

Chapter 1

1.1

Exercise 1A

- | | | | |
|------------------------|-------------------|-----------------|-------------------|
| 1 a x^7 | b $6x^5$ | c $2p^2$ | d $3x^{-2}$ |
| e k^5 | f y^{10} | g $5x^8$ | h p^2 |
| i $2a^3$ | j $2p^{-7}$ | k $6a^{-9}$ | l $3a^2b^{-2}$ |
| m $27x^8$ | n $24x^{11}$ | o $63a^{12}$ | p $32y^6$ |
| q $4a^6$ | r $6a^{12}$ | | |
| 2 a x^5 | b x^{-2} | c x^4 | |
| d x^3 | e x^5 | f $12x^0 = 12$ | |
| g $3x^{\frac{1}{2}}$ | h $5x$ | i $6x^{-1}$ | |
| 3 a ± 5 | b ± 9 | c 3 | d $\frac{1}{16}$ |
| e $\pm \frac{1}{3}$ | f $\frac{1}{125}$ | g 1 | h ± 6 |
| i $\pm \frac{125}{64}$ | j $\frac{9}{4}$ | k $\frac{5}{6}$ | l $\frac{64}{49}$ |

1.2

Exercise 1B

- | | | |
|---------------------------|---------------------------|-------------------------|
| 1 $2\sqrt{7}$ | 2 $6\sqrt{2}$ | 3 $5\sqrt{2}$ |
| 4 $4\sqrt{2}$ | 5 $3\sqrt{10}$ | 6 $\sqrt{3}$ |
| 7 $\sqrt{3}$ | 8 $6\sqrt{5}$ | 9 $7\sqrt{2}$ |
| 10 $12\sqrt{7}$ | 11 $-3\sqrt{7}$ | 12 $9\sqrt{5}$ |
| 13 $23\sqrt{5}$ | 14 2 | 15 $19\sqrt{3}$ |
| 16 $\frac{\sqrt{5}}{5}$ | 17 $\frac{\sqrt{11}}{11}$ | 18 $\frac{\sqrt{2}}{2}$ |
| 19 $\frac{\sqrt{5}}{5}$ | 20 $\frac{1}{2}$ | 21 $\frac{1}{4}$ |
| 22 $\frac{\sqrt{13}}{13}$ | 23 $\frac{1}{3}$ | |

1.3

Exercise 1C

- | | |
|-------------------------------|-------------------------------|
| 1 a $\log_4 256 = 4$ | b $\log_3 (\frac{1}{9}) = -2$ |
| c $\log_{10} 1\,000\,000 = 6$ | d $\log_{11} 11 = 1$ |
| 2 a $2^4 = 16$ | b $5^2 = 25$ |
| c $9^{\frac{1}{2}} = 3$ | d $5^{-1} = 0.2$ |
| e $10^5 = 100\,000$ | |
| 3 a 3 | b 2 |
| e 6 | f $\frac{1}{2}$ |
| c 7 | d 1 |
| g -1 | h 10 |

- | | | | |
|----------|---------|----------|----------|
| 4 a 625 | b 9 | c 7 | d 2 |
| 5 a 1.30 | b 0.602 | c 3.85 | d -0.105 |
| 6 a 1.04 | b 1.55 | c -0.523 | d 3.00 |

1.4

Exercise 1D

- | | | |
|--|----------------------------|---------------|
| 1 a $\log_2 21$ | b $\log_2 9$ | c $\log_5 80$ |
| d $\log_6 (\frac{68}{81})$ | e $\log_{10} 120$ | |
| 2 a $\log_2 8 = 3$ | b $\log_6 36 = 2$ | |
| c $\log_{12} 144 = 2$ | d $\log_8 2 = \frac{1}{3}$ | |
| e $\log_{10} 10 = 1$ | | |
| 3 a $3 \log_a x + 4 \log_a y + \log_a z$ | | |
| b $5 \log_a x - 2 \log_a y$ | | |
| c $2 + 2 \log_a x$ | | |
| d $\log_a x + \frac{1}{2} \log_a y - \log_a z$ | | |
| e $\frac{1}{2} + \frac{1}{2} \log_a$ | | |

1.5

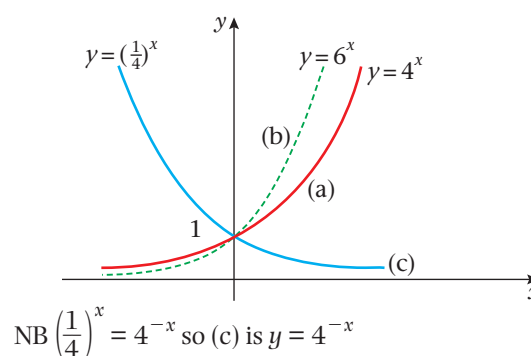
Exercise 1E

- | | | |
|------------------------|-------------------------------|---------|
| 1 a 2.460 | b 3.465 | c 4.248 |
| d 0.458 | e 0.774 | |
| 2 a 1.27 | b 2.09 | c 0.721 |
| a $\frac{1}{2}, 512$ | e $\frac{1}{16}, \frac{1}{4}$ | |
| 3 a 6.23 | b 2.10 | c 0.431 |
| d 1.66 | | |
| 4 a $\frac{1}{2}, 512$ | b $\frac{1}{16}, \frac{1}{4}$ | c 2.52 |

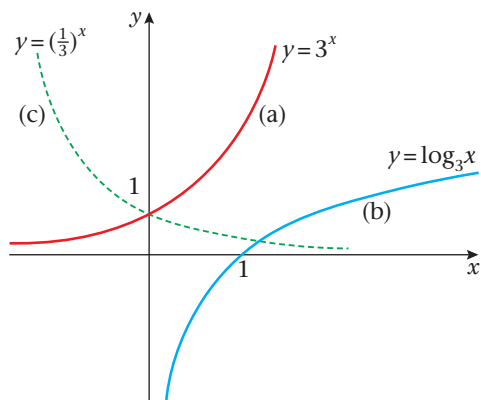
1.6

Exercise 1F

1



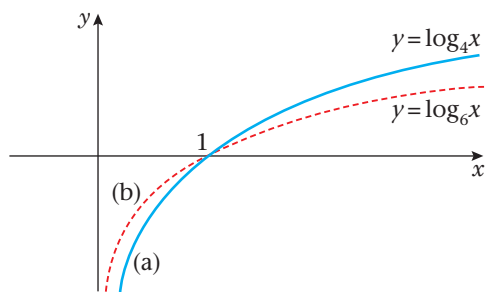
2



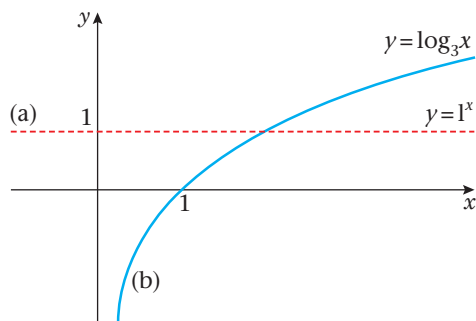
NB $y = \log_3 x$ is a reflection of $y = 3^x$ in the line $y = x$.

$$y = \left(\frac{1}{3}\right)^x \text{ is } y = 3^{-x}$$

3



4



a $y = 1^x = 1$

c $y = \log_3 x = 1$

$$\Rightarrow x = 3^1 = 3$$

So coordinates of intersection are (3, 1)

Exercise 1G

1 a y^8 **b** $6x^7$ **c** 32^x **d** $12b^9$

2 a $3x^6$ **b** ± 2 **c** $6x^2$ **d** $\frac{1}{2}x^{\frac{1}{3}}$

3 a $\frac{4}{9}$ **b** $\frac{3375}{4913}$

4 a $\frac{\sqrt{7}}{7}$ **b** $4\sqrt{5}$

5 a $\frac{\sqrt{3}}{3}$ **b** $\frac{15}{\sqrt{5}}$

6 a $2 \log_d p + \log_d q$ **b** $\log_d p = 4, \log_d q = 1$

7 a $5^x = 80$

So $x = \log_5 80$

$$\left(= \frac{\log_{10} 80}{\log_{10} 5} \right)$$

i.e. $x = 2.72270 \dots$

$$= 2.72 \text{ 3sf}$$

b $7^x = 123$

$\Rightarrow x = \log_7 123$

$$\left(= \frac{\log_{10} 123}{\log_{10} 7} \right)$$

i.e. $x = 2.47297 \dots$

$$= 2.47 \text{ 3sf}$$

8 a 9

b 12

c $\frac{1}{9}, 9$

9 $\frac{1}{3}, 9$

10 $-\frac{1}{3}, -2$

Chapter 2

2.1

Exercise 2A

1 $x(x + 4)$

3 $(x + 8)(x + 3)$

5 $(x + 8)(x - 5)$

7 $(x + 2)(x + 3)$

9 $(x - 5)(x + 2)$

11 $(2x + 1)(x + 2)$

13 $(5x - 1)(x - 3)$

15 $(2x - 3)(x + 5)$

17 $(x + 2)(x - 2)$

19 $(2x + 5)(2x - 5)$

21 $4(3x + 1)(3x - 1)$

23 $2(3x - 2)(x - 1)$

2 $2x(x + 3)$

4 $(x + 6)(x + 2)$

6 $(x - 8)(x - 2)$

8 $(x - 6)(x + 4)$

10 $(x + 5)(x - 4)$

12 $(3x - 2)(x + 4)$

14 $2(3x + 2)(x - 2)$

16 $2(x^2 + 3)(x^2 + 4)$

18 $(x + 7)(x - 7)$

20 $(3x + 5y)(3x - 5y)$

22 $2(x + 5)(x - 5)$

24 $3(5x - 1)(x + 3)$

2.2

Exercise 2B

1 $(x + 2)^2 - 4$

3 $(x - 8)^2 - 64$

5 $(x - 7)^2 - 49$

7 $3(x - 4)^2 - 48$

9 $5(x + 2)^2 - 20$

11 $3(x + \frac{3}{2})^2 - \frac{27}{4}$

2 $(x - 3)^2 - 9$

4 $(x + \frac{1}{2})^2 - \frac{1}{4}$

6 $2(x + 4)^2 - 32$

8 $2(x - 1)^2 - 2$

10 $2(x - \frac{5}{4})^2 - \frac{25}{8}$

12 $3(x - \frac{1}{6})^2 - \frac{1}{12}$

2.3

Exercise 2C

- 1 $x = 0$ or $x = 4$
- 2 $x = 0$ or $x = 25$
- 3 $x = 0$ or $x = 2$
- 4 $x = 0$ or $x = 6$
- 5 $x = -1$ or $x = -2$
- 6 $x = -1$ or $x = -4$
- 7 $x = -5$ or $x = -2$
- 8 $x = 3$ or $x = -2$
- 9 $x = 3$ or $x = 5$
- 10 $x = 4$ or $x = 5$
- 11 $x = 6$ or $x = -1$
- 12 $x = 6$ or $x = -2$
- 13 $x = -\frac{1}{2}$ or $x = -3$
- 14 $x = -\frac{1}{3}$ or $x = \frac{3}{2}$
- 15 $x = -\frac{2}{3}$ or $x = \frac{3}{2}$
- 16 $x = \frac{3}{2}$ or $x = \frac{5}{2}$
- 17 $x = \frac{1}{3}$ or $x = -2$
- 18 $x = 3$ or $x = 0$
- 19 $x = 13$ or $x = 1$
- 20 $x = 2$ or $x = -2$
- 21 $x = \pm \frac{\sqrt{5}}{3}$
- 22 $x = 3 \pm \sqrt{13}$
- 23 $x = 1 \pm \frac{\sqrt{11}}{3}$
- 24 $x = 1$ or $x = -\frac{7}{6}$
- 25 $x = -\frac{1}{2}$ or $x = \frac{7}{3}$
- 26 $x = 0$ or $x = -\frac{11}{62}$
- 27 $x = -3 \pm 2\sqrt{2}$
- 28 $x = -6 \pm \sqrt{33}$
- 29 $x = 5 \pm \sqrt{30}$
- 30 $x = -2 \pm \sqrt{6}$
- 31 $x = \frac{3}{2} \pm \frac{\sqrt{29}}{2}$
- 32 $x = 1 \pm \frac{3}{2}\sqrt{2}$
- 33 $x = \frac{1}{8} \pm \frac{\sqrt{129}}{8}$
- 34 No real roots
- 35 $x = -\frac{3}{2} \pm \frac{\sqrt{39}}{2}$
- 36 $x = -\frac{4}{5} \pm \frac{\sqrt{26}}{5}$

2.4

Exercise 2D

- 1 $b^2 - 4ac = (-2)^2 - 4 \times 1 = 0$ so equal roots
 $x^2 - 2x + 1 = 0 \Rightarrow (x - 1)^2 = 0 \quad \therefore x = 1$
- 2 $b^2 - 4ac = (-2)^2 - 4(-1) = 8$ so two real roots
 $x = \frac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$ or 2.41, -0.414 3sf
- 3 $b^2 - 4ac = (-3)^2 - 4(-2) = 17$ so two real roots
 $x = \frac{3 \pm \sqrt{17}}{2} = \underline{3.56 \text{ or } -0.562}$ 3sf
- 4 $b^2 - 4ac = (-3)^2 - 4 \times 4 = 9 - 16 = -7$ so no real roots
- 5 $b^2 - 4ac = (1)^2 - 4 \times 2 \times (-2) = 17$ so two real roots
 $x = \frac{-1 \pm \sqrt{17}}{4} = \underline{0.781, -1.28}$ 3sf
- 6 $b^2 - 4ac = (-1)^2 - 4 \times 3 \times 3 = -35$ so no real roots

- 7 $3x^2 = 7 - x$
 $\Rightarrow 3x^2 + x - 7 = 0$
 $b^2 - 4ac = 1^2 - 4 \times 3 \times (-7) = 1 + 84 = 85$
 so two real roots
 $x = \frac{-1 \pm \sqrt{85}}{6}$
 $x = \underline{1.37 \text{ or } -1.70}$ 3sf
- 8 $2x^2 = x + 4$
 $\Rightarrow 2x^2 - x - 4 = 0$
 $b^2 - 4ac = (-1)^2 - 4 \times (2) \times (-4) = 33$
 so two real roots
 $x = \frac{1 \pm \sqrt{33}}{4}$
 $x = \underline{1.69 \text{ or } -1.19}$ 3sf
- 9 $-3 \pm \frac{\sqrt{5}}{2}, -0.38 \text{ or } -2.62$
- 10 $\frac{+3 \pm \sqrt{17}}{2}, -0.56 \text{ or } 3.56$
- 11 $-3 \pm \sqrt{3}, -1.27 \text{ or } -4.73$
- 12 $\frac{5 \pm \sqrt{33}}{2}, 5.37 \text{ or } -0.37$
- 13 $5 \pm \frac{\sqrt{31}}{3}, -3.52 \text{ or } 0.19$
- 14 $\frac{1 \pm \sqrt{2}}{2}, 1.21 \text{ or } -0.21$
- 15 $9 \pm \frac{\sqrt{53}}{14}, -0.12 \text{ or } -1.16$
- 16 $\frac{-2 \pm \sqrt{19}}{5}, 0.47 \text{ or } -1.27$
- 17 $2 \text{ or } -\frac{1}{4}$
- 18 $\frac{-1 \pm \sqrt{78}}{11}, 0.71 \text{ or } -0.89$

2.5

Exercise 2E

- 1 $x^2 + 5x + 2 = 0$
 $\alpha + \beta = -5$
 $\alpha\beta = 2$
- a $2\alpha + 1 + 2\beta + 1 = 2(\alpha + \beta) + 2 = -10 + 2 = -8$
 $(2\alpha + 1)(2\beta + 1) = 4\alpha\beta + 2(\alpha + \beta) + 1$
 $= 8 - 10 + 1 = -1$
 $\therefore$ new equation is $x^2 + 8x - 1 = 0$
- b $\alpha\beta + \alpha^2\beta^2 = \alpha\beta(1 + \alpha\beta) = 2(1 + 2) = 6$
 $(\alpha\beta)(\alpha^2\beta^2) = (\alpha\beta)^3 = 2^3 = 8$
 $\therefore$ new equation is $x^2 - 6x + 8 = 0$

- 2** $x^2 + 6x + 1 = 0$
 $\alpha + \beta = -6$
 $\alpha\beta = 1$
- a** $(\alpha + 3) + (\beta + 3) = (\alpha + \beta) + 6 = 0$
 $(\alpha + 3)(\beta + 3) = \alpha\beta + 3(\alpha + \beta) + 9$
 $= 1 - 18 + 9 = -8$
 $\therefore$ new equation is $x^2 - 8 = 0$
- b** $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$
 $= \frac{36 - 2}{1} = 34$
 $\frac{\alpha}{\beta} \times \frac{\beta}{\alpha} = 1$
 $\therefore$ new equation is $x^2 - 34x + 1 = 0$
- 3** $x^2 - x + 3 = 0$
 $\alpha + \beta = 1$
 $\alpha\beta = 3$
- a** $(\alpha + 2) + (\beta + 2) = (\alpha + \beta) + 4 = 1 + 4 = 5$
 $(\alpha + 2)(\beta + 2) = \alpha\beta + 2(\alpha + \beta) + 4$
 $= 3 + 2 + 4 = 9$
 $\therefore$ new equation is $x^2 - 5x + 9 = 0$
- b** $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
 $= 1^2 - 6 = -5$
 $\alpha^2 \times \beta^2 = (\alpha\beta)^2 = 3^2 = 9$
 $\therefore$ new equation is $x^2 + 5x + 9 = 0$
- 4** $x^2 + x - 1 = 0$
 $\alpha + \beta = -1$
 $\alpha\beta = -1$
- a** $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{-1}{-1} = 1$
 $\frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{-1} = -1$
 $\therefore$ new equation is $x^2 - x - 1 = 0$
- b** $\frac{\alpha}{\alpha\beta} + \frac{\beta}{\alpha + \beta} = \frac{\alpha + \beta}{\alpha + \beta} = 1$
 $\frac{\alpha}{\alpha + \beta} \times \frac{\beta}{\alpha + \beta} = \frac{\alpha\beta}{(\alpha + \beta)^2} = \frac{-1}{1} = -1$
 $\therefore$ new equation is also $x^2 - x - 1 = 0$

Mixed Exercise 2F

- 1 a** $x(3x + 4)$ **b** $2y(2y + 5)$
c $x(x + y + y^2)$ **d** $2xy(4y + 5x)$
- 2 a** $(x + 1)(x + 2)$ **b** $3x(x + 2)$
c $(x - 7)(x + 5)$ **d** $(2x - 3)(x + 1)$
e $(5x + 2)(x - 3)$ **f** $(1 - x)(6 + x)$
- 3 a** $y = -1$ or -2 **b** $x = \frac{2}{3}$ or -5
c $x = -\frac{1}{5}$ or 3 **d** $5 \pm \frac{\sqrt{7}}{2}$

