NUMBER 1 – BASIC SKILLS EXERCISE

1 (a) 27
(b) 56
(c) 26
(d) 12

2 $4 \frac{1}{3} = \frac{13}{3} \times \frac{52}{12} = \frac{4 \times 13}{4 \times 3} = \frac{13}{3}$,

$\frac{6.5}{1.5} = \frac{65}{15} = \frac{5 \times 13}{5 \times 3} = \frac{13}{3}$

3 (a) $\frac{1}{3}$
(b) $\frac{10}{21}$
(c) $3 \frac{5}{12}$
(d) $3 \frac{5}{7}$
(e) $\frac{1}{35}$
(f) $\frac{5}{9}$

4 $3 \frac{1}{2} \div 2 \frac{1}{2} = \frac{7}{2} \div \frac{7}{3}$

$= \frac{7}{2} \times \frac{3}{7}$

$= \frac{3}{2}$

$= 1 \frac{1}{2}$

5 $4 \frac{2}{3} - 2 \frac{1}{2} + 1 \frac{3}{4} = \frac{14}{3} - \frac{5}{2} + \frac{7}{4}$

$= \frac{56}{12} - \frac{30}{12} + \frac{21}{12}$

$= \frac{47}{12}$

$= 3 \frac{11}{12}$

6 $\frac{0.12}{32} \div 0.024 = \frac{12}{3200} \div \frac{24}{7200}$

$= \frac{12}{3200} \times \frac{7200}{24}$

$= \frac{9}{8}$

7 $\frac{1}{4} - \left( \frac{1}{4} \times \frac{1}{4} \right) + \left( \frac{1}{4} \div \frac{1}{4} \right)$

$= \frac{1}{4} - \left( \frac{1}{4} \times \frac{1}{4} \right) + \left( \frac{1}{4} \times \frac{4}{1} \right)$

$= \frac{1}{4} - \frac{1}{16} + 1$

$= \frac{4}{16} - \frac{1}{16} + \frac{16}{16}$

$= \frac{19}{16}$

$= 1 \frac{3}{16}$

8 $\frac{4}{2 + \frac{2}{3 + 4}} = \frac{4}{\frac{2}{3 + 4}}$

$= \frac{4}{\frac{2}{7}}$

$= \frac{4}{\frac{14}{7}} + \frac{2}{7}$

$= \frac{4}{16} + \frac{2}{7}$

$= \frac{28}{16}$

$= 1 \frac{3}{4}$
9 (a) 8  
(b) –16  
(c) –48  
(d) \(-\frac{1}{3}\)  
(e) 48  

10 (a) 3  
(b) 16  
(c) 8  
(d) 38  
(e) 48  

11 (a) 12300  
(b) 12400  
(c) 12300  
(d) 4390000  
(e) 5500000  
(f) 0.0130  
(g) 1.01  
(h) 0.010000  

12 (a) 1.294  
(b) 1.295  
(c) 1.295  
(d) 1.200  
(e) 0.100  
(f) 340.005  
(g) 1.000  
(h) 0.000499  

**NUMBER 1 – EXAM PRACTICE EXERCISE**  

1 (a) \(\frac{2}{3} \div \frac{5}{9} - \frac{1}{8} = \frac{14}{9} - \frac{32}{9} - \frac{11}{8}\)  
\[= \frac{14}{9} \times \frac{9}{32} - \frac{11}{8}\]  
\[= \frac{21}{16} - \frac{22}{16} = -\frac{1}{16}\]  

(b) \(\frac{1}{4} = \frac{7}{28}, \frac{2}{7} = \frac{8}{28}, \frac{3}{14} = \frac{6}{28}\), so Karim eats the most.  

(c) \(\frac{1}{4} + \frac{2}{7} + \frac{3}{14} = \frac{7 + 8 + 6}{28}\)  
\[= \frac{21}{28} = \frac{3}{4}\]  
leaving \(1 - \frac{3}{4} = \frac{1}{4}\) uneaten.  

2 (a) (i) 0.0018548 = 0.002 to 3 d.p.  
the 8 rounds the 1 up to 2  
(ii) 0.00185/48 = 0.00185 to 3 s.f.  
the 4 does not round anything up  
(iii) 0.00185/48 = 0.00 to 2 d.p.  
the 1 does not round anything up  
(iv) 0.00185/48 = 0.0019 to 2 s.f.  
the 5 rounds the 8 up to 9  

(b) (i) One of the following:  
Lowest common multiple of 2 and 10 is 10, not 2 – 10  
The numerator (top) of the first fraction has not been multiplied by anything to keep the value of the fraction correct.  
(ii) \(\frac{9}{2} - \frac{25}{10} = \frac{45}{10} - \frac{25}{10} = \frac{20}{10} = 2\)  

3 (a) \((5^2 \div 4 - 6 \times 3^2 \div 2^3) = \frac{25}{4} - \frac{6 \times 9}{8}\)  
\[= \frac{25}{4} - \frac{27}{4} = -\frac{2}{4} = -\frac{1}{2}\]  
so \(1 \div 2 \times (5^2 \div 4 - 6 \times 3^2 \div 2^3)\)  
\[= 1 \div 2 \times -\frac{1}{2} = \frac{1}{2} \times -\frac{1}{2} = -\frac{1}{4}\]  

(b) \(u = \frac{8}{3} \Rightarrow \frac{1}{u} = \frac{3}{8}\); \(v = \frac{6}{5} \Rightarrow \frac{1}{v} = \frac{5}{6}\)  
so \(\frac{1}{u} + \frac{1}{v} = \frac{3}{8} + \frac{5}{6} = \frac{3 \times 3 + 5 \times 4}{24} = \frac{29}{24}\)  
\[\Rightarrow f = \frac{29}{24}\]  

4 (a) 187 \(\frac{1}{2} = \frac{375}{2}\); 3 \(\frac{1}{8} = \frac{25}{2}\)  
number of presses = \(\frac{375}{2} \div \frac{25}{8}\)  
\[= \frac{375}{2} \times \frac{8}{25} = \frac{15}{1} \times \frac{4}{1} = 60\]  

(b) Time from Granada to Antequera is 72 minutes  
Time from Granada to Sevilla is 168 minutes  
So fraction is \(\frac{72}{168} = \frac{6}{14} = \frac{3}{7}\)
### ALGEBRA 1 – BASIC SKILLS EXERCISE

1. \(2xy + 2xz\)
2. \(2xy\)
3. \(10a + 5\)
4. \(7b\)
5. \(9ab\)
6. \(7a^5\)
7. \(5a^6\)
8. \(72a^5\)
9. \(12a - 6b\)
10. \(12a + 16b\)
11. \(-a - b\)
12. \(6b - 2a\)
13. \(5\)
14. \(-2\)
15. \(-3\)
16. \(-9\)
17. \(49\)
18. \(1\)
19. \(-2\)
20. \(-1\)
21. \(3/2\)
22. \(1.8\)
23. \(-1\)
24. \(2\)
25. \(3\)
26. \(-1\)
27. \(2\)
28. \(-1\)
29. \(3\)
30. \(2\)

### ALGEBRA 1 – EXAM PRACTICE EXERCISE

1. The numbers are \(x, x + 2\) and \(x + 4\), so \(x + (x + 2) + (x + 4) = 648\)
   \[3x + 6 = 648, 3x = 642\]
   \[x = 214\] so the numbers are 214, 216 and 218

2. Interior angle at \(B\) is
   \[180 - (145 - 6x) = 35 + 6x\]
   Angle \(C\) is \(35 + 6x\) as it is an isosceles triangle
   Angle sum of a triangle is 180°, so
   \[35 + 6x + 35 + 6x + 70 - 4x = 180\]
   \[8x + 140 = 180, 8x = 40\]
   \[x = 5\]
   so angles are 65°, 65° and 50°

3. The width of the screen is \(x - 0.5\)
The length of the phone is \(2x\) so the length of the screen is \(2x - 3\)
The perimeter of the screen is 32
so \(2(x - 0.5) + 2(2x - 3) = 32\)
\[2x - 1 + 4x - 6 = 32\]
\[6x = 39\]
\[x = 6.5\]
So the screen measures 6 cm by 10 cm and the area is 6 \(\times\) 10 = 60 cm²

4. Let \(x\) be the diameter of the circle.
The side of the square is \(x\) so the perimeter of the square is \(4x\)
The circumference of the circle is \(\pi x\)
So \(4x + \pi x = 30, x(4 + \pi) = 30\)
\[x = \frac{30}{4 + \pi}\]
\[x = 4.20\) (3 s.f.)
So lengths are 4 \(\times\) 4.20 = 16.8 cm and \(\pi \times 4.20 = 13.2\) cm (3 s.f.)

5. (a) Let \(x\) be the number of minutes after 12:00
In 60 minutes the minute hand moves 360° or 6° per minute
In \(x\) minutes the minute hand moves 6\(x\)° from the vertical.
The hour hand moves at \(\frac{1}{12}\) of the speed of the minute hand
In \(x\) minutes the hour hand moves \(\frac{6x}{12} = \frac{x}{2}\) degrees from the vertical.
Angle between hands must equal
\[90° \Rightarrow 6x - \frac{x}{2} = 90 \Rightarrow \frac{11x}{2} = 90 \Rightarrow x = 16.36\]
\[ x = 16 \text{ minutes 22 secs so time is 12:16:22 to the nearest second.} \]

(b) Let \( x \) be the number of minutes after 12:00.

In \( x \) minutes the minute hand moves \( 6x^\circ \) from the vertical.

The time will be after 01:00. At 01:00 the minute hand has moved 360\(^\circ\), so the angle of the minute hand will be \( 6x - 360 \) degrees from the vertical.

The hour hand will have moved \( \frac{x}{2} \) degrees from the vertical.

\[ \Rightarrow 6x - 360 = \frac{x}{2} \Rightarrow \frac{11x}{2} = 360 \Rightarrow x = 65.45. \]

\[ x = 1 \text{ hour 5 minutes and 27 seconds so time is 01:05:27 to the nearest second.} \]

OR

The time will be after 01:00.

Let \( y \) be the number of minutes after 01:00.

In \( y \) minutes the minute hand moves \( 6y^\circ \) from the vertical.

At 01:00 the hour hand has moved 30\(^\circ\) from the vertical so \( y \) minutes later it has moved \( 30 + \frac{y}{2} \) degrees.

\[ \Rightarrow 6y = 30 + \frac{y}{2} \Rightarrow \frac{11y}{2} = 30 \Rightarrow y = 5.45. \text{ or 5 mins 27 secs to nearest second} \]

So time is 01:05:27 to the nearest second.

**GRAPHS 1 – BASIC SKILLS EXERCISE**

1. (a) \( 2 \)
   (b) \( -\frac{1}{2} \)
2. \( 45 \) m
3. \( \frac{2}{3} \) m
4. \( 28.6 \)
5. \( p = -4 \)
6. No. Gradient of \( AB \) is 2, gradient of \( BC \) \( \neq 2 \), it is 2.02 to 3 s.f.
7. \( -63 \)
8. \( 3 \)
9. \( A \) and \( D \)
10. The second point in the table should be (0, 3).
11. | \( x \) | \( -3 \) | \( a = -2 \) | \( 0 \) | \( 1 \) | \( 3 \) | \( c = 4 \) |
    | \( y \) | \( 11 \) | \( 8 \) | \( 2 \) | \( -1 \) | \( b = -7 \) | \( -10 \) |
12. (a)
    | \( x \) | \( -2 \) | \( 0 \) | \( 2 \) | \( 4 \) |
    | \( y = 3 - 2x \) | \( 7 \) | \( 3 \) | \( -1 \) | \( -5 \) |
    | \( 2y - x + 2 = 0 \) | \( -2 \) | \( -1 \) | \( 0 \) | \( 1 \) |

**GRAPHS 1 – EXAM PRACTICE EXERCISE**

1. (a) \( \frac{1}{x} = \frac{1}{3} \Rightarrow x = 3 \) so width is 6 m

(b) Gradient = \( \frac{(5p - 9) - (p - 9)}{(p + 7) - (p - 1)} = \frac{5p - p}{7 + 1} = \frac{4p}{8} = \frac{p}{2} \Rightarrow p = 1 \)
2 Sketching the position of the points shows that if it is a parallelogram $AB \parallel DC$ and $AD \parallel BC$.

(a) Gradient of $AB = \frac{20 - 10}{29 - 16} = \frac{10}{13} = \frac{2}{3}$

Gradient of $DC = \frac{-12 - 42}{45 - 0} = \frac{-54}{45} = \frac{2}{3}$

$\Rightarrow AB$ is parallel to $DC$ as the gradients are the same.

Gradient of $AD = \frac{-42 - 10}{0 - 16} = \frac{-52}{-16} = \frac{13}{4}$

Gradient of $BC = \frac{-12 - 20}{45 - 29} = \frac{-32}{16} = -2$

$\Rightarrow AD$ is parallel to $BC$ as the gradients are the same.

$\Rightarrow ABCD$ is a parallelogram as it has two pairs of opposite parallel sides.

(b) Gradient of $AO = \frac{0 - 10}{0 - 16} = \frac{10}{16} = \frac{5}{8}$

As the gradient of $AO \neq$ gradient of $AB$, $O$ does not lie on $AB$.

3 (a)

<table>
<thead>
<tr>
<th>$t$ (min)</th>
<th>0</th>
<th>300</th>
<th>1000</th>
<th>1800</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$ (£)</td>
<td>20</td>
<td>20</td>
<td>34</td>
<td>50</td>
</tr>
</tbody>
</table>

(b) Gradient of graph after 300 minutes is

$$\frac{50 - 20}{1800 - 300} = \frac{30}{1500} = 0.02$$

This is 0.02 £/min or 2p per minute

4 (a) The area is decreasing by $86000 \text{ km}^2$ per year, so the formula must be of the form $A = -86000y + c$

When $y = 0$, $A = 7.7 \times 10^6$, so the formula is $A = 7.7 \times 10^6 - 86000y$

(b)

<table>
<thead>
<tr>
<th>$x$ (year after 1980)</th>
<th>0</th>
<th>20</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$ (area in $\text{km}^2$)</td>
<td>$7.7 \times 10^6$</td>
<td>$5.98 \times 10^6$</td>
<td>$4.26 \times 10^6$</td>
</tr>
</tbody>
</table>

(c) (i) $600\,000 \text{ km}^2$

(ii) 2011

5 (a)

<table>
<thead>
<tr>
<th>Weight, $W$ (g)</th>
<th>200</th>
<th>1200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length, $L$ (m)</td>
<td>0</td>
<td>325</td>
</tr>
</tbody>
</table>

(b) Reading from graph gives

(i) 231 m

(ii) 503 g

(Note as you are reading from a graph, your answers might not be the same, but they should be within ±5 units.)

(c) Gradient is $\frac{330}{1000} = 0.33$ so equation is of the form $L = 0.33W + c$

Substituting in one of the known points, say $(200, 0)$, gives $c = -66$

$\Rightarrow$ equation is $L = 0.33W - 66$
1. \( x = 36^\circ, \ y = 106^\circ, \ z = 38^\circ \)
2. \( x = 60^\circ, \ y = 30^\circ \)
3. \( x = 33^\circ, \ y = 33^\circ, \ z = 83^\circ \)
4. \( x = 75^\circ \)

5. (a) Exterior angles sum to 360
   \[ 22x - 80 = 360, \ x = 20 \]
   (b) Angle \( A = 70^\circ \), angle \( B = 70^\circ \),
   angle \( C = 40^\circ \) so isosceles with \( AC = BC \)

6. (a) Interior angles sum to 360°
   \[ x + \frac{2}{8}x + \frac{29}{24}x + \frac{2}{3}x = 360 \]
   \[ \frac{15}{4}x = 360, \ x = 96 \]
   (b) Substituting \( x = 96 \) gives interior angle
   \( A = 64^\circ, \ B = 96^\circ, \ C = 84^\circ \) and \( D = 116^\circ \)
   \( A + D = 180^\circ \) so \( AB \) is parallel to \( DC \)
   (corresponding angles)
   or
   \( B + C = 180^\circ \) so \( AB \) is parallel to \( DC \)
   (corresponding angles)

7. Acute angle between 12 and 8 is 120°
   \[ \frac{1}{3} \times 360^\circ \]
   Angle between minute hand and 12 is
   \[ x = \frac{6}{60} \times 360 = 36^\circ \]
   Whole turn is 60 mins so 6 mins is \( \frac{6}{60} \) of 360°
   Hour hand travels at \( \frac{1}{12} \) speed of minute hand.
   \[ \Rightarrow y = \frac{1}{12} \times 36^\circ = 3^\circ \]
   \[ \Rightarrow \text{acute angle between hands is} \]
   \[ 120^\circ + x - y = 153^\circ \]

8. \( x = 100^\circ, \ y = 75^\circ, \ z = 135^\circ \)
9. Mark all the equal sides.

10. (a) Exterior angle of a regular pentagon is
    \[ 360 \div 5 = 72 \]
    angle \( A = 180 - 72 = 108^\circ \)
    \( \Delta ABE \) is isosceles,
    so \( ABC = (180 - 108) \div 2 = 36^\circ \)
    Angle \( B = 108^\circ \) so \( EBC = 108 - 36 = 72^\circ \)
    \( \Delta BCE \) is isosceles so \( B\hat{C}E = 72^\circ \)
    By angle sum of \( \Delta BCE, \ x = 36^\circ \)
    (b) \( A\hat{B}E = B\hat{E}C \) so \( AB \) is parallel to \( CE \)
        (alternate angles)
    or
    \( A\hat{B}C + B\hat{C}E = 180 \) so \( AB \) is parallel to \( CE \)
        (corresponding angles)
    or
    \( B\hat{A}E + A\hat{E}C = 180 \) so \( AB \) is parallel to \( CE \)
        (corresponding angles)

11. Sum of interior angles is \( 180(n - 2) = 3060 \)
    so \( n = 19 \)

12. (a) Triangle shown is isosceles so interior angle is 156°, exterior angle is 24° and
    number of sides is \( 360 \div 24 = 15 \)
    (b) \( 15 \times 156 = 2340^\circ \)

13. \( a = 4.5 \)
14. \( a = 6, \ b = 4.5 \)
15. \( a = 4.5, \ b = 2.5 \)
16 (a) and (b)

(c) 4.36 cm

17 (a) and (b)

(c) 7.21 cm

18 (a) and (b) CP = 4.9 so CP = 9.8 m

SHAPE AND SPACE 1 – EXAM PRACTICE EXERCISE

1 (a) Using isosceles triangle properties and angle sum on a straight line gives the angles shown in the diagram.

5x = 180 (angle sum of PRS)

x = 36°

(b) Draw out the similar triangles as shown.

20

21

AB^2 = 20^2 + 21^2 => AB = 29

Pythagoras

\frac{x}{21} = \frac{20}{29}

x = \frac{420}{29}

2 (a) 35 km = 7 cm, 28 km = 5.6 cm.

Draw HN, then construct a 60° angle at H. Bisect this to give 30° and measure 7 cm to find B. From B construct the perpendicular to HN and measure 5.6 cm to find F.
(b) $FH$ measures 6.4 cm and angle $FHN$ measures $14^\circ$

So bearing is $180 - 14 = 166^\circ$

$FH$ represents $6.4 \times 5 = 32$ km

So it took $32 \div 4 = 8$ h

3 (a) The interior angle at $D$ is $360 - 4x$

Due to the rotational symmetry, all the interior angles are the same.

The sum of the interior angles of a 10-sided polygon is $(10 - 2) \times 180 = 1440^\circ$

$5x + 5(360 - 4x) = 1440$

$5x + 1800 - 20x = 1440$

$15x = 360$

$x = 24^\circ$

Let the angle in the yellow pentagon at $C$ be $x$

The angle in the yellow pentagon at $D$ is $x$

by symmetry

$2x + 130 + 130 + 80 = 540$

$x = 100^\circ$

The angle in the blue pentagon at $C$ is also $100^\circ$

The interior angle of the polygon is

$360 - 100 - 100 = 160^\circ$

exterior angle of polygon $= 20^\circ$

number of sides $= 360 \div 20 = 18^\circ$

5 Add marks showing equal sides

Mark in $60^\circ$ angles in equilateral triangle $ABE$ is isosceles, so angle at $A$ is $x$ and angle at $B$ is $180 - 2x$ (angle sum of triangle)

Angles at $B$ and $C$ sum to $180^\circ$ ($AB$ parallel to $CD$, complementary angles)

$180 - 2x + 60 + 60 + y = 180$

$y = 2x - 120$

Triangle $CDE$ is isosceles

$2z + 2x - 120 = 180$

$z = 150 - x$

Angles at $E$ sum to $360$ so

$AED + x + 60 + 150 - x = 360$

$AED = 150^\circ$

4 The interior angles of a pentagon sum to $3 \times 180 = 540$

By symmetry the angle in the yellow pentagon at $G$ is $130^\circ$
SETS 1 – BASIC SKILLS EXERCISE

1. (a) Any multiples of 3
   (b) Any negative integers
   (c) Any sport
   (d) Any make of car

2. (a) \{multiples of 3\}
   (b) \{negative integers\}
   (c) \{sports\}
   (d) \{makes of car\}

3. (a) \{2, 4, 6, 8\}
   (b) \{4, 9, 16\}
   (c) \{January, June, July\}
   (d) \{Red, Amber (or Orange), Green\}

4. (a) True
   (b) False
   (c) False
   (d) True

5. a, b and d

6. (a) \{2, 4, 6, 8\}
   (b) \{1, 3, 5, 7, 9, 11\}, odd numbers between 1 and 11 inclusive OR odd numbers between 0 and 12
   (c) 8
   (d) Yes. All multiples of 4 are also multiples of 2

7. (a) Because 10 is not a member of \(\xi\)
   (b) \{5, 15\}
   (c) 2 Factors are 1 and 5
   (d) \{5\}

8. (a) \(\emptyset\)
   (b) \(\emptyset\)

9. (a) \(W = \{A, E, I, U\}, W' = \{O\}, S = \{O, U\}, S' = \{A, E, I\}\)
   (b) \(\emptyset\)
   (c) (i) \{A, E, I, O, U\} or \(\emptyset\)
       (ii) \{U\}

10. (a) \(\emptyset\)
    (b) 15

SETS 1 – EXAM PRACTICE EXERCISE

1. (a) (i) False
    (ii) True
    (iii) False
    (iv) False
   (b) (i) \(A \cap C = \emptyset\)
       (ii) \(C \cup D = C\)
       (iii) \(A \cap B \neq \emptyset\)

2. (a) \(\emptyset\)
   (b) \(A \cup B = \emptyset\)
(b) (i) 8
(ii) 2
(iii) In the Venn diagram, \( x \) can be any number between 0 and 9

\[
\begin{array}{c}
\text{C} \\
\text{D} \\
\text{9} - x
\end{array}
\]

3 (a) 

\[
\begin{array}{c}
\text{A} \\
\text{B} \\
\text{2} \\
\text{20} \\
\text{5} \\
\text{3}
\end{array}
\]

(b) 20

4 (a) 

\[
\begin{array}{c}
\text{I} \\
\text{R}
\end{array}
\]

(b) \( I \cap R \) is the set of isosceles right-angled triangles, so angles are 90°, 45° and 45°

(c) 

\[
\begin{array}{c}
\text{E} \\
\text{R}
\end{array}
\]

All equilateral triangles are isosceles, so \( E \) is a subset of \( I \)

5 (a) 

\[
\begin{array}{c}
\text{A} \\
\text{B} \\
\text{2} \\
\text{x} \\
\text{3}
\end{array}
\]

As \( n(A) = x \) and \( n(A \cap B) = \frac{x}{2} \) then

\[
n(A \cap B') = \frac{x}{2}
\]

\[
\frac{3x}{2} + 2x + 7 + 17 - x = 33
\]

\[
\frac{3x}{2} = 9
\]

\[
x = 6
\]

\( n(B) = 19, n(A \cap B) = 3 \)

\( n(A' \cap B) = 19 - 3 = 16 \)

(b) (i) 

\[
\begin{array}{c}
\text{B} = \frac{3}{4} \\
\text{I}
\end{array}
\]

Fraction using \( B \) and \( I \) is

\[
\frac{3}{4} + \frac{5}{24} = \frac{23}{24}
\]

2 students are \( \frac{1}{24} \) of the group

number of students is \( 24 \times 2 = 48 \)

(ii) Turn the fractions into numbers and fill in the Venn diagram

\[
\begin{array}{c}
\text{A} \\
\text{B} \\
\text{2} \\
\text{22} \\
\text{14} \\
\text{10}
\end{array}
\]

From the Venn diagram \( n(B \cap I) = 14 \)

**NUMBER 2 – BASIC SKILLS EXERCISE**

1 (a) \( 1.45 \times 10^5 \)
(b) \( 1.23 \times 10^8 \)
(c) \( 1 \times 10^6 \)
(d) \( 1 \times 10^9 \)

2 (a) \( 1.38 \times 10^5 \)
(b) \( 9.74 \times 10^8 \)
(c) \( 3.13 \times 10^3 \)
(d) \( 3.16 \times 10^4 \)

3 (a) 350
(b) 5750
(c) 1250000
(d) 93 210

4 \( 6 \times 10^{199} \)
5 \( 4 \times 10^1 \)
6 \( 2.8 \times 10^{100} \)
7 \( 3.2 \times 10^{100} \)
8 \( 2 \times 10^{50} \)
9 \( 4 \times 10^{200} \)
10 \( 2.89 \times 10^{50} \)

11 (a) \( 1 \times 10^{-1} \)
(b) \( 5 \times 10^{-3} \)
(c) \( 2.5 \times 10^{-1} \)
(d) \( 7 \times 10^{-6} \)
12 (a) $1.23 \times 10^{-2}$  
(b) $1.24 \times 10^{-2}$  
(c) $1.60 \times 10^{-4}$  
(d) $8.89 \times 10^{-3}$

13 (a) $0.035$  
(b) $0.00575$  
(c) $0.00000125$  
(d) $0.00009321$

14 $6 \times 10^{-199}$

15 $4 \times 10^{-1}$

16 $-1.7 \times 10^{-99}$

17 $2.3 \times 10^{-99}$

18 $2 \times 10^{-50}$

19 $4 \times 10^{-200}$

20 $1.85 \times 10^{-6}$

21 $2.66 \times 10^{-22}$

22 $2.41 \times 10^{-6}$

23 $1.62 \times 10^{-3}$

24 $1.38 \times 10^{-2}$

25 (a) $4.08 \times 10^{-7}$  
(b) $1.76 \times 10^{-9}$  
(c) $3.87 \times 10^{-11}$  
(d) $4.83 \times 10^{-4}$

26 $2.90 \times 10^{-5}$ km

27 (a) $3.02 \times 10^{30}$ mm$^2$  
(b) $1.69 \times 10^{-8}$

28 $6.32 \times 10^{-13}$ km/s

29 $9.46 \times 10^{-12}$ km/year

30 (a) $\frac{1}{4}$  
(b) $\frac{1}{10}$  
(c) $\frac{3}{4}$  
(d) $\frac{3}{5}$  
(e) $\frac{7}{20}$

31 150 m

32 $\$360$

33 42 g

34 8%

35 0.01 %

36 loss of 37.5% or -37.5% increase

37 12%

38 1650 m

39 528 kg

40 $\$912$

41 21 250 cm$^2$

42 $\$90$

43 €897

44 £56.25

45 (a) 56.7 s  
(b) 19% improvement

46 (a) 1.39 m  
(b) 14%

47 (a) €11 880  
(b) €120 000 \left(1 + \frac{x}{100}\right) \left(1 - \frac{R}{100}\right)$

48 $x \times\left(1 + \frac{y}{100}\right)$

49 $x \times\left(1 - \frac{y}{100}\right)$

50 +44%

**Number 2 – Exam Practice Exercise**

1 (a) (i) Let $V$ be volume of all five planets:  
$V = 1.43 \times 10^{24} + 8.27 \times 10^{23} + 1.08 \times 10^{21} + 1.63 \times 10^{20} + 7.15 \times 10^{18}$  
$= 2.2582... \times 10^{24}$ m$^3$  
Thus,  $V = 2.26 \times 10^{24}$ m$^3$ (3 s.f.)

(ii) Volume of Jupiter − Volume of Pluto = \[1.43 \times 10^{24} - 7.15 \times 10^{18}\]  
= $1.4299... \times 10^{24}$ m$^3$  
= $1.43 \times 10^{24}$ m$^3$

(b) $\text{Volume Mars} \times k = \text{Volume Earth}$  
$1.63 \times 10^{20} \times k = 1.08 \times 10^{21}$  
$k = \frac{1.08 \times 10^{21}}{1.63 \times 10^{20}} = 7$  
(nearest integer)

2 (a) (i) DNA molecule width − Water molecule width = $2.15 \times 10^{-9} - 2.70 \times 10^{-10}$  
= $0.000 \times \frac{001}{88}$ m  
= $1.88 \times 10^{-9}$ m

(ii) Grain of sand width − Human hair width = $5.25 \times 10^{-4} - 7.50 \times 10^{-5}$  
= $0.000 \times 45$  
= $4.50 \times 10^{-4}$ m

(b) Width Covid-19 virus : Human hair width = $1.60 \times 10^{-7} : 7.50 \times 10^{-5}$  
= $\frac{1.60 \times 10^{-7}}{7.50 \times 10^{-5}}$  
= 468.75  
= 1 : $n$ where $n = 469$ to the nearest integer

3 1 January 2022:  
Maira account = $15,000 \times 1.08 - (0.08 \times 15,000 \times 0.40) = 15,720$
(Multiply by 1.08 to increase by 8%. Multiply by 0.08 to find 8% of $15 000. Multiply by 0.40 to find 40% of the profit.)

Jurgen account = $18 000 × 0.88 = $15 840

Jurgen has $15 840 − $15 720 = $120 more in his account than Maira on 1 Jan 2022.

(Multiply by 0.88 to decrease by 12%)

4 (a) London’s population in 1900 as a percentage of its population in 2000

\[ \frac{5}{7.27} \times 100 = 68.8\% \]

(x as a % of y = \( \frac{x}{y} \times 100 \))

(b) (i) Percentage change in London’s population from 1950 to 2000

\[ \frac{7.27 - 8.2}{8.2} \times 100 = -11.3\% \]

(ii) Percentage change in London’s population from 1900 to 2020

\[ \frac{9.3 - 5}{5} \times 100 = +86.0\% \]

(% change = \( \frac{\text{change}}{\text{original}} \times 100 \))

5 (a) Percentage of female doctors by country:

England: \( \frac{98 974}{107 221} \times 100 = 91\% \)

Scotland: \( \frac{11 012}{97 66} \times 100 = 11\% \)

Wales: \( \frac{4 711}{5 331} \times 100 = 88\% \)

Northern Ireland: \( \frac{3 337}{3 207} \times 100 = 104\% \)

Scotland has the highest percentage of female doctors in 2019.

(b) Number of male doctors = 125 725

New number of male doctors = 130 754

\[ 125 725 \times p = 130 754 \]

so \( p = \frac{130 754}{125 725} = 1.04 \), so \( k = 4 \)

ALGEBRA 2 – BASIC SKILLS EXERCISE

1 \( \frac{4}{x} \)

2 \( 2a \)

3 \( \frac{2x}{y} \)

4 \( 4x^2 \)

5 \( \frac{5ad}{7b} \)

6 \( 4 \)

7 \( z \)

8 \( 1 \)

9 \( 4bc \)

10 \( b \)

11 \( \frac{17x}{12} \)

12 \( \frac{7}{6x} \)

13 \( \frac{14z}{15} \)

14 \( \frac{3}{5x} \)

15 \( \frac{4(x - y^2)}{6} \)

16 \( \frac{x^2}{6} \)

17 \( a^2 + b^2 \)

18 \( \frac{a + b^3}{b} \)

19 \( \pm 4 \)

20 \( \pm 4 \)

21 \( \pm 3 \)

22 3

23 \( \pm 2 \)

24 4

25 9

26 4

27 9

28 4

29 3\( ^{\frac{10}{10}} \)

30 \( a^6 \)

31 5\( ^7 \)

32 \( x^2 \)

33 4\( ^6 \)

34 \( y^{\frac{18}{18}} \)

35 7\( ^4 \)

36 \( z^3 \)

37 (a) \( 2 > -2 \)

(b) \( -2 > -5 \)

(c) \( 20\% < \frac{1}{4} \)

(d) \( -0.3 > -\frac{1}{3} \)

38 (a)

(b)

(c)

39 (a) \(-2 \leq x < 2 \)

(b) \( x \leq -1 \) or \( x > 2 \)
40 $x \leq 2$
41 $x > 4$
42 $x < -2$
43 $-4 < x \leq 0$

**ALGEBRA 2 – EXAM PRACTICE EXERCISE**

1 (a) $\frac{12x^3y^2z}{5x^3y^2} \div \frac{15x^3y^2}{9x^2}$

$= \frac{12x^3y^2z}{5x^3y^2} \times \frac{9x^2}{15x^3y^2}$

To divide, ‘turn upside down and multiply’

$= \frac{12x^3y^2z}{5x^3y^2} \times \frac{9x^2}{15x^3y^2}$

‘Cancelling’ $x$

$= \frac{12y}{5}$

‘Cancelling’ $y$

$= \frac{z^2}{2x^2}$

‘Cancelling’ the numbers

(b) $\left(\frac{1}{x} - \frac{3y}{x}\right) = \frac{1}{x} - \frac{3y}{x} = \frac{-2}{x}$

Deal with $\left(\frac{1}{x} - \frac{3y}{x}\right)$ first (BIDMAS)

$\Rightarrow \frac{1}{x} \div \left(\frac{1}{x} - \frac{3y}{x}\right) = \frac{1}{x} \times \frac{x}{x-3} = \frac{1}{2}$

To divide, turn fraction upside down and multiply

$\Rightarrow 1 - \frac{1}{x} \div \left(\frac{1}{x} - \frac{3y}{x}\right) = 1 - \frac{1}{2x}$

$= \frac{2x}{2x} + \frac{1}{2x} = \frac{2x+1}{2x} \Rightarrow a = 2.$

2 (a) Ava’s age $= y$, Ben’s age $= y - 4$,
Charlie’s age $= 2(y - 4)$
Sum of ages $= y + y - 4 + 2(y - 4) = 4y - 12$
$4y - 12 > 27$ and $4y - 12 < 41$
(or $27 < 4y - 12 < 41$)

(b) $4y - 12 < 41$

$4y < 53 \Rightarrow y < 13.25$ $y$ is an integer
Ava is 13, Ben is 9 and Charlie is 18.
(c) $4y - 12 > 27$

$4y > 39$

$y > 9.75$ $y$ is an integer
Ava is 10, Ben is 6 and Charlie is 12

3 (a) Method 1: $a^4 \div a^3 = a^1$ Subtracting indices rule

$\sqrt{x+1} = 4$

$x + 1 = 16$

$x = 15$

Method 2: $\frac{a^{x+1}}{a^3} = a$ Multiplying both sides by $a^3$

$\sqrt{x+1} = 4$

$x + 1 = 16$

$x = 15$

(b) (i) $1 + \frac{1}{a} = \frac{a}{a} + \frac{1}{a} = \frac{a+1}{a}$

Simplify denominator first

$\frac{1}{1 + \frac{1}{a}} = 1 \times \frac{a}{a+1}$

$= \frac{a}{a+1}$

(ii) $\frac{1}{1 + \frac{1}{a}} = \frac{1}{1 + \frac{1}{a}}$ Using result from part i

$\Rightarrow 1 + \frac{a}{a+1} = \frac{a+1}{a+1} = \frac{2a+1}{a+1}$

Simplify denominator first

$\frac{1}{1 + \frac{a}{a+1}} = 1 \times \frac{2a+1}{a+1}$

$= \frac{2a+1}{a+1}$

4 First third of journey is $x$ km at a speed of 60 km/h

Time taken for the first third of the journey is $\frac{x}{60}$ hours. Time $= \frac{\text{distance}}{\text{speed}}$

Remaining two-thirds of journey is 2x km at 40 km/h

Time taken for remaining two-thirds is $\frac{2x}{40}$ hours. Time $= \frac{\text{distance}}{\text{speed}}$

$\frac{x}{60} + \frac{2x}{40} = \frac{3}{2}$

total journey time is 1.5 hours or $\frac{3}{2}$ hours

$\frac{x}{60} + \frac{2x}{40} = \frac{3}{2}$
\[ \frac{2x}{120} + \frac{6x}{120} = \frac{180}{120} \]

multiplying both sides by 120

\[ 2x + 6x = 180 \]

\[ x = \frac{180}{8} \]

\[ 3x = \frac{3 \times 180}{8} \]

3x is total length of journey

\[ = 67.5 \text{ km} \]

5 (a) \( \frac{1}{R} = \frac{b}{ab} + \frac{a}{ab} = \frac{b + a}{ab} \implies R = \frac{ab}{a + b} \)

(b) \( a \) becomes \( a + 1 \), \( b \) becomes \( b - 1 \)

Substitute these values into \( R = \frac{ab}{a + b} \)

\[ R_{\text{new}} = \frac{(a + 1)(b - 1)}{a + 1 + b - 1} = \frac{(a + 1)(b - 1)}{a + b} \]

Change in \( R \) is \( R_{\text{new}} - R = \frac{(a + 1)(b - 1)}{a + b} \)

\[ \frac{ab}{a + b} = \frac{ab + b - a - 1}{a + b} = \frac{b - a - 1}{a + b} \]

\% change = \( \frac{b - a - 1}{a + b} \times \frac{ab}{a + b} \times 100 \)

\% change is \( \frac{\text{Change in } R}{\text{Original } R} \times 100 \)

\[ = \frac{b - a - 1}{a + b} \times \frac{a + b}{ab} \times 100 \]

\[ = \frac{b - a - 1}{ab} \times 100 \]

14 \( \frac{1}{4}, -1 \)

\( y = -2x + 12 \)

10 \( y = \frac{x}{3} + \frac{5}{3} \)

11 \( y = -\frac{x}{2} - 1 \)

12 \( y = 3x - 5 \)

13 3, 2

**GRAPHS 2 – BASIC SKILLS EXERCISE**

1 \( y = 3x - 1 \)

2 \( y = -\frac{1}{4} x + 2 \)

3 \( y = \frac{5}{3} \)

4 \( y = 2x + 1 \)

5 \( y = -\frac{1}{3} x + 4 \)

6 \( y = 4x - 2 \)

7 \( y = -0.4x + 1 \)

8 \( y = 0.2x \)
16. \((-1, 2)\)

17. \((3, 0), (0, 12)\)

18. \((15, 0), (0, 5)\)

19. \((10, 0), 0, -2)\)

20. \(\left(\frac{1}{2}, 0\right), \left(0, -\frac{1}{2}\right)\)

21. \((1.5, 4.5)\)

\[
\begin{array}{c|ccc}
 x & 0 & 3 & 6 \\
\hline
 y = x + 3 & 3 & 6 & 9 \\
 y = 6 - x & 6 & 3 & 0 \\
\end{array}
\]
22 \((\frac{2}{3}, \frac{2}{3})\)

23 \((2, 1)\)

24 \((4, 2)\)

25 \((5.3, 3.3)\)

26 \((3.2, -0.4)\)

GRAPHS 2 – EXAM PRACTICE EXERCISE

1

(a) $y = 2x - 4$ has gradient 2 so $L_2$ must be $y = 2x + c$

$L$ intersects the $x$-axis at $(4, 0)$

Substituting $x = 4, y = 0$ into $y = 2x + c$ gives

$c = -8$ so $M$ is $y = 2x - 8$

(b) $L$ intersects the $y$-axis at $(0, 3)$, $M$ intersects the $y$-axis at $(0, -8)$ and they both intersect the $x$-axis at $(4, 0)$

Area $= \frac{1}{2} \times 11 \times 4 = 22$ square units

2 (a) Rearrange the equations as

$A: y = \frac{1}{2}x + 2$, $B: y = -2x + 2$

$C: y = \frac{1}{2}x - \frac{1}{2}$, $D: y = -2x + 7$
A sketch helps you to understand and answer the question.

\[ P \]

\[ y \]

\[ x \]

\[ A \]

\[ B \]

\[ C \]

\[ D \]

\[ P \]

\[ Q \]

\[ R \]

\[ S \]

\[ T \]

\[ U \]

\[ V \]

\[ W \]

\[ X \]

\[ Y \]

\[ Z \]

\[ A\] and \( C \) have the same gradient of \( \frac{1}{2} \) so they are parallel and one pair of opposite sides.

\[ B \] and \( D \) have the same gradient of \(-2\) so they are parallel and the other pair of opposite sides.

