $v = 2w^3$ **b** 54 **c** 4
4 **a** $m = 10\sqrt{\text{n}}$ **b** 20 **c** 25
5 **a** $y = 5t^2$ **b** 45 m **c** $\sqrt{20} \simeq 4.5\,s$
6 **a** $\text{P} = \frac{h^3}{20}$ **b** £86.40 **c** 8 cm

Exercise 123*

1

g	2	4	6
f	12	48	108

2

n	1	2	5
m	4	32	500

3 **a** $R = \left(\frac{5}{256}\right)s^2$ **b** 113 km/h
4 **a** $\text{H} = 1.53\sqrt[3]{y}$ **b** 2 years old
5 $x = 10\sqrt{2}$
6 79.4 cm

Exercise 124

1 **a** $y = \frac{12}{x}$ **b** $y = 6$ **c** $x = 4$
2 **a** $d = \frac{250}{\text{t}}$ **b** $d = 125$ **c** $t = 5$
3 **a** $m = \frac{36}{n^2}$ **b** $m = 9$ **c** $n = 6$
4 **a** $V = \frac{100}{w^3}$ **b** $V = 100$ **c** $w = 5$
5 **a** $I = \frac{4 \times 105}{d^2}$ **b** 0.1 candle power
6 **a** $I = \frac{1}{4d^2}$ **b** 25 days

Exercise 124*

1

b	2	5	10
a	50	8	2

2 **a** $r \propto \frac{1}{t}$, in fact $r = \frac{20}{t}$

t	1	4	5	10
r	20	5	4	2

3 **a** $R = \frac{2}{r^2}$ **b** $\frac{2}{9}$ ohm
4 **a** $C = \frac{5000}{t}$, **b** £277.78 **c** 12.5 °C
5 **a**

Day	***N***	***T***
Mon	400	25
Tues	447	20
Wed	500	16

b 407 approx.
6 4.5 s

Exercise 125 (Revision)

1 **a** $y = 6x$ **b** $y = 42$ **c** $x = 11$
2 **a** $p = 5q^2$ **b** $p = 500$ **c** $q = 11$
3 **a** $c = \frac{3}{4}a^2$ **b** \$675 **c** 28.3 m^2
4

a	1	2	4	**8**
t	80	**40**	**20**	10

5 $a = 20b$

b	10	15	30
a	200	300	600

6 **a** $d = 5t^2$ **b** $d = 20\,\text{m}$ **c** $t \approx 4.24\,\text{s}$
7 **a** $C = 1.5d$ **b** \$97.50 **c** 53.3 cm
8 **a** $p = 1.5n^2$ **b** $p = \$216$ **c** $n = 20$

Exercise 125* (Revision)

1 **a** $y^2 = 50z^2$ **b** 56.6 **c** 5.85
2 **a** $m = \frac{8839}{\sqrt{n}}$ **b** 3.23×10^5 **c** 7.81×10^{-5}
3 1500 m

4

x	0.25	1	4	25
y	**20**	10	5	**2**

5

b	125	8	1
a	**2**	5	10

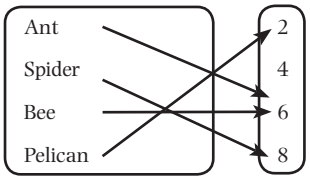
6 **a** $e = 0.5v^2$ **b** $e = 1250$ kJ **c** $v = 1414$ m/s

7 **a** $n = \frac{250}{t^2}$ **b** $n = 62.5$ **c** $t = 15.8$ yrs

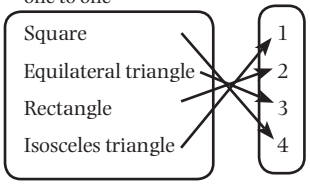
8 **a** $v = 3.1\sqrt{d}$ **b** **(i)** $v = 21.9$ m/s **(ii)** $v = 98.0$ m/s
c 0.104 m **d** $d = 5011$ m

Exercise 126

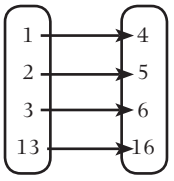
1 many to one



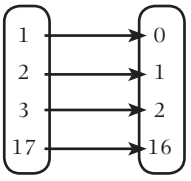
2 one to one



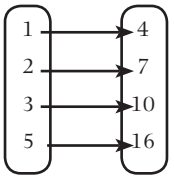
3



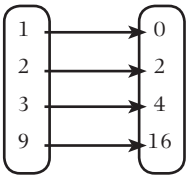
4



5



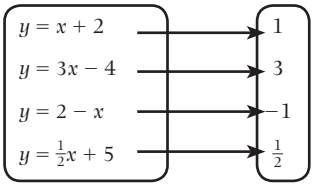
6



7 $a = 5, b = 4$, add 3 **8** $a = 7, b = 5$, add 5
9 $a = 1, b = 9$, subtract 4 **10** $a = 11, b = 13$, subtract 2
11 $a = -2, b = -1, c = 0, d = 1$ **12** $a = 4, b = 3, c = 2, d = 1$
13 $a = 5, b = 2, c = 1, d = 2$ **14** $a = -1, b = 2, c = 3, d = 2$

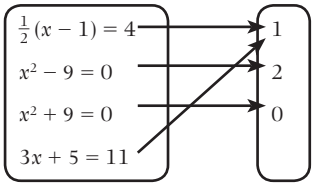
Exercise 126*

1



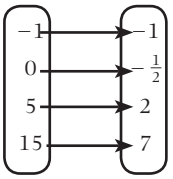
One to one

2

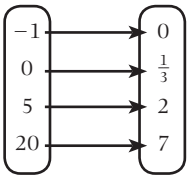


Many to one

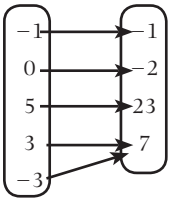
3



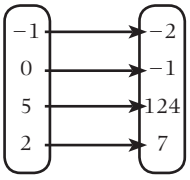
4



5



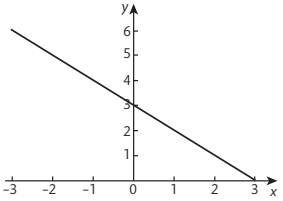
6

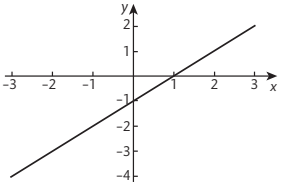


7 $a = 4, b = 5, x \to 2x$ **8** $a = 3, b = 21, x \to 3x$
9 $a = 1, b = 18, x \to \frac{x}{3}$ **10** $a = -2, b = 0, x \to \frac{x}{2}$
11 $a = 0, b = 3, c = 1, d = 0$ **12** $a = 0, b = 0, c = 2, d = 0$
13 $a = 0, b = \pm 1, c = \pm 2, d = \pm 3$
14 $a = \pm 6, b = \pm 3, c = \pm 2, d = \pm 1.5$

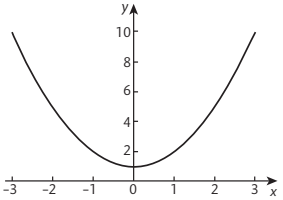
Exercise 127

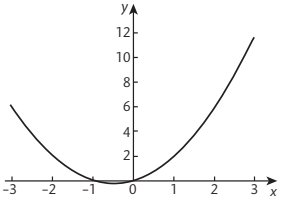
1 Many to one, function **2** Many to many, not a function
3 Function **4** Function





5 Function **6** Function

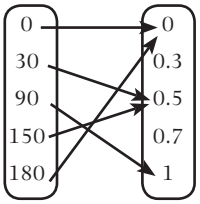




7 Function, any vertical line intersects at one point
8 Function, any vertical line intersects at one point
9 Not a function, most vertical lines intersect at two points
10 Not a function, any vertical line intersects at two points

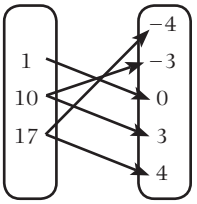
Exercise 127*

1



$x \to \sin x°$
Many to one
Function

2



$x \to \sqrt{(x - 1)}$
One to many
Not a function

3 Function **4** Function
5 Function **6** Not a function
7 Not a function, most vertical lines intersect at two points
8 Function, any vertical line intersects at no more than one point

Exercise 128

1 **a** 5 **b** -5 **c** 2 **d** 1
2 **a** 4 **b** -11 **c** $-\frac{1}{2}$ **d** -2
3 **a** 8 **b** 3 **c** $1\frac{1}{4}$ **d** 0
4 **a** 9 **b** -26 **c** $1\frac{1}{8}$ **d** 1
5 **a** 2 **b** 0 **c** 10
6 **a** $\frac{1}{5}$ **b** -1 **c** $\frac{1}{(1+2a)}$
7 **a** $1\frac{1}{2}$ **b** $2\frac{1}{2}$ **c** $2-\frac{1}{y}$
8 3
9 3
10 3
11 **a** $1-2x$ **b** $2x+5$ **c** $2x+3$
12 **a** $4x+1$ **b** $8x-3$ **c** $8x-6$

Exercise 128*

1 **a** 12 **b** 7 **c** 8 **d** $8-2p$
2 **a** 11 **b** $2\frac{1}{4}$ **c** 3 **d** p^2-2p+3
3 **a** 0 **b** $1\frac{1}{4}$ **c** 0 **d** $p(p+2)$
4 **a** -6 **b** $2\frac{1}{8}$ **c** 2 **d** p^3+2
5 **a** $-\frac{1}{3}$ **b** $\frac{1}{5}$ **c** $-\frac{1}{195}$
6 **a** $\sqrt{8}$ **b** 0 **c** $2\sqrt{(a^2+a)}$
7 **a** -4 **b** $\frac{2}{3}$ **c** $\frac{(9y+2)}{(3y-4)}$
8 1
9 $-2, 3$
10 $\frac{1}{2}$
11 **a** $2+x$ **b** $2+2x$ **c** $2x-4$
12 **a** x^2+4x+5 **b** x^2+3 **c** x^2+1

Exercise 129

1 **a** $0 \leqslant y \leqslant 2$ **b** $-2 \leqslant y \leqslant 2$
2 **a** $1 \leqslant y \leqslant 4$ **b** $-2 \leqslant y \leqslant 4$
3 **a** $-1 \leqslant y \leqslant 0$ **b** $-2 \leqslant y \leqslant 0$
4 **a** $0 \leqslant y \leqslant 4$ **b** $0 \leqslant y \leqslant 8$
5 **a** $-0.5 \leqslant y \leqslant 1.5$ **b** $-4.5 \leqslant y \leqslant 5.5$

Exercise 129*

1 **a** $\{8, 5, 2, -1\}$ **b** $\{\mathbb{R}\}$
2 **a** $\{0, 0, 8, 24\}$ **b** $\{y: y \geqslant 0, y \text{ a real number}\}$
3 **a** $\{27, 11, 3, 3\}$ **b** $\{x: x \geqslant 2\ x \in \mathbb{R}\}$
4 **a** $\{-10, 0, 10, 68\}$ **b** $\{y: y \geqslant 2, y \text{ a real number}\}$
5 **a** $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right\}$ **b** $\{x: 0 < x \leqslant 1\}$

Exercise 130

1 -1 **2** 1
3 $\{x: x < 2, x \text{ a real number}\}$
4 $\{x: x > 2, x \text{ a real number}\}$
5 0 **6** 0 **7** None
8 None
9 $\{x: x > -2, x < 2, x \text{ a real number}\}$
10 $\{x: x > -3, x < 3, x \text{ a real number}\}$

Exercise 130*

1 $\frac{1}{2}$ **2** $\frac{3}{4}$
3 $\{x: x > 9, x \text{ a real number}\}$ **4** $\{x: x < -4, x \text{ a real number}\}$
5 -1 **6** 1
7 ± 1 **8** None
9 $\{x: x \leqslant -2, x \text{ a real number}\}$ **10** $\{x: x \geqslant 2, x \text{ a real number}\}$

Exercise 131

1 $fg(3) = 6, gf(3) = 6$ **2** $fg(1) = 9, gf(1) = 3$
3 $fg(4) = 5, gf(4) = \frac{4}{5}$
4 **a** $x-1$ **b** $x-1$ **c** $x-8$ **d** $x+6$
5 **a** $2x+4$ **b** $2x+2$ **c** $4x$ **d** $x+4$
6 **a** $(x+2)^2$ **b** x^2+2 **c** x^4 **d** $x+4$
7 **a** x **b** x **c** $x-12$ **d** $x+12$
8 **a** 7 **b** 6

Exercise 131*

1 $fg(-3) = 19, gf(-3) = 8$ **2** $fg(2) = 82, gf(2) = 36$
3 $fg(-3) = -4\frac{1}{2}, gf(-3) = -\frac{3}{7}$
4 **a** $x-2$ **b** $x-4$ **c** $\frac{(x-12)}{4}$ **d** $4x$
5 **a** $2(x-2)^2$ **b** $2x^2-2$ **c** $8x^4$ **d** $x-4$
6 **a** $\frac{1}{x}$ **b** $\frac{1}{(x-2)}+2$ **c** $\frac{(x-2)}{(5-2x)}$ **d** $x+4$
7 **a** $4\sqrt{\left(\frac{x}{4}+4\right)}$ **b** $\sqrt{(x+4)}$
c $16x$ **d** $\sqrt{4+\frac{1}{4}\sqrt{4+\frac{x}{4}}}$
8 **a** -12 b 12
9 **a** $\{x: x \neq 5, x \text{ a real number}\}$ **b** $\{x: x \neq \pm 2, x \text{ a real number}\}$
10 **a** $\{x: x \geqslant -1, x \text{ a real number}\}$ **b** $\{x: x \geqslant -2, x \text{ a real number}\}$

Exercise 132

1 7 **2** 4 **3** $\frac{(x-4)}{6}$ **4** $27-3x$
5 $\frac{x}{3}+6$ **6** $\frac{1}{3}\left(\frac{1}{x}-4\right)$ **7** $\frac{3}{(4-x)}$ **8** $\sqrt{(x-7)}$
9 **a** 4 **b** $\frac{5}{2}$ **c** 1
10 $x = -5$

Exercise 132*

1 17 **2** 3.2 **3** $\frac{4}{3}-\frac{x}{24}$
4 $\frac{2-3}{2x}$ **5** $\frac{7}{(4-x)}$ **6** $\sqrt{(x^2-7)}$
7 $\sqrt{\frac{(x-16)}{2}}$ **8** $\frac{(4x-3)}{(x+2)}$
9 **a** 4 **b** 7 **c** 0
10 $x = 2$ **11** $x = 1$ or $x = 2$

Exercise 133 (Revision)

1 **a** $x = 6\ y = 7, \quad y = x + 1$ **b** $x = 4\ y = 0, \quad y = 2x$
2 $-1 \rightarrow -3$ {range $-3, -1, 1, 3$}
$0 \rightarrow -1$
$1 \rightarrow 1$
$2 \rightarrow 3$
3 Many possible answers
4 mango $\rightarrow$ 5, coconut $\rightarrow$ 7, yam $\rightarrow$ 3, pumpkin $\rightarrow$ 7, guava $\rightarrow$ 5, yes it is a function
5 **a** function, for each x value there is only one y value
b not function
6 **a** 13 **b** -2 **c** 7
7 **a** $x = \frac{-1}{2}$ **b** $x = \frac{5}{4}$
8 **a** $5x-1$ **b** $5x+3$
9 **a** $x = 1$ **b** $x = \frac{1}{2}$ **c** $x < -1$ **d** $x < 2$
10 **a** $f(x) \in \mathbb{R}$ **b** $g(x) \geqslant 1$ **c** $h(x) \geqslant 0$ **d** $f(x) \in \mathbb{R}$
11 **a** **(i)** $1+\frac{1}{x^2}$ **(ii)** $\frac{1}{1+x^2}$
b **(i)** $x = 0$ **(ii)** none if domain is real
c x
12 **a** $p^{-1}(x) = \frac{1}{2}\left(\frac{x}{4}-3\right)$ **b** $q^{-1}(x) = 7-x$
c $r^{-1}(x) = \frac{1}{x}-3$
13 **a** x **b** $g(x)$ is the inverse of $f(x)$
c $\sqrt{3}$

Exercise 133* (Revision)

1 **a** $x = 4\ y = 4,\quad y = x^2$ **b** $x = 2\ y = 9,\quad y = 2x + 1$

2 $-2 \rightarrow 4$ it is a function {range $-4, -2, 4$}
$-1 \rightarrow -2$
$0 \rightarrow -4$
$1 \rightarrow -2$
$2 \rightarrow 4$

3 Many possible answers

4 tetrahedron → 4, cube → 6, octahedron → 8, triangular prism → 5, dodecahedron → 12, it is a function, each value of x has a unique value of y, this is always true

5 **a** not function
b function, each value of x has a unique value of y

6 **a** ± 4 **b** ± 3 **c** 0

7 **a** $3, -2$ **b** $8, -7$

8 **a** $4 - 2x$ **b** $7 - 2x$

9 **a** $x = \frac{4}{3}$ **b** $x = -2$ **c** $x < -\frac{2}{5}$ **d** $-3 < x < 3$

10 **a** $f(x) \geqslant 3$ **b** $g(x) \geqslant 0$ **c** $h(x) \in \mathbb{R} \geqslant 0$ **d** $f(x) \in \mathbb{R}$

11 **a** **(i)** $\frac{1}{(x-8)^3}$ **(ii)** $\frac{1}{x^3 - 8}$
b **(i)** $x = 8$ **(ii)** $x = \sqrt[3]{8}$
c $\frac{x-8}{65-8x}$

12 **a** $p^{-1}(x) = \frac{1}{2}\left(1 - \frac{x}{4}\right)$ **b** $q^{-1}(x) = 4 - \frac{3}{2-x}$
c $r^{-1}(x) = \frac{1}{2}(x^2 + 3)$
d $s^{-1}(x) = \sqrt{x} + 2$

13 **a** x **b** $p(x)$ is the inverse of $q(x)$, $p(x) = q^{-1}(x)$
c $\sqrt{7}$

Exercise 134

1 100° **2** 30° **3** 45° **4** 10°
5 280° **6** 140° **7** 60° **8** 60°
9 ADB and BCA are angles in the same segment
10 ABC + ADC = 180°

Exercise 134*

1 140° **2** 60° **3** 110° **4** 54°
5 76° **6** 22° **7** $3x$ **8** Correct proof
9 BEC = CDB (angles in the same segment), so CEA = BDA
10 Correct proof

Exercise 135

1 70° **2** 30° **3** 80°
4 30° **5** 100°
6 **a** 90° **b** 60° **c** 60° **d** 60°
7 **a** NTM = NPT (Alternate segment)
b PLT = NTM (Corresponding angles)
8 **a** <ATC = <ABT (alternate segment)
b <ABT = <BTD (AB parallel to CD)

Exercise 135*

1 65° **2** 140°
3 ATE = 55° (alternate segment)
TBC = 125° (angles on straight line)
BTC = 35° (angle sum of triangle)
ATB = 90° (angles on straight line)
AB is a diameter
4 **a** 56° **b** 68°
5 **a** 55° **b** 35° **c** CBD = 55°, BCD = 35°; isosceles
6 20°
7 **a** $A\hat{C}G = A\hat{B}F$ (angles of chords in the same segment), $A\hat{G}C = 90°$, hence $A\hat{C}G = 180° - 90° - 75° = 15°$.
b Since $F\hat{C}G = A\hat{C}G$ and $F\hat{B}G = A\hat{B}F$, from previous result $F\hat{C}G = F\hat{B}G$, so angles in same segment and points CFGB concyclic
8 **a** EOC = $2x°$ (angle at centre twice angle at circumference)
CAE = $(180 - 2x)°$, AEB = $x°$, isosceles
b true by alternate segment theorem

Exercise 136

1 22.5 cm **2** 10.5 cm **3** 18 cm
4 4 cm **5** 12 cm **6** 9 cm

Exercise 136*

1 3 cm **2** $x^2 - 22x + 120 = 0$, $x = 10$ or 12 cm²
3 8 cm **4** 24 cm **5** 4.5 cm
6 60 mm, 40 mm

Exercise 137

1 18.8 cm, 28.3 cm² **2** 33.6 cm, 58.9 cm²
3 22.3 cm, 30.3 cm² **4** 49.1 cm, 146 cm²
5 50.8 cm, 117 cm² **6** 46.8 cm, 39.5 cm²
7 37.7 cm, 37.7 cm²

	Radius in cm	Circumference in cm	Area in cm²
8	0.955	6	2.86
9	2.11	13.3	14
10	8.28	52	215
11	5.17	32.5	84

12 2.07 km
13 7.54 km
14 3.77 km

Exercise 137*

1 20.5 cm, 25.1 cm² **2** 47.0, 115
3 43.7 cm, 99.0 cm² **4** 66.8 cm, 175 cm²
5 37.7 cm, 56.5 cm² **6** 29.7 cm, 63.3 cm²
7 $r = 3.19$ cm, $P = 11.4$ cm **8** 16.0 m
9 546 m² **10** 2 cm
11 **a** 40 100 km **b** 464 m/s
12 6.28 **13** $r = 1.79$ cm, $A = 7.55$ cm²

Exercise 138

1 8.62 cm **2** 25.6 cm **3** 38.4 cm **4** 63.6 cm
5 34.4° **6** 115° **7** 14.3 cm **8** 10.6 cm

Exercise 138*

1 11.0 cm **2** 38.3 cm **3** 25.1°
4 121° **5** 13.4 cm **6** 117 cm
7 33.0 cm **8** 15.5 cm **9** 6.06 cm

Exercise 139

1 12.6 cm² **2** 61.4 cm² **3** 170 cm² **4** 11.9 cm²
5 76.4° **6** 129° **7** 5.86 cm **8** 8.50 cm

Exercise 139*

1 15.8 cm² **2** 625 cm² **3** 53.3°
4 103° **5** 4.88 cm **6** 19.7 cm
7 11.5 cm **8** 5.08 cm **9** 7.31 cm²
10 12.4 cm² **11** 2.58 cm²

Exercise 140 (Revision)

1 62° **2** 55° **3** 124° **4** 132°
5 54° **6** 66° **7** 30° **8** 276°
9 12 **10** 3 **11** 8 **12** 8
13 4 **14** 3 **15** 5 **16** 4
17 **a** 80° **b** 100° **c** 50° **d** 50°
18 **a** A = 21.86, P = 27.4 **b** A = 1.93, P = 10.7
c A = 3.43, P = 12.6 **d** A = 17.7, P = 22.3
19 **a** A = 19.5, P = 20.9 **b** A = 43.3, P = 29.6
c A = 6.98, P = 11.0 **d** A = 13.1, P = 28.2

Exercise 140* (Revision)

1 20° **2** 120° **3** 65° **4** 45°

5 **a** 55° **b** 35°
c ∠TDC = 35° (Angles in the same segment)
∴ ∠EDC = 90° and EC is the diameter

6 **a** 40° **b** 50°
c ∠ZXT = ∠WVT (Angle in alternate segment)
∴ XZ is parallel to WV (Alternate angles)

7 6 **8** 3 **9** 8 **10** 5
11 4 **12** 4 **13** $\sqrt{2}$ **14** 4.85

15 **a** A = 96, P = 49.1 **b** A = 113, P = 49.7
c A = 5.37, P = 25.7 **d** A = 49.1, P = 28.6

16 **a** $x = 45.8°$, A = 10 **b** $x = 251°$, P = 39

17 **a** Since the chords are in the same segment $C\hat{A}Y = C\hat{B}Y$ and $B\hat{A}Y = B\hat{C}Y$. Since the line AY is the bisector of angle CÂB, $C\hat{A}Y = B\hat{A}Y$, hence $C\hat{B}Y = B\hat{C}Y$ and the triangle BCY is isosceles
b The angle $A\hat{B}Y = 180 - B\hat{A}Y - A\hat{Y}B$.
The angle $B\hat{X}Y = 180 - C\hat{B}Y - A\hat{Y}B$. Since in the same segment, and the fact that AY bisects $C\hat{A}B$, $C\hat{B}Y = B\hat{A}Y$, hence $A\hat{B}Y = B\hat{X}Y$