- 4 a** $\frac{-5 \pm \sqrt{17}}{2}, -0.44$ or -4.56
b $2 \pm \sqrt{7}, 4.65$ or -0.65
c $\frac{-3 \pm \sqrt{29}}{10}, 0.24$ or -0.84
d $\frac{5 \pm \sqrt{73}}{6}, 2.25$ or -0.59
- 5 a** $p = 3, q = 2, r = -7$ **b** $-2 \pm \frac{\sqrt{7}}{3}$
- 6** ± 4
- 7 a** $\alpha\beta = t, \alpha^2 + \beta^2 = 2t(2t - 1)$
b $t = 1 + \frac{\sqrt{577}}{2}$
- c** $x^2 - 2\sqrt{577}x + 1 = 0$
- 8** $2x^2 - 7x + 3 = 0$
 $x^2 - \frac{7}{2}x + \frac{3}{2} = 0$
 $\alpha + \beta = \frac{7}{2}$
 $\alpha\beta = \frac{3}{2}$
- a** $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \frac{49}{4} - 2 \times \frac{3}{2} = \frac{37}{4}$
b $\alpha - \beta = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta} = \sqrt{\frac{49}{4} - 3} = \sqrt{\frac{37}{4}} = \frac{\sqrt{37}}{2}$
c $\alpha^3 - \beta^3 = (\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2)$
 $= \frac{\sqrt{37}}{2} \left(\frac{37}{4} + \frac{3}{2} \right) = \frac{\sqrt{37}}{2} \times \frac{43}{4} = \frac{43\sqrt{37}}{8}$
- 9** $x^2 - 2tx + t = 0$
- a** $\alpha + \beta = 2t$
 $\alpha\beta = t$
 $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 4t^2 - 2t$
 $= 2t(2t - 1)$
- b** $\alpha + \beta = 2t$
 $\alpha - \beta = 24$
 $\Rightarrow 2\alpha = 2t + 24$
 $\alpha = t + 12, \beta = t - 12$
 $\therefore \alpha\beta = t \Rightarrow (t + 12)(t - 12) = t$
i.e. $t^2 - 144 = t$ or $0 = t^2 - t - 144$
 $\therefore t = \frac{1 \pm \sqrt{1 + 576}}{2} \therefore t = \frac{1 + \sqrt{577}}{2} (t > 0)$
- c** $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{2t(2t - 1)}{t} = 2(2t - 1)$
 $\frac{\alpha}{\beta} \times \frac{\beta}{\alpha} = 1$
 $\therefore$ equation is $x^2 - 2(2t - 1)x + 1 = 0$
or $x^2 - 2\sqrt{577}x + 1 = 0$

Chapter 3

3.1

Exercise 3A

- 1 **a** $x^2 + 5x + 3$ **b** $x^2 + x - 9$
c $x^2 - 3x + 7$ **d** $x^2 - 3x + 2$
e $x^2 - 3x - 2$
- 2 **a** $6x^2 + 3x + 2$ **b** $3x^2 + 2x - 2$
c $2x^2 - 2x - 7$ **d** $-3x^2 + 5x - 7$
e $-5x^2 + 3x + 5$
- 3 **a** $2x^3 + 5x^2 - 5x + 1 = (2x - 1)(x^2 + 3x - 1)$
 $\therefore \frac{2x^3 + 5x^2 - 5x + 1}{2x - 1} = x^2 + 3x - 1$
- b** $3x^3 + 2x^2 - 3x - 2 = (3x + 2)(x^2 - 1)$
 $\therefore \frac{3x^3 + 2x^2 - 3x - 2}{(3x + 2)} = x^2 - 1$
- c** $6x^3 + x^2 - 7x + 2 = (3x - 1)(2x^2 + x - 2)$
 $\therefore \frac{6x^3 + x^2 - 7x + 2}{3x - 1} = 2x^2 + x - 2$
- d** $4x^3 + 4x^2 + 5x + 12 = (2x + 3)(2x^2 - x + 4)$
 $\therefore \frac{4x^3 + 4x^2 + 5x + 12}{2x + 3} = 2x^2 - x + 4$
- e** $2x^3 + 7x^2 + 7x + 2 = (2x + 1)(x^2 + 3x + 2)$
 $\therefore \frac{2x^3 + 7x^2 + 7x + 2}{2x + 1} = x^2 + 3x + 2$

3.2 Exercise 3B

- 1 $(x - 1)(x + 3)(x + 4)$
- 2 $(x + 1)(x + 7)(x - 5)$
- 3 $(x - 5)(x - 4)(x + 2)$
- 4 $(x - 2)(2x - 1)(x + 4)$
- 5 **a** $(x + 1)(x - 5)(x - 6)$
b $(x - 2)(x + 1)(x + 2)$
c $(x - 5)(x + 3)(x - 2)$
- 6 **a** $(x - 1)(x + 3)(2x + 1)$
b $(x - 3)(x - 5)(2x - 1)$
c $(x + 1)(x + 2)(3x - 1)$
d $(x + 2)(2x - 1)(3x + 1)$
e $(x - 2)(2x - 5)(2x + 3)$
- 7 2
- 8 -16
- 9 $p = 3, q = 7$

3.3

Exercise 3C

- 1 **a** 27 **b** -6 **c** 0 **d** 1
- 2 27
- 3 18
- 4 30
- 6 -9
- 7 $8\frac{8}{27}$
- 8 $a = 5, b = -8$
- 9 $p = 8, q = 3$

3.4

Exercise 3D

- 1 **a** $x = 5, y = 6$ or $x = 6, y = 5$
b $x = 0, y = 1$ or $x = \frac{4}{5}, y = \frac{3}{5}$
c $x = -1, y = -3$ or $x = 1, y = 3$
d $x = 4\frac{1}{2}, y = 4\frac{1}{2}$ or $x = 6, y = 3$
e $a = 1, b = 5$ or $a = 3, b = -1$
f $u = 1\frac{1}{2}, v = 4$ or $u = 2, v = 3$
- 2 $(-11, -15)$ and $(3, -1)$
- 3 $(-1\frac{1}{6}, -4\frac{1}{2})$ and $(2, 5)$
- 4 **a** $x = -1\frac{1}{2}, y = 5\frac{3}{4}$ or $x = 3, y = -1$
b $x = 3, y = \frac{1}{2}$ or $x = 6\frac{1}{3}, y = -2\frac{5}{6}$
- 5 **a** $x = 3 + \sqrt{13}, y = -3 + \sqrt{13}$ or $x = 3 - \sqrt{13}, y = -3 - \sqrt{13}$
b $x = 2 - 3\sqrt{5}, y = 3 + 2\sqrt{5}$ or $x = 2 + 3\sqrt{5}, y = 3 - 2\sqrt{5}$

3.5

Exercise 3E

- 1 **a** $x < 4$ **b** $x \geq 7$ **c** $x > 2\frac{1}{2}$
d $x \leq -3$ **e** $x < 11$ **f** $x < 2\frac{3}{5}$
g $x > -12$ **h** $x < 1$ **i** $x \leq ??$
j 8 **k** $x > 1\frac{1}{7}$
- 2 **a** $x \geq 3$ **b** $x < 1$ **c** $x \leq -3\frac{1}{4}$
d $x < 18$ **e** $x > 3$ **f** $x \geq 4\frac{2}{5}$
g $x < 4$ **h** $x > -7$ **i** $x \leq -\frac{1}{2}$
j $x \geq \frac{3}{4}$
- 3 **a** $x > 2\frac{1}{2}$ **b** $2 < x < 4$ **c** $2\frac{1}{2} < x < 3$
d No values **e** $x = 4$

3.6

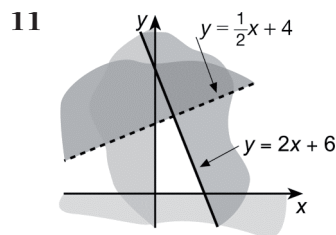
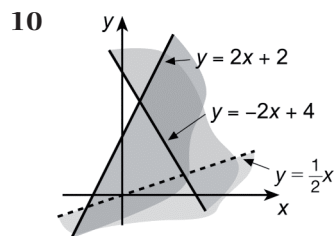
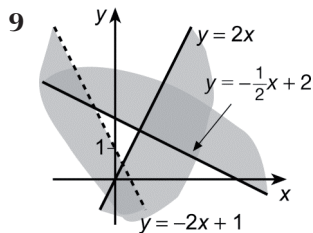
Exercise 3F

- 1 a $3 < x < 8$ b $-4 < x < 3$
 c $x < -2, x > 5$ d $x \leq -4, x \geq -3$
 e $-\frac{1}{2} < x < 7$ f $x < -2, x > 2\frac{1}{2}$
 g $\frac{1}{2} < x < 1\frac{1}{2}$ h $x < \frac{1}{3}, x > 2$
 i $-3 < x < 3$ j $x < -2\frac{1}{2}, x > \frac{2}{3}$
 k $x < 0, x > 5$ l $-1\frac{1}{2} < x < 0$
 2 a $-5 < x < 2$ b $x < -1, x > 1$
 c $\frac{1}{2} < x < 1$ d $-3 < x < \frac{1}{4}$

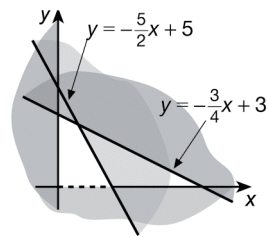
3.7

Exercise 3G

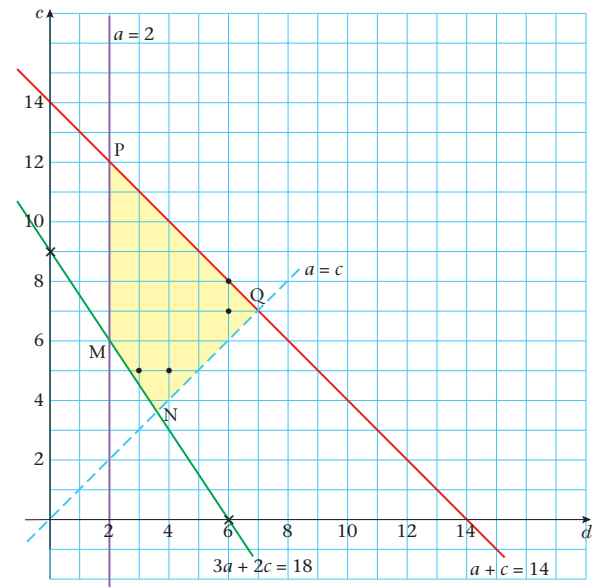
- 1 $-3 \leq x < 4$
 2 $y < 2$ or $y \geq 5$
 3 $2y + x \geq 10$ or $2y + x \leq 4$
 4 $-2 \leq 2x - y \leq 2$
 5 $4x + 3y \leq 12, y \geq 0$ and $y < 2x + 4$
 6 $y > \leq \frac{3x}{4} - 3, y \leq 0$ and $y \geq -\frac{3x}{2} - 3$
 7 $x \geq 0, y \geq 0, y < -\frac{3x}{2+9}$ and $y \leq -\frac{2x}{3+6}$
 8 $y \geq 0, y \leq x + 2, y \leq 2x - 2$ and $y \leq 18 - 2x$



12

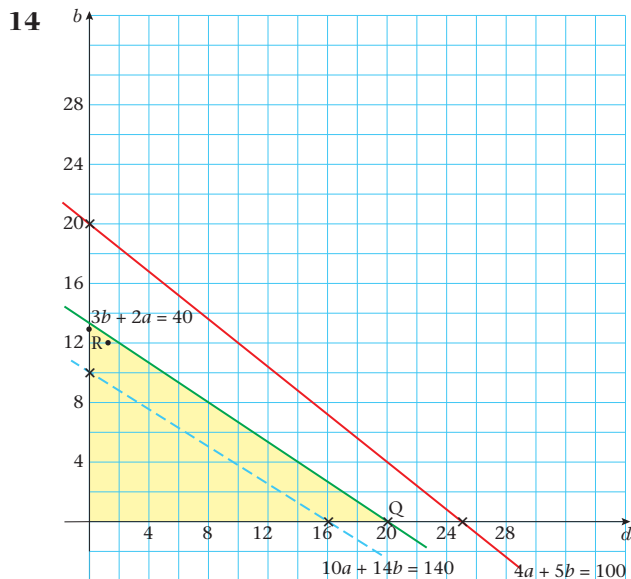


- 13 a Let a = no. of adults, and c = no. of children.
 $a + c \leq 14$ (no more than 14 passengers)
 $12a + 8c \geq 72$ (money raised must cover cost of £72)
 $c > a$ (more children than adults)
 $a \geq 2$ (at least 2 adults)



NB $12a + 8c \geq 72$ requires line $3a + 2c = 18$

- b To find smallest sized group you need to consider points close to M and N
 M(2, 6) is 2 adults and 6 children
 Points close to N are (3, 5) and (4, 5)
 So the smallest sized group is 8: 2 adults and 6 children or 3 adults and 5 children.
- c To find the maximum amount of money that can be made you need to consider points close to P and Q
 P(2, 12) raises $2 \times 12 + 12 \times 8 = \text{£}120$
 Q(7, 7) is not in the region ($\because c > a$) but (6, 8) is on d
 (6, 8) raises $6 \times 12 + 8 \times 8 = \text{£}136$
 So the maximum amount available for refreshments is £64 from taking 6 adults and 8 children



Let a = no. of machine A
 b = no. of machine B

$$4a + 5b \leq 100 \quad (\text{floor area is } 100\text{m}^2)$$

$$2a + 3b \leq 40 \quad (\text{no. of operators is } 40)$$

Profit would be $P = 100a + 140b$
 (this is parallel to $10a + 14b$)

The maximum profit will run along RQ
 Nearest to Point R (0, 13) will raise £1820
 Point (1, 12) will raise £1780
 Point Q (20, 0) will raise £2000
 So maximum profit is from 20 Machine A

15 Let x = no. of ornament A
 y = no. of ornament B

$$3x + 2y \leq 48 \quad (\text{machine time is } 48 \text{ h})$$

$$1.5x + 2.5y \leq 45 \quad (\text{craftsman's time is } 45 \text{ h})$$

Profit $P = 12x + 15y$ (e.g. $12x + 15y = 120$ is drawn)



To find maximum profit drag the profit line towards the edges R, S, T. It will first cross at T, then R and finally S.

T (16, 0) gives a profit of £192
 R (0, 24) gives a profit of £270
 S is not a point giving whole numbers for x and y but the nearby points are (6, 14) and (7, 13)
 (6, 14) gives a profit of $6 \times 12 + 14 \times 15 = £282$
 (7, 13) gives a profit of $7 \times 12 + 13 \times 15 = £279$
 So maximum profit is £282 from making 6 ornament A and 14 ornament B.

Mixed Exercise 3H

- 1 $x = 4, y = 3\frac{1}{2}$
- 2 (3, 1) and $(-2\frac{1}{5}, -1\frac{3}{5})$
- 3 $x = -1\frac{1}{2}, y = 2\frac{1}{4}$ and $x = 4, y = -\frac{1}{2}$
- 4 a $x > 10\frac{1}{2}$ b $x < -2, x > 7$
- 5 $3 < x < 4$
- 6 a $x < 2\frac{1}{2}$
 b $\frac{1}{2} < x < 5$
 c $\frac{1}{2} < x < 2\frac{1}{2}$
- 7 $x < 0, x > 1$
- 8 $A = 2, B = 4, C = -5$
- 9 $p = 1, q = 3$
- 10 $(x - 2)(x + 4)(2x - 1)$
- 11 7
- 12 $7\frac{1}{4}$
- 13 a $p = 1, q = -15$ b $(x + 3)(2x - 5)$
- 14 a $r = 3, s = 0$ b $1\frac{13}{27}$
- 15 a $(x - 1)(x + 5)(2x + 1)$ b $-5, -\frac{1}{2}, 1$
- 16 -2
- 17 -18