(b) \( A \) and \( B \) have a common point, \( P \), \((0, 2)\) so this is a vertex

\( B \) and \( C \) have a common point, \( Q \), \((1, 0)\) so this is a vertex.

\[ PQ^2 = 1^2 + 2^2 = 5 \quad \text{Pythagoras’ Theorem} \]

\[ PQ = \sqrt{5} \text{ so perimeter} = 4\sqrt{5} \]

3 (a)

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>Mia</th>
<th>Priya</th>
</tr>
</thead>
<tbody>
<tr>
<td>Now</td>
<td>( x )</td>
<td>( y )</td>
</tr>
<tr>
<td>10 years ago</td>
<td>( x - 10 )</td>
<td>( y - 10 )</td>
</tr>
<tr>
<td>10 years’ time</td>
<td>( x + 10 )</td>
<td>( y + 10 )</td>
</tr>
</tbody>
</table>

Ages 10 years ago were \( x - 10 \), \( y - 10 \)

\[ x - 10 = 6(y - 10) \]
\[ x - 10 = 6y - 60 \]
\[ x + 50 = 6y \]

Ages in 10 years’ time will be \( x + 10 \), \( y + 10 \)

\[ x + 10 = 2(y + 10) \]
\[ x + 10 = 2y + 20 \]
\[ x - 10 = 2y \]

(b) Some points for \( x + 50 = 6y \) are \((10, 10), (22, 12)\) and \((46, 16)\)

Some points for \( x - 10 = 2y \) are \((10, 0), (30, 10)\) and \((50, 20)\)

or rearrange equations as \( y = \frac{x}{6} + \frac{50}{6} \)

and \( y = \frac{x}{2} - 5 \) and make a table of values using 3 widely spaced values of \( x \)

4 (a) 200 minutes on the phone costs \( 200p \) cents and 200 texts cost \( 200t \) cents.

\[ 200p + 200t = 2800 \]

Cost of $28 must be expressed in cents

\[ p + t = 14 \]

100 minutes on the phone costs \( 100p \) cents and 300 texts cost \( 300t \) cents.

\[ 100p + 300t = 2200 \]

Cost of $22 must be expressed in cents

\[ p + 3t = 22 \]

(b) Table of values for \( p + t = 14 \)

<table>
<thead>
<tr>
<th>( p )</th>
<th>( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
</tr>
</tbody>
</table>

Table of values for \( p + 3t = 22 \)

<table>
<thead>
<tr>
<th>( p )</th>
<th>( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>22</td>
<td>0</td>
</tr>
</tbody>
</table>

(e) The graphs intersect at \( p = 10, t = 4 \), so 150 minutes on the phone will cost \( 150 \times 10 \) cents, 250 texts will cost \( 250 \times 4 \) cents.

Total cost is 2500 cents or $25.

5 (a) Let equation of \( L \) be \( y = mx + c \), then equation of \( K \) is \( y = 2mx + c \)

Gradient of \( K \) is twice gradient of \( L \) and they both have the same \( y \) intercept.
**L:** Substituting \((-2, 4)\) gives \(4 = -2m + c\)

**K:** Substituting \((4, -1)\) gives 
\(-1 = 2m \times 4 + c\) so 
\(-1 = 8m + c\)

(b) Subtract the two equations:
\[
4 - (-1) = -2m - 8m \\
5 = -10m \\
m = -\frac{1}{2} \\
and \ c = 3
\]

(c) Equation of \(L\) is \(y = -\frac{x}{2} + 3\)

\(L\) intersects the \(x\)-axis at \((6, 0)\) and the \(y\)-axis at \((0, 3)\)

\(K\) intersects the \(x\)-axis at \((3, 0)\) and the \(y\)-axis at \((0, 3)\)

\[
A = \frac{1}{2} \times 3 \times 3 = 4.5 \text{ square units}
\]

**SHAPE AND SPACE 2 – BASIC SKILLS EXERCISE**

1. \(a = 6.40\)
2. \(b = 4.47\)
3. \(c = 15.0\)
4. \(AC = 36.6\)
5. \(a = 5.39\)
6. \(a = 5.20\)
7. (a) \(r = 11.7\)
   (b) \(a = 18.7\)
8. (a) \(XC = 2\) cm
    (b) \(AC = 4.47\) cm
9. \(k = 3\) Using Pythagoras’, horizontal distance is 3 and vertical distance is 15 gives hypotenuse of \(3\sqrt{26}\)
10. Using Pythagoras’, (length of square)\(^2\) = \((a - c)\(^2\) + (b - d)\(^2\). But area is (length of square)\(^2\)
    So \(A = (a - c)\(^2\) + (b - d)\(^2\)
11. \(a = 5, b = 17\) or vice-versa. Use Pythagoras’ with horizontal distance 20 and vertical distance 90
    \(20^2 + 90^2 = 8500\) so \(OP = 10\sqrt{85}\). 5 and 17 are prime numbers which multiply to give 85.

**SHAPE AND SPACE 2 – EXAM PRACTICE EXERCISE**

1. (a) \(r = 11.7\)
   (b) \(a = 18.7\)
8. (a) \(XC = 2\) cm
    (b) \(AC = 4.47\) cm
9. \(k = 3\) Using Pythagoras’, horizontal distance is 3 and vertical distance is 15 gives hypotenuse of \(3\sqrt{26}\)
10. Using Pythagoras’, (length of square)\(^2\) = \((a - c)\(^2\) + (b - d)\(^2\). But area is (length of square)\(^2\)
    So \(A = (a - c)\(^2\) + (b - d)\(^2\)
11. \(a = 5, b = 17\) or vice-versa. Use Pythagoras’ with horizontal distance 20 and vertical distance 90
    \(20^2 + 90^2 = 8500\) so \(OP = 10\sqrt{85}\). 5 and 17 are prime numbers which multiply to give 85.

(a) Let \(O\) be the centre of the circle, so triangle \(AOB\) is a right-angled triangle.
If \(AO = BO = r\), then from Pythagoras’ Theorem: \(7^2 = r^2 + r^2 = 2r^2\), so \(r^2 = \frac{49}{2}\)
area of the shaded segments = circle
area – square area = \(\pi \frac{49}{2} - \frac{25}{2}\)

\[= 49\left(\frac{\sqrt{5} - 1}{2}\right)\]
(a) Area of one of the original circle shaded segments $= \frac{49}{4} (\pi - 2) \pi = \frac{49}{8} (\pi - 2) \Rightarrow a$

Let segment of the enlarged circle be $A$, so if the scale factor of enlargement = 4

$A = 4^2 \times a = 16 \times \frac{49}{8} (\pi - 2) = 98 (\pi - 2)$

$= n (\pi - 2), n = 98$

(b) Area of one of the original circle shaded segments $= \frac{49}{4} (\pi - 2)$

$= \frac{49}{8} (\pi - 2) \Rightarrow m$

$m = \frac{100}{\pi}$

Angle $HDE = 180^\circ - 0^\circ = 120^\circ$

($GHDE$ is a cyclic quadrilateral: Opposite angles in a cyclic quadrilateral sum to $180^\circ$)

$\therefore \angle HDP = \frac{1}{3} \times 120^\circ = 40^\circ$

($\angle HDE$: $\angle EDP = 1:2$)

Angle $HDP = \angle HFP = 40^\circ$

(Both angle $HDP$ and angle $HFP$ are formed in the same segment off chord $HP$: Angles in the same segment are equal).

Angle $LOP = 2 \times 48^\circ = 96^\circ$

(Angle at centre $= 2 \times$ angle at circumference off the same chord in the same segment)

Angle $OLP = \angle OPL = \frac{180^\circ - 96^\circ}{2} = 42^\circ$

(Triangle $OLP$ is isosceles, so the base angles are equal)

Angle $MPL = \frac{2}{3} \times 42^\circ = 28^\circ$

Angle $MLP = 180^\circ - 48^\circ - 28^\circ = 104^\circ$

(Angle sum of a triangle $= 180^\circ$)
(a) Angle $OCA = 90^\circ$ (Radius $OC$ is perpendicular to tangent $AD$.)
Angle $AOC = 180^\circ - 90^\circ - 36^\circ = 54^\circ$ (Angle sum of a triangle = $180^\circ$)
Angle $COG = 180^\circ - 54^\circ = 126^\circ$ (Sum of angles in a straight line = $180^\circ$)
Angle $OCG = \frac{180^\circ - 126^\circ}{2} = 27^\circ$ (Triangle $OCG$ is isosceles, so the base angles are equal.)

(b) Angle $BEO = 360^\circ - 70^\circ - 90^\circ - 54^\circ = 146^\circ$ (Angle sum of a quadrilateral = $360^\circ$)
Angle $FEO = 180^\circ - 146^\circ = 34^\circ$ (Sum of angles in a straight line = $180^\circ$)
Angle $FGO = 180^\circ - 90^\circ - 34^\circ = 56^\circ$ (Angle sum of a triangle = $180^\circ$. Angle in a semi-circle is $90^\circ$. $AB$ is the diameter of the circle.)

(c) Triangle $ECG$ is a right-angled triangle as $EG$ is a diameter of the circle.

Using Pythagoras’: $EG^2 = EC^2 + CG^2$,

$EC^2 = 4r^2 - 16s^2 = 4(r^2 - 4s^2) = 4(r + 2s)(r - 2s)$ – using a difference of squares

$EC = \sqrt{(r + 2s)(r - 2s)}$ as required.

HANDLING DATA 1 – BASIC SKILLS EXERCISE

1  
(a) Phone make – categorical
(b) Number of goals scored – discrete
(c) Height of a horse – continuous
(d) Number of coins – discrete
(e) Time to eat a pizza – continuous
(f) Hair colour – categorical

2  
(a)  
<table>
<thead>
<tr>
<th>Number</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
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<td></td>
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<tr>
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<td></td>
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</tr>
<tr>
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<tr>
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<td></td>
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<td>5</td>
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</table>

(b)  
<table>
<thead>
<tr>
<th>Number of books</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 to 5</td>
<td>6</td>
</tr>
<tr>
<td>6 to 10</td>
<td>3</td>
</tr>
<tr>
<td>11 to 15</td>
<td>2</td>
</tr>
<tr>
<td>16 to 20</td>
<td>7</td>
</tr>
<tr>
<td>21 to 25</td>
<td>9</td>
</tr>
<tr>
<td>26 to 30</td>
<td>3</td>
</tr>
</tbody>
</table>

(c) There are many students with a lot of revision guides, quite a few with not many guides, but not many students with a middling number of guides.
4 (a) | January–April | May–August | September–December | Total |
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Year 1</td>
<td>64</td>
<td>42</td>
<td>72</td>
<td>178</td>
</tr>
<tr>
<td>Year 2</td>
<td>55</td>
<td>24</td>
<td>37</td>
<td>116</td>
</tr>
<tr>
<td>Total</td>
<td>119</td>
<td>66</td>
<td>109</td>
<td>294</td>
</tr>
</tbody>
</table>

(b) TV sales

(c) Year 2 sales were worse overall, especially from April–December

5 (a) 7.5% had a rabbit
   Percentages must sum to 100° or % must sum to 100
(b) Angle for cat = \( \frac{55}{100} \times 360° = 198° \)
    Angle for bird = \( \frac{30}{100} \times 360° = 108° \)
    Angle for rabbit = \( \frac{7.5}{100} \times 360° = 27° \)
    Angle for fish = \( \frac{5}{100} \times 360° = 18° \)
    Angle for other = \( \frac{2.5}{100} \times 360° = 9° \)

6 Orange juice is 56°
   \( \frac{56}{14} = 4 \) so 4° corresponds to 1 person
   Coffee \( \frac{88}{4} = 22 \) people

Tea \( \frac{104}{4} = 26 \) people
Milk \( \frac{72}{4} = 18 \) people
Other \( \frac{40}{4} = 10 \) people

7 Data in order is 0, 1, 2, 3, 4, 5, 6, 7, 7, 7, 9
   mean = 4.83, median = 4.5 and mode = 7

8 Data in order:
   0 0 0 0 0 0 1 1 1 1 2 2 2 3 3 3 3 4 4 4 4 5 5 5 6 6
   (a) Sum of data = 73
      mean is \( \frac{73}{30} = 2.43 \) (3 s.f.)
      median is 2 and mode is 0
   (b) She would probably use the mode as this is the lowest average.

9 Total sent over the week = \( 7 \times 32 = 224 \)
   Total sent over the first 6 days = 176
   Number sent on seventh day = 224 – 176 = 48

10 Data in order: 0, 0, 0, 0, 0, 1, 1, 1, 2, 3
   (a) Mean = 0.8, median is 0.5 and mode is 0
   (b) Goals scored this season = \( 12 \times 1.25 = 15 \),
      goals last season = 8
      Total goals = 23, total matches = 22,
      mean = \( 23 \div 22 = 1.05 \) (3 s.f.)

11 Total points needed: \( 857 \times 7 = 5999 \)
   Total so far: \( 6 \times 840 = 5040 \)
   Points needed: 5999 – 5040 = 959

12 | State | A | B | C | Total |
<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Population in millions</td>
<td>21</td>
<td>26</td>
<td>18</td>
<td>65</td>
</tr>
<tr>
<td>Mean</td>
<td>0.1</td>
<td>0.09</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>Number infected in millions</td>
<td>2.1</td>
<td>2.34</td>
<td>2.7</td>
<td>7.14</td>
</tr>
</tbody>
</table>

Mean for whole country = \( \frac{7.14}{65} = 0.11 \) (2 d.p.)

13 \( x = 56, y = 73 \)

14 \( x = 7, y = 42 \)

15 Sum of the first 19 prime numbers squared
   \( 19 \times 1314 = 24966 \)
   mean is \( \frac{24966 + 71^2}{20} = 1500.35 \)
1 (a) Data in order are 29, 32, 32, 35, 83, 86, 95
Mean $= \frac{392}{7} = 56$, median is 35 and 
mode = 32
(b) Mode cannot be calculated as frequencies are not known.
Median cannot be calculated as the distribution of sales is not known.
Mean can be calculated. Total sales for 
the first week are 392.
Total sales for the next three weeks 
(21 days) are $60 \times 21 = 1260$.
Total sales for the four-week period of 
28 days are $392 + 1260 = 1652$
mean is $\frac{1652}{28} = 59$
2 (a) Two of the following:
The vertical axis does not start at zero.
The vertical axis has no zig-zag line to 
show it does not start at zero.
The bars are not of equal widths.
The size of the type is not the same.
(b) 
\[
\begin{array}{c|c|c|c|c|c}
\text{Song} & \text{Apricot} & \text{Burn} & \text{Tablet sales} \\
\hline
1 & 0 & & & \\
2 & & & & \\
3 & & & & \\
4 & & & & \\
5 & & & & \\
\end{array}
\]
Note: Bars must not touch each other and 
the vertical scale must be linear.
3 $\theta = 50^\circ$ (Angles must sum to 360)
$50^\circ$ corresponds to 40 pupils so $10^\circ$

\[\frac{10^\circ}{360^\circ} = \frac{10}{360} = \frac{1}{36}\]

\[\begin{align*}
50^\circ & \rightarrow 40 \text{ pupils} \\
100^\circ & \rightarrow 80 \text{ pupils} \\
60^\circ & \rightarrow 48 \text{ pupils} \\
70^\circ & \rightarrow 56 \text{ pupils} \\
80^\circ & \rightarrow 64 \text{ pupils}
\end{align*}\]
4 (a) New mean is $\frac{12a + 16a + 21a + 27a}{4}$ 
$= \frac{a(12 + 16 + 21 + 27)}{4} = a \times 19$
The mean has been multiplied by $a$.
(b) Let $s$ be the total of the numbers in set
$A$, then $x = \frac{s}{n}$ and $s = xn$
Let $t$ be the total of the numbers in set
$B$, then $y = \frac{t}{m}$ and $t = my$
The total of all $n + m$ numbers in set $C$
is $s + t = nx + my$ so the mean is $\frac{nx + my}{n + m}$
5 (a) Range is $d - 6 = 15$ so $d = 25$
Median is the mean of $b$ and 14 so $b = 12$
Mode is 8 so $a = 8$
Mean = 14 so the sum of the numbers is 
$8 \times 14 = 112$
Sum of the numbers is $94 + c$ so $c = 18$
$a = 8, b = 12, c = 18$ and $d = 25$
(b) Mean = 17 so 
\[
\frac{w + x + y + z}{4} = 17
\]
$w + x + y + z = 17 \times 4 = 68$ (equation 1)
Range = 8 so $z - w = 8$ (equation 2)
Median = 16 so $\frac{x + y}{2} = 16$ so 
$x + y = 16 \times 2 = 32$ (equation 3)
Substitute equation 3 into equation 1
$w + 32 + z = 68$ so 
$w + z = 68 - 32 = 36$ (equation 4)
Add equations 2 and 4 to get 
$2z = 44$
$z = 22$ and $w = 14$
$x + y = 32$ so $x = 15$ and $y = 17$ since 
the median is 16
Integers are 14, 15, 17 and 22.
6 (a) \[60 = 2^2 \times 3 \times 5, \quad 70 = 2 \times 5 \times 7\] therefore \[HCF = 2 \times 5 = 10\]
LCM = \[2^2 \times 3 \times 5 \times 7 = 420\]
(b) \[140 = 2^2 \times 5 \times 7, \quad 84 = 2^2 \times 3 \times 7\] therefore \[HCF = 2^2 \times 7 = 28\]
LCM = \[2^2 \times 3 \times 5 \times 7 = 420\]
(c) \[525 = 3 \times 5^2 \times 7, \quad 441 = 3^2 \times 7^2\] therefore \[HCF = 1\]
LCM = \[2 \times 3 \times 5 \times 7^2 = 88200\]
7 HCF = 3, LCM = \[2^4 \times 3^2 \times 5^2 \times 7^2 \times 11^2\]
8 15, 21
9 (a) \[HCF = 5pq, \quad LCM = 140 pq\]
(b) \[HCF = 2xyz, \quad LCM = 12\]
(c) \[HCF = \frac{a}{2} \times \frac{b}{3} \times \frac{c}{2}, \quad LCM = \frac{3a}{2} \times \frac{2b}{3} \times \frac{c}{2}\]
10 (a) 1, 2, 3, 4, 6, 12
(b) Factors of 12
11 \[48 = 2^4 \times 3, \quad 45 = 3^2 \times 5\] therefore LCM = \[2^4 \times 3 \times 5 = 720\] s or 12 minutes
12 \[88 = 2^3 \times 11, \quad 110 = 2 \times 5 \times 11\] therefore LCM = \[2^3 \times 5 \times 11 = 440\] so 4.4 seconds
13 (a) \[5 : 11\]
(b) \[1 : 5 : 17\]
(c) \[3 : 4 : 70\]
(d) \[x : 4y : 20\]
14 (a) \[1 : 24\]
(b) \[3 : 1440 = 1 : 480\]
(c) \[4 : 1000 = 1 : 250\]
(d) \[10 : \frac{900000}{60 \times 60} = 1 \div 25\]
15 \[\frac{3}{x} = \frac{x}{27} \text{ so } x^2 = 81 \text{ so } x = 9 \text{ or } x = -9\]
16 (a) \[\frac{360}{9} = 40 \text{ so } 160 : 200\]
(b) \[\frac{133}{17} = 19 \text{ so } 19 : 38 : 76\]
(c) \[\frac{1000}{10} = 100 \text{ so } 100 : 200 : 300 : 400\]
(d) \[\frac{352}{11} = 32 \text{ so } 64 : 96 : 192\]
17 \[\frac{3450}{15} = 230 \text{ so shares are } £460, £1380 \text{ and } £1610 \text{ so difference is } £1150\]
18 \[\frac{9}{7} \times 12 = 27 \text{ therefore Petra is 27 years old}\]
19 \[x : y = 175 : 100 \div 7 : 4\]
20 \[\frac{9}{7} \times 3.5 = 4.5 \text{ therefore longest charging time is 4 h 30 mins}\]
21 \[\frac{3}{103} \times 51.5 = 1.5 \text{ therefore the distance swum is 1.5 km}\]
22 \[\frac{5}{8} - \frac{3}{8} = \frac{1}{4} \text{ therefore } \frac{1}{4} \text{ of the weight of both sloths is } 6 \text{ kg and the total weight is } 24 \text{ kg hence } 9 \text{ kg and } 15 \text{ kg}\]
23 \[\frac{3}{10} - \frac{7}{10} = \frac{3}{10} \text{ of total length is } 120 \text{ cm and the total length is } 400 \text{ cm}\]
24 Total no of parts is 16.
Sugar : milk : flour is \[4 : 5 : 7\]
Therefore \[\frac{5}{16} \times 4 = \frac{1}{16} \times 160 = 60 \text{ g so the total weight } = 60 \times 16 = 960 \text{ g}\]
Therefore the weight of flour = \[\frac{7}{16} \times 960 = 420 \text{ g}\]
25 LCM of 2 and 9 is 18, \[1 : 2 = 9 : 18\] and \[9 : 5 = 18 : 10\] so \[a : b : c = 9 : 18 : 10\]
26 \[a : b = 2 : 3 = 8 : 12, b : c = 12 : 15\]
So \[a : c = 8 : 15\]
27 Angles \(A\) and \(C\) are \[\frac{2}{3}\] of the angle sum of the triangle
\[\frac{8}{8 + x} \div 2\]
\[24 = 16 + 2x\]
x = 4
28 LCM of 3 and 7 is 21, \[2 : 3 = 14 : 21\]
\[7 : 5 = 21 : 15\] so \[P : W : R = 14 : 21 : 15\].
\[14 + 21 + 15 = 50 = \text{ therefore the fraction of roses that are red is } \frac{15}{50} \div \frac{3}{10}\]
And hence the number is \[\frac{3}{10} \text{ r}\]
29 Area of the circle is \[\pi r^2\]
Area of square is \[4 \times \frac{1}{2} r^2 = 2r^2\]
The square is made up of 4 right-angled triangles with base and height of \(r\).
Alternatively calculate the length of the side of the square using Pythagoras’ Theorem, length \[\sqrt{r^2 + r^2} = \sqrt{2r^2}\]
Area = length \times length = \(2r^2\).
30 Let \( F \) be father’s age and \( S \) be son’s age
\[ F = 3S \text{ and } F + 12 = 2(S + 12) \]
\[ 3S + 12 = 2S + 24 \]
\[ S = 12, \quad F = 36 \]
In 36 years, \( F = 72, \quad S = 48 \)
So \( 3 : 2 \)

**NUMBER 3 – EXAM PRACTICE EXERCISE**

1 (a) \( 10^6 = 2^6 \times 5^6 \) therefore the greatest is \( 5^6 = 15,625 \) An odd factor cannot contain any power of 2
(b) \( 525 = 3 \times 5^2 \times 7, \quad 21 = 3 \times 7, \quad 3150 = 2 \times 3^2 \times 5^2 \times 7 \)
\[ x = 2 \times 3^2 \times 7 = 126 \]

2 (a) \( 60 = 2^2 \times 3 \times 5 \) \( x = 2^2 \times 3 = 12, \quad y = 3 \times 5 = 15 \)
\( x \) and \( y \) must share the factor 3, but not the factors 2 or 5
(b) \( 45 \times 360 = 2^4 \times 3^4 \times 5 \times 7, \quad 36 = 2^2 \times 3^2 \)
There are two possible cases, shown in the Venn diagrams.

3 (a) \( 1 + 3 = 4 \) and \( 11 + 13 = 24 \), so multiply the first ratio by 6 so they can be directly compared, i.e. \( 6 : 18 \) and \( 11 : 13 \).

The diagram shows that \( BC = 5, \) so \( AB : BC : CD = 6 : 5 : 13 \)
(b) (i) \( 11 : 3 = 22 : 6 \) and \( 5 : 2 : 15 : 6 \) so men : women : children = \( 22 : 15 : 6 \)
(ii) \( 22 + 15 + 6 = 43 \) so the fractional difference of men and women is \( \frac{22}{43} - \frac{15}{43} = \frac{7}{43} \)

Total population is \( 42 \times \frac{43}{7} = 258 \) therefore the number of children is \( 6 \times \frac{43}{7} \times 258 = 36 \)

4 Number of V, C and M is \( 0.6 \times 140 = 84 \)
60% are not Mushroom
Ratio of 2 : 5 : 7 is a total of 14 parts
number of Meat is \( \frac{7}{14} \times 84 = 42 \)
number of Veggie is \( \frac{2}{14} \times 84 = 12 \)
difference is \( 42 - 12 = 30 \) pizzas

5 \( 720 = 2^4 \times 3^2 \times 5, \quad 1260 = 2^2 \times 3^3 \times 5 \times 7, \quad 1800 = 2^3 \times 3^2 \times 5^2 \)
HCF of 720, 1260 and 1800 is \( 2^2 \times 3 \times 5 = 180 \) so 180 parcels
Number of pints of milk is \( \frac{720}{180} = 4 \)
Number of loaves of bread is \( \frac{1260}{180} = 7 \)
Number of cans of beans is \( \frac{1800}{180} = 10 \)
180 parcels each with 4 pints of milk, 7 loaves of bread and 10 cans of beans

**ALGEBRA 3 – BASIC SKILLS EXERCISE**

1 \[ a(7 - a) \]
2 \[ 3x(1 - 4x) \]
3 \[ ab(a + b) \]
4 \[ 4xy(1 - 2xy) \]
5 \[ \frac{1}{3} pqr (qr^2 + 2p^2q) \]
6 \[ (x + 1)(2x - 5) \]
7. $x + 4$
8. $1 + a$
9. $2x$
10. $\frac{5}{xy}$
11. $\frac{2(x + 2)}{y}$
12. $x^2y$
13. 6
14. $\frac{3}{5}$
15. 11
16. 4
17. 14
18. 15
19. $\frac{1}{5}$
20. $\frac{2}{3}$
21. $49$
22. $\frac{3}{4}$
23. $\pm 4$
24. $\pm 8$
25. $1 \frac{1}{5}$
26. 6

27. Let £x be Zazoo’s winnings ⇒ Yi’s winnings are £3x and Xavier’s are £$\frac{3x}{2}$

$\Rightarrow x + 3x + \frac{3x}{2} = 11000 \Rightarrow \frac{11x}{2} = 11000$

$\Rightarrow \frac{x}{2} = 1000 \Rightarrow x = 2000$

$\Rightarrow$ Zazoo gets £2000, Yi gets £6000 and Xavier gets £3000.

28. (4, 1)
29. (3, 2)
30. (1, 2)
31. (1, 1)
32. (1, 4)
33. (1, 8)
34. (3, 1)
35. (6, 2)
36. (2, 1)
37. (2, 1)
38. ($-1, 5$)
39. ($-0.4, -9.2$)
40. $x = 24, y = 15$
41. 6 stools

**ALGEBRA 3 – EXAM PRACTICE EXERCISE**

1. (a) (i) $\frac{2}{3}p^2r^2(r - 2p)$
   (ii) $(x - 1)(3 - x)$

(b) (i) $\frac{10x^2 + 5x}{5x} - 1 = \frac{5x}{5x}(2x + 1) - 1$

$\Rightarrow (2x + 1) - 1 = 2x$

(ii) $\frac{3xy + 9x^2y^2}{x^2 + 3xy^3} = \frac{3xy(x + 3y)}{x(x + 3y)}$

$\Rightarrow 3xy$

(c) $\frac{x^2 - xy}{xy^3 + y^2} + \frac{x^2 - y^2}{x^2 + xy}$

$\Rightarrow x(x - y)\frac{x^2 + xy^2}{y^2(x + y)}$

$\Rightarrow \left(\frac{x}{y}\right)^3$ so $n = 3$

2. (a) (i) $\frac{2(x + 1)}{5} - \frac{3(x + 1)}{10} = x$

$\Rightarrow 4x + 4 - 3x - 3 = 10x$

$\Rightarrow x + 1 = 10x$

$\Rightarrow x = \frac{1}{9}$

(ii) $\frac{36}{x} - 4x = 0$

$\Rightarrow 36 - 4x^2 = 0$

$\Rightarrow 36 = 4x^2$

$\Rightarrow x^2 = 9$

$\Rightarrow x = \pm 3$

(b) Let $C$ be the amount Carla gets.
Then Bobbie gets $1.4C$

To increase by 40%, multiply by 1.4

And Anna gets $0.7C + 500$

$0.7C + 500 + 1.4C + C = 16000$

$3.1C = 15500$

$\Rightarrow C = 5000$

Anna gets $4000$, Bobbie gets $7000$ and Carla gets $5000$.

3. $4y - 2x - 15 = 2x + y$ simplifies to $3y - 4x = 15$

$x : y = 3 : 5$ so $5x = 3y$

Substituting gives

$5x - 4x = 15$

$\Rightarrow x = 15$

$\Rightarrow y = 25$

$\Rightarrow$ angle $B = angle C = 55^\circ$ so angle $A = 70^\circ$

Ratio angle $A : angle B = 70 : 55 = 14 : 11$
4 (a) \[ 12 = 2^2 + 2a + b \]
\[ -6 = (-1)^2 - a + b \]
\[ 2a + b = 8 \]
\[ b - a = -7 \]
\[ 3a = 15 \]
\[ a = 5 \]
\[ b = -2 \]

equation of curve is \( y = x^2 + 5x - 2 \)

(b) Let \( a \) be number of ash trees and \( b \) be the number of beech trees.
\[ 20a + 30b = 3100 \]
\[ 30a + 20b = 2900 \]
\[ 2a + 3b = 310 \]
\[ 3a + 2b = 290 \]
\[ 6a + 9b = 930 \]
\[ 6a + 4b = 580 \]
\[ 5b = 350 \]
\[ b = 70 \]
\[ a = 50 \]
Ash costs $50, beech costs $70

5 \( (x + 100) : (y + 100) = 4 : 3 \)
\[ \frac{x + 100}{y + 100} = \frac{4}{3} \]
\[ 3(x + 100) = 4(y + 100) \]
\[ 3x - 4y = 100 \] equation 1
\[ (x - 100) : (y - 100) = 11 : 7 \]
\[ \frac{x - 100}{y - 100} = \frac{11}{7} \]
\[ 7(x - 100) = 11(y - 100) \]
\[ 7x - 11y = -400 \] equation 2
\[ 21x - 28y = 700 \] equation 1 \( \times 7 \)
\[ 21x - 33y = -1200 \] equation 2 \( \times 3 \)
\[ 5y = 1900 \Rightarrow y = 380 \]
\[ x = 540 \]
Bike costs $540, laptop costs $380

**GRAPHS 3 – BASIC SKILLS EXERCISE**

1 8 m/s
2 -4 m/s
3 (a) 15 m/s  
(b) 4 s  
(c) -9 m/s
4 (a)

5 (a)

6

7 5 m/s²
8 -0.5 m/s²

9 (a) 3 m/s²  
(b) 0 m/s²  
(c) 2 m/s²  
(d) 43.3 m/s

10 (a)  

(b) (i) 0.5 m/s²  
(ii) -1 m/s²  
(iii) 7.5 m/s
11 (a) 

\[ V \]

(b) 

\[ V \]

(c) 

12 (a) 

\[ V \]

(b) 

\[ V \]

(c) 

\[ V \]

GRAPHS 3 – EXAM PRACTICE EXERCISE

1 Gradient of a distance–time graph = velocity

(a) 

(b) 11:40 (Return speed = 30 km/h, \( t = \frac{20}{3} \) hr = 40 min)

2 \( d \) (km)

Mrs Lam is on time

3 (a) Area under a speed–time graph = distance travelled

Gradient of a speed–time graph = acceleration

(b) (i) 1 km = 1000 m

Area under graph = Distance travelled

\[ 1000 = \frac{1}{2} (t + 300)s \]

\[ 400 = t + 300 \]

\[ t = 100 \text{ s} \]

(ii) Acceleration, \( a = -\frac{5}{300-110} \)

\[ a = -0.0263 \text{ m/s}^2 \text{ (3 s.f.)} \]

4 (a) 

(b) Initial acceleration = 2 m/s\(^2\), so final retardation = 6 m/s\(^2\)

(c) Mean speed of the hawk = 

\[ \text{area under the speed – time graph} \]

\[ \text{time of travel} \]

Area under graph = \( 0.5 \times 6 \times 12 + 0.5 \times (12 + 18) \times 3 + 0.5 \times (10 + 13) \times 18 \)

\[ = 288 \text{ m} \]
Mean speed of the hawk = \( \frac{288}{22} = 13.1 \text{ m/s} \)

5 (a) Be careful to work in consistent units.
1 km = 1000 m
1 hour = 3600 s

(i) \( \frac{400}{62.5} = 6.4 \text{ m/s} \)
(ii) \( 0.4 \times 60 \times 60 \div 62.5 = 23.04 \approx 23.0 \text{ km/h} \)

(b) Area under speed-time graph = 400 m
400 = 0.5 \times (50.5 + 62.5) \times S_{\text{max}},
so \( S_{\text{max}} = 7.08 \text{ m/s} \)

(c) Initial acceleration = gradient of first phase = \( \frac{7.0796…}{12} = 0.58997… = 0.590 \text{ m/s}^2 \)

SHAPE AND SPACE 3 – BASIC SKILLS EXERCISE

1. \( x = 8.39 \text{ m}, y = 3.53 \text{ m} \)
2. \( x = 12.1 \text{ cm}, y = 5.12 \text{ cm} \)
3. \( x = 6.53 \text{ cm}, y = 1.55 \text{ cm} \)
4. \( x = 34.6 \text{ cm}, y = 29.1 \text{ cm} \)
5. \( \theta = 43.6° \)
6. \( \theta = 5.20° \)
7. \( \theta = 59.6° \)
8. \( \theta = 16.1° \)
9. \( 71.6° \)
10. 144 cm
11. \( x = 7.71, y = 7.96 \)
12. \( x = 14.3, y = 24.9 \)
13. \( x = 17.3, y = 34.6 \)
14. \( x = 17.3, y = 6 \)
15. \( \theta = 16.8° \)
16. \( \theta = 59.5° \)
17. \( \theta = 50.2° \)
18. \( \theta = 11.0° \)
19. 3.95 m
20. (a) 4.5 km
(b) 2.25 km
(c) 3.90 km

SHAPE AND SPACE 3 – EXAM PRACTICE EXERCISE

1 (a)

Area of triangle = \( \frac{1}{2} \times 10 \times h \)
= \( \frac{1}{2} \times 10 \times (10 \times \sin 60°) = 25\sqrt{3} \)

Area of circle = \( \pi r^2 = 25\sqrt{3}, r^2 = \frac{25\sqrt{3}}{\pi} \)
so \( r = \frac{5\sqrt{3}}{\pi} \)

Circumference of circle = \( 2\pi r = \frac{10}{\pi} \times \frac{5\sqrt{3}}{\pi} = 10\sqrt{3} \times 10 \times \frac{1}{\pi^2} \times 3^2 = 10 \times \pi^a \times 3^b \)
\( a = \frac{1}{2}, b = \frac{1}{4} \)

Triangle \( OAB \):
\( \sin(35°) = \frac{OB}{7}, \) so \( OB = 7 \times \sin(35°) = 4.0150…\text{cm} \)
\( \sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}} \)

Area of circle = \( \pi \times 4.0150^2 = 50.6440…\text{cm}^2 \)
area of circle = \( \pi r^2 \)
Area of triangle $BCD = \frac{1}{2} \times BC \times DC$

(Angle $BCD = 90^\circ$ as angles in a semi-circle are right-angles)

Triangle $BCD$:

$\sin(70^\circ) = \frac{CD}{2 \times 4.01503}$,

so $CD = 2 \times 4.01503 \times \sin(70^\circ) = 7.5458\ldots\text{cm}$

$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$

$\cos(70^\circ) = \frac{BC}{2 \times 4.01503}$,

so $BC = 2 \times 4.01503 \times \cos(70^\circ) = 2.7464\ldots\text{cm}$

$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$

Area of triangle $BCD$:

$= \frac{1}{2} \times 2.7464 \times 7.5458 = 10.362\ldots\text{cm}^2$

or

Area $= \frac{1}{2} \times BD \times BC \times \sin70^\circ$

Area in circle and outside of triangle $BCD$:

$= 50.644 - 10.362 = 40.282\text{ cm}^2$

% of whole circle not occupied by triangle $BCD$:

$= \frac{40.282}{50.644} \times 100 = 79.540\ldots\%$

So value of $p = 79.5$ (3 s.f.)

(b) Let area of the pentagon be $A$.

$A = \text{area of triangle } OAB + \text{area of triangle } OBC + \text{area of triangle } OCD$

Area of triangle $OAB$

$= \frac{1}{2} \times n \times 3\sqrt{n}$

$= \frac{3\sqrt{3}}{2} n^2$

Area of triangle $OBC = \frac{1}{2} \times BC \times OC$

$= \frac{1}{2} \times (\sqrt{3} n \times \sin(30^\circ)) \times \frac{3}{2} \sqrt{n}$

$= \frac{3\sqrt{3}}{8} n^2$

Area of triangle $OCD = \frac{1}{2} \times CD \times OD$

$= \frac{1}{2} \times \frac{3}{4} n \times \left( \frac{3}{2} n \times \cos(30^\circ) \right)$

$= \frac{9\sqrt{3}}{32} n^2$

So $A = \frac{3\sqrt{3}}{2} n^2 + \frac{3\sqrt{3}}{8} n^2 + \frac{9\sqrt{3}}{32} n^2$

$= \frac{37\sqrt{3}}{32} n^2$

If $A = \frac{37\sqrt{3}}{2} = \frac{37\sqrt{3}}{32} n^2$, $n^2 = 16$, $n = 4$

Pentagon $PQRSTU$ is regular so each triangle is equilateral.