Exercise 141

1 $\mathbf{p}+\mathbf{q} = \begin{pmatrix}6\\8\end{pmatrix}$; $\mathbf{p}-\mathbf{q} = \begin{pmatrix}-2\\-2\end{pmatrix}$; $2\mathbf{p}+3\mathbf{q}\begin{pmatrix}16\\21\end{pmatrix}$

2 $\mathbf{u}+\mathbf{v}+\mathbf{w} = \begin{pmatrix}-1\\0\end{pmatrix}$; $\mathbf{u}+2\mathbf{v}-3\mathbf{w} = \begin{pmatrix}-13\\23\end{pmatrix}$; $3\mathbf{u}-2\mathbf{v}-\mathbf{w} = \begin{pmatrix}9\\5\end{pmatrix}$

3 $\begin{pmatrix}4\\6\end{pmatrix}$; $\mathbf{p}-\mathbf{q} = \begin{pmatrix}-2\\-2\end{pmatrix}$; $2\mathbf{p}+5\mathbf{q} = \begin{pmatrix}17\\24\end{pmatrix}$

4 $\mathbf{s}+\mathbf{t}+\mathbf{u} = \begin{pmatrix}7\\-5\end{pmatrix}$; $2\mathbf{s}-\mathbf{t}+2\mathbf{u} = \begin{pmatrix}8\\-19\end{pmatrix}$; $2\mathbf{u}-3\mathbf{s} = \begin{pmatrix}5\\-1\end{pmatrix}$

5 $\mathbf{v}+\mathbf{w} = \begin{pmatrix}4\\5\end{pmatrix}, \sqrt{41}$; $2\mathbf{v}-\mathbf{w} = \begin{pmatrix}5\\-2\end{pmatrix}, \sqrt{29}$; $\mathbf{v}-2\mathbf{w} = \begin{pmatrix}1\\-7\end{pmatrix}, \sqrt{50}$

6 $\mathbf{p}+\mathbf{q} = \begin{pmatrix}5\\4\end{pmatrix}, \sqrt{41}$

$3\mathbf{p}+\mathbf{q} = \begin{pmatrix}9\\2\end{pmatrix}, \sqrt{85}$

$\mathbf{p}-3\mathbf{q} = \begin{pmatrix}-7\\-16\end{pmatrix}, \sqrt{305}$

Exercise 141*

1 $\mathbf{p}+\mathbf{q} = \begin{pmatrix}5\\0\end{pmatrix}$, 5, 090°

$\mathbf{p}-\mathbf{q} = \begin{pmatrix}-1\\2\end{pmatrix}, \sqrt{5}$, 333°

$2\mathbf{p}-3\mathbf{q} = \begin{pmatrix}-5\\5\end{pmatrix}, \sqrt{50}$, 315°

2 $2(\mathbf{r}+\mathbf{s}) = \begin{pmatrix}10\\-4\end{pmatrix}, \sqrt{116}$, 112°

$3(\mathbf{r}-2\mathbf{s}) = \begin{pmatrix}-21\\-15\end{pmatrix}, \sqrt{666}$, 234°

$(4\mathbf{r}-6\mathbf{s})\sin 30° = \begin{pmatrix}-10\\-9\end{pmatrix}, \sqrt{181}$, 228°

3 $m = -1, n = -2$

4 $m = -4\frac{1}{2}, n = 2\frac{2}{3}$

5 **a** $\begin{pmatrix}-5\\8.7\end{pmatrix}$ km **b** $\begin{pmatrix}-8.5\\-3.1\end{pmatrix}$ km

6 **a** $m = 1, n = 3$ **b** $p = 3, q = 10$

Exercise 142

1 **a** 2×1 **b** 1×2

2 $\begin{pmatrix}4\\6\end{pmatrix}$

3 $\begin{pmatrix}3&4\\-4&4\\6&18\end{pmatrix}$ **4** $\begin{pmatrix}-11&17\\34&22\end{pmatrix}$

5 $\begin{pmatrix}2\\2\end{pmatrix}$ **6** $\begin{pmatrix}3&6\\-8&-4\\12&4\end{pmatrix}$

7 Not possible **8** $\begin{pmatrix}10\\14\end{pmatrix}$

Exercise 142*

1 $p = 2$ **2** $r = 2$ **3** $t = -1$

4 $X = (-4\ -6)$ **5** $a = 12, b = 10$

6 **a** $a = 10, b = 2$ **b** $c = 7, d = 5$ **c** St Mary's won

Exercise 143

1 $\begin{pmatrix}1\\2\end{pmatrix}$ **2** $2\times1, \begin{pmatrix}1\\2\end{pmatrix}$ **3** $\begin{pmatrix}0&1&3\\2&2&0\end{pmatrix}$

4 $2\times2, \begin{pmatrix}10&10\\2&6\end{pmatrix}$ **5** $\begin{pmatrix}8&2\\15&-8\end{pmatrix}$ **6** $\begin{pmatrix}3&-4&1\\-13&16&-3\\-9&8&1\end{pmatrix}$

7 $\begin{pmatrix}13&24\\18&61\end{pmatrix}$ **8** $3\times3, \begin{pmatrix}3&-4&1\\-16&16&-3\\-9&8&1\end{pmatrix}$

Exercise 143*

1 $x = 10, y = 30$ **2** $x = 3, y = 10$

3 $x = 2, y = 1$ **4** $x = 8, y = 4$

5 $x = 10, y = 60$ **6** $x = -1, y = 14$

7 $m = -2$

8 **a** 3×1 **b** $x = 4$
c Jamaica 18, Mexico 23, Cuba 23

Exercise 144

1 $\frac{1}{7}\begin{pmatrix}1&1\\-4&3\end{pmatrix}$ **2** $\frac{1}{6}\begin{pmatrix}5&1\\-1&1\end{pmatrix}$

3 $-\frac{1}{4}\begin{pmatrix}-2&-1\\2&3\end{pmatrix}$ **4** $\frac{1}{13}\begin{pmatrix}1&-7\\1&6\end{pmatrix}$

5 $\frac{1}{12}\begin{pmatrix}-1&1\\-7&5\end{pmatrix}$ **6** $\frac{1}{2}\begin{pmatrix}-1&0\\-1&2\end{pmatrix}$

7 $-1\begin{pmatrix}2&1\\5&2\end{pmatrix}$ **8** $\frac{1}{-6}\begin{pmatrix}3&1\\0&-2\end{pmatrix}$

9 $\frac{1}{222}\begin{pmatrix}9&12\\-11&10\end{pmatrix}$ **10** $\frac{1}{1000}\begin{pmatrix}30&10\\-40&20\end{pmatrix}$

Exercise 144*

1 $x = 1.5$ **2** $y = 16$

3 Reflection in x axis **4** Enlargement SF = $\frac{1}{5}$ about 0

5 $\begin{pmatrix}3&1\\5&2\end{pmatrix}$ **6** $\mathbf{P} = \begin{pmatrix}2&-1\\-5&3\end{pmatrix}$

7 $\begin{pmatrix}-7&-2\\-4&-1\end{pmatrix}$ **8** $\mathbf{Y} = \begin{pmatrix}9&2\\4&1\end{pmatrix}$

9 Correct proof

10 Correct proof

Exercise 145 (Revision)

1 $\mathbf{p}+\mathbf{q} = \begin{pmatrix}1\\5\end{pmatrix}, \sqrt{26}$; $\mathbf{p}-\mathbf{q} = \begin{pmatrix}5\\3\end{pmatrix}, \sqrt{34}$; $2\mathbf{p}-2\mathbf{q} = \begin{pmatrix}13\\10\end{pmatrix}, \sqrt{269}$

2 $\mathbf{r}+\mathbf{s} = \begin{pmatrix}5\\-1\end{pmatrix}, \sqrt{26}$; $\mathbf{r}-\mathbf{s} = \begin{pmatrix}-1\\-9\end{pmatrix}, \sqrt{82}$; $3\mathbf{s}-2\mathbf{r} = \begin{pmatrix}5\\22\end{pmatrix}, \sqrt{509}$

3 **a** $\begin{pmatrix}4\\1\end{pmatrix}$ **b** $\begin{pmatrix}6\\-4\end{pmatrix}$ **c** $\sqrt{29}$ **d** $v = 1, w = 2$

4 **a** $n = 1, m = -3$ **5** $m = -2, n = 5$

6 **a** $\begin{pmatrix}2&-3\\0&2\end{pmatrix}$ **b** $\begin{pmatrix}2&5\\6&0\end{pmatrix}$ **c** $\begin{pmatrix}8&-28\\-12&12\end{pmatrix}$ **d** $\begin{pmatrix}4&34\\30&-6\end{pmatrix}$

7 **a** $\begin{pmatrix}-1&5\\2&-7\end{pmatrix}$ **b** $\begin{pmatrix}-9&-3\\2&1\end{pmatrix}$ **c** $\begin{pmatrix}3&0\\0&3\end{pmatrix}$ **d** $\begin{pmatrix}1&4\\0&1\end{pmatrix}$

Exercise 145* (Revision)

1 **a** $\begin{pmatrix}-1\\5\end{pmatrix}, \sqrt{26}$ **b** $\begin{pmatrix}3\\-1\end{pmatrix}, \sqrt{10}$ **c** $\begin{pmatrix}-4\\13\end{pmatrix}, \sqrt{185}$ **d** $\begin{pmatrix}7\\0\end{pmatrix}$, 7

2 **a** $\begin{pmatrix}5\\2\end{pmatrix}, \sqrt{29}$ **b** $\begin{pmatrix}1\\0\end{pmatrix}$, 1 **c** $\begin{pmatrix}12\\1\end{pmatrix}, \sqrt{145}$ **d** $\begin{pmatrix}-2\\-42\end{pmatrix}, 2\sqrt{442}$

3 **a** $\begin{pmatrix}-7\\19\end{pmatrix}$ **b** $\sqrt{140}$ 340° **c** 5.1 km/h

4 **a** **(i)** $\begin{pmatrix}6\\-3\end{pmatrix}$ **(ii)** $\begin{pmatrix}3\\9\end{pmatrix}$ **b** $m = 1, n = 3$
c $r = 2, s = 9$ **d** $v = 2, u = -1$

5 $n = -1, m = 2$

6 **a** $\begin{pmatrix}9&0\\-31&-28\end{pmatrix}$ **b** $\begin{pmatrix}-\frac{8}{21}&-\frac{1}{7}\\\frac{1}{42}&\frac{1}{14}\end{pmatrix}$

c $-\frac{1}{42}\begin{pmatrix}-6&-9\\8&19\end{pmatrix}$ **d** $\begin{pmatrix}41&15\\75&86\end{pmatrix}$

7 **a** $\begin{pmatrix} -x & 1-y & -3 \\ 2x & -1+2y & 4 \\ 5x & 3+5y & -1 \end{pmatrix}$ **b** $\begin{pmatrix} -7 & -8 \\ 3+x-y & 5-x+2y \end{pmatrix}$

c $x = -2, y = 3$

Multiple choice 6

1	C	**2**	D	**3**	B	**4**	C	**5**	A
6	A	**7**	D	**8**	A	**9**	B	**10**	D

Self-assessment 6

1 \$546

2 ¥4098 : R1596 : R9222

3 **a** $y = 8x$ **b** $y = 80$ **c** $x = 5$

4 **a** $y = \frac{1}{64}x^3$ **b** $y = 512$

5 **a** $y = \frac{48}{x}$ **b** $y = 6$ **c** $x = 4$

6 **a** $N = \frac{9000}{d^2}$ **b** $N = 2250$ **c** $d = 3$

7 **a** $A = 15h^2$ **b** $A = 135\,\text{m}^2$ **c** $h = 6\,\text{m}$

8 **a** $x = 60°, y = 60°, z = 55°$ **b** $x = 40°, y = 70°, z = 40°$

9 **a** BP = 1.6 cm **b** CQ = 4 cm

10 20.1 cm, 30.2 cm^2

11 **a** 28.6° **b** 3.02 cm

12 {22, 17, 12, 7, 2}

13 **a** 263 **b** 598 **c** $3 - 2x(3 - x)$

d $(3 - 2x)(6 - 2x)$

14 **a** $\frac{1-x}{3}$ **b** $\frac{1}{x}$

c x is equal to either of the answers below

$x = -2.30278, x = 1.30278$

d $\left(-1 - \frac{3}{x}\right)^2$

15 **a** 99 **b** 4 **c** $x = 20$

16 **a** $\begin{pmatrix} -1 \\ 5 \end{pmatrix}$ **b** $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ **c** $\sqrt{41}$

17 $a = 1$ $b = -3$

18 **a** $\begin{pmatrix} 2 & -8 \\ 4 & 2 \end{pmatrix}$ **b** $\begin{pmatrix} -7 & -8 \\ 4 & -7 \end{pmatrix}$ **c** $\frac{1}{9}\begin{pmatrix} 1 & 4 \\ -2 & 1 \end{pmatrix}$

Exercise 146

1	$\frac{1}{9}$	**2**	$\frac{1}{10}$	**3**	$\frac{1}{8}$	**4**	$\frac{1}{8}$	**5**	9
6	$\frac{1}{64}$	**7**	3	**8**	2	**9**	$\frac{1}{16}$	**10**	$\frac{1}{9}$
11	$\frac{1}{16}$	**12**	$\frac{1}{2}$	**13**	16	**14**	625	**15**	2.84
16	0.167	**17**	0.0123	**18**	46 700				
19	0.0370	**20**	0.111	**21**	64				
22	a	**23**	c^{-3}	**24**	e				
25	a^{-2}								

Exercise 146*

1	$\frac{1}{64}$	**2**	5	**3**	1	**4**	512
5	12.5	**6**	4	**7**	10	**8**	8
9	$\frac{1}{2}$	**10**	$-\frac{1}{36}$	**11**	0.364	**12**	1.92
13	0.001 37	**14**	a^2	**15**	$2c^{-4}$	**16**	$3a^{-2}$
17	$12a^{-1}$	**18**	a	**19**	c^{-1}	**20**	2
21	2	**22**	-3	**23**	$k = 2\frac{1}{3}$	**24**	a^{-3}
25	b	**26**	$2a^{-2}$	**27**	$5a^{-2}$	**28**	$\frac{1}{3}$
29	$-9a^6$	**30**	$\frac{b}{a}$				

Exercise 147

1	$\frac{1}{3}$	**2**	$\frac{1}{4}$	**3**	$\frac{1}{9}$	**4**	$\frac{1}{4}$
5	27	**6**	$\frac{1}{27}$	**7**	0.224	**8**	0.376
9	36.5	**10**	0.0540	**11**	b^{-1}	**12**	d
13	f^{-1}						

Exercise 147*

1	$\frac{1}{6}$	**2**	6	**3**	16	**4**	$\frac{1}{16}$
5	1.29	**6**	0.894	**7**	2.85	**8**	0.351
9	a^{-2}	**10**	c^2	**11**	e	**12**	9
13	$\frac{1}{36}$						

Exercise 148 (Revision)

1	$\frac{1}{16}$	**2**	10	**3**	4	**4**	8	**5**	5
6	5	**7**	$\frac{1}{64}$	**8**	$\frac{1}{4}$	**9**	$\frac{1}{11}$	**10**	9

11 $\frac{1}{81}$ **12** 27 **13** $36^{-\frac{1}{2}} = \frac{1}{36^{\frac{1}{2}}} = \frac{1}{\sqrt{36}} = \frac{1}{6}$

14 $9^{\frac{3}{2}} = \left(9^{\frac{1}{2}}\right)^3 = \left(\sqrt{9}\right)^3 = 3^3 = 27$ **15** a^2 **16** a^4

17 d^{-2} **18** $b^{-\frac{3}{2}}$ **19** $b^{\frac{5}{2}}$ **20** c

Exercise 148* (Revision)

1	$\frac{1}{32}$	**2**	6	**3**	1	**4**	3
5	4	**6**	$\frac{1}{7}$	**7**	64	**8**	8
9	$\frac{1}{9}$	**10**	$\frac{1}{625}$				

11 $(0.125)^{-\frac{2}{3}} = \left(\frac{1}{8}\right)^{-\frac{2}{3}} = \left(\frac{8}{1}\right)^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$

12 $\left(4^{\frac{1}{3}}\right)^{-1\frac{1}{2}} = \left(4^{\frac{1}{3}}\right)^{-\frac{3}{2}} = 4^{-\frac{1}{2}} = \frac{1}{4^{\frac{1}{2}}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$

13	$3c$	**14**	$5b^{-2}$	**15**	a^2	**16**	$3c^{-2}$
17	$2a^{-1}$	**18**	$d^{\frac{7}{6}}$	**19**	$-9a^6$	**20**	2

Exercise 149

1	$x = -2$ or $x = -1$	**2**	$x = -3$ or $x = 2$	**3**	$x = -5$ or $x = -2$
4	$x = -3$ or $x = 5$	**5**	$x = 3$	**6**	$x = -6$ or $x = 2$
7	$x = -1$ or $x = 0$	**8**	$x = 0$ or $x = 4$	**9**	$x = \pm 2$
10	$x = -6$ or $x = 6$				

Exercise 149*

1	$x = -5$ or $x = -1$	**2**	$x = -1$ or $x = 4$	**3**	$x = -8$ or $x = -7$
4	$x = -5$ or $x = 9$	**5**	$x = 7$	**6**	$x = -5$ or $x = 8$
7	$x = 0$ or $x = 13$	**8**	$x = 0$ or $x = -17$	**9**	$x = \pm 9$
10	$x = -11$ or $x = 11$				

Exercise 150

1	$x = \pm\frac{7}{2}$	**2**	$x = \frac{\pm 9}{4}$	**3**	$x = -2$ or $x = 0$
4	$x = 0$ or $x = 1$	**5**	$x = 2$ or $x = 3$	**6**	$x = 0.5$ or $x = 2$
7	$x = -1.5$ or $x = -1$	**8**	$x = -2$ or $x = -1$	**9**	$x = -3$ or $x = 3$
10	$x = 0$ or $x = 2$	**11**	$x = -2$ or $x = -\frac{1}{3}$	**12**	$x = -\frac{1}{3}$ or $x = 2$
13	$x = -2$ or $x = 3$	**14**	$x = -2$ or $x = -\frac{2}{3}$	**15**	$x = -4$ or $x = \frac{2}{3}$

Exercise 150*

1	$x = \pm\frac{5}{7}$	**2**	$x = \pm\frac{8}{3}$	**3**	$x = -2$ or $x = 0$
4	$x = 0$ or $x = \frac{3}{2}$	**5**	$x = 1$ or $x = 2$	**6**	$x = 1.5$ or $x = 2$
7	$x = -9$ or $x = -\frac{4}{3}$	**8**	$x = -\frac{1}{3}$ or $x = 1.5$	**9**	$x = -\frac{1}{2}$ or $x = -\frac{1}{4}$
10	$x = \frac{2}{5}$ = or $x = 5$	**11**	$x = 0.8$ or $x = 1.5$	**12**	$x = -\frac{4}{3}$ or $x = 7$
13	$x = -4$ or $x = 4$	**14**	$x = 0$ or $x = 3$		
15	$x = -5$ (repeated root)	**16**	$x = 0.25$ or $x = 7$		
17	$x = \frac{2}{3}$ or $x = 1.5$	**18**	$x = \frac{5}{3}$ (repeated root)		

Exercise151

1	$x = -3.45$ or $x = 1.45$	**2**	$x = -1.65$ or $x = 3.65$
3	$x = -5.46$ or $x = 1.46$	**4**	$x = -8.16$ or $x = -1.84$
5	$x = -14.2$ or $x = 0.21$	**6**	$x = -1.53$ or $x = 21.5$
7	$x = -2.90$ or $x = 6.90$	**8**	$x = -7.04$ or $x = 17.0$
9	$x = -3.56$ or $x = 0.561$	**10**	$x = -0.541$ or $x = 5.54$

Exercise 151*

1	$x = 0.171$ or $x = 5.83$	**2**	$x = 0.190$ or $x = 15.8$
3	$x = -7.58$ or $x = 1.58$	**4**	$x = -1.69$ or $x = 7.69$
5	$x = -0.41$ or $x = 2.41$	**6**	$x = 0.258$ or $x = 7.74$
7	$x = -1.00$ or $x = 3.50$	**8**	$x = -1.86$ or $x = -0.537$
9	$x = 0.27$ or $x = -2.77$	**10**	$x = -0.522$ or $x = 1.09$

Exercise 152

1	$x = -3.45$ or $x = 1.45$	**2**	$x = -1.65$ or $x = 3.65$
3	$x = -5.46$ or $x = 1.46$	**4**	$x = 1.84$ or $x = 8.16$

5 $x = -14.2$ or $x = 0.211$
6 $x = 0.258$ or $x = 7.74$
7 $x = -2.90$ or $x = 6.90$
8 $x = -7.04$ or $x = 17.0$
9 $x = -3.56$ or $x = 0.561$
10 $x = 0.522$ or $x = 1.09$
11 $x = -3.37$ or $x = 2.37$
12 $x = -1.83$ or $x = 3.83$

Exercise 152*

1 $x = 0.171$ or $x = 5.83$
2 $x = 0.190$ or $x = 15.8$
3 $x = -7.58$ or $x = 1.58$
4 $x = -12.7$ or $x = -0.315$
5 $x = 1.59$ or $x = 4.41$
6 $x = -\frac{1}{3}$ or $x = 2$
7 $x = -1.69$ or $x = 7.69$
8 $x = -0.414$ or $x = 2.41$
9 $x = 0.258$ or $x = 7.74$
10 $x = 3.11$ or $x = -1.61$
11 $x = 0.105$ or $x = 5.37$
12 $x = -2.16$ or $x = 1.16$

Exercise 153

1 1 **2** 2 **3** 0 **4** 1 **5** 0
6 1 **7** 2 **8** 2 **9** 0 **10** 2

Exercise 153*

1 2 **2** 0 **3** 2 **4** 4 **5** 1
6 0 **7** 2 **8** 0 **9** 2 **10** 1

Exercise 154 (Revision)

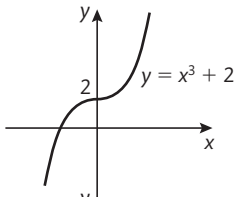
1 **a** $x = \pm 4$ **b** $x = \pm 5$
c $x = 0$ or -4 **d** $x = 0$ or 3
2 **a** $x = 1$ or 3 **b** $x = -3$ or -4
c $x = -4$ or 3 **d** $x = -2$ or 4
3 **a** $x = -2$ or 3 **b** $x = -1$ or -2
c $x = -1$ or $\frac{2}{3}$ **d** $x = 0$ or -1
4 **a** $x = -3.73$ or -0.27 **b** $x = 0.7$ or 4.30
c $x = -1.24$ or 3.24 **d** $x = -2.77$ or 0.27
5 **a** $x = -2$ or 5 **b** $x = -3$ or 2
c $x = 0$ or 4 **d** $x = 0.23$ or 1.43

Exercise 154* (Revision)

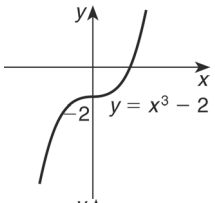
1 **a** $x = \pm 13$ **b** $x = \pm 2$
c $x = 0$ or 9 **d** $x = -36$ or $x = 0$
2 **a** $x = -9$ or 8 **b** $x = -1$ or 4
c $x = -6$ or 8 **d** $x = -6$ or 3
3 **a** $x = -3$ or -4 **b** $x = 2$
c $x = -4$ or 6 **d** $x = -\frac{5}{2}$ or $\frac{3}{4}$
4 **a** $x = -0.44$ or 3.44 **b** $x = -1.82$ or 0.823
c $x = -0.57$ or 2.91 **d** $x = -4.46$ or 0.46
5 **a** $x = -7$ or 2 **b** $x = -2$ or 4
c $x = \frac{1}{3}$ or 2 **d** $x = -0.38$ or 3.30

Exercise 155

1



2



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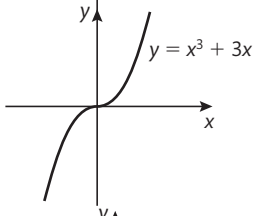