18 $2, -\frac{3}{2} \pm \frac{\sqrt{5}}{2}$

19 $\frac{1}{2}, 3$

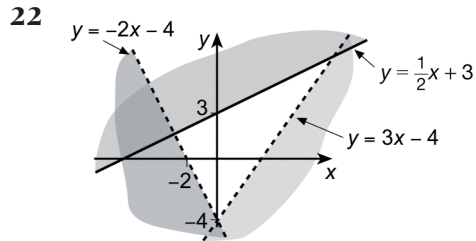
20 A: $y > 2x + 2, 4y + 3x + 12 > 0$

B: $y < 2x + 2, 4y + 3x + 12 > 0$

C: $y < 2x + 2, 4y + 3x + 12 < 0$

D: $y > 2x + 2, 4y + 3x + 12 < 0$

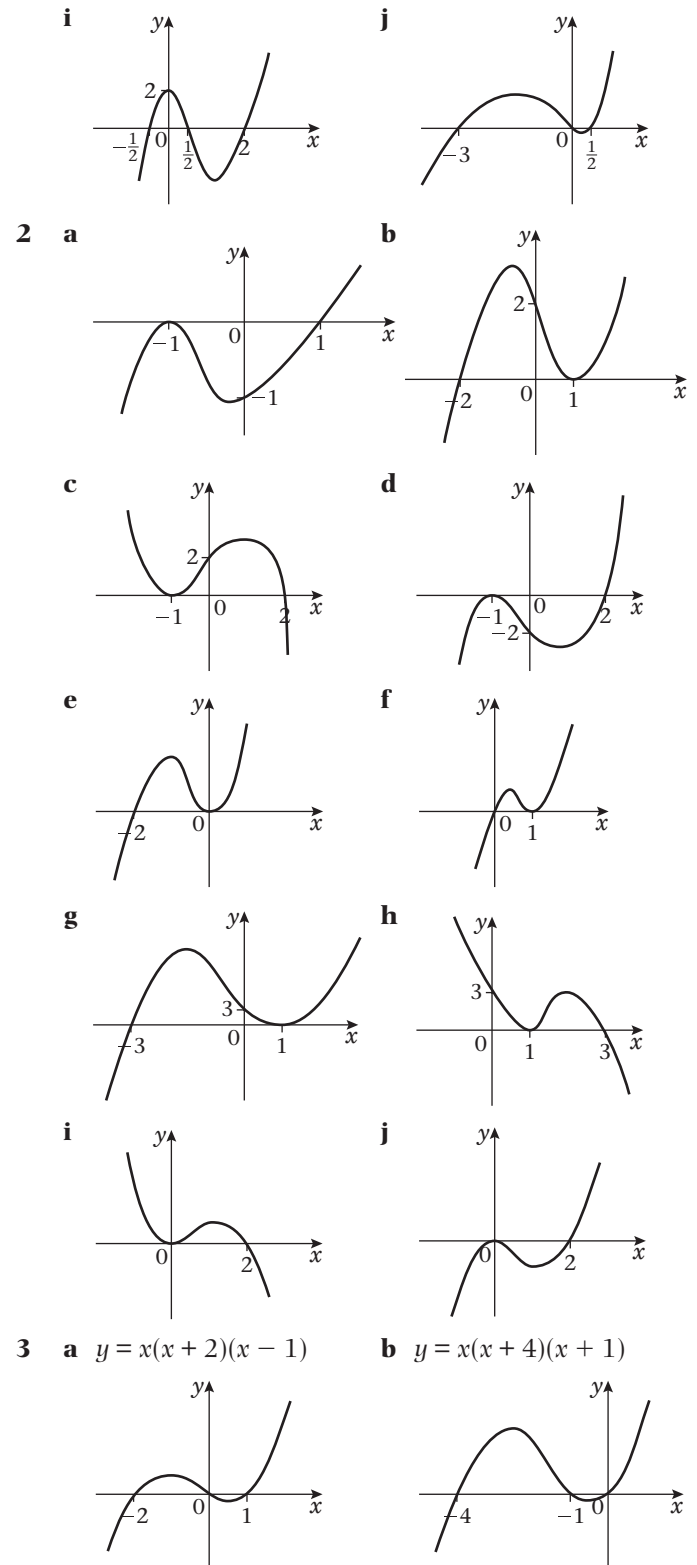
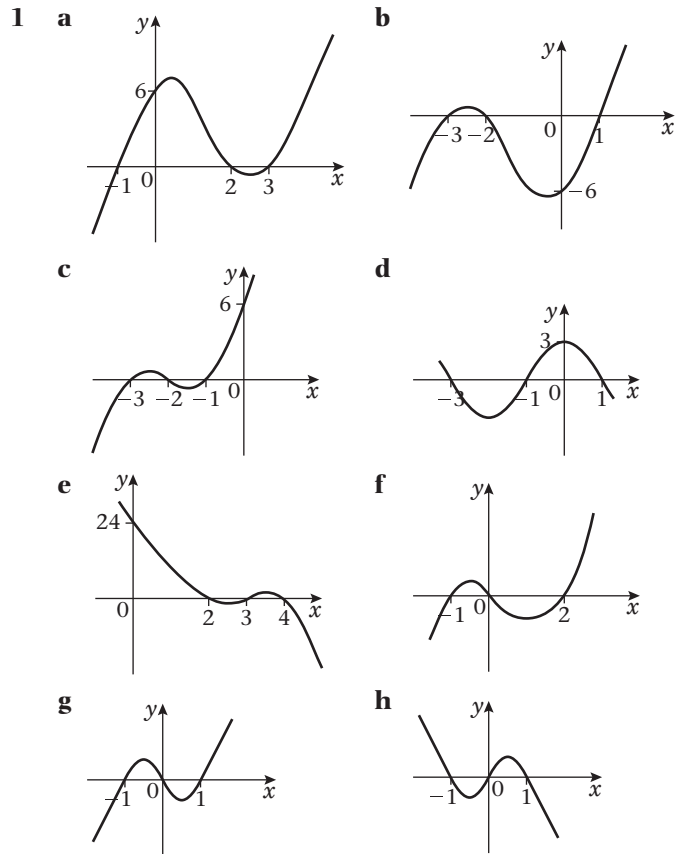
21 $\{y \geq 3 \text{ and } 2x + y \leq 6\}$



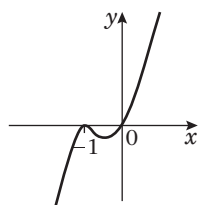
Chapter 4

4.1

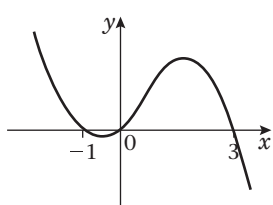
Exercise 4A



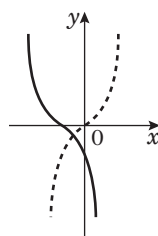
c $y = x(x + 1)^2$



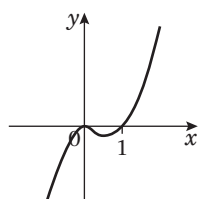
d $y = x(x + 1)(3 - x)$



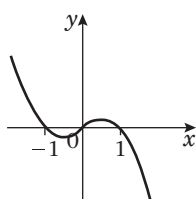
e



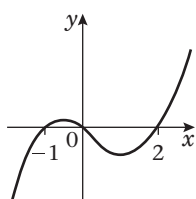
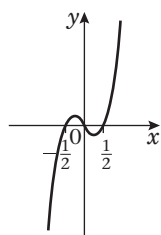
e $y = x^2(x - 1)$



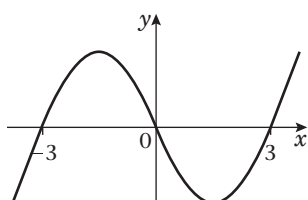
f $y = x(1 - x)(1 + x)$



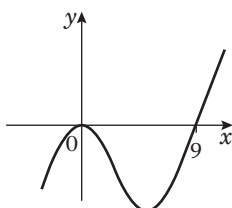
g $y = 3x(2x - 1)(2x + 1)$ **h** $y = x(x + 1)(x - 2)$



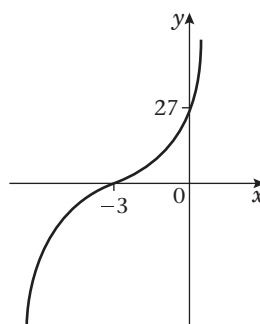
i $y = x(x - 3)(x + 3)$



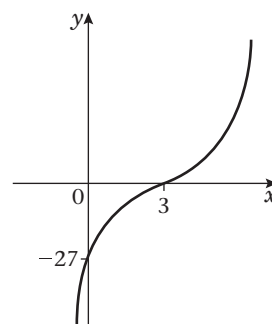
j $y = x^2(x - 9)$



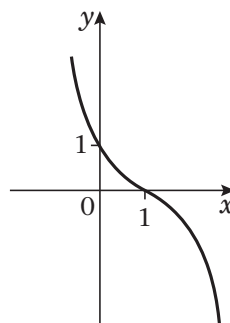
2 a



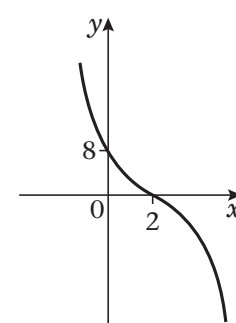
b



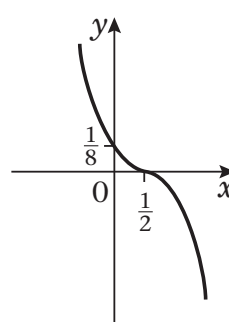
c



d



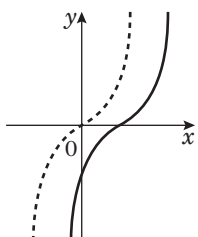
e



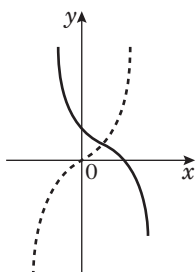
4.2

Exercise 4B

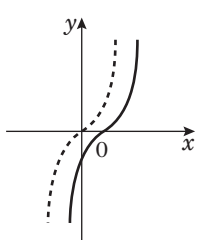
1 a



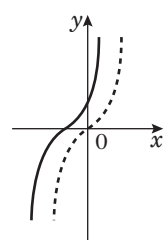
b



c



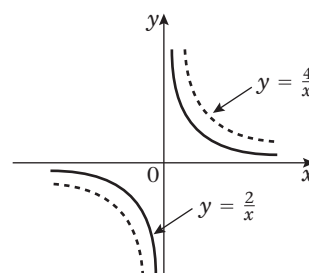
d



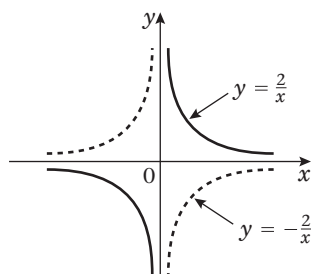
4.3

Exercise 4C

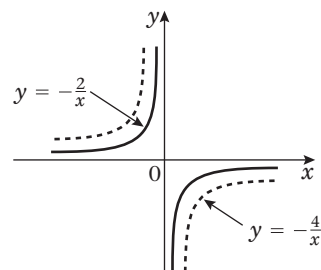
1



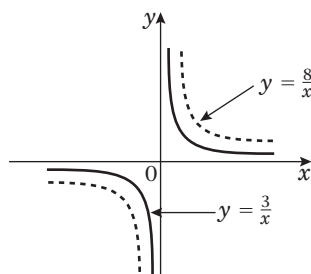
2



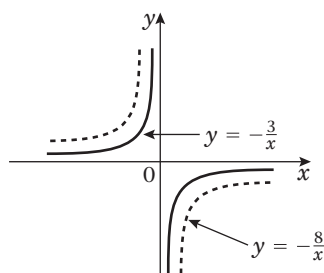
3



4



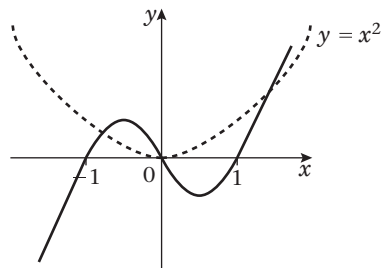
5



4.4

Exercise 4D

1 a i

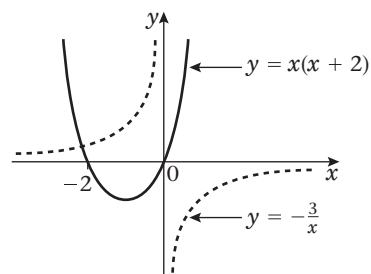


$$y = x(x^2 - 1)$$

ii 3

$$\text{iii } x^2 = x(x^2 - 1)$$

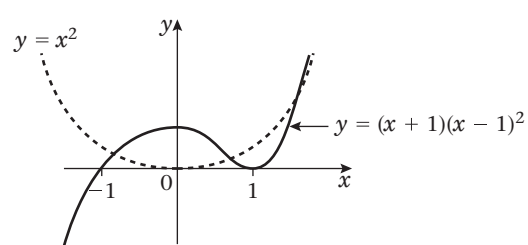
b i



ii 1

$$\text{iii } x(x + 2) = -\frac{3}{x}$$

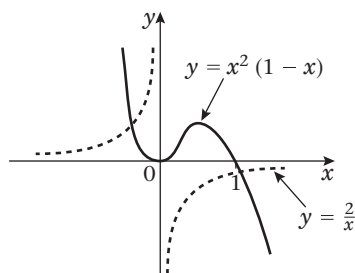
c i



ii 3

$$\text{iii } x^2 = (x + 1)(x - 1)^2$$

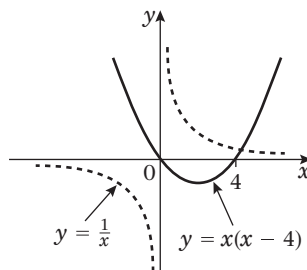
d i



ii 2

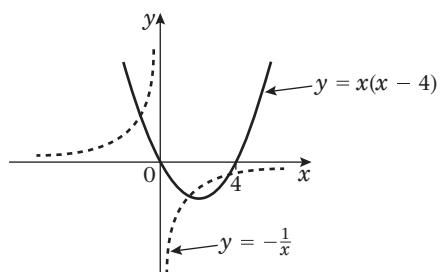
$$\text{iii } x^2(1 - x) = -\frac{2}{x}$$

e i

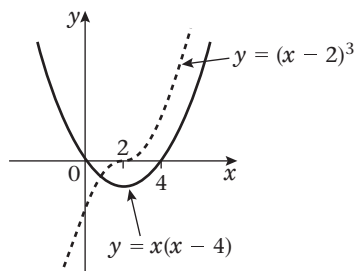


ii 1

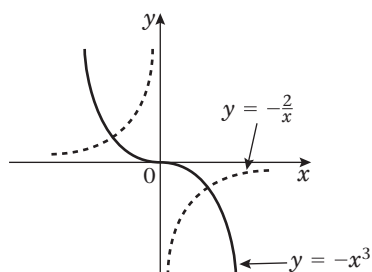
$$\text{iii } x(x - 4) = \frac{1}{x}$$

f i**ii** 3

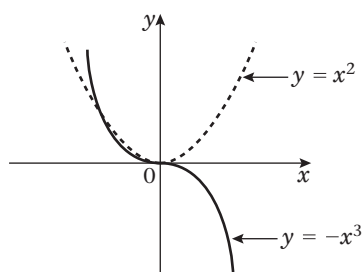
iii $x(x - 4) = -\frac{1}{x}$

g i**ii** 1

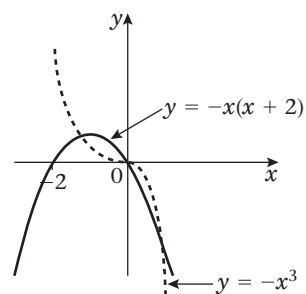
iii $x(x - 4) = (x - 2)^3$

h i**ii** 2

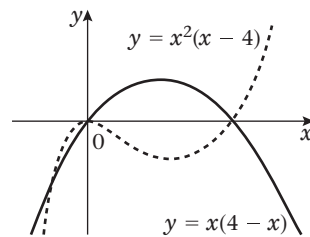
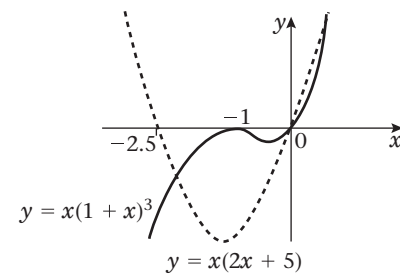
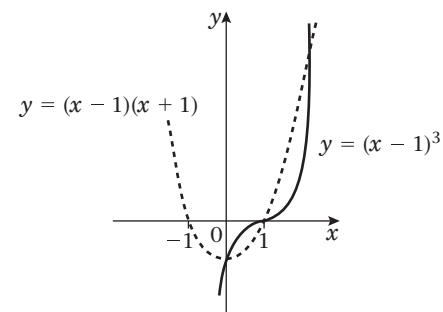
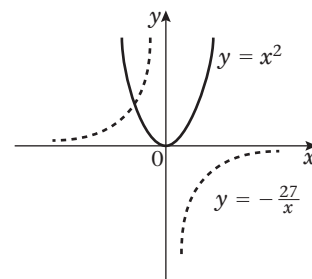
iii $-x^3 = -\frac{2}{x}$

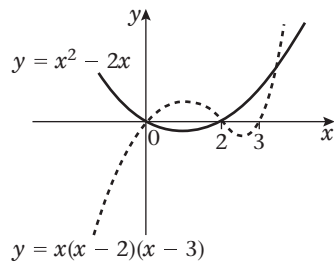
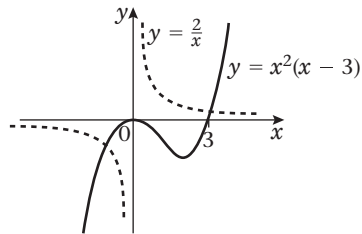
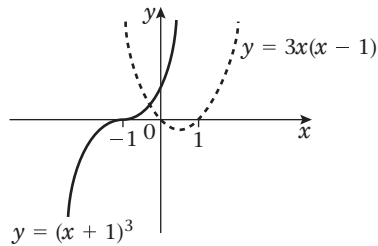
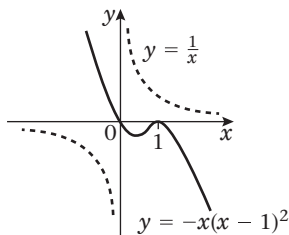
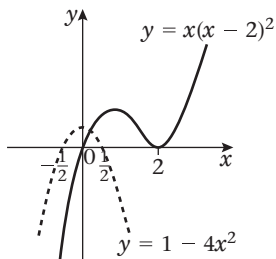
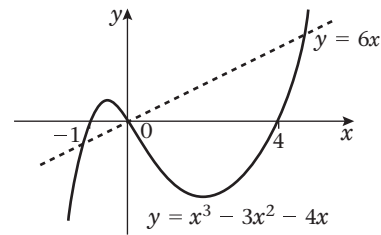
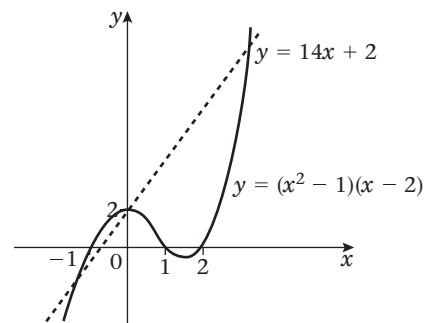
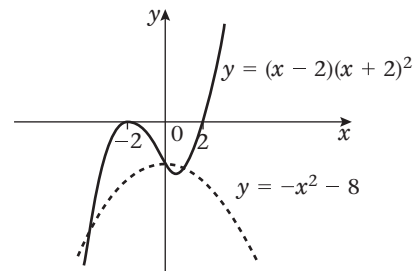
i i**ii** 2

iii $-x^3 = x^2$

j i**ii** 3

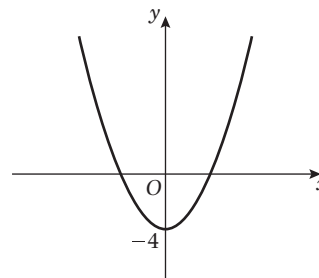
iii $-x^3 = -x(x + 2)$

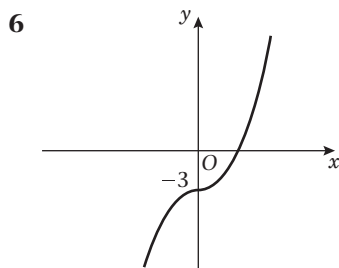
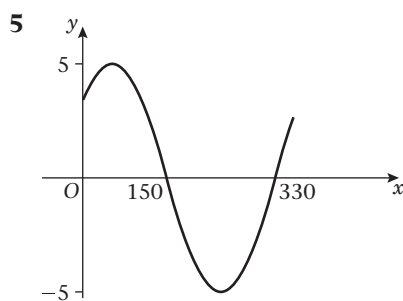
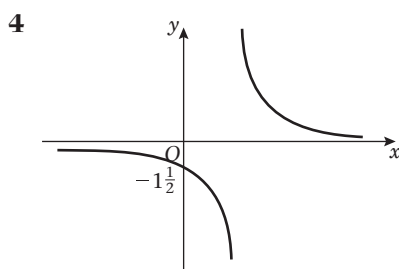
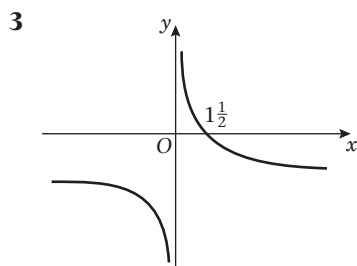
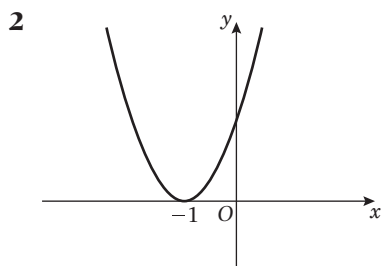
2 a**b** (0, 0); (4, 0); (-1, -5)**3 a****b** (0, 0); (2, 18); (-2, -2)**4 a****b** (0, -1); (1, 0); (3, 8)**5 a****b** (-3, 9)