Area of triangle $OST = \frac{1}{2} \times a \times (a \times \sin 60^\circ)$

$= \frac{3}{4} a^2$

Area of pentagon $= 6 \times \frac{3}{4} a^2 = \frac{3\sqrt{3}}{2} a^2$ as required.
Pythagoras’ theorem: Let height of equilateral triangle = \( h \)

\[ 2^2 = h^2 + 1^2, \quad h = \sqrt{3} \]

If area of equilateral triangle = area of regular pentagon

\[ \frac{1}{2} \times 2 \times \sqrt{3} = \frac{3\sqrt{3}}{2} \quad a^2, \quad 1 = \frac{3}{2} \quad a^2, \quad a^2 = \frac{2}{3}, \]

\[ a = \sqrt[3]{\frac{2}{3}} \]

\[ P = 6a = 6 \times \sqrt[3]{\frac{2}{3}} = 3 \times 2 \times \sqrt[3]{\frac{2}{3}} = \sqrt[3]{2} \times 2 \times \frac{1}{2} \]

\[ = \sqrt[3]{2} \times \frac{1}{2} = \sqrt[3]{2}, \quad \text{so} \quad k = \frac{3}{2} \]

**HANDLING DATA 2 – BASIC SKILLS EXERCISE**

1. (a)

<table>
<thead>
<tr>
<th>Time</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 ( \leq t &lt; 45 )</td>
<td>12</td>
</tr>
<tr>
<td>45 ( \leq t &lt; 50 )</td>
<td>5</td>
</tr>
<tr>
<td>50 ( \leq t &lt; 55 )</td>
<td>8</td>
</tr>
<tr>
<td>55 ( \leq t &lt; 60 )</td>
<td>4</td>
</tr>
<tr>
<td>60 ( \leq t &lt; 65 )</td>
<td>2</td>
</tr>
<tr>
<td>65 ( \leq t &lt; 70 )</td>
<td>10</td>
</tr>
<tr>
<td>70 ( \leq t &lt; 75 )</td>
<td>4</td>
</tr>
<tr>
<td>75 ( \leq t &lt; 80 )</td>
<td>5</td>
</tr>
</tbody>
</table>

Time to solve puzzle

- Frequency
- Time in seconds*

(b) 58%

2. (a) 85

(b)

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>50–60</td>
</tr>
<tr>
<td>20</td>
<td>60–70</td>
</tr>
<tr>
<td>30</td>
<td>70–80</td>
</tr>
<tr>
<td>40</td>
<td>80–90</td>
</tr>
</tbody>
</table>

* Upper bounds are not included in groups

(c) 23.5%

3. (a)

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Weight (g)*</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>490–495</td>
</tr>
<tr>
<td>10</td>
<td>495–500</td>
</tr>
<tr>
<td>15</td>
<td>500–505</td>
</tr>
<tr>
<td>20</td>
<td>505–510</td>
</tr>
<tr>
<td>25</td>
<td>510–515</td>
</tr>
<tr>
<td>30</td>
<td>515–520</td>
</tr>
<tr>
<td>35</td>
<td>520–525</td>
</tr>
</tbody>
</table>

* Upper bounds are not included in groups

(b) Evidence suggests the mean is between 505 and 510 so 500 is probably minimum weight.

4. (a) 126

(b)

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Speed s (km/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>20–30</td>
</tr>
<tr>
<td>10</td>
<td>30–40</td>
</tr>
<tr>
<td>15</td>
<td>40–50</td>
</tr>
<tr>
<td>20</td>
<td>50–60</td>
</tr>
<tr>
<td>25</td>
<td>60–70</td>
</tr>
</tbody>
</table>

* Upper bounds are not included in groups

(c) Speed limit is probably 50 km/h as there is a sharp cut off at that speed.

5. (a) 15

(b)

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Time (h)*</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0–2</td>
</tr>
<tr>
<td>10</td>
<td>2–4</td>
</tr>
<tr>
<td>15</td>
<td>4–6</td>
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<td>20</td>
<td>6–8</td>
</tr>
<tr>
<td>25</td>
<td>8–10</td>
</tr>
<tr>
<td>30</td>
<td>10–12</td>
</tr>
<tr>
<td>35</td>
<td>12–14</td>
</tr>
</tbody>
</table>

* Upper bounds are not included in groups

(c) 38.3 – assuming that the 4 is included and not the 10
6. (a) 50 lessons
   (b) 16–20
   (c) 15.3

7. (a) 24
   (b) $4 < m \leq 6$
   (c) 4.75 kg

8. (a) 24
   (b) $4 < m \leq 6$
   (c) 15.3

9. (a) 7
   (b) $14 \leq w < 15$
   (c) 14.3
   (d) Decrease as 13 is below the mean

10. (a) mean = 1.69, median = 1.72
    (b) new mean = 1.76

11. (a) 14.2 m
    (b) 1.40 m

12. (a) mean = 7.45, median = 7.45
    (b) new mean = 8.2

13. 1.74 m

14. (a) 83 cm
    (b) 9.37 cm

15. 21 m 56 s

16. (a) 126 s
    (b) 129 s

HANDLING DATA 2 – EXAM PRACTICE EXERCISE

1. Treat the number of calls as continuous data as we only have the data in class intervals.
   (a) February is the only month with 28 days.
   (b) 16–20 calls

2. Treat the length of words as continuous data as we only have the data in class-intervals
   (a) 1000 students
   (b) 601–800 words

(c) Number of Words | Frequency ($f$) | Midpoint ($x$) | $f \times x$  
----------------------|---------------|---------------|----------
 401–600              | 150           | 500.5         | 75075    
 601–800              | 425           | 700.5         | 297712.5 
 801–1000             | 350           | 900.5         | 315175   
 1001–1200            | 75            | 1100.5        | 82537.7  

Estimated mean = $\frac{770500.5}{1000} = 770.5$ words

(Estimated mean = $\frac{\sum f \times x}{\sum f}$, where $x$ is the mid-point of each class interval.)

3. (a) 540 Munros
    (b) 3000–3300 ft

(c) Height (h feet) | Frequency ($f$) | Midpoint ($x$) | $f \times x$  
---------------------|---------------|---------------|----------
 3000 < h ≤ 3300     | 300           | 3150          | 945000   
 3300 < h ≤ 3600     | 135           | 3450          | 465750   
 3600 < h ≤ 3900     | 80            | 3750          | 300000   
 3900 < h ≤ 4200     | 20            | 4050          | 81000    
 4200 < h ≤ 4500     | 5             | 4350          | 21750    

Estimated mean = $\frac{1813500}{540} = 3358.3$...

= 3360 ft (nearest 10 ft)

(Estimated mean = $\frac{\sum f \times x}{\sum f}$, where $x$ is the mid-point of each class interval.)
4 (a) \( \sum f = 50 = x + 23 + p + 5 = 28 + x + p \),
so \( p = 22 - x \)

(b) 

<table>
<thead>
<tr>
<th>Speed (s mph)</th>
<th>Frequency (f)</th>
<th>Midpoint (x)</th>
<th>( f \times x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>90 ≤ s &lt; 100</td>
<td>x</td>
<td>95</td>
<td>( x \times 95 = 95x )</td>
</tr>
<tr>
<td>100 ≤ s &lt; 110</td>
<td>23</td>
<td>105</td>
<td>( 23 \times 105 = 2415 )</td>
</tr>
<tr>
<td>110 ≤ s &lt; 120</td>
<td>( 22 - x )</td>
<td>115</td>
<td>( (22 - x) \times 115 )</td>
</tr>
<tr>
<td>120 ≤ s &lt; 130</td>
<td>5</td>
<td>125</td>
<td>( 5 \times 125 = 625 )</td>
</tr>
</tbody>
</table>

\( \Sigma f = 50 \)

Estimated mean = \( \frac{5570 - 20x}{50} \)

107.8 × 50 = 5570 – 20x, so \( x = 9 \)

(Estimated mean = \( \frac{\Sigma fx}{\Sigma f} \), where \( x \) is the midpoint of each class interval.)

5 (a) 

(b)

<table>
<thead>
<tr>
<th>Delay (d mins)</th>
<th>Midpoint (x)</th>
<th>Frequency (f)</th>
<th>( f \times x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 ≤ d &lt; 30</td>
<td>15</td>
<td>10</td>
<td>( 10 \times 15 = 150 )</td>
</tr>
<tr>
<td>30 ≤ h &lt; 60</td>
<td>45</td>
<td>14</td>
<td>( 14 \times 45 = 630 )</td>
</tr>
<tr>
<td>60 ≤ h &lt; 90</td>
<td>75</td>
<td>16</td>
<td>( 16 \times 75 = 1200 )</td>
</tr>
<tr>
<td>90 ≤ h &lt; 120</td>
<td>105</td>
<td>11</td>
<td>( 11 \times 105 = 1155 )</td>
</tr>
<tr>
<td>120 ≤ h &lt; 150</td>
<td>135</td>
<td>8</td>
<td>( 8 \times 135 = 1080 )</td>
</tr>
<tr>
<td>150 ≤ h &lt; 180</td>
<td>165</td>
<td>1</td>
<td>( 1 \times 165 = 165 )</td>
</tr>
</tbody>
</table>

\( \Sigma f = 60 \)

\( \frac{\Sigma fx}{\Sigma f} = 4380 \)

Estimated mean = \( \frac{4380}{60} = 73 \) mins

(Estimated mean = \( \frac{\Sigma fx}{\Sigma f} \), where \( x \) is the midpoint of each class interval.)

(c) Estimate because midpoints used as there are no exact values.

(d) Median class interval is 60 ≤ d < 90
as this is where the 30th value must be placed.
18 \[ \frac{24}{120} \times 100 = \$20 \]

19 \[ \frac{2125}{85} \times 100 = €2500 \]

20 80 s

21 €600

22 12

23 $4329

24 €1811.59

25 £409.36

26 €2573.53

27 $1283.76

28 $888 889

29 €6863.56

30 £2326.24

31 Let \( Q \) be the factor of depreciation in the first year, then \( Q - 0.1 \) is the factor for the second year.

\[ 60000(Q)(Q + 0.1) = 30000 \]
\[ 60000Q(Q - 0.1) = 30000 \]
\[ Q^2 + 0.1Q - 0.5 = 0 \]
\[ Q^2 - 0.1Q - 0.5 = 0 \]
\[ Q = 0.75987... \]
\[ R = (1 - 0.759... \times 100 = 24.1\% \text{ (3 s.f.)} \]

32 Final radius is \( \sqrt{\frac{730}{\pi}} = 15.24356... \)

Original area was \( \frac{730}{1.15} = 634.7826... \)

therefore original radius is \( \sqrt{\frac{634.78...}{\pi}} = 14.2146959... \)

Fractional increase in radius is \( \frac{15.24356... - 14.21469...}{14.21469...} = 0.7238... \) therefore percentage increase is 7.24%

33 (a) \[ \frac{120000}{0.9875^2} = 219482 \text{ so the lost area is} \]

\[ 219482 - 120000 = 99500 \text{ hectares} \]

(b) \[ 120000 \times 0.9875^2 = 65608 \text{ so the lost area is} \]

\[ 120000 - 65608 = 54400 \text{ hectares} \]

34 (a) \[ \frac{412}{1.005^{30}} = 373 \text{ ppm} \]

(b) \[ 412 \times 1.005^{30} = 455 \text{ ppm} \]

NUMBER 4 – EXAM PRACTICE EXERCISE

1 Total amount in account after

5 years = $50000 \times 1.035^5 = $59384.32

Interest after 35% deduction of interest = ($59384.32 – $50000) \times 0.65 = $6099.81

Percentage gained from original investment = \( \frac{6099.81}{50000} \times 100 = 12.2\% \text{ (3 s.f.)} \)

2 Total amount in account after first

3 years = €12000 \times 1.0325^3 = €13208.44

Total amount in account after final 7 years = €13208.44 \times 1.0225^7 = €15434.57

Total interest gained after 10 years = €15434.57 – €12000 = €3434.57

Percentage gained from original investment = \( \frac{3434.57}{12000} \times 100 = 28.6\% \text{ (3 s.f.)} \)

3 Let \( Q \) be the factor of depreciation each year. After three years’ depreciation

£50000 \times Q^3 = £25000

So, \( Q^3 = 0.5, Q = \sqrt[3]{0.5} = 0.7937... \)

Therefore, \( \% \) depreciation each year = \( (1 - 0.7937) \times 100 = 20.6\% \text{ (3 s.f.)} \)

4 (a) Let \( H \) be the height of the tree.

\[ H \times (1.075)^3 = 12, \text{ so} \]

\[ H = \frac{12}{1.075^3} = 9.66 \text{ m (3 s.f.)} \]

(b) \[ x \times 1.075 \times 1.05 \times 1.025 = 1.8 \]

\[ x = 1.56 \text{ metres (3 s.f.)} \]

5 (a) Let required price be £\( P \)

\[ P = £46800 \times (1 - 0.152) = £39686.40 \]

\[ P = £39686 \text{ (nearest £)} \]

(b) Let required price be €\( Q \)

18% = €3848, so 1% = €213.78, so 100% = €21377.78

\[ Q = €21378 \text{ (nearest €)} \]

ALGEBRA 4 – BASIC SKILLS EXERCISE

Note that answers can be correct but look different to the answer given. For example, the two answers given for Q1 and Q6 are just different rearrangements of the same expression.

1 \[ \frac{b - c^2}{a} \text{ or} \]

\[ \frac{ab - c^2}{a} \]

2 \[ \frac{cd - b}{a} \]

3 \[ \frac{a + c}{bd} \]
ANSWERS 287

4 \sqrt{a(b + c)}

5 \frac{a}{c - b}

6 \left(\frac{c}{a} + b\right)^2 \text{ or } \left(\frac{ab + c}{a}\right)^2

7 \left(\frac{c}{a}\right)^2 + b

8 \frac{c - f}{d + e}

9 \frac{a - bc}{c - 1} \text{ or } \frac{bc - a}{1 - c}

10 \frac{ab + cd}{a + c}

11 t(p^2 - s)

12 \frac{ex}{r + s}

13 h = \frac{3W}{\pi r^2}

14 r = \frac{3V}{4\pi}

15 s = \frac{v^2 - u^2}{2a}

16 \frac{\pi A}{2mr} - r

17 a = \frac{\pi}{n} \cdot \frac{d(n - 1)}{2}

18 a = \frac{2(s - ut)}{t}

19 a = \frac{8(1 - r)}{1 - r}

20 x = a\sqrt{1 - \frac{y^2}{b^2}} \text{ or } \frac{a}{b} \sqrt{b^2 - y^2}

21 t = g\left(\frac{v}{2}\right)^2

22 r = \sqrt{GmM_F}

23 d = \left(\frac{F}{k}\right)^3

24 r = \frac{6a}{5m - 1}

25 b = \frac{2A}{\sin C}, b = 4

26 a = \frac{2A - bh}{h}, a = 8

27 \sin A = \frac{a \sin B}{b}, A = 48.6^\circ \ (3 \text{ s.f.)}

28 \cos A = \frac{b^2 + c^2 - a^2}{2bc} \text{ or } \frac{a^2 - b^2 - c^2}{-2bc}, \cos A = 0.7

29 v = \frac{fu}{u - f}, v = 6 \frac{2}{3}

30 c = \frac{b^2 - (2ax + b)^2}{4a}, c = -3

ALGEBRA 4 – EXAM PRACTICE EXERCISE

1 (a) Perimeter of room is 18 m so the wall area is 18 \times 2.6 = 46.8 \text{ m}^2.

N = 2 + 0.4 \times 46.8 = 20.72 so she needs to buy 21 rolls.

(b) Making A the subject of the formula:

\[ A = \frac{N - 2}{0.4}. \]

Substituting N = 15 gives A = 32.5 m\^2.

If A is just less than 32.5 m\^2, say 32 m\^2, Juan will still need 15 rolls as A \leq 32.5 m\^2.

Substituting N = 14 gives A = 30 m\^2 and if A is greater than 30 then 15 rolls are needed.

\[ 30 < A \leq 32.5 \]

2 (a) The diagram shows \( n \) cubes joined together.

There are two end faces each with an area of 1 cm\^2 so the end face area = 2 cm\^2.

Each cube has 4 side faces exposed, each of area 1 cm\^2 so each cube’s side face area = 4 cm\^2.

\( n \) cubes have a total side face area of 4\( n \) cm\^2.

\[ A = 4n + 2 \]

(b) Making \( n \) the subject of the formula gives \( n = \frac{A - 2}{4} \).

Substituting A = 214 gives \( n = 53 \).

(c) \( n \) is an integer. In the formula \( n = \frac{A - 2}{4} \), if A is odd then A - 2 is also odd and an odd number divided by 4 (an even number) can never be an integer.
3 (a) Substitute \( C = \frac{100N}{33}\) into \( F = \frac{2C + 160}{5}\) gives
\[9C = 9 \times \frac{100N}{33} = \frac{300N}{11}\]
\[F = \frac{\frac{300N}{11} + 160}{5}\]
\[= \frac{60N}{11} + 32\]
\[= \frac{60N + 11 \times 32}{11}\]
\[= \frac{60N + 352}{11}\]

3 (b) Let the temperature where they read the same be \( T \)
then \( T = \frac{60T + 352}{11} \)
Substituting \( T \) for \( F \) and \( N \)
in \( F = \frac{60N + 352}{11} \) gives
\[11T = 60T + 352\]
\[49T = -352\]
\[T = -7.2 \text{ (1 d.p.)}\]

4 (a) \( t = 2\pi \sqrt{\frac{l}{9.8}} \)
\[\frac{t}{2\pi} = \sqrt{\frac{l}{9.8}}\]
\[(\frac{t}{2\pi})^2 = \frac{l}{9.8}\]
\[l = 9.8 \left(\frac{t}{2\pi}\right)^2\]
\[(l = \frac{9.8t^2}{4\pi^2} \text{ is also correct})\]

(b) Method 1:
Actual length = \(1.05 \times 9.8 \left(\frac{1}{2\pi}\right)^2\)
\[= 0.26065 \text{ m (to 5 s.f.)}\]
\[t = 2\pi \sqrt{\frac{0.26065}{9.8}} = 1.0247 \text{ s (5 s.f.)}\]
Increase is 0.0247 s \(\Rightarrow\) % increase
\[= \frac{0.0247}{1} \times 100 = 2.47 \approx 2.5\% \text{ (2.s.f.)}\]

Method 2: Let \( l \) be the length of the second pendulum; then the length of the manufactured pendulum is \(1.05l\). Let \( t_1 \) and \( t_2 \) be the times of swings respectively (note \( t_1 = 1 \))
\[\frac{t_2}{t_1} = \left(2\pi \sqrt{\frac{1.05l}{9.8}}\right) \div \left(2\pi \sqrt{\frac{l}{9.8}}\right)\]
\[= \sqrt{1.05}\]
\[= 1.0247\]
So the % increase is 2.5\% (2 s.f.)

1.025 is multiplier for 2.5\% increase

5 (a) The heights of each trapezium are calculated by substituting \( x = 0, 1 \) and \( 2 \) respectively into \( y = ax^2 + bx + c \)

When \( x = 0 \), \( y = c \)
When \( x = 1 \), \( y = a + b + c \)
When \( x = 2 \), \( y = 4a + 2b + c \)

The area of a trapezium = \(\frac{1}{2}(a + b)h\)

In this case \( h = 1 \)
Area of first trapezium
\[= \frac{1}{2}(c + a + b + c)\]
\[= \frac{1}{2}(a + b + 2c)\]
Area of second trapezium
\[= \frac{1}{2}(a + b + c + 4a + 2b + c)\]
\[= \frac{1}{2}(5a + 3b + 2c)\]
Total area
\[= \frac{1}{2}(a + b + 2c + 5a + 3b + 2c)\]
\[= \frac{1}{2}(6a + 4b + 4c) = 3a + 2b + 2c\]
(b) Difference in areas
\[= (3a + 2b + 2c) - \left( \frac{8a}{3} + 2b + 2c \right) = \frac{a}{3} \]
Percentage error\[= \frac{\frac{a}{3}}{\frac{8a}{3} + 6b + 6c} \times 100 \]
\[= \frac{a}{8a + 6b + 6c} \times 100 = \frac{100a}{8a + 6b + 6c} \% \]

### GRAPHS 4 – BASIC SKILLS EXERCISE

#### 1

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
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</thead>
<tbody>
<tr>
<td>( y )</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>9</td>
<td>15</td>
</tr>
</tbody>
</table>

#### 2

<table>
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<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-3</td>
<td>-5</td>
<td>-5</td>
<td>-3</td>
<td>1</td>
<td>7</td>
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</tbody>
</table>

#### 3

<table>
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<th>( x )</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>10</td>
<td>7</td>
<td>6</td>
<td>7</td>
<td>10</td>
<td>15</td>
<td>22</td>
<td>31</td>
</tr>
</tbody>
</table>

#### 4

<table>
<thead>
<tr>
<th>( x )</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-4</td>
<td>-6</td>
<td>-6</td>
<td>-4</td>
<td>0</td>
<td>6</td>
<td>14</td>
<td>24</td>
</tr>
</tbody>
</table>

#### 5

<table>
<thead>
<tr>
<th>( x )</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>10</td>
<td>4</td>
<td>0</td>
<td>-2</td>
<td>-2</td>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

### GRAPHS 4 – EXAM PRACTICE EXERCISE

#### 1

<table>
<thead>
<tr>
<th>( t )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>0</td>
<td>5</td>
<td>20</td>
<td>45</td>
<td>80</td>
<td>125</td>
</tr>
</tbody>
</table>

(b) (i) 61 m
(ii) 2.2 s

#### 2

Substitute \( x = 0 \) into formula for \( y \) to produce \( y = 5 \) and hence solve for \( p = 5 \).
(a) \( p = 5 \)
(b) \( x \)
<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>5</td>
<td>0</td>
<td>-1</td>
<td>2</td>
<td>9</td>
</tr>
</tbody>
</table>

(c) \( x = 1 \) or 2.5

#### 3

Substitute \( x = 0 \) and \( y = -3 \), \( x = 4 \) and \( y = 13 \) to produce simultaneous equations:
\[-3 = q \]
\[13 = 32 + 4p + q \]
Solve them to find \( p \) and \( q \).
(a) \( p = -4, q = -3 \)
(b) \( x \)
<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>13</td>
<td>3</td>
<td>-3</td>
<td>-5</td>
</tr>
</tbody>
</table>

(c) \( x \approx -0.6 \) or 2.6
4  Substitute $t = 1$ into formula for $p$ to produce $p = 7$ to solve for $k = 3$.  
(a) $k = 3$  
(b) $t$ $0$ $1$ $2$ $3$ $4$  
   $p$ $0$ $7$ $8$ $7$ $-8$  
(c) (i) £8333 @ $t = 1.7$ months  
(ii) $t > 3.3$ months

SHAPE AND SPACE 4 – BASIC SKILLS EXERCISE

1  
(a) 22.4 cm  
(b) 26.4 cm  
(c) 32.1°

2  
(a) 70.7 m  
(b) 60.4 m  
(c) 41.8°  
(d) 9038 m $^2$

3  
(a) 16.7°  
(b) $DF = 94.3$ m  
(c) 9.03°  
(d) 2.19 m/s

4  79.2 m

5  Show by clear working that area = 600 cm $^2$, by Pythagoras’ theorem.

6  
The diagram shows a tetrahedron.  
$AD$ is perpendicular to both $AB$ and $AC$.  
$AB = 10$ cm.  $AC = 8$ cm.  $AD = 5$ cm.  
Angle $BAC = 90^\circ$  
Let angle $BDC = \theta$  
Area of triangle $BDC = \frac{1}{2} \times BD \times CD \times \sin (\theta)$  

(Area of a triangle = $\frac{1}{2} absinC$)  
Triangle $ACD$:  
$CD^2 = 5^2 + 8^2 = 89$, $CD = 9.4340...$cm  
Triangle $ABD$:  
$BD^2 = 5^2 + 10^2 = 125$, $BD = 11.180...$cm  
Triangle $ABC$:  
$BC^2 = 8^2 + 10^2 = 164$, $BD = 12.806...$cm  
Triangle $BCD$: (Cosine Rule : $a^2 = b^2 + c^2 \ – 2bc \cos A$)  
$164 = 125 + 89 \ – 2 \times 11.180 \times 9.4340 \times \cos \theta$  
(Rearrange to make $\cos \theta$ the subject and find $\theta$)  
$\cos \theta = 0.23703...$, so $\theta = 76.289...^\circ$
Area of triangle $BDC$

$= \frac{1}{2} \times 11.180 \times 9.4340 \times \sin(76.289^\circ)$

$= 51.2 \text{ cm}^2 \text{ (3 s.f.)}$

Triangle $AXC$:

$\cos(\theta) = \frac{XC}{AC} = \frac{XC}{25} \left( \cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}} \right)$

$XC = 21.813\ldots \text{ m}$

$XM : MC = 3 : 1 = 16.360 : 5.453$

Triangle $TXM$:

Calculate angle $XMT = \alpha$

(Alternate angles)

$\tan(\alpha) = \frac{TX}{XM} = \frac{10}{16.360}$

$\alpha = 31.435\ldots^\circ = 31.4^\circ \text{ (3 s.f.)}$

But angle of depression is angle $XTM = 45^\circ - 31.4^\circ = 13.6^\circ$

This could have been calculated immediately using $\tan(\theta) = \frac{XM}{TX}$

3 (a)

Triangle $ABC$:

Let angle $ACX = \theta$

$\tan(\theta) = \frac{AB}{AC} = \frac{14}{25} \left( \tan \theta = \frac{\text{opposite side}}{\text{adjacent side}} \right)$

$\theta = 29.249\ldots^\circ$
\[ LA^2 = 14^2 + 12^2, \quad LA = 18.439\ldots \text{cm} \]

Triangle \( KLA \):

\[ KA^2 = LA^2 + KL^2, \quad KA = 20.518\ldots \text{cm} \]

Now consider triangle \( AKB \):

Let angle \( AKB = \theta \)

(Cosine rule: \( a^2 = b^2 + c^2 - 2bc \cos A \))

\[ 21^2 = 20.518^2 + 16.553^2 - 2 \times 20.518 \times 16.553 \times \cos \theta \]

(Re-arrange to make \( \cos \theta \) the subject and find \( \theta \))

\[ \cos \theta = 0.37392\ldots, \quad \text{so} \quad \theta = 68.043^\circ, \quad \text{so} \quad \text{angle } AKB = 68.0^\circ \quad (3 \text{ s.f.}) \]

(b) Let required angle \( KAL = \alpha \)

Triangle \( KAL \):

\[ \tan(\alpha) = \frac{KL}{AL} = \frac{9}{18.439} \]

\[ \tan \theta = \frac{\text{opposite side}}{\text{adjacent side}} \]

\[ \alpha = 26.017\ldots, \quad \text{so angle } KAL = 26.0^\circ \quad (3 \text{ s.f.}) \]

4

(4) Triangle \( OWT \):

\[ \tan(25^\circ) = \frac{TW}{OW} = \frac{2000}{OW} \]

\[ OW = 2000 \div \tan(25^\circ) = 4289.0\ldots \text{m} \]

= 4290 m (3 s.f.)

Triangle \( OWS \):

Angle \( WOS = 360 - 310 = 50^\circ \)

(Plan view on base triangle \( OWS \))

\[ \cos(50^\circ) = \frac{OS}{OW} = \frac{OS}{4289.0\ldots} \]

\[ OS = 4289.0 \times 4 \cos(50^\circ) = 2757.0\ldots \text{m} = 2760 \text{ m} \quad (3 \text{ s.f.}) \]

(b) Triangle \( ROS \):

Let required angle \( ROS = \theta \)

\[ \tan(\theta) = \frac{RS}{OS} = \frac{2000}{2757.0\ldots} \]

\[ \tan \theta = \frac{\text{opposite side}}{\text{adjacent side}} \]

\[ \theta = 35.958\ldots = 36.0^\circ \quad (3 \text{ s.f.}) \]

(c) Speed = \frac{\text{distance}}{\text{time}} = \frac{2757.0\ldots \text{m}}{1 \text{ min}} = 197.13\ldots \text{km/hr}

= 197 km/h (3 s.f.)

5

(a) The base of the pyramid is a regular pentagon, so angle \( DOC = \frac{360^\circ}{5} = 72^\circ \)

Triangle \( OCM \):

\[ \tan(36^\circ) = \frac{CM}{OM} = \frac{4}{OM} \]

\[ \tan \theta = \frac{\text{opposite side}}{\text{adjacent side}} \]
OM = \frac{4}{\tan(36°)} = 5.5055...\text{ cm}

Triangle \textit{POM}:

Required angle is \textit{PMO} = \alpha

\tan (\alpha) = \frac{PO}{OM} = \frac{10}{5.5055} = 1.8164...,

so angle \textit{PMO} = 61.2° (3 s.f.)

(b) Weight of pyramid + weight of water = 1000 g

Area of pyramid base = $5 \times $area of triangle \textit{ODC}$

\hspace{1cm} = 5 \times \frac{1}{2} \times 8 \times 5.5055

\hspace{1cm} = 110.11...\text{ cm}^2

(Volume of pyramid = $\frac{1}{3} \times $base area $\times$ perpendicular height)

Volume of pyramid = $\frac{1}{3} \times 110.11 \times 10$

\hspace{1cm} = 367.03...\text{ cm}^3

(Density = \text{Mass} \div \text{Volume})

1000 kg/m$^3$ = 1 g/cm$^3$

Weight of water = density $\times$ volume

\hspace{1cm} = 1 \times 367.03 \text{ g}.

So $w + 367.03 = 1000$

(working in units of g)

\hspace{1cm} w = 633 \text{ g} (3 \text{ s.f.})

**HANDLING DATA 3 – BASIC SKILLS EXERCISE**

1. **Median** | $Q_1$ | $Q_3$ | Range | IQR
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>1</td>
<td>7</td>
<td>8</td>
<td>6</td>
</tr>
</tbody>
</table>

2. **Median** | $Q_1$ | $Q_3$ | Range | IQR
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1</td>
<td>6</td>
<td>9</td>
<td>5</td>
</tr>
</tbody>
</table>

3. **Median** | $Q_1$ | $Q_3$ | Range | IQR
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1.5</td>
<td>7</td>
<td>11</td>
<td>5.5</td>
</tr>
</tbody>
</table>

4. **Median** | $Q_1$ | $Q_3$ | Range | IQR
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>62</td>
<td>48</td>
<td>83</td>
<td>69</td>
<td>35</td>
</tr>
</tbody>
</table>

5. **Median** | $Q_1$ | $Q_3$ | Range | IQR
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>5</td>
<td>8</td>
<td>9</td>
<td>3</td>
</tr>
</tbody>
</table>

6. **Median** | $Q_1$ | $Q_3$ | Range | IQR
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3.9</td>
<td>2.1</td>
<td>9.0</td>
<td>9</td>
<td>6.0</td>
</tr>
</tbody>
</table>

7. **Median** | $Q_1$ | $Q_3$ | Range | IQR
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>67</td>
<td>42</td>
<td>77</td>
<td>84</td>
<td>35</td>
</tr>
</tbody>
</table>

8. **Median** | $Q_1$ | $Q_3$ | Range | IQR
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.56</td>
<td>0.46</td>
<td>0.68</td>
<td>0.7</td>
<td>0.22</td>
</tr>
</tbody>
</table>

9. **Median** | $Q_1$ | $Q_3$ | Range | IQR
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>

10. **Median** | $Q_1$ | $Q_3$ | Range | IQR
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>2.5</td>
<td>5</td>
<td>1.5</td>
</tr>
</tbody>
</table>

11. (a) **Time (t mins)** | C.F.
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &lt; t ≤ 10</td>
<td>3</td>
</tr>
<tr>
<td>10 &lt; t ≤ 20</td>
<td>10</td>
</tr>
<tr>
<td>20 &lt; t ≤ 30</td>
<td>25</td>
</tr>
<tr>
<td>30 &lt; t ≤ 40</td>
<td>52</td>
</tr>
<tr>
<td>40 &lt; t ≤ 50</td>
<td>71</td>
</tr>
<tr>
<td>50 &lt; t ≤ 60</td>
<td>80</td>
</tr>
</tbody>
</table>

(b) **Cumulative Frequency**

(c) $Q_2 = 36$, $Q_1 = 27$, $Q_3 = 43$, IQR = 16

(d) 50 students

12. (a) **Speed s (m.p.h)** | C.F.
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$s \leq 55$</td>
<td>0</td>
</tr>
<tr>
<td>55 &lt; $s \leq 60$</td>
<td>6</td>
</tr>
<tr>
<td>60 &lt; $s \leq 65$</td>
<td>25</td>
</tr>
<tr>
<td>65 &lt; $s \leq 70$</td>
<td>71</td>
</tr>
<tr>
<td>70 &lt; $s \leq 75$</td>
<td>85</td>
</tr>
<tr>
<td>75 &lt; $s \leq 80$</td>
<td>90</td>
</tr>
</tbody>
</table>

(c) $Q_2 = 67$, $Q_1 = 64.5$, $Q_3 = 69.5$, IQR = 5

(d) 18%
13 (a)  

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>w ≤ 2.0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2.0 &lt; w ≤ 2.5</td>
<td>14</td>
<td>0</td>
</tr>
<tr>
<td>2.5 &lt; w ≤ 3.0</td>
<td>43</td>
<td>3</td>
</tr>
<tr>
<td>3.0 &lt; w ≤ 3.5</td>
<td>66</td>
<td>23</td>
</tr>
<tr>
<td>3.5 &lt; w ≤ 4.0</td>
<td>80</td>
<td>74</td>
</tr>
<tr>
<td>4.0 &lt; w ≤ 4.5</td>
<td>80</td>
<td>80</td>
</tr>
</tbody>
</table>

(c) Country A: $Q_2 = 2.95$, $Q_1 = 2.62$, $Q_3 = 3.37$, IQR = 0.75  
Country B: $Q_2 = 3.65$, $Q_1 = 3.45$, $Q_3 = 3.78$, IQR = 0.33  

(d) Babies heavier in country B, more variation in weight in country A

14 (a)  

<table>
<thead>
<tr>
<th>Time t (milliseconds)</th>
<th>Cum. freq. before drink</th>
<th>Cum. freq. after drink</th>
</tr>
</thead>
<tbody>
<tr>
<td>t ≤ 160</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>160 &lt; t ≤ 180</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>180 &lt; t ≤ 200</td>
<td>45</td>
<td>0</td>
</tr>
<tr>
<td>200 &lt; t ≤ 220</td>
<td>76</td>
<td>8</td>
</tr>
<tr>
<td>220 &lt; t ≤ 240</td>
<td>80</td>
<td>49</td>
</tr>
<tr>
<td>240 &lt; t ≤ 260</td>
<td>80</td>
<td>74</td>
</tr>
<tr>
<td>260 &lt; t ≤ 280</td>
<td>80</td>
<td>80</td>
</tr>
</tbody>
</table>

(c) Before drink $Q_2 = 197$, $Q_1 = 189$, $Q_3 = 206$, IQR = 17  
After drink $Q_2 = 237$, $Q_1 = 229$, $Q_3 = 245$, IQR = 16

(d) Drink lengthens reaction times by approximately 40 milliseconds, but doesn’t after the spread. This possibly means that everybody is equally affected.

HANDLING DATA 3 – EXAM PRACTICE EXERCISE

1 Median = 23.5, $Q_1 = 17.5$, $Q_3 = 31$, range = 28, IQR = 13.5

2 (a)  

<table>
<thead>
<tr>
<th>Weight w (g)</th>
<th>Cum. freq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>66 &lt; w ≤ 68</td>
<td>5</td>
</tr>
<tr>
<td>68 &lt; w ≤ 70</td>
<td>18</td>
</tr>
<tr>
<td>70 &lt; w ≤ 72</td>
<td>36</td>
</tr>
<tr>
<td>72 &lt; w ≤ 74</td>
<td>46</td>
</tr>
<tr>
<td>74 &lt; w ≤ 76</td>
<td>54</td>
</tr>
<tr>
<td>76 &lt; w ≤ 78</td>
<td>60</td>
</tr>
</tbody>
</table>

(e) $Q_3 = 71.4$, IQR = 4.1
(d) 15%

3 (a)  

<table>
<thead>
<tr>
<th>Time t (min)</th>
<th>Cum. freq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &lt; t ≤ 20</td>
<td>3</td>
</tr>
<tr>
<td>20 &lt; t ≤ 40</td>
<td>10</td>
</tr>
<tr>
<td>40 &lt; t ≤ 60</td>
<td>21</td>
</tr>
<tr>
<td>60 &lt; t ≤ 80</td>
<td>31</td>
</tr>
<tr>
<td>80 &lt; t ≤ 100</td>
<td>52</td>
</tr>
<tr>
<td>100 &lt; t ≤ 120</td>
<td>77</td>
</tr>
<tr>
<td>120 &lt; t ≤ 140</td>
<td>100</td>
</tr>
</tbody>
</table>

(c) $Q_3 = 98$, $Q_1 = 68$, $Q_3 = 119$, IQR = 51
(d) 35%

4 (a)  

<table>
<thead>
<tr>
<th>Diameter d (cm)</th>
<th>Frequency Short Wood</th>
<th>Frequency Waley Wood</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &lt; d ≤ 10</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>10 &lt; d ≤ 20</td>
<td>15</td>
<td>5</td>
</tr>
<tr>
<td>20 &lt; d ≤ 30</td>
<td>27</td>
<td>12</td>
</tr>
<tr>
<td>30 &lt; d ≤ 40</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>40 &lt; d ≤ 50</td>
<td>71</td>
<td>92</td>
</tr>
<tr>
<td>50 &lt; d ≤ 60</td>
<td>85</td>
<td>98</td>
</tr>
<tr>
<td>60 &lt; d ≤ 70</td>
<td>95</td>
<td>100</td>
</tr>
<tr>
<td>70 &lt; d ≤ 80</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

(c) Short Wood $Q_3 = 40$, $Q_1 = 29$, $Q_3 = 53$, IQR = 24  
Waley Wood $Q_3 = 40$, $Q_1 = 36$, $Q_3 = 46$, IQR = 10

(d) Waley Wood is much more uniform in size. This possibly means it is a plantation with all the trees having been planted at the same time. Short Wood is more diverse in size, possibly a wild wood.
NUMBER 5 – BASIC SKILLS EXERCISE

1.  \( 1 \times 10^8 \)
2.  \( 4 \times 10^{13} \)
3.  \( 5 \times 10^{15} \)
4.  \( 1 \times 10^{11} \)
5.  \( 2 \times 10^2 \)
6.  \( 2 \times 10^3 \)
7.  \( 5 \times 10^7 \)
8.  \( 5 \times 10^8 \)
9.  \( 3 \times 10^{14} \)
10.  \( 3 \times 10^{15} \)
11.  \( 1 \times 10^3 \)
12.  \( 2 \times 10^{10} \)
13.  \( 2 \times 10^6 \)
14.  \( 4 \times 10^{-3} \)
15.  \( 2 \times 10^{-12} \)
16.  \( 5 \times 10^7 \)
17.  \( 2 \times 10 \)
18.  \( 8 \times 10^{-3} \)
19.  upper bound = 3.52, lower bound = 2.93
20.  upper bound = 4.08, lower bound = 3.15
21.  upper bound = 2.33, lower bound = 1.73
22.  upper bound = 1.71, lower bound = 1.50
23.  upper bound = 2.29, lower bound = 2.15
24.  upper bound = 0.159, lower bound = 0.09832
25.  upper bound = 3.38, lower bound = 2.59
26.  (a) upper bound = 15.9 m², lower bound = 9.62 m²
    (b) upper bound = 14.1 m, lower bound = 11.0 m
27.  upper bound = 12.1, lower bound = 11.7
28.  upper bound = 104, lower bound = 78
29.  upper bound = 17 N/cm², lower bound = 15 N/cm²
30.  upper bound = 75 000 mm², lower bound = 64 000 mm²

NUMBER 5 – EXAM PRACTICE EXERCISE

1.  \( a_{\text{max}} = 2.5 \text{ m}, b_{\text{max}} = 100.5 \text{ m}, \alpha_{\text{max}} = 40.5^\circ \)
    \( a_{\text{min}} = 1.5 \text{ m}, b_{\text{min}} = 99.5 \text{ m}, \alpha_{\text{min}} = 39.5^\circ \)
    (a) upper bound of \( h \)
        \[ h = a_{\text{max}} + (b_{\text{max}} \times \tan(\alpha_{\text{max}})) \]
        \[ \tan \theta = \frac{\text{opposite side}}{\text{adjacent side}} \]
        \[ = 2.5 + 100.5 \times \tan(40.5^\circ) = 88.335... \text{ m} \]
        \[ = 88.3 \text{ m} \ (3 \text{ s.f.}) \]
    (b) lower bound of \( h \)
        \[ h = a_{\text{min}} + (b_{\text{min}} \times \tan(\alpha_{\text{max}})) \]
        \[ \tan \theta = \frac{\text{opposite side}}{\text{adjacent side}} \]
        \[ = 1.5 + 99.5 \times \tan(39.5^\circ) = 83.521... \text{ m} \]
        \[ = 83.5 \text{ m} \ (3 \text{ s.f.}) \]

2.  Let area of the base be \( A \text{ cm}^2 \) and force be \( F \text{ N} \).
    \( A_{\text{max}} = 12.5 \times 12.5 = 156.25 \text{ cm}^2, \ F_{\text{max}} = 55 \text{ N} \)
    \( A_{\text{min}} = 11.5 \times 11.5 = 132.25 \text{ cm}^2, \ F_{\text{min}} = 45 \text{ N} \)
    (a) upper bound of \( P \)
        \[ P = \frac{F_{\text{max}}}{A_{\text{min}}} = \frac{55}{132.25} \]
        \[ = 0.41587... \text{ N/cm}^2 \]
        \[ = 0.416 \text{ N/cm}^2 \]
    (b) lower bound of \( P \)
        \[ P = \frac{F_{\text{min}}}{A_{\text{max}}} = \frac{45}{156.25} \]
        \[ = 0.288 \text{ N/cm}^2 \]

3.  \( d_{\text{max}} = 2.55 \text{ m}, t_{\text{max}} = 2.75 \text{ s} \)
    \( d_{\text{min}} = 2.45 \text{ m}, t_{\text{min}} = 2.25 \text{ s} \)
    (speed = \( \frac{\text{distance}}{\text{time}} \), Circumference of a circle = \( 2\pi r \))
    (a) upper bound of \( V \)
        \[ V = \frac{d_{\text{max}}}{t_{\text{min}}} = \frac{2 \times \pi \times 0.5 \times 2.55}{2.25} \]
        \[ = 3.5604... \text{ m/s} \]
        \[ = 3.6 \text{ m/s} \ (2 \text{ s.f.}) \]
    (b) lower bound of \( V \)
        \[ V = \frac{d_{\text{min}}}{t_{\text{max}}} = \frac{2 \times \pi \times 0.5 \times 2.45}{2.75} \]
        \[ = 2.7988... \text{ m/s} \]
        \[ = 2.8 \text{ m/s} \ (2 \text{ s.f.}) \]

4.  \( R_{\text{max}} = 12.85 \text{ cm}, r_{\text{max}} = 10.35 \text{ cm} \)
    \( R_{\text{min}} = 12.75 \text{ cm}, r_{\text{min}} = 10.25 \text{ cm} \)
    (Area of a circle = \( \pi r^2 \))
    \( A_{\text{max}} = \pi (R_{\text{max}}^2 - r_{\text{min}}^2) = \pi (12.85^2 - 10.25^2) = 60.06\pi \text{ cm}^2 \)
    \( A_{\text{min}} = \pi (R_{\text{min}}^2 - r_{\text{max}}^2) = \pi (12.75^2 - 10.35^2) = 55.44\pi \text{ cm}^2 \)
So, if $p\pi \leq A < q\pi$

55.44 $\pi \leq A < 60.06\pi$

$p = 55, q = 60$ (2 s.f.)