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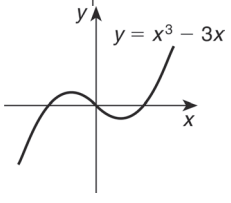

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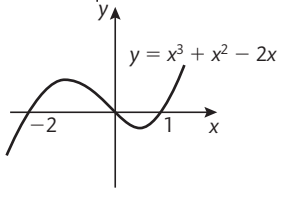

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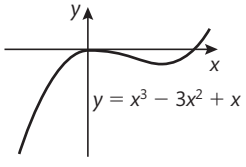
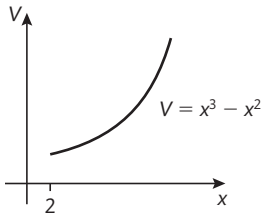


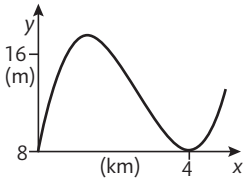
7 **a** $V = x^2(x - 1) = x^3 - x^2$
b



c 48 m²
d 4.6 m × 4.6 m × 3.6 m

8 **a**

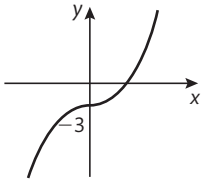
x	0	1	2	3	4	5
y	8	17	16	11	8	13



b 9.5m

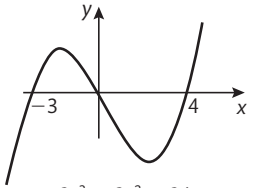
Exercise 155*

1



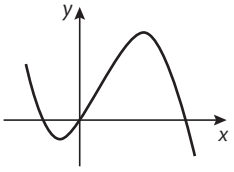
$y = 2x^3 - x^2 + x - 3$

2



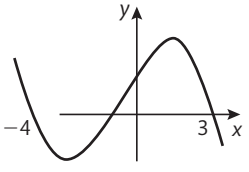
$y = 2x^3 - 2x^2 - 24x$

3



$y = -2x^3 + 3x^2 + 4x$

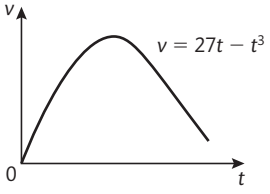
4



$y = -x^3 - 2x^2 + 11x + 12$

5 **a**

t	0	1	2	3	4	5
v	0	26	46	54	44	10

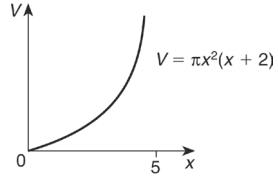


b $v_{max} = 54$ m/s and occurs at $t = 3$ s
c $v \geqslant 30$ m/s when $1.2 \geqslant t \geqslant 4.5$ so for about 3.3 s

6 **a** $V = \pi x^2 + \frac{1}{3}\pi x^2 6 = \pi x^3 + 2\pi x = \pi x^2(x + 2)$

b

x	0	1	2	3	4	5
V	0	3π	16π	45π	96π	175π

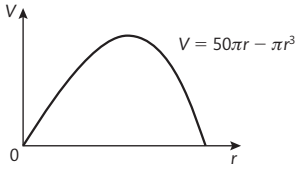


c When $x = 3.5$ cm, $V \cong 212$ cm³
d When $V = 300$ cm³, $x \cong 4$ cm $\rightarrow A \cong 100.5$ cm³

7 **a** $A = 100\pi = 2\pi r^2 + 2\pi rh$
$\rightarrow 100\pi - 2\pi r^2 = 2\pi rh$
$\rightarrow \frac{50}{r} - r = h$
$\rightarrow V = \pi r^2 h = \pi r^2\left(\frac{50}{r} - r\right) = 50\pi r - \pi r^2$

b

r	0	1	2	3	4	5
V	0	153.9	289.0	386.4	427.3	392.7

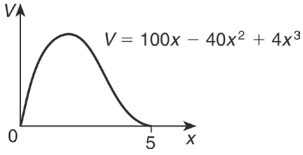


c $V_{max} = 427.3\text{ cm}^3$ **d** $d = 8.16\text{ cm}$, $h = 8.2\text{ cm}$

8 **b** $V = (10 - 2x)(10 - 2x)x = 100x - 40x^2 - 4x^3$

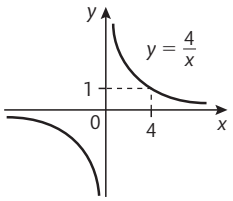
c

x	0	1	2	3	4	5
V	0	64	72	48	16	0



d $V_{max} = 74\text{ cm}^3$, $1.67 \times 6.66 \times 6.66 \times 6.66$ cm

Exercise 156

1


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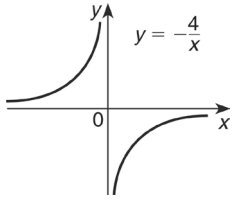

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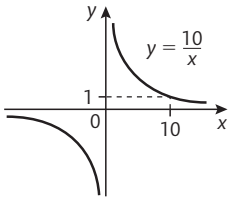
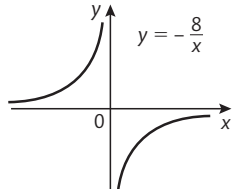
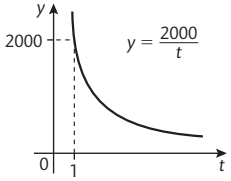


4


5 **a**

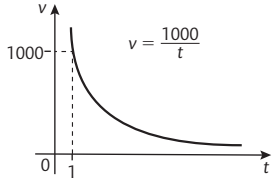
t (months)	1	2	3	4	5	6
y	2000	1000	667	500	400	333

b


c 3.3 months **d** 2.7 months approx.

6 **a**

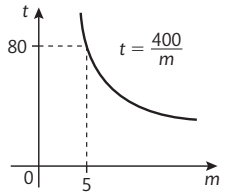
t (hours)	1	5	10	15	20
v (m^3)	1000	200	100	67	50

b


c 4 hours **d** 62.5 m^3

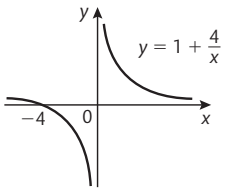
7 **a** $k = 400$

m (min)	5	6	7	8	9	10
t (°C)	80	67	57	50	44	40

b


c 53 (°C) **d** 6 min 40 s **e** $5.3 \leqslant m \leqslant 8$

Exercise 156*

1


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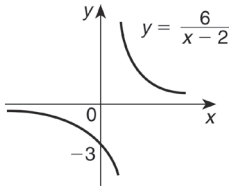

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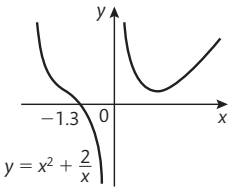

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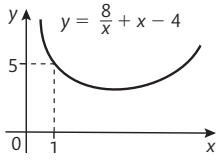
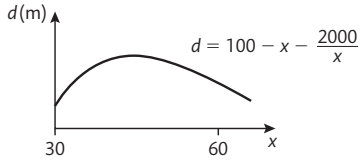


5 **a**

x (°)	30	35	40	45	50	55	60
d (m)	3.3	7.9	10	10.6	10	8.6	6.7



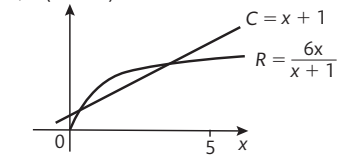
b 10.6 m at $x = 45°$

c $37° \leqslant x \leqslant 54°$

6 **a**

x	0	1	2	3	4	5
R	0	3	4	4.5	4.8	5

x	1	3	5
C	2	4	6

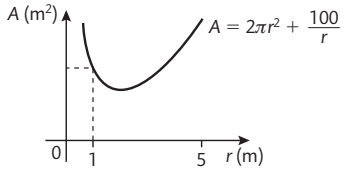


b $P > 0$ when $0.27 < x < 3.7$ so between 27 and 370 boards per week

c £1100 when $x = 1.45$ so 145 boards hired out

7 **b**

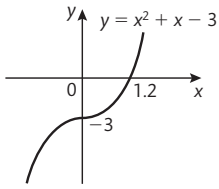
r (m)	1	2	3	4	5
A (m^2)	106	75	90	126	177



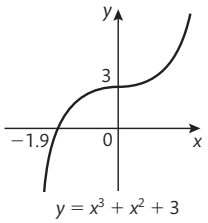
c $A = 75\text{ m}^2$ at $r = 2.0$ m

Exercise 157 (Revision)

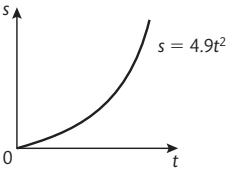
1



2

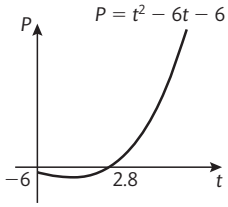


3 **a**



b 30.6 m **c** 3.19 s

4 **a**

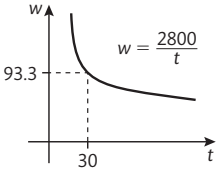


b After 2.85 years **c** 5.15 years

5 **a** $k = 2800$

b

***t* (weeks)**	30	32	34	36	38	40
***x* (kg)**	93	88	82	78	74	70

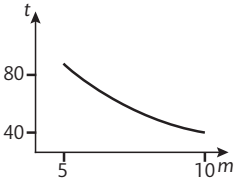


c 35 weeks

d Clearly after 500 weeks, Nick cannot weigh 5.6 kg. So there is a domain over which the equation fits the situation being modelled.

6 **a**

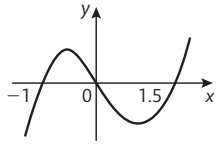
m	5	6	7	8	9	10
t	85	70.8	60.7	53.1	47.2	42.5



b $6 \leqslant m \leqslant 8.5$

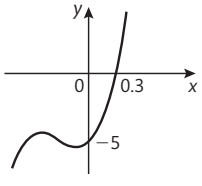
Exercise 157* (Revision)

1



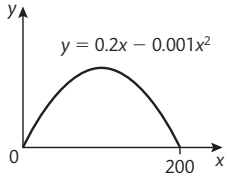
$y = 2x^3 - x^2 + 3x$

2



$y = 3x(x + 2)^2 - 5$

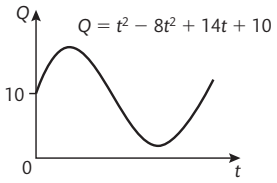
3 **a**



b 10 m **c** $29 \leqslant x \leqslant 170$

4 **a**

t	0	1	2	3	4	5
Q	10	17	14	7	2	5

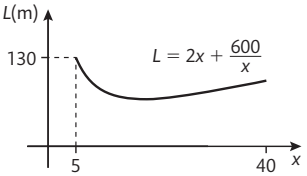


b $Q_{max} = 17.1\ m^3/s$ at 01.06 **c** Between midnight and 02:35

5 **a** $\frac{600}{x}$

c

x	5	10	15	20	25	30	35	40
L	130	80	70	70	74	80	87	95



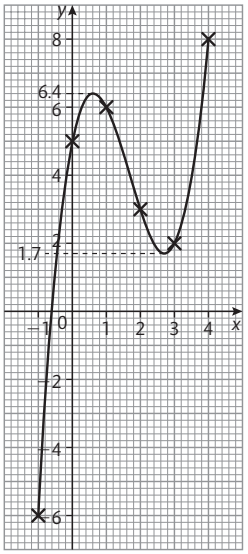
d 69.3 m at $x = 17.3$ m

e $11.6 < x < 25.9$

6 **b**

x	−1	0	1	2	3	4
y	−6	5	6	3	2	9

c



d R(0.6, 6.4), S(2.7, 1.7)

Exercise 158

1 120 cm³ **2** 104 cm³

3 48 cm², 108 cm² **4** 248 cm², 240 cm³

5 1.57 m³, 7.85 m²

6 Volume = 452 cm³ (3 s.f.); Area = 509 cm² (3 s.f.)

7 800 m³ **8** 11.25 m³

9 18.0 m **10** $16\frac{2}{3}$ cm

Exercise 158*

1 4800 cm³

2 Volume = 120 000 cm³; Surface area = 18 400 cm²

3 3800 cm³, 6238 cm² **4** 229 cm³, 257 cm²

5 7151 cm³, 2260 cm² **6** 5 mm

7 1.37 m³ **8** 405 m³, 426 m²

9 approx 30 m **10** 0.004 cm

Exercise 159

1 $2.57 \times 10^6\ m^3$ **2** $9820\ cm^3$, $2410\ cm^2$
3 Volume = $7069\ cm^3$; Surface area = $1414\ cm^2$
4 $25.1\ cm^3$ **5** $396\ m^3$, $311\ m^2$
6 12 cm, $1810\ cm^2$ **7** 61 cm

Exercise 159*

1 $83\frac{1}{3}\ mm^3$ **2** $98.2\ cm^3$ **3** $2150\ cm^3$, $971\ cm^2$
4 $2.92 \times 10^5\ m^3$ **5** $1089\ cm^3$ **6** $5.12 \times 10^8\ km^2$
7 0.42 cm **8** 137 **9** 12 cm
10 4.5×10^{-4} mm

Exercise 160

1 $16\ cm^2$
2 **a** Angles are the same
b $8.55\ cm^2$
3 $213\ cm^2$ **4** $84.4\ cm^2$
5 6 cm **6** $3\ cm^2$
7 3 cm **8** 13.3 cm

Exercise 160*

1 $675\ cm^2$ **2** $45\ cm^2$ **3** 7.5 cm **4** 10 cm
5 $1000\ cm^2$ **6** 44% **7** 19% **8** $75\ cm^2$

Exercise 161

1 $800\ cm^3$ **2** $23.4\ cm^3$ **3** $14.1\ cm^3$ **4** $25.3\ cm^3$
5 15.1 cm **6** 9.64 cm **7** 5.06 cm **8** 27.8 cm

Exercise 161*

1 $135\ cm^3$ **2** $31.25\ cm^2$ **3** 33.4 cm
4 18.6 cm **5** 72.8% **6** \$33.75
7 **a** 270 g **b** 16 cm
8 **a** 270 g **b** 180 cm
9 **a** 3000 **b** 22.5 m
c $810\ cm^2$ **d** 240 g

Exercise 162 (Revision)

1 **a** SA = 207, V = 226 **b** SA = 152, V = 96
2 $r = 1.91$ cm, $A = 11.5\ cm^2$ **3** $r = 4.57$ cm, $SA = 263\ cm^2$
4 P = 20, A = 18.75 **5** $x = 12$
6 V = 246, A = 62.5 **7** $h = 10$, SA = 339

Exercise 162* (Revision)

1 **a** SA = 452, V = 509 **b** SA = 223, V = 192
2 $r = 4.67$ cm, $A = 34.2\ cm^2$ **3** $r = 2.84$ cm, $V = 48.0\ cm^3$
4 P = 20, A = 31.36 **5** $x = 4.5$
6 V = 2048, SA = 1875
7 Diameter of Moon = 3479 km, SA Earth = $5.09 \times 10^8\ km^2$

Exercise 163

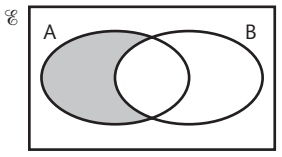
1 6 **2** 93
3 22 **4** 41
5 **a** 10 **b** 8 **c** 5
6 18

Exercise 163*

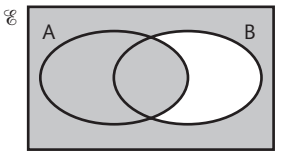
1 23 **2** 100 **3** 100
4 3 **5** $8 \leqslant x \leqslant 14, 0 \leqslant y \leqslant 6$ **6** $40 \leqslant p \leqslant 65$

Exercise 164

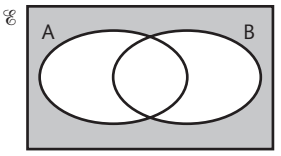
1



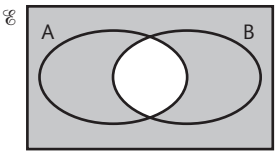
A ∩ B′



A ∪ B′

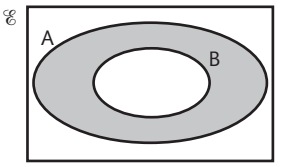


A′ ∩ B′

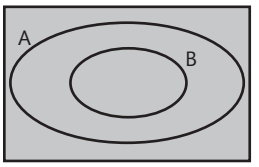


A′ ∪ B′

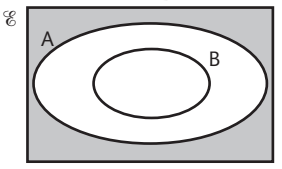
2



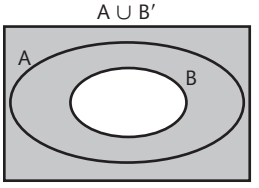
A ∩ B′



A ∪ B′

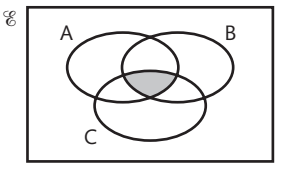


A′ ∩ B′

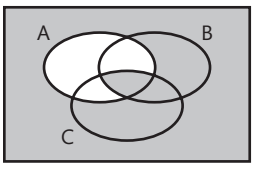


A′ ∪ B′

3

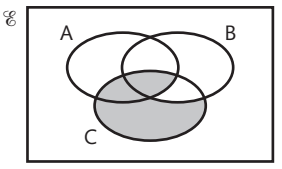


A ∩ B ∩ C

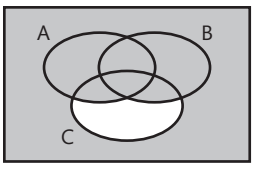


A′ ∪ B ∩ C

4



(A ∪ B′) ∩ C

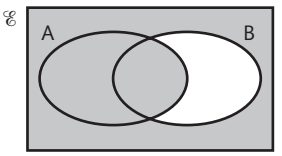


A ∪ B ∪ C

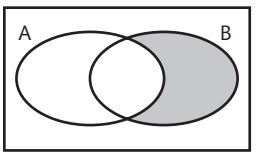
5 A ∩ B′, A′ ∩ B′, A′ ∪ B′
6 A′ ∪ B, (A′ ∩ B′) ∪ (A ∩ B), A ∩ B′

Exercise 164*

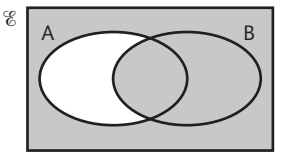
1



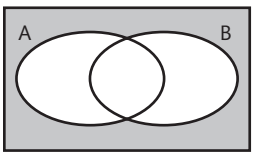
(A′ ∩ B′)



(A ∪ B′)′

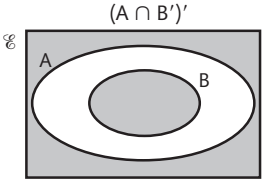


(A ∩ B′)′

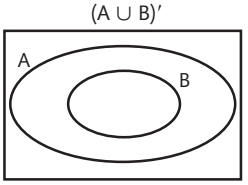


(A ∪ B)′

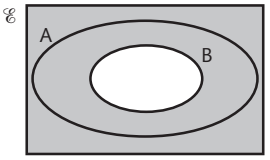
2



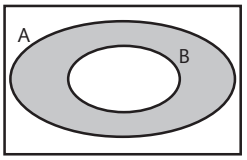
(A′ ∩ B′)′



(A ∪ B′)′

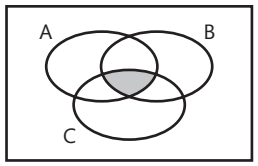


(A ∩ B)′

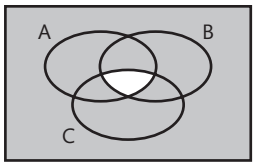


(A′ ∪ B)′

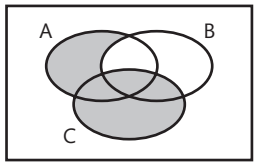
3



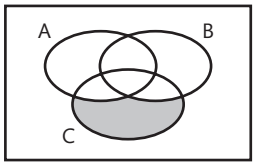
$A \cap B \cap C$



$(A \cap B \cap C)'$

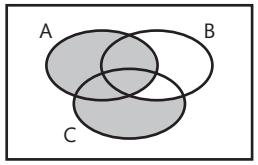


$(A \cap B') \cup C$

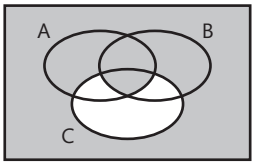


$(A \cup B)' \cap C$

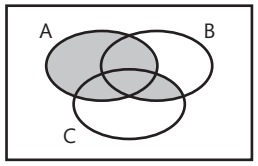
4



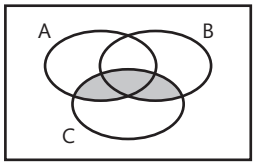
$A \cup (B' \cap C)$



$(A \cap B) \cup C'$



$A \cup (B \cap C)$



$(A \cup B) \cap C$

5 $B \cap (A \cup C)'$, $(B' \cap C)$, $(B' \cap C) \cup (A \cap B \cap C')$

6 $A \cap C \cap B'$, $A' \cap (B \cup C)$, $[A' \cap (B \cup C)] \cup [A \cap (B \cup C)']$

7 Correct proof

Exercise 165

1 **a** {Tuesday, Thursday} **b** {Red, Amber, Green}
c {1, 2, 3, 4, 5, 6} **d** {−1, 0, 1, 2, 3, 4, 5, 6}

2 **a** {Africa, Antarctica, Asia, Australia, Europe, North America, South America}
b {All Mathematics teachers in the school}
c {1, 2, 3, 4, 5} **d** {−3, −2, −1, 0, 1, 2}

3 **a** $\{x : x < 7, x \in \mathbb{N}\}$ **b** $\{x : x > 4, x \in \mathbb{N}\}$
c $\{x : 2 \leqslant x \leqslant 11, x \in \mathbb{N}\}$ **d** $\{x : -3 < x < 3, x \in \mathbb{N}\}$
e $\{x : x$ is odd, $x \in \mathbb{N}\}$ **f** $\{x : x$ is prime$\}$

4 **a** $\{x : x > -3, x \in \mathbb{N}\}$ **b** $\{x : x \leqslant 9, x \in \mathbb{N}\}$
c $\{x : 5 < x < 19, x \in \mathbb{N}\}$ **d** $\{x : -4 \leqslant x \leqslant 31, x \in \mathbb{N}\}$
e $\{x : x$ is a multiple of 5, $x \in \mathbb{N}\}$ or $\{x : x = 5y, y \in \mathbb{N}\}$
f $\{x : x$ is a factor of 48, $x \in \mathbb{N}\}$

Exercise 165*

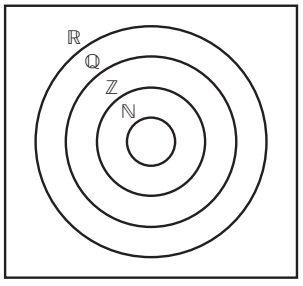
1 **a** {2, 4, 6, 8, 10, 12} **b** {3, 7, 11, 15, 19, 23}
c {2, 4, 6} **d** {Integers between 1 and 12 inclusive}

2 **a** {0, 1, 4} **b** $\{\frac{1}{4}, \frac{1}{2}, 1, 2, 4\}$ **c** {1} **d** {(1, 1), (2, 2)}

3 **a** ∅ **b** $(1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16})$
c {2} **d** {−3, 2}

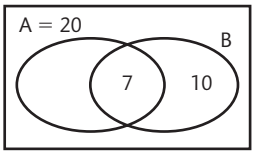
4 **a** ∅ **b** {1, 2, 4, 8, 16, 32}
c ∅ **d** $\{-1 + \sqrt{7}, -1 - \sqrt{7}\}$

5



Exercise 166 (Revision)

1 **a**

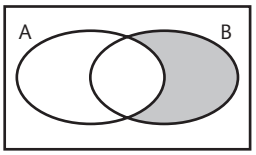


b 17 **c** 30

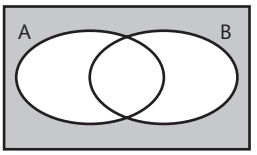
2 **a** 6 **b** 2 **c** 10

3 **a** 17% **b** 52% **c** 31%

4 **a**



b



5 $A' \cup B'$

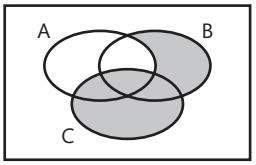
6 **a** {−2, −1, 0, 1, 2, 3} **b** {1, 2, 3, 4} **c** ∅

7 **a** $\{x : x$ is even, $x \in \mathbb{N}\}$ **b** $\{x : x$ is a factor of 24, $x \in \mathbb{N}\}$
c $\{x : -1 \leqslant x \leqslant 4, x \in \mathbb{N}\}$

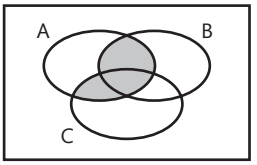
Exercise 166* (Revision)

1 34 **2** 10 **3** 2

4 **a**



b



5 **a** $(A \cup B') \cap C$ **b** $A \cup B \cup C'$

6 **a** {−1, 1} **b** {0, −4} **c** ∅

7 **a** $\{x : x > -5, x \in \mathbb{N}\}$ **b** $\{x : 4 < x < 12, x \in \mathbb{N}\}$
c $\{x : x$ is a multiple of 3, $x \in \mathbb{N}\}$ or $\{x : x = 3y, y \in \mathbb{N}\}$

Multiple choice 7

1 C **2** C **3** B **4** A **5** D
6 D **7** A **8** A **9** D **10** D

Self-assessment 7

1 **a** x^7 **b** x^3 **c** x^{10} **d** x^{-3}

2 **a** $\frac{1}{9}$ **b** 2 **c** $\frac{1}{2}$ **d** $\frac{1}{25}$ **e** $\frac{1}{16}$

3 **a** b **b** c^{-3} **c** b^{-2} **d** c **e** 1

4 **a** $x = 7$ **b** $x = -1$ **c** $x = \frac{1}{2}$ **d** $x = \frac{1}{3}$

5 **a** $x = 3$ or 4 **b** $x = -5$ or -2
c $x = 0$ or 7 **d** $x = 0$ or 5

6 **a** $x = \frac{1}{2}$ or 2 **b** $x = -4$ or $\frac{-2}{3}$
c $x = \frac{2}{3}$ or -1 **d** $x = \frac{-5}{2}$ or 2

7 **a** $x = -2$ or 8 **b** $x = -5.46$ or 1.46
c $x = 0.84$ or 7.16 **d** $x = -3$ or 1

8 **a** $x = -0.79$ or 2.12 **b** $x = 0.31$ or 1.29
c $x = -3.91$ or 1.41 **d** $x = 0.21$ or 4.79

9 $a = 15$

10 $a = 60$ $b = 3$

11 1560 cm² (3 s.f.)