6 a**b** $(0, 0); (2, 0); (4, 8)$ **7 a****b** Only 2 intersections**8 a****b** Only 1 intersection**9 a****b** Graphs do not intersect**10 a****b** 1, since graphs only cross once**11 a****b** $(0, 0); (-2, -12); (5, 30)$ **12 a****b** $(0, 2); (-3, -40); (5, 72)$ **13 a****b** $(0, -8); (1, -9); (-4, -24)$

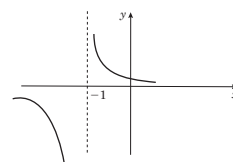
4.5

Exercise 4E

1

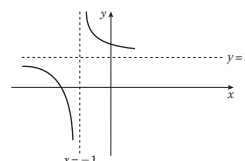


Transformation $f(x+1)$ gives



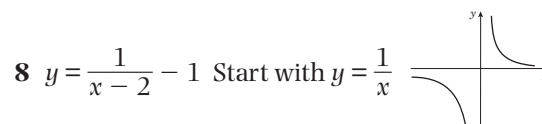
$$y = \frac{1}{x+1}$$

Finally transformation $f(x) + 3$ gives

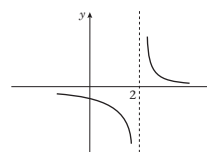


$$y = 3 + \frac{1}{x+1}$$

Asymptotes $x = -1, y = 3$

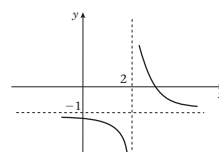


Transformation $f(x-2)$ gives



$$y = \frac{1}{x-2}$$

Finally $f(x) - 1$ gives



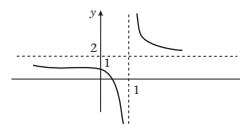
$$y = \frac{1}{x-2} - 1$$

Asymptotes $x = 2, y = -1$

9 $y = 2 + \frac{1}{x-1}$

Vertical asymptote is $x = 1$
(put denominator = 0)

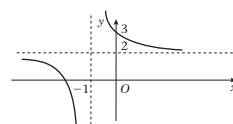
Horizontal asymptote is $y = 2$ (let $x \Rightarrow \infty$)



$$x = 0, y = 1$$

Asymptotes $x = 1$ and
 $y = 2$

10 $y = \frac{3+2x}{1+x} = \frac{2(1+x)+1}{1+x} = 2 + \frac{1}{1+x}$



$$x = 0, y = 3$$

Asymptotes $x = -1, y = 2$

11 $y = 2 + \frac{3}{1-x}$

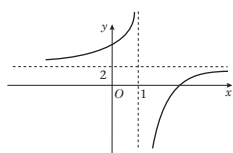
Vertical asymptote $x = 1$

Horizontal asymptote $y = 2$

Let $x = 0$ and $y = 2 + 3 = 5$

NB This is a series of

transformations of $y = -\frac{3}{x}$



12 $y = 5 - \frac{2}{3+x}$

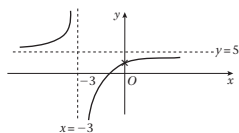
Vertical asymptote $x = -3$

Horizontal asymptote $y = 5$

Let $x = 0$ and $y = 5 - \frac{2}{3} = 4\frac{1}{3}$

Again this is a series of

transformations of $y = -\frac{2}{x}$



13 $y = \frac{3}{2-x} - 4$

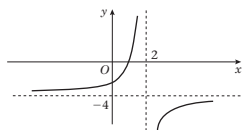
Vertical asymptote $x = 2$

Horizontal asymptote $y = -4$

Let $x = 0$ and $y = \frac{3}{2} - 4 = -2\frac{2}{3}$

This is a series of transformations

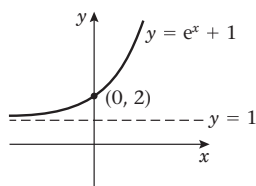
of $y = -\frac{3}{x}$



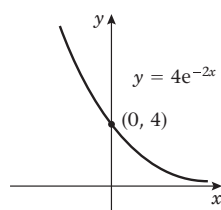
4.6

Exercise 4F

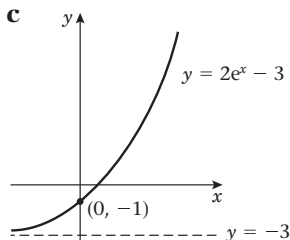
1 a



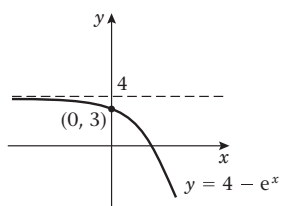
b



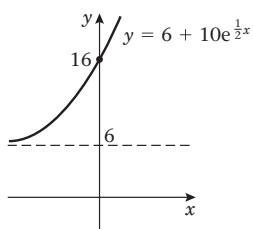
c



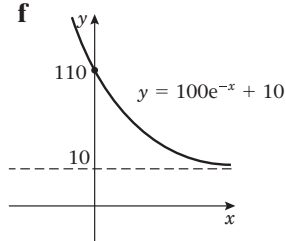
d



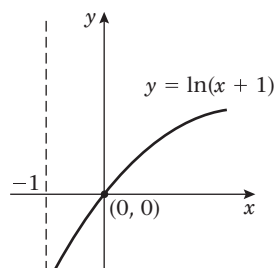
e



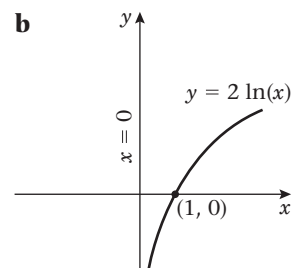
f



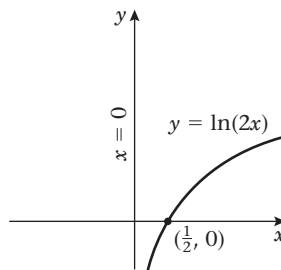
2 a



b



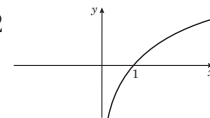
c



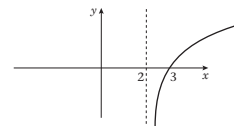
d

$y = 3\ln(x-2) \quad x > 2$

Start with $y = \ln x$

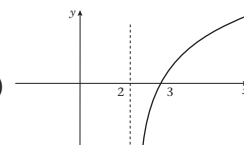


Use transformation $f(x-2)$

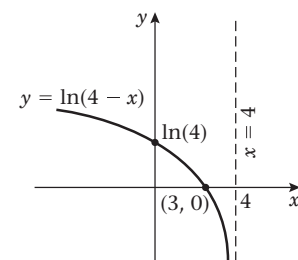


Now use transformation $3f(x)$

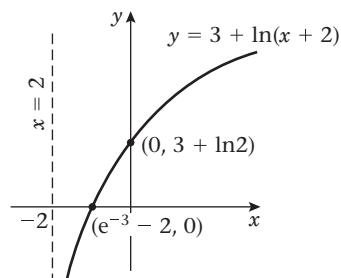
$y = 3\ln(x-2)$



e



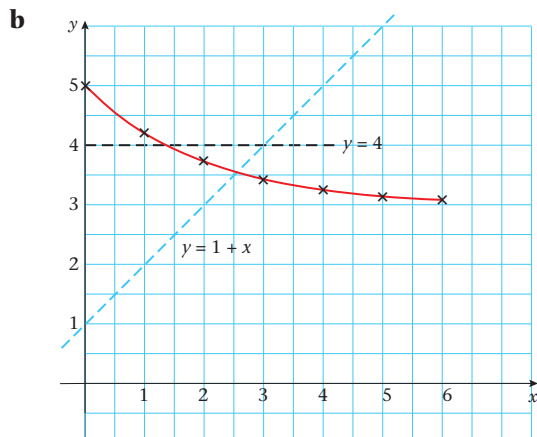
f



4.7

Exercise 4G

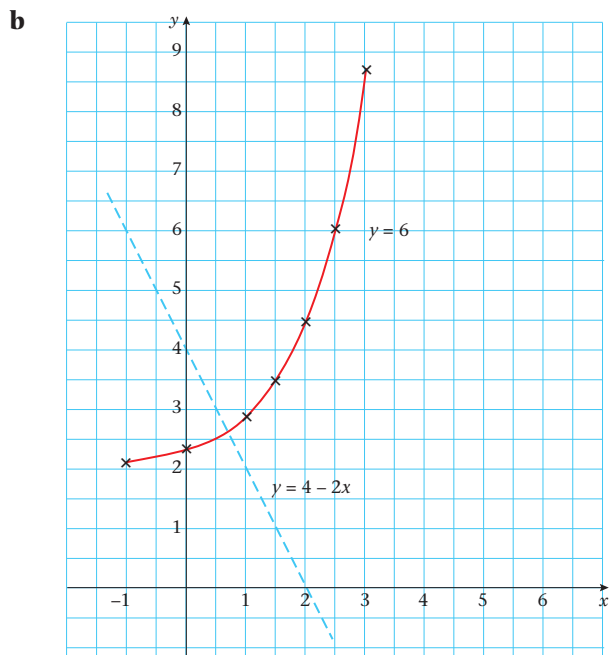
- 1 a $x = 1, y = 4.21; x = 5, y = 3.16$



- c $e^{-\frac{1}{2}x} = 0.5 \Rightarrow 3 + 2e^{-\frac{1}{2}x} = 3 + 2 \times 0.5 = 4$
Draw $y = 4$ and intersection is at ≈ 1.35

- d $x = -2 \ln \left(\frac{x-2}{2} \right)$
 $\Rightarrow e^{-\frac{x}{2}} = \frac{x-2}{2}$
 $\Rightarrow 2e^{-\frac{x}{2}} = x - 2 \therefore 3 + 2e^{-\frac{1}{2}x} = 1 + x$
Draw $y = 1 + x$ and intersection at $x \approx 2.55$

- 2 a $x = 0, y = 2 + \frac{1}{3} = 2.33; x = 2.5, y = 6.06$



- c $e^x = 12 \Rightarrow 2 + \frac{1}{3}e^x = 2 + \frac{1}{3} \times 12 = 6$
Draw line $y = 6$ and intersection is at ≈ 2.45

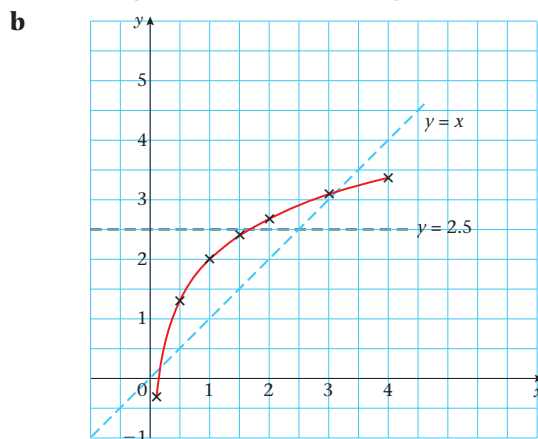
- d $x = \ln(6 - 6x)$
 $\Rightarrow e^x = 6 - 6x$

$$\Rightarrow \frac{1}{3}e^x = 2 - 2x$$

$$\therefore 2 + \frac{1}{3}e^x = 4 - 2x \text{ so draw } y = 4 - 2x$$

and intersection at ≈ 0.65 or 0.7 to 1sf

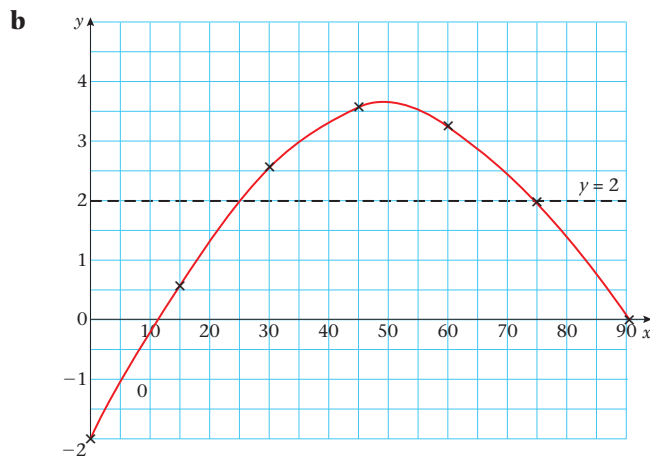
- 3 a $x = 1, y = 2 + \ln 1 = 2; x = 4, y = 3.39$



- c $\ln x = 0.5$
 $\Rightarrow 2 + \ln x = 2.5$ so draw $y = 2.5$
Intersection ≈ 1.60

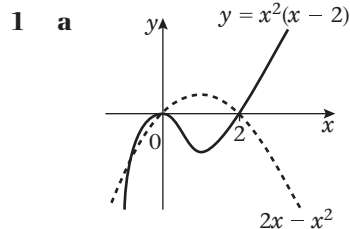
- d $x = e^{x-2}$
 $\Rightarrow \ln x = x - 2$
 $\Rightarrow 2 + \ln x = x$ so draw $y = x$
Intersection ≈ 3.1 to 1sf

- 4 a $x = 30, y = 2.60; x = 75, y = 1.98$

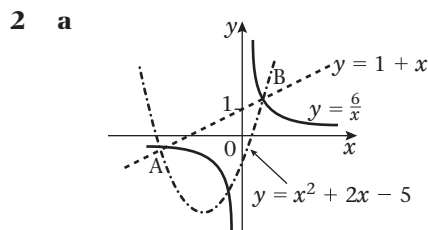


- c $2 + 2 \cos x = 5 \sin 2x$
 $\Rightarrow 2 = 5 \sin 2x - 2 \cos x$
Draw $y = 2$
and intersections at ≈ 25.1 and 74.6
or 25 and 75 to 2sf

Mixed Exercise 4H



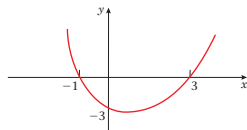
b $x = 0, -1, 2$; points $(0, 0)$, $(2, 0)$, $(-1, -3)$



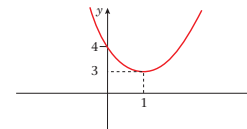
b $A(-3, -2)$ $B(2, 3)$

c $y = x^2 + 2x - 5$

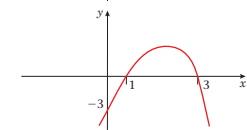
3 a $y = x^2 - 2x - 3$
 $= (x - 3)(x + 1)$



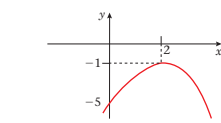
b $y = x^2 - 2x + 4$
 $= (x - 1)^2 + 3$



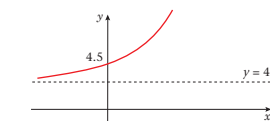
4 a $y = -(x^2 - 4x + 3)$
 $= (3 - x)(x - 1)$



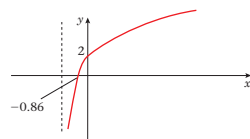
b $y = -(x^2 - 4x + 5)$
 $= -[(x - 2)^2 + 1]$
 $= -1 - (x - 2)^2$



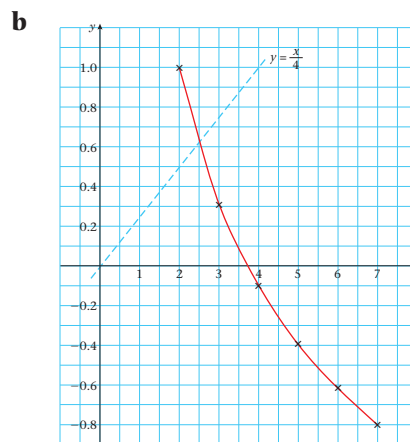
5 a $y = \frac{1}{2}e^x + 4$
 $(x = 0, y = 4.5)$



b $y = \ln(x + 1) + 2$
 $(x = 0, y = 2)$
 $y = 0 \Rightarrow \ln(x + 1) = -2$
 $\Rightarrow x + 1 = e^{-2}$
 $\therefore x = e^{-2} - 1 \approx -0.86$



6 a $x = 5$ $y = 1 - \ln 4 = -0.39$
 $x = 6$ $y = 1 - \ln 5 = -0.61$



c $\ln(x - 1) = 0$
 $\Rightarrow 1 - \ln(x - 1) = 1$ so draw $y = 1$
Intersection at $x = 2$

d $x = 1 + e^{1 + \frac{x}{4}}$
 $\ln(x - 1) = 1 + \frac{x}{4}$
 $\Rightarrow x - 1 = e^{1 + \frac{x}{4}}$
 $\Rightarrow 1 - \ln(x - 1) = 1 - (1 + \frac{x}{4}) = -\frac{x}{4}$ so draw $y = -\frac{x}{4}$
and intersection at ≈ 2.5