5 $a_{\text{max}} = 12.55, b_{\text{max}} = 3.255, c_{\text{max}} = 1.755$ and

$d_{\text{max}} = 3.855$

$a_{\text{min}} = 12.45, b_{\text{min}} = 3.245, c_{\text{min}} = 1.745$ and

$d_{\text{min}} = 3.845$

(a) $Z_{\text{max}} = \frac{a_{\text{max}} - b_{\text{max}}}{c_{\text{max}} + d_{\text{max}}} = \frac{12.55 - 3.245}{1.755 + 3.845} = 1.6645$, upper bound of $T = 10z_{\text{max}}$

= 46 (2 s.f.)

(b) $Z_{\text{min}} = \frac{a_{\text{min}} - b_{\text{min}}}{c_{\text{min}} + d_{\text{min}}} = \frac{12.45 - 3.255}{1.745 + 3.855}$

= 1.6390 . . . , lower bound of $T = 10z_{\text{min}}$

= 44 (2 s.f.)

**ALGEBRA 5 – BASIC SKILLS EXERCISE**

1 $x^2 - 3x - 10$

2 $x^2 + 16x + 64$

3 $x^2 - 10x^2 + 25$

4 $8x^2 + 2x - 6$

5 $10x^4 + x^2 - 2x$

6 $x^3 - 4x^2 - 3x + 18$

7 $9x^3 - 18x^2 - 25x + 50$

8 $8x^3 - 36x^2 + 54x - 27$

9 $x^2 - 5$

10 $-9$

For questions 11–14, there is no need to multiply out the brackets.

11 $(x^2 + 3)$ is a common factor.

$(x^2 + 3)(2x + 1) + (x^2 + 3)(1 - 2x)$

= $(x^2 + 3)[2x + 1 + 1 - 2x] = 2(x^2 + 3)$

12 $(4x + 1)$ is a common factor.

$(4x + 1)^2 - (4x + 1)(x + 1)$

= $(4x + 1)[(4x + 1) - (x + 1)]$

= $(4x + 1)[4x + 1 - x - 1] = 3x(4x + 1)$

13 $(4x + 3)$ is a common factor.

$\pi(4x + 3) - 3(4x + 3) = (4x + 3)(\pi - 3)$

14 $(1 - \cos x)$ is a common factor.

$2x(1 - \cos x) - 3(1 - \cos x)$

= $(1 - \cos x)(2x - 3)$

15 $(x - 3)(x - 4) = 0, x = 3$ or $4$

16 $(x - 1)(x + 9) = 0, x = 1$ or $-9$

17 $x(x - 11) = 0, x = 0$ or $11$

18 $(x - 4)(x + 5) = 0, x = 4$ or $-5$

19 $(3x - 5)(x + 2) = 0, x = -\frac{5}{3}$ or $-2$

20 $(4x + 3)(x - 2) = 0, x = -\frac{3}{4}$ or $2$

21 $(2x - 3)(3x - 2) = 0, x = \frac{3}{2}$ or $-\frac{2}{3}$

22 $(5x - 8)(2x + 5) = 0, x = \frac{8}{5}$ or $-\frac{5}{2}$

23 $(x - 4)(x + 6) = 0, x = 4$ or $-6$

24 $2(x - 1)(x + 3) = 0, x = 1$ or $-3$

25 $2(2x + 1)(x - 5) = 0, x = \frac{1}{2}$ or $5$

26 $2x(2x + 3)(x + 2) = 0, x = 0, -\frac{3}{2}$ or $-2$

27 $(x^2 - 9)(x^2 - 4) = 0, x = \pm 3$ or $\pm 2$

28 $7(x + 2) = 0$ so $x = -2$

29 $(x - 4)(x - 1)$ so $x = 4$ or $x = 1$

30 $(3x + 1)(2x - 1) = 0$ so $x = \frac{1}{2}$ or $-\frac{1}{3}$

31 $\frac{1}{2} \times 2x \times x = 3x(x - 3)$

The areas are the same.

$x^2 = 3x^2 - 9x$

$2x^2 - 9x = 0$\n
$x(2x - 9) = 0$\n
$x = 0$ or $4.5$

$x = 0$ is not a real-life solution so $x = 4.5$

32 $x(x + 2) = 15$

$x^2 + 2x + 15 = 0$\n
$(x + 5)(x - 3)$\n
$x = 3$\n
$x = -5$ is not a real-life solution

33 The coconut hits the ground when $x = 14$ so

$5t^2 + 3t = 14$

$5t^2 + 3t - 14 = 0$

$(5t - 7)(t + 2) = 0$ so $t = \frac{7}{5}$

$t = -2$ is not a real-life solution
34 Let one integer be \( x \), then the other integer is \( x + 4 \) (\( x - 4 \) is also correct – see later)

\[
x^2 + (x + 4)^2 = 208
\]

\[
x^2 + x^2 + 8x + 16 = 208
\]

\[
2x^2 + 8x - 192 = 0
\]

\[
x^2 + 4x - 96 = 0
\]

\[
(x + 12)(x - 8) = 0
\]

\[
x = -12 \text{ or } 8 \Rightarrow \text{numbers are } -12 \text{ and } -8 \text{ or } 8 \text{ and } 12
\]

If using \( x - 4 \):

\[
x^2 + (x - 4)^2 = 208
\]

\[
x^2 + x^2 - 8x + 16 = 208
\]

\[
2x^2 - 8x - 192 = 0
\]

\[
x^2 - 4x - 96 = 0
\]

\[
(x - 12)(x + 8) = 0
\]

\[
x = 12 \text{ or } -8 \Rightarrow \text{numbers are } 12 \text{ and } 8 \text{ or } -8 \text{ and } -12
\]

35 \((4x - 3)^2(x + 1)^2 + (2x + 4)^2\)

\[
16x^2 - 24x + 9 + x^2 + 2x + 1 + 4x^2 + 16x + 16
\]

\[
11x^2 - 42x - 8 = 0
\]

\[
(11x + 2)(x - 4) = 0
\]

\[
x = 4 \text{ as } x = -\frac{2}{11} \text{ is not a real-life solution}
\]

triangle sides are 5, 12 and 13 so area is \( \frac{1}{2} \times 5 \times 12 = 30 \text{ cm}^2 \)

36 (a) \((a + b)(a - b)\)

(b) \(2^{24} - 1^2 = (2^{12} + 1)(2^{12} - 1)\)

so suitable integers are \(2^{12} + 1 = 4097\) and \(2^{12} - 1 = 4095\)

ALGEBRA 5 – EXAM PRACTICE EXERCISE

1 Internal angle sum of a quadrilateral is 360°

\[
8x^2 - 32 + 22x - 16 + 20x + 4 + 6x^2 + 12 = 360
\]

\[
14x^2 + 42x - 392 = 0
\]

\[
x^2 + 3x - 28 = 0 \Rightarrow \text{Dividing by 14}
\]

\[
(x - 4)(x + 7) = 0 \Rightarrow x = 4
\]

\[
x = -7 \text{ is not a real-life solution}
\]

angles are \( A = 96\°, \ B = 72\°, \ C = 84\° \) and \( D = 108\° \)

it is cyclic as \( A + C = 180\° \) or \( B + D = 180\° \)

2 (a) \( \frac{1}{2} (3x + 5 + x + 1) \times (x + 3) = 35 \)

See formula sheet for area of a trapezium.

\( (2x + 3)(x + 3) = 35 \)

\( 2x^2 + 9x - 26 = 0 \)

\( (x - 2)(2x + 13) = 0 \)

\( x = 2 \quad x = -\frac{13}{2} \) is not a real-life solution.

(b) \((x - 2)^2 + (2x + 6)^2 = (3x - 2)^2\)

\[\text{Pythagoras’ theorem.}\]

\[x^2 - 4x + 4 + 4x^2 + 24x + 36 = 9x^2 - 12x + 4 \]

\[4x^2 - 32x - 36 = 0 \]

\[x^2 - 8x - 9 = 0 \quad \Rightarrow \text{Dividing equation by 4} \]

\[x = 9 \quad x = -1 \text{ is not a real-life solution} \]

Sides are 7, 24 and 25 so perimeter is 56 cm

3 (a) \((x + 2)(x + a) = x^2 + px + 6\)

\[
x^2 + ax + 2x + 2a = x^2 + px + 6
\]

\[
a = 3
\]

factors are \((x + 2)\) and \((x + 3)\)

\[
(x + 2)(x + 3) = x^2 + 5x + 6
\]

\[
p = 5
\]

(b) (i) \((2x + 3)(x - 7)\)

\[
2\left(x + \frac{1}{2}\right)^2 - 11\left(x + \frac{1}{2}\right) = 21
\]

\[
2\left(x + \frac{1}{2}\right)^2 - 11\left(x + \frac{1}{2}\right) - 21 = 0
\]

Compare this with \(2x^2 - 11x - 21 = 0\)

so replace \( x \) by \( \left(x + \frac{1}{2}\right) \) in b.

This gives \(2\left(x + \frac{1}{2}\right) + 3\left(\left(x + \frac{1}{2}\right) - 7\right) = 0\)

\[
(2x + 4)\left(x - 6\frac{1}{2}\right) = 0
\]

\[
x = -2 \text{ or } x = 6\frac{1}{2}
\]

4 (a) \((x - 5)\left(x + \frac{2}{3}\right) = 0\)

\[
(x - 5)(3x + 2) = 0 \quad \text{Multiplying by 3}
\]

\[
3x^2 - 13x - 10 = 0 \quad \text{Any multiple of this equation is correct} \]
(b) To have one solution, when factorised the equation must be \((x - a)(x - a) = 0\)
\((x - a)^2 = 0\)
\(x^2 - 2ax + a^2 = 0\)
\(2a = 6\)
\(a = 3\)
\(p = 9\)

The equation is \((x - 3)^2 = 0\) so the solution is \(x = 3\)

5 (a) Let \(x\) be the number of jars she bought so each jar costs \(\frac{2000}{x}\) cents.

In the other shop, each jar costs \(\frac{2000}{x} - 20\) and she could have bought \(x + 5\) jars.

\(\left(x + 5\right)\left(\frac{2000}{x} - 20\right) = 2000\)
\(2000 - 20x + \frac{10000}{x} - 100 = 2000\)
\(\frac{10000}{x} - 20x - 100 = 0\)
\(10000 - 20x^2 - 100x = 0\)  Multiplying equation by \(x\)
\(20x^2 + 100x - 10000 = 0\)  Multiplying by \(-1\) and re-arranging
\(x^2 + 5x - 500 = 0\)  Dividing by 20

(b) \(x^2 + 5x - 500 = 0\)
\((x + 25)(x - 20) = 0\)
\(x = 20\)  \(x = -25\) is not a real-life solution

Priti bought 20 jars.

(c) Each jar costs \(\frac{2000}{x}\) = 100 cents or $1

---

**GRAPHS 5 – BASIC SKILLS EXERCISE**

1. \(y > -3\)
2. \(x < 2\) or \(x \geq 4\)
3. \(x + y \geq 5\)
4. \(y < 2x + 2\)
5. \(x \geq 0, y \geq 0\) and \(x + 2y < 6\)
6. \(x + y \geq -4, y - 3x > -4\) and \(3y - x \leq 4\)
A sketch will help you to answer Q11–Q25

11 (a) $\frac{3}{4}, \frac{4}{3}$
   (b) $\frac{6}{7}, -\frac{7}{6}$

12 a and d, b and c

13 Sketch shows right angle is at $A$.

   Gradient of $AB = \frac{1}{3}$, gradient of $AC = -3$,
   $\frac{1}{3} \times -3 = -1$ hence $AB$ is perpendicular to $AC$

14 Gradient of $L$ is $-\frac{3}{7}$, gradient of $M$ is

   \[
   \frac{12 - (-9)}{5 - (-4)} = \frac{21}{9} = \frac{7}{3}
   \]

   $\frac{3}{7} \times \frac{7}{3} = -1$ so $L$ is perpendicular to $M$

15 $9y + 5x = 18$

16 $-5$

17 (a) $(-2, -1\frac{1}{2})$

   (b) $(11\frac{1}{2}, -13)$

18 $(-2, 3)$

19 Sketch shows diagonals are $AC$ and $BD$.

   Midpoint of $AC$ is $\left(\frac{7 - 4}{2}, \frac{1 + 1}{2}\right) = (1\frac{1}{2}, 1)$,
   Midpoint of $BD$ is $\left(\frac{4 - 1}{2}, \frac{3 - 1}{2}\right) = (1\frac{1}{2}, 1)$

   As midpoint is the same, diagonals bisect each other.

20 Gradient of $AB$ is $\frac{1}{3}$ so a perpendicular gradient is $-3$

   The midpoint of $AB$ is $(1,0)$
   Equation is $y = -3x + 3$

21 $A$ lies on $2y = x + 2$ so the median passes through $A$ and midpoint of $BC$.

   The midpoint of $BC$ is $(\frac{5 - 1}{2}, \frac{1 + 3}{2}) = (2, 2)$

   which lies on $2y = x + 2$ hence it is a median.

22 (a) 13
   (b) 15

23 Centre of circle is $C (3, 0)$. $AC = 5$ so the radius is 5. $CP = \sqrt{2^2 + 2^2} = 5$ and $P$ lies on circle.

24 $AB^2 = (2\sqrt{3})^2 + 2^2 = 12 + 4 = 16$

   $AB = 4$. $BC = 1 - 3 = 4$

   $AC^2 = (2\sqrt{3})^2 + (-2)^2 = 12 + 4 = 16$

   $AC = 4$

   As all sides are equal, triangle is equilateral.

25 $CT^2 = 3^2 + 2^2$ so the radius is $\sqrt{13}$

   The gradient of $CT$ is $-\frac{2}{3}$ and the gradient of the tangent is $\frac{3}{2}$

   The equation of tangent is $y = \frac{3y}{2} + c$

   Substitute $(2, -1)$ hence $c = -4$ and the tangent is $2y = 3x - 8$

26 Let the coordinates of $B$ be $(2k, k)$

   $B$ lies on $2y = x$

   $\frac{2k - 2}{1 - (-1)} = \frac{k - 1}{\frac{3}{2}}$

   $2k - 2 = \frac{3k - 3}{2}$

   Multiply both sides by 2

   $2(2k - 2) = 3k - 3$

   $4k - 4 = 3k - 3$

   $k = 1$
\[ AB^2 = (k - 1)^2 + (2k - 2)^2 \]
\[ 20 = k^2 - 2k + 1 + 4k^2 - 8k + 4 \]
\[ 5k^2 - 10k - 15 = 0 \]
\[ k^2 - 2k - 3 = 0 \]
\[ (k + 1)(k - 3) = 0 \]
\[ k = -1 \text{ or } 3 \]

Coordinates of \( B \) are \((-2, -1) \) or \((6, 3)\)

**GRAPHS 5 – EXAM PRACTICE EXERCISE**

1. **The gradient of the line passing through \((-2, 0)\) and \((0, 4)\) is 2**
The equation of the line is \( y = 2x + 4 \)
One required inequality is \( y \leq 2x + 4 \)
\leq as the line is solid
The gradient of the line passing through \((0, -1)\) and \((2, 0)\) is \( \frac{1}{2} \)
The equation of the line is \( y = \frac{x}{2} - 1 \) or
\( 2y + 2 = x \)
One required inequality is \( 2y + 2 \geq x \)
\geq as the line is solid
The gradient of the line passing through \((0, 6)\) and \((8, 0)\) is \( -\frac{3}{4} \)
The equation of the line is \( y = -\frac{3}{4}x + 6 \) or
\( 4y + 3x = 24 \)
One required inequality is \( 4y + 3x < 24 \)
< as the line is dotted
The three inequalities are:
\( y \leq 2x + 4, 2y + 2 \geq x, 4y + 3x < 24 \)

2. **A sketch with a guess for \( k \) will help you understand the problem. Axes need to be equal aspect so that the two lines look as though they are at right angles**

The gradient of \( L \) is \(-\frac{2}{5}\)
Rearranging \( L \) gives \( y = -\frac{2}{5}x + \frac{23}{5} \)
The gradient of \( M \) is \( \frac{5}{2} \)
\( -\frac{2}{5} \times \frac{5}{2} = -1 \)
The gradient of line joining the two points is \( \frac{10 - k}{k - 3} \)
\( \frac{k - 10}{3 - k} \) is also correct and will give the same answer.
\( \frac{10 - k}{k - 3} = \frac{5}{2} \)
\( 2(10 - k) = 5(k - 3) \)
\( 20 - 2k = 5k - 15 \)
\( 7k = 35 \)
\( k = 5 \)

3. **A sketch shows that \( AB \) and \( AC \) will be the equal sides.**

(a) \( AB^2 = (21 + 3)^2 + (3 + 4)^2 = 625 \)
\( AB = 25 \)
\( AC^2 = (17 + 3)^2 + (11 + 4)^2 = 625 \)
\( AC = 25 \)
So \( ABC \) is an isosceles triangle.

(b) \( M \) is \( \left( \frac{3 + 11}{2}, \frac{21 + 17}{2} \right) = (7, 19) \)
Gradient of \( AM \) is \( \frac{19 + 3}{7 + 4} = 2 \)
Gradient of \( BC \) is \( \frac{17 - 21}{11 - 3} = -\frac{1}{2} \)
Product of gradients is \( 2 \times -\frac{1}{2} = -1 \)
So \( AM \) is perpendicular to \( BC \)
4 Let the point R be \((k, k)\).
R lies on \(y = x\) so coordinates are equal.
A rough sketch helps.

![Graph showing point R on line y = x](image)

Gradient of \(PR\) is \(\frac{k - 1}{k - 2}\) = \(\frac{k - 1}{k + 2}\)

\(\frac{1 - k}{2 - k}\) is also correct

Gradient of \(QR\) is \(\frac{k - 4}{k - 8}\) = \(\frac{k + 4}{k - 8}\)

\(\frac{-4 - k}{8 - k}\) is also correct

As \(PR\) is perpendicular to \(QR\)
\(\frac{k - 1}{k + 2} \times \frac{k + 4}{k - 8} = -1\)

\((k - 1)(k + 4) = -(k + 2)(k - 8)\)
\(k^2 - 3k - 4 = -k^2 + 6k + 16\)
\(2k^2 - 3k - 20 = 0\)
\(2k + 5)(k - 4) = 0\)
\(k = 4\) or \(k = -\frac{5}{2}\)

R is \((4, 4)\) or \((-2.5, -2.5)\)

5 Shortest distance is along the perpendicular
to the road passing through \((7, -2)\).

Gradient of the line (road) is \(\frac{3}{4}\)

Rearranging \(4y + 4 = 3x\) gives \(y = \frac{3}{4}x - 1\)

The gradient of perpendicular is
\(-\frac{4}{3} - \frac{4}{3} \times \frac{3}{4} = -1\)

The equation of perpendicular is
\(y = -\frac{4}{3}x + c\) or \(3y + 4x = d\)

Substituting \(x = 7, y = -2\) gives \(d = 22\) so
equation of perpendicular is \(3y + 4x = 22\)

Intersection is given by solving \(3y + 4x = 22\) and \(4y + 4 = 3x\) simultaneously.
\(3y + 4x = 22\) (1)
\(4y + 4 = 3x\) (2)
\(9y + 12x = 66\) (3)

(3) is (1) multiplied by 3

\(16y - 12x = -16\) (4)

(4) is (2) rearranged and multiplied by 4

Add (3) and (4)

\(y = 2, x = 4\) so the lines intersect at \((4, 2)\)

Distance from \((7, -2)\) to \((4, 2)\) is
\(\sqrt{(7 - 4)^2 + (-2 - 2)^2} = \sqrt{25} = 5\)

Kyle must walk 500 m.
12 Rotation 90° clockwise about centre (6, 8)
13 (2, 4)
14 (9, 12)
15 100
16 40
17 (a) (3, -4)
    (b) (-3, 4)
    (c) (4, -3)
    (d) (10, -2)
18 (a) (-3, -5)
    (b) (3, 5)
    (c) (-5, -3)
    (d) (-8, 8)
19
20
(c) Rotation of 90° clockwise around 0
(e) Translation along \( \begin{pmatrix} 4 \\ 0 \end{pmatrix} \)

SHAPE AND SPACE 5 – EXAM PRACTICE EXERCISE

1. \( x = -11, y = -1 \)
   Perform the inverse operation of each translation in reverse order on point (4, 8).
   1. Translate along vector \( \begin{pmatrix} -3 \\ 2 \end{pmatrix} \)
   2. Rotation of 90° in an anticlockwise direction about 0
   3. Reflection in x-axis

2. \( A(1, -5), B(-1, -5), C(1, -9) \)
   Perform the inverse operation of each translation in reverse order on triangle JKL.
   1. Translate along vector \( \begin{pmatrix} 3 \\ -4 \end{pmatrix} \)
   2. Rotation of 90° in a clockwise direction about 0
   3. Reflection in y-axis

3. \( A \text{ translates to } C \text{ along vector } \begin{pmatrix} -2 \\ 2 \end{pmatrix} \)
(e) Rotation of $90^\circ$ clockwise about centre $(0, -1)$

(f) Enlargement of scale factor $+2$ about centre $(-1, -4)$

5 (a) The image of $A$ after the translation along vector $\begin{pmatrix} 6 \\ 6 \end{pmatrix}$ is at $(7, 6 + \sqrt{3})$

(b) The image hexagon would have a new perimeter of $6 \times 12 = 72$.

Each triangle within the original hexagon is an equilateral triangle of side 2.

Area of whole hexagon image $= 6 \times$ area of an equilateral triangle side 12 $= 6 \times A_1$

$A_1 = \frac{1}{2} \times 12 \times 12 \times \sin(60^\circ)$

(Area of triangle $= \frac{1}{2} \text{absinC}$)

$= \frac{1}{2} \times 12 \times 12 \times \frac{\sqrt{3}}{2}$

$= 36 \sqrt{3}$

$= 2^2 \times 3^2 \times \frac{1}{2} = 2^2 \times 3^2$ $\times \frac{5}{2}$

So the total hexagon area $= 6 \times A_1$

$= (3 \times 2) \times 2^2 \times 3^2 = 2^3 \times 3^2$

$a = 3, b = \frac{7}{2}$

HANDLING DATA 4 – BASIC SKILLS EXERCISE

1 0.8

2 (a) $\frac{1}{2}$

(b) $\frac{5}{6}$

(c) 1

(d) 0

3 (a) $\frac{3}{29}$

(b) $\frac{9}{58}$

(c) $\frac{17}{29}$

(d) 0

4 (a) 0

(b) $\frac{9}{25}$

(c) $\frac{16}{25}$

(d) $\frac{9}{25}$

5 (a) $\frac{1}{13}$

(b) $\frac{2}{13}$

(c) $\frac{3}{4}$

(d) $\frac{3}{26}$

6 (a) $\frac{1}{8}$

(b) $\frac{1}{2}$

(c) $\frac{3}{4}$

(d) $\frac{5}{8}$

7 (a) RG GR GG

(b) $\frac{2}{3}$
8 (a) 

<table>
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<tr>
<th>×</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<tbody>
<tr>
<td>1</td>
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<td>12</td>
<td>18</td>
<td>24</td>
<td>30</td>
<td>36</td>
</tr>
</tbody>
</table>

(b)  
(i) \(\frac{1}{36}\)  
(ii) 0  
(iii) \(\frac{11}{36}\)  
(iv) \(\frac{2}{9}\)

9 (a) 

<table>
<thead>
<tr>
<th></th>
<th>10</th>
<th>30</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>10</td>
<td>30</td>
<td>50</td>
</tr>
<tr>
<td>16</td>
<td>16</td>
<td>30</td>
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<td>25</td>
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<td>50</td>
</tr>
<tr>
<td>36</td>
<td>36</td>
<td>36</td>
<td>50</td>
</tr>
</tbody>
</table>

(b)  
(i) \(\frac{1}{12}\)  
(ii) 0  
(iii) \(\frac{11}{12}\)  
(iv) \(\frac{1}{3}\)

10 (a) \(\frac{25}{28}\)

(b) \(\frac{1}{6}\)

(c) Number not cured =  
\[\frac{1}{9} \times 90 + \frac{3}{14} \times 84 + \frac{1}{10} \times 99 = 10 + 18 + 9.9 
\approx 38 \text{ horses}\]

11 (a)  

<table>
<thead>
<tr>
<th></th>
<th>△</th>
<th>○</th>
<th>☆</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue Box</td>
<td>□</td>
<td>□△</td>
<td>□○</td>
</tr>
<tr>
<td>Pink Box</td>
<td>□</td>
<td>□△</td>
<td>□○</td>
</tr>
<tr>
<td></td>
<td>☆</td>
<td>☆△</td>
<td>☆○</td>
</tr>
</tbody>
</table>

(b)  
(i) \(\frac{5}{9}\)  
(ii) \(\frac{4}{9}\)

12 10

13 \(\frac{3}{5}\)

14 40

15 5

16 \(\frac{1}{245}\)

17 (a)  

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>15</td>
<td>16</td>
<td>18</td>
<td>20</td>
<td>24</td>
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<td>17</td>
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<td>31</td>
<td>32</td>
<td>34</td>
<td>36</td>
<td>40</td>
</tr>
</tbody>
</table>

(b) P(number with a zero) = \(\frac{5}{25}\), expected numbers with a zero = \(50 \times \frac{5}{25}\) = 10

18 72 letters with a line of symmetry

**HANDLING DATA 4 – EXAM PRACTICE EXERCISE**

1 (a)  
\[8x = 1\]
\[x = \frac{1}{8}\]
\[\text{so } P(\text{black}) = \frac{3}{8}\]

(Sum of all probabilities = 1)

(b) P(not red or green) = \(6x = \frac{6}{8}\)
P(not red or green) = \(\frac{3}{4}\)

(c) If the spinner lands 25 times on one colour its probability must be \(\frac{1}{4}\) which leads to the conclusion that it is most likely to have been blue as P(blue) = \(\frac{1}{4}\).

2 (a) P(4) = 0.35

P(prime) = p (2 or 3 or 5) = 0.1 + 0.05 + 0.2

P(prime) = 0.35

(b) Li throws two numbers a total number of 40 times from 100. The probability of this event must = 0.4
The only two numbers which have a probability sum of 0.4 is 3 and 4.
(P(3) = 0.05, P(4) = 0.35)

Most likely numbers are 3 and 4.

3 (a)

<table>
<thead>
<tr>
<th></th>
<th>Milkshake</th>
<th>Orange juice</th>
<th>Tea</th>
<th>Coffee</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 10</td>
<td>2</td>
<td>10</td>
<td>7</td>
<td>5</td>
<td>24</td>
</tr>
<tr>
<td>Year 11</td>
<td>3</td>
<td>12</td>
<td>4</td>
<td>7</td>
<td>26</td>
</tr>
<tr>
<td>Total</td>
<td>5</td>
<td>22</td>
<td>11</td>
<td>12</td>
<td>50</td>
</tr>
</tbody>
</table>

(b) (i) \( P(\text{year 10, milkshake}) = \frac{2}{50} = \frac{1}{25} \)

(ii) \( P(\text{year 11, not tea or coffee}) = \frac{3 + 12}{50} = \frac{15}{50} = \frac{3}{10} \)

(c) \( P(\text{not milkshake/year 10}) = \frac{22}{24} = \frac{11}{12} \)

4 (a)

<table>
<thead>
<tr>
<th></th>
<th>35</th>
<th>42</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>5</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>63</td>
<td>7</td>
<td>21</td>
<td>3</td>
</tr>
<tr>
<td>96</td>
<td>1</td>
<td>6</td>
<td>24</td>
</tr>
</tbody>
</table>

(b) (i) \( P(\text{even}) = \frac{4}{9} \)

(ii) \( P(\text{prime or triangular}) = \frac{2}{3} \)

(Triangle numbers are 1, 3, 6, 10, 15, 21…)

(c) \( P(\text{odd}) = \frac{5}{9} \), therefore the expected number of odds from 90 choices

\( = \frac{5}{9} \times 90 = 50 \)

5 Let the number of purple jellyfish be \( x \)

\( P(\text{purple}) = \frac{x}{60} \)

\( P(\text{white}) = \frac{60 - x}{60} \)

If 40 white jellyfish are added:

\( P(\text{purple}) = \frac{x}{100} \cdot P(\text{white}) = \frac{100 - x}{100} \)

\( \frac{100 - x}{100} = 3 \times \frac{60 - x}{60} \)

\( 20(100 - x) = 100(60 - x) \)

\( 100 - x = 300 - 5x \)

\( 4x = 200, x = 50 \)

\( P(\text{purple}) = \frac{50}{100} = \frac{1}{2} \)

6 (a)

<table>
<thead>
<tr>
<th>Number of people</th>
<th>50</th>
<th>80</th>
<th>160</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost in £</td>
<td>125</td>
<td>200</td>
<td>400</td>
<td>500</td>
</tr>
</tbody>
</table>

(b) \( \frac{50}{125} \times 875 = 350 \) people

7 Between 0–5 mins temperature drop is 1° C/min, between 5–15 mins it is 0.9° C/min and between 15–30 mins it is 0.867° C. Temperature drop is not constant so Kit is wrong.

8 1 square contains \( \frac{30}{100} \times \frac{1}{20} \times 85 = 1.275 \) g of solids.

15 g of solids needs \( \frac{15}{1.275} = 11.76 \) squares so at least 12 squares are needed.

9 \( x \times y = \text{constant if } x \text{ and } y \text{ are in inverse proportion} \)

\[
\begin{array}{lcccccc}
\hline
x & 2 & 3 & 4 & 6 \\
y & 18 & 12 & 10 & 6 \\
xy & 36 & 36 & 40 & 36 \\
\hline
\end{array}
\]

(4, 10) is not in inverse proportion.
10 Number of scarves × temperature = constant as they are in inverse proportion. Constant = 32 × 15 = 480

<table>
<thead>
<tr>
<th>Number of scarves</th>
<th>120</th>
<th>60</th>
<th>40</th>
<th>32</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (°C)</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>15</td>
<td>20</td>
</tr>
</tbody>
</table>

11 (a) \( \frac{24}{64} = 0.375 \) seconds

(b) \( \frac{15}{0.25} = 60 \) Mbs

12 Number of cleaners × days = constant as they are in inverse proportion

Constant = 3 × 8 = 24

(a) 2 × 12 = 24 so 12 days

(b) 4 × 6 = 24 so 4 window cleaners

13 Number of desks × time = constant as they are in inverse proportion. \( \frac{3}{4} \) h = 45 min,

constant = 3 × 45 = 135. Easier to work in minutes.

(a) 5 × 27 = 135 so 27 minutes

(b) \( n \times 15 = 135 \) so \( n = 9 \) desks

14 Time × temperature = constant as they are in inverse proportion

Constant = 20 × 9 = 180

(a) \( t \times 15 = 180 \) so \( t = 12 \) minutes

(b) \( 24 \times T = 180 \) so \( T = 7.5°C \)

15 Number of waiters × time = constant as they are in inverse proportion

Constant = 12 × 3 = 36 (for 60 people)

(a) 18 × 2 = 36 therefore 18 waiters

(b) 24 × 1.5 = 36 therefore 24 waiters are needed to serve 60 people in 1.5 minutes

90 secs = 1.5 mins

24 × \( \frac{100}{60} = 40 \) waiters needed to serve 100 people in 90 seconds

16 (a) 150 km of flight produces 1 g so

\( 150 \times 1000 = 150000 \) km produces 1 kg

\( 1000 \) g = 1 kg mean distance per bee is

\( 150000 \div 10000 = 15 \) km

(b) 12 × 150000 km will produce 12 kg number of bees = 12 × 150000 \( \div 45 = 40000 \)

17 \( \frac{1}{25} \)

18 \( \frac{1}{27} \)

19 \( \frac{1}{16} \)

20 \( \frac{1}{49} \)

21 \( \frac{1}{5} \)

22 \( \frac{1}{108} \)

23 \( \frac{9}{8} \) or \( 1 \frac{1}{8} \)

24 1

25 9

26 \( \frac{2}{3} \)

27 \( \frac{1}{7} \)

28 \( \frac{1}{16} \)

29 \( \frac{8}{125} \)

30 \( \frac{8}{27} \)

31 \( \frac{9}{4} \)

32 \( \frac{5}{3} \)

33 1

34 \( \frac{1}{3} \)

35 16

36 9

37 4

38 0

39 \( \frac{1}{2} \)

40 \( \frac{2}{3} \)

41 0

42 \( \frac{3}{4} \)

43 \( \frac{1}{2} \)

44 1 or \( -2 \)
1. One machine produces $\frac{3000}{20} = 600$ shoes in 20 days.
   One machine produces $\frac{6000}{20} = 300$ shoes every day.
   Five machines working for 6 days produce $5 \times 6 \times 300 = 9000$ shoes.
   To complete the order, 27000 shoes must be produced in 10 days.
   $36000 - 9000 = 27000$.
   1 machine will produce $300 \times 10 = 3000$ shoes in 10 days.
   To produce 27000 shoes in 10 days will take $\frac{3000}{27000} = 9$ machines.
   Kiko must order an extra $9 - 5 = 4$ machines.

2. Temperature decreases by 10°C in 500 m; hence 2°C in 100 m and 14°C in 700 m.
   So the temperature at 700 m is $20 - 14 = 6°C$.
   Below 700 m, let $T$ be temperature in °C and $d$ be depth in metres.
   $T \times d = \text{constant}$ as $T$ and $d$ are in inverse proportion.
   $T \times d = k$.
   $k = 6 \times 700 = 4200$.
   $T \times d = 4200$.
   If $T = 4$, then $d = \frac{4200}{4} = 1050$ m.