12 158 cm (3 s.f.)

13 **a** SA = 452, V = 509 **b** SA = 223, V = 192

14 **a**

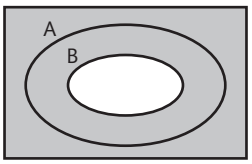
m	5	6	7	8	9	10
t	80	66.7	57.1	50	44.4	40

c 53°C **d** 6.7 min **e** 5.3−8 min

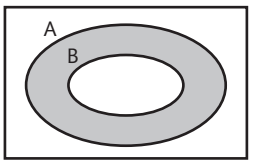
15 **a** 135 cm³ **b** 10.8 cm **c** 12 cm³, 55.7 cm³

16 **a** 9 **b** 5 **c** 7

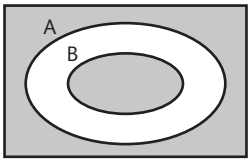
17 **a**



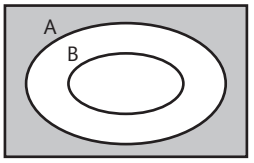
b



c



d



Exercise 167

1 2, 4, 6, 8
2 −9, −6, −3, 0
3 15, 10, 5, 0
4 2, 4, 8, 16
5 12, 6, 3, 1.5
6 Add 4; 19, 23, 27
7 Subtract 5; −7, −12, −17
8 Double; 48, 96, 192
9 Halve; 4, 2, 1
10 Add 0.3; 1.4, 1.7, 2

Exercise 167*

1 −1, 0.5, 2, 3.5
2 3, 1.75, 0.5, −0.75
3 1, 2.5, 6.25, 15.625
4 3, $-1, \frac{1}{3}, -\frac{1}{9}$
5 1, 1, 2, 3, 5, 8
6 Add $2\frac{1}{2}$; 13, $15\frac{1}{2}$, 18
7 Divide by 3; 3, 1, $\frac{1}{3}$
8 Square; 65 536, 4.3×10^9, 1.8×10^{19}
9 Multiply by $-\frac{1}{2}, \frac{1}{16}, -\frac{1}{32}, \frac{1}{64}$
10 Double then add 1; 63, 127, 255

Exercise 168

1 3, 5, 7, 9
2 4, 9, 14, 19
3 30, 27, 24, 21
4 2, 0, −2, −4
5 3, 9, 27, 81
6 −4, −8, −12, −16
7 8
8 7
9 7
10 8th

Exercise 168*

1 −1, 4, 9, 14
2 97, 94, 91, 88
3 $1, \frac{3}{2}, 2, \frac{5}{2}$
4 $1, \frac{4}{3}, \frac{5}{3}, 2$
5 2, 5, 10, 17
6 $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$
7 8
8 12
9 10
10 51

Exercise 169

1 17, 20, 23
2 29, 34, 39
3 −7, −10, −13
4 −9, −13, −17
5 56, 76, 99
6 52, 68, 86
7 0, 8, 19
8 4, 10, 18
9 −4, −11, −20
10 3, 0, −4

Exercise 169*

1 58, 78, 101
2 74, 100, 130
3 1, −6, −15
4 8, 5, 1
5 1, 8, 17
6 −6, −4, −1
7 71, 101, 139
8 150, 215, 297
9 −6, −19, −38
10 −54, −90, −139

Exercise 170

1 $3n + 1$
2 $2n + 3$
3 $34 - 4n$
4 $29 - 3n$
5 **a** 1, 3, 5, 7, 9, 11, ...
b $c = 2l - 1$
c c is always odd, 50 layers.
6 **a** 8, 10, 12, 14, 16, 18, ...
b $p = 2n + 6$
c 47th

Exercise 170*

1 $4n - 1$
2 $3n - 2$
3 $9 - 3n$
4 $13 - 4n$
5 **a** 6, 10, 14, 18, 22, 26, ...
b $s = 4n + 2$
c 202
6 **a** 10, 16, 22, 28, 34, 40, ...
b $p = 6n + 4$
c 32nd

Exercise 171 (Revision)

1 −2, 43, 493
2 5, 8, 11
3 **a** $15 - 3n$
b $6n + 4$
4 **a** 1800 m
b $800 + 200n$
c 36 days
5 **a** Odd numbers
b 4, 9, 16, 25
c 121
d n^2
e 29
6 **a** $n + n + 1$
b $2n + 1$
c $n + n + 1 = 2n + 1$
d $2n$ is always even, so $2n + 1$ is odd

Exercise 171* (Revision)

1 5, −58, −688
2 −2, −5, −8
3 $4n - 10$
4 $18 - 5n$
5 **a** £2.90
b $50 + 20n$
c 12 years
6 **a** 17, 21, 25, 29
b 7, 13
c 1, 3, 7, 13, ...; 21st term
7 **a** $n(n + 1)$
b $n^2 + n$
c $n(n + 1) = n^2 + n$
d Either n or $n + 1$ is even, so their product is even.
8 **a** $n + 1$
b $n - 1$
c $(n + 1)(n - 1)$
d $(n + 1)(n - 1) = n^2 - 1$

Exercise 172

1 $f(-2) = 0, f(4) = 0$
2 $f(-4) = 0, f(-6) = 0$
3 $f(1) \neq 0$ so no
4 $f(-2) = 0$ so yes
5 $f(6) = 0, (x - 3)$
6 $f(-3) = 0, (x - 7)$
7 $f(1) = 0, f(-9) = 0, (x - 1)(x + 9)$
8 $f(-2) = 0, f(8) = 0, (x + 2)(x - 8)$
9 $f(-3) = 0 \Rightarrow p = -1$
10 $f(2) = 0 \Rightarrow q = 5$

Exercise 172*

1 $f(-9) \neq 0$ so no
2 $f(8) \neq 0$ so no
3 $f(-7) \neq 0$ so no
4 $f(14) = 0$ so yes
5 $f(2) = 0, f(19) = 0, (x - 2)(x - 19)$
6 $f(-1) = 0, f(23) = 0, (x + 1)(x - 23)$
7 $f(-12) = 0 \Rightarrow p = 36$
8 $f(15) = 0 \Rightarrow q = -6$
9 $f(-5) = 0 \Rightarrow r = -35$
10 $f(8) = 0 \Rightarrow s = 104$

Exercise 173

1 $p = 3$
2 $q = 5$
3 $r = 24$
4 $x = -2$, 3 or −4
5 $x = -1, -5$, or 4
6 Other factor is $(x - 1)$, $x = -2, -3$ or 1
7 Other factor is $(x + 1)$, $x = 2, -4, -1$
8 Other factors are $(x - 1), (x + 1)$, $x = -5$, 1 or −1
9 Other factors are $(x + 1), (x - 2)$, $x = 4, -1$ or 2
10 $(x + 1)(x - 1)(x + 3)$, $x = -1$, 1 or −3

Exercise 173*

1 $p = -24$
2 $q = 0$
3 $p = 7, q = -15$
4 $r = -10, s = 8$
5 Other factor is $(x - 2)$, $x = -3$, 4 or 2
6 Other factor is $(x - 1)$, $x = -4$, 5 or 1
7 Other factors are $(x - 2)$ and $(x - 3)$, $x = -6$, 2 or 3
8 Other factors are $(x + 1)$ and $(x + 2)$, $x = 7, -1, -2$
9 $(x + 1)(x + 2)(x - 2)$, $x = -1, -2$ or 2
10 $(x + 3)(x - 3)(x + 2)$, $x = -3$, 3 or −2

Exercise 174

1 $x(x + 1)(x - 2)$
2 $x(x - 3)(x - 1)$
3 $x(x - 1)(x - 2)$
4 $(x + 1)^3$
5 $2(x + 1)(x + 2)(x + 3)$
6 $3(x - 2)(x - 1)(x + 3)$
7 $p = 0$, $(x - 1)(x + 3)(2x + 3)$, $x = 1, -3$ or −1.5
8 $q = -12$, $(x + 2)(x - 2)(3x + 1)$, $x = -2$, 2 or $-\frac{1}{3}$

Exercise 174*

1 $x(2x + 1)(x - 3)$
2 $x(3x - 2)(x - 2)$
3 $(x + 1)(x - 2)^2$
4 $(x + 2)(x + 3)^2$
5 $3(x - 3)(x + 1)^2$
6 $(2x - 1)(x + 2)(x + 3)$
7 $p = 4$, $(x + 4)(x - 2)(3x - 2)$, $x = -4$, 2 or 2/3
8 $q = -14$, $(x + 3)(x + 1)(2x - 5)$, $x = -3$, −1 or 2.5

Exercise 175

1 12, 8 or −8, −12 **2** 9, 7 or −7, −9 **3** 2.61
4 4.4 cm **5** 3.22 **6** 6 cm
7 8, 7 or −7, −8 **8** −10, −9 or 9, 10

Exercise 175*

1 4 **2** 6 m × 8 m
3 8, 9 or −9, −8 **4** 7 and 9 or −9 and −7
5 4.40 m by 11.6 m
6 **a** 1414 **b** 446 terms too small, 447 terms too big
7 **a** 38 sides **b** 30 sides too few, 31 sides too many
8 6 days **9** 14.8 cm

Exercise 176 (Revision)

1 $p = 0$
2 $(x - 3), (x + 1)$; $x = -3$, 3 or −1
3 $(x + 2)(x - 2)(x - 1)$, $x = -2$, 2 or 1
4 $x(x + 4)(x - 7)$; $x = 0, -4, 7$
5 $p = 5$, $(x + 4)(x - 1)(2x + 1)$, $x = -4$, 1 or $\frac{1}{2}$
6 2.5 cm
7 $\frac{1}{2}x(x + 3) = 25$, 5.73 cm
8 $x^2 + (x + 1)^2 = (x + 2)^2$, $x = 3$
9 $x^2 + (x + 1)^2 = 145$, 8 and 9 or −8 and −9
10 $x(30 - x) = 210$, 11.1 cm by 18.9 cm

Exercise 176* (Revision)

1 $p = 3, q = -6$
2 $(x + 5)(x - 1)(x + 3)$; $x = -5$, 1 or −3
3 $x = 7$, −1 or 2
4 $x = 1$ or 3
5 $p = -1$, $(x + 5)(x + 2)(2x - 3)$, $x = -5$, −2 or $\frac{3}{2}$
6 4.63 cm
7 6 m × 5 m
8 5, 12, 13
9 9 and 11
10 $x^2 + (5 - x)^2 = 16$, 1.18 cm and 3.82 cm
11 $\pi(x + 2)^2 - \pi 2^2 = \pi 2^2$, $x = 0.828$ m
12 $\left(\frac{1200}{x} - 20\right)(x + 3) = 1200$, $x = 12$ days

Exercise 177

1 **a** 2 **b** 3 **c** −1 **d** −2 **e** $x = 1$
2 **a** 2 **b** 2 **c** 0.1 **d** 5.8 **e** $x = 0.18$ or 1.8
3 **a** 4 **b** 2 **c** −4 **d** $x = 3$
4 **a** −4 **b** 0 **c** 2 **d** $x = 4$

Exercise 177*

1 **a** 2 **b** 4 **c** −1 **d** −3 **e** $x = 1.5$
2 **a** −2.75 **b** 1.25 **c** −0.75 **d** −4
e $x = -1$ or 0.33
3 **b**

***x* coordinate**	−4	−3	−2	−1	0	1	2	3	4
Gradient	−8	−6	−4	−2	0	2	4	6	8

4

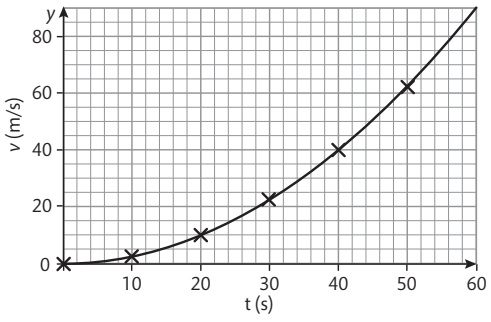
x	0	1	2	3	4	5
y	1	2	4	8	16	32

b 1.4 and 5.5
c 4.1

Exercise 178

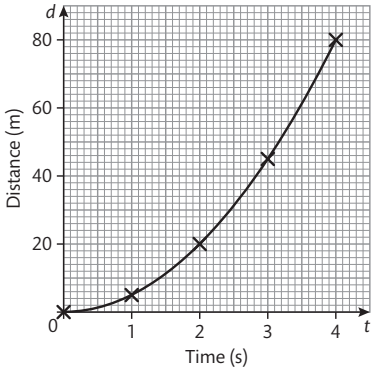
1 **a** **(i)** 1 m/s **(ii)** 0 m/s **(iii)** 2 m/s
b 0−20 s gradually increased speed then slowed down to a stop
20−30 s stationary
30−40 s speed increasing
40−50 s travelling at a constant speed of 2 m/s
50−60 s slowing down to a stop

2 **a**



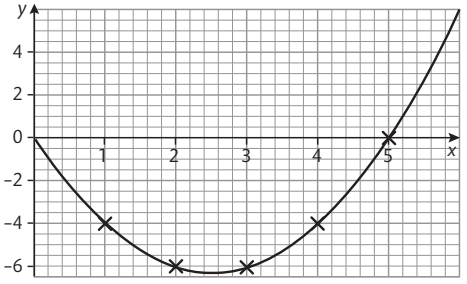
b **(i)** 0.5 m/s² **(ii)** 1.5 m/s² **(iii)** 2.5 m/s²

3 **a**



b **(i)** 10 m/s **(ii)** 20 m/s **(iii)** 30 m/s

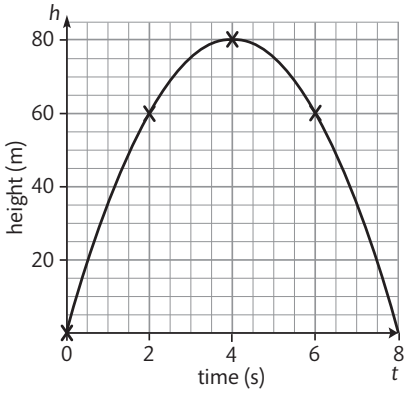
4 **a**



b −3, 0, 5

Exercise 178*

1 **a**



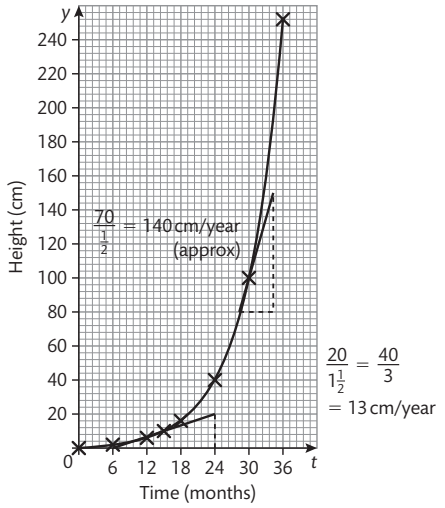
b

***t* (s)**	0	1	2	3	4	5	6	7	8
***V* (m/s)**	40	30	20	10	0	−10	−20	−30	−40

c Straight line graph passing through (0, −40) and (8, −40)
d Acceleration is constant (−10 m/s), i.e. constant deceleration (10 m/s)

2 **a** 2 **b** $y = 2x + 1$

3 **a**



b $t = 15$, 13 cm/year (approx.) $t = 30$, 140 cm/year (approx.)

4 **a** **(i)** -9.44 **(ii)** 0.075 **(iii)** 1.56

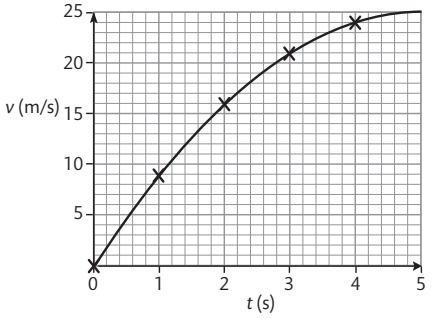
b max $t = 1.7$=, -11.7 m/s

Exercise 179 (Revision)

1 **a** **(i)** 1 **(ii)** -2 **(iii)** 3

b $x = -2$ **c** $y = 2x - 2$

2 **a**



b **(i)** 8 ms^{-2} **(ii)** 4 ms^{-2} **c** 2.5 s

3 **a** **(i)** 2 ms^{-1} **(ii)** 0 ms^{-1} **(iii)** -1.6 ms^{-1}

b 23.5 s at -2.8 ms^{-1}

c Increases velocity for 5 secs, slows down for 5 secs, on flower for 10 secs, returns faster than outward journey.

4 **a**

Time, t (years)	0	10	20	30	40	50	60	70
Number, N, of eagles	100	150	225	338	506	759	1139	1709

b **(i)** 13.7 **(ii)** 30.8 **c** After 39 years

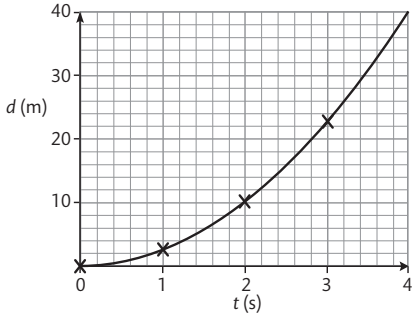
Exercise 179* (Revision)

1 **a** **(i)** 1 **(ii)** -2 **(iii)** 3

b -10 **c** Negative of each other.

d $y = 2x + 6$ **e** ± 1.414

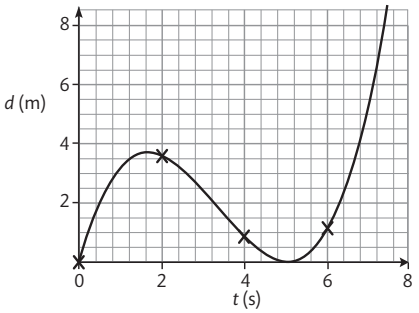
2 **a**



b **(i)** 5 **(ii)** 10 **(iii)** 15 **(iv)** 20 **(v)** 0

c Should be $v = 5t$ **d** Constant acceleration of 5 ms^{-2}

3 **a**



b Runs off for about 1.7 secs, returns more slowly, at 5 secs runs off again

c **(i)** 1.6 ms^{-1} **(ii)** -1.6 ms^{-1} **d** 6.9 s

4 **a**

t (days)	0	1	2	3	4	5	6	7	8
d (cm)	20	17	14.45	12.28	10.44	8.87	7.54	6.41	5.45

b 4.27 days **c** 2.35 cm/day **d** 0.0511 cm/hour

e 3.25 cm/hour at $t = 0$

Exercise 180

1 $x = 17.5°, 163°$ **2** $x = 44.4°, 136°$ **3** $x = 72.5°$

4 $x = 53.1°$ **5** $x = 71.6°$ **6** $x = 63.4°$

7 $x = 107°$ **8** $x = 127°$ **9** no solution in range

10 no solution in range **11** $x = 108°$ **12** $x = 117°$

Exercise 180*

1 $x = 27.2°, 153°$ **2** $x = 40.9°, 139°$ **3** $x = 207°, 333°$

4 $x = 221°, 319°$ **5** $x = 75.7°, 284°$ **6** $x = 50.0°, 310°$

7 $x = 112°, 248°$ **8** $x = 147°, 213°$ **9** $x = 67.9°, 248°$

10 $x = 34.2°$ **11** $x = 125°, 305°$ **12** $x = 144°$

Exercise 181

1 $x = 5.94$ **2** $y = 11.1$ **3** MN = 39.0 cm

4 RT = 8.75 cm **5** AC = 37.8 cm **6** YZ = 33.0 cm

7 $x = 37.3°$ **8** $y = 37.8°$ **9** $\angle ABC = 38.8°$

10 $\angle XYZ = 26.0°$ **11** $\angle ABC = 62.2°$ **12** $\angle DCE = 115°$

13 13.5 km

Exercise 181*

1 $x = 29.7$ **2** $\angle LMN = 67.4°, 113°$

3 EF = 10.4 cm, $\angle DEF = 47.5°$, $\angle FDE = 79.0°$

4 MN = 10.8 cm, $\angle ABC = 68.3°$, $\angle ABC = 49.7°$

5 13.5 km **6** 1090 km **7** BC = 261 m

8 XY = 3.26 km **9** PR = 115 m, 112 m **10** YT = 333 m, 180 m

Exercise 182

1 $x = 7.26$ **2** $b = 8.30$ **3** AB = 39.1 cm

4 XY = 32.9 cm **5** RT = 24.2 cm **6** MN = 6.63 cm

7 $Y = 70.5°$ **8** $\angle ABC = 92.9°$ **9** $\angle XYZ = 109.6°$

Exercise 182*

1 $x = 9.34$ **2** $\angle XYZ = 95.5°$ **3** $\angle BAC = 81.8°$

4 $\angle CAB = 81.8°$

5 QR = 4.18 cm, $\angle PQR = 39.2°$, $\angle QRP = 62.8°$

6 LM = 11.4 cm, $\angle NLM = 34.6°$, $\angle LMN = 28.4°$

7 11.6 km

8 **a** $\angle VWU = 36.3°$ **b** 264°

Exercise 183

1 **a** 9.64 **b** 38.9°

2 **a** 6.6 **b** 49.3°

3 **a** 54.9° **b** 88.0° **c** 37.1°

4 **a** 24.1° **b** 125.1° **c** 30.8°

5 **a** 4.1 **b** 4.9

6 **a** 44.7° **b** 4.1

7 **a** 16.8 km **b** 168°

8 **a** 8.9 km **b** 062.9°

Exercise 183*

1 247 km, 280° **2** 14.7 km/h, 089°
3 **a** 50.4° **b** 7.01 m **c** 48.4°
4 5.3 cm, 8.7 cm **5** BC = 23.4 km, 186.3°
6 CS = 2.64 km, 040° **7** **a** 38.1° **b** 29.4 cm