Chapter 5

5.1

Exercise 5A

1 Arithmetic sequences are **a, b, c, h, i**

2 a $23, 2n + 3$

b $32, 3n + 2$

c $-3, 27 - 3n$

d $35, 4n - 5$

e $10x, nx$

f $a + 9d, a + (n - 1)d$

3 a $\pounds 5800$

b $\pounds(3800 + 200m)$

4 a 22

b 40

c 39

d 46

e 18

f n

5.2

Exercise 5B

1 a $78, 4n - 2$

b $42, 2n + 2$

c $23, 83 - 3n$

d $39, 2n - 1$

e $-27, 33 - 3n$

f $59, 3n - 1$

g $39p, (2n - 1)p$

h $-71x, (9 - 4n)x$

2 a 30

b 29

c 32

d 31

e 221

f 77

3 $d = 6$

4 $a = 36, d = -3, 14\text{th term}$

5 24

6 $x = 5; 25, 20, 15$

7 $7 \times \frac{1}{2}, x = 8$

5.3

Exercise 5C

- 1 **a** 820 **b** 450 **c** -1140
d -294 **e** 1440 **f** 1425
g -1155 **h** $21(11x + 1)$
- 2 **a** 20 **b** 25 **c** 65
d 4 or 14 (2 answers)
- 3 2550
- 4 **i** £222 500 **ii** £347 500
- 5 1683, 32674
- 6 £9.03, 141 days
- 7 $d = -\frac{1}{2}, -5.5$
- 8 $a = 6, d = -2$

5.4

Exercise 5D

- 1 **a** $\sum_{r=1}^{10} (3r + 1)$ **b** $\sum_{r=1}^{30} (3r - 1)$
c $\sum_{r=1}^{11} 4(11 - r)$ **d** $\sum_{r=1}^{16} 6r$
- 2 **a** 45 **b** 210
c 1010 **d** 112
- 3 19
- 4 49

5.5

Exercise 5E

- 1 **a** Geometric $r = 2$ **b** Not geometric
c Not geometric **d** Geometric $r = 3$
e Geometric $r = \frac{1}{2}$ **f** Geometric $r = -1$
g Geometric $r = 1$ **h** Geometric $r = \frac{1}{4}$
- 2 **a** 135, 405, 1215 **b** -32, 64, -128
c 7.5, 3.75, 1.875 **d** $\frac{1}{64}, \frac{1}{256}, \frac{1}{1024}$
e p^3, p^4, p^5 **f** $-8x^4, 16x^5, -32x^6$
- 3 **a** $3\sqrt{3}$ **b** $9\sqrt{3}$

5.6

Exercise 5F

- 1 **a** 486, 39 366, $2 \times 3^{n-1}$
b $\frac{25}{8}, \frac{25}{128}, \frac{100}{2^{n-1}}$
c -32, -512, $(-2)^{n-1}$
d 1.610 51, 2.357 95, $(1.1)^{n-1}$

- 2 10, 6250
- 3 $a = 1, r = 2$
- 4 $\pm \frac{1}{8}$
- 5 -6 (from $x = 0$), 4 (from $x = 10$)

5.7

Exercise 5G

- 1 **a** 255 **b** 63.938 (3 dp)
c -728 **d** $546\frac{2}{3}$
e 5460 **f** 19 680
g 5.994 (3 dp) **h** 44.938 (3 dp)
- 2 $\frac{5}{4}, -\frac{9}{4}$
- 3 $264 - 1 = 1.84 \times 1019$
- 4 **a** £49 945.41 **b** £123 876.81
- 5 **a** 2.401 **b** 48.8234
- 6 19 terms
- 7 22 terms
- 8 26 days, 98.5 miles on the 25th day
- 9 25 years

5.8

Exercise 5H

- 1 **a** $\frac{10}{9}$ **b** Doesn't exist
c $6\frac{2}{3}$ **d** Doesn't exist
e Doesn't exist **f** $4\frac{1}{2}$
g Doesn't exist **h** 90
i $\frac{1}{1} - r$ if $|r| < 1$ **j** $\frac{1}{1} + 2x$ if $|x| < \frac{1}{2}$
- 2 $\frac{2}{3}$
- 3 $-\frac{2}{3}$
- 4 20
- 5 $\frac{40}{3} = 13\frac{1}{3}$
- 6 $\frac{23}{99}$
- 7 4
- 8 40 m
- 9 $r < 0$ because $S_\infty < S_3, a = 12, r = -\frac{1}{2}$
- 10 $r = \pm \sqrt{\frac{2}{3}}$

Mixed Exercise 5I

- 1 **a** Add 6 to the previous term, i.e. $U_{n+1} = U_n + 6$
(or $U_n = 6n - 1$)
b Add 3 to the previous term, i.e. $U_{n+1} = U_n + 3$
(or $U_n = 3n$)
c Multiply the previous term by 3, i.e. $U_{n+1} = 3U_n$
(or $U_n = 3^{n-1}$)

- d** Subtract 5 from the previous term,
i.e. $U_{n+1} = U_n - 5$ (or $U_n = 15 - 5n$)
- e** The square numbers ($U_n = n^2$)
- f** Multiply the previous term by 1.2, i.e. $U_{n+1} = 1.2U_n$
(or $U_n = (1.2)^{n-1}$)

Arithmetic sequences are:

- a** $a = 5, d = 6$
b $a = 3, d = 3$
c $a = 10, d = -5$
- 2 a** 81 **b** 860
3 32
4 a £13 780 **b** £42 198
5 a $a = 25, d = -3$ **b** -3810
6 a 26 733 **b** 53 467
7 a 5 **b** 45
8 a $d = 5$ **b** 59
9 b $11k - \frac{9}{3}$ **c** 1.5 **d** 415
10 a Not geometric **b** Geometric $r = 1.5$
b Geometric $r = \frac{1}{2}$ **c** Geometric $r = -2$
d Not geometric **e** Geometric $r = 1$
11 a 0.8235 (4 dp), $10x(0.7)^{n-1}$
b $640, 5 \times 2^{n-1}$
c $-4, 4 \times (-1)^{n-1}$
d $\frac{3}{128}, 3 \times (-\frac{1}{2})^{n-1}$
12 a 4092 **b** 19.98 (2 dp)
c 50 **d** 3.33 (2 dp)
13 a 9 **b** $\frac{8}{3}$
c Doesn't converge **d** $\frac{16}{3}$
14 b 60.72 **c** 182.25
d 3.16
15 b 200 **c** $333\frac{1}{3}$ **d** 8.95×10^{-4}
16 a 76, 60.8 **b** 0.876
c 367 **d** 380
17 a $1, \frac{1}{3}, -\frac{1}{9}$
18 a 0.8 **b** 10
c 50 **d** 0.189 (3sf)
19 a $-\frac{1}{2}$ **b** $\frac{3}{4}, -2$ **c** 14

Chapter 6

6.1

Exercise 6A

- 1** $1 + 8x + 28x^2 + 56x^3$
2 $1 - 12x + 60x^2 - 160x^3$
3 $1 + 5x + \frac{45}{4}x^2 + 15x^3$
4 $1 - 15x + 90x^2 - 270x^3$

- 5 a** $p = 5$ **b** -10 **c** -80
6 $1 - 0.6x + 0.15x^2 - 0.02x^3$, 0.94148, accurate to 5 dp
7 a $-20x^3$ **b** $120x^3$ **c** $1140x^3$
8 $b = -2$

6.2

Exercise 6B

- 1 a** $1 + 6x + 12x^2 + 8x^3$, valid for all x
b $1 + x + x^2 + x^3, |x| < 1$
c $1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3, |x| < 1$
d $1 - 6x + 24x^2 - 80x^3, |x| < \frac{1}{2}$
e $1 - x - x^2 - \frac{5}{3}x^3, |x| < \frac{1}{3}$
f $1 - 15x + \frac{75}{2}x^2 + \frac{125}{2}x^3, |x| < \frac{1}{10}$
g $1 - x + \frac{5}{8}x^2 - \frac{5}{16}x^3, |x| < 4$
h $1 - 2x^2 + \dots, |x| < \frac{\sqrt{2}}{2}$
- 2** $|x| < \frac{1}{2}$
- 3** $1 + \frac{3x}{2} - \frac{9}{8}x^2 + \frac{27}{16}x^3$, 10.148 891 88, accurate to 6 d.p.
- 4** $a = \pm 8, \pm 160x^3$
- 6** $1 - \frac{9x}{2} + \frac{27x^2}{8} + \frac{27x^3}{16}, x = 0.01, 955.339(1875)$

Mixed Exercise 6C

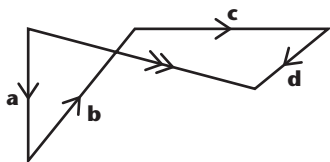
- 1 a** $p = 16$ **b** 270 **c** -1890
- 2 a** $1 - 20x + 180x^2 - 960x^3$
b 0.817 04, $x = 0.01$
- 3 a** $n = 8$ **b** $\frac{35}{8}$
- 4 a** $1 + 24x + 264x^2 + 1760x^3$
b 1.268 16
c 1.268 241 795
d 0.006 45% (3 sf)
- 5 a** $1 - 12x + 48x^2 - 64x^3$, all x
b $1 + 2x + 4x^2 + 8x^3, |x| < \frac{1}{2}$
c $1 - 2x + 6x^2 - 18x^3, |x| < \frac{1}{3}$
- 6** $1 - \frac{x}{4} - \frac{x^2}{32} - \frac{x^3}{128}$
- 7** $1 - \frac{3}{2}x^2 + \frac{27}{8}x^4 - \frac{135}{16}x^6$
- 8** $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}, \frac{1145}{512}$
- 9 a** $n = -2, a = 3$
b -108
c $|x| < \frac{1}{3}$

Chapter 7

7.1

Exercise 7A

1



2 25

3 $\sqrt{569} \approx 23.9$

4 **a** $\mathbf{d} - \mathbf{a}$ **b** $\mathbf{a} + \mathbf{b} + \mathbf{c}$
c $\mathbf{a} + \mathbf{b} - \mathbf{d}$ **d** $\mathbf{a} + \mathbf{b} + \mathbf{c} - \mathbf{d}$

7.2

Exercise 7B

1 **a** $2\mathbf{a} + 2\mathbf{b}$ **b** $\mathbf{a} + \mathbf{b}$ **c** $\mathbf{b} - \mathbf{a}$

2 **a** $\mathbf{b} - \frac{1}{2}\mathbf{a}$ **b** $\mathbf{b} - 3\mathbf{a}$
c $\frac{3}{2}\mathbf{a} - \mathbf{b}$ **d** $2\mathbf{a} - \mathbf{b}$

3 **a** Yes ($\lambda = 2$) **b** Yes ($\lambda = 4$) **c** No
d Yes ($\lambda = -1$) **e** Yes ($\lambda = -3$) **f** No

4 **a** $\lambda = \frac{1}{2}, \mu = -3$ **b** $\lambda = -2, \mu = 1$
c $\lambda = \frac{1}{4}, \mu = 5$ **d** $\lambda = -2, \mu = -1$
e $\lambda = 4, \mu = 8\frac{1}{2}$

5 **a** $\mathbf{b} - \mathbf{a}, \frac{5}{6}(\mathbf{b} - \mathbf{a}), \frac{1}{6}\mathbf{a} + \frac{5}{6}\mathbf{b}$

b $-\frac{1}{6}\mathbf{a} + (\lambda - \frac{5}{6})\mathbf{b}$

c $-\mu\mathbf{a} + (\mu - \lambda)\mathbf{b}$

d $\lambda = \frac{1}{2}, \mu = \frac{1}{6}$

6 **a** $\mathbf{i} - \mathbf{a} + \mathbf{b}$ **ii** $\frac{2}{3}\mathbf{a} - \frac{7}{4}\mathbf{b}$

b $(\frac{2}{3}\lambda - \mu)\mathbf{a} + (\frac{3}{4} - \frac{7}{4}\lambda + \mu)\mathbf{b} = 0$

d $\frac{6}{13}$

e $\frac{6}{13}\mathbf{a} + \frac{7}{13}\mathbf{b}$

f $\frac{13}{10}$

7 **a** $-\mathbf{a} + \mathbf{b}$ **b** $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

c $\frac{3}{8}\mathbf{a} + \frac{3}{8}\mathbf{b}$ **d** $-\frac{5}{8}\mathbf{a} + \frac{3}{8}\mathbf{b}$

e $-\mathbf{a} + k\mathbf{b}$ **f** $5:3, k = 35$

8 **a** $\frac{1}{3}\mathbf{a}$

b $\frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$

c $-\frac{1}{12}\mathbf{a} + \frac{3}{4}\mathbf{b}$

d $2\mathbf{b}$

e $-\frac{1}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$

f $-\frac{1}{4}\mathbf{a} + \frac{9}{4}\mathbf{b}$

g $1:3$

h $-\frac{1}{3}\mathbf{a} + \mathbf{b}, -\mathbf{a} + 3\mathbf{b}, \overrightarrow{AG} = 3\overrightarrow{EB} \Rightarrow \text{parallel}$

7.3

Exercise 7C

1 $\frac{5}{6}\mathbf{a} + \frac{1}{6}\mathbf{b}$

2 $-\frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b} + \mathbf{c}$

3 $\overrightarrow{OC} = -2\mathbf{a} + 2\mathbf{b}, \overrightarrow{OD} = 3\mathbf{a} + 2\mathbf{b}, \overrightarrow{OE} = -2\mathbf{a} + \mathbf{b}$

7.4

Exercise 7D

1 **a** $\begin{pmatrix} 12 \\ 3 \end{pmatrix}$ **b** $\begin{pmatrix} -1 \\ 16 \end{pmatrix}$ **c** $\begin{pmatrix} -21 \\ -29 \end{pmatrix}$

2 **a** $3\mathbf{i} - \mathbf{j}, 4\mathbf{i} + 5\mathbf{j}, -2\mathbf{i} + 6\mathbf{j}$

b $\mathbf{i} + 6\mathbf{j}$

c $-5\mathbf{i} + 7\mathbf{j}$

d $\sqrt{40} = 2\sqrt{10}$

e $\sqrt{37}$

f $\sqrt{74}$

3 **a** $\frac{1}{5}\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ **b** $\frac{1}{13}\begin{pmatrix} 5 \\ -12 \end{pmatrix}$

c $\frac{1}{25}\begin{pmatrix} -7 \\ 24 \end{pmatrix}$ **d** $\frac{1}{\sqrt{10}}\begin{pmatrix} 1 \\ -3 \end{pmatrix}$

4 -7 or -23

Mixed Exercise 7E

1 $m = 3, n = 1$

2 $m = -2, n = 5$

3 **a** $\overrightarrow{XM}\begin{pmatrix} -1 \\ 3 \end{pmatrix}$

$\overrightarrow{XZ} = \begin{pmatrix} -10 \\ 6 \end{pmatrix}$

b $v\begin{pmatrix} 7 \\ 3 \end{pmatrix}$

c $\begin{pmatrix} 8 \\ 0 \end{pmatrix} + w\begin{pmatrix} -10 \\ 6 \end{pmatrix}$

d $v = \frac{2}{3}, w = \frac{1}{3}$

4 **a** $\overrightarrow{AC} = x + y; \overrightarrow{BE} = \frac{1}{3}y - x$

b **i** $\overrightarrow{BF} = v(\frac{1}{3}y - x)$

ii $\overrightarrow{AF} = x + \overrightarrow{BF} = x + v(\frac{1}{3}y - x)$

iii $v = \frac{3}{4}$

5 **v** $+ \mathbf{w} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}, \sqrt{41}$

$2\mathbf{v} - \mathbf{w} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}, \sqrt{29}$

$\mathbf{v} - 2\mathbf{w} = \begin{pmatrix} 1 \\ -7 \end{pmatrix}, \sqrt{50}$

$$6 \quad p + q = \left(\frac{5}{4}\right), \sqrt{41}$$

$$3p + q = \left(\frac{9}{2}\right), \sqrt{85}$$

$$p - 3q = \left(-\frac{7}{16}\right), \sqrt{305}$$

$$7 \quad \mathbf{a} \quad \text{Chloe } \left(\frac{5}{7}\right); \text{Leo } \left(\frac{4}{5}\right); \text{Max } \left(\frac{3}{2}\right)$$

b Chloe: 74 km, 2.9 km/h

Leo: 41 km, 2.1 km/h

Max: 13 km, 1.2 km/h

Chapter 8

8.1

Exercise 8A

- | | | | |
|----------------------------------|----------------------------|-----------------------------|-----------------------------|
| 1 a -2 | b -1 | c 3 | d $\frac{1}{3}$ |
| e $\frac{2}{3}$ | f $\frac{5}{4}$ | g $\frac{1}{2}$ | h 2 |
| i $\frac{1}{2}$ | j $\frac{1}{2}$ | k -2 | l $\frac{3}{2}$ |
| 2 a 4 | b -5 | c $-\frac{2}{3}$ | d 0 |
| e $\frac{7}{5}$ | f 2 | g 2 | h -2 |
| i -9 | j -3 | k $\frac{3}{2}$ | l $-\frac{1}{2}$ |
| 3 a $4x - y + 3 = 0$ | b $3x - y - 2 = 0$ | c $6x + y - 7 = 0$ | d $4x - 5y - 30 = 0$ |
| e $5x - 3y + 6 = 0$ | f $7x - 3y = 0$ | g $14x - 7y - 4 = 0$ | h $27x + 9y - 2 = 0$ |
| i $18x + 3y + 2 = 0$ | j $2x + 6y - 3 = 0$ | k $4x - 6y + 5 = 0$ | l $6x - 10y + 5 = 0$ |
| 4 $y + 5x + 3$ | | | |
| 5 $2x + 5y + 20 = 0$ | | | |
| 6 $y + -\frac{1}{2}x + 7$ | | | |
| 7 $y = \frac{2}{3}x$ | | | |
| 8 $(3, 0)$ | | | |
| 9 $(\frac{5}{3}, 0)$ | | | |
| 10 $(0, 5), (-4, 0)$ | | | |

8.2

Exercise 8B

- | | | | |
|--------------------------|--|------------------------|------------------------|
| 1 a $\frac{1}{2}$ | b $\frac{1}{6}$ | c $\frac{3}{5}$ | d 2 |
| e -1 | f $\frac{1}{2}$ | g $\frac{1}{2}$ | h 8 |
| i $\frac{2}{3}$ | j -4 | k $\frac{1}{3}$ | l $\frac{1}{2}$ |
| m 1 | n $\frac{q^2 - p^2}{q - p} = q + p$ | | |
| 2 7 | | | |

$$3 \quad 12$$

$$4 \quad 4\frac{1}{3}$$

$$5 \quad 2\frac{1}{4}$$

$$6 \quad \frac{1}{4}$$

$$7 \quad 26$$

$$8 \quad -5$$

8.3

Exercise 8C

- | | |
|----------------------------------|-----------------------------------|
| 1 a $y = 2x + 1$ | b $y = 3x + 7$ |
| c $y = -x - 3$ | d $y = -4x - 11$ |
| e $y = \frac{1}{2}x + 12$ | f $y = \frac{2}{3}x - 5$ |
| g $y = 2x$ | h $y = -\frac{1}{2}x + 2b$ |
| 2 $y = 3x - 6$ | |
| 3 $y = 2x + 8$ | |
| 4 $2x - 3y + 24 = 0$ | |
| 5 $-\frac{1}{5}$ | |
| 6 $y = \frac{2}{3}x + 3$ | |
| 7 $2x + 3y - 12 = 0$ | |
| 8 $\frac{8}{5}$ | |
| 9 $y = \frac{4}{3}x - 4$ | |
| 10 $6x + 15y - 10 = 0$ | |