3. (a) $(5^3)^{\frac{1}{3}} \times (5^{10})^{\frac{1}{2}} = 100^{\frac{1}{2}} = 5^{\frac{1}{2}} 5^{\frac{1}{2}} + 10^{\frac{1}{2}}$
   $= \frac{1}{2} \times 10$
   $= \frac{5}{2} \times 10$
   $= 2$

(b) $27\sqrt{27} = 3^1 \times (3^1)^{\frac{1}{2}}$
   $= 3^1 \times 3^{1 \times \frac{1}{2}}$
   $= 3^1 \times 3^{\frac{1}{2}}$
   $= 3^4 \times 3^{\frac{1}{2}}$
   $= 81\sqrt{3}$

(c) $27\sqrt{27} = (3^3)^{\frac{1}{2}}$
   $3^3 \times 3^{\frac{3}{2}} = 3^{2k}$
   $\frac{9}{2} = 3^{2k}$
   $k = \frac{9}{4}$

4. (a) $2^3 = (2^3)^k$ so $2k = 3$ and $k = \frac{3}{2}$

(b) $2\sqrt[3]{32} = 4^k$
   $2^1 \times (2^3)^{\frac{1}{3}} = (2^3)^k$
   $\frac{2}{3} = 2^k$
   $k = \frac{7}{4}$

(c) $\frac{1}{32} = 8^k$
   $32^{-1} = (2^3)^k$
   $(2^3)^{-1} = 2^k$
   $2^{-\frac{1}{3}} = 2^k$
   $k = -\frac{5}{3}$

5. (a) (i) $6 = 2 \times 3$
   $= (2^1)^{\frac{1}{2}} \times (3^1)^{\frac{1}{2}}$
   $= x^{\frac{1}{2}} \times y^{\frac{1}{2}}$

(ii) $4\sqrt[3]{3} = 2^2 \times 3^{\frac{1}{3}}$
   $= (2^3)^{\frac{1}{3}} \times (3^1)^{\frac{1}{3}}$
   $= x^{\frac{1}{3}} \times y^{\frac{1}{3}}$

(iii) $\frac{1}{3\sqrt{2}} = 3^{-1} \times 4^{-\frac{1}{2}}$
   $= 3^{-1} \times (2^2)^{-\frac{1}{2}}$
   $= 3^{-1} \times 2^{-1}$
   $= (3^3)^{\frac{1}{2}} \times (2^1)^{-\frac{1}{2}}$
   $= x^{-\frac{1}{3}} \times y^{-\frac{1}{2}}$

(b) $\frac{3\sqrt{6}}{8} = 3 \times \sqrt[3]{2} \times \sqrt[3]{3} \times 2^{-3}$
   $= 3 \times 2^\frac{1}{3} \times 3^{\frac{1}{3}} \times 2^{-3}$
   $= 2^\frac{5}{3} \times 3^{\frac{2}{3}}$
   $2^a \times 3^b = 2^{\frac{5}{3}} \times 3^{\frac{2}{3}}$
   $a = -\frac{5}{2}, b = \frac{3}{2}$
ALGEBRA 6 – BASIC SKILLS EXERCISE

1 (a) \( y = 9x \)  
(b) \( y = 90 \)  
(c) \( x = 5 \)  

2 \( a = 20b \)  

<table>
<thead>
<tr>
<th>( b )</th>
<th>10</th>
<th>5</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>200</td>
<td>300</td>
<td>600</td>
</tr>
</tbody>
</table>

3 (a) \( y = 8x \)  
(b) \( y = 80 \)  
(c) \( x = 5 \)  

4 (a) \( y = \left(\frac{x}{4}\right)^3 \)  
(b) \( y = 512 \)  

5 (a) \( y = 0.4x^2 \)  
(b) \( y = 90 \)  
(c) \( x \approx 6.12 \)  

6 (a) \( p = 20\sqrt{q} \)  
(b) \( p = 160 \)  
(c) \( q = 6.25 \)  

7 (a) \( d = 5t^2 \)  
(b) \( d = 20m \)  
(c) \( t \approx 4.24\,s \)  

8 (a) \( p = 1.5n^2 \)  
(b) \( p = \€216 \)  
(c) \( n = 20 \)  

9 (a) \( y = 4x^3 \)  
(b) \( y = 256 \)  
(c) \( x = 6 \)  

10 (a) \( e = 0.5y^2 \)  
(b) \( e = 1250\,kJ \)  
(c) \( v = 1414\,m/s \)  

11 (a) \( A = 15h^2 \)  
(b) \( A = 135\,m^2 \)  
(c) \( h = 6\,m \)  

12 (a) \( y = \frac{48}{x} \)  
(b) \( y = 6 \)  
(c) \( x = 4 \)  

13 (a) \( p = \frac{50}{q} \)  
(b) \( p = 2.5 \)  
(c) \( q = 2.5 \)  

14 (a) \( y = \frac{80}{x^2} \)  
(b) \( y = 5 \)  
(c) \( x = 12.6 \)  

15 (a) \( p = \frac{2500}{\sqrt{q}} \)  
(b) \( p = 250 \)  
(c) \( q = 2500 \)  

16 (a) \( p^2 = \frac{800}{q^2} \)  
(b) \( p = 3.54 \)  
(c) \( q = 3.17 \)  

17  

<table>
<thead>
<tr>
<th>( b )</th>
<th>125</th>
<th>8</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>2</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

18 (a) \( N = \frac{9000}{d^2} \)  
(b) \( N = 2250 \)  
(c) \( d = 3 \)  

19 \( \frac{1}{2} \)  

20 \( -1 \)  

21 \( -2 \)  

22 \( -3 \)  

23 \( \frac{1}{3} \)  

24 \( -\frac{1}{2} \)  

25 \( -\frac{1}{3} \)  

26 \( -\frac{1}{4} \)  

27 \( \frac{1}{2} \)  

28 \( 2 \)  

29 \( 3 \)  

30 \( 3 \)  

31 \( -5 \)  

32 \( -4 \)  

33 \( -4 \)  

34 \( -3 \)  

35 \( 1 \)
2 (a) \( n \propto \frac{1}{r^3} \), \( n = \frac{k}{r^3} \)

If \( n = 10^3 \), \( t = 0.5 \)

\[ \begin{align*}
10^3 &= \frac{k}{0.5^3} \\
k &= 2.5 \times 10^2 \\
n &= \frac{2.5 \times 10^2}{r^3}
\end{align*} \]

(b) If \( t = 2 \)

\[ \begin{align*}
n &= \frac{2.5 \times 10^2}{2^3} = 62.5
\end{align*} \]

(c) If \( n = 1 \)

\[ \begin{align*}
1 &= \frac{2.5 \times 10^2}{r^3}, t = \sqrt[3]{2.5 \times 10^2} = 15.8 \text{ yrs}
\end{align*} \]

3 (a) \( v \propto \sqrt{d} \), \( v = k \sqrt{d} \)

If \( d = 10 \text{ m} \), \( v = 9.8 \text{ m/s} \)

\[ v = \frac{9.8 \sqrt{d}}{\sqrt{10}} = 9.8 \sqrt{\frac{d}{10}} \]

(b) (i) If \( d = 50 \text{ m} \), \( v = 9.8 \sqrt{\frac{50}{10}} = 21.9 \text{ m/s (3 s.f.)} \)

(ii) If \( d = 1000 \text{ m} \), \( v = 9.8 \sqrt{\frac{1000}{10}} = 98 \text{ m/s} \)

(c) If \( v = 1 \text{ m/s} \)

\[ \begin{align*}
1 &= 9.8 \sqrt{\frac{d}{10}}, \quad \frac{1}{9.8} = \sqrt{\frac{d}{10}}, \quad \left(\frac{1}{9.8}\right)^2 = \frac{d}{10}
\end{align*} \]

\[ d = 10 \times \left(\frac{1}{9.8}\right)^2 = 0.104 \text{ m (3 s.f.)} \]

(d) 790 km/h = \( \frac{790 \times 1000}{60 \times 60} = 219.4 \text{ m/s} \)

(Convert 790 km/h into m/s)

\[ \left(\frac{219.4}{9.8}\right)^2 = \frac{d}{10}, \quad d = 10 \times \left(\frac{219.4}{9.8}\right)^2 = 5010 \text{ m (3 s.f.)} \]

4 \( x = k_1 z^3 \) and \( x = k_2 y^2 \), where \( k_1 \) and \( k_2 \) are constants

So \( x = k_2 y^2 \) therefore \( k_1 z^3 = k_2 y^2 \),

\[ \begin{align*}
z^3 &= \frac{k_2}{k_1} y^2, \quad 50^3 = \frac{k_2}{k_1} \times 25^2, \quad \frac{k_2}{k_1} = 200
\end{align*} \]

\[ z^3 = 200 \times 10^2, \text{ so } z^3 = 20 000, \text{ } z = 27.1 \text{ (3 s.f.)} \]
5. \(ab = 125\) implies that \(5^m \times 5^n = 5^3\) so \(m + n = 3\) \(\text{(1)}\)

\(a^b \cdot 2 = 5^{-9}\) implies that \(5^{4m} \times 5^{-2n} = 5^{-9}\) so \(4m - 2n = -9\) \(\text{(2)}\)

Solving equations (1) and (2):

(1): \(m = 3 - n\)

substituting into (2) gives

\(4(3 - n) - 2n = -9\)

\(12 - 4n - 2n = -9\)

\(21 = 6n\)

\(n = 3.5\)

substituting into (1) gives

(1): \(m + 3.5 = 3\)

\(m = -0.5\)

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SEQUENCES 1 – BASIC SKILLS EXERCISE

1. 16, 19.5, 23 (add 3.5)
2. 0.2, 0.2 (subtract 0.2)
3. \(\frac{1}{16} \times \frac{1}{32} \times \frac{1}{64} \times \frac{1}{256}\)
4. 0.32, 0.064, 0.0128 (divide by 5)
5. -9, 27, -81 (multiply by -3)
6. 35, 48, 63 \(n^2 - 1\)
7. -2, 4, 10, 16, ...
8. 80, 76, 72, 68
9. \(-\frac{1}{3}, 0, \frac{2}{3}\)
10. 2, \(\frac{5}{2}, \frac{10}{3}, \frac{17}{4}\)
11. 3, 10, 21, 36
12. \(\frac{1}{3}, -\frac{1}{4}, -\frac{3}{5}, -\frac{5}{6}\)
13. 56, 76, 99
14. 2, -8, -21
15. 4, 10, 18
16. 74, 100, 130
17. 8, 5, 1
18. -6, -4, -1
19. \(2n + 3\)
20. \(29 - 3n\)
21. \(\frac{n-1}{n+1}\)
22. \(3n - 2\)
23. \(13 - 4n\)
24. \(2^{n-1}\)
25. No, \(78 - 3n = -218 \Rightarrow n = 98.67\) ...
26. \(-23 + 5n > 1000 \Rightarrow n > 204.6\) 205th term is 1002
27. First sequence is \(4n - 7\), second sequence is \(3n + 9\) \(4n - 7 = 3n + 9\) so \(n = 16\)
28. 2nd sequence is \(11n + 8\) so \(n^2 - 4 = 11n + 8\)

\((n + 1)(n - 12) = 0\)

\(n = 12\)

The term is 140
29. 17th term = \(\frac{1}{2^{17}}\) or 0.0192...
30. \(n = 1\) gives \(1 + a + b = 3\)

\(n = 2\) gives \(4 + 2a + b = 3\) so \(a = -3, b = 5\)
31. \(a = 6, d = 5, n = 20\) so \(S_{20} = 1070\)
32. \(a = -2, d = -4, a + (n - 1)d = -46\)

\(n = 12\)

\(S_{12} = -288\)
33. \(S_{38} = \frac{38}{2} (1 + 297) = 5662\)
34. \(a = 4, d = 4, n = 100\)

\(S_{100} = 20 200\)
35. First term = \(461 - 7 \times 67 = -8\)

\(S_{68} = \frac{68}{2} (-16 + 67 \times 7) = 15 402\)
36. \(a = 120 = \frac{10}{2} [6 + (10 - 1)d]\) so \(d = 2\)

(b) \(a = 120 = \frac{10}{2} [2a + (10 - 1)3]\) so \(a = -\frac{3}{2}\)
37. \(a + 6d = 37, a + 17d = 92\) so \(a = 7, d = 5\)

\(S_{20} - S_9 = \frac{20}{2} (14 + 19 \times 5) - \frac{9}{2}\)

\((14 + 8 \times 5) = 847\)
38. \(270 = \frac{n}{2} [12 + (n - 1)3]\)

\(n^2 + 3n - 180 = 0\)

\((n + 15)(n - 12) = 0\)

\(n = 12\)
39 Sum of first 80 even numbers = \( \frac{80 \times (4 + 79 \times 2)}{2} \)
\( = 6480 \)

Even multiples of 3 are multiples of 6. Need to subtract \( 6 + 12 + 18 + \ldots + 156 \)
Number of terms given by \( 6 + (n - 1) \)
\( 6 = 156 \) so \( n = 26 \)
Sum of \( 6 + 12 + 18 + \ldots + 156 = \frac{26}{2} \)
\( (12 + 25 \times 6) = 2106 \)
answer is \( 6480 - 2106 = 4374 \)

40 \( 935 = \left( \frac{7}{2} \right) (a + 103) \) so \( a = 7 \) and
\( d = \frac{103 - 7}{17 - 1} = 6 \)

41 \( a = -11, d = 2 \) gives
\( 540 = \left( \frac{a}{2} \right) [-22 + (n - 1)2] \)
\( n^2 - 12n - 540 = 0 \)
\( (n - 30)(n + 18) = 0 \)
n = 30
last term is -11 + 29 \times 2 = 47

42 \( 145 = \left( \frac{10}{2} \right) (2a + 9d) \) (1)
Sum of first 20 terms is 145 + 645 = 790
\( 790 = \left( \frac{20}{2} \right) (2a + 19d) \) (2)
Solving (1) and (2) simultaneously gives
\( a = -8, d = 5 \)

43 \( 64 = a + 11d \) and \( 504 = \left( \frac{12}{2} \right) (2a + 11d) \)
so \( a = 20, d = 4 \)
\( S_{24} = \frac{24}{2} (40 + 23 \times 4) = 1584 \)

44 First term \( (k = 1) \) is 3, common difference is 4
\( S_n = \frac{n}{2} [6 + (n - 1)4] = n(2n + 1) \)

45 (a) \( 0 = 48 + (k - 1)(-3) \) so \( k = 17 \)
(b) After the 17th term, terms are negative and thus reducing the sum of the series.
Largest sum is \( S_{17} = \frac{17}{2} [96 + 16 \times (-3)] \)
\( = 408 \)
Note \( S_{15} \) is also correct as the 17th term is zero.

### SEQUENCES 1 – EXAM PRACTICE EXERCISE

1 (a) The common difference is 4 so the sequence continues as 21, 25, 29, \( n \)th term is \( 4n - 3 \)
(b) 25 is a square number and is 7th in the sequence.
Next square number is 36. \( 4n - 3 \) is always odd so 36 is not a member of the sequence.
or \( 4n - 3 = 36 \) so \( n = 9.75 \) and hence 36 is not a member of the sequence.
Next square number is 49 so \( 4n - 3 = 49 \)
and \( n = 13 \)
(c) \( T \) is the sequence 1, 3, 7, 13, ...
Table of differences shows 2nd difference is constant and equal to 2
\[
\begin{array}{cccc}
1 & 3 & 7 & 13 \\
2 & 4 & 6 & 8 \\
2 & 2 & 2 & 2 \\
\end{array}
\]
Extending the table gives (shown in red)
\[
\begin{array}{cccc}
1 & 3 & 7 & 13 & 21 & 31 \\
2 & 4 & 6 & 8 & 10 & 12 \\
2 & 2 & 2 & 2 & 2 & 2 \\
\end{array}
\]
\( T \) is 1, 3, 7, 13, 21, 31, ...
The sixth square number in \( S \) is in the 31st position.
Substituting \( n = 31 \) into \( 4n - 3 \) gives the value as 121 (or \( 11^2 \))

2 (a) \( \frac{n + 1}{2n + 1} \)
(b) \( n + 1 = 99 \) so \( n = 98 \)
\( 2n + 1 = 195 \)
\( n = 97 \)
\( \frac{99}{195} \) is not a member of the sequence
\( \frac{n + 1}{2n + 1} \)
(c) \( \frac{n + 1}{2n + 1} = 0.52 \implies n + 1 = (2n + 1)(0.52) \)
\( = 1.04n + 0.52 \)
\( 0.04n = 0.52 \)
\( n = 12 \)
so 13th term is the first with a value less than 0.52.
12th term equals 0.52 so is not less than 0.52.
3 Subtracting two consecutive terms gives $d$

$(10x - 9) - (4x + 10) = d$

$6x - 19 = d$

$(12x - 10) - (10x - 9) = d$

$2x - 1 = d$

$6x - 19 = 2x - 1$

$4x = 18$

$x = 4.5$

$d = 8$ and $a = 28$

Sum from the 20th to the 30th terms

$= S_{30} - S_{19}$

Sum includes the 20th term

$S_{30} = \frac{30}{2} (56 + 29 \times 8) = 4320$

$S_{19} = \frac{19}{2} (56 + 18 \times 8) = 1900$

$S_{30} - S_{19} = 4320 - 1900 = 2420$

5th term is $a + 4d$

$\frac{5}{2} (2a + 4d) = 300$

$S_5 = \frac{5}{2} [2a + (5 - 1)d]$

$a + 2d = 60$

Solving (1) and (2) simultaneously gives

$a = 64$, $d = -2$

$S_n : S_6 = 6 : 11$

$S_n = \frac{11}{6} \times 300 = 550$

$\frac{n}{2} [128 - 2 (n - 1)] = 550$

$64n - n^2 + n = 550$

$n^2 - 65n + 550 = 0$

$(n - 55) (n - 10) = 0$

$n = 55$ or $n = 10$

(a) $1 + 2 + 3 + \ldots + 99 + 100 = 5050$

Smallest share is $\frac{1}{5050} \times 10100 = £2$

Largest share is $\frac{100}{5050} \times 10100 = £200$

(b) $1 + 2 + 3 + 4 + \ldots + n = \frac{n}{2} [2 + (n - 1) \times 1] = \frac{n}{2} (n + 1) = \frac{n(n + 1)}{2}$

Smallest share is $1 + \left\lfloor \frac{n(n + 1)}{2} \right\rfloor$ of the

amount $= A \times \frac{2}{n(n + 1)} = \frac{2A}{n(n + 1)}$

Largest share is $n + \left( \frac{n(n + 1)}{2} \right)$ of the

amount $= A \times n \times \frac{2}{n(n + 1)} = \frac{2A}{n + 1}$
29 42°
30 228°
31 44°
32 (a) $x = 60^\circ$, $y = 60^\circ$, $z = 55^\circ$
(b) $x = 40^\circ$, $y = 70^\circ$, $z = 40^\circ$

SHAPE AND SPACE 6 – EXAM PRACTICE EXERCISE

1

(a) $PB \times PA = PC \times PD$ (intersecting chords theorem)
$PB = \frac{15 \times 8}{10} = 12$ cm, so $AB = 12 - 10 = 2$ cm
(b) If angle $DPA = 40^\circ$, $BC$ can be found from the cosine rule.
$(\text{cosine rule: } a^2 = b^2 + c^2 - 2bc \cos(A))$
$BC = PC^2 + PB^2 - 2 \times PC \times PB \times \cos(40^\circ)$
$= 15^2 + 12^2 - 2 \times 15 \times 12 \times \cos(40^\circ)$
$BC = 9.6553 \ldots$ cm = 9.66 cm (3 s.f.)
(c) Angle $BCD$ can be found from the sine rule.
$(\text{sine rule: } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C})$
$\frac{BC}{\sin(DPA)} = \frac{PB}{\sin(BCD)}$
$\frac{9.6553}{\sin(40^\circ)} = \frac{12}{\sin(BCD)}$
$\sin(BCD) = \frac{12 \times \sin(40^\circ)}{9.6553} = 0.79888 \ldots$
$BCD = 53.0^\circ$ (2 s.f.)

2

(a) $PT \times RT = ST \times QT$ (intersecting chords theorem)
$PT \times 6 = 7 \times 3$
$PT = \frac{7 \times 3}{6} = 3.5$ cm, so $PR = 3.5 + 6 = 9.5$ cm
(b) If the angle $OTR = 74^\circ$, consider triangle $RST$ and use the cosine rule
$(\text{cosine rule: } a^2 = b^2 + c^2 - 2bc \cos(A))$
$RS^2 = RT^2 + ST^2 - 2 \times RT \times ST \times \cos(74^\circ)$
$= 6^2 + 7^2 - 2 \times 6 \times 7 \times \cos(74^\circ)$
$RS = 7.8643 \ldots$ cm = 7.86 cm (3 s.f.)
(c) $\angle QPT = \angle TSR$
(Angles in the same segment are equal.)
So consider triangle $RST$ to find angle $TSR$ hence finding angle $QPT$.
$(\text{cosine rule: } a^2 = b^2 + c^2 - 2bc \cos(A))$
$RT^2 = RS^2 + ST^2 - 2 \times RS \times ST \times \cos(RST)$
$= 61.846 + 7^2 - 2 \times 61.846 \times 7 \times \cos(RST)$
$\cos(RST) = \frac{61.846 + 7^2 - 6^2}{2 \times 61.846 \times 7}$, so angle $RST = 85.041^\circ \ldots$
So angle $RST = \angle QPT = 85.0^\circ$ (3 s.f.)

3

$\angle EOD = 68^\circ$

(Intersecting chords theorem)
(a) angle \( BAO = 68^\circ \) (alternate segment theorem)

angle \( BAO = \) angle \( ABO = 68^\circ \) (triangle \( ABO \) is isosceles)

angle \( AOB = 180^\circ - 2 \times 68^\circ = 44^\circ \) (angle sum of triangle = \( 180^\circ \))

angle \( BOD = 180^\circ - 44^\circ = 136^\circ \) (angle sum of a straight line \( = 180^\circ \))

(b) angle \( ABD = 90^\circ \) (angles in a semi-circle \( = 90^\circ \) at the circumference)

So triangle \( ABD \) is a right-angled triangle.

\[ AD^2 = AB^2 + BD^2, \quad (2r)^2 = (2s)^2 + BD^2 \]

\[ BD = \sqrt{4(r^2 - s^2)} = 2 \sqrt{(r + s)(r - s)} \]

as required

4

(a) angle \( PLK = 62^\circ \) (alternate segment theorem)

angle \( PLK = \) angle \( PMJ = 62^\circ \) (angles in the same segment are equal)

angle \( MLP = \) angle \( PKJ = 21^\circ \) (angles in the same segment are equal)

Triangle \( GLJ: \)

angle \( LJJ = 180^\circ - (21^\circ + 78^\circ) = 81^\circ \) (angle sum of a triangle = \( 180^\circ \))

angle \( LJK = 180^\circ - (81^\circ + 62^\circ) = 37^\circ \) (angle sum of a straight line = \( 180^\circ \))

(b) angle \( LMK = \) angle \( KJL = 37^\circ \) (angles in the same segment are equal)

angle \( GMJ = 180^\circ - (37^\circ + 62^\circ) = 81^\circ \) (angle sum of a straight line = \( 180^\circ \))

angle \( GMK = \) angle \( GMJ + \) angle \( PMJ = 81^\circ + 62^\circ = 143^\circ \)

5

(a) (i) Draw line \( AB \) on diagram as shown.

angle \( BOA = 100^\circ \) (angle sum in a circle = \( 360^\circ \))

angle \( ACB = 50^\circ \) (angle at centre of circle = \( 2 \times \) angle at circumference off the same chord)

angle \( CAO = 360^\circ - (50^\circ + 260^\circ + 30^\circ) = 20^\circ \)

(ii) angle \( ABO = \frac{180^\circ - 100^\circ}{2} = 40^\circ \) (triangle \( ABO \) is isosceles so base angles are equal.)

angle \( ABC = \) angle \( ABO + \) angle \( OBC = 40^\circ + 30^\circ = 70^\circ \)

(b) angle \( EAB = \) angle \( ACB = 50^\circ \) (alternate segment theorem)

angle \( FAB = 25^\circ \) (line \( FA \) bisects angle \( BAE \))

angle \( AFB = 180^\circ - 50^\circ = 130^\circ \) (opposite angles in a cyclic quadrilateral = \( 180^\circ \))

angle \( FBA = 180^\circ - 130^\circ - 25^\circ = 25^\circ \) (angle sum of a triangle = \( 180^\circ \))

SETS 2 - BASIC SKILLS EXERCISE

1 (a) \( \angle A \)

(b) \( \angle B \)
2 (a) \( A \cap B \)
(b) \( A \cup B \) or \( \emptyset \)
(c) \( A \)
(d) \( B \)

3 (a) \( A \cap B \cap C \)
(b) \( A \cup B \cup C \)
(c) \( A \)
(d) \( B \)

4 (a) \( A \cap B \)
(b) \( A \cup B \)
(c) \( A \cap B \)
(d) \( A \cap B \cap C \)

5 (a) \( A' \cap B \)
(b) \( (A' \cap B) \cup (A \cap B') \)

6 (a) \( A' \cup B \)
(b) \( (A \cap B') \cup C \)

7 (a) There are no tabby cats over 10 years old.
(b) There are some non-tabby cats under 10 years old.

8 \( 30 \) play neither.
9 \( \varepsilon = 11 \)

\[ B = 9 \quad G = 3 \]

\[ 7 \quad 2 \quad 1 \]

1 have both.

10 \( \varepsilon = 39 \)

\[ 5G = 18 \quad P = 12 \]

\[ 14 \quad 4 \quad 8 \]

13

8 cannot.

11 \( \varepsilon = 31 \)

\[ S = 18 \quad B \]

\[ 5 \quad 8 \]

31 animals are in the field.

12 (a) 21
(b) 13

\[ 5 \quad 4 \quad 7 \]

3 (b) have both.

13 \( I = \) Isosceles triangles
\( E = \) Equilateral triangles
\( R = \) Right-angled triangles

\( \varepsilon \)

\[ 5 \quad 2 \quad 7 \]

\[ 3 \quad 6 \]

14 \( \varepsilon \)

\[ B = 40 \quad G = 55 \]

\[ 23 \quad 7 \quad 30 \]

\[ 5 \quad 13 \]

\( T = 30 \)

Number of pensioners =

\[ 23 + 7 + 30 + 5 + 5 + 13 + 7 = 90 \]

15 \( \varepsilon = 40 \)

\[ W = 22 \]

\[ 3 + x \quad 12 - x \quad 5 + x \]

\[ S = 23 \]

\[ x \quad 7 - x \quad 6 - x \]

\[ D = 17 \]

\((3 + x) + (12 - x) + (5 + x) + (7 - x) + (6 - x) + (4 + x) + x = 40\)

\(x = 3\) so 3 teenagers enjoy all three sports.

16 (a) \( A = \{-1, 0, 1\} \)
(b) \( B = \{3, 4, 5\} \)
(c) \( C = \{-1, 0, 5\} \)
(d) \( D = \{1, 2, 3, 4\} \)
(e) \( E = \{-2, 2\} \)
(f) \( F = \{0, 1, 2, 3, 4, 5, 6\} \)

17 (a) \( A = \{x: x < 5\} \)
(b) \( B = \{x: x \geq -8\} \)
(c) \( C = \{x: -2 < x < 4\} \)
(d) \( D = \{x: 3 \leq x \geq 8\} \)
(e) \( E = \{x: -2 \leq x \leq 2\} \) or \( C = \{x: -3 < x < 3\} \)
(f) \( F = \{x: x = 2y \) and \( 2 \leq y \leq 4\} \) or \( D = \{x: x = 2y \) and \( 1 < y \leq 5\} \)

SETS 2 – EXAM PRACTICE EXERCISE

1 (a) (i) \( (A \cup B') = \)

\[ 5 \quad 2 \quad 7 \]

so \( (A \cup B')' = \)

\[ 5 \quad 2 \quad 7 \]
(ii) \( A' \cap B' = \)

(b) (i) Largest intersection is 39\% when swimming is a subset of jogging.

(b) (ii) Smallest intersection is when the percentage not in \( J \) or \( S \) is 0\%.
Let the \( \% \) in \( J \cap S \) be \( x \), then \( 68 - x \) + \( x \) + \( 39 - x \) = 100
\( \Rightarrow \) \( x \) = 7\% \( \Rightarrow \) smallest percentage is 7\%.

2 (a) Suzie has some green T shirts.
(b) All Suzie’s dresses are green.
(c) \( \%

3 The Venn diagram shows \( H \) representing horses, \( D \) representing donkeys and \( NM \) representing non-microchipped \( H \) and \( D \) do not intersect.

Let \( x \) be the number of horses with microchips so \( 2x \) is the number donkeys with microchips
\( \Rightarrow \) \( 34 - x \) is the number of horses without microchips, \( 24 - 2x \) is the number of donkeys without microchips

Putting these into the Venn diagram gives:

4 Let \( B \) represent blue cars, \( G \) green cars and \( S \) soft tops.
The numbers represent the numbers of cars. \( B \) and \( G \) do not intersect.
The total number of cars is 50
\( 19 - x + x + 4 - x + 2 + 5 + 23 = 50 \)
\( x = 3 \)

5 (a) Of the 6 that like peppermint and chocolate, some might like toffee.
Of the 9 that like chocolate and toffee, some might like peppermint.
Let \( P \) represent peppermint lovers, \( C \) chocolate lovers and \( T \) toffee lovers.
The numbers represent the numbers of teenagers.

(b) \( 6 - x \) from the Venn diagram
(c) All the numbers must sum to 60.
\( 10 + (6 - x) + x + 4 + 7 + (9 - x) + 12 + 15 = 60 \)
\( x = 3 \) so 3 teenagers like all three.
NUMBER 7 – BASIC SKILLS EXERCISE

1. 2.64
2. 4.37
3. 0.245
4. 6.75
5. 5.94
6. 0.314
7. 27.3
8. 2,400,000
9. 26,200
10. 15.6
11. 755,000
12. 25.5
13. 862,000
14. 2.79
15. 2.31
16. 5.68
17. 14.7
18. 0.104
19. $\frac{1}{9}$
20. $\frac{2}{9}$
21. $\frac{1}{3}$
22. $\frac{4}{11}$
23. $\frac{7}{11}$
24. $\frac{17}{99}$
25. $\frac{71}{99}$
26. 1
27. $\frac{4}{33}$
28. $\frac{34}{99}$
29. $\frac{1}{45}$
30. $\frac{7}{90}$
31. $\frac{25}{333}$
32. $\frac{41}{333}$
33. $\frac{1,234}{9,999}$
34. $\frac{2,468}{9,999}$
35. $\frac{11}{15}$
36. $\frac{17}{45}$
37. $\frac{2}{1125}$
38. $\frac{211}{9000}$

NUMBER – 7 EXAM PRACTICE EXERCISE

1. (a) $6.99 \times 10^4$
   (b) $7.85 \times 10^7$
   (c) $2.90 \times 10^3$
2. (a) $3.16 \times 10^0$
   (b) $1.11 \times 10^1$
   (c) $2.87 \times 10^7$
3. (a) 0.773
   (b) 0.992
   (c) 0.129
4. Let $x = 0.02\bar{3}$
   $10x = 0.2323\ldots$
   $1000x = 23.2323\ldots$
   $990x = 23$, so $x = \frac{23}{990}$
   Let $y = 0.1\bar{7}$
   $10y = 1.777\ldots$
   $100y = 17.777\ldots$
   $90y = 16$, so $y = \frac{16}{90}$
   
   $x + y = \frac{23}{990} + \frac{16}{90} = \frac{199}{990}$, so
   
   $p = 199$, $q = 990$
5 (a) Let \( p = 0.xxyx\ldots \)
\[ 100p = xy.xxyx\ldots \]
\[ 99p = xy = 10x + y, \text{ so } p = \frac{10x + y}{99}, \]
as required.
(The number \( xy \) means there are 10 \( x \)'s and 1 \( y \))

(b) Let \( p = 5.xxyzx\ldots \)
\[ 1000p = 5xxyz.xxyz\ldots \]
(The number \( xyz \) means there are 100 \( x \)'s, 10 \( y \)'s and 1 \( z \))
\[ 999p = 5xyz - 5 = 5000 + 100x + 10y + z - 5 = 4995 + 100x + 10y + z \]
so \( p = \frac{4995 + 100x + 10y + z}{999}, \) as required.

### ALGEBRA 7 – BASIC SKILLS EXERCISE

1 5, 6
2 -1, 5
3 -1, 4
4 -4, -3
5 2
6 2, 0
7 -3, 5
8 -4, 8
9 -3 ± √3
10 \( \frac{-3 ± √3}{2} \)
11 3 ± \( \sqrt{\frac{3}{2}} \)
12 2 ± √3
13 -1 ± \( \frac{\sqrt{3}}{3} \)
14 3 ± \( \sqrt{\frac{3}{5}} \)
15 1.59, 4.41

16 -0.257, 2.59
17 3.11, -1.61
18 -57.9, -4.29
19 -3.30, 0.379
20 -2.45, -0.147
21 \( x < -5 \) or \( x > 3 \)
22 -2 ≤ \( x \) ≤ 7
23 \( -\frac{1}{3} < x < \frac{1}{2} \)
24 \( x < -3.45 \) or \( x > 1.45 \)
25 \( 2 - \sqrt{2} < x < 2 + \sqrt{2} \)
26 -4 ≤ \( x \) ≤ -2 or 3 ≤ \( x \) ≤ 6 or \( x \) ≥ 6 or \( x \) ≤ 4 or -1.65 ≤ \( x \) ≤ 3.64
27 \( b = 6, c = -7 \)
28 \( (x + 3\pi)(x - \pi) = 0, x = -3\pi \) or \( x = \pi \)
29 \( (x + p)(x + q) = 0, x = -p \) or \( x = -q \)
30 9 - \( (x - 3)^2 \)
31 \( b = -2, c = -4 \)
32 \( x^3 + (x + 2)^2 = 202, 9 \) and 11 or -11 and -9
33 \( x(x + 3) = 9 \times 5, x = 5.37 \)
34 (a) \( (x - 2)^2 + 5 \)
(b) \( x = 2 \)
(c) 5
35 \( b = -5, c = 6 \)
36 8 m by 5 m
37 \( x^3 + (5 - x)^2 = 4^2 \) so the sides are 1.18 cm and 3.82 cm (3 s.f.)
38 \( \pi(w + 2)^2 - \pi2^2 = \pi2^2 \)
The width of the path is 0.828 m (3 s.f.)
39 \( A + B + C + D = 360^\circ \)
\[ 3x^2 + 24x - 315 = 0 \]
\[ x^2 + 8x - 105 = 0 \]
\[ (x - 7)(x + 15) = 0 \]
\( x = 7 \)
The angles are \( A = 60^\circ, B = 120^\circ, C = 135^\circ \) and \( D = 45^\circ \)
\( A + B = 180^\circ \) therefore a trapezium
(or \( C + D = 180^\circ \))
40  $600 < 3x(x + 5) < 700$, $11.9 < x < 13.0$

$(x - 3)(2x + 3) > \frac{1}{2}(x + 2)(2x - 6)$

$x^2 - 2x - 3 > 0$

$x > 3$

41 Area of Rectangle $= (2x + 3)(x - 3)$

Area of Triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

$x^2 - x - 6 > 0$

$(x + 1)(x - 3) > 0$

Solving, $x < -1$ or $x > 3$. As $x$ must be greater than zero then $x > 3$ for the area of the rectangle to be greater than the area of the triangle.

**ALGEBRA 7 – EXAM PRACTICE EXERCISE**

1 (a) $f(x) = -3 \left[ x^2 + 4x - 1 \right] = -3[(x + 2)^2 - 5]$

$= -3(x + 2)^2 + 15$

(b) $-3(x + 2)^2 + 15 = 0$

$3(x + 2)^2 = 15$

$(x + 2)^2 = 5$

$x = -2 \pm \sqrt{5}$

(c)

2 (a) Formula sheet: Curved surface area of cylinder $= 2\pi rh$, surface area of sphere $= 4\pi r^2$

Surface area of cylinder $= 2\pi r \times 20 = 40\pi r$

The two hemispherical ends have a surface area equal to the surface area of a sphere.

Surface area of ends $= 4\pi r^2$

$4\pi r^2 + 40\pi r \leq 800\pi$

$r^2 + 10r - 200 \leq 0$

Dividing both sides by $4\pi$ and rearranging.

$(r + 20)(r - 10) \leq 0$

$0 < r \leq 10$ as $r > 0$

(b) Maximum volume is when $r = 10$

Volume of cylinder $= \pi \times 10^2 \times 20$

$= 2000\pi$

Volume of two hemispheres

$= \frac{4}{3} \times \pi \times 10^3 = \frac{4000}{3} \pi$

Volume of a sphere $= \frac{4}{3} \pi r^3$

Total volume $= \frac{10000}{3} \pi = 10500$ cm$^3$ to 3 s.f.

3 (a) The sum of the internal angles $= 360^\circ$

$2x^2 + 50 + 3x^2 - 18 + 3x + 40 + 12x + 18 = 360$

$5x^2 + 15x - 270 = 0$

$x^2 + 3x - 54 = 0$

$(x + 9)(x - 6) = 0$

$x = 6$ or $x = -9$

When $x = -9$ the angle $D = -90^\circ$ so $x \neq -9$

Substituting $x = 6$ gives $A = 122^\circ$, $B = 90^\circ$, $C = 58^\circ$ and $D = 90^\circ$

Since $B + D = 180^\circ$, $ABCD$ is cyclic (or $A + C = 180^\circ$)

(b) Since $B = 90^\circ$, $AC$ is a diameter.

By Pythagoras’ theorem in triangle $ABC$

$(d - 3)^2 + (d - 2)^2 = d^2$

$d^2 - 6d + 9 + d^2 - 4d + 4 = d^2$

$d^2 - 10d + 13 = 0$
Using the quadratic formula gives

\[ d = \frac{10 \pm \sqrt{100 - 4 \times 17}}{2} \]

\[ d = \frac{10 \pm \sqrt{100 - 4 \times 17}}{2} \]

\[ d = \frac{10 \pm \sqrt{16 \times 3}}{2} \]

\[ d = \frac{10 \pm 4\sqrt{3}}{2} \]

\[ d = 5 \pm 2\sqrt{3} \]

\[ d = 5 - 2\sqrt{3} \]

1.5 means \( AB \) and \( BC \) are negative so \( d = 5 + 2\sqrt{3} \) cm

4 (a) Area of garden = 6 \times 8 = 48 m²

Total area of the flower beds

= \((6 - x)(8 - x) = 48 - 14x + x² \)

⇒ area of path = \( 48 - (48 - 14x - x²) = 14x - x² \)

Or: area of one path is 6x, area of the other path is 8x. When added together the overlap of area \( x² \) is counted twice, so area is 6x + 8x - \( x² = 14x - x² \).

(b) \((14x - x²) : 48 = 1 : 4 \Rightarrow \frac{14x - x²}{48} = \frac{1}{4} \)

\[ \Rightarrow 14x - x² = 12 \]

Using the formula to solve \( x² - 14x + 12 = 0 \),

\[ a = 1, \ b = -14, \ c = 12 \]

\[ \Rightarrow x = \frac{14 \pm \sqrt{196 - 4 \times 12}}{2} \Rightarrow x = 7 \pm \sqrt{37} \]

\[ x < 6 \Rightarrow x = 0.917 \text{ (3 s.f.)} \]

If \( x > 6 \) m path takes up more than the width of the garden.

5 (a) Volume of sweet is \( [\pi(r + 5)² - \pi r²] \times 3 \)

Volume of hole is \( \pi r² \times 3 \)

Ratio of volumes is 4 : 1

so \([\pi(r + 5)² - \pi r²] \times 3 = 4\pi r² \times 3 \)

\[ r² + 10r + 25 - r² = 4r² \]

Dividing both sides by \( \pi \) and 3 and simplifying

\[ 4r² - 10r - 25 = 0 \]

(b) \[ r = \frac{10 \pm \sqrt{100 + 4 \times 4 \times 25}}{2 \times 4} \]

\[ = \frac{10 \pm \sqrt{500}}{8} \]

\[ = \frac{5 \pm 5\sqrt{3}}{4} \]

The positive value of \( r = 4.0451 \ldots \)

Volume of sweet is \( 4 \times \pi \times 4.0451² \times 3 \)

Volume of sweet is 4 times the volume of the hole

Volume of sugar is \( 0.6 \times 4 \times \pi \times 4.0451² \times 3 \)

= 370 mm³ (3 s.f.)

370 mm³ = 0.370 cm³ (3.s.f.)
6 Substitute \( x = 6 \) into formula for \( y \) to find value of \( k = 12 \)

(a) \( k = 12 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>12</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>2.4</td>
<td>2</td>
</tr>
</tbody>
</table>

(b) 

![Graph](image)

(c) \( x \approx -1.1, 2.2 \) or 3.8

4 (a) \( a = -7, b = 8 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-9</td>
<td>8</td>
<td>11</td>
<td>6</td>
<td>-1</td>
<td>-4</td>
<td>3</td>
<td>26</td>
</tr>
</tbody>
</table>

(b) 

![Graph](image)

(c) \( x \approx -0.6, 2.8 \) or 4.7

5 (a) 

<table>
<thead>
<tr>
<th>( t )</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>20</td>
<td>10</td>
<td>5</td>
<td>3.3</td>
<td>2.5</td>
<td>2</td>
<td>1.7</td>
</tr>
</tbody>
</table>

(b) 

![Graph](image)

(c) \( x = 1 \), \( y = 8 \)

8 (a) 

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>9</td>
<td>252</td>
<td>294</td>
<td>284</td>
<td>237</td>
<td>155</td>
<td>34.3</td>
</tr>
</tbody>
</table>

(b) (i) \( 1 \frac{2}{7} \) months

(ii) \$2222

(c) \( x = 80 \)
(b) By inspection $y_{\text{max}} \approx 295$

(ii) $x^3 + \frac{200}{x} + \frac{200}{x^2} = 210,$

$-x^3 - \frac{200}{x} - \frac{200}{x^2} + 410 = 200$

Draw $y = 200$ and $x$ values in the domain are $x \approx 1.6$ or $5.5$

3 (a) Substitute $t = 0$ into formula for $T$ to find value of $b$, then substitute $t = 5$ into formula for $T$ to find value of $a$.