Exercise 184 (Revision)

1 9.22 cm **2** 18.0 cm **3** 15.6 cm **4** 13.5 cm
5 11.9 cm **6** 19.4 cm **7** 46.5° **8** 38.2°
9 55.1° **10** 36.2° **11** 20.7° **12** 59.0°

Exercise 184* (Revision)

1 BH = 506 m
2 6.32 cm and 9.74 cm
3 **a** 60°, 50°, 70° **b** 4.9 km
4 22.3°
5 **a** C = 42.2°, a = 6.96 m **b** C = 44.7°, a = 5.84 m
6 **a** 25.5 km **b** 022.7° **c** 202.7°
7 **a** 42.2 km **b** 022.7° **c** 14.1 km/h
8 82.8°, 41.4°, 55.8°

Exercise 185

1 **a** $\frac{1}{36}$ **b** $\frac{25}{36}$ **c** $\frac{5}{36}$ **d** $\frac{5}{18}$
2 **a** $\frac{4}{25}$ **b** $\frac{9}{25}$ **c** $\frac{6}{25}$ **d** $\frac{12}{25}$
3 **a** $\frac{4}{25}$ **b** $\frac{9}{25}$ **c** $\frac{6}{25}$ **d** $\frac{12}{25}$
4 **a** $\frac{9}{49}$ **b** $\frac{24}{49}$
5 **a** $\frac{4}{9}$ **b** $\frac{4}{9}$ **c** $\frac{1}{9}$
6 **a** $\frac{13}{28}$ **b** $\frac{15}{28}$
7 **a** $\frac{1}{169}$ **b** $\frac{1}{4}$ **c** $\frac{30}{169}$ **d** $\frac{1}{8}$
8 **a** $\frac{4}{15}$ **b** $\frac{11}{15}$ **c** 8 days

Exercise 185*

1 **a** $\frac{4}{15}$ **b** $\frac{8}{15}$ **c** $\frac{3}{5}$
2 **a** $\frac{9}{56}$ **b** $\frac{31}{56}$ **c** $\frac{47}{56}$
3 **a** $\frac{1}{9}$ **b** $\frac{4}{9}$ **c** $\frac{5}{9}$
4 **a** $\frac{1}{7}$ **b** $\frac{10}{21}$ **c** $\frac{6}{7}$
5 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{15}{32}$ **d** $\frac{3}{16}$
6 **a** $\frac{1}{3}$ **b** $\frac{2}{3}$
7 **a** $\frac{1}{5}$ **b** $\frac{13}{35}$ **c** $\frac{4}{5}$
8 **a** Top: $\frac{1}{3}$, Bottom: $\frac{1}{4}$ **b** $\frac{5}{9}$

Exercise 186

1 **a** $\frac{1}{9}$ **b** $\frac{4}{9}$
2 **a** $\frac{1}{5}$ **b** $\frac{1}{5}$
c Let X be the number of kings dealt in the first three cards:
$\text{p}(X \geqslant 1) = 1 - \text{p}(X = 0) = 1 - \frac{16}{20} \times \frac{15}{19} \times \frac{14}{18} = \frac{29}{57}$
3 **a** $\frac{43}{63}$ **b** $\frac{20}{63}$ **c** $\frac{2}{7}$
d Let X be the number of beads added to the box:
$$\text{p}(W_2) = \frac{2}{7} \times \frac{(2+X)}{(7+X)} + \frac{5}{7} \times \frac{2}{(7+X)}$$
$$= \frac{2}{[7(7+X)]} \times [(2+X)+5] = \frac{2}{7} \quad \text{Therefore true!}$$
4 **a** 0.0459 **b** 0.3941

Exercise 186*

1 **a** 0.0034 **b** 0.0006 **c** 0.0532
2 **a** $\frac{9}{16}$ **b** $\frac{27}{64}$ **c** $\frac{29}{128}$
3 **a** $\frac{1}{8}$ **b** $\frac{8}{15}$ **c** $\frac{13}{30}$
4 **a** $\frac{5}{18}$
b Let event X be 'clock is slow at noon on Wednesday':
$\text{p}\overline{X} = 1 - \text{p}(X) = 1 - \frac{7}{24} = \frac{47}{54}$
5 **a** $\text{p}(H_1) = \frac{1}{4}$
b $\text{p}(H_2) = \text{p}(HH) + \text{p}(\overline{H}H) = \frac{1}{4} \times \frac{12}{51} + \frac{3}{4} \times \frac{13}{51} = \frac{1}{4}$
c $\text{p}(H_3) = \text{p}(HHH) + \text{p}(\overline{H}\,\overline{H}HH) + \text{p}(\overline{H}HH) + \text{p}(\overline{H}\,\overline{H}H)$
$= \frac{1}{4} \times \frac{12}{51} \times \frac{11}{50} + \frac{1}{4} \times \frac{39}{51} \times \frac{12}{50} + \frac{3}{4} \times \frac{13}{51} \times \frac{12}{50} + \frac{3}{4} \times \frac{38}{51} \times \frac{13}{50}$
$= \frac{1}{4}$
6 Let event A be that two people share the same birth date in class.
$$\text{p}(A) = 1 - \text{p}(\overline{A})$$
$$= 1 - 1 \times \frac{364}{365} \times \frac{363}{365} \times \frac{362}{365} \times \frac{361}{365} \times \ldots \times \frac{343}{365}$$
$$= 0.507 \approx 50\%$$

Exercise 187 (Revision)

1 **a** $\frac{9}{25}$ **b** $\frac{12}{25}$
2 **a** $\frac{1}{16}$ **b** $\frac{3}{8}$
3 **a** $\frac{9}{25}$ **b** $\frac{12}{25}$
4 **a** $\frac{4}{5}$ **b** $\frac{6}{25}$
5 **a** $\frac{1}{36}$ **b** $\frac{5}{18}$ **c** $\frac{5}{9}$
6 **a** 0.1 **b** 0.7 **c** 0.15

Exercise 187* (Revision)

1 **a** $\frac{1}{6}$ **b** $\frac{5}{18}$ **c** $\frac{13}{18}$
2 **a** $\frac{2}{9}$ **b** $\frac{8}{45}$ **c** $\frac{2}{45}$ **d** $\frac{43}{45}$
3 **a** $\frac{1}{32}$ **b** $\frac{5}{16}$ **c** $\frac{3}{16}$
4 $\frac{n}{25} \times \frac{(n-1)}{24} = 0.07$
$n^2 - n - 42 = 0 \rightarrow n = 7$
$\text{p(diff colours)} = \frac{7}{25} \times \frac{18}{24} + \frac{18}{25} \times \frac{7}{24} = \frac{21}{50}$
5 **a** 1:3:5 **b** $\frac{4}{45}$
c **(i)** $\frac{16}{2025}$ **(ii)** $\frac{164}{2025}$ **(iii)** $\frac{344}{2025}$
6 **a** $\frac{6}{25}$ **b** $\frac{19}{25}$ **c** $\frac{12}{43}$ **d** $\frac{31}{43}$ **e** 0.320

Multiple choice 8

1 B **2** B **3** D **4** A **5** B
6 D **7** C **8** D **9** A **10** A

Self-assessment 8

1 **a** $4n + 1$ **b** 401 **c** 50
2 **a** $18 - 6n$ **b** −582 **c** 25
3 $f(2) = 0$
4 $p = 6$
5 $(x+1)(x-1)(x-5)$; $x = -1$, 1 or 5
6 $a = -2$ or 2
7 $p = -14$, $(x+3)(x-2)(3x+4)$; $x = -3$, 2 or $-\frac{4}{3}$
8 **a** $x(x+2) = 6$ **b** $x = 1.65$
9 2.19 × 3.19 m
10 4 cm
11 4.83 (3 s.f.)
12 11 and 12 or −11 and −12
13 **a** 1.2 **b** −3.5 **c** 0.67
14 **a** −1.8, 1.1 **b** −2.8, 2.4 **c** −2.4, 1.7
15 (1.1, 1.8) (−1.8, −3.2)
16 **a** 31.3°, 5.55 cm **b** 45.1°, 13.4 cm
17 **a** 82.2 km **b** 045° **c** 225°
18 **a** 36.1 km **b** 064.6° **c** 34 km/h
19 **a** 0.2 **b** 0.1625 **c** 0.09725
20 **a** 0.1 **b** **(i)** 0.8 **(ii)** 0.5 **c** 0.9

Exercise 188

1 $\frac{57}{100}$ **2** $\frac{47}{99}$ **3** 7
4 Irrational **5** $\frac{3}{1}$ **6** Rational, $\frac{1}{1}$
7 Irrational **8** Rational, $\frac{0}{1}$ **9** e.g. 2.5
10 e.g. $\sqrt{53}$ **11** $\frac{2}{\pi}$ **12** $\frac{3}{\sqrt{\pi}}$

Exercise 188*

1 Irrational **2** $\frac{2}{5}$ **3** $\frac{3}{5}$
4 $\frac{3}{2}$ **5** Irrational **6** $\frac{1}{1}$

7 Irrational **8** e.g. 3.5 **9** e.g. $\sqrt{7}$

10 e.g. $\sqrt{2} \times \sqrt{8}$ **11** $\frac{9}{\pi}$

12 **a** e.g. 3:4:5 **b** e.g. $\sqrt{2} : \sqrt{2} : 2$

c e.g. $1:2:\sqrt{5}$ **d** e.g. $1:\sqrt{3}:2$

Exercise 189

1 $6\sqrt{5}$ **2** $4\sqrt{3}$ **3** 32 **4** 20 **5** 8

6 105 **7** $2\sqrt{2}$ **8** 4 **9** 8

Exercise 189*

1 $5\sqrt{11}$ **2** $4\sqrt{7}$ **3** 99 **4** 96 **5** $56\sqrt{7}$

6 $120\sqrt{2}$ **7** 9 **8** 4 **9** 6

Exercise 190

1 $2\sqrt{3}$ **2** $3\sqrt{2}$ **3** $4\sqrt{3}$ **4** $3\sqrt{5}$

5 $3\sqrt{3}$ **6** $6\sqrt{2}$ **7** $\sqrt{50}$ **8** $\sqrt{27}$

9 $\sqrt{54}$ **10** $\frac{1}{2}$ **11** $\frac{2}{5}$ **12** $\frac{2}{3}$

13 $12, 10\sqrt{2}, \sqrt{26}$

Exercise 190*

1 $2\sqrt{7}$ **2** $3\sqrt{11}$ **3** $4\sqrt{5}$ **4** $3\sqrt{13}$

5 $5\sqrt{3}$ **6** $5\sqrt{3}$ **7** $\sqrt{75}$ **8** $\sqrt{80}$

9 $\sqrt{63}$ **10** $\frac{1}{6}$ **11** $\frac{9}{10}$ **12** $\frac{7}{13}$

13 $8\text{ cm}^2, 4\sqrt{8}\text{ cm}$

Exercise 191

1 $\frac{\sqrt{3}}{3}$ **2** $\frac{\sqrt{5}}{5}$ **3** $\sqrt{3}$ **4** $2\sqrt{2}$ **5** $\frac{2\sqrt{3}}{3}$

6 $\frac{3\sqrt{2}}{2}$ **7** $\frac{3\sqrt{2}}{4}$ **8** $\frac{5\sqrt{3}}{6}$ **9** $\frac{(2+\sqrt{2})}{2}$ **10** $\frac{(3+\sqrt{3})}{3}$

Exercise 191*

1 $\frac{\sqrt{13}}{13}$ **2** $\frac{\sqrt{11}}{11}$ **3** $\sqrt{a}$ **4** $b\sqrt{b}$ **5** $2\sqrt{3}-1$

6 $2\sqrt{2}-1$ **7** $\frac{\sqrt{3}}{3}$ **8** $\frac{\sqrt{6}}{3}$ **9** $2+\sqrt{5}$ **10** $3+2\sqrt{7}$

Exercise 192 (Revision)

1 $0.\dot{3}$ and $\sqrt{25}$ **2** 2, for example (answers may vary)

3 e.g. $\sqrt{11}$ **4** $\sqrt{45}$ **5** $5\sqrt{3}$ **6** $\sqrt{3}$

7 18 **8** 1.5 **9** $2\sqrt{2}$ **10** $3\sqrt{7}$

11 $3\sqrt{3}$ **12** $\frac{2}{3}$ **13** $\sqrt{5}$ **14** $2\sqrt{3}$

15 $\frac{3}{2}$ **16** $1+\sqrt{3}$ **17** $16\sqrt{2}, 30, 2\sqrt{17}$

Exercise 192* (Revision)

1 $\sqrt{3^2}$ and $0.2\dot{3}\dot{3}$ **2** 3, for example (answers may vary)

3 e.g. $\sqrt{41}$ **4** $\sqrt{176}$ **5** $8\sqrt{5}$ **6** $2\sqrt{5}$

7 75 **8** $\frac{5}{3}$ **9** $4\sqrt{3}$ **10** $11\sqrt{2}$

11 $19\sqrt{6}$ **12** $\frac{3}{5}$ **13** $\frac{\sqrt{5}}{10}$ **14** $2\sqrt{6}$

15 $2+\sqrt{6}$ **16** 2 **17** $\frac{(2+3\sqrt{2})}{4}$ **18** $5\sqrt{3}, 18$

19 $\sqrt{3}, \frac{1}{2}, \frac{\sqrt{3}}{2}$

Exercise 193

1 $\frac{3}{2}$ **2** $\frac{4}{3}$ **3** $\frac{x}{y}$ **4** $\frac{y}{x}$ **5** $x+2$

6 $x-7$ **7** $\frac{1}{(x+2)}$ **8** $\frac{1}{(x+2)}$ **9** $\frac{(x+y)}{(x-y)}$

Exercise 193*

1 $\frac{3}{5}$ **2** $\frac{7}{3}$ **3** $\frac{x}{y}$ **4** $\frac{1}{y}$ **5** $\frac{(x-4)}{(x+3)}$

6 $\frac{(x-3)}{(x+5)}$ **7** $\frac{(x+3)}{(x+4)}$ **8** $\frac{(x-2)}{(x-5)}$ **9** $\frac{(r-3)}{(r+1)}$

Exercise 194

1 $\frac{5x+3}{6}$ **2** $\frac{5x+4}{6}$ **3** $\frac{x-3}{12}$ **4** $\frac{8x+12}{15}$ **5** $\frac{5x+3}{4}$

6 $\frac{4x-2}{3}$ **7** $\frac{3-x}{2}$ **8** $\frac{7x+2}{12}$ **9** $\frac{5x-1}{6}$

Exercise 194*

1 $\frac{9x+13}{10}$ **2** $\frac{11x-5}{12}$ **3** $\frac{1-12x}{15}$ **4** $\frac{16-3x}{14}$ **5** $\frac{2x+9}{8}$

6 $\frac{x-230}{15}$ **7** $\frac{84-7x}{24}$ **8** $\frac{x-6}{72}$ **9** $\frac{7x+1}{100}$

Exercise 195

1 $\frac{5}{6x}$ **2** $\frac{1}{4x}$ **3** $\frac{(x-4)}{2(x-2)}$

4 $\frac{2x}{(x-1)(x+1)}$ **5** $\frac{(x+8)}{(x-1)(x+2)}$ **6** $\frac{(x-15)}{(x+1)(x-3)}$

7 $\frac{(x^2+x+2)}{x(x+2)}$ **8** $\frac{(x^2+x-3)}{x(x-3)}$ **9** $\frac{(2x+3)}{(x+2)(x+1)}$

10 $\frac{2(x+5)}{(x+3)(x+1)}$

Exercise 195*

1 $\frac{17}{15x}$ **2** $\frac{x-1}{(x+3)(x+2)}$ **3** $\frac{x^2+x+1}{1+x}$ **4** $\frac{x}{1+x}$

5 $\frac{1}{(x+1)}$ **6** $\frac{1}{x}$ **7** $\frac{2x+3}{(x+1)(x+2)}$ **8** $\frac{1}{(x-1)}$

9 $\frac{-3x-2}{(x+1)^2}$ **10** $\frac{8}{(x-3)(x+1)}$

Exercise 196

1 $2x$ **2** 6 **3** $\frac{y}{x}$ **4** $\frac{2a}{b}$ **5** $\frac{(p-1)}{(p-2)}$

6 $\frac{(r+2)}{(r-1)}$ **7** $\frac{(x-2)}{(x-4)}$ **8** $\frac{(x-3)}{(x-5)}$ **9** $\frac{(x+3)}{(x+4)}$ **10** $\frac{(x+2)}{(x+5)}$

Exercise 196*

1 $\frac{2(x-3)}{(x+2)}$ **2** $\frac{2(x-4)}{(x-3)}$ **3** $\frac{1}{(x+1)}$ **4** $x+2$ **5** $\frac{(x+2)}{(x-2)}$

6 $\frac{(x-4)}{(x+2)}$ **7** $\frac{(p+4)}{(p-5)}$ **8** $\frac{(q+2)}{(q+6)}$ **9** $\frac{y}{(x+3y)}$ **10** $\frac{(x-y)}{x}$

Exercise 197

1 21 **2** 2 **3** $\frac{1}{2}$ **4** 3 **5** -8

6 $-\frac{2}{3}$ **7** $\frac{1}{2}$ **8** 230 **9** -6 **10** 0

Exercise 197*

1 $\frac{2}{5}$ **2** 15 **3** $\frac{1}{3}$ **4** 0 **5** 15

6 7 **7** 1 **8** $\frac{7}{5}$ **9** $-\frac{8}{3}$ **10** $-\frac{5}{13}$

11 6 km **12** 30 km

Exercise 198

1 $-7, 2$ **2** $1, 2$ **3** 6 **4** $-\frac{2}{3}$

5 $-4, 5$ **6** $0, 6$ **7** $-1, 4$ **8** $-\frac{1}{3}, 2$

Exercise 198*

1 $-2.77, 1.27$ **2** $-4, 1$ **3** 4

4 $-6, 6$ **5** $-\frac{7}{3}, 2$ **6** $-0.768, 0.434$

7 $-2, 6$ **8** 60 **9** 10.5

10 -2.5 **11** $\frac{2}{5}, 2$ **12** $-\frac{2}{3}, 5$

Exercise 199 (Revision)

1 3 **2** $x+2$ **3** $\frac{(x+3)}{(x-3)}$

4 $\frac{(x-1)}{(x+3)}$ **5** $\frac{(x-3)}{6}$ **6** $\frac{3x-21}{20}$

7 $\frac{-3}{(x+1)(x-2)}$ **8** $\frac{x+5}{(x-1)(x+1)}$ **9** $\frac{x^2+2x-4}{x(x-2)}$

10 $\frac{4x+10}{(x+2)(x+4)}$ **11** $\frac{x(x-1)}{(x+1)}$ **12** $\frac{x-2}{x-3}$

13 $\frac{8}{3}$ **14** -3 **15** -7

16 -2 **17** $-2, 1$ **18** $\frac{1}{2}$

19 $-8, 2$ **20** 1

Exercise 199* (Revision)

1 $\frac{2}{3}$ **2** $\frac{(x-11)}{(x+5)}$ **3** $\frac{(x+4)}{(x-7)}$

4 $\frac{(x-1)}{(x+1)}$ **5** $\frac{x-11}{12}$ **6** $\frac{5x-13}{18}$

7 $\frac{5x+7}{12}$ **8** $\frac{7x+6}{10}$ **9** $\frac{2x+3}{(x+1)(x+2)}$

10 $\frac{-1}{(x-4)(x+1)}$ **11** $x+1$ **12** 1

13 -1 **14** 5 **15** $\frac{15}{61}$

16 5 **17** $\frac{1}{3}$ or 2 **18** $-\frac{2}{3}$ or 1

19 -0.464 or 6.46 **20** -9.16 or 3.16

Exercise 200

1 $\frac{dy}{dx} = 3x^2$ **2** $\frac{dy}{dx} = 4x^3$ **3** $\frac{dy}{dx} = 5x^4$

4 $\frac{dy}{dx} = 6x^5$ **5** $\frac{dy}{dx} = 8x^7$ **6** $\frac{dy}{dx} = 7x^6$

7 $\frac{dy}{dx} = 10x^9$ **8** $\frac{dy}{dx} = 11x^{10}$ **9** $\frac{dy}{dx} = -x^{-2}$

10 $\frac{dy}{dx} = -2x^{-3}$ **11** $\frac{dy}{dx} = -x^{-2}$ **12** $\frac{dy}{dx} = -2x^{-3}$

13 4 **14** 12 **15** 32

16 80 **17** 448 **18** 192

Exercise 200*

1 $\frac{dy}{dx} = 12x^{11}$ **2** $\frac{dy}{dx} = 9x^8$ **3** $\frac{dy}{dx} = 1$

4 $\frac{dy}{dx} = 0$ **5** $\frac{dy}{dx} = -3x^{-4}$ **6** $\frac{dy}{dx} = -4x^{-5}$

7 $\frac{dy}{dx} = -6x^{-7}$ **8** $\frac{dy}{dx} = -5x^{-6}$ **9** $\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$

10 $\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}}$ **11** $\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$ **12** $\frac{dy}{dx} = 0$

13 $-\frac{3}{16}$ **14** $-\frac{1}{8}$ **15** $-\frac{1}{4}$

16 $-\frac{1}{4}$ **17** $\frac{1}{4}$ **18** $\frac{1}{12}$

19 0 **20** 1

Exercise 201

1 $\frac{dy}{dx} = 5x^4 + 2x$ **2** $\frac{dy}{dx} = 4x^3 + 3x^2$ **3** $\frac{dy}{dx} = 6x^2 + 4$

4 $\frac{dy}{dx} = 20x^3 - 3$ **5** $\frac{dy}{dx} = 9x^2 + 8x^3$ **6** $\frac{dy}{dx} = 10x - 3x^2$

7 $\frac{dy}{dx} = 4x$ **8** $\frac{dy}{dx} = 18x^2$ **9** $\frac{dy}{dx} = -x^{-2}$

10 $\frac{dy}{dx} = -2x^{-3}$ **11** $\frac{dy}{dx} = -6x^{-4}$ **12** $\frac{dy}{dx} = -20x^{-5}$

13 $\frac{dy}{dx} = 2x - 2x^{-3}$ **14** $\frac{dy}{dx} = 3x^2 - x^{-2}$ **15** $\frac{dy}{dx} = 3x^2 + 3x^{-4}$

16 $\frac{dy}{dx} = 1 + x^{-2}$

Exercise 201*

1 $\frac{dy}{dx} = 3x^2 + 4x$ **2** $\frac{dy}{dx} = 3x^2 + 2$ **3** $\frac{dy}{dx} = 2x + 4$

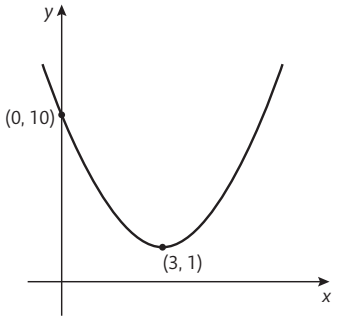
4 $\frac{dy}{dx} = 2x + 6$ **5** $\frac{dy}{dx} = 2x + 6$ **6** $\frac{dy}{dx} = 2x + 10$

7 $\frac{dy}{dx} = 8x - 4$ **8** $\frac{dy}{dx} = 18x - 30$ **9** $\frac{dy}{dx} = 8x - 2x^{-3}$

10 $\frac{dy}{dx} = 2x - 2x^{-3}$ **11** $\frac{dy}{dx} = 5$ **12** $\frac{dy}{dx} = 7$