8.4

Exercise 8D

- | | |
|------------------------------------|---|
| 1 a Perpendicular | b Parallel |
| c Neither | d Perpendicular |
| e Perpendicular | f Parallel |
| g Parallel | h Perpendicular |
| i Perpendicular | j Parallel |
| k Neither | l Perpendicular |
| 2 $y = -\frac{1}{3}x$ | |
| 3 $4x - y + 15 = 0$ | |
| 4 a $y = -2x + \frac{1}{2}$ | b $y = \frac{1}{2}x$ |
| c $y = -x - 3$ | d $y = \frac{1}{2}x - 8$ |
| 5 a $y = 3x + 11$ | b $y = -\frac{1}{3}x + \frac{13}{3}$ |
| c $y = \frac{2}{3}x + 2$ | d $y = -\frac{3}{2}x + \frac{17}{2}$ |
| 6 $3x + 2y - 5 = 0$ | |
| 7 $7x - 4y + 2 = 0$ | |

8.5

Exercise 8E

- | | | |
|----------------|-----------------|-----------------|
| 1 10 | 2 13 | 3 5 |
| 4 $\sqrt{5}$ | 5 $2\sqrt{10}$ | 6 $\sqrt{106}$ |
| 7 $\sqrt{113}$ | 8 $a\sqrt{53}$ | 9 $3b\sqrt{5}$ |
| 10 $5c$ | 11 $d\sqrt{61}$ | 12 $2e\sqrt{5}$ |

8.6

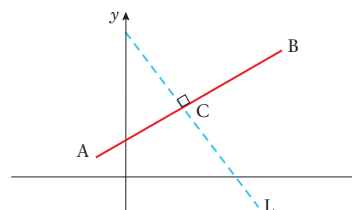
Exercise 8F

- 1 a (0, 6); (4, 10) is ratio 3 : 1
 $\left(\frac{3 \times 4 + 1 \times 0}{4}, \frac{3 \times 10 + 1 \times 6}{4}\right) = (3, 9)$
- b (1, 5); (-2, 8) is ratio 1 : 2
 $\left(\frac{2 \times 1 + 1 \times (-2)}{3}, \frac{2 \times 5 + 1 \times 8}{3}\right) = (0, 6)$
- c (3, -7); (-2, 8) is ratio 3 : 2
 $\left(\frac{2 \times 3 + 3 \times (-2)}{5}, \frac{2 \times (-7) + 3 \times 8}{5}\right) = (0, 2)$
- d (-2, 5); (5, 2) is ratio 4 : 3
 $\left(\frac{4 \times 5 + 3 \times (-2)}{7}, \frac{4 \times 2 + 3 \times 5}{7}\right) = (2, 3\frac{2}{7})$
- 2 a (4, 2); (6, 8) midpoint $\left(\frac{4+6}{2}, \frac{2+8}{2}\right) = (5, 5)$
- b (0, 6); (12, 2) midpoint $\left(\frac{0+12}{2}, \frac{6+2}{2}\right) = (6, 4)$
- c (2, 2); (-4, 6) midpoint $\left(\frac{2-4}{2}, \frac{2+6}{2}\right) = (-1, 4)$
- d (-6, 4); (6, -4) midpoint $\left(\frac{-6+6}{2}, \frac{4-4}{2}\right) = (0, 0)$

Mixed Exercise 8G

- | | |
|--|---|
| 1 a $y = -3x + 14$ | b (0, 14) |
| 2 a $y = -\frac{1}{2}x + 4$ | b $y = -\frac{1}{2}x + \frac{3}{2}, (1, 1)$ |
| 3 a $y = \frac{1}{7}x + \frac{12}{7}, y = -x + 12$ | b (9, 3) |
| 4 a $y = -\frac{5}{12}x + \frac{11}{6}$ | b -22 |
| 5 a $y = \frac{3}{2}x - \frac{3}{2}$ | b (3, 3) |
| 6 $11x - 10y + 19 = 0$ | |
| 7 a $y = -\frac{1}{2}x = 3$ | b $y = \frac{1}{4}x + \frac{9}{4}$ |
| 8 a $y = \frac{3}{2}x - 2$ | b (4, 4) |
| 9 a $2x + y = 20$ | b $y = \frac{1}{3}x + \frac{4}{3}$ |
| 10 a $\frac{1}{2}$ | b 6 |
| | c $2x + y - 16 = 0$ |

- 11 a (-1, 1); (4, 11) in ratio 3 : 2
 is $\left(\frac{2 \times (-1) + 3 \times 4}{5}, \frac{2 \times 1 + 3 \times 11}{5}\right)$
 so $c = (2, 7)$
- b $M_{AB} = \frac{11-1}{4-(-1)} = \frac{10}{5} = 2 \therefore m_l = -\frac{1}{2}$
 Equation of l is $y - 7 = -\frac{1}{2}(x - 2)$
 $2y - 14 = -x + 2$
 or $x + 2y - 16 = 0$
- c $x = 0 \Rightarrow y = 8 \therefore D$ is (0, 8)



- d $\triangle ABD$
 $CD = \sqrt{2^2 + 1^2} = \sqrt{5}$
 $AB = \sqrt{(4 - (-1))^2 + (11 - 1)^2} = \sqrt{5^2 + 10^2} = 5\sqrt{5}$
 Area of $\triangle ABD = \frac{1}{2} AB \cdot CD = \frac{1}{2} \times \sqrt{5} \times 5\sqrt{5} = \frac{25}{2}$

- 12 a $y = \frac{1}{3}x + \frac{1}{3}$

Chapter 9

9.1

Exercise 9A

- | | | |
|---------------------------------|-----------------------------------|-----------------------------------|
| 1 $7x^6$ | 2 $8x^7$ | 3 $4x^3$ |
| 4 $\frac{1}{3}x^{-\frac{2}{3}}$ | 5 $\frac{1}{4}x^{-\frac{4}{3}}$ | 6 $\frac{1}{3}x^{-\frac{2}{3}}$ |
| 7 $-3x^{-4}$ | 8 $-4x^{-5}$ | 9 $-2x^{-3}$ |
| 10 $-5x^{-6}$ | 11 $-\frac{1}{3}x^{-\frac{4}{3}}$ | 12 $-\frac{1}{2}x^{-\frac{3}{2}}$ |
| 13 $-2x^{-3}$ | 14 1 | 15 $3x^2$ |
| 16 $9x^8$ | 17 $5x^4$ | 18 $3x^2$ |

9.2

Exercise 9B

- | | | |
|-------------------------------------|--|-----------------------|
| 1 a $4x^3 - x^{-2}$ | b $-x^{-3}$ | c $-x^{-\frac{3}{2}}$ |
| 2 a 0 | b $11\frac{1}{2}$ | |
| 3 a $(2\frac{1}{2}, -6\frac{1}{4})$ | b (4, -4) and (2, 0) | |
| c (16, -31) | d $(\frac{1}{2}, 4), (-\frac{1}{2}, -4)$ | |

- 4 a $x^{-\frac{1}{2}}$ b $-6x^{-3}$ c $-x^{-4}$
 d $\frac{4}{3}x^3 - 2x^2$ e $-6x^{-4} + \frac{1}{2}x^{-\frac{1}{2}}$
 f $\frac{1}{3}x^{-\frac{2}{3}} - \frac{1}{2}x^{-2}$ g $-3x^{-2}$ h $3 + 6x^{-2}$
 i $5x^{\frac{3}{2}} + \frac{3}{2}x^{-\frac{1}{2}}$ j $3x^2 - 2x + 2$
 k $12x^3 + 18x^2$ l $24x - 8 + 2x^{-2}$
 5 a 1 b $\frac{2}{9}$ c -4 d 4

9.3

Exercise 9C

- 1 a $y = e^{2x} \Rightarrow \frac{dy}{dx} = 2e^{2x}$
 b $y = e^{-6x} \Rightarrow \frac{dy}{dx} = -6e^{-6x}$
 c $y = e^x + 3x^2 \Rightarrow \frac{dy}{dx} = e^x + 6x$
 d $y = \sin 2x \Rightarrow \frac{dy}{dx} = 2 \cos 2x$
 e $y = \cos 3x \Rightarrow \frac{dy}{dx} = -3 \sin 3x$
 f $y = 3 \sin 4x + 4 \cos 3x \Rightarrow \frac{dy}{dx} = 12 \cos 4x - 12 \sin 3x$
 2 a $y = \sin 5x \Rightarrow \frac{dy}{dx} = 5 \cos 5x$
 b $y = 2 \sin \frac{1}{2}x \Rightarrow \frac{dy}{dx} = 2 \times \frac{1}{2} \cos \frac{1}{2}x = \cos \frac{1}{2}x$
 c $y = \sin 8x \Rightarrow \frac{dy}{dx} = 8 \cos 8x$
 d $y = 6 \sin \frac{2}{3}x \Rightarrow \frac{dy}{dx} = 6 \times \frac{2}{3} \cos \frac{2}{3}x = 4 \cos \frac{2}{3}x$
 e $y = 2 \cos x \Rightarrow \frac{dy}{dx} = -2 \sin x$
 f $y = 6 \cos \frac{5}{6}x \Rightarrow \frac{dy}{dx} = 6 \times \frac{5}{6} \sin \frac{5}{6}x = -5 \sin \frac{5}{6}x$
 g $y = \cos 4x \Rightarrow \frac{dy}{dx} = -4 \sin 4x$
 h $y = 4 \cos \left(\frac{x}{2}\right) \Rightarrow \frac{dy}{dx} = -4 \times \frac{1}{2} \sin \frac{x}{2} = -2 \sin \frac{x}{2}$
 3 $y = 2e^{-x} \Rightarrow \frac{dy}{dx} = -2e^{-x} @ (0, 2)$
 4 $y = 3 \sin x \Rightarrow \frac{dy}{dx} = 3 \cos x @ \left(\frac{\pi}{3}, x\right)$

Remember x is in radians

$$\therefore m = 3 \cos \left(\frac{\pi}{3}\right) = \frac{3}{2} \left[\cos \frac{\pi}{3} = \cos 60^\circ = \frac{1}{2}\right]$$

- 5 $y = 4 \cos 2x \Rightarrow \frac{dy}{dx} = -8 \sin 2x @ \left(x = \frac{\pi}{4}\right)$
 $\therefore m = -8 \sin \left(2 \times \frac{\pi}{4}\right)$

$$= -8 \sin \frac{\pi}{4} \left[\sin \frac{\pi}{2} = \sin 90^\circ = 1\right]$$

$$= -8$$

9.4

Exercise 9D

- 1 a $y = (1 + 2x)^4 \Rightarrow y' = 4(1 + 2x)^3 \times 2 = 8(1 + 2x)^3$
 b $y = (1 + x^2)^3 \Rightarrow y' = 3(1 + x^2)^2 \times 2x = 6x(1 + x^2)^2$
 c $y = (3 + 4x)^{\frac{1}{2}} \Rightarrow y' = \frac{1}{2}(3 + 4x)^{-\frac{1}{2}} \times 4 = \frac{2}{\sqrt{3 + 4x}}$
 d $y = (x^2 + 2x)^3 \Rightarrow y' = 3(x^2 + 2x)^2 \times (2x + 2)$
 $= 6(x + 1)(x^2 + 2x)^2$
 2 a $y = 4e^{3x^2} \Rightarrow y' = 4e^{3x^2} \times 6x = 24xe^{3x^2}$
 b $y = 9e^{3-x} \Rightarrow y' = 9e^{3-x} \times -1 = -9e^{3-x}$
 c $y = e^{-6x} \Rightarrow y' = e^{-6x} \times -6 = -6e^{-6x}$
 d $y = e^{x^2 + 2x} \Rightarrow y' = e^{x^2 + 2x} \times (2x + 2)$
 $= 2(x + 1)e^{x^2 + 2x}$
 3 a $y = \sin(2x + 1) \Rightarrow y' = \cos(2x + 1) \times 2 = 2 \cos(2x + 1)$
 b $y = \cos(2x^2 + 4) \Rightarrow y' = -\sin(2x^2 + 4) \times 4x \Rightarrow -4x \sin(2x^2 + 4)$
 c $y = \sin^3 x \Rightarrow y' = 3 \sin^2 x \times \cos x$
 $= 3 \sin^2 x \cos x$
 d $y = \cos^2 2x \Rightarrow y' = 2 \cos 2x \times (-\sin 2x) \times 2$
 $= -4 \sin 2x \cos 2x$
 4 a $y = x(1 + 3x)^5 \Rightarrow y' = (1 + 3x)^5 + x \cdot 5(1 + 3x)^4 \times 3$
 $= (1 + 3x)^4 [1 + 3x + 15x]$
 $= (1 + 3x)^4 (1 + 18x)$
 b $y = 2x(1 + 3x^2)^3 \Rightarrow y' = 2(1 + 3x^2)^3$
 $+ 2x \times 3(1 + 3x^2)^2 \times 6x$
 $= 2(1 + 3x^2)^2 [1 + 18x^2]$
 c $y = x^3(2x + 6)^4 \Rightarrow y' = 3x^2(2x + 6)^4$
 $+ x^3 \times 4(2x + 6)^3 \times 2$
 $= x^2(2x + 6)^3 [6x + 18 + 8x]$
 $= 2x^2(2x + 6)^3 (7x + 9)$
 5 a $y = xe^{2x} \Rightarrow y' = e^{2x} + x \cdot 2e^{2x} = e^{2x}(1 + 2x)$
 b $y = (x^2 + 3)e^{-x} \Rightarrow y' = 2x \times e^{-x} + (x^2 + 3)(-e^{-x})$
 $= e^{-x}(2x - x^2 - 3)$
 c $y = (3x - 5)e^{x^2} \Rightarrow y' = 3 \times e^{x^2} + (3x - 5)e^{x^2} \times 2x$
 $= e^{x^2}(6x^2 - 10x + 3)$
 6 a $y = x \sin x \Rightarrow y' = \sin x + x \cos x$
 b $y = \sin^2 x \cos x$
 $\Rightarrow y' = 2 \sin x \cos x \times \cos x + \sin^2 x (-\sin x)$

$$y' = \sin x (2 \cos^2 x - \sin^2 x)$$

$$\begin{aligned} \text{c } y = e^x \cos x &\Rightarrow y' = e^x \cos x - e^x \sin x \\ &= e^x (\cos x - \sin x) \end{aligned}$$

$$\begin{aligned} 7 \text{ a } y = \frac{5x}{x+1} &\Rightarrow y' = \frac{(x+1) \times 5 - 5x \times 1}{(x+1)^2} \\ &= \frac{5x + 5 - 5x}{(x+1)^2} \\ &= \frac{5}{(x+1)^2} \end{aligned}$$

$$\begin{aligned} \text{b } y = \frac{2x}{3x-2} &\Rightarrow y' = \frac{(3x-2) \times 2 - 2x \times 3}{(3x-2)^2} \\ &= \frac{6x - 4 - 6x}{(3x-2)^2} \\ &= \frac{-4}{(3x-2)^2} \end{aligned}$$

$$\begin{aligned} \text{c } y = \frac{3x^2}{(2x-1)^2} &\Rightarrow y' = \frac{(2x-1)^2 \times 6x - 3x^2 \times 2(2x-1) \times 2}{(2x-1)^4} \\ &= \frac{6x(2x-1)[2x-1-2x]}{(2x-1)^4} \\ &= \frac{-6x}{(2x-1)^3} \end{aligned}$$

$$8 \text{ a } y = \frac{x}{e^{2x}} \Rightarrow y' = \frac{e^{2x} \times 1 - x \times 2e^{2x}}{e^{4x}} = \frac{1-2x}{e^{2x}}$$

$$\text{b } y = \frac{e^x}{x+1} \Rightarrow y' = \frac{(x+1)e^x - e^x \times 1}{(x+1)^2} = \frac{xe^x}{(x+1)^2}$$

$$\text{c } y = \frac{e^{x^2}}{x} \Rightarrow y' = \frac{x \times 2xe^{x^2} - e^{x^2} \times 1}{x^2} = \frac{e^{x^2}(2x^2 - 1)}{x^2}$$

$$\begin{aligned} 9 \text{ a } y = \frac{\sin x}{x} &\Rightarrow y' = \frac{x \cos x - \sin x \times 1}{x^2} \\ &= \frac{x \cos x - \sin x}{x^2} \end{aligned}$$

$$\begin{aligned} \text{b } y = \frac{e^x}{\cos x} &\Rightarrow y' = \frac{\cos x \times e^x + e^x \sin x}{\cos^2 x} \\ &= \frac{e^x(\sin x + \cos x)}{\cos^2 x} \end{aligned}$$

$$\begin{aligned} \text{c } y = \frac{\sin^2 x}{e^{2x}} &\Rightarrow y' = \frac{e^{2x} \times 2 \sin x \cos x - \sin^2 x \times 2e^{2x}}{e^{4x}} \\ &= \frac{2 \sin x e^{2x} (\cos x - \sin x)}{e^{4x}} \\ &= \frac{2 \sin x (\cos x - \sin x)}{e^{2x}} \end{aligned}$$

$$\begin{aligned} 10 \text{ a } y = x^2 (3x-1)^3 &\Rightarrow y' = 2x(3x-1)^3 \\ &\quad + x^2 \times 3(3x-1)^2 \times 3 \text{ when } x=1 \\ m &= 2 \times 2^3 + 1 \times 9 \times 2^2 = 16 + 36 = 52 \end{aligned}$$

$$\begin{aligned} \text{b } y = (2x+3)e^{2x} &\Rightarrow y' = 2(2x+3)e^{2x} + 2 \times e^{2x} \\ \text{when } x=0 \\ m &= 6e^0 + 2e^0 = 8 \end{aligned}$$

$$\begin{aligned} \text{c } y = 3 \sin^2 x &\Rightarrow y' = 6 \sin x \cos x \text{ when } x = \frac{\pi}{4} \\ m &= 6 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 3 \end{aligned}$$

$$\begin{aligned} \text{d } y = x \cos x &\Rightarrow y' = \cos x - x \sin x \text{ when } x = \frac{\pi}{2} \\ m &= \cos \frac{\pi}{2} - \frac{\pi}{2} \sin \frac{\pi}{2} = 0 - \frac{\pi}{2} \times 1 \\ &= -\frac{\pi}{2} \end{aligned}$$