(a) $a = -6, \ b = 10$

(b) $23.1 \text{ m depth at 00:54}$

(c) $02:54 \leq t \leq 05:36$
(i) 571 approx 
(ii) 15 March approx

5 Substitute $t = 1$ and $t = 10$ into formula for $V$ to form two simultaneous equations:

\[25 = a + b\] \[52 = 10a + \frac{b}{10}\] 

Solve these equations to find values of $a$ and $b$.

(a) $a = 5, b = 20$

(b) $t \geq 4$. Time for speed to be at least 25 m/s is: $12:04 \leq \text{time} \leq 12:10$

SHAPE AND SPACE 7 – BASIC SKILLS EXERCISE

1 (a) $A = 9.72 \text{ cm}^2$, $P = 14.3 \text{ cm}$
(b) $A = 49.1 \text{ cm}^2$, $P = 28.6 \text{ cm}$
(c) $A = 13.1 \text{ cm}^2$, $P = 28.2 \text{ cm}$

2 $r = 4.67 \text{ cm}$, $A = 34.2 \text{ cm}^2$

3 $x = 100^\circ$, $P = 18 \text{ cm}$

4 $A = 92.47 \text{ cm}^2$

5 18.5 cm²

6 $P = 153 \text{ mm} A = 625 \text{ mm}^2$

7 $r = 6 \text{ cm}$, $V = 144\pi$ or 452 cm³

8 4 : 5

9 (a) $A = 2369 \text{ mm}^2$
(b) $V = 8247 \text{ mm}^3$

10 (a) $V = 320 \text{ cm}^3$
(b) $A = 341 \text{ cm}^2$

11 (a) $x = 25.5^\circ$
(b) $V = 126 \text{ cm}^3$
(c) $A = 190 \text{ cm}^2$

12 $V = 24 \text{ cm}^3$

13 (a) $P = 20 \text{ cm}$
(b) $A = 31.4 \text{ cm}^2$

14 $x = 4.5 \text{ cm}$

15 $\frac{4a}{27} \text{ cm}^2$

16 (a) $V = 2048 \text{ cm}^3$, (b) $A = 1875 \text{ cm}^2$

17 $n = 3$

18 (a) Diameter of Moon = 3480 km
(b) surface area of Earth = $5.09 \times 10^8 \text{ km}^2$

SHAPE AND SPACE 7 – EXAM PRACTICE EXERCISE

1 Let $r$ be the radius of the circle so $\pi r^2 = k\pi$
hence $r = \sqrt{k}$

Area of red triangle is $\frac{1}{2} \times r \times r \times \sin120^\circ$

Area of a triangle $= \frac{1}{2} ab \sin C$, $\sin60^\circ = \frac{\sqrt{3}}{2}$

The area of the equilateral triangle is $
6 \times \frac{3}{4} r^2 = 3\frac{3}{4} k$

OR

The base of the green triangle is $r \cos30^\circ = \frac{\sqrt{3}}{2} r$

Height of the the green triangle is $r \sin30^\circ = \frac{r}{2}$

The area of the green triangle is

$\frac{1}{2} \times \frac{\sqrt{3}}{2} \times \frac{r}{2} = \frac{3\sqrt{3}}{8} r^2$

The area of the equilateral triangle is

$6 \times \frac{3}{8} r^2 = 3\frac{3}{4} r^2 = 3\frac{3}{4} k$

Six green triangles make up the equilateral triangle.
There are other equally valid ways of calculating the area of the triangle.
The blue area is $k\pi - \frac{3\sqrt{3}}{4}k$

$$= k\left(\pi - \frac{3\sqrt{3}}{4}\right) \text{ cm}^2$$

2 (a) Curved surface area of a cylinder

$$= 2\pi rh$$

Inside height is $d = 2r \implies$ Curved surface area $= 2\pi r \times 2r = 4\pi r^2$

Total inside surface area

$$= \pi r^2 + 4\pi r^2 = 250$$

Base is $\pi r^2$

$$5\pi r^2 = 250$$

$$r = \frac{25}{\sqrt{5}} = 3.989\ldots \text{ cm}$$

Volume of a cylinder $= \pi r^2h$

Volume $= \pi r^2 \times 2r = 2\pi r^3 = 400 \text{ cm}^3$ (2 s.f.)

(b) Area scale factor $= \frac{360}{250} = \frac{36}{25}$

Length scale factor $= \frac{6}{5}$

Volume scale factor $= \left(\frac{6}{5}\right)^3 = \frac{216}{125}$

or $216 : 215$

3 The maximum number of pieces is when the cube has maximum volume, the cylinder has minimum outside diameter and maximum inside diameter and the pieces have minimum length.

Therefore, the cube side length is 3.05 cm, the cylinder has outside diameter 4.95 mm and inside diameter 3.05 mm and piece length of 5.95 mm.

Working in mm

$$30.5^3 = \frac{1}{4} (4.95^2 - 3.05^2) \times l$$

$$l = 2376.65\ldots$$

number of pieces $= 2376.65\ldots \div 5.95 = 399.4\ldots$

Number of pieces must be an integer so number is 399.

4 (a) Flat end area is $\pi(kr)^2 - \pi r^2 = \pi r^2 (k^2 - 1)$

$$A = 2\pi r^2 (k^2 - 1)$$

Remember there are two flat end faces.

Curved outer area is $2\pi kr \times r = 2\pi kr^2$

Curved area $=$ circumference $\times$ height

Curved inner area is $2\pi r \times r = 2\pi r^2$

$$B = 2\pi kr^2 + 2\pi r^2$$

$$= 2\pi r^2 (k + 1)$$

$$A : B = \frac{2\pi r^2 (k^2 - 1)}{2\pi r^2 (k + 1)}$$

$$= \frac{(k + 1)(k - 1)}{k - 1}$$

$$= k - 1$$

$$k^2 - 1 = (k + 1)(k - 1)$$

‘difference of two squares’

(b) Length scale factor $= 2$ so the area scale factor $= 4$.

Both areas will be 4 times larger so the ratio will not change.

5 (a) The cone that is removed is similar to the original cone.

The area scale factor is $= \frac{a}{4} \div a = \frac{1}{4}$

The length scale factor $= \sqrt[4]{\frac{7}{8}} = \frac{1}{2}$

i.e. removed cone is half the height of the original cone.

Volume scale factor $= \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

So the removed cone has a volume $= \frac{1}{8}V$

Therefore, the volume of truncated cone $= \frac{7}{8}V \text{ cm}^3$.

(b) The total surface area of the removed cone $= \frac{A}{4}$

Area scale factor is $\frac{1}{4}$

Curved surface area of removed cone

$$= \frac{A}{4} - \frac{a}{4}$$

Base area of removed cone $= \frac{a}{4}$

Surface area of truncated cone is surface area of original cone less curved surface of removed cone plus surface area of top of truncated cone.

Surface area of truncated cone

$$A - \left(\frac{A}{4} - \frac{a}{4}\right) + \frac{a}{4} = A - \frac{A}{4} + \frac{a}{4} + \frac{a}{4}$$

$$= \frac{3A}{4} + \frac{a}{2}$$
SETS 3 – BASIC SKILLS EXERCISE

1. (a) \( n(\Omega) = 30 \)

(b) (i) \( \frac{16}{30} = \frac{8}{15} \)

(ii) \( \frac{11}{30} \)

2. (a) \( n(\Omega) = 50 \)

(b) (i) \( P(B) = \frac{14}{50} = 0.28 \)

(ii) \( P(A \cup B) = \frac{44}{50} = 0.88 \)

(iii) \( P(A \cap B) = \frac{8}{50} = 0.16 \)

3. \( n(\Omega) \)

(a) \( P(A \cap B) = 0.2 \)

(b) \( P(A \cup B') = 0.7 \)

4. (a) 45

(b) (i) 29

(ii) 11

(c) (i) \( \frac{20}{45} = \frac{4}{9} \)

(ii) \( \frac{7}{45} \)

(iii) \( \frac{16}{45} \)

5. (a) \( P(X \cup Y) = 0.69 \)

(b) \( P(Y \cap Z') = 0.21 \)

(c) \( P(X \cup (Y \cup Z')) = 0.36 \)

6. \( n(\Omega) \)

(a) \( P(B) = 0.2 \)

(b) \( P(B|A) = \frac{0.1}{0.4} = \frac{1}{4} = 0.25 \)

7. \( n(\Omega) = 50 \)

(a) \( P(A|B) = \frac{50}{50} = 1 \)

(b) \( P(B|A) = \frac{50}{70} = \frac{5}{7} \)
9 (a) \( n(\mathbb{E}) = 100 \)

(b) \( P((B \cup R)^\prime) = \frac{14}{100} = 0.14 \)

(c) \( P(R|B) = \frac{4}{50} = 0.08 \)

10 \( n(\mathbb{E}) = 100 \)

(a) \( P(B) = 0.55 \)

(b) \( P(A'|B) = \frac{0.3}{0.35} = \frac{6}{11} \)

11 \( n(\mathbb{E}) = 200 \)

(a) \((2x + 1) + (3x - 6) + (4x - 3) + 100 = 200 \)
\( x = 12 \)

(b) \( P(B) = \frac{75}{200} = \frac{3}{8} = 0.375 \)

(c) \( P(B|G) = \frac{30}{55} = \frac{6}{11} \)

(d) \( P(G|B) = \frac{30}{75} = \frac{2}{5} = 0.4 \)

12 \( n(\mathbb{E}) = 1000 \)

(a) \( \frac{370}{1000} = \frac{37}{100} \)

(b) \( \frac{370}{570} = \frac{37}{57} \)

(c) \( \frac{200}{530} = \frac{20}{53} \)

13 \( n(\mathbb{E}) = 100 \)

\( P((A \cap C)|B) = \frac{1}{8} = \frac{x}{12} \) so \( x = 4 \)

\( P(A \cap B \cap C) = \frac{4}{100} = \frac{1}{25} = 0.04 \)

14 (a) \( n(\mathbb{E}) = 12 \)

(b) (i) \( P(R \cap E) = \frac{1}{12} \)

(ii) \( P(R \cap E \cap O) = 0 \)

(iii) \( P((R \cup E)^\prime) = \frac{2}{12} = \frac{1}{6} \)

(c) \( P(R|O) = \frac{4}{6} = \frac{2}{3} \)

15 (a) Let \( x = n(B \cap S \cap F) \)

Summing, putting expression = 48 and solving gives \( x = 9 \)
(b) \( V \)

\[
\begin{align*}
B & \quad 0 \\
3 & \quad 12 \\
6 & \quad 9 \\
12 & \quad 5 \\
& \quad 1 \\
& \quad F
\end{align*}
\]

(i) \( \frac{9}{48} = \frac{3}{16} \)

(ii) \( \frac{23}{48} \)

(c) \( \frac{27}{36} = \frac{3}{4} \)

---

**SETS 3 – EXAM PRACTICE EXERCISE**

1  (a) \( 2x(x + 1) + 10x + x^2 + 6x + 160 = 1000 \)
Number of spectators sums to 1000
\( 3x^2 + 18x - 840 = 0 \)
\( x^2 + 6x - 240 = 0 \)
\( (x + 20)(x - 14) = 0 \)
\( x = 14 \) or use quadratic formula \( x = -20 \) is not possible

Venn diagram becomes (numbers represent number of spectators)

\( n(H) = 1000 \)

\[
\begin{array}{ccc}
420 & 140 & 280 \\
160 & & \\
\end{array}
\]

(b) (i) \( \frac{420 + 140 + 280}{1000} = \frac{840}{1000} = 0.84 \)

or \( \frac{1000 - 160}{1000} = \frac{840}{1000} = 0.84 \)

(ii) \( \frac{420 + 280}{1000} = \frac{700}{1000} = 0.7 \)

(c) \( \frac{140}{140 + 280} = \frac{140}{420} = \frac{1}{3} \)

---

2  (a) \( n(H) = 100 \)

\( n(W) = 35 \)

\( n(S) = 26 \)

\( n(K) = 34 \)

(b) (i) \( \frac{34}{100} = \frac{17}{50} = 0.34 \)

(ii) \( \frac{12}{100} = \frac{3}{25} = 0.12 \)

(iii) \( \frac{7}{100} = 0.07 \)

(c) \( P(S|R) = \frac{15}{34} \)

---

3  (a) \( P(A|B) = 0.2 \)
\( \frac{x + 3 + y}{70} = 0.2 \)
\( P(C|A) = 0.32 \)
\( \frac{2x + y}{50} = 0.32 \)

Simplifying gives:
\( x + y = 11 \)
\( 2x + y = 16 \)

Solving simultaneously gives
\( x = 5 \) and \( y = 6 \)

The Venn diagram can now be filled in.

\( n(H) = 136 \)

Total number of elements = 136
(b) \( P(A \cap C) = \frac{16}{136} = \frac{2}{17} \)

(c) \( P(B|C) = \frac{20}{60} = \frac{1}{3} \)

4 (a) \[ \frac{x}{42} = \frac{4}{21} \]
\[ x = 8 \]
\[ P((E \cap S)|E) = \frac{3}{8} \]
\[ \frac{y + 8}{64} = \frac{3}{8} \]
\[ y = 16 \]
\[ n(E \cap M) = 40 \]
\[ z + 8 = 40 \]
\[ z = 32 \]
\[ n(M \cap S) = 10 \]
\[ w + 8 = 10 \]
\[ w = 2 \]
\[ n(E) = 64 \]
\[ p = 8 \]
\[ n(S) = 42 \]
\[ q = 16 \]
\[ n(M) = 60 \]
\[ r = 18 \]

(b) (i) \[ \frac{18}{60} = \frac{3}{10} = 0.3 \]
(ii) \[ \frac{w + x + y + z}{100} = \frac{16 + 8 + 32 + 2}{100} \]
\[ = \frac{58}{100} = \frac{29}{50} = 0.58 \]
(c) \[ \frac{y + z}{64} = \frac{48}{64} = \frac{3}{4} = 0.75 \]

5 (a) \[ n(D) = 0.1 \times 200 = 20 \]
\[ n(ND) = 200 - 20 = 180 \]
\[ \frac{x}{20} = 0.9 \Rightarrow x = 18 \]
\[ \frac{y}{180} = 0.95 = \frac{y}{180} \]
\[ h \Rightarrow y = 171 \]

6 (a) \[ 5 \times 10^5 \text{ cm} \]
(b) \[ 8 \times 10^{-3} \text{ km} \]
(c) \[ 200 \text{ km} \]
(d) \[ 0.6 \text{ mm} \]

(b) \[ \frac{9}{9 + 18} = \frac{1}{3} \]
(c) \[ \frac{2}{2 + 171} = \frac{2}{173} \]

NUMBER 8 – BASIC SKILLS EXERCISE

1 (a) \[ 5 \times 10^5 \text{ cm} \]
(b) \[ 8 \times 10^{-3} \text{ km} \]
(c) \[ 200 \text{ km} \]
(d) \[ 0.6 \text{ mm} \]

2 \[ 9 \times 10^{-6} \text{ km} \]

3 \[ 2 \times 10^5 \text{ micrometres} \]

4 Number of thicknesses of paper = \( 2^{50} \)
height = \( 2^{50} \times 0.004 \times 25.4 \times 10^{-6} \)
= \( 1.14 \times 10^8 \text{ km} \)

5 (a) \[ 5 \times 10^2 \text{ m}^2 \]
(b) \[ 4 \times 10^7 \text{ cm}^2 \]
(c) \[ 4 \times 10^6 \text{ cm}^2 \]
(d) \[ 2 \text{ km}^2 \]

6 \[ 1.96 \times 10^{-4} \text{ km}^2 \]

7 \[ 1.75 \times 10^{25} \text{ cm}^2 \]

8 \[ 5.40 \times 10^{-13} \text{ km}^2 \]

9 (a) \[ 2 \times 10^9 \text{ ml} \]
(b) \[ 4 \times 10^9 \text{ m}^3 \]
(c) \[ 80 \text{ m}^3 \]
(d) \[ 7 \times 10^9 \text{ mm}^3 \]

10 (a) \[ 980 \text{ ml} \]
(b) \[ 0.98 \text{ litres} \]
(c) \[ 9.8 \times 10^{-13} \text{ km}^3 \]

11 Volume of rain drop = \( \frac{4}{3} \times \pi \times 0.75^3 \text{ mm}^3 \),
volume of reservoir = \( 1.24 \times 10^8 \times 10^6 \text{ mm}^3 \).
The number of raindrops = \[ \frac{(1.24 \times 10^8 \times 10^6)}{\left( \frac{4}{3} \times \pi \times 0.75^3 \right)} = 7.02 \times 10^{16} \]
12 1 litre of a chemical contains \(3 \times 10^{22} \times 10^3 = 3 \times 10^{25}\) molecules.

Volume of ocean is \(1.15 \times 10^{10} \times 10^3 = 1.5 \times 10^{21}\) litres

Number of molecules per litre is \((3 \times 10^{25}) \div (1.5 \times 10^{21}) = 2 \times 10^4\) or 20,000

13 1230 km/h

14 133 mm

15 0904

16 4.5 \times 10^{-3}\) seconds

17 2.92 mm

18 Time taken is 19 h 19 mins,

average speed = 880 km/h = 244 m/s

19 50 m³

20 £361.18

21 1.87 g/cm³

22 4.45 g/cm³

23 Density of A is 0.911 (floats), of B is 1.1 (sinks), of C is 0.802 (floats)

24 \(\frac{a + 0.001b}{2}\)

25 \(3 \times 10^{-4}\) N

26 20 400 N/m²

27 1 000 000 cm²

28 4500 cm³

29 102 N

30 \(1.76 \times 10^5\) N/m²

31 Cylinder volume = \(\pi \times 6^2 \times 10 = 360\pi\) cm³

Dimensions changed to cm.

12 tonnes/m³ = \(\frac{12 \times 1000}{10^3} = 1.2 \times 10^{-2}\) kg/cm³

Mass of cylinder = \(360\pi \times 1.2 \times 10^{-2} = \frac{108}{25} \pi\) kg.

This exerts a force of \(10 \times \frac{108}{25} \pi = \frac{216}{5} \pi\) N on the table.

Area in contact with the table = \(\pi \times 6^2 = 36\pi\) cm²

The pressure is \(\frac{216}{5} \pi \div 36\pi = \frac{6}{5}\) N/cm² or 1.2 N/cm²

**NUMBER 8 – EXAM PRACTICE EXERCISE**

1 (a) mass = density \times volume

3 cm³ of gold has a mass of \(3 \times 19.3 = 57.9\) g

1 cm³ of silver has a mass of 10.5 g.

4 cm³ of the alloy has a mass of 57.9 + 10.5 = 68.4 g

Density of alloy is \(\frac{68.4}{4} = 17.1\) g/cm³

(b) \(x\) cm³ of gold has a mass of \(x \times 19.3\) g

1 cm³ of silver has a mass of 10.5 g

1 + \(x\) cm³ of the alloy has a mass of 19.3\(x\) + 10.5 g

Density of alloy is \(\frac{19.3x + 10.5}{1 + x}\)

= 18.1 g/cm³

Solving for \(x\) gives

19.3\(x\) + 10.5 = 18.1(1 + \(x\))

19.3\(x\) + 10.5 = 18.1 + 18.1\(x\)

1.2\(x\) = 7.6

\(x = \frac{19}{3}\)

2 (a) The candle uses \(\frac{1}{15} \times 60 = 4\) g of wax every hour

The wax has a density of \(900 \times \frac{1000}{10^9} = 9 \times 10^{-1}\) g/mm³

4 g of wax has a volume of \(\frac{4}{9 \times 10^{-1}} = 4444.4...\) mm³

Let the distance between marks be \(x\) mm

Volume of cylinder is \(\pi r^2 h\) and \(r = 10\) mm

\(\Rightarrow x \times \pi \times 10^2 = 4444.4... \Rightarrow x = 14.147...\)

or 14.1 mm to 3 s.f.

(b) Volume of cone = \(\frac{1}{3} \pi r^2 h\), top half has

\(r = 20\) mm and \(h = 100\) mm

OR work out volume of whole cone,

top half has \(\left(\frac{1}{2}\right)^3 = \frac{1}{8}\) of volume by similarity

Volume of top half of cone

\(\left(\frac{1}{3}\right) \times \pi \times 20^2 \times 100 = \frac{40000\pi}{3}\) mm³
4 (a) Area of hose = $\pi \times 6^2$ mm$^2$
16 m/s = 16000 mm
Volume of water that comes out of hose in one second
= $\pi \times 6^2 \times 16000$ mm$^3$
= $(\pi \times 6^2 \times 16000) \div 1000$ cm$^3$
= 576$\pi$ cm$^3$
1000 cm$^3$ = 1 litre so 576$\pi$ cm$^3$
= 0.576$\pi$ litres/s
Time taken to fill pond = $\frac{20000}{0.576 \times \pi \times 60}$ minutes
= 184.207… minutes = 3 hours 4 minutes (and 12.4 seconds)

(b) Area of the hose = $\pi \times \frac{d^2}{4}$ mm$^2$
16 m/s = 16000 mm/s
Volume of water that comes out of the hose every second is = $\pi \times \frac{d^2}{4} \times 16000$
= 4000$\pi$d$^2$/mm$^3$ = 4000$\pi$d$^2$/1000 litres
= 4$\pi$d$^2$/litres
2 hours is 2 $\times$ 60 $\times$ 60 seconds = 7200 seconds
7200 = 15000 $\div$ 0.004$\pi$d$^2$ $\Rightarrow$ $d^2$ = $\frac{15000}{7200 \times 0.004\pi}$ = 165.78…
$d$ = 12.9 (3 s.f.)

5 Volume of A = $\pi \times r^2 \times h$ cm$^3$ so the mass of A = $\pi r^2 \times h$ kg
Force A exerts on table = $10\pi r^2$ h N
Each kg exerts a force of 10N
Area of A in contact with table = $\pi r^2$ cm$^2$
pressure exerted by A on table = $\frac{10\pi r^2 \times h}{\pi r^2} = 10hd$ N/cm$^2$
Volume of B = $\pi \times R^2 \times h$ cm$^3$ so the mass of B = $\pi R^2 \times h$ kg
Force B exerts on table = $10\pi R^2$ h N
Each kg exerts a force of 10N
Area of B in contact with table = $\pi R^2$ cm$^2$
The pressure exerted by B on table = $\frac{10\pi R^2 \times h}{\pi R^2} = 10hd$ N/cm$^2$
The pressure exerted by each cylinder is the same and equal to 10hd N/cm$^2$
ALGEBRA 8 – BASIC SKILLS EXERCISE

1 (a) Function as any vertical line only cuts the graph once
(b) Not a function as a vertical line can cut the graph more than once
(c) Not a function as there is no value when \( x = 0 \)
(d) Not a function as a vertical line can cut the graph more than once

2 (a)

(b) It is a function as it is a many-to-one mapping.

3 \( f: x \rightarrow 4x^2 + 12x + 9 \)

4 (a) 1
(b) 11
(c) 7
(d) \( 7 - 2y^2 \)

5 (a) (i) 1.5
(ii) 2.5
(b) 0 as \( 1 ÷ 0 \) is undefined

6 (a) 2
(b) 0

7 (a) 2, 3
(b) \(-1, 6\)

8 (a) \( 2x + 2 \)
(b) \( 2x + 1 \)

9 (a) \( 8 + 6x \)
(b) \( 4 + 6x \)

10 \( f(x) = x^2 + 6x \)

11 \( f(x) = 4x^2 \)

12 \(-5\)

13 1 or \(-\frac{2}{3}\)

14 \( a = -2, b = 4 \)

15 \( f(a) = a^2 + b, f(b) = ab + b \)
\[ f(a) = f(b) \]
\[ a^2 + b = ab + b \]
\[ a^2 - ab = 0 \]
\[ a(a - b) = 0 \]
\[ a = 0 \text{ or } a = b \]

16 (a) \( x = \frac{1}{2} \)
(b) \( x = 0 \)
(c) \( x < -3 \)
(d) \( x \geq 3 \)

17 (a) \(-3 < x < 3 \)
(b) None
(c) \( x = 180n, n \text{ is an integer} \)
(d) \( x = 90 + 180n, n \text{ is an integer} \)

18 (a) All real numbers
(b) \( g(x) \geq -1 \)
(c) \( h(x) \geq 0 \)

19 (a) range = \{\(-1, 0, 3\)\}
(b) Range is all real numbers \( \geq 0 \)
(c) Range is all real numbers \( \geq -4 \)

20 (a) 12
(b) 3
(c) 16
(d) \(-1\)

21 \( x = \frac{3}{4} \)

22 \( k = 1 \) or \( k = -\frac{1}{3} \)

23 \(-\frac{1}{2}\)

24 \( k = 2 \) or \( k = -1 \)

25 \( 2 - 3x^2 \)

26 (a) \( 2 - \frac{x}{7} \)
(b) \( x^2 - 5 \)
(c) \( \frac{4}{x+2} \)

27 \( f: x \rightarrow \frac{\frac{x+2}{1-x}} \)

28 7

29 \( f(x) = x \)

30 (a) \( f^{-1}(x) = \frac{x}{x-1} \)
(b) Self inverse
31 \( f(x) = \frac{x+3}{4} \)
32 \( f(x) = \frac{1}{1-x} \)
33 (a) (i) \( x \)
(ii) \( x \)
(b) Inverse of each other
(c) \( \pi \)
34 (a) (i) \( \sqrt{3x + 3} \)
(ii) \( \sqrt{(x + 1) + 2} \) or \( \sqrt{x + 5} \)
(b) (i) \( x < -1 \)
(ii) \( x < 0 \)
35 \( g(x) = \frac{5x + 4}{3} \) \( g(x) \) is the inverse of \( f(x) \)
36 (a) (i) \( 1 + 3x \)
(ii) \( 5 + 3x \)
(b) \( (fg)^{-1}(x) = g^{-1} f^{-1}(x) = \frac{x-1}{3} \) because \( g^{-1}(x) = \frac{x+1}{3} \) and \( f^{-1}(x) = x - 2 \)
37 \( a = -2, b = 4 \)
38 \( a = \frac{1}{10} \)
39 \( g(x) = 3x + 1 \) \( g(x) = [(6x - 5) + 7] / 2 \)
40 (a) Range is \( \{ y : y \geq 0 \} \)
(b) \( y = \sqrt{x} \), Restriction on domain \( x \geq 0 \)
41 (a) Range is \( \{ y : y \geq -2 \} \)
(b) \( y = \sqrt{x} + 2 \), Restriction on domain \( x \geq -2 \)
42 (a) \( f(x) = (x + 1)^2 - 1 \)
(b) \( \{ y : y \geq -1 \} \)
(c) \( y = (x + 1)^2 - 1 \Rightarrow (x + 1)^2 = y + 1 \Rightarrow x = \sqrt{y + 1} - 1 \Rightarrow f^{-1}(x) = \sqrt{y + 1} - 1 \)
Restriction on domain \( x \geq -1 \)
43 (a) \( f(x) = (x - 1)^2 - 1 + 4 = (x - 1)^2 + 3 \)
(b) Function is a positive quadratic, minimum value at \((1, 3)\)
Cuts the \( y \) axis at \((0, 4)\)
Range is \( \{ y : y \geq 3 \} \)
(c) \( y = (x - 1)^2 + 3 \Rightarrow y - 3 = (x - 1)^2 \Rightarrow x = \sqrt{y - 3} \Rightarrow f^{-1}(x) = \sqrt{y - 3} + 1 \)
Restriction on domain \( x \geq 3 \)
44 (a) \( f(x) = 2[x^2 - 2x + 3] \)
\( = 2((x - 1)^2 + 2) = 2(x - 1)^2 + 4 \)
(b) \( \{ y : y \geq 4 \} \)
(c) \( y = 2(x - 1)^2 + 4 \Rightarrow \frac{y - 4}{2} = (x - 1)^2 \Rightarrow x = \sqrt{\frac{y - 4}{2}} + 1 \Rightarrow f^{-1}(x) = \sqrt{\frac{y - 4}{2}} + 1 \)
Restriction on domain \( x \geq -4 \)
1 (a) (i) \( y = 7 - x \)
\( x = 7 - y \)
\( f^{-1}(x) = 7 - x \)

(ii) \( g(x) \) and \( g^{-1}(x) \) are self inverse

(b) \((x - 1)^2 < 7 - x \)
\( x^2 - x - 6 < 0 \)
\( (x - 3)(x + 2) < 0 \)
\(-2 < x < 3 \)

A sketch of \( y = (x + 2)(x - 3) \) is a positive quadratic passing through \((-2, 0)\) and \((3, 0)\) showing \( x < 0 \) for \(-2 < x < 3 \)

2 (a) \( fg(x) = f(2x + 1) \)
\( = (2x + 1 - 1)^2 \)
\( = 4x^2 \)
\( g(f(x)) = g((x - 1)^2) \)
\( = 2((x - 1)^2) + 1 \)
\( = 2(x^2 - 2x + 1) + 1 \)
\( = 2x^2 - 4x + 3 \)
\( 2g(f(x)) = g(f(x)) \)
\( 8x^2 = 2x^2 - 4x + 3 \)
\( 6x^2 + 4x - 3 = 0 \)

(b) \( 2g(k) = g(k) \)
\( 6k^2 + 4k - 3 = 0 \)
Solving for \( k \) using the quadratic formula
\( k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)
\( k = \frac{-4 \pm \sqrt{4^2 - 4 \times 6 \times -3}}{2 \times 6} \)
\( = \frac{-4 \pm \sqrt{32}}{12} \)
\( = \frac{-4 \pm 4\sqrt{2}}{12} \)
\( = \frac{-2 \pm 2\sqrt{2}}{6} \)
So \( p = -2, q = 22 \) and \( r = 6 \)

3 \( f(3) = 5 \)
\( 5 = 3a + b \) \( \text{Equation 1} \)
\( g(4) = f(2) \) so \( g(4) = 2a + b \)
\( g^{-1}(x) = 3x - 5 \)
\( y = 3x - 5 \)
\( 3x = y + 5 \)
\( x = \frac{y + 5}{3} \)
\( g(x) = \frac{x + 5}{3} \)
\( g(4) = 3 \)
\( 3 = 2a + b \) \( \text{Equation 2} \)

Solving Equation 1 and Equation 2 simultaneously
\( 5 = 3a + b \) \( (1) \)
\( 3 = 2a + b \) \( (2) \)
\( 2 = a \) \( \text{Subtracting (2) from (1)} \)
\( b = -1 \) \( \text{Subtracting } a = 2 \text{ into either (1) or (2)} \)
\( f(x) = 2x - 1 \)
\( 7 = 2x - 1 \)
\( x = 4 \)
\( f^{-1}(7) = 4 \)
Or
\( f^{-1}(x) = \frac{x + 1}{2} \)
\( f^{-1}(7) = 4 \)

4 Answers will differ as read from a graph.
(a) (i) \( fg(2) = 3.8 \)
(ii) \( gf(4) = 6.2 \)
(iii) \( gf^{-1}(3) = 0.57 \)
(b) \( k > 0.4 \)
(c) \(-0.6 \) or \( 2.5 \)

5 (a) Let \( f(x) \) be the radius and \( x \) the area.
\( f(x) \) is the output of a function, \( x \) the input
\( \pi[f(x)]^2 = x \)
\( \pi r^2 = A \)
\( [f(x)]^2 = \frac{x}{\pi} \)
\( f(x) = \sqrt{\frac{x}{\pi}} \)

(b) Let \( g(x) \) be the circumference and \( x \) the radius
\( f(x) \) is the output of a function, \( x \) the input
\( g(x) = 2\pi x \)
\( C = 2\pi r \)
\( g(x) = g[f(x)] = 2\pi \sqrt{\frac{x}{\pi}} \)
\( = 2\pi \sqrt{x} \)
\( g(f(x)) \) gives the circumference of a circle when the area is the input

(d) \( a = 2\pi \sqrt{x} \)
\( a^2 = 4\pi x \)
\( a^2 - 4\pi a = 0 \)
\( a(a - 4\pi) = 0 \)
\( a = 0 \) or \( a = 4\pi \)
\( a = 0 \) is not a real-world solution, so \( a = 4\pi \)
\( 4\pi \) is the only value where the area and circumference are numerically the same
GRAPHS 7 – BASIC SKILLS EXERCISE

1 (a)

(b) (i) $-0.79, 3.8, y = 4$
(ii) $-0.24, 4.2, y = x + 2$
(iii) $-1.3, 2.3, y = 4 - 2x$

2 (a) $y = 2$
(b) $y = 1$
(c) $y = x$
(d) $y = 4 - x$

3 (a) $5x^2 - 3x + 17 = 0$
(b) $4x^2 - 4x - 7 = 0$
(c) $x^2 - 7x + 3 = 0$
(d) $3x^2 - 3x + 5 = 0$

4 (a) No solutions if $x^2 - 4x + 3 < -1$
   \[ \Rightarrow x^2 - 4x < -4 \Rightarrow k < -4 \]
(b) $x^2 - 3x = p$
   \[ x^2 - 4x + 3 = 3 + p - x \]
   There is one solution if $y = -x + 3 + p$ is a tangent to the curve.
   $y = -x + 3 + p$ is a straight line with gradient $-1$ and intercept $3 + p$.
   Using a ruler, the intercept is approximately 0.75 so $p = -2.25$

5 (a) (i) $-1.7, 0, 1.7, y = -x$
(iii) $-1.325, y = x + 1$
   (iii) $-1.8, -0.45, 1.2, y = x^2 - 1$
(b) $k < -1.1$ or $k > 1.1$

6 (a) (i) $-1, 1, y = 2x$
(ii) $-3.4, -0.59, y = \frac{x}{2} - 2$
   (iii) $-2.7, 0.27, 1.4, y = 4 - x^2$
(b) $-2 < k < 2$

7 (a) $-0.6 < k < 2.1$
(b) $3x^3 - 15x^2 + 19x - 3 = 0$
   $3x^3 - 15x^2 + 18x = 3 - x$
   $x^3 - 5x^2 + 6x = 1 - \frac{x}{3}$, find intersections
   with $y = 1 - \frac{x}{3}$, $x = 0.2, 1.8, 3$

8 $(-1.3, 3.3), (2.3, -0.3)$

9 $(-1.5, 0.9), (-0.3, -2.5), (1.9, -1.3)$

10 $(-1.7, -1.3), (0.3, -0.6), (1.8, 2.5)$
1 (a) $\begin{array}{c|cccccccc} x & -2 & -1 & 0 & 1 & 2 & 3 & 4 \\ \hline y & 6 & 1 & -2 & -3 & -2 & 1 & 6 \end{array}$

(b) $\begin{align*}
    x^2 - x &= 3 \\
    x^2 - 2x + x - 2 + 2 &= 3 \\
    x^2 - 2x - 2 &= -x + 1 \\
    \text{The line to plot is } y &= -x + 1 \\
    \text{Solutions are } x &= -1.3 \text{ or } 2.3
\end{align*}$

(c) $\begin{align*}
    x^2 - 3x + k &= 0 \\
    x^2 - 2x - x + k - 2 + 2 &= 0 \\
    x^2 - 2x - 2 &= x - k - 2 \\
    x^2 - 2x - 2 &= x - (k + 2) \\
    \text{The line to plot is } y &= x - (k + 2) \\
    \text{This is a straight line with gradient } 1 \\
    \text{and } y \text{ intercept of } -(k + 2). \\
\end{align*}$

(d) $\begin{align*}
    x^2 - 3x + k &= 0 \\
    x^2 - 2x - x + k - 2 + 2 &= 0 \\
    x^2 - 2x - 2 &= x - k - 2 \\
    x^2 - 2x - 2 &= x - (k + 2) \\
    \text{The line to plot is } y &= x - (k + 2) \\
    \text{This line must be a tangent to the graph} \\
    \text{of } y &= x^2 - 2x - 2 \\
\end{align*}$

Place a ruler on the graph at a gradient of 1, move it until it is a tangent to the curve.

The intercept is $-4.25 = -(k + 2)$

so $k = 2.25$

$2 \leq k \leq 2.5$ is acceptable providing a tangent is drawn on the graph.

2 (a) Draw two horizontal lines on the graph as shown, one touching the maximum point, the other the minimum point. Any horizontal line drawn between these two lines will intersect the graph at 3 points giving three solutions.

(b) $\begin{align*}
    x^3 - 8x + 2 &= 0 \\
    -x^3 + 8x - 2 &= 0 \\
    -x^3 + 7x + x - 6 + 4 &= 0 \\
    -x^3 + 7x - 6 &= -x - 4 \\
    \text{Solutions are the } x \text{ values of the intersections of } y &= -x^3 + 7x - 6 \\
    \text{and } y &= -x - 4 \\
    x &= -2.9 \text{ or } x = 0.3 \text{ or } x = 2.7
\end{align*}$

3 (a) The $x$ values of the intersection of the two graphs are given by

$\begin{align*}
    x^3 - 3x + \frac{1}{x} = \frac{x}{2} - 1 \\
    x^4 - 3x^2 + 1 &= \frac{x^4}{2} - x
\end{align*}$

Multiplying both sides by $x$

$2x^4 - 6x^2 + 2 = x^2 - 2x$

Multiplying both sides by 2

$2x^4 - 7x^2 + 2x + 2 = 0$
(b) Adding the line \( y = \frac{x}{2} - 1 \) to the graph shows there are 4 solutions. Reading from the graph, \( x = -1.9, -0.4, 0.8, 1.6 \) to 1 d.p.