13 $(3, -9)$ **14** $(0.5, 4)$

15



16 $A = (0.5, 4)$, $B = (-0.5, -4)$

Exercise 202

1 **a** $2x + 4$ **b** $(-2, 6)$

2 **a** $2x + 2$ **b** $(-1, -9)$

3 **a** $x = 3$ **b** $y = 24$

4 **a** $x = -5$ **b** $y = 31$

5 **a** 15 **b** 6 **c** $y = 6x + 9$

6 **a** 7 **b** 8 **c** $y = 7x - 14$

7 **a** $8t + 8$ **b** 16° per minute **c** 48° per minute

8 **a** $200t + 200$ **b** 400 voters per month
c 1000 voters per month

9 **a** $3x^2 - 24x$ **b** $x = 0$ or 8 **c** $(0, 5), (8, -251)$
d Maximum: $(0, 5)$, minimum: $(8, -251)$;

10 **a** $3x^2 - 12x$ **b** $x = 0, 4$ **c** $(0, 10), (4, -22)$
d Maximum: $(0, 10)$, minimum: $(4, -22)$

11 **a** $6 - 2x$ **b** 20

12 **a** $2x - 10$ **b** -15

13 **a** $8 + 2x$ **b** -1

14 **a** $-2 - 2x$ **b** 9

Exercise 202*

1 $6x - 7$ **b** $y = 5x - 7$

2 **a** $2x + 4x^{-2} + 1$ **b** $y = 3x + 7$

3 Minimum at $(-3, -54)$, maximum at $(3, 54)$

4 Maximum at $(0, 0)$, minimum at $(4, -32)$

5 **a** 11 150 **b** 305 per year

6 **a** \$490 000 **b** \$22 000 per month

7 **a** $80 - 40t$ **b** $t = 2$ **c** 350°
d 120° per hour

8 **a** $6 - \frac{t}{2}$ **b** 12 minutes **c** 24 m per minute

9 **a** $\frac{dy}{dx} = 6x^2 - 2x - 4$ **b** $x = \frac{-2}{3}$
c $(1, 7)\left(\frac{-2}{3}, 11.63\right)$
d $\left(\frac{-2}{3}, 11.63\right)$ maximum, $(1, 7)$ minimum

10 **a** $6x^2 + 4x - 16$
b $x = \frac{4}{3}$ **c, d** $\left(\frac{4}{3}, -25.0\right)$ minimum, $(-2, 12)$ maximum

11 **a** $2x - 16x^{-2}$ **b** $x = 2$ **c** $(2, 12)$

12 **a** $2x - 2x^{-3}$ **b** $x = -1, 1$
c $(-1, 4), (1, 4)$

13 **a** $\frac{dC}{dt} = 3 - \frac{27}{t^2}$ **b** 18°C **c** 1.92°C per month

14 **a** $6t - 48t^{-2}$ **b** 36 **c** 21 per year

Exercise 203

1 $10t$ **2** $48 - 32t$

3 **a** $40 + 10t$ **b** 70 m/s

4 **a** $30 - 10t$ **b** 0 m/s

5 32

6 -32

7 **a** $v = 3t^2 + 8t - 5$ **b** $a = 6t + 8$ **c** $v = 6$ m/s; $a = 14$ m/s^2
8 **a** $v = 3t^2 - 4t + 3$ **b** $a = 6t - 4$ **c** $v = 7$ m/s; $a = 8$ m/s^2
9 **a** $2t + 10$ **b** 14 m/s
10 **a** $6 - 2t$ **b** 2 m/s

Exercise 203*

1 **a** $v = 8t + \frac{2}{t^2}$ **b** $a = 8 - \frac{4}{t^3}$
2 **a** $v = 6 - \frac{4}{t^2}$ **b** $a = \frac{8}{t^3}$
3 **a** $20 + 10t$ **b** 30 m/s, 40 m/s, 50 m/s **c** 10 m/s^2
4 **a** $8t + 5$ **b** 13 m/s, 21 m/s, 29 m/s **c** 8 m/s^2
5 **a** $40 - 10t$ **b** 30 m/s, 20 m/s, 10 m/s, 0 m/s **c** 80 m
6 **a** $25 - 2t$ **b** 12.5 **c** 156.25 m
7 **a** $\frac{ds}{dt} = -\frac{36}{t^3}$; $\frac{dv}{dt} = \frac{36}{t^3}$
b 3 s **c** 28 m
8 **a** $\frac{ds}{dt} = -t + \frac{8}{t^2}$; $\frac{dv}{dt} = -1 - \frac{16}{t^3}$
b 7 m/s **c** 2 s **d** 44 m
9 **a** 8 s **b** 3 s **c** 170 m
10 **a** 5 s **b** 50 m

Exercise 204 (Revision)

1 **a** $\frac{dy}{dx} = 3$ **b** $\frac{dy}{dx} = 0$ **c** $\frac{dy}{dx} = 3x^2$ **d** $\frac{dy}{dx} = 4x^3$
e $\frac{dy}{dx} = 5x^4$ **f** $\frac{dy}{dx} = 12x^5$ **g** $\frac{dy}{dx} = 15x^4$ **h** $\frac{dy}{dx} = 160x^7$
2 **a** $\frac{dy}{dx} = 6x^2 + 10x$ **b** $\frac{dy}{dx} = 14x - 3$ **c** $\frac{dy}{dx} = 15x^2$
d $\frac{dy}{dx} = 12x^3 - 10x$ **e** $\frac{dy}{dx} = 3x^2 + 10x$ **f** $\frac{dy}{dx} = 2x + 2$ **g** $\frac{dy}{dx} = 4x - 9$
h $\frac{dy}{dx} = 2x + 4$
3 **a** $\frac{dy}{dx} = 4$ **b** $\frac{dy}{dx} = -5$ **c** $\frac{dy}{dx} = 26$ **d** $\frac{dy}{dx} = 19$
4 **a** $\frac{dQ}{dt} = 3t^2 - 16t + 14$
b **(i)** 14 m^3/s^2 **(ii)** -6 m^3/s^2 **(iii)** 9 m^3/s^2
5 **a** $\frac{dy}{dx} = 3x^2 - 3$ **b** $(1, 0), (-1, 4)$
c $(-1, 4)$ is max, $(1, 0)$ is min.
6 **a** $\frac{dy}{dx} = 3x^2 + 6x - 9$ **b** $(-3, 28), (1, -4)$
c $(-3, 28)$ is max, $(1, -4)$ is min.
7 **a** $v = 24t$ m/s, 48 m/s **b** $a = 24$ m/s^2, 24 m/s^2
8 **a** $v = 3t^2 + 8t - 3$ m/s, 377 m/s **b** $a = 6t + 8$ m/s^2, 68 m/s^2

Exercise 204* (Revision)

1 **a** $\frac{dy}{dx} = -x^{-2} = -\frac{1}{x^2}$ **b** $\frac{dy}{dx} = -2x^{-3} = -\frac{2}{x^3}$
c $\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$ **d** $\frac{dy}{dx} = -3x^{-4} = -\frac{3}{x^4}$
e $\frac{dy}{dx} = -16x^{-5} = -\frac{16}{x^5}$ **f** $\frac{dy}{dx} = -x^{-3} = -\frac{1}{x^3}$
g $\frac{dy}{dx} = 4x + 3 - 4x^{-2}$ **h** $\frac{dy}{dx} = 2 + 3x^{-2}$
2 **a** -2 **b** 2 **c** 0 **d** $-2\frac{1}{8}$
3 $4y = 3x + 4$
4 $y = -3x, y = 3x - 9$
5 **a** $y = 4x - 7$ **b** $x = 0$ is a max, $x = \frac{4}{3}$ is a min.
6 $p = 1$
7 **a** $\frac{dC}{dt} = 4 - 16t^{-2} = 4 - \frac{16}{t^2}$ **b** $t = 2, C = 16$
c -12 °C/month
8 **a** $\frac{dP}{dt} = 10t - 10\,000t^{-2} = 10t - \frac{10\,000}{t^2}$
b $t = 10, P = 1500$
9 $t = 5, s = 125$ m
10 **a** $v = \frac{ds}{dt} = 3t^2 - 300$ km/s, $a = \frac{dv}{dt} = 6t$ km/s^2
b at $t = 5$, $v = -225$ km/s, $a = 30$ km/s^2
c $t = 10$ s

Exercise 205

1 7.39 cm^2 **2** 29.7 cm^2 **3** 36.2 cm^2
4 8.46 cm^2 **5** 121 cm^2 **6** 77.0 cm^2

Exercise 205*

1 173 cm^2 **2** 48.1 cm **3** 16.5 cm
4 51.4° **5** 53.5 cm^2 **6** 65.8 cm

Exercise 206

1 **a** 11.7 cm **b** 14.2 cm **c** 34.4°
2 **a** 18.6 cm **b** 28.1 cm **c** 48.6°
3 **a** 14.1 cm **b** 17.3 cm **c** 35.4°
4 **a** 28.3 cm **b** 34.6 cm **c** 35.1° **d** 19.5°
5 **a** 4.47 m **b** 4.58 m **c** 29.2° **d** 12.6°
6 **a** 407 m **b** 402 m **c** 8.48° **d** 13.3°
7 **a** 43.3 cm **b** 68.7 cm **c** 81.2 cm
8 **a** 28.9 cm **b** 75.7 cm **c** 22.4°

Exercise 206*

1 **a** 16.2 cm **b** 67.9° **c** 55.3 cm^2
2 **a** 26.5 cm **b** 61.8° **c** 1530 cm^2
3 **a** 30.3° **b** 31.6° **c** 68.9°
4 **a** 36.9° **b** 828 cm^2
5 **a** 15 m **b** 47.7° **c** €91 300
6 **a** 66.4° **b** 32.9°
7 46.5 m
8 **a** OW = 4290 m, OS = 2760 m
b 36.0° **c** 197 km/h

Exercise 207

1 5.08 cm^2 **2** 1.85 cm^2 **3** 24.0 cm^2
4 14.4 cm^2 **5** 411 cm^2 **6** 569 cm^2

Exercise 207*

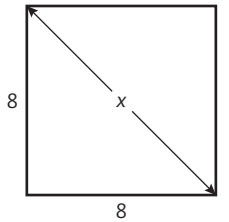
1 6.97 cm^2 **2** 12.8 cm **3** 11.5 cm^2
4 8.68 cm **5** 7.96×10^{-1} m^3 **6** 165 cm

Exercise 208 (Revision)

1 148 cm^2
2 **a** AC = 42.4 cm **b** 33.9 cm **c** 68.0° **d** 58.0°
3 **a** 18.4° **b** 500 m **c** 11.3°
4 **a** 68.9 m^2 **b** 120 m^2
5 **a** 22.4 cm **b** 26.4 cm **c** 32.1° **d** 35.0°
6 **a** 14.1 cm^2 **b** 67.8 cm^2

Exercise 208* (Revision)

1 Pythagoras proof
2 4.68 m^2
3 **a** AC = 70.7 cm **b** 98.7 cm
c 27.9° **d** 216 000 cm^2
4 **a** x is the length of the diagonal of the square that is the bottom face of the cube.
Using Pythagoras
$x^2 = 8^2 + 8^2$
$= 128$
$x = \sqrt{128} = \sqrt{64 \times 2} = \sqrt{64} \times \sqrt{2} = 8\sqrt{2}$



b 50.5°
5 92.1 m
6 **a** 70.7 m **b** 60.4 m **c** 41.4° **d** 67.5° **e** 9038 m^2
7 14.3 cm
8 5.2 cm/s

Exercise 209

1 **a** $\overrightarrow{XY} = X$ **b** $\overrightarrow{EO} = 4Y$ **c** $\overrightarrow{WC} = -8Y$ **d** $\overrightarrow{TP} = -4X$

2 **a** $\overrightarrow{KC} = 2X - 4Y$ **b** $\overrightarrow{VC} = X - 8Y$ **c** $\overrightarrow{CU} = -2X + 8Y$ **d** $\overrightarrow{AS} = 3X + 6Y$

3 **a** $\overrightarrow{HJ}$ **b** $\overrightarrow{HN}$ **c** $\overrightarrow{HL}$ **d** $\overrightarrow{HO}$

4 **a** $\overrightarrow{HT}$ **b** $\overrightarrow{HP}$ **c** $\overrightarrow{HD}$ **d** $\overrightarrow{HY}$

5 **a** $\overrightarrow{DC} = \mathbf{x}$ **b** $\overrightarrow{DB} = \mathbf{x} + \mathbf{y}$ **c** $\overrightarrow{BC} = -\mathbf{y}$ **d** $\overrightarrow{AC} = \mathbf{x} - \mathbf{y}$

6 **a** $\overrightarrow{AC} = 2\mathbf{x} - \mathbf{y}$ **b** $\overrightarrow{DB} = \mathbf{x} + \mathbf{y}$ **c** $\overrightarrow{BC} = \mathbf{x} - \mathbf{y}$ **d** $\overrightarrow{CB} = \mathbf{y} - \mathbf{x}$

7 **a** $\overrightarrow{DC} = \mathbf{x}$ **b** $\overrightarrow{AC} = \mathbf{x} + \mathbf{y}$ **c** $\overrightarrow{BD} = \mathbf{y} - \mathbf{x}$ **d** $\overrightarrow{AE} = \frac{1}{2}(\mathbf{x} + \mathbf{y})$

8 **a** $\overrightarrow{BD} = \mathbf{y} - \mathbf{x}$ **b** $\overrightarrow{BE} = \frac{1}{2}(\mathbf{y} - \mathbf{x})$ **c** $\overrightarrow{AC} = (\mathbf{x} + \mathbf{y})$ **d** $\overrightarrow{AE} = \frac{1}{2}(\mathbf{x} + \mathbf{y})$

9 **a** $\overrightarrow{AB} = \mathbf{x} - \mathbf{y}$ **b** $\overrightarrow{AD} = 3\mathbf{x}$ **c** $\overrightarrow{CF} = 2\mathbf{y} - 3\mathbf{x}$ **d** $\overrightarrow{CA} = \mathbf{y} - 3\mathbf{x}$

10 **a** $\overrightarrow{PQ} = \mathbf{x} - \mathbf{y}$ **b** $\overrightarrow{PC} = 2\mathbf{x} - \mathbf{y}$ **c** $\overrightarrow{QB} = 2\mathbf{y} - \mathbf{x}$ **d** $\overrightarrow{BC} = 2\mathbf{x} - 2\mathbf{y}$

11 **a** $2\mathbf{x} + 4\mathbf{y}$ **b** $4\mathbf{y} - 2\mathbf{x}$ **c** $2\mathbf{y} - 2\mathbf{x}$ **d** $3\mathbf{x} - 4\mathbf{y}$

12 **a** $\overrightarrow{OE} = \mathbf{x} + \mathbf{z}$ **b** $\overrightarrow{OB} = \mathbf{x} + \mathbf{y}$ **c** $\overrightarrow{OF} = \mathbf{x} + \mathbf{y} + \mathbf{z}$
d $\overrightarrow{EC} = -\mathbf{z} + \mathbf{y} - \mathbf{x}$

Exercise 209*

1 **a** $\overrightarrow{AB} = Y - X$ **b** $\overrightarrow{AM} = \frac{1}{2}(Y - X)$ **c** $\overrightarrow{OM} = \frac{1}{2}(X + Y)$

2 **a** $\overrightarrow{AB} = Y - X$ **b** $\overrightarrow{AM} = \frac{1}{3}(Y - X)$ **c** $\overrightarrow{OM} = \frac{1}{3}(2X + Y)$

3 **a** $\overrightarrow{AB} = Y - X$; $\overrightarrow{OD} = 2X$; $\overrightarrow{DC} = 2Y - 2X$
b DC = 2AB and they are parallel lines

4 **a** $\overrightarrow{AB} = Y - X$; $\overrightarrow{OD} = 3X$; $\overrightarrow{DC} = 2Y - 3X$ **b** $\overrightarrow{OM} = \frac{1}{2}(3X - Y)$

5 **a** $\overrightarrow{AB} = Y - X$; $\overrightarrow{OC} = -2X$; $\overrightarrow{OD} = -2Y$; $\overrightarrow{DC} = 2Y - 2X$
b DC = 2AB and they are parallel lines

6 **a** $\overrightarrow{AB} = \mathbf{y} - \mathbf{x}$; $\overrightarrow{OD} = \frac{1}{3}(\mathbf{x} - \mathbf{y})$; $\overrightarrow{BD} = -\frac{1}{3}(\mathbf{x} + 2\mathbf{y})$; $\overrightarrow{DE} = -\frac{1}{6}(\mathbf{x} + 2\mathbf{y})$; $\overrightarrow{OE} = -\frac{1}{2}\mathbf{x}$

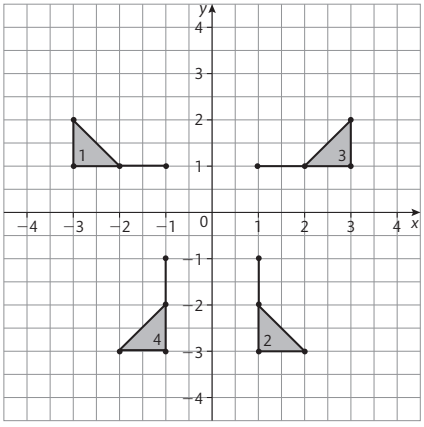
7 $\overrightarrow{AB} = \mathbf{y} - \mathbf{x}$; $\overrightarrow{BC} = \mathbf{y} - 2\mathbf{x}$; $\overrightarrow{AD} = 2\mathbf{y} - 4\mathbf{x}$; $\overrightarrow{BD} = \mathbf{y} - 3\mathbf{x}$

8 **a** $\overrightarrow{OP} = 2\mathbf{x}$; $\overrightarrow{AB} = \mathbf{y} - \mathbf{x}$; $\overrightarrow{BP} = 2\mathbf{x} - \mathbf{y}$
b $\overrightarrow{OX} = \mathbf{x} + \frac{1}{2}\mathbf{y}$

9 **a** $\overrightarrow{MA} = \frac{3}{5}\mathbf{x}$; $\overrightarrow{AB} = \mathbf{y} - \mathbf{x}$; $\overrightarrow{AN} = \frac{3}{5}(\mathbf{y} - \mathbf{x})$; $\overrightarrow{MN} = \frac{3}{5}\mathbf{y}$
b OB and MN are parallel; $\overrightarrow{MN} = \frac{3}{5}$ OB

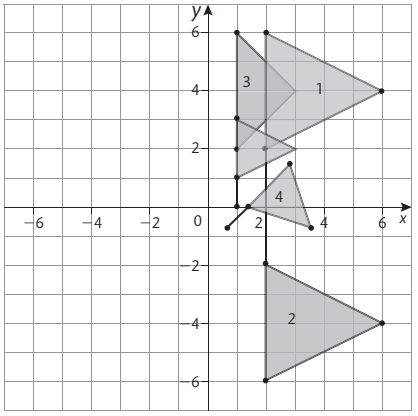
10 **a** $\overrightarrow{AB} = \mathbf{y} - \mathbf{x}$; $\overrightarrow{MN} = \frac{2}{3}\mathbf{x}$ **b** OA and MN are parallel and MN = $\frac{2}{3}$ OA

Exercise 210



1 Rotation of 90° anticlockwise about the origin.
2 Reflection in the x axis
3 Reflection in $y = x$
4 Rotation of 180° about the origin
5 **a** (3, 4), (5, 7), (5, 6)
b $\begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix}$
c Back to original shape
6 (1, −1), (1, 2), (−1,1)

Exercise 210*



1 Enlargement scale factor 2 centre the origin
2 Reflection in x axis and enlargement scale factor 2 centre the origin
3 Stretch parallel to y axis scale factor 2.
4 Rotation of 45° clockwise centre the origin
5 **a** (2, 1) (−3, 4) (−4, −2) **b** $\frac{1}{11}\begin{pmatrix} 3 & 5 \\ 1 & -2 \end{pmatrix}$
c Back to original shape
6 (1, −1), (0, 1), (−1, 1)

Exercise 211

1 **a** $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ **b** $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$
c Rotation 180° centre (0, 0)

2 **a** $\mathbf{X} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $\mathbf{Y} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ **b** $\mathbf{XY} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ $\mathbf{YX} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$
c Rotation of 180°

3 **a** $\mathbf{R} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ $\mathbf{X} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ **b** $\mathbf{RX} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ $\mathbf{XR} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$
c Reflection in $y = -x$

4 **a** 270° clockwise rotation about the origin
b $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ **c** $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

5 **a** $\mathbf{TS} = \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}$ $\mathbf{ST} = \begin{pmatrix} 0 & -1 \\ -2 & 0 \end{pmatrix}$ **b** A′(2, 1) B′(2, 3) C′(4, 1)
c F(−2, −2) G(−1, −2) H(1, 1)

6 77.0 cm^2

Exercise 211*

1 **a** $\mathbf{R} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ $\mathbf{Q} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
b $\mathbf{RQ} = \mathbf{QR} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ **c** Rotation 180°

2 **a** $\mathbf{X} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\mathbf{Y} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$
b $\mathbf{XY} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ $\mathbf{YX} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ **c** Rotation by 180°

3 **a** $\mathbf{P} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ $\mathbf{Q} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ $\mathbf{R} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
b $\mathbf{PQR} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ **c** Reflection in x axis
d $\mathbf{(PQR)^{-1}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

4 **b** $\begin{pmatrix} -0.5 & 0 \\ 0 & -0.5 \end{pmatrix}$ **c** $\begin{pmatrix} -0.5 & 0 \\ 0 & -0.5 \end{pmatrix}$

5 **a** $\mathbf{RS} = \begin{pmatrix} -2 & -2 \\ -2 & 0 \end{pmatrix}$ $\mathbf{SR} = \begin{pmatrix} 0 & -2 \\ -2 & -2 \end{pmatrix}$
b A′(−10, −4) B′(12, 2) C′(4, 8) **c** F$\left(-\frac{1}{2}, -1\right)$ G$\left(2, \frac{1}{2}\right)$ H(−3, 2)

Exercise 212 (Revision)

1 **a** $\overrightarrow{AB} = 3\mathbf{y} + \mathbf{x}$ **b** $\overrightarrow{AC} = 2\mathbf{y} + 2\mathbf{x}$ **c** $\overrightarrow{CB} = -\mathbf{x} + \mathbf{y}$

2 **a** $\overrightarrow{AB} = \mathbf{w} - \mathbf{v}$ **b** $\overrightarrow{AM} = \frac{1}{2}(\mathbf{w} - \mathbf{v})$ **c** $\overrightarrow{OM} = \frac{1}{2}(\mathbf{v} + \mathbf{w})$

3 **a** $\overrightarrow{AB} = \mathbf{y} - \mathbf{x}$ **b** $\overrightarrow{FB} = 2\mathbf{y} - \mathbf{x}$ **c** $\overrightarrow{FD} = \mathbf{y} - 2\mathbf{x}$

4 **a** $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ $\overrightarrow{OQ} = 2\mathbf{b}$ $\overrightarrow{AB} = \mathbf{b} = \mathbf{a}$ $\overrightarrow{PQ} = 2\mathbf{b} - 2\mathbf{a}$

b AB parallel to PQ, PQ = 2AB

5 **a** $\overrightarrow{AB} = 2\mathbf{y} - 2\mathbf{x}$ **b** $\overrightarrow{AM} = \mathbf{y} - \mathbf{x}$ **c** $\overrightarrow{OM} = \mathbf{x} + \mathbf{y}$

6 **a** **(i)** $\overrightarrow{MP} = \frac{2}{5}\mathbf{p}$ **(ii)** $\overrightarrow{PQ} = \mathbf{q} - \mathbf{p}$

(iii) $\overrightarrow{PN} = \frac{2}{5}(\mathbf{q} - \mathbf{p})$ **(iv)** $\overrightarrow{MN} = \frac{2}{5}\mathbf{q}$

b OQ parallel to MN, $MN = \frac{2}{5}OQ$

7 **a** A′(2, −1) B′(−15, 9) C′(−8, 5)

b $\begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$ **c** A′B′C′ → ABC

8 **a** A′(1, −5) B′(2, −7) C′(−1, 5) D′(−3, 7)

b E = (−6, −5), F = (3, 2), G = (6, 5), H = (−22, −17)