9.5

Exercise 9E

$$\begin{aligned} 1 \text{ a } y + 3x - 6 &= 0 \\ \text{b } 4y - 3x - 4 &= 0 \\ \text{c } y &= x \\ 2 \text{ a } 7y + x - 48 &= 0 \quad \text{b } 17y + 2x - 212 = 0 \end{aligned}$$

$$3 \text{ } y = -8x + 10, 8y - x - 145 = 0$$

$$4 \text{ } y = 2ex - \frac{e}{2}$$

$$5 \text{ } y = \frac{1}{3}e$$

$$\begin{aligned} 6 \text{ } y = x \sin x &\Rightarrow y' = \sin x + x \cos x @ (\pi, 0) \\ \therefore m &= \sin \pi + \pi \cos \pi = -\pi \end{aligned}$$

$\therefore$ equation of tangent:

$$y - 0 = -\pi(x - \pi)$$

$$\text{or } y = \pi^2 - x\pi$$

$$7 \text{ } y = 2 \cos^2 x \Rightarrow y' = 4 \cos x (-\sin x) @ (\frac{\pi}{4}, 1)$$

$$\therefore m = -4 \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = -2$$

$$\text{normal has gradient} = \frac{1}{2}$$

$\therefore$ equation of normal is:

$$y - 1 = \frac{1}{2} \left(x - \frac{\pi}{4} \right)$$

$$\text{or } 8y - 8 = 4x - \pi$$

$$\text{or } 8y - 4x = 8 - \pi$$

9.6

Exercise 9F

$$1 \text{ } \frac{1}{4}x^4 + x^2 + c$$

$$2 \text{ } -2x^{-1} + 3x + c$$

$$3 \quad 2x^{\frac{5}{2}} - x^3 + c$$

$$4 \quad \frac{4}{3}x^{\frac{3}{2}} - 4x^{\frac{1}{2}} + 4x + c$$

$$5 \quad x^4 + x^{-3} + rx + c$$

$$6 \quad t^3 + t^{-1} + c$$

$$7 \quad \frac{2}{3}t^3 + 6t^{-\frac{1}{2}} + t + c$$

$$8 \quad \frac{1}{2}x^2 + 2x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} + c$$

$$9 \quad \frac{p}{5x^5} + 2tx - 3x^{-1} + c$$

$$10 \quad \frac{p}{4}t^4 + q^2t + px^3t + c$$

$$11 \quad \text{a} \quad \frac{1}{2}x^4 + x^3 + c$$

$$\text{b} \quad 2x - \frac{3}{x} + c$$

$$\text{c} \quad \frac{4}{3}x^3 + 6x^2 + 9x + c$$

$$\text{d} \quad \frac{2}{3}x^3 + \frac{1}{2}x^2 - 3x + c$$

$$\text{e} \quad \frac{4}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + c$$

$$12 \quad \text{a} \quad \int 2 \sin 3x \, dx = -\frac{2}{3} \cos 3x + c$$

$$\text{b} \quad \int 3e^{4x} \, dx = \frac{3}{4} e^{4x} + c$$

$$\text{c} \quad \int 2 \cos 3x \, dx = \frac{2}{3} \sin 3x + c$$

$$\text{d} \quad \int 2e^{-x} \, dx = -2e^{-x} + c$$

$$13 \quad \text{a} \quad 5e^x + 4 \cos x + \frac{x^4}{2} + c$$

$$\text{b} \quad -2 \cos x - 2 \sin x + x^2 + C$$

$$\text{c} \quad 5e^x + 4 \sin x + \frac{2}{x} + c$$

$$\text{d} \quad e^x - \cos x + \sin x + C$$

9.7

Exercise 9G

$$1 \quad 10t$$

$$2 \quad 48 - 32t$$

$$3 \quad \text{a} \quad 40 + 10t \quad \text{b} \quad 70 \text{ m/s}$$

$$4 \quad \text{a} \quad 30 - 1t \quad \text{b} \quad 0 \text{ m/s}$$

$$5 \quad \text{a} \quad a = 32$$

$$\text{b} \quad v = 32t + 100$$

$$\Rightarrow s = 16t^2 + 100t + d; \text{ when } t = 0, s = 0 \Rightarrow d = 0$$

$$\therefore s = 16t^2 + 100t$$

$$6 \quad \text{a} \quad a = -32$$

$$\text{b} \quad v = 160 - 32t$$

$$\Rightarrow s = 160t - 16t^2 + d;$$

$$t = 0, s = 384 \Rightarrow d = 384$$

$$\therefore s = 384 + 160t - 16t^2$$

$$\text{c} \quad s = 0 \Rightarrow 16(t^2 - 10t - 24) = 0$$

$$\text{i.e. } 16(t - 12)(t + 2) = 0$$

$$\therefore \text{ passes through origin when } t = 12$$

$$7 \quad \text{a} \quad 3t^2 + 8t - 5 \quad \text{b} \quad 6t + 8$$

$$\text{c} \quad v = 6 \text{ m/s}, a = 14 \text{ m/s}^2$$

$$8 \quad \text{a} \quad v = 3t^2 - 4t + 3 \quad \text{b} \quad a = 6t - 4$$

$$9 \quad \text{a} \quad a = 2t + 10 \quad \text{b} \quad a = 14 \text{ m/s}^2$$

$$\text{c} \quad v = t^2 + 10t + 5$$

$$\Rightarrow s = \frac{t^3}{3} + 5t^2 + 5t + d; s = 0 \text{ when } t = 0 \Rightarrow d = 0$$

$$\therefore \text{ when } t = 2 \quad s = \frac{8}{3} + 20 + 10 = 32\frac{2}{3}$$

$$10 \quad \text{a} \quad 6 - 2t \quad \text{b} \quad 2 \text{ m/s}$$

$$\text{c} \quad v = 24 + 6t - t^2$$

$$\Rightarrow s = 24t + 3t^2 - \frac{t^3}{3} + d$$

$$s = 100, t = 3 \Rightarrow 100 = 24 \times 3 + 3 \times 9 - \frac{27}{3} + d$$

$$\text{i.e. } 100 = 72 + 27 - 9 + d \quad \therefore d = 10$$

$$\therefore s = 10 + 24t + 3t^2 - \frac{t^3}{3}$$

9.8

Exercise 9H

$$1 \quad \text{a} \quad -28 \quad \text{b} \quad -17 \quad \text{c} \quad -\frac{1}{5}$$

$$2 \quad \text{a} \quad 10 \quad \text{b} \quad 4 \quad \text{c} \quad 12.25$$

$$3 \quad \text{a} \quad \left(-\frac{3}{4}, -\frac{9}{4}\right) \quad \text{b} \quad \left(\frac{1}{2}, 9\frac{1}{4}\right)$$

$$\text{c} \quad \left(-\frac{1}{3}, 1\frac{5}{27}\right), (1, 0) \quad \text{d} \quad (3, -18), \left(-\frac{1}{3}, \frac{14}{27}\right)$$

$$\text{e} \quad (1, 2), (-1, -2) \quad \text{f} \quad (3, 27)$$

$$5 \quad \left(\frac{3\pi}{8}, \frac{1}{\sqrt{2}}e^{\frac{3\pi}{4}}\right) \text{ maximum, } \left(\frac{7\pi}{8}, -\frac{1}{\sqrt{2}}e^{\frac{7\pi}{4}}\right) \text{ minimum}$$

9.9

Exercise 9I

$$6 \quad \text{a} \quad 8 \quad \text{b} \quad 9\frac{3}{4}$$

$$\text{c} \quad 19\frac{2}{3} \quad \text{d} \quad 21$$

$$\text{e} \quad 8\frac{5}{12}$$

$$7 \quad \text{a} \quad \text{i} \quad 2 \ln 2 \quad \text{ii} \quad 2\pi$$

$$\text{b} \quad \text{i} \quad \frac{8}{3} \quad \text{ii} \quad \frac{64}{15}\pi$$

$$8 \quad \text{a} \quad A(1, 3), B(3, 3) \quad \text{b} \quad 1\frac{1}{3}$$

$$9 \quad 6\frac{2}{3}$$

$$10 \quad \text{a} \quad (2, 12) \quad \text{b} \quad 13\frac{1}{3}$$

$$11 \quad 3\frac{3}{8}$$

9.10

Exercise 9J

$$1 \quad \frac{8}{9}\pi$$

$$2 \quad 6\pi$$

$$3 \quad 15e^2$$

$$4 \quad y = 5x^4 \Rightarrow \frac{dy}{dx} = 20x^3$$

$$\therefore \delta y \approx 20x^3 \delta x = 20x^3 \times \frac{x}{200} \quad \left[0.5\% \text{ of } x = \frac{x}{200}\right]$$

$$\text{i.e. } \delta y = \frac{20x^4}{200} = \frac{x^4}{10}$$

$$\therefore \% \text{ change in } y = \frac{\delta y}{y} \times 100 = \frac{x^4}{10 \times 5x^4} \times 100 = 2\%$$

$$5 \quad y = 3x^2 \Rightarrow \frac{dy}{dx} = 6x$$

$$\therefore \delta y \approx 6x \delta x = 6x \times \frac{x}{100} = \frac{6x^2}{100}$$

$$\therefore \% \text{ change in } y = \frac{\delta y}{y} \times 100 = \frac{6x^2}{3x^2 \times 100} \times 100 = 2\%$$

$$6 \quad \text{For a sphere: } V = \frac{4}{3}\pi r^3 = \frac{dv}{dr} = 4\pi r^2$$

$$\delta r \approx 0.02 \text{ cm}$$

$$\delta v \approx 4\pi r^2 \delta r$$

$$\text{Use } r = 1 \Rightarrow$$

$$\delta v \approx 4\pi \times 0.02$$

$$= 0.25 \text{ cm}^3$$

Exercise 9K

- 1 $20 \text{ m} \times 40 \text{ m}; 800 \text{ m}^2$
- 2 $2000\pi \text{ cm}^2$
- 3 40 cm
- 4 $\frac{800}{4 + \pi} \text{ cm}^2$
- 5 27216 mm^2

Mixed Exercise 9L

- 1 **a** $x = 4, y = 20$
b $\frac{d^2y}{dx^2} = \frac{15}{8} > 0 \therefore \text{minimum}$
- 2 $(1, -11)$ and $(\frac{7}{3}, -12\frac{5}{27})$
- 3 **a** $7\frac{31}{32}$
b $\frac{x^3}{3} - 2x - \frac{1}{x} - 2\frac{2}{3}$
c $f'(x) = \left(\frac{x-1}{x}\right)^2 > 0$ for all values of x
- 4 $(1, 4)$
- 5 **a** $y = 1 - \frac{x}{2} - \frac{\pi x}{4}$
c $\frac{2}{4 + \pi} \text{ m}^2 (0.280 \text{ m}^2)$
- 6 **b** $\frac{10}{3}$ **c** $\frac{d^2v}{dx^2} < 0 \therefore \text{maximum}$
d $\frac{2300\pi}{27}$ **e** $22\frac{2}{9}\%$
- 7 **a** $\frac{250}{x^2} - 2x$
b $(5, 125)$
- 8 **b** $x = \pm 2\sqrt{2}$, or $x = 0$

$$\text{c } OP = 3; f''(x) > 0 \text{ so minimum when } x = \pm 2\sqrt{2}$$

$$(\text{maximum when } x = 0)$$

- 9 **b** A is $(-1, 0)$; B is $(\frac{5}{3}, 9\frac{13}{27})$
- 10 $3x^2 \cos 3x + 2x \sin 3x$
- 12 $\frac{x \cos x - \sin x}{x^2}$
- 13 **b** $y = 2x + 1$
- 14 **a** $-(x^3 - 2x)e^x + (3x^2 - 2)e^x$
- 15 $\frac{56\pi}{5}$

Chapter 10

10.1

Exercise 10A

- | | | |
|------------------------------|---------------------------|----------------------------|
| 1 a 9° | b 12° | c 75° |
| d 90° | e 140° | f 210° |
| g 225° | h 270° | i 540° |
| 2 a 26.4° | b 57.3° | c 65.0° |
| d 99.2° | e 143.2° | f 179.9° |
| g 200° | | |
| 3 a 0.479 | b 0.156 | c 1.74 |
| d 0.909 | e -0.897 | |
| 4 a $\frac{2\pi}{45}$ | b $\frac{\pi}{18}$ | c $\frac{\pi}{8}$ |
| d $\frac{\pi}{6}$ | e $\frac{\pi}{4}$ | f $\frac{\pi}{3}$ |
| g $\frac{5\pi}{12}$ | h $\frac{4\pi}{9}$ | i $\frac{5\pi}{8}$ |
| j $\frac{2\pi}{3}$ | k $\frac{3\pi}{4}$ | l $\frac{10\pi}{9}$ |
| m $\frac{4\pi}{3}$ | n $\frac{3\pi}{2}$ | o $\frac{7\pi}{4}$ |
| p $\frac{11\pi}{6}$ | | |
| 5 a 0.873 | b 1.31 | c 1.75 |
| d 2.79 | e 4.01 | f 5.59 |

10.2

Exercise 10B

- | | | |
|-----------------------------------|--|----------------------------|
| 1 a i 2.7 | ii 2.025 | iii $7.5\pi (23.6)$ |
| b i $16\frac{2}{3}$ | ii 1.8 | iii 3.6 |
| c i $1\frac{1}{3}$ | ii 0.8 | iii 2 |
| 2 $\frac{10\pi}{3} \text{ cm}$ | | |
| 3 2π | | |
| 4 $5\sqrt{2} \text{ cm}$ | | |
| 5 a 10.4 cm | b $1\frac{1}{4}$ | |
| 6 7.5 | | |
| 7 0.8 | | |
| 8 a $\frac{\pi}{3}$ | b $(6 + \frac{4\pi}{3}) \text{ cm}$ | |
| 9 6.8 cm | | |
| 10 a $(R - r) \text{ cm}$ | c 2.43 | |

10.3

Exercise 10C

- 1 **a** 19.2 cm^2 **b** $6.75\pi \text{ cm}^2$
c $1.296\pi \text{ cm}^2$ **d** 38.3 cm^2
e $5\frac{1}{3}\pi \text{ cm}^2$ **f** 5 cm^2
2 **a** 4.47 **b** 3.96
c 1.98
3 12 cm^2
4 **b** 120 cm^2
5 $40\frac{2}{3} \text{ cm}$
6 **a** 12 **c** 1.48 cm^2
7 8.88 cm^2
8 **a** 1.75 cm^2 **b** 25.9 cm^2 **c** 25.9 cm^2
9 4.5 cm^2
10 **b** 28 cm
11 78.4 cm
12 **b** 28 cm

10.4

Exercise 10D

- 1 **a** $\frac{\sqrt{2}}{2}$ **b** $-\frac{\sqrt{3}}{2}$ **c** $-\frac{1}{2}$
d $\frac{\sqrt{3}}{2}$ **e** $\frac{\sqrt{3}}{2}$ **f** $-\frac{1}{2}$
g $\frac{1}{2}$ **h** $-\frac{\sqrt{2}}{2}$ **i** $-\frac{\sqrt{3}}{2}$
j $-\frac{\sqrt{2}}{2}$ **k** -1 **l** -1
m $\frac{\sqrt{3}}{3}$ **n** $-\sqrt{3}$ **o** $\sqrt{3}$

10.5

Exercise 10E

- 1 **a** $x = 84, y = 6.32$
b $x = 13.5, y = 16.6$
c $x = 85, y = 13.9$
d $x = 80, y = 6.22$ (Isosceles $\triangle$)
e $x = 6.27, y = 7.16$
f $x = 4.49, y = 7.49$ (right-angled)
2 **a** 48.1 **b** 45.6 **c** 14.8
d 48.7 **e** 86.5 **f** 77.4
3 **a** $x = 74.6, y = 65.4$
 $x = 105, y = 34.6$
b $x = 59.8, y = 48.4$
 $x = 120, y = 27.3$
c $x = 56.8, y = 4.37$

$$x = 23.2, y = 2.06$$

- 4 **a** 3.19 cm **b** $1.73 \text{ cm } (\sqrt{3} \text{ cm})$
c 9.85 cm **d** 4.31 cm
e 6.84 cm (isosceles) **f** 9.80 cm
5 **a** $108(.2)^\circ$ **b** 90° **c** 60°
d 52.6° **e** 137° **f** 72.2°
6 **a** 23.7 cm^3 **b** 4.31 cm^3 **c** 20.2 cm^3
7 **a** 155° **b** 13.7 cm
8 **a** $x = 49.5$, area = 1.37 cm^2
b $x = 55.2$, area = 10.6 cm^2
c $x = 117$, area = 6.66 cm^2
9 6.50 cm^2
10 **a** 36.1 cm^3 **b** 12.0 cm^3

10.6

Exercise 10F

- 1 **a** 11.7 cm **b** 14.2 cm **c** 34.4°
d 63.4°
2 **a** 18.6 cm **b** 28.1 cm **c** 48.6°
3 **a** 14.1 cm **b** 17.3 cm **c** 35.4°
4 **a** 28.3 cm **b** 34.6 cm **c** 35.1°
c 19.5°
5 **a** 4.47 m **b** 4.58 m **c** 29.2°
d 12.6° **e** 26.6°
6 **a** 407 m **b** 402 m **c** 8.57°
c 13.3°
7 **a** 43.3 cm **b** 68.7 cm **c** 81.2 cm
8 **a** 28.9 cm **b** 75.7 cm **c** 22.4°
9 **a** 16.2 cm **b** 67.9° **c** 55.3 cm^2
d 71.6°
10 **a** 26.5 cm **b** 61.8° **c** 1530 cm^2
11 **a** 30.3° **b** 31.6° **c** 68.9°
12 **a** 36.9° **b** 828 cm^2
13 **a** 15 m **b** 47.7° **c** €91 300
14 **a** 66.4° **b** 32.9°
15 46.5 m
16 **a** OW = 4290 m, OS = 2760 m
b 36.0° **c** 197 km/h