4 (a) The \( y \) values of the points of intersection of the graphs \( x^2 + y^2 = 16 \) and \( y = x^2 - 5 \) are given by eliminating \( x \) between the two equations.

\[
\begin{align*}
x^2 &= y + 5 \\
y^2 + x^2 &= 16 \\
y^2 &= 11
\end{align*}
\]

(b) Reading the \( y \) values from the points of intersection gives \( y = -3.8 \) (3.9) or \( y = 2.8 \) (2.9)

(c) \( y = x^2 + k \) is a parabola with vertex on the \( y \)-axis at \((0, k)\). If the vertex is on the \( y \)-axis and within the circle, then there will be exactly two intersections with the circle. \(-4 < k < 4\)

5 (a) \( \sin x - 2 \cos x = 1 \) so \( \sin x - 1 = 2 \cos x \) and the \( x \) values of the points of intersection of the two graphs are the solutions. \( x = -270^\circ, -145^\circ, 90^\circ, 215^\circ \)

Solutions within \( 5^\circ \) are acceptable

\[
\begin{align*}
\cos x + \frac{x}{180} &= \frac{1}{2} \\
2 \cos x + \frac{x}{90} &= 1 \\
2 \cos x &= -\frac{x}{90} + 1 \\
L \text{ is } y &= -\frac{x}{90} + 1
\end{align*}
\]

(c) \( p + q = \left( \frac{6}{-3} \right) = \left( 2 \right) \)

\[
\begin{align*}
\text{(ii) } 2p - q &= \left( \frac{6}{4} \right) - \left( \frac{3}{-5} \right) = \left( \frac{9}{9} \right)
\end{align*}
\]

(b) \( mp + nq = \left( \frac{12}{-13} \right) = m \left( \frac{3}{2} \right) + n \left( \frac{3}{-5} \right) \)

\[
\begin{align*}
m + n &= 4 \\
2m - 5n &= -13 \\
3m + 3n &= 12 \\
m + n &= 4 \rightarrow (1) \\
2m - 5n &= -13 \rightarrow (2)
\end{align*}
\]

(1): \( m = 4 - n \rightarrow (2) \)

(2): \( 2(4 - n) - 5n = -13 \)

\[
\begin{align*}
8 - 2n - 5n &= -13 \\
-7n &= -21 \\
\end{align*}
\]

\( n = 3, m = 1 \)

(c) \( p + rq = \left( \frac{s}{-8} \right) = s \left( \frac{3}{2} \right) + r \left( \frac{3}{-5} \right) \)

\[
\begin{align*}
3 + 3r &= 5 \\
3 - 5r &= -8 \\
15 + 15r &= 5s \\
6 - 15r &= -24 \\
(3) + (4): 21 &= 5s - 24 \\
4s &= 5s, s = 9, r = 2
\end{align*}
\]

(d) \( u(p + q) + r(2p - q) = r \left( \frac{0}{21} \right) \)

\[
\begin{align*}
u \left( \frac{6}{-3} \right) + v \left( \frac{3}{9} \right) &= \left( \frac{0}{21} \right)
\end{align*}
\]

\[
\begin{align*}
6u + 3v &= 0 \\
-3u + 9v &= 21 \\
-6u + 18v &= 42
\end{align*}
\]

(1) + (3): \( 21v = 42 \)

\( v = 2, u = -1 \)

2 (a) \( p = \left( \frac{5}{0} \right), \sqrt{29} \)

(b) \( q = \left( \frac{1}{0} \right), 1 \)

(c) \( r = \left( \frac{12}{1} \right), \sqrt{145} \)

(d) \( s = \left( \frac{-2}{-42} \right), \sqrt{1768} \)
4. \( m = 2, n = -1 \)

5. (a) \( \overrightarrow{AB} = -a + b \)
   (b) \( \overrightarrow{AM} = -\frac{1}{2}(b - a) \)
   (c) \( \overrightarrow{OM} = -\frac{1}{2}(a + b) \)

6. (a) \( \overrightarrow{AB} = -2x + 2y \)
   (b) \( \overrightarrow{AM} = y - x \)
   (c) \( \overrightarrow{OM} = x + y \)

7. (a) \( \overrightarrow{ED} = p \)
   (b) \( \overrightarrow{DE} = -p \)
   (c) \( \overrightarrow{AC} = p + q \)
   (d) \( \overrightarrow{AE} = 2q - p \)

8. (a) \( \overrightarrow{RS} = -r + s \)
   (b) \( \overrightarrow{OP} = \frac{3}{2}r \)
   (c) \( \overrightarrow{PQ} = -\frac{1}{2}r + 2s \)
   (d) \( \overrightarrow{OM} = s + \frac{3}{4}r \)

9. (a) (i) \( \overrightarrow{PQ} = q - p \)
    (ii) \( \overrightarrow{PR} = \frac{1}{3}(q - p) \)
    (iii) \( \overrightarrow{OR} = p + \frac{1}{3}(q - p) = p + \frac{1}{3}q - \frac{1}{3}p = \frac{2}{3}p + \frac{1}{3}q = \frac{1}{3}(2p + q) \)

(b) (i) \( \overrightarrow{OS} = k \overrightarrow{OR} \)
    \[ = \frac{3}{5} \overrightarrow{OR} = -k = \frac{3}{5} \]
    (ii) \( \overrightarrow{OS} = \frac{3}{5} \times \frac{1}{3}(2p + q) = \frac{1}{5}(2p + q) \)

10. (a) (i) \( \overrightarrow{MP} = \frac{2}{5}p \)
    (ii) \( \overrightarrow{PQ} = q - p \)
    (iii) \( \overrightarrow{PN} = \frac{2}{3}(q - p) \)
    (iv) \( \overrightarrow{MN} = \overrightarrow{MP} + \overrightarrow{PN} \)
    \[ = \frac{2}{5}p + \frac{2}{3}(q - p) = \frac{2}{5}q \]

(b) \( \overrightarrow{OQ} = q \)
    \[ \overrightarrow{MN} = \frac{2}{3}q \]
    OQ is parallel to MN

11. (a) (i) \( a \)
    (ii) \( \frac{1}{2}(a + b) \)
    (iii) \( a - b \)
    (iv) \( \frac{1}{2}(a + b) \)

12. (a) \( w = \begin{pmatrix} -7 \\ 19 \end{pmatrix} \)
    (b) \( \triangle TUV, 339.8^\circ \)
    (c) 5.06 km/h

13. (a) \( \begin{pmatrix} 1 \\ 5 \end{pmatrix} \)
    (b) \( \begin{pmatrix} 13 \\ 10 \end{pmatrix} \)
    (c) 5.10
    (d) 052.4°

14. (a) \[ t \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \]
   \[ r \begin{array}{cccccc} \frac{1}{5} & \frac{3}{4} & \frac{5}{3} & \frac{7}{2} & \frac{9}{1} & \frac{11}{0} \end{array} \]

(b) \[ \text{8050 km/h, 117°} \]

SHAPE AND SPACE 8 – EXAM PRACTICE EXERCISE

1. (a) \( ma - nb = \begin{pmatrix} 8 \\ -11 \end{pmatrix} = c, \) so
   \[ 2m + n = 8 \] (1)
   \[ -m - 4n = -11 \] (2)
   \[ (2) \times 2: -2m - 8n = -22 \] (3)
   \[ (1) + (3): -7n = -14, \text{ so } n = 2 \text{ and } m = 3 \]
   (b) If \( m = 1, n = 1 \text{ and } c = \begin{pmatrix} 3 \\ -5 \end{pmatrix} \).
   (c) \[ \text{45°} \]
   \[ \text{10} \]
   \[ \text{D} \]
   \[ \text{45°} \]
   \[ \text{C} \]
   \[ \text{x} \]
   \[ \text{y} \]
10^2 = 2x^2
x^2 = 50
x = \sqrt{50}
= 5\sqrt{2}

So vector \( \mathbf{d} = \left( \frac{3}{5} \right) + \left( 5\sqrt{2} \right) \)

\[ \begin{align*}
2 (a) \quad \overrightarrow{OP} &= \overrightarrow{OA} + \overrightarrow{AP} \\
&= \overrightarrow{OA} + \frac{m}{m+n} \overrightarrow{AB} \\
&= \mathbf{a} + \frac{m}{m+n} (\mathbf{b} - \mathbf{a}) = \frac{(m+n)\mathbf{a} + m(\mathbf{b} - \mathbf{a})}{m+n} \\
&= \frac{m\mathbf{a} + mb}{m+n}
\end{align*} \]

(b) \[ \overrightarrow{OP} = \frac{2\left( \frac{4}{3} \right) + 3\left( \frac{2}{1} \right)}{3+2} = \left( \frac{-7}{9} \right) \]

\[ \overrightarrow{OP} = \frac{1}{5} \sqrt{(-2)^2 + 9^2} = \frac{1}{5} \sqrt{85} = \frac{1}{5} \sqrt{5 \times 17} = \frac{17}{5} \]

as required

3 (a) (i) \( t = 0, \mathbf{r} = \left( \frac{1}{2} \right) \)

(ii) \( t = 4, \mathbf{r} = \left( \frac{-3}{10} \right) \)

(b) (Each hour that passes, the boat travels along vector \( \left( -\frac{1}{2} \right) \) km, so the length of this vector is the distance travelled in 1 h)

Speed = \( \sqrt{(-1)^2 + 2^2} = \sqrt{5} \) km/h

(c) At 15:00, \( t = 3, \mathbf{r} = \left( \frac{-2}{8} \right) \) km

Bearing of boat from \( L = 180^\circ + \theta^\circ \)

\[ \tan(\theta) = \frac{6}{3} = 2, \text{ so } \theta = 63.43^\circ \ldots \]

Bearing of boat from \( L = 180^\circ + 63.43^\circ \ldots = 243^\circ \) (3 s.f.)

4 \[ \overrightarrow{AQ} = k \overrightarrow{AB} = k(\mathbf{b} - \mathbf{a}) \]

\[ \overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AM} = \mathbf{a} + \frac{3}{4} \left( \frac{1}{2} \mathbf{b} - \mathbf{a} \right) = \frac{1}{4} \mathbf{a} + \frac{3}{8} \mathbf{b} \]

\[ \overrightarrow{OQ} = \mathbf{a} + \lambda (\mathbf{b} - \mathbf{a}) = \mathbf{a} + \lambda \mathbf{b} - \lambda \mathbf{a} = (1 - \lambda) \mathbf{a} + \lambda \mathbf{b} \]

(If vector \( \mathbf{mp} \) is collinear with vector \( \mathbf{nq} \), then \( \frac{m}{n} = \text{a constant} \))

Now \( OPQ \) are collinear so:

\[ \frac{\lambda}{1-\lambda} = \frac{3}{8} = \frac{3}{2} \]

\[ 2\lambda = 3 - 3\lambda \]

\[ 5\lambda = 3 \]

\[ \lambda = \frac{3}{5} \]

\[ \overrightarrow{AQ} = \overrightarrow{AO} + \overrightarrow{OQ} = -\mathbf{a} + \left( 1 - \frac{3}{5} \right) \mathbf{a} + \frac{3}{5} \mathbf{b} = -\frac{3}{5} \mathbf{a} + \frac{3}{5} \mathbf{b} = \frac{3}{5} (\mathbf{b} - \mathbf{a}) = \frac{3}{5} \overrightarrow{AB} \]

\( AQ: QB = 3 : 2 \)
5 (a) \[ \overrightarrow{CD} = \overrightarrow{CB} + \overrightarrow{BA} + \overrightarrow{AD} \]
\[ = -c - b + 3e = 2c - b \]
(b) If \( BPD \) are collinear \( \overrightarrow{BP} = k \overrightarrow{BD} \)
\[ \overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{CD} = c + 2c - b = 3c - b \]
Now \( \overrightarrow{BP} = \overrightarrow{BA} + \lambda \overrightarrow{AC} = -b + \lambda (b + c) \)
\[ = (\lambda - 1)b + \lambda c \quad (1) \]
(If vector \( m\mathbf{p} \) is collinear with vector \( n\mathbf{q} \), then \( \frac{m}{n} = \text{a constant} \))
\[ \frac{\lambda}{\lambda - 1} = \frac{3}{1}, \quad \lambda = \frac{3}{4}, \quad \text{so from (1)} \]
\[ \overrightarrow{BP} = \frac{1}{4}(3c - b) \text{ so } \overrightarrow{BP} = \frac{1}{4} \overrightarrow{BD} \]
\[ \overrightarrow{AP} = \overrightarrow{AB} + \overrightarrow{BP} = b + \frac{1}{4} (3c - b) \]
\[ = \frac{3}{4}(b + c) = \frac{3}{4} \overrightarrow{AC} \]
\[ \overrightarrow{AP} : \overrightarrow{PC} = 3 : 1 \]
(c) \( \overrightarrow{CD} = 2c - b = 2\begin{pmatrix} \frac{3}{0} \end{pmatrix} - \begin{pmatrix} \frac{4}{1} \end{pmatrix} = \begin{pmatrix} \frac{2}{-1} \end{pmatrix} \)
\[ |\overrightarrow{CD}| = \sqrt{(2)^2 + (-1)^2} = \sqrt{5} \]

HANDLING DATA 5 – BASIC SKILLS EXERCISE

1 (a) \( \frac{1}{12} \)
(b) \( \frac{5}{24} \)
(c) \( \frac{7}{24} \)

2 \( P(O_1E_2 \text{ or } E_1O_2) = P(O_1E_2) + P(E_1O_2) \)
\[ = \frac{4}{7} \times \frac{3}{6} + \frac{3}{6} \times \frac{4}{6} = \frac{24}{42} = \frac{4}{7} \]
\( P(A \text{ or } B) = P(A) + P(B) \text{ if } A \text{ and } B \text{ are mutually exclusive} \)

3 Either all 3 discs are even or 1 is even and two are odd to sum an even number.
\( P(E_1E_2E_3 \text{ or } E_1O_2O_3 \times 3) \)
\[ = P(E_1E_2E_3) + P(E_1O_2O_3) \times 3 \]
\[ = \frac{4}{9} \times \frac{3}{8} \times \frac{2}{7} + \frac{4}{9} \times \frac{5}{8} \times \frac{4}{7} \times 3 \]
\[ = \frac{264}{504} \]
\[ = \frac{11}{21} \]
\( P(A \text{ or } B) = P(A) + P(B) \text{ if } A \text{ and } B \text{ are mutually exclusive} \)

4 Let \( Z \) be the number of \( Z \)'s that are revealed
\[ P(Z \geq 1) + P(Z = 0) = 1 \]
\[ P(Z \geq 1) = 1 - P(Z = 0) \]
\[ = 1 - \left[ P(Z_1'Z_2'Z_3') \right] \]
\[ = 1 - \frac{6}{9} \times \frac{5}{8} \times \frac{4}{7} \]
\[ = 1 - \frac{5}{21} \]
\[ = \frac{16}{21} \]
Scarlett is correct.

6 (a) \( \frac{1}{4} \)
(b) \( \frac{1}{2} \)
(c) \( \frac{1}{4} \)

7 (a) \( \frac{1}{15} \)

(b) (i) \( \frac{1}{225} \)
(ii) \( \frac{196}{225} \)
(iii) \( \frac{28}{225} \)
8 (a)

Box A

Good Apple

Rotten Apple

Good Lemon

Bad Lemon

Box B

(b) \( \frac{3}{7} \)

(c) \( \frac{1}{10} \)

9 (a)

Cough

Cold

No Cough

(b) (i) \( \frac{1}{21} \)

(ii) \( \frac{3}{14} \)

(iii) \( \frac{13}{42} \)

HANDLING DATA 5 – EXAM PRACTICE EXERCISE

1 (a) Let event \( F \) be a female baby giant panda.

Let event \( M \) be a male baby giant panda.

(b) \( P(M_1M_2) = P(M_1) \times P(M_2) \)

\[ = \frac{3}{7} \times \frac{3}{7} = \frac{9}{49} \]

P(\( A \) and \( B \)) = \( P(A) \times P(B) \)

(c) \( P(M_1F_2 \text{ or } F_1M_2) \)

\[ = P(M_1F_2) + P(F_1M_2) \]

\[ = \frac{3}{7} \times \frac{4}{7} + \frac{4}{7} \times \frac{3}{7} = \frac{24}{49} \]

\( P(A \text{ or } B) = P(A) + P(B) \) if \( A \) and \( B \) are mutually exclusive

2 (a)

<table>
<thead>
<tr>
<th>1st Shot</th>
<th>2nd Shot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hits</td>
<td>( \frac{2}{3} )</td>
</tr>
<tr>
<td>Misses</td>
<td>( \frac{1}{3} )</td>
</tr>
</tbody>
</table>

Table completed as \( P(E) + P(E') = 1 \)

Let event \( H \) be a hit of the bullseye.

Let event \( M \) be a miss of the bullseye.
1st Shot | 2nd Shot
---|---
\[ \frac{2}{3} \] | \[ \frac{4}{5} \]
\[ \frac{1}{3} \] | \[ \frac{4}{5} \]
\[ \frac{2}{3} \] | \[ \frac{1}{5} \]
\[ \frac{1}{3} \] | \[ \frac{1}{5} \]

(b) (i) \[ P(H_1 H_2) = P(H_1) \times P(H_2) \]
\[ = \frac{2}{3} \times \frac{4}{5} = \frac{8}{15} \]
\[ P(A \text{ and } B) = P(A) \times P(B) \]

(ii) \[ P(H = 1) = P(H_1 M_2 \text{ or } M_1 H_2) \]
\[ = P(H_1 M_2) + P(M_1 H_2) \]
\[ = \frac{2}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{4}{5} = \frac{6}{15} = \frac{2}{5} \]
\[ P(A \text{ or } B) = P(A) + P(B) \text{ if } A \text{ and } B \text{ are mutually exclusive} \]

(iii) \[ P(H \geq 1) + P(H = 0) = 1 \]
\[ P(H \geq 1) = 1 - P(H = 0) \]
\[ = 1 - P(M_1 M_2) \]
\[ P(E) + P(E') = 1 \]
\[ = 1 - \frac{1}{3} \times \frac{1}{5} \]
\[ = 1 - \frac{1}{15} \]
\[ = \frac{14}{15} \]

3 (a) Let event T be a teddy bear is selected. Let event K be a kangaroo is selected.

1st Pick | 2nd Pick
---|---
\[ \frac{3}{5} \] | \[ \frac{1}{2} \]
\[ \frac{3}{5} \] | \[ \frac{1}{4} \]
\[ \frac{2}{5} \] | \[ \frac{1}{4} \]

(b) (i) \[ P(T_1 T_2) = P(T_1) \times P(T_2) \]
\[ = \frac{3}{5} \times \frac{1}{2} = \frac{3}{10} \]
\[ P(A \text{ and } B) = P(A) \times P(B) \]

(ii) \[ P(T_1 K_2 \text{ or } K_1 T_2) \]
\[ = P(T_1 K_2) + P(K_1 T_2) \]
\[ = \frac{3}{5} \times \frac{1}{2} + \frac{2}{5} \times \frac{3}{4} \]
\[ = \frac{12}{20} = \frac{3}{5} \]
\[ P(A \text{ or } B) = P(A) + P(B) \text{ if } A \text{ and } B \text{ are mutually exclusive} \]

(iii) \[ P(K \geq 1) + P(K = 0) = 1 \]
\[ P(K \geq 1) = 1 - P(K = 0) \]
\[ P(E) + P(E') = 1 \]
\[ = 1 - \frac{3}{10} = \frac{7}{10} \]

4 (a)

\[ \frac{3}{5} \]
Pass theory
\[ \frac{2}{3} \]
Fill practical
\[ \frac{3}{5} \]
Pass practical

(b) \[ P(FT \text{ or } PT,FP) = P(FT) + P(PT,FP) \]
\[ = \frac{2}{5} + \frac{3}{5} \times \frac{2}{3} = \frac{4}{5} \]
\[ P(A \text{ or } B) = P(A) + P(B) \text{ if } A \text{ and } B \text{ are mutually exclusive} \]

(c) \[ P(PT,FP,PA) = \frac{3}{5} \times \frac{1}{3} \times \frac{1}{4} = \frac{1}{20} \]
\[ P(A \text{ and } B) = P(A) \times P(B) \]

5 (a) Let d be the number of diamonds in the box
Let r be the number of rubies in the box
\[ P(D_1 D_2) = P(D_1) \times P(D_2) \]
\[ = \frac{d}{20} \times \frac{d-1}{19} = \frac{21}{38} \]
\[ P(A \text{ and } B) = P(A) \times P(B) \]
\[ d(d - 1) = 210 \]
\[ d^2 - d - 210 = 0 \]
\[ (d - 15)(d + 14) = 0 \]
\[ d = 15 \text{ and } r = 5 \]
(b) \( P(D_1R_2 \text{ or } R_1D_2) = P(D_1R_2) + P(R_1D_2) \)

\[ = \frac{15}{20} \times \frac{5}{19} + \frac{5}{20} \times \frac{15}{19} = \frac{30}{76} = \frac{15}{38} \]

\( P(A \text{ or } B) = P(A) + P(B) \) if \( A \) and \( B \) are mutually exclusive

**NUMBER 9 – BASIC SKILLS EXERCISE**

1. $660
2. $625
3. Yes. Saskia has been over charged by £4.
4. €4140
5. 15 hours
6. $700
7. €1233.75
8. 2 h 30 mins
9. $885 (Aus)
10. £892.86
11. 44.32 yuan
12. 5200 reais
13. 804.74 yuan
14. £10 050.25
15. Australia: £2127.12
   Brazil: £2000
   Spain: £2200
   Cheapest purchase is in Brazil.
   811.85 ringitts, 14 740.74 rupees, 3920 yuan
17. A: 33.3 g/£, B: 33.3 g/£ so same value!
18. Square: €25/m², Octagonal: €24/m², Octagonal are better value.
19. Everamp: 20 h/£, Dynamo: 24 h/£, Dynamo is better value.
20. kg bag $1.90 per kg, 21 kg bag $1.75 per kg
   so 12 kg bag better value per kg.

**NUMBER 9 – EXAM PRACTICE EXERCISE**

1. Let \( x \) be the amount that Aria receives.
   Aria: \( 0.25x \)
   Blake: \( 1.25x \)
   Chloe: \( x \)
   So, \( 1750 = x + 0.25x + 1.25x = 2.5x \)
   \( x = 700 \)
   So Aria: €700 Blake: €175 Chloe: €875

2. Total amount paid by Kofi
   = \( 24 \times €36 = €864 \)
   Money received by Kofi
   = \( \frac{2}{3} \times 24 \times 30 \times €1.80 = €864 \)
   Kofi’s overall profit = \( 0.25 \times €864 = €216 \)
   Number of bottles left
   = \( \frac{1}{3} \times 24 \times 30 = 240 \)
   So price of each bottle to secure 25% profit
   = \( \frac{€216}{240} = €0.90 \) per bottle

3. (a) Total parts = 6, so 1 part of £7200
   = £1200
   Malaysia: £1200 = 5.48 × £1200
   = 6576 ringitts
   India: £2400 = 99.50 × £2400
   = 238 800 rupees
   China: £3600 = 8.82 × £3600
   = 31 752 yuan
   (b) Return journey: 25% of 31752 yuan
   = 7938 yuan
   If 1€ = 8 yuan, 1 yuan = €\( \frac{1}{8} \) euros
   = €992.25

4. Tangerine cost = \( $25 + $0.35 \times 60 \times 24 \)
   = $529 \times 1.175 = $621.58
   Aardvark cost = \( $90 + $0.30 \times 60 \times 24 \)
   = $522 \times 1.175 = $613.35
   So Aardvark is cheaper by $8.23 and is therefore better value assuming the quality of service is the same.
   \( p = 8.23 \)

5. (a) \( g = 1.025 \times £44 940, \) so \( g = £46 063.50 \)
   (b) \( s \times 0.96 = £600, \) so \( s = £625 \)
   (c) Price of 1 kg of gold on 1 Jan 2022
   = \( 0.975 \times £44 940 = £43 816.50 \)
   Price of 1 kg of silver on 1 Jan 2022
   = \( 1.04 \times £625 = £650 \)
   500 g of gold = \( 0.5 \times £43 816.50 = £21 908.25 \)
   500 g of silver = \( 0.5 \times £650 = £325 \)
   Total price = £22 233.25
**ALGEBRA 9 – BASIC SKILLS EXERCISE**

1. \((-4, 12), (3, 5)\)
2. \((3, 2), (-3, -2)\)
3. \((3, 1), (9, 4)\)
4. \((2, 3), (3, 2)\)
5. \((1, 1)\)
6. \((-4.16, -6.16), (2.16, 0.162)\)
7. \((3.92, 1.08), (-1.92, 6.92)\)
8. \((3.24, 1.24), (-1.24, -3.24)\)
9. \((91.65, 18.33)\)
10. Equation of top of egg cup is \(y = 4\).
11. Eliminating \(y\) gives \(x^2 - 4x + 5 = 0\). Using quadratic formula \(b^2 - 4ac = 16 - 20 = -4\)
12. Substituting \(y = k - x\) into \(x^2 + y^2 = 25\)
gives \(2x^2 - 2kx + (k^2 - 25) = 0\). This has one solution if, using quadratic formula, \(b^2 - 4ac = 0\). Substituting values gives \(4k^2 = 4 \times 2 \times (k^2 - 25)\)
\(k^2 = 50\)
\(k = \pm 5\sqrt{2}\)
13. \(p = 4, 2^p - 1 = 15\)
14. \(x = 0\)
15. \(n = 5\) gives 99 which is not prime.
16. False if \(a\) and \(b\) are of opposite signs; for example, \((-2)^3 = (2)^3\) but \(-2 \neq 2\)
17. \((2m + 1) - (2n + 1) = 2(m - n)\) which is even.
18. \((2n - 1) + (2n + 1) + (2n + 3) = 6n + 3\) this leaves remainder 3 when divided by 6
19. \(S = \frac{n}{2}[2 + (n - 1)2] = \frac{n}{2}[2n] = n^2\)
20. Sum = \(\frac{n}{2}[2a + (n - 1)].\) Let \(n = 2p + 1\)
\(S = (2p + 1)\left[a + \frac{2p + 1 - 1}{2}\right] = (2p + 1)(a + p)\) which is divisible by \(n = (2p + 1)\)

21. \((2n - 1)(2n + 1)(2n + 3)\)
\[= 8n^3 + 12n^2 - 2n - 3\]
\[= 2(4n^3 + 6n^2 - n - 1) - 1\]
22. \(n(n + 1) + (n + 1) = n^2 + 2n + 1 = (n + 1)^2\)
23. \((2m + 1)^2 = 4n^2 + 4n + 1 = 4(n^2 + n) + 1\)
24. \((2m + 1)^2 - (2n + 1)^2 = 4(m^2 + m - n^2 - n)\)
\[= 4[m(m + 1) - n(n + 1)]\]
Either \(m\) or \(m + 1\) is even with a factor of 2, likewise \(n\) and \(n + 1\) hence 8 is a factor.
25. \((n + 1)^2 - n^2 = 2n + 1 = (n + 1) + n\)
26. \((x - 4)^2 + 1\)
27. \(-x - 3\)
28. \(2x^2 - 24x + 73 = 2(x - 6)^2 + 1 > 0\)
29. \((2x + 1)^2 \geq 0\)
30. \(x^2 + 14x + c = (x + 7)^2 - 49 + c\)
\(c - 49 \geq 0\)
\(c \geq 49\)
31. \(x^2 + bx + 4 = \left(x + \frac{b}{2}\right)^2 - \frac{b^2}{4} + 4\)
\[4 - \frac{b^2}{4} \geq 0\]
\(b^2 \leq 16\)
\(-4 \leq b \leq 4\)
32. \(\sqrt{3a(\sqrt{18a} + \sqrt{2a})} = \sqrt{36a^2}\)
\[+ 2a = 6a + 2a = 8a\]
33. \(2^{127} - 2 = 2(2^{126} - 1)\)
\(2^{127} - 2\) has a factor of 2.
\(2^{127} - 2, 2^{127} - 1, 2^{127}\) are three consecutive integers. \(2^{127}\) is even, \(2^{127} - 1\) is prime, \(2^{127} - 2\) has a factor of 3 as one of any three consecutive integers has a factor of 3. \(2^{127} - 2\) has a factor of 6.
34. Using the quadratic formula gives \(b^2 - 4ac\)
\[= (2\sqrt{c})^2 - 4c = 0\] therefore there is only one solution.
35. \(abc = 100a + 10b + 5 = 5(20a + 2b + 1)\)
36. \(100a + 10b + c = 100a + 10(a + c) + c = 110a + 11c = 11(10a + c)\)
37. \([(n + 1)^2 - 6(n + 1) + 10] - [n^2 - 6n + 10]\)
\[= n^2 + 2n + 1 - 6n - 10 - n^2 + 6n - 10\]
\[= 2n - 5\] which is an odd number.
Midpoint of $AB$ is $\left(\frac{-1+2}{2}, \frac{5+2}{2}\right) = \left(\frac{1}{2}, \frac{7}{2}\right)$

Midpoint is mean of the coordinates.

Gradient of $AB$ is $-1$

Gradient of perpendicular is $1$

Equation of $CD$ is $y = x + 3$

For perpendicular lines, product of gradients $= -1$

Substituting midpoint gives $\frac{7}{2} = \frac{1}{2} + C$

$C = 3$ so equation of $CD$ is $y = x + 3$

Therefore, the coordinates of $C$ and $D$ are given by the solutions to the simultaneous equations:

\[
y = x^2 - 2x + 2 \quad \text{and} \quad y = x + 3
\]

\[
x^2 - 2x + 2 = x + 3
\]

\[
x^2 - 3x - 1 = 0
\]

Using the quadratic formula with $a = 1$, $b = -3$ and $c = -1$ gives

\[
x = \frac{3 \pm \sqrt{9 + 4}}{2} = \frac{3 \pm \sqrt{13}}{2}
\]

Substituting into $y = x + 3$ gives

\[
C = \left(\frac{3 - \sqrt{13}}{2}, \frac{9 - \sqrt{13}}{2}\right)
\]

\[
D = \left(\frac{3 + \sqrt{13}}{2}, \frac{9 + \sqrt{13}}{2}\right)
\]
3 (a) Coordinates of $A$ and $B$ are given by the solutions to the simultaneous equations

\[4x^2 + y^2 - 4y = 0 \text{ and } 2x + y = 3\]

\[2x + y = 3\]
\[y = 3 - 2x\]
\[y^2 = 9 - 12x + 4x^2\]

Substituting into $4x^2 + y^2 - 4y = 0$ gives

\[4x^2 + 9 - 12x + 4x^2 - 12 + 8x = 0\]
\[8x^2 - 4x - 3 = 0\]

Solving using the quadratic formula gives $x = \frac{1 + \sqrt{7}}{4}$,

When $x = \frac{1 + \sqrt{7}}{4}$,
\[y = 3 - 2 \times \frac{1 + \sqrt{7}}{4} = \frac{5 - \sqrt{7}}{2}\]

so the coordinates of $B$ are \(\left(\frac{1 + \sqrt{7}}{4}, \frac{5 - \sqrt{7}}{2}\right)\)

When $x = \frac{1 - \sqrt{7}}{4}$,
\[y = 3 - 2 \times \frac{1 - \sqrt{7}}{4} = \frac{5 + \sqrt{7}}{2}\]

so the coordinates of $A$ are \(\left(\frac{1 - \sqrt{7}}{4}, \frac{5 + \sqrt{7}}{2}\right)\)

Distance in the $x$ direction between $A$ and $B$ is $\frac{1 + \sqrt{7}}{4} - \frac{1 - \sqrt{7}}{4} = \sqrt{7}$

Distance in the $y$ direction between $A$ and $B$ is $\frac{5 + \sqrt{7}}{2} - \frac{5 - \sqrt{7}}{2} = \sqrt{7}$

\[AB^2 = \left(\frac{\sqrt{7}}{2}\right)^2 + \left(\frac{\sqrt{7}}{2}\right)^2 = \frac{7}{4} + 7 = \frac{35}{4}\]
\[AB = \frac{\sqrt{35}}{2} \text{ cm}\]

(b) \(a^2 - b^2 = (a + b)(a - b)\)

(i) As $a^2 - b^2$ is a prime number then one of the factors $(a + b)$ or $(a - b)$ must equal 1.

$a + b = 1$ so $a = 1 - b$ implies that $a$ is negative as $b > 0$.

But $a > 0$ so $a + b \neq 1$

$a - b = 1$ so $a = 1 + b$ implies that $a$ and $b$ are consecutive integers.

(ii) $a^2 - b^2 = (a + b) \times 1 = a + b$

$= a^2 - b^2$ which is prime, so $a + b$ is also prime.

5 (a) From the formula sheet, the sum to $n$ terms is given by $S = \frac{a}{2}[2a + (n - 1)d]$

where $a$ is the first term and $d$ the common difference.

$S = \frac{n}{2} [2 \times 2 + (n - 1)4] = \frac{n}{2}[4 + 4(n - 1)]$

$= \frac{4n}{2}[1 + n - 1] = 2n^2$ which is double a square number.

(b) The $n$th term of $S$ is $a + (n - 1)d$

$= 2 + 4(n - 1) = 4n - 2$

$n$th term squared $= (4n - 2)^2$

$= 16n^2 - 16n + 4$

$n$th term squared + 12 $= 16n^2 - 16n + 4 + 12 = 16(n^2 - n + 1)$ which is divisible by 16.

GRAPHS 8 – BASIC SKILLS EXERCISE

1 (a) (i) $-1.2$

(ii) $1.8$

(b) (i) $(3.2, -1.6)$ and $(0.8, 1.6)$

(ii) $(5.1, -1.0)$ and $(-1.1, 1.0)$

(iii) $(4.0, -2.1)$ and $(0, 2.1)$

2 (a) $-1.2$

(b) $y = -1.2x + 2.1$

3 $1.76$ and $-1.5$

4 $2.4$ and $1.5$
5 (a) \[ \begin{array}{c|cccc}
 t & 0 & 1 & 2 & 3 & 4 \\
 \hline
d & 0 & 0.25 & 1 & 2.25 & 4 \\
\end{array} \]

(b) Gradient gives velocity
(i) 0 m/s
(ii) 0.5 m/s
(iii) 1 m/s
(iv) 1.5 m/s
(v) 2 m/s

(c) Gradient gives velocity
(i) 1.6 m/s
(ii) −1.6 m/s

(d) 6.9 s

6 (a) (i) \[ \begin{array}{c|cccccccc}
 t & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
 \hline
d & 0 & 3.2 & 3.6 & 2.4 & 0.8 & 0 & 1.2 & 5.6 & 14.4 \\
\end{array} \]

(ii) Gradient gives acceleration
Acceleration is constant and equal to 0.5 m/s²

7 (a) \[ \begin{array}{c|cccccccc}
 t & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
 \hline
r & 0 & 10 & 20 & 15 & 10 & 5 & 0 & 5 & 10 \\
\end{array} \]

(b) Gradient gives acceleration
(i) 8 m/s²
(ii) 4 m/s²

(c) 2.5 s

8 (a) (i) \[ \begin{array}{c|cccccccccccc}
 t \text{(days)} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
 \hline
d \text{(cm)} & 20 & 17 & 14.5 & 12.3 & 10.4 & 8.87 & 7.54 & 6.41 & 5.45 \\
\end{array} \]

(ii) \[ \begin{array}{c|cccccccccccc}
 t & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
 \hline
20 & 17 & 14.5 & 12.3 & 10.4 & 8.87 & 7.54 & 6.41 & 5.45 \\
\end{array} \]

(b) Approximately 4.3 days
(c) 2.3 cm/day
(d) 61.2 cm/day = 0.05 cm/h

9 (a) (1, 4)
(b) \((-1, 2)\)

(c) \((1, 4)\)

(d) \((0.5, 2)\)

15 (a) \((0, k)\)
(b) \((0, -2k)\)
(c) \((0, k - 2)\)

16 (a) \(k\)
(b) \(2k\)
(c) \(2k\)
(d) \(k\)

17 (a) \(-m\)
(b) \(-m\)
(c) \(\frac{m}{2}\)
(d) \(m\)

18 (a) \((0, p - 3)\)
(b) \((0, p)\)
(c) \((0, 3p)\)
(d) \((0, p)\)

19 (a) 3
(b) 2
(c) -1

20 (a) \((-6, 4)\)
(b) \((-1, -3)\)

**GRAPHS 8 – EXAM PRACTICE EXERCISE**

10 (a) \((-2, 2)\)
(b) \((2, -2)\)
(c) \((-2, -1)\)
(d) \((-4, -2)\)

11 \(y = f(x + 2)\) and \(y = f(-x)\)

12 \(y = -x^2 + 2x + 3\)

13 \(y = -[\sin(x) + 2] = -\sin(x) - 2\)

14 (a) \(y = x^2 + 4x + 4\)
(b) \(y = x^2 - 4x + 4\)

(a) Gradient is approximately \(-0.0175\)
(b) Gradient \(-m\) at \((330^\circ, 0.5)\)
(c) See graph. \(x = 215^\circ\) or \(325^\circ\)
(d) The cosine curve is periodic with period $360^\circ$

$$x = 215 + 360 = 575^\circ \text{ or }$$

$$x = 325 + 360 = 685^\circ$$

2

(a) $C = \text{approximately } 2.5 \text{ or }$

$C = \text{approximately } -2.5 \text{ (see graph)}$

(b) $m = \text{approximately } 1.1 \text{ or }$

$m = \text{approximately } -1.1 \text{ (see graph)}$

3

(a) See graph. Gradient is $-2$

(b) (i) Stretch with scale factor $\frac{1}{2}$ parallel to the $x$-axis

so $P$ becomes $\left(\frac{3}{a}, 2\right)$, gradient

is $a \times -2 = -2a$

(ii) Translation of $\left(-\frac{a}{2}, 0\right)$ so $P$ becomes

$(3-a, 2)$, gradient is unchanged

$= -2$

(iii) Stretch scale factor $a$ parallel to the $y$-axis so $P$ becomes $(3, 2a)$, gradient is $a \times -2 = -2a$

(iv) Reflection in the $y$-axis so $P$ becomes $(-3, 2)$, gradient becomes $2$. 

4

(a) $t(s)$  1  2  3  4  5

$v(m/s)$  3  6  9  12  15

(b) (c) Gradient is equal to 3 therefore the acceleration is $3 \text{ m/s}^2$

5

(a) (i) Translation of $\left(\begin{array}{c} 2 \\ 0 \end{array}\right)$

(ii) Stretch scale factor 2 parallel to $x$-axis

(iii) Reflection in the $x$-axis
(b) \( g(x) \) has been reflected in the \( y \)-axis then translated by \( \left( \begin{array}{l} 0 \\ -2 \end{array} \right) \).

The reverse of this is translated by \( \left( \begin{array}{l} 0 \\ 2 \end{array} \right) \) then reflect in the \( y \)-axis.

\((5, 3)\) translated by \( \left( \begin{array}{l} 0 \\ -2 \end{array} \right) \) gives \((5, 1)\).

\((5, 1)\) reflected in the \( y \)-axis gives \((-5, 1)\) \(\Rightarrow\) turning point is \((-5, 1)\).

SHAPE AND SPACE 9 – BASIC SKILLS EXERCISE

1. \( A' \) (90°, 10)
2. \( A' \) (180°, 9)
3. \( A' \) (180°, 10)
4. \( x = 60°, 120° \)
5. \( x = 60°, 300° \)
6. \( x = 45°, 225° \)
7. 9.22 cm
8. 18.0 cm
9. 15.6 cm
10. 13.5 cm
11. 11.9 cm
12. 19.4 cm
13. 46.5°
14. 38.2°
15. 55.1°
16. 36.2°
17. 20.7°
18. 59.0°
19. (a) \( C = 42.2°, a = 6.96 \) m
   (b) \( C = 44.7°, a = 5.84 \) m
20. (a) 68.9 m²
   (b) 120 m²
21. 92.1 m

SHAPE AND SPACE 9 – EXAM PRACTICE EXERCISE

1. (a) (i) \( 2f(x) = 2\sin(x°) \)
   
   \( (2f(x)) \) stretches the function by scale factor 2 parallel to \( y \)-axis.

   \( P \) is transformed to (90°, 2), \( Q \) is transformed to (180°, 0).

   (ii) \( -f(x) = -\sin(x°) \)

   \( (-f(x)) \) reflects the function in the \( x \)-axis.

   \( P \) is transformed to (90°, -1), \( Q \) is transformed to (180°, 0).

   (iii) \( -2f(2x) + 2 = -2\sin(2x°) + 2 \)

   The function \(-2f(2x) + 2\) (i) stretches the function by scale factor \( 1 \) parallel to \( x \)-axis.

   (ii) stretches the function by scale factor 2 parallel to the \( y \)-axis.

   (iii) reflects the function in the \( x \)-axis.

   (iv) translates the function along \( \left( \begin{array}{l} 0 \\ 2 \end{array} \right) \)

   \( P \) is transformed to (45°, 0), \( Q \) is transformed to (90°, 2).