9 **a** $\mathbf{PQ} = \begin{pmatrix} 2 & 2 \\ 0 & 2 \end{pmatrix}$ $\mathbf{QP} = \begin{pmatrix} 2 & 2 \\ 0 & 2 \end{pmatrix}$

b A′(0, 2) B′(10, 6) C′(−2, −4) **c** $F\left(1, -\frac{1}{2}\right)$ $G\left(-\frac{1}{2}, 2\right)$ H(−1, 0)

10 **a** $\mathbf{RS} = \begin{pmatrix} -2 & 1 \\ -1 & 2 \end{pmatrix}$ **b** A′(3, 3) B′(−3, 3) C′(8, 10)

c E = (1, 1), F = (−1, 1), G = $\frac{1}{3}$(8, 10)

Exercise 212* (Revision)

1 **a** $\overrightarrow{XM}\begin{pmatrix} -1 \\ 3 \end{pmatrix}$

$\overrightarrow{XZ} = \begin{pmatrix} -10 \\ 6 \end{pmatrix}$

b $v\begin{pmatrix} 7 \\ 3 \end{pmatrix}$ **c** $\begin{pmatrix} 8 \\ 0 \end{pmatrix} + w\begin{pmatrix} -10 \\ 6 \end{pmatrix}$ **d** $v = \frac{2}{3}$, $w = \frac{1}{3}$

2 **a** $\overrightarrow{AC} = \mathbf{x} + \mathbf{y}$; $\overrightarrow{BE} = \frac{1}{3}\mathbf{y} - \mathbf{x}$

b **(i)** $\overrightarrow{BF} = v\left(\frac{1}{3}\mathbf{y} - \mathbf{x}\right)$

(ii) $\overrightarrow{AF} = \mathbf{x} + \overrightarrow{BF} = \mathbf{x} + v\left(\frac{1}{3}\mathbf{y} - \mathbf{x}\right)$

(iii) $v = \frac{3}{4}$

3 **a** $\overrightarrow{RS} = -\mathbf{r} + \mathbf{s}$ **b** $\overrightarrow{OP} = \frac{3}{2}\mathbf{r}$

c $\overrightarrow{PQ} = -\frac{3}{2}\mathbf{r} + 2\mathbf{s}$ **d** $\overrightarrow{OM} = \frac{3}{4}\mathbf{r} + \mathbf{s}$

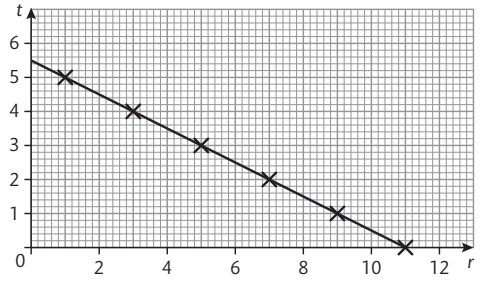
4 **a** **(i)** $\overrightarrow{PQ} = -\mathbf{p} + \mathbf{q}$ **(ii)** $\overrightarrow{PR} = \frac{1}{3}(\mathbf{q} - \mathbf{p})$ **(iii)** $OR = \frac{1}{3}(2\mathbf{p} + \mathbf{q})$

b **(i)** $k = \frac{3}{5}$ **(ii)** $OS = \frac{1}{5}(2\mathbf{p} + \mathbf{q})$

5 **a**

t	0	1	2	3	4	5
r	$\begin{pmatrix} 1 \\ 5 \end{pmatrix}$	$\begin{pmatrix} 3 \\ 4 \end{pmatrix}$	$\begin{pmatrix} 5 \\ 3 \end{pmatrix}$	$\begin{pmatrix} 7 \\ 2 \end{pmatrix}$	$\begin{pmatrix} 9 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 11 \\ 0 \end{pmatrix}$

b



c 8050 km/h, 117°

6 **a** **(i)** $\mathbf{r} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \mathbf{s} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$ **(ii)** $\mathbf{r} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \mathbf{s} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$

b $\begin{pmatrix} 1 \\ 3 \end{pmatrix}; \sqrt{10}$ **c** $\sqrt{5}; \sqrt{10}$

7 **a** A′(4, 3) B′(2, 1) C′(−1, −1)

b $\begin{pmatrix} -2 & 3 \\ 3 & -4 \end{pmatrix}$ **c** A′B′C′ → ABC

8 **a** A′(−21, 16) B′(−5, 4) C′(−153, 116) D′(−47, 36)

b E′(0, 1) F′(1, 2) G′(−3, 2) H′(1, 4)

9 **a** $\mathbf{GH} = \begin{pmatrix} 4 & 2 \\ 2 & 2 \end{pmatrix}$ $\mathbf{HG} = \begin{pmatrix} 2 & 2 \\ 2 & 4 \end{pmatrix}$

b A′(−12, −4) B′(8, 10) C′(2, −4)

c F = (−5, 3) G = $\frac{1}{2}$(−7, 13) H = $\frac{1}{2}$(8, −13)

10 **a** $\mathbf{MQ} = \begin{pmatrix} 4 & 2 \\ -3 & 0 \end{pmatrix}$ **b** A′(−26, 24) B′(−6, 0) C′(26, −24)

c E(−1, −2) F(1, −2) G(1, 2)

Multiple choice 9

1	D	**2**	C	**3**	C	**4**	B	**5**	B
6	D	**7**	B	**8**	A	**9**	B	**10**	C

Self-assessment 9

1 Rational **a**, **c** and **e**

2 **a** e.g. 2.3 **b** e.g. 2.8

3 **a** e.g. $\sqrt{17}$ **b** e.g. $\sqrt{23}$

4 **a** 6 **b** a **c** 18 **d** 6 **e** $\frac{2}{3}$

5 **a** $\sqrt{27}$ **b** $\sqrt{28}$ **c** $\sqrt{80}$ **d** $\sqrt{18}$

6 **a** $4\sqrt{5}$ **b** $12\sqrt{5}$ **c** 160 **d** 2

7 $p = 20$

8 $\frac{1}{2}\sqrt{2}$

9 $2\sqrt{3}$

10 **a** $x + 3$ **b** $\frac{(x+2)}{(x-2)}$ **c** $\frac{(x-1)}{(x+2)}$

11 **a** $\frac{1}{2}x$ **b** $\frac{5}{(x-2)(x+3)}$ **c** $\frac{-1}{(x+1)(x-4)}$

12 **a** $\frac{2(x+1)}{x}$ **b** $(x-2)(2x-1)$

13 **a** $\frac{dy}{dx} = 6x^2 - 12x$ **b** $\frac{dy}{dx} = 6x^2 - 2x + 2$ **c** $\frac{dy}{dx} = 2x - 1 - 2/x^3$

14 **a** $\frac{dy}{dx} = x^2 + 2x - 7$ **b** $\left(-4, 17\frac{2}{3}\right), \left(2, -12\frac{1}{3}\right)$

15 **a** $160t - 4t^2$

b **(i)** 1600 hairs/yr **(ii)** 2000 hairs/yr

c 40 yrs

16 **a** $v = 40 - 10t$ **b** $t = 4$ s **c** 80 m

17 **a** 16.7° **b** DF = 94.3 m

c 9.03° **d** 2.19 m/s

18 **a** 31.9 cm **b** 80.5 cm

19 **a** **(i)** **a** **(ii)** $\frac{1}{2}(\mathbf{a} + \mathbf{b})$ **(iii)** $\mathbf{a} - \mathbf{b}$ **(iv)** $\frac{1}{2}(\mathbf{a} + \mathbf{b})$

b Parallel and equal lengths

c PQRS is a parallelogram

20 **a** $\mathbf{a} - \mathbf{b}$

b **(i)** $(\mu - 1)\mathbf{a}$ **(ii)** $(\lambda - 1)\mathbf{b}$

c **(i)** $\mathbf{a} + (\lambda - 1)\mathbf{b}$ **(ii)** $(\mu - 2)\mathbf{a} + 2\mathbf{b}$

d $\mu = 2$ $\lambda = 3$

21 **b** Q points (0, 0) (3, −4) (10, −5)

c 90 degrees clockwise about the origin

d R points (0, 0) (−4, −3) (−5, −10)

(0.8, −4.4) rotation of 53.1° anticlockwise about the origin

Consolidation exercise 1

1 **a** $\frac{7}{12}$ **b** $1\frac{1}{3}$ **c** 24 **d** 12

2 **a** 9 **b** $1\frac{22}{35}$

3 14.235, 14.25, 14.3, 14.532

4 $3^3 \times 5 \times 7$

5 **a** $504 = 2^3 \times 3^2 \times 7$ **b** 210, 3

6 **a** $24 = 2^3 \times 3$ **b** $90 = 2 \times 3^2 \times 5$

c 6 **d** 360

7 **a** $\frac{34}{99}$ **b** 0.375 **c** \$1.12 **d** 2

8 $\frac{7}{33}$

9 **a** 35% **b** 1 : 2.5

10 **a** 72 s **b** 4.17 cents **c** 16.7%

11 **a** 5625 **b** 1×10^{10} **c** 8.25×10^{-5} **d** 8.66×10^{-3}

12 **a** 37.625 **b** 37.62 **c** 1.398 **d** 1.40

13 **a** 52.4 **b** 0.57 **c** 38 500 **d** 0.0026

14 **a** 64×10^4 **b** 5×10^1

15 **a** 0.351 **b** 5.93 **c** 1.52

16 **a** 10, 12, 14, 16 **b** $b = 2w + 6$

c 56 **d** 60

17 **a** 8, 13, 19, 26 **b** find a formula for the number of bolts

c 12

18 **a** 2^6 **b** 2^{21}

19 **a** 3^6 **b** 3^2 **c** 3^{3m} **d** 3^{10}
20 **a** $4\sqrt{3}$ **b** 1 **c** 12
21 **a** 9.39 billion people **b** 1.40%
22 3.2 kg
23 **a** 329 blocks **b** 820 tonnes **c** 12.5 years
24 **a** 10.5 days **b** 4.55 km
25 **a** 8565 gallons/s **b** 39 m^2/s
26 597 dominos **27** \$403.13 **28** \$408
29 **a** \$89.70 **b** \$71.07
30 £76.80

Consolidation exercise 1*

1 $\frac{2}{3} = \frac{24}{36}, \frac{13}{18} = \frac{26}{36}, \frac{3}{4} = \frac{27}{36}, \frac{7}{9} = \frac{28}{36}$
2 **a** $3\frac{1}{3}$ **b** **(i)** 12 **(ii)** $6\frac{7}{12}$
3 cement 10 kg sand 45 kg
4 **a** $180 = 2^2 \times 3^2 \times 5$, $84 = 2^2 \times 3 \times 7$ **b** 1260, 12
5 **a** a^4, $a = 2$ **b** $2a^{-2}$, $a = \frac{1}{4}$
6 **a** 3^4 **b** $2^{-3} \times 5^2$
7 $\frac{3}{2}$
8 for example, $\sqrt{5}$ or $\sqrt{6}$ or $\sqrt{7}$ or $\sqrt{8}$
9 1.95
10 2.4×10^1
11 **a** 150 cm^3 **b** 320 cm^3
12 **a** 3.7×10^{10} **b** 0.000 075 **c** 2×10^{-10}
13 30 tonnes
14 48 km/h
15 **a** 33.3% **b** **(i)** 38.5 °C **(ii)** 37°C
16 9 metres
17 **a** 16.4 cm/day **b** 4.17×10^{10} m^3 **c** 438 cm
18 112.5
19 **a** 19 230 769 **b** 2.28×10^{10} m
20 **a** 185, 181, 177 **b** 5 **c** $205 - 4n$
21 **a** 62, 82, 105 **b** $a = 1, b = 91$
22 **a** 11, 16, 21, $w = 5c + 1$
b c: 3, 6, 10; w: 15, 27, 60; $r + c$: 2, 5, 9, 14, 20
c $w = 3(r + c)$ **d** $w = \frac{3}{2}(3r + r^2)$ **e** 195

Consolidation exercise 2

1 **a** x^{10} **b** x^2
c y^2 **d** $4x^6y^4$
2 **a** $x^2 + 2x - 3$ **b** $2x^2 + 7x - 4$
3 **a** $z^2 + 3z$ **b** $14 - x$
4 **a** $4(p - 2)$ **b** $x(x + 3)$ **c** $3ab(b + 2a)$
5 **a** $(x + 2)(x - 2)$ **b** $(x + 1)(x + 2)$
6 **a** $\frac{7x}{12}$ **b** $\frac{(x+1)}{6}$ **c** $\frac{4}{(x-3)}$
7 **a** $v - I/m$ **b** $\sqrt{(12/\pi h)}$
8 **a** -14 **b** $\frac{(v-u)}{t}$
9 **a** 3.14 **b** $g\left(\frac{T}{2\pi}\right)^2$
10 **a** 114 **b** $C = 30 + 0.15t$ **c** $t = \frac{C - 30}{0.15}$
d 650 minutes
11 **a** 3 **b** 2 **c** 2
12 **a** 3 **b** 5
13 **a** $5(x + 3) - 8 = 42$ **b** 7
14 15
15 $x = 6, y = 3$; 216 cm^2
16 **a** $y = 4x$ **b** 8 **c** 2
17 **a** $y = 3x^2$ **b** 48 **c** 3
18 **a** $y = \frac{24}{x}$ **b** 12 **c** 4
19 **a** $y = \frac{36}{\sqrt{x}}$ **b** 12 **c** 16
20 (3, 5)
21 **a** $x + y = 17, 4x + 2y = 58$ **b** 5
22 **a** $x = -4$ or 4 **b** $x = 0$ or 16
23 **a** $x = -1$ or 3 **b** $x = 2$ or 4
24 **a** $x = 0.59$ or 3.41 **b** $x = -1.78$ or 0.28
25 **b** $-8, 4$ **c** 24
26 $x = 5$, area 48
27 **a** $x > 1$ **b** $x \geqslant 3$
28 $-2, -1, 0, 1$
31 $p = -3$ $(x + 3)(x - 2)(x - 4)$
32 $q = -2$ $(2x + 1)(x + 2)(x - 1)$ $x = -\frac{1}{2}, -2$ or 1
33 **a** $x = 7$ **b** $x = 11$
34 **a** width $= 6x$, height $= 4x + 28$
b **(i)** $6x = 4x + 28$ **(ii)** $x = 14$

Consolidation exercise 2*

1 **a** $\frac{11a}{12}$ **b** $\frac{3}{8}$ **c** b **d** $\frac{(21 - 2x)}{15}$
2 **a** $x + 1$ **b** x
3 **a** $(x - 9)(x + 8)$ **b** $\frac{(x+9)}{(x+8)}$
4 **a** $(3x - 1)(x + 11)$ **b** $\frac{(3x-1)}{(x-11)}$
5 **a** 27.6 **b** $\sqrt{\frac{3I - Mb^2}{M}}$
6 **a** 3024 **b** $\frac{2(S - an)}{n(n-1)}$
7 **a** 2 **b** $\frac{fv}{(v-f)}$ **c** 12
8 -1
9 $\frac{x + 123}{x + 456} = \frac{1}{2}$, 210
10 **a** $(160° - 3x)$ **b** $x = 20°, 32°, 35°$
11 **a** $x^2 + 25 = I^2$ **b** $(x + 2)^2 + 16 = I^2$ **c** 1.25 m
12 **a** $v = 2\sqrt{d}$ **b** 14 m/s **c** 25 m
13 **a** $y = \frac{40}{x^2}$ **b** 640 **c** 2
14 **a** $T = \frac{120}{\sqrt{m}}$ **b** 40 **c** 5.8 min
15 $x = 2$ $y = 1$
16 **a** $2x + 3y = 180$ **b** $2x + 2y = 140$
c $x = 30, y = 40$
17 **a** $x + y = 14, 12x + 18y = 204$ **b** $x = 8, y = 6$
18 **a** $x = -10$ or 10 **b** $x = 0$ or 100
19 **a** $x = -9$ or 4 **b** $x = 2$ or 5
20 **a** $x = 0.697$ or 4.30 **b** $x = -1.88$ or 0.884
21 **a** $x^2 - 2x - 3 = 0$ **b** $x = -1$ or $x = 3$, ∴ 3, 4, 5 triangle
22 **b** $-8, 2$ **c** 6
23 **b** $-6, 4$ **c** 5
24 $-1, 0, 1, 2$
25 **a** $x > -\frac{1}{3}$ **b** $x \leqslant 2$
26 **a** $2.5x + 3 > 3x$, $4x + 1 > 1.5x + 2$, $4.5x + 2 > x + 1$
b $\frac{2}{5} < x < 6$
27 4.7 cm
28 $\frac{x + 4}{x - 4}$
29 $\frac{x + 7}{x + 6}$
30 $p = -6, q = -1, x = -2, 5$ or 3
31 $p = -7$, $(4x + 1)(x - 4)(x + 2)$ $x = -\frac{1}{4}, 4$ or -2
32 **a** $x = 2$ **b** $x = 2$ or -2
33 **b** 1.58 cm

Consolidation exercise 3

1 **a** **(i)** 5 **(ii)** 4 **(iii)** $\frac{2}{5}$
b $x = -3$ **c** $x = -1$
2 **a** No
b **(i)** $\mathbb{R}$ **(ii)** $g(x) \geqslant 0$
c **(i)** 3 **(ii)** $x^2 - 1$ **d** $x = 0$ or 3

3 **a** **(i)** -1 **(ii)** $\frac{1}{7}$

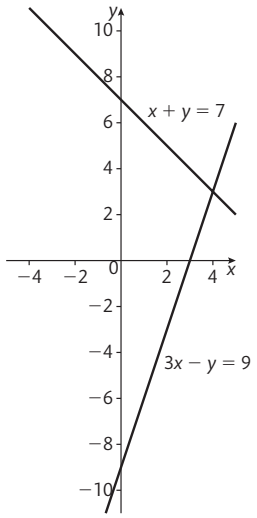
b **(i)** $\frac{3}{x} - 2$ **(ii)** 0 **c** ± 1

4 **a** i **b** iii **c** iv **d** ii

5 A **(v)** B **(ii)** C **(vi)** D **(i)** E **(iv)** F **(iii)**

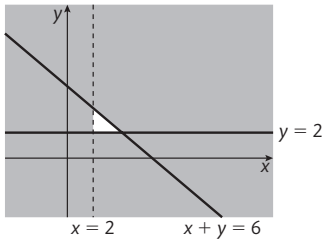
6 $x > 2$

7 **a**



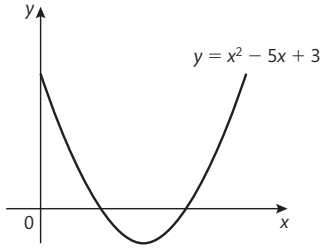
b $x = 4, y = 3$

8



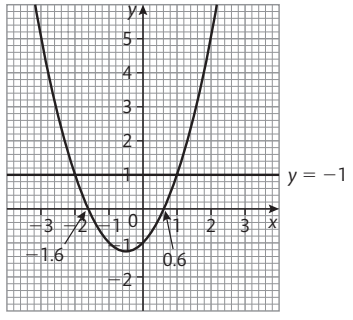
9 **a** $\frac{3}{4}$ **b** $(3, -1.5)$

10 **a**



b $x \approx 0.7, x \approx 4.3$

11 **a**



b $x = -1.6$ or 0.6

c $x = -2, x = 1$ **d** -1.25

12 **a** $y = 1$ **b** $y = x + 2$

13 **a** $x^2 - 4x + 2 = 0$ **b** $x = 0.6$ or $x = 3.4$

14 **a** $x = 2, y = 6, y = 2x$ **b** $x = 1, y = 4, y = x^2$

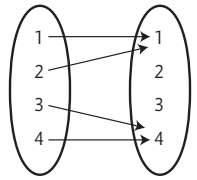
15 $-1 \rightarrow -1$ range $\{-1, 1, 3, 5\}$

$0 \rightarrow 1$

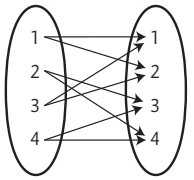
$1 \rightarrow 3$

$2 \rightarrow 5$

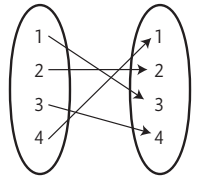
16 **a** e.g.



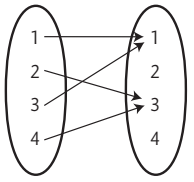
b e.g.



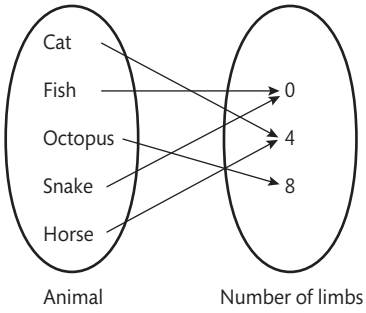
c e.g.



d e.g.



17

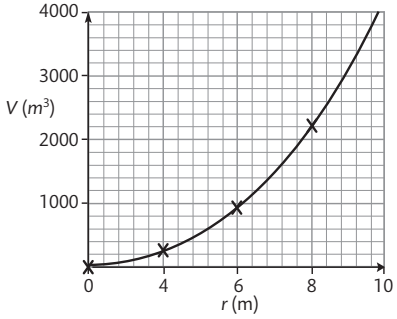


Yes it is a function

18 **a** not a function, one to many **b** a function, many to one

19 **a** 268, 905, 2145, 4189

b

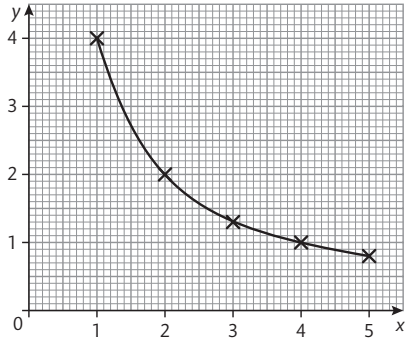


c **(i)** $1440\,m^3$ **(ii)** 4.9 m

20 **a**

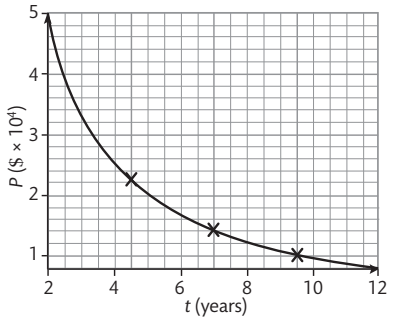
x	1	2	3	4	5
y	4	2	$1\frac{1}{3}$	1	0.8

b



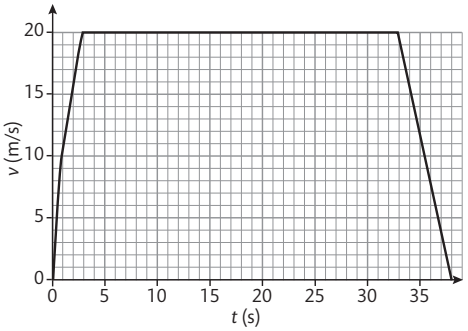
21 **a** 2.5, 1.67, 1.25, 1

b



c **(i)** $\$3.3 \times 10^4$ **(ii)** 7.1 years

22 **a**

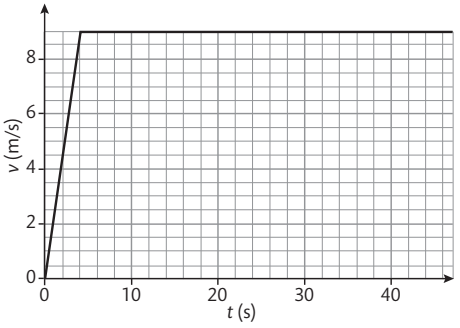


b 4

c **(i)** 10 m/s **(ii)** $5\ \text{m/s}^2$ **(iii)** 18.0 m/s

23 **a** **(i)** 8.65 m/s **(ii)** 31.1 km/h

b



c 9.04 m/s **d** $2.26\ \text{m/s}^2$

24 **a** $(1, -5)$ **b** $-1.2, 3.2$

c Line is $y = x + 2$, solutions 4.4 or -1.4

d **(i)** $\frac{dy}{dx} = 2x - 2$ **(ii)** Gradient is 10

25 **b** **(i)** $28 - 4x$ **(ii)** $x = 7$ **(iii)** Graph is a negative parabola

c $98\ \text{m}^2$

26 **a** $v = 3 - 2t - \frac{8}{t^2}$ **b** $a = -2 + \frac{16}{t^3}$

Consolidation exercise 3*

1 **a** 9 **b** 19 **c** $\frac{x+1}{2}$ **d** $2x$

2 **a** 20 **b** 10^{11} **c** $10^x + 10$ **d** 10^{x+10}

3 $x = -4$ or 3

4 $x = -10$ or 5

5 **a** 2 **b** $y = 2x - 1$

c $y = 2x \pm c$ (c positive integer)

6 **a** m gradient, c intercept **b** $y = x - 3$

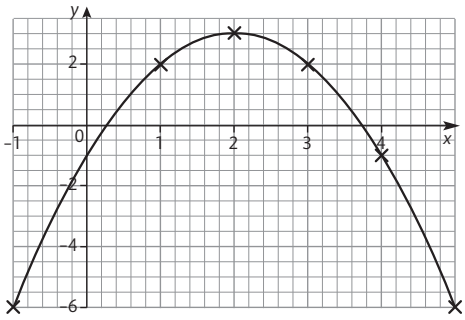
7 $x + y = 5$ and $y - 2x = 2$

8 **a**

x	0	4	8	12	16	20
y	0	8.8	11.2	7.2	−3.2	−20

b 31.3 m **c** 15 m

9 **a**



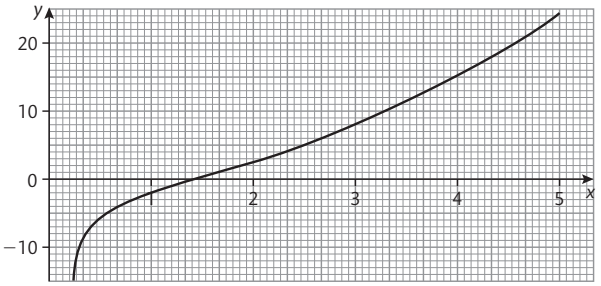
b $x = -0.23$ or $x = 4.24$ **c** $x = 0.70$ or $x = 4.30$

10 **a** $y = 3$ **b** $y = 6 + 6x - 2x^2$

11 **a**

x	0.5	1	1.5	2	3	4	5
y	−5.75	−2	0.25	2.5	8	15.25	24.4

b



c $x = 1.4$ **d** $y = 2x$, $x = 2.5$

12 **a** **(i)** 1.5 **(ii)** 0.75 **b** $x = 1$

c **(i)** $ff(x) = x$ **(ii)** it is its own inverse

13 **a** $x = -1$ $y = 8$ $y = x^3$

b $x = 3$ $y = 9$ $y = 2x - 1$

14 $-2 \to 0$ range $\{0, 3, 4\}$

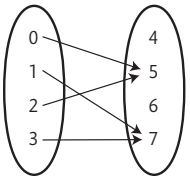
$-1 \to 3$

$0 \to 4$

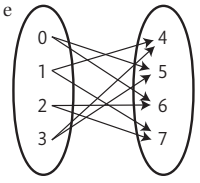
$1 \to 3$

$2 \to 0$

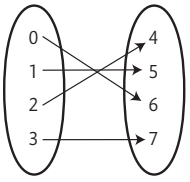
15 **a** e.g.