10.7

Exercise 10G

- 1 **a** $\frac{\sin^2 \theta}{2}$ **b** 5 **c** $-\cos^2 A$
d $\cos \theta$ **e** $\tan x^0$ **f** $\tan 3A$
g 4 **h** $\sin^2 \theta$ **i** 1
2 **a** LHS = $\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta$
 $= 1 + 2 \sin \theta \cos \theta$
= RHS

$$\begin{aligned} \text{b LHS} &= \frac{1 - \cos^2 \theta}{\cos \theta} = \frac{\sin^2 \theta}{\cos \theta} = \sin \theta \times \frac{\sin \theta}{\cos \theta} \\ &= \sin \theta \tan \theta = \text{RHS} \end{aligned}$$

$$\begin{aligned} \text{c LHS} &= \frac{\sin x^\circ}{\cos x^\circ} + \frac{\cos x^\circ}{\sin x^\circ} = \frac{\sin^2 x^\circ + \cos^2 x^\circ}{\sin x^\circ \cos x^\circ} \\ &= \frac{1}{\sin x^\circ \cos x^\circ} = \text{RHS} \end{aligned}$$

$$\begin{aligned} \text{d LHS} &= \cos^2 A - (1 - \cos^2 A) = 2 \cos^2 A - 1 \\ &= 2(1 - \sin^2 A) - 1 = 1 - 2 \sin^2 A = \text{RHS} \end{aligned}$$

$$\begin{aligned} \text{e LHS} &= (4 \sin^2 \theta - 4 \sin \theta \cos \theta + \cos^2 \theta) \\ &\quad + (\sin^2 \theta + 4 \sin \theta \cos \theta + 4 \cos^2 \theta) \\ &= 5(\sin^2 \theta + \cos^2 \theta) = 5 = \text{RHS} \end{aligned}$$

$$\begin{aligned} \text{f LHS} &= 2 - (\sin 2\theta - 2 \sin \theta \cos \theta + \cos^2 \theta) \\ &= 2(\sin^2 \theta + \cos^2 \theta) \\ &\quad - (\sin^2 \theta - 2 \sin \theta \cos \theta + \cos^2 \theta) \\ &= \sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta \\ &= (\sin \theta + \cos \theta)^2 = \text{RHS} \end{aligned}$$

$$\begin{aligned} \text{g LHS} &= \sin^2 x(1 - \sin^2 y) - (1 - \sin^2 x) \sin^2 y \\ &= \sin^2 x - \sin^2 y = \text{RHS} \end{aligned}$$

$$\begin{array}{lll} 3 \text{ a } \sin 35^\circ & \text{b } \sin 35^\circ & \text{c } \cos 210^\circ \\ \text{d } \tan 31^\circ & \text{e } \cos \theta & \text{f } \cos 7\theta \\ \text{g } \sin 3\theta & \text{h } \tan 5\theta & \text{i } \sin A \\ \text{j } \cos 3x & & \end{array}$$

$$\begin{array}{lll} 4 \text{ a } 1 & \text{b } 0 & \text{c } \frac{\sqrt{3}}{2} \\ \text{d } \frac{\sqrt{2}}{2} & \text{e } \frac{\sqrt{2}}{2} & \text{f } -\frac{1}{2} \\ \text{g } \sqrt{3} & \text{h } \frac{\sqrt{3}}{3} & \text{i } 1 \\ \text{j } \sqrt{2} & & \end{array}$$

$$\begin{aligned} 5 \text{ a LHS} &= \sin A \cos 60^\circ + \cos A \sin 60^\circ \\ &\quad + \sin A \cos 60^\circ - \cos A \sin 60^\circ \\ &= 2 \sin A \cos 60^\circ \\ &= 2 \sin A \left(\frac{1}{2}\right) = \sin A = \text{R.H.S.} \end{aligned}$$

$$\begin{aligned} \text{b LHS} &= \frac{\cos A \cos B - \sin A \sin B}{\sin B \cos B} = \frac{\cos(A+B)}{\sin B \cos B} \\ &= \text{R.H.S.} \end{aligned}$$

$$\begin{aligned} \text{c LHS} &= \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y} \\ &= \frac{\sin x \cos y}{\cos x \cos y} + \frac{\cos x \sin y}{\cos x \cos y} \\ &= \tan x + \tan y = \text{R.H.S.} \end{aligned}$$

$$\begin{aligned} \text{d LHS} &= \frac{\cos x \cos y - \sin x \sin y}{\sin x \sin y} + 1 \\ &= \cot x \cot y - 1 + 1 = \cot x \cot y = \text{R.H.S.} \end{aligned}$$

$$\begin{aligned} \text{e LHS} &= \cos \theta \cos \frac{\pi}{3} - \sin \theta \sin \frac{\pi}{3} + \sqrt{3} \sin \theta \\ &= \frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta + \sqrt{3} \sin \theta \\ &= \frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta \end{aligned}$$

$$\begin{aligned} &= \sin \frac{\pi}{6} \cos \theta + \cos \frac{\pi}{6} \sin \theta \\ &= \sin\left(\frac{\pi}{6} + \theta\right) = \text{R.H.S.} \end{aligned}$$

10.8

Exercise 10H

- a 270° b $60^\circ, 240^\circ$
c $60^\circ, 300^\circ$ d $15^\circ, 165^\circ$
e $140^\circ, 220^\circ$ f $135^\circ, 315^\circ$
g $90^\circ, 270^\circ$ h $230^\circ, 310^\circ$
i $45.6^\circ, 134.4^\circ$ j $135^\circ, 225^\circ$
- a $-120, -60, 240, 300$ b $-171, -8.63$
c $-144, 144$ d $-327, -32.9$
e $150, 330, 510, 690$ f $251, 431$
- a $-\pi, 0, \pi, 2\pi$ b $-\frac{4\pi}{3}, -\frac{2\pi}{3}, \frac{2\pi}{3}$
c $-\frac{7\pi}{4}, -\frac{5\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$ d $-0.14, 3.00, 6.14$
- a $0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ, 225^\circ, 270^\circ, 315^\circ, 360^\circ$
b $60^\circ, 180^\circ, 300^\circ$
c $22\frac{1}{2}^\circ, 112\frac{1}{2}^\circ, 202\frac{1}{2}^\circ, 292\frac{1}{2}^\circ$
d $30^\circ, 150^\circ, 210^\circ, 330^\circ$
e 300° f $225^\circ, 315^\circ$
g $90^\circ, 270^\circ$ h $50^\circ, 170^\circ$
i $165^\circ, 345^\circ$
- a $-\frac{7\pi}{12}, -\frac{\pi}{12}$ b $1.48, 5.85$

10.9

Exercise 10I

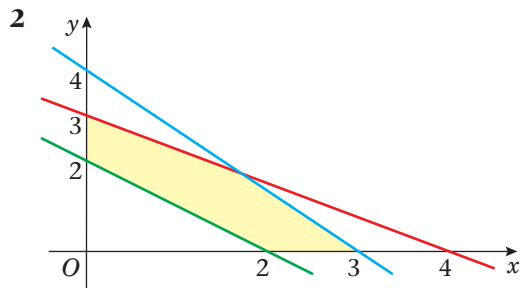
- a $30^\circ, 210^\circ$
b $135^\circ, 315^\circ$
- a $\pi, 2\pi$
b $0.59, 3.73$
- a $60^\circ, 120^\circ, 240^\circ, 300^\circ$
b $0^\circ, 180^\circ, 199^\circ, 341^\circ, 360^\circ$
c $60^\circ, 300^\circ$
d $30^\circ, 60^\circ, 120^\circ, 150^\circ, 210^\circ, 240^\circ, 300^\circ, 330^\circ$
e 270°
f $0^\circ, 18.4^\circ, 180^\circ, 198^\circ, 360^\circ$
g $194^\circ, 270^\circ, 346^\circ$
- a $\frac{5\pi}{12}, \frac{11\pi}{12}, \frac{17\pi}{12}, \frac{23\pi}{12}$
b $0.841, \frac{2\pi}{3}, \frac{4\pi}{3}, 5.44$
c $4.01, 5.41$

Mixed Exercise 10J

- 1 **a** $\cos^2 \theta \sin^2 \theta$
b $\sin^4 3\theta$
c 1
- 2 **a** 1 **b** $\tan y = \frac{4 + \tan x}{2 \tan x - 3}$
- 3 **a** $2 \sin 2\theta = \cos 2\theta \Rightarrow 2 \sin 2\theta \backslash \cos 2\theta = 1$
 $\Rightarrow 2 \tan 2\theta = 1 \Rightarrow \tan 2\theta = 0.5$
b 13.3, 103.3, 193.3, 283.3
- 4 **a** 225, 345
b 22.2, 67.8, 202.2, 247.8
- 5 **a** $\frac{11\pi}{12}, \frac{23\pi}{12}$ **b** $\frac{2\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6}$
- 6 $0^\circ, 131.8^\circ, 228.2^\circ$
- 7 $0, \pi, 2\pi$
- 8 **a** Max 1, $\theta = 100^\circ$; Min $= -1$, $\theta = 280^\circ$
b Max 1, $\theta = 330^\circ$; Min $= -1$, $\theta = 150^\circ$
- 9 **a** **i** $\frac{1}{2}$ **ii** $\frac{1}{2}$ **iii** $\sqrt{3} \backslash 3$
b $23.8^\circ, 203.8^\circ$

Review exercise

- 1 **a** $A = 5, B = -\frac{5}{2}, C = -28\frac{1}{4}$
b $f_{\min} = -\frac{113}{4}, x = \frac{5}{2}$



- 3 **a** 3 **b** $2\frac{1}{2}$ **c** $125\frac{1}{2}$
- 4 814
- 5 **a** - **b** $P = 126.8^\circ, Q = R = 26.6^\circ$
- 6 **a** **i** $\frac{1}{4}\mathbf{b} - \mathbf{a}$ **ii** $\frac{1}{3}\mathbf{a} + \frac{1}{6}\mathbf{b}$ **iii** $\frac{1}{3}\mathbf{a} - \frac{5}{6}\mathbf{b}$
b $\frac{2}{5}$ **c** $\frac{2}{3}$
- 7 **a** - **b** $54.5^\circ, 234.5^\circ$
- 8 $4\frac{1}{2}\text{ m}$
- 9 $21.8^\circ, 38.2^\circ, 120^\circ$
- 10 **a** 3.18, 6.69, 13.04 **b** -
c 2.3 **d** 0.6 (0.5 acceptable)
- 11 $x^6 - 18x^4 + 135x^2$
- 12 $-3 < p < 2$
- 13 -
- 14 **a** (3.14), (5, 24) **b** $1\frac{1}{3}$
- 15 **a** - **b** 12 cm **c** $6\sqrt{3}$
d 54.7° **e** 109.5°
- 16 $\frac{1}{2}x\%$
- 17 **a-b** $(-3, -\frac{5}{3}), (2, \frac{5}{2})$ **c** $y = -\frac{6}{5}(x - 1)$
d -
- 18 **a** 0.253, 2.89 **b** 1.11, 2.68
c 1.91, 2.30
- 19 **a** 625 **b** 2, 64
c $x = 2, y = 3$
- 20 -
- 21 82.8°
- 22 **a** $8\mathbf{i} - \mathbf{j}$ **b** -
- 23 $449\frac{2}{5}\pi$
- 24 **a** $\cos 2\theta = 2 \cos^2 \theta - 1$ **b** $\sin 2\theta = 2 \sin \theta \cos \theta$
c - **d** 0.767, 1.33, 2.86 **e** 4
- 25 **a** -6 **b** 50 **c** 17
- 26 **a** (2, 4), (5, 16) **b** $x \leq 2, x \geq 5$
- 27 **a** $-\mathbf{i} - 11\mathbf{j}$ **b** $\frac{13}{5}\mathbf{i} - \frac{2}{5}\mathbf{j}$
- 28 73.9°
- 29 **a** - **b** 12 791
- 30 15 200
- 31 **a** $r = \frac{1}{2}, r = -3$ **b** 10
- 32 $46.5^\circ, 133.5^\circ$
- 33 $\frac{1}{2}$
- 34 **a** $2y + x = 25$ **b** (25, 0) **c** (10, 0)
- 35 **a** 4 m/s^2 **b** $25\frac{1}{3}\text{ m}$
- 36 **a** $4y = x + 23$ **b** $y = -4x + 26$
c (-3, 38) **d** $6\frac{16}{17}$
e $12\frac{28}{51}$
- 37 **a** $-2p + q = 28, 3p + q = 18$
b -2, 24
c $(x + 2)(x - 3)(x - 4)$
d -
e $-\frac{12}{5}, 2$
f $7\frac{7}{12}$
- 38 -
- 39 **a** $2y = 3x - 18$ **b** $3y = -2x + 51$
c 156 **d** 216π
- 40 **a** $1 + \frac{x}{2p} - \frac{x^2}{8p^2} + \frac{x^3}{16p^3}$
b $p = \pm \frac{1}{2}$
- 41 **a** - **b** - **c** 4.76
d - **e** 408
- 42 **a** - **b** 4000
- 43 **a** $1 - 2x - 4x^2$ **b** 2.76132
c 0.087% **d** $a = 1, b = -5, c = 8$

e $|x| < \frac{1}{6}$

44 $2y = x - 2$

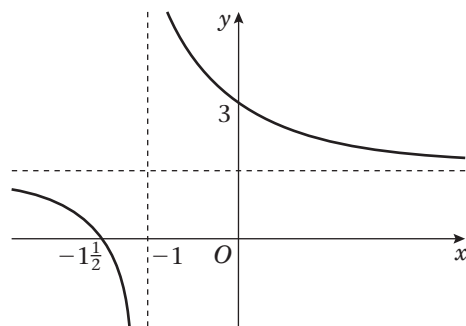
45 $\pi[\frac{1}{4}e^8 + 4e^4 + 27\frac{3}{4}]$

46 a $-$ b $-\frac{3}{8}$
c 15, 75, 105, 165 d $\frac{3}{8}$

47 a i $y = 2$ ii $x = -1$

b $(0, 3), (-\frac{3}{2}, 0)$

c



48 a 4 m/s^2 b 90 m

49 a $\frac{1}{2}b - \frac{1}{3}a$ b $2b - \frac{4}{3}a$ c $-$

50 a $\frac{dy}{dx} = 10x \cos 3x - 15x^2 \sin 3x$

b $\frac{dy}{dx} = \frac{3e^{3x}(x^2 + 3) - 2xe^{3x}}{(x^2 + 3)^2}$

51 0.212 m/s

52 a 1.39 b 28.7°

53 $p < -5, p > 2$

54 a 1, 3.75, 5.89, 6.92 b $-$
c 0.79 d 2.1

55 a $A = -\frac{7}{2}, B = -\frac{9}{4}$ b $-\frac{9}{4}, x = \frac{7}{2}$
c $(1, 4), (7, 10)$ d $(2, 0), (5, 0)$
e $-$ f 24

56 $(-2, 1), (-1, 3)$

57 a i $\frac{p^2}{4} + 6$ ii 9 b $p = \pm 4$
c $x^2 - 10x + 9 = 0$

58 a $-\frac{9}{11}, 5$ b 2
c 4 d 16 380

59 a $\frac{dy}{dx} = 10xe^{2x} + 2(5x^2 - 2)e^{2x}$

b $\frac{dy}{dx} = \frac{2x^3 - x^4 + 4x - 2}{(x - x^2)^2}$

60 $\ln 4$

61 91.1°

62 $23\frac{2}{5}$

63 a $1 + \frac{x}{12} - \frac{x^2}{144}$ b $1 + \frac{x}{12} - \frac{x^2}{72}$

c $|x| < 4$

d $1 + \frac{x}{6} - \frac{x^2}{72}$

e 0.308

64 a $\cos 2A = 2 \cos^2 A - 1$

b $\sin 2A = 2 \sin A \cos A$

c $-$

d $17.7^\circ, 102.3^\circ, 137.7^\circ$

e $\frac{3\sqrt{3}}{8}$

65 $(6, -1), (1, 4)$

66 a $p^5 + 5p^4qx + 10p^3q^2x + 10p^2q^3x + 5pq^4x^4 + q^5x^5$

b $p = \frac{6}{5}, q = \frac{12}{5}$ or $p = -2, q = 4$

67 a 3 b $q = 20$

c $a = 2, b = 1$ d 9

68 a i $\sqrt{20}$ ii $\sqrt{40}$ iii $\sqrt{20}$

b $\angle A = 90^\circ, \angle B = \angle C = 45^\circ$

c $(5, 5)$ d $\sqrt{10}$

69 $\pm 60^\circ$

70 67.4°

71 a 2 b $\log p$

c $r = n - 1, s = n$

72 a $2x^2 - 5x + 2 = 0$ b $x^2 - \frac{12}{p}x + \frac{12}{p} = 0$

c $\frac{8}{3}$ d $\frac{3}{2}$

73 $-4 < p < 3$

Practice examination papers

Paper 1

1 80.4° or 99.6°

2 a $-$ b $p = -10, q = 33$

3 $20 \text{ cm}^2/\text{s}$

4 $x = 2, y = 3, x = 3, y = 2$

5 a $p = 6, q = -4$ b $5\mathbf{i} + \frac{4}{3}\mathbf{j}$

6 a $(\frac{1}{3}, 2\sqrt{\frac{5}{3}})$ b $\frac{25}{3}\pi$

7 a 5 b $\frac{7r}{2} + \frac{3}{2}$ c $3\frac{1}{2}$ d 11

8 a $a^6 + 6a^5bx + 15a^4b^2x^2 + 20a^3b^3x^3 + 15a^2b^4x^4 + 6ab^5x^5 + b^6x^6$

b $a = 2, b = \frac{4}{3}, a = -2, b = -\frac{4}{3}$

9 a 5 b 28 c 9, 3

10 a $(2, 4)$ b $y = 4x - 4$

c $y = 4$ b 8 units²

11 a 11.0 cm b 11.9 cm c 40.1°

d 101.4° e 61.9°

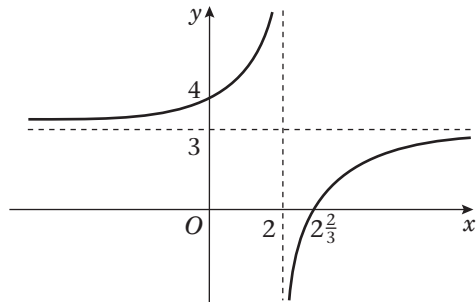
Paper 2

1 $2e^{2x} \sin 3x + 3e^{2x} \cos 3x$

2 a 37.0° b 17.2 cm^2

3 a i $y = 3$ ii $x = 2$ b i $(\frac{2}{3}, 0)$ ii $(0, 4)$

c



4 a	x	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
	y	1	0.649	-1.28	-4.52	-8.61	-12.8	-15.9	-15.9	-9.40

b graph drawn

c i 1.9 ii 1.3

5 a $0, 3, 4$ b $-$ c 11.8 m

6 a $\pm \frac{1}{2}$ b $a = 120x$ c 4

7 a $(\ln 3, 36), (0, 4)$ b 32.02 c 82.2 units^2

8 a $-$ b 2.71 c $-$ d 138

9 a $-$ b -280 c 37

d $46\sqrt{37}$ e $9x^2 + 280 + 3 = 0$

10 a $-$ b i $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ ii $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

c $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$

d $\sqrt{2} - 1$

e $\frac{20}{29}$