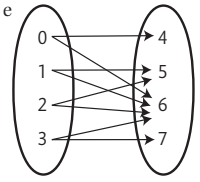
b e



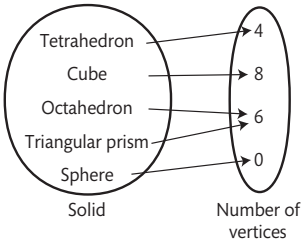
c e.g.



d e



16



Yes it is a function

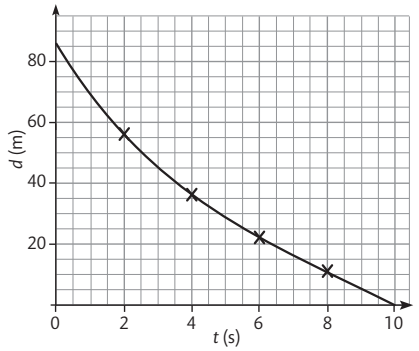
17 **a** 7.04 m/s **b** 8.20 m/s **c** 2.05 m/s

d No. Sidd takes 15 s to run 100 m so gets beaten by 0.8 s.

18 **a**

t (s)	0	2	4	6	8	10
d (m)	86.6	55.5	36.3	22.3	10.6	0

b



Graph shows that hare's speed is very gradually decreasing as it reaches the bush.

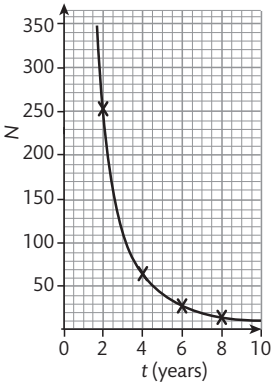
c Gradient at $t = 6$ s is approx. -6.27 m/s: this is the velocity.

19 **a** 1.2 m/s^2 **b** 2.4 m/s^2 **c** 15 m/s

20 $v = -2(t-2)^2 + 18 \therefore v_{max} = 18$

21 **a** $k = 1000$; 250, 62.5, 27.8, 15.6

b



c $t = 5$

22 **a** $\frac{dy}{dx} = 5000 - 1250x$ **b** (4, 10 000)

c **(i)** max **(ii)** second derivative is negative

d **(i)** £4 **(ii)** highest profit

Consolidation exercise 4

1 **b** 215 km

2 (3, −5) **b** (5, −3) **c** (−1, 9) **d** (7, 5)

3 **a** (−1, 5), (−3, 5), (−3, 6)

b (−4, 3), (−4, 5), (−3, 5)

c Rotation +90° about the point (−3.5, 5.5)

4 **a** 026.6° **b** 206.6°

5 $p = 5.40$ cm, $q = 11.9$ cm, $r = 65.4°$, $s = 41.8°$

6 $x = 15$; $y = 4$

7 **a** $160° - 7x$

b $160° - 11x = 0$, $140° - 10x = 0$, $20° - x = 0$

c $x = 14°$, 14.5°, 20°

8 **a** 45° **b** 1080°

c each interior angle has size 135°, $\frac{360}{135}$ does not give a whole number

9 **a** 2.4 cm **b** $1\frac{1}{3}$ cm

10 **a** $\frac{h + OT}{OT} = \frac{2r}{r}$ $\therefore 2 \times OT = h + OT$ and $OT = h$

b $V = \left(\frac{1}{3} \times \pi 4r^2 \times 2h\right) - \left(\frac{1}{3} \times \pi r^2 \times h\right)$

$= \frac{8\pi r^2 h}{3} - \frac{\pi r^2 h}{3} = \frac{7\pi r^2 h}{3}$

c $V = 7.330r^2h$ **d** $r = 1.68$ cm

11 $y = 4$ cm

12 6200 cm^3

13 **a** $x = 100°$, $y = 90°$ **b** $x = 70°$, $y = 76°$

c $x = 110°$, $y = 35°$ **d** $x = 65°$, $y = 35°$

14 **a** $x = 25°$, $y = 65°$, $z = 115°$ **b** triangle ECF

15 **a** $\angle TPQ = 66°$ (triangle TPQ is isosceles)

$\angle OPT = 90°$ (TP is tangent)

$\therefore \angle QPO = 24°$

b OPS = 42° (isosceles triangle)

$\therefore \angle QPS = 66°$

c $\angle QRS = 114°$ (opposite angles of a cyclic quadrilateral)

16 $x = 8.7$; $y = 43.4°$, $z = 73.4°$

17 **a** SA = 2.63 km, BS = 6.12 km **b** 28.08 minutes

18 **a** PQ = 8.7 m **b** QR = 5 m **c** RS = 8.4 m

d 63.6 m^2

19 **a** 40 m **b** 26.5 m **c** 4.68 m

20 **a** 25.5 cm **b** 27.3 cm **c** 21.4°

21 28.5 cm^2

Consolidation exercise 4*

1 **a** PQ = 10.7 m **b** QR = 9.0 m **c** RS = 8.1 m

d Area PQRS = 104.8 m^2 = 105 m^2 (to 3 s.f.)

2 **a** $x = 29.7°$, $y = 1.2$ **b** $x = 49.4°$, $y = 23.2$

c $x = 40.3°$, $y = 16.4$

3 **a** CP = 30 m **b** QB = 4 m **c** 10 m

4 **a** BC = 1.81 m; 10.1 m

b No, because the gap when $b = 35°$ is 5.43 m

5 $x = 6\frac{2}{3}$, $y = 9$

6 **a** OBA, OAC **b** 2.08 cm **c** 13

7 **b** **(i)** (1, −3), (1, −5), (2, −3)

(ii) (3, −1), (5, −1), (3, −2)

(iii) (2, 2), (2, 6), (4, 2)

(iv) (−1, 1), (−1, 5), (1, 1)

c 90 degrees clockwise about (1, −1)

d Scale factor 2 centre (3, 1)

8 **a** $V = \pi R^2 h - \pi r^2 h = \pi h(R^2 - r^2) = \pi h(R - r)(R + r)$

b 5280 cm^3 **c** $h = 4.5$ cm

9 Perimeter $= 18(1 + \sqrt{3})$ cm)

10 **a** 12π **b** 6 cm

c 8 cm **d** $r = \sqrt[3]{52}$ cm

11 386 cm^2

12 $x = y = 51°$, $z = 78°$

13 **a** $AB = 2r$ **b** $\frac{s}{360} \times 2\pi r$, so $s = 114.6°$

c $0.545r^2$

14 **a** 20 **b** 270

15 **a** **(i)** 100° **b** 80°

(ii) $\angle TQX = \angle QPX$ (angles in alternate segments)

$\therefore \angle XQA = \angle QPB$

$\angle QAB = \angle QPB$ (angles in same segment)

$\therefore$ XQ is parallel to AB

16 60 cm

17 **a** CD = 14.4 m **b** CE = 142 m **c** 12.7 m/s

18 $x = 3$ cm $y = 12$ cm

19 **a** (2, −5) **b** (−5, −2)

c (−1, 12) **d** (−2, 24)

20 **a** $h = 2$ m **b** π m/s

c $\frac{1}{2}\pi$ m/s, so 50% decrease

21 **a** $AP = \frac{10\sqrt{3}}{3}$ $PQ = \frac{5\sqrt{3}}{3}$ $OQ = 5\sqrt{3}$

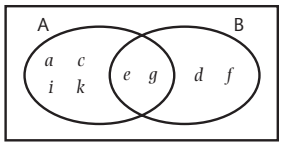
b 54.7° **c** 70.5°

Consolidation exercise 5

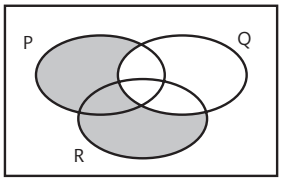
1 **a** **(i)** $A \cap B = \{e, g\}$

(ii) $A \cup B = \{a, c, d, e, f, g, i, k\}$

b

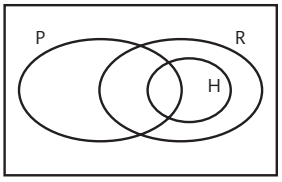


2



3 **a** 4 **b** 14

4



5 **a** $\frac{1}{3}$ **b** $\frac{2}{3}$ **c** $\frac{1}{12}$ **d** $\frac{1}{3}$

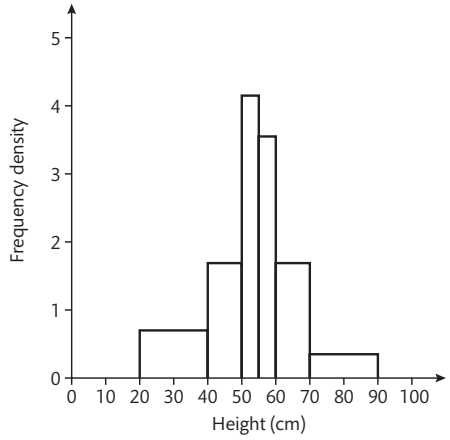
6 **a** $\frac{1}{9}$ **b** $\frac{7}{9}$ **c** $\frac{4}{9}$

7 **a** 24 years **b** 24.6 years

8 **a** 60 matches **b** 149 points **c** 0.6

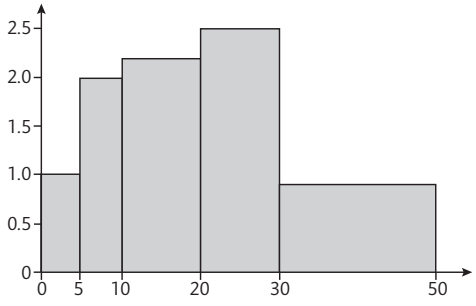
9 **a** 0.75, 1.8, 4.4, 3.8, 1.8, 0.4

b



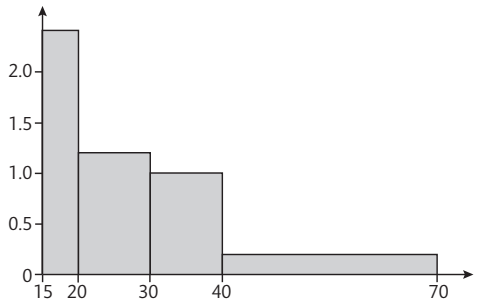
c 53.2 cm

10 **a**



b 7 **c** 22.0 cm

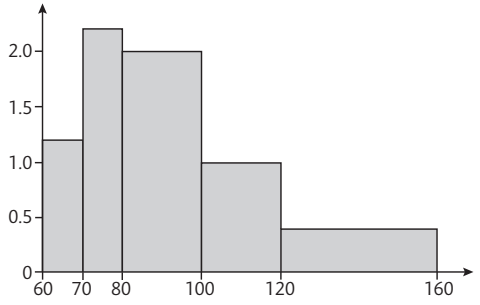
11 **a**



b 2 **c** 17

12 **a** 12, 16

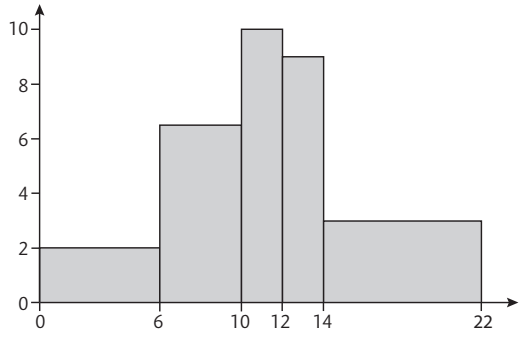
b



c 41

13 **a** 12, 20

b

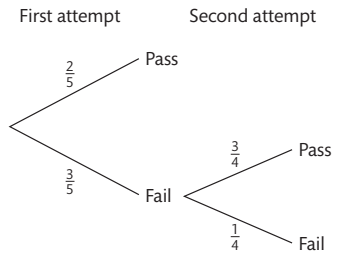


c 57 **d** 11 min

14 **a** 0.57 **b** 30

15 **a** $\frac{1}{6}$ **b** $\frac{4}{15}$

16 **a**



b $\frac{17}{20}$

17 **a** 5 **b** 53.1°

18 $\begin{pmatrix} 6\cos 30° \\ 6\sin 30° \end{pmatrix}$

19 $2\mathbf{v} = \begin{pmatrix} -2 \\ 10 \end{pmatrix}$ $\mathbf{u} + \mathbf{v} = \begin{pmatrix} 1 \\ 8 \end{pmatrix}$ $\mathbf{u} - \mathbf{v} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ $\sqrt{130}$

20 **a** $\mathbf{A} + \mathbf{B} = \begin{pmatrix} 3 & -3 \\ -2 & 4 \end{pmatrix}$ **b** $\mathbf{A} - \mathbf{B} = \begin{pmatrix} 3 & 5 \\ 8 & 0 \end{pmatrix}$

c $2(\mathbf{A} + 2\mathbf{B}) = \begin{pmatrix} 6 & -14 \\ -14 & 12 \end{pmatrix}$ **d** $3(\mathbf{A} + 4\mathbf{B}) = \begin{pmatrix} 9 & 51 \\ 69 & -18 \end{pmatrix}$

21 **a** $\begin{pmatrix} 2 & 5 \\ 3 & 1 \end{pmatrix}$ **b** $\begin{pmatrix} -1 & -3 \\ -3 & 4 \end{pmatrix}$ **c** $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ **d** $\frac{1}{13}\begin{pmatrix} 1 & 4 \\ -3 & 1 \end{pmatrix}$

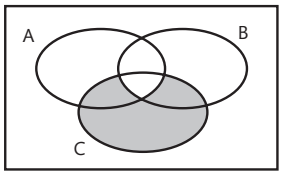
22 **a** $\overrightarrow{AB} = \mathbf{y} - \mathbf{x}$ $\overrightarrow{OM} = \frac{1}{2}(\mathbf{x} + \mathbf{y})$ $\overrightarrow{ON} = \frac{1}{2}(\mathbf{y} - \mathbf{x})$ $\overrightarrow{MN} = -\mathbf{x}$
MN is parallel to OA and the same length.

23 **a** $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ **b** $\overrightarrow{PQ} = 3(\mathbf{b} - \mathbf{a})$
c AB is parallel to PQ; PQ = 3AB

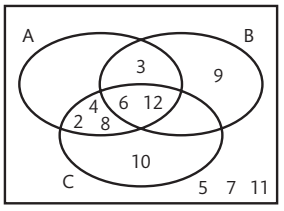
24 **a** $\mathbf{T} = \begin{pmatrix} 2 & -1 \\ -4 & -5 \end{pmatrix}$ **b** $\mathbf{T}^{-1} = \frac{1}{14}\begin{pmatrix} 5 & -1 \\ -4 & -2 \end{pmatrix}$
c A $\left(\frac{-2}{7}, \frac{3}{7}\right)$ B $\left(\frac{23}{14}, \frac{-12}{7}\right)$ C $\left(\frac{-3}{7}, \frac{1}{7}\right)$

Consolidation exercise 5*

1



2



3 17

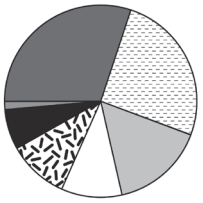
4 **a** 35 **b** 32.3

5 **a**

size (kb)	1	2	3	4	5	6	7
f	18	16	9	6	6	4	1

b mean = 2.7 KB, median = 2 KB, mode = 1 KB

c



size (KB)	1	2	3	4	5	6	7
Angle size	108°	96°	54°	36°	36°	24°	6°

6 **a** $\frac{8}{24} = \frac{1}{3}$ **b** $\frac{2}{3}$ **c** $\frac{4}{24} = \frac{1}{6}$ **d** $\frac{12}{24} = \frac{1}{2}$

7 **a**

	1	**2**	**3**	**4**	**5**	**6**
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

b $\frac{10}{36} = \frac{5}{18}$ **c** $\frac{12}{36} = \frac{1}{3}$

8 1.74 m

9 **a** $\frac{1}{2}$ **b** $\frac{5}{6}$ **c** 1 **d** 0

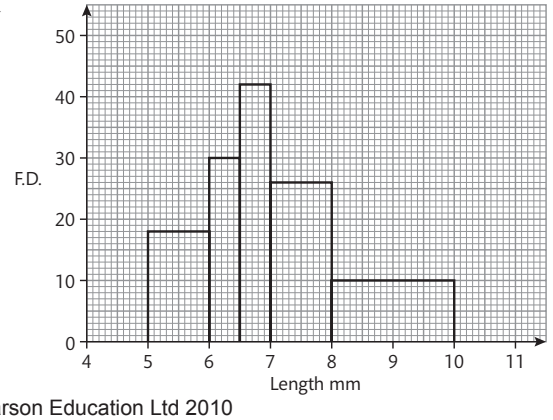
10 **a** 0 **b** $\frac{9}{25}$ **c** $\frac{16}{25}$ **d** $\frac{9}{25}$

11 **a** 0.4 **b** 0.35

12 **a** $3n$ **b** 21 people

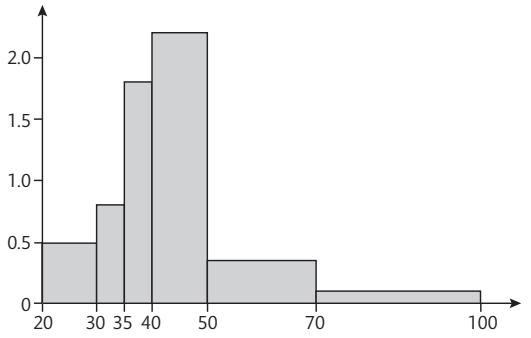
13 **a** 18 red **b** 12 red

14 **a**



b 76 **c** 7.10 mm

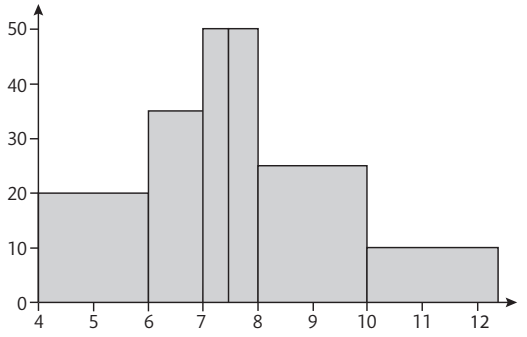
15 **a**



b 71.2% **c** 43.2 mins

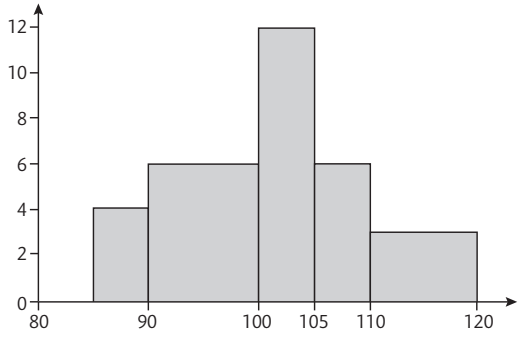
16 **a** 40, 35, 25, 25, 50, 25 **b** $x = 12.5$

c



17 **a** 20, 60, 60, 30, 30

b



c 0.6

18 **a** $\sqrt{13}$ **b** $p = 1, q = 2$

19 **a** $\begin{pmatrix} 4\cos 20° \\ 4\sin 20° \end{pmatrix}$ **b** $\begin{pmatrix} 3\cos 50° \\ 3\sin 50° \end{pmatrix}$
c $\begin{pmatrix} 4\cos 20° + 3\cos 50° \\ 4\sin 20° + 3\sin 50° \end{pmatrix}$ **d** 6.77 units 32.8°

20 **a** $\overrightarrow{PU} = \mathbf{a} - \mathbf{b}$ **b** $\overrightarrow{TR} = 3\mathbf{b} + \mathbf{a}$ **c** $\overrightarrow{US} = \frac{1}{2}(\mathbf{a} - \mathbf{b})$
d PUS is a straight line, PU = 2US

21 **a** $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ **b** $\overrightarrow{CD} = \lambda\mathbf{b} - \mu\mathbf{a}$
c $\lambda = 3$ $\mu = 3$ **d** CD parallel to AB; CD = 3AB

22 **a** $\frac{1}{10}\begin{pmatrix} -4 & 2 \\ 1 & -3 \end{pmatrix}$ **b** $\begin{pmatrix} 4 & 1 \\ 3 & 7 \end{pmatrix}$
c $\begin{pmatrix} 5 & 5 \\ 5 & 11 \end{pmatrix}$ **d** $\begin{pmatrix} 19 & 11 \\ 33 & 52 \end{pmatrix}$

23 **a** $\begin{pmatrix} -1 & -2 & -3 \\ 2 & 5 & 4 \\ 5 & 18 & -1 \end{pmatrix}$ **b** $\begin{pmatrix} -7 & 8 \\ 1 & 10 \end{pmatrix}$ **c** not commutative

24 **a** $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ reflection in x axis
b $a = -3$ $b = -1$ $c = -2$ $d = 2$

25 **a** $\begin{pmatrix} 2 & -1 \\ -4 & -5 \end{pmatrix}$ **b** A $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ B $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ C $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$