   (b) \( R \) is at (30°, 0.5)
(a) Triangle $OAB$ at 05:00 angle $AOB$  
\[ \angle AOB = 5 \times 30^\circ = 150^\circ \]
\[ AB^2 = 10^2 + 7^2 - 2 \times 10 \times 7 \times \cos(150^\circ) \]
\[ AB = 16.4 \text{ cm (3 s.f.)} \]

(b) Triangle $OAB$ at 17:50 angle $AOB$  
\[ \angle AOB = 4 \times 30^\circ + (30^\circ - \frac{50}{60} \times 30) = 125^\circ \]
\[ AB^2 = 10^2 + 7^2 - 2 \times 10 \times 7 \times \cos(125^\circ) \]
\[ AB = 15.1 \text{ cm (3 s.f.)} \]

Let required area $LMNP =$  
Area of triangle $LNP$ + Area of triangle $LMN$  
Area of triangle $LNP$  
\[ = \frac{1}{2} \times 15.6 \times 4.3 \times \sin(72^\circ) = 31.898\ldots \text{cm}^2 \]

(area of triangle = $\frac{1}{2} \times ab \times \sin C$)

Triangle $LNP$:  
\[ LN^2 = 15.6^2 + 4.3^2 - 2 \times 15.6 \times 4.3 \times \cos(72^\circ) \]
\[ LN = 14.846\ldots \text{cm} \]

Triangle $LMN$:  
\[ \frac{\sin(MLN)}{13.7} = \frac{\sin(58^\circ)}{14.846} \]
\[ \sin(MLN) = 0.78259\ldots, \]
\[ \angle MLN = 51.498^\circ\ldots \]
\[ \angle MNL = 180^\circ - 58^\circ - 51.498^\circ = 70.502^\circ\ldots \]
\[ \text{(angle sum of a triangle =180°)} \]

Area triangle $LMN$  
\[ = \frac{1}{2} \times 13.7 \times 14.846 \times \sin(70.502^\circ) \]
\[ = 95.863 \text{ cm}^2\ldots \]

Area $LMNP = 31.898 + 95.863 = 127.761 \text{ cm}^2\ldots = 128 \text{ cm}^2$ (3 s.f.)

ABDE is a rectangle in which $AB = 2BD$  
Triangle $BCD$:  
\[ BD^2 = (x - 3)^2 + (x - 2)^2 - 2(x - 3)(x - 2) \cos(120^\circ) \]
\[ = (x^2 - 6x + 9) + (x^2 - 4x + 4) + (x^2 - 5x + 6) \]
\[ = 3x^2 - 15x + 19 \]

Alternative:  
\[ AB \times BD = 14 \text{ so } 2(BD)^2 = 14 \]
\[ \text{and } BD^2 = 7 \]
\[ 3x^2 - 15x + 19 = 7 \text{ etc} \]

Area $ABDE = 14 = AB \times BD = 2BD \times BD = 2(BD)^2 = 2(3x^2 - 15x + 19) = 6x^2 - 30x + 38 \]
\[ = 6x^2 - 30x + 24 \]
\[ = 3x^2 - 5x + 4 \]
\[ = (x - 4)(x - 1) \]
\[ x = 4 \text{ or } x = 1 \]

Discard $x = 1$ as the sides lengths are > 0.

If $x = 4$, $BC = 1$, $DC = 2$, $BD^2 = 7$,  
$BD = \sqrt{7}$, $AB = 2\sqrt{7}$  
Required perimeter of pentagon  
\[ ABCDE = 2AB + AE + BC + CD \]
\[ = 4\sqrt{7} + \sqrt{7} + 1 + 2 = 5\sqrt{7} + 3 \text{ cm} \]

Perimeter of triangle $PQR = 42 \text{ cm} $  
$PS = PU = 7 \text{ cm} $  
$QS = QT = 6 \text{ cm} $  
$RT = RU = 8 \text{ cm} $  
Triangle $PQR$  
\[ 15^2 = 13^2 + 14^2 - 2 \times 13 \times 14 \times \cos(SQT) \]
\[ \text{(Cosine rule: } a^2 = b^2 + c^2 - 2bc \cos A) \]
$\cos(SQT) = \frac{13^2 + 14^2 - 15^2}{2 \times 13 \times 14} = \frac{5}{13}$,

angle $SQT = 67.380^\circ$...

Let required area be $A$ cm$^2$.

$A = \text{Area of triangle } PQR - \text{Area of circle}$

Area of triangle $PQR$

$= \frac{1}{2} \times 13 \times 14 \times \sin(67.380^\circ) = 84,000...cm^2$

(area of triangle $= \frac{1}{2} \times \text{absinC}$)

Consider circle centre $O$ and triangle $QSO$.

Angle $OSQ = 90^\circ$, so triangle $QSO$ is a right-angled triangle.

$OS = \text{radius of circle } r$

$\tan\left(\frac{1}{2} \times 67.380^\circ\right) = \frac{r}{6}$

$r = 4 \text{ cm}$

Area of circle $= \pi \times 4^2 = 50.265...cm^2$

$A = 84,000 - 50.265 = 33.7 \text{ cm}^2$ (3 s.f.)

**HANDLING DATA 6 – BASIC SKILLS EXERCISE**

frequency density $= \frac{\text{frequency}}{\text{class width}}$

Be careful with the class widths when the data is continuous (i.e. time, weight, length, volume...)

1

<table>
<thead>
<tr>
<th>Time, $t$ (min)</th>
<th>Frequency</th>
<th>Frequency density</th>
</tr>
</thead>
<tbody>
<tr>
<td>$15 &lt; t \leq 20$</td>
<td>7</td>
<td>0.7</td>
</tr>
<tr>
<td>$20 &lt; t \leq 25$</td>
<td>15</td>
<td>3</td>
</tr>
<tr>
<td>$25 &lt; t \leq 30$</td>
<td>25</td>
<td>5</td>
</tr>
<tr>
<td>$30 &lt; t \leq 40$</td>
<td>18</td>
<td>1.2</td>
</tr>
<tr>
<td>$40 &lt; t \leq 50$</td>
<td>12</td>
<td>0.7</td>
</tr>
<tr>
<td>$50 &lt; t \leq 70$</td>
<td>8</td>
<td>0.4</td>
</tr>
<tr>
<td>$70 &lt; t \leq 95$</td>
<td>5</td>
<td>0.2</td>
</tr>
</tbody>
</table>

2 (a)

<table>
<thead>
<tr>
<th>Length, $l$ (mm)</th>
<th>Frequency density</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5 &lt; l \leq 6$</td>
<td>18</td>
</tr>
<tr>
<td>$6 &lt; l \leq 6.5$</td>
<td>30</td>
</tr>
<tr>
<td>$6.5 &lt; l \leq 7$</td>
<td>42</td>
</tr>
<tr>
<td>$7 &lt; l \leq 8$</td>
<td>26</td>
</tr>
<tr>
<td>$8 &lt; l \leq 10$</td>
<td>10</td>
</tr>
</tbody>
</table>

(b) 76

(c) 7.10 mm

3 (a)

<table>
<thead>
<tr>
<th>Time, $t$ (min)</th>
<th>Frequency density</th>
</tr>
</thead>
<tbody>
<tr>
<td>$15 &lt; t \leq 20$</td>
<td>2.4</td>
</tr>
<tr>
<td>$20 &lt; t \leq 30$</td>
<td>1.2</td>
</tr>
<tr>
<td>$30 &lt; t \leq 40$</td>
<td>1</td>
</tr>
<tr>
<td>$40 &lt; t \leq 70$</td>
<td>0.2</td>
</tr>
</tbody>
</table>

(b) 2

(c) 17 (16.8)

4 (a)

<table>
<thead>
<tr>
<th>Length, $l$ (mm)</th>
<th>Frequency density</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; l \leq 5$</td>
<td>1</td>
</tr>
<tr>
<td>$5 &lt; l \leq 10$</td>
<td>2</td>
</tr>
<tr>
<td>$10 &lt; l \leq 20$</td>
<td>2.2</td>
</tr>
<tr>
<td>$20 &lt; l \leq 30$</td>
<td>2.5</td>
</tr>
<tr>
<td>$30 &lt; l \leq 50$</td>
<td>0.9</td>
</tr>
</tbody>
</table>

(b) 7

(c) 22.0 mm

5 (a)

<table>
<thead>
<tr>
<th>Time, $t$ (min)</th>
<th>Frequency density</th>
</tr>
</thead>
<tbody>
<tr>
<td>$25 &lt; t \leq 50$</td>
<td>2.5</td>
</tr>
<tr>
<td>$50 &lt; t \leq 75$</td>
<td>1.5</td>
</tr>
<tr>
<td>$75 &lt; t \leq 100$</td>
<td>0.5</td>
</tr>
</tbody>
</table>
HANDLING DATA 6 – EXAM PRACTICE EXERCISE

Frequency density = \( \frac{\text{frequency}}{\text{class-width}} \)

Be careful with the class widths when the data is continuous (i.e. time, weight, length, volume...)

1. (a) 
<table>
<thead>
<tr>
<th>Mass, ( m ) (g)</th>
<th>Frequency</th>
<th>Frequency density</th>
</tr>
</thead>
<tbody>
<tr>
<td>60 &lt; ( m ) ≤ 70</td>
<td>12</td>
<td>1.2</td>
</tr>
<tr>
<td>70 &lt; ( m ) ≤ 80</td>
<td>22</td>
<td>2.2</td>
</tr>
<tr>
<td>80 &lt; ( m ) ≤ 100</td>
<td>40</td>
<td>2</td>
</tr>
<tr>
<td>100 &lt; ( m ) ≤ 120</td>
<td>20</td>
<td>1</td>
</tr>
<tr>
<td>120 &lt; ( m ) ≤ 160</td>
<td>16</td>
<td>0.4</td>
</tr>
</tbody>
</table>

(b) 71.2%
(c) 43.2 mins

2. (a)
<table>
<thead>
<tr>
<th>Time, ( t ) (min)</th>
<th>Frequency</th>
<th>Frequency density</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &lt; ( t ) ≤ 6</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>6 &lt; ( t ) ≤ 10</td>
<td>26</td>
<td>6.5</td>
</tr>
<tr>
<td>10 &lt; ( t ) ≤ 12</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>12 &lt; ( t ) ≤ 14</td>
<td>18</td>
<td>9</td>
</tr>
<tr>
<td>14 &lt; ( t ) ≤ 22</td>
<td>24</td>
<td>3</td>
</tr>
</tbody>
</table>

(b) \( x = 166 \)
(c) 41

3. (a) 
<table>
<thead>
<tr>
<th>Life, ( t ) (h)</th>
<th>Frequency</th>
<th>Frequency density</th>
</tr>
</thead>
<tbody>
<tr>
<td>60 &lt; ( t ) ≤ 80</td>
<td>24</td>
<td>1.2</td>
</tr>
<tr>
<td>80 &lt; ( t ) ≤ 90</td>
<td>26</td>
<td>2.6</td>
</tr>
<tr>
<td>90 &lt; ( t ) ≤ 95</td>
<td>20</td>
<td>4</td>
</tr>
<tr>
<td>95 &lt; ( t ) ≤ 100</td>
<td>25</td>
<td>5</td>
</tr>
<tr>
<td>100 &lt; ( t ) ≤ 115</td>
<td>54</td>
<td>3.6</td>
</tr>
<tr>
<td>115 &lt; ( t ) ≤ ( x )</td>
<td>51</td>
<td>1</td>
</tr>
</tbody>
</table>

(b) \( x = 166 \)
(c) 57%
(d) 11.2 min

4. (a) 
<table>
<thead>
<tr>
<th>Mass, ( m ) kg</th>
<th>Frequency</th>
<th>Frequency density</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 &lt; ( m ) ≤ 3</td>
<td>52</td>
<td>52</td>
</tr>
<tr>
<td>3 &lt; ( m ) ≤ 3.25</td>
<td>34</td>
<td>136</td>
</tr>
<tr>
<td>3.25 &lt; ( m ) ≤ 3.5</td>
<td>30</td>
<td>120</td>
</tr>
<tr>
<td>3.5 &lt; ( m ) ≤ 4</td>
<td>60</td>
<td>120</td>
</tr>
<tr>
<td>4 &lt; ( m ) ≤ 4.75</td>
<td>24</td>
<td>32</td>
</tr>
</tbody>
</table>

(b) 
(c) 0.83
(d) 3.37 kg

5. (a) 
<table>
<thead>
<tr>
<th>Consumption, ( m ) (ml)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &lt; ( m ) ≤ 50</td>
<td>9</td>
</tr>
<tr>
<td>50 &lt; ( m ) ≤ 100</td>
<td>42</td>
</tr>
<tr>
<td>100 &lt; ( m ) ≤ 125</td>
<td>30</td>
</tr>
<tr>
<td>125 &lt; ( m ) ≤ 150</td>
<td>25</td>
</tr>
<tr>
<td>150 &lt; ( m ) ≤ 200</td>
<td>25</td>
</tr>
<tr>
<td>200 &lt; ( m ) ≤ 300</td>
<td>22</td>
</tr>
<tr>
<td>300 &lt; ( m ) ≤ 500</td>
<td>27</td>
</tr>
</tbody>
</table>
15 \( 4\sqrt{ab} \)
16 \( a = 4, b = 12 \)
17 (a) \( \sqrt{3} \)
    (b) 2
    (c) 2
    (d) \( 9\sqrt{3} \)
18 (a) \( \frac{22(7 - \sqrt{3})}{7} \)
    (b) \( 3 + 3\sqrt{3} \)
    (c) \( \sqrt{7} + \sqrt{3} \)
19 (a) \( 2a + 1 + \sqrt{2} (a + 1) \)
    (b) \( \sqrt{a} \)
    (c) \( 4\sqrt{a} + 3 \)
20 \( \frac{3\sqrt{7}}{8} \)
21 \( \frac{\sqrt{a}(a^2 + a + 1)}{a^2} \)
22 All are equal to \( \frac{3\sqrt{33}}{14} \)
23 Radius = \( \sqrt{12} = 2\sqrt{3} \) ⇒
    Perimeter = \( 2 \times 2\sqrt{3} + 2\pi \sqrt{3} \)
24 \( x\sqrt{12} + x\sqrt{75} = 21 \)
    \( x(2\sqrt{3} + 5\sqrt{3}) = 21 \)
    \( x = \frac{21(2\sqrt{3})}{(7\sqrt{3}) \times \sqrt{3}} = \sqrt{3} \)
25 \( x2\sqrt{2} - \frac{3x}{\sqrt{2}} = 5 \)
    \( 4x - 3x = 5\sqrt{2} \)
    \( x = 5\sqrt{2} \)
26 \( 4\sqrt{2} \)
27 \( n = 4 \)
28 \( x = \frac{2 + \sqrt{10}}{2} \)
29 \( x = 2\sqrt{3} \) or \( x = \frac{\sqrt{3}}{2} \)
30 \( 7(4a + 2\sqrt{2a^2}) \)
31 \( \sqrt{a} \)
23/12/21   3:40 PM

32. Let $h$ be the third side of the triangle

$$\sqrt{2} + \sqrt{14} = h^2 + (2 + \sqrt{7})^2$$

**Pythagoras’ theorem**

$$16 + 2\sqrt{2} \cdot \sqrt{14} = h^2 + 11 + 4\sqrt{7}$$

$$5 + 4\sqrt{7} = h^2 + 4\sqrt{7} \Rightarrow \sqrt{14} = \sqrt{2}\sqrt{7}$$

$h^2 = 5$

$h = \sqrt{5}$

Area of a triangle $= \frac{1}{2} \text{base} \times \text{height}$

Area $= \left(\frac{2 + \sqrt{7}}{2}\right) \sqrt{5}$, so $p = \sqrt{5}$

---

**NUMBER 10 – EXAM PRACTICE EXERCISE**

1. (a) (i)

$$\frac{a - \sqrt{a}}{a + \sqrt{a}} = \frac{(a - \sqrt{a})(a - \sqrt{a})}{(a + \sqrt{a})(a - \sqrt{a})}$$

$$= \frac{a^2 - 2a\sqrt{a} + a}{a^2 - a^2}$$

$$= \frac{a(a - 2\sqrt{a} + 1)}{a(a - 1)}$$

$$= \frac{a + 1 - 2\sqrt{a}}{a - 1}$$

(ii)

$$\frac{\sqrt{a} + a}{\sqrt{a}} + \frac{\sqrt{a}}{\sqrt{a} - 1} = \frac{\sqrt{a} + 1}{\sqrt{a}} + \frac{1}{\sqrt{a} - 1}$$

$$= \frac{a + \sqrt{a} + 1}{\sqrt{a}}$$

$$= \frac{\sqrt{a}(a + \sqrt{a} + 1)}{\sqrt{a} \cdot \sqrt{a}}$$

$$= \frac{a + \sqrt{a} + 1}{a}$$

$$= \frac{a^2 - \sqrt{a} + a + \sqrt{a}}{a^2}$$

$$= \frac{a\sqrt{a} + a + \sqrt{a}}{a^2}$$

$$= \frac{a + \sqrt{a}(a + 1)}{a^2}$$

(b) $r = 25s$

$$\frac{p}{q} = \frac{\sqrt{125} + \sqrt{255}}{\sqrt{5} + \sqrt{5}}$$

$$= \frac{5\sqrt{5} + 5\sqrt{5}}{\sqrt{5} + \sqrt{5}}$$

$$= \frac{5\sqrt{5} + \sqrt{5}}{\sqrt{5} + \sqrt{5}}$$

$$= \frac{5\sqrt{5} + \sqrt{5}}{\sqrt{5} + \sqrt{5}}$$

$$= 5$$

$p : q = 5 : 1$

---

2. Total surface area is two ends plus curved surface area $= 2\pi r^2 + 2\pi rh$

$$2\pi(3\sqrt{2})^2 + 2\pi(3\sqrt{2}) (a\sqrt{3} + b\sqrt{3}) = 48\sqrt{6}$$

Divide both sides by $\pi$

$$36 + 2(6a + 3b\sqrt{6}) = 48\sqrt{6}$$

$$36 + 12a + 6b\sqrt{6} = 48\sqrt{6}$$

equating rational numbers and irrational numbers separately

$$36 + 12a = 0 \quad \text{and} \quad 6b = 48$$

$a = -3$ and $b = 8$

3. (a) Using the quadratic formula with

$$a = 1, \quad b = -(1 + 2p) \quad \text{and} \quad c = p + \sqrt{p}$$

$$b^2 - 4ac = (1 + 2p)^2 - 4(p + \sqrt{p})$$

$$= 1 + 4p + 4p^2 - 4p - 4\sqrt{p} = 1$$

$$x = \frac{1 + 2p \pm 1}{2}$$

$$x = \frac{2 + 2p}{2} \quad \text{or} \quad x = \frac{2\sqrt{p}}{2}$$

$$x = 1 + \sqrt{p} \quad \text{or} \quad x = \sqrt{p}$$

(b) $(a + \sqrt{b})(a + \sqrt{b}) = a^2 + b + 2a\sqrt{b}$$

$a^2 + b = 7(1)$ and $2a\sqrt{b} = \sqrt{48}(2)$

equating rational numbers and irrational numbers separately

(1) $a^2 = 7 - b$, (2) $a^2b = 12$

Substituting (1) into (2) gives

$(7 - b)b = 12$

$b^2 - 7b + 12 = 0$

$(b - 4)(b - 3) = 0$

$b = 4 \quad \text{or} \quad b = 3$

$a^2 = 3$ or $a^2 = 4$

$a = \pm\sqrt{3} \quad \text{or} \quad a = \pm2$

$\sqrt{7} + \sqrt{48} = \pm2 \pm\sqrt{3}$

The positive possible values are $2 + \sqrt{3}$ or $2 - \sqrt{3}$

$(2 + \sqrt{3})^2 = 7 + 4\sqrt{48} = 7 + \sqrt{48}$ \quad \text{Correct}

$(2 - \sqrt{3})^2 = 7 - 4\sqrt{3} = 7 - \sqrt{48}$ \quad \text{Incorrect}

Positive value of $\sqrt{7} + \sqrt{48} = 2 + \sqrt{3}$
4 Let $w$ be the width

$$w(a + \sqrt{2}) = (1 + a) \sqrt{2} + (a + 2)$$

$$w = \frac{(1 + a)\sqrt{2} + (a + 2)}{a + \sqrt{2}}$$

$$= \frac{(1 + a)\sqrt{2} + (a + 2)(a - \sqrt{2})}{(a + \sqrt{2})(a - \sqrt{2})}$$

$$= \frac{a^2\sqrt{2} + a^2 + 2a + 2 - 2a - a\sqrt{2} - 2\sqrt{2}}{a^2 - 2}$$

$$= \frac{a^2(1 + \sqrt{2}) - 2(1 + \sqrt{2})}{a^2 - 2}$$

$$= 1 + \sqrt{2} \text{ cm}$$

5 Let $h$ be the height of the triangle

$$\frac{1}{2} (3 - \sqrt{2}) \times h = \sqrt{6} + \frac{\sqrt{3}}{2}$$

$$\sqrt{6} + \frac{\sqrt{3}}{2} = 2\sqrt{6} + \frac{\sqrt{3}}{2}$$

$$h = \frac{2\sqrt{6} + \sqrt{3}}{3 - \sqrt{2}}$$

$$= \frac{(2\sqrt{6} + \sqrt{3})(3 + \sqrt{2})}{(3 - \sqrt{2})(3 + \sqrt{2})}$$

$$= \frac{6\sqrt{6} + 3\sqrt{3} + 2\sqrt{2}\sqrt{6} + \sqrt{3}}{9 - 2}$$

$$= 2\sqrt{6}, 6 = 2\sqrt{3}, 3$$

$$= 4\sqrt{3}$$

$$h = \frac{7\sqrt{6} + 7\sqrt{3}}{7}$$

$$= \sqrt{6} + \sqrt{3} \text{ cm}$$

Let $H$ be the hypotenuse of triangle.

$$H^2 = (3 - \sqrt{2})^2 + (\sqrt{6} + \sqrt{3})^2$$

$$= 11 - 6\sqrt{2} + 9 + 2\sqrt{3}\sqrt{6}$$

$$= 2\sqrt{6}, 6 = 2\sqrt{3}, 3 = 6\sqrt{2}$$

$$= 20$$

$$H = \sqrt{20}$$

Perimeter = $(3 - \sqrt{2}) + (\sqrt{6} + \sqrt{3}) + \sqrt{20}$

$$= \sqrt{3}, \sqrt{3} - \sqrt{2} + \sqrt{2}, \sqrt{3} + \sqrt{3} + \sqrt{4}, \sqrt{3}$$

$$= \sqrt{2}(\sqrt{3} - 1) + (\sqrt{3}(\sqrt{3} + 1) + 2\sqrt{3} \text{ cm}$$
26 \( x = -2.427 \).  \( x = 2 \)
28 \( x = -8 \)
29 \( x = -\frac{1}{2} \) or \(-4 \)
30 \( x = -6 \) or \( x = 4 \)
31 \( x = \pm 2 \)
32 \( x = 2.4 \)
33 \( x = -2 \)
34 \( x = 2 \) or \( x = 3 \)
35 \( \frac{1}{x - 1} \)
36 \( \frac{2}{3} \)
37 \( 3 - 2x \)
38 \( \frac{x^2 - 3x - 4}{x + 3} + \frac{x + 1}{x^2 - x - 12} = \frac{(x - 4)(x + 1)}{x + 3} \times \frac{(x - 4)(x + 3)}{x + 1} = (x - 4)^2 \geq 0 \)

**ALGEBRA 10 – EXAM PRACTICE EXERCISE**

1 (a) \( \frac{3y}{x} - \frac{y}{x + 3} = y\left(\frac{3}{x} - \frac{1}{x + 3}\right) \)
\[= y\left(\frac{3x + 3 - x}{x(x + 3)}\right) = y\left(\frac{2x + 9}{x(x + 3)}\right) \]
\[\frac{3y}{x} - \frac{y}{x + 3} = 2x + 9 \]
\[y = \frac{(2x + 9)}{(x(x + 3))} = 2x + 9 \]
\[y = \frac{x(x + 3)}{(2x + 9)} \]
\[y = \frac{x^2 + 3x}{(2x + 9)} \]
\[y = \frac{(x + \frac{3}{2})^2 - \frac{9}{4}}{4} \]

(b) (i) \( \left(-\frac{3}{2}, \frac{9}{2}\right) \), (ii) a minimum
\( y \) is a positive quadratic

2 (a) \( x = \frac{t + 3}{t - 1} \)
\[x(t - 1) = t + 3 \]
\[xt - x = t + 3 \]
\[t(x - 1) = x + 3 \]
\[t = \frac{x + 3}{x - 1} \]

Substituting gives \( y = \left(\frac{x + 3}{x - 1}\right)^2 - 4\left(\frac{x + 3}{x - 1}\right) \)
\[y = \frac{(x + 3)^2 - 4(x - 1)(x + 3)}{(x - 1)^2} \]
\[y = \frac{x^2 + 6x + 9 - 4x^2 - 8x + 12}{(x - 1)^2} \]
\[y = \frac{-3x^2 - 2x + 21}{(x - 1)^2} \]
\[y = -\frac{3x^2 - 2x + 21}{(x - 1)^2} \] or \( \frac{(7 - 3x)(x + 3)}{(x - 1)^2} \)

(b) \( \left(\frac{x + 3}{x - 1}\right)^2 \div y \)
\[= \left(\frac{x + 3}{x - 1}\right) \times \frac{(x - 1)^2}{(7 - 3x)(x + 3)} \]
\[= \frac{x - 1}{7 - 3x} \times \frac{(x + 3)^2}{(x - 1)^2} \]
\[y = (x - 1) \times (7 - 3x) \]

3 (a) \( \frac{2x^2 + 5x - 12}{4x^2 - 9} + \frac{x^2 + 2x - 8}{2x^5 + 5x + 3} \)
\[= \frac{2x^2 + 5x - 12}{4x^2 - 9} \times \frac{x^2 + 5x + 3}{2x^5 + 2x - 8} \]
\[= \frac{(2x - 3)(x + 4)}{(2x + 5)(2x - 3)} \times \frac{(x + 3)(x + 1)}{(x + 4)(x - 2)} \]
\[= \frac{x + 1}{x - 2} \]

(b) \( \frac{2x^2 + 5x - 12}{4x^2 - 9} \times \frac{x^2 + 2x - 8}{2x^5 + 5x + 3} \)
\[= 1 + \frac{x}{x + 2} \]
\[\frac{x + 1}{x - 2} = 1 + \frac{x}{x + 2} \]

Using the result from part a.
\[\frac{x + 1}{x - 2} = 1 + \frac{x}{x + 2} \]
\[(x + 1)(x + 2) = (x - 2) + x(x + 2) + x(x - 2) \]
Multiplying both sides by \((x - 2)(x + 2)\)
\[x^2 + 3x + 2 = x^2 - 4 + x^2 - 2x \]
\[x^2 - 5x - 6 = 0 \]
\[(x + 1)(x - 6) = 0 \]
\[x = -1 \text{ or } x = 6 \]
4 Time taken to travel $x$ km at $x + 10$ km/h is
\[ \frac{x}{x + 10} \text{ h} \]
Time taken to travel $70 - x$ km at $x - 20$ km/h is
\[ \frac{70 - x}{x - 20} \text{ h} \]
Multiply both sides by $(x + 10)(x - 20)$
\[ x(x - 20) + (x + 10)(70 - x) = (x + 10)(x - 20) \]
\[ x^2 - 20x - x^2 + 60x + 700 = x^2 - 10x - 200 \]
\[ x^2 - 50x - 900 = 0 \]
Solving using the quadratic formula gives
\[ x = 25 + \frac{5}{\sqrt{61}} \text{ or } 64.1 \text{ to 3 s.f.} \]

5 (a) $y = \frac{8x^2 + 16x - 10}{3x^3 - 3}$
\[ = \frac{2(2x - 1)(2x + 5)}{3(x - 1)(x + 1)} \times \frac{3(x - 1)}{(2x + 5)} \]
\[ = \frac{2(2x - 1)}{x + 1} \]

(b) Half perimeter is $\frac{2x + 5}{3x - 3} + \frac{2(2x - 1)}{x + 1} = 5$
Multiply both sides by $3(x - 1)(x + 1)$
\[ (2x + 5)(x + 1) + 2(2x - 1)3(x - 1) = 5 \times 3(x - 1)(x + 1) \]
\[ 2x^2 + 7x + 5 + 12x^2 - 18x + 6 = 15x^2 - 15 \]
\[ x^2 + 11x - 26 = 0 \]
\[ (x - 2)(x + 13) = 0 \]
\[ x = 2 \] sides are $\frac{4 + 5}{6} = 3$ and $\frac{2(4 - 1)}{3} = 2$
area = $3 \times 2 = 6 \text{ cm}^2$

3 (a) $\frac{dy}{dx} = -x^{-2} = -\frac{1}{x^2}$

(b) $\frac{dy}{dx} = -2x^{-3} = -\frac{2}{x^3}$

(c) $\frac{dy}{dx} = \frac{1}{2}x^{-\frac{3}{2}} = \frac{1}{2\sqrt{x}}$

(d) $\frac{dy}{dx} = -3x^{-4} = -\frac{3}{x^4}$

(e) $\frac{dy}{dx} = -16x^{-5} = -\frac{16}{x^5}$

(f) $\frac{dy}{dx} = -x^{-3} = -\frac{1}{x^3}$

(g) $\frac{dy}{dx} = 4x + 3 - 4x^{-2}$

(h) $\frac{dy}{dx} = 2 + 3x^{-2}$

4 (a) $\frac{dy}{dx} (at \ x = 1) = 4$

(b) $\frac{dy}{dx} (at \ x = 1) = -5$

(c) $\frac{dy}{dx} (at \ x = 1) = 26$

(d) $\frac{dy}{dx} (at \ x = 2) = 19$

5 (a) $-2$

(b) 2

(c) 0

(d) $-\frac{17}{8}$

6 $4y = 3x + 4$

7 $y = -3x, y = 3x - 9$

8 $p = 1$
9 \[ p = 3, \ q = -2, \text{ so } p^3 + q^3 = 27 - 8 = 19 \]

10 (a) \[ \frac{dy}{dx} = 3x^2 - 3 \]
(b) \((1, 0), (-1, 4)\)
(c) \((-1, 4)\) is max, \((1, 0)\) is min.

11 (a) \[ dy = 3x^2 + 6x - 9 \]
(b) \((-3, 28), (1, -4)\)
(c) \((-3, 28)\) is max, \((1, -4)\) is min.

12 (a) \((0, 1), (4/3, -0.185185...)\)
(b) \(x = 0\) is a max, \(x = \frac{4}{3}\) is a min.

13 (a) \[ v = 24t \text{ m/s} \]
(b) \(48 \text{ m/s}\)
(c) \(a = 24 \text{ m/s}^2\)
(d) \(24 \text{ m/s}^2\)

14 (a) \[ v = 3t^2 + 8t - 3 \text{ m/s} \]
(b) \(377 \text{ m/s}\)
(c) \(a = 6t + 8 \text{ m/s}^2\)
(d) \(68 \text{ m/s}^2\)

15 (a) \(s = 125 \text{ m}\)
(b) \(t = 5\)

16 (a) \[ v = \frac{dx}{dt} = 3t^2 - 300 \text{ km/s} \]
(b) at \(t = 5\), \(v = -225 \text{ km/s}\)
(c) \(a = \frac{dc}{dt} = 6t \text{ km/s}^2\)
(d) at \(t = 5\), \(a = 30 \text{ km/s}^2\)
(e) \(t = 10 \text{ s}\)

17 (a) \[ v = 24t \text{ m/s} \]
(b) \(72 \text{ m/s}\)
(c) \(a = 24 \text{ m/s}^2\), (constant)
(d) \(24 \text{ m/s}^2\)

18 (a) \[ v = 3t^2 - 500 \text{ km/s} \]
(b) at \(t = 10 \text{ s}\), \(v = -200 \text{ km/s}\)
(c) \(a = 6t \text{ km/s}^2\)

19 (a) \[ \frac{dQ}{dt} = 3t^2 - 16t + 14 \]
(b) (i) \(14 \text{ m}^3/\text{s}^2\)
(ii) \(-6 \text{ m}^3/\text{s}^2\)
(iii) \(9 \text{ m}^3/\text{s}^2\)

20 (a) \[ \frac{dc}{dr} = 4 - 16r^2 = 4 - \frac{16}{r^2} \]
(b) \(t = 2, C = 1\)
(c) \(-12^\circ C/\text{month}\)

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**GRAPHS 9 – EXAM PRACTICE EXERCISE**

1 (a) \[ P = 5t^2 + \frac{10000}{t} + 10 \]

(Please note the correction in the differentiation of the function.)

\[ P = 5t^2 + 10000t^{-1} + 10 \]

(Rewrite \(P\) in index form so differentiation is easier.)

\[ \frac{dP}{dt} = 10t - 10000t^{-2} \text{ flowers/year} \]

(Now equate this to zero and solve for \(t\).)

\[ \frac{dP}{dt} = 0 = 10t - \frac{10000}{t^2} \]

(Multiply by \(t^2\).)

\[ 0 = 10t^3 - 10000 \]

\[ 10000 = 10t^3 \]

\[ t = 10 \text{ yrs} \]

(From the graph, the gradient of the curve is zero at \(t = 10 \text{ yrs}\).)

At \(t = 10\), \(P = 1510\)

(Substitute \(t = 10\) into equation for \(P\).)

2 (a) \[ y = x^2 - 3x \] (Differentiate to find the gradient function.)

\[ \frac{dy}{dx} = 2x - 3 \]

(Find the gradient to the curve at \(x = 4\).)

At \(x = 4\), \(\frac{dy}{dx} = 5\)

Gradient of normal = \(-\frac{1}{5}\)

(If \(m_1\) is gradient of tangent and \(m_2\) is gradient of normal, \(m_1 \times m_2 = -1\))

Equation of normal at \(x = 4\) is

\[ y = -\frac{1}{5}x + c \]
(Equation of a straight line is \( y = mx + c \))

At \( x = 4, y = 4 \) (The \( y \)-value is found by substituting \( x = 4 \) into \( y = x^2 - 3x \))

\[ 4 = \frac{-1}{2} \times 4 + c \]  \( \text{Point (4, 4) is on the normal so it must satisfy the equation.} \)

\[ c = 4 \]  \( \text{so equation of normal is} \)

\[ y = -\frac{1}{5}x + \frac{4}{5} \]

\[ \text{4} = -\frac{1}{5} \times 4 + c \]

Due to the shape of the cubic curve, the smallest \( x \) value is the maximum point on the function. So \( A(1, 5) \) max point and \( B(3, 1) \) min point.

\[ 4 = \frac{-1}{5} \times 4 + c \]

\[ c = 4 \]

\[ \text{so equation of normal is} \]

\[ y = -\frac{1}{5}x + \frac{4}{5} \]

\[ x + 5y = 24 \]  \( \text{Multiply the equation} \)

\[ y = -\frac{1}{5}x + \frac{4}{5} \]

\[ \text{by 5 and re-arrange.} \]

\[ a = 1, \quad b = 5, \quad c = 24 \]  \( \text{(Differentiate velocity function to get acceleration function)} \)

\[ t = 0, \quad a = 3, \quad so \quad 3 = k - 10 \times 0, \]

\[ k = 3 \]

\[ \text{(Gradient of curve is flat at } V_{max} \text{)} \]

\[ r = 0 \text{ or } 0.6 \text{ s} \]

\[ a = 3 - 10 \times 0.6 = 3 - 6 = -3 \text{ m/s}^2 \]  \( \text{as required} \)

\[ V = \pi r^2 h \]

\[ 12 : 4 = h : (4 - r) \]

\[ h = 3 (4 - r) \]

\[ V = \pi r^2 (4 - r) = 3 \pi r (4 - r) \]

\[ \text{as required} \]

\[ V = 12 \pi r^2 - 3 \pi r^3 \]

\[ \frac{dV}{dr} = 24 \pi r - 9 \pi r^2 = 3 \pi r (8 - 3r) \]

\[ 0 = 3 \pi r (8 - 3r) \]

\[ r = 0 \text{ or } r = \frac{8}{3} \text{ cm} \]

\[ (\text{Ignore } r = 0 \text{ as it does not fit the model}) \]

\[ (\text{Negative cubic curves will have this shape}) \]

\[ (\text{Gradient = 0 at turning points}) \]

\[ 0 = (x - 1)(x - 3) \]

\[ x = 1 \text{ or } x = 3, \text{ so } y = 5 \text{ or 1 so } A(1, 5), \]

\[ B(3, 1) \]

\[ \text{Due to the shape of the cubic curve, the smallest } x \text{ value is the maximum point on the function. So } A(1, 5) \text{ max point and } B(3, 1) \text{ min point.} \]
SHAPE AND SPACE 10 – BASIC SKILLS EXERCISE

1 36.4 m  
2 21.4 m  
3 18.4°  
4 0.828 m  
5 10.8°  
6 10.39 m  
7 1.79 m  
8 82.7%  
9 (a) 32.6°  
   (b) 38.5 units²  
10 (a) 243°  
   (b) 63.4°  
11 125°  
12 (a) 2250 m  
   (b) 3897 m  
   (c) 4500 m  
13 (a) 13.9 km  
   279.7°  
   (b) He travels at 7.2 km/h so he does arrive by 18:00  
14 2.5 km  
15 (a) 25.5 km  
   (b) 022.7°  
   (c) 203°  

SHAPE AND SPACE 10 – EXAM PRACTICE EXERCISE

1 Let cliff height \( WZ \) be \( h \) metres  
   Speed = \( \frac{\text{Distance}}{\text{Time}} \)  
   \[ 0.75 = \frac{ZX - ZY}{60} \]  
   Triangle \( WZX \):  
   \[ \tan(60°) = \frac{ZX}{h} \text{, so } ZX = h \tan(60°) \]  
   Triangle \( WZY \):  
   \[ \tan(40°) = \frac{ZY}{h} \text{, so } ZY = h \tan(40°) \]  

\[ 0.75 = \frac{h \tan(60°) - h \tan(40°)}{60} \]  
So \( 0.75 \times 60 = h [\tan(60°) - \tan(40°)] \)  
\[ h = \frac{45}{\tan(60°) - \tan(40°)} \]  
\[ h = 50.4 \text{ m (3 s.f.)} \]
**3**

**4(a)**

(a) Angle $OPA = 80° - 45° = 35°$  
(alternate angles, North to South = 180°)  
(cosine rule  (SSSA condition so cosine rule)  
$OA^2 = 28^2 + 62^2 - 2 \times 28 \times 62 \times \cos(35°) = 1783.9041...$  
$OA = 42.2363...km$  
$OA = 42.2\,\text{km (3 s.f.)}$

(b) Bearing of $O$ from $A$:  
(sine rule  (SASA condition so sine rule)  
Let angle $OAP = \theta$  
\[
\frac{\sin \theta}{28} = \frac{\sin(35°)}{42.2363}
\]

so $\sin \theta = \frac{\sin(35°) \times 28}{42.2363} = 0.38024...$  
$\theta = 22.349°...$  
Angle $AOP = 180° - 35° - 22.349° = 122.651°...$  
Bearing of $O$ from $A = (80° + 122.651°) = 202.651°$  
(half angles)  
Bearing of $O$ from $A = 022.7°$ (3 s.f.)

(c) speed = \frac{\text{distance}}{\text{time}}  
balloon: $30 = \frac{28 + 62}{t}$, so $t = \frac{90}{30} = 3\,\text{h}$  
truck: speed = \frac{42.2363}{3} = 14.0787 km/h...  
Speed = 14.1 km/h (3 s.f.)

**4(b)**

Angle $BOA = 240° - 130° = 110°$  
(cosine rule  (SSSA condition so cosine rule)  
$AB^2 = 20^2 + 30^2 - 2 \times 20 \times 30 \times \cos(110°) = 1710.4241...$km  
$AB = 41.3573...$km  
$AB = 41.4\,\text{km (3 s.f.)}$

(c) Bearing of $A$ from $B$:  
(sine rule  (SASA condition so sine rule)  
Let angle $OBA = \theta$  
\[
\sin \theta = \frac{\sin(110°)}{41.3573} \times 30 = 0.68163...\]  
$\theta = 42.9719°...$  
Bearing of $A$ from $B = 060° + 042.9719°$  
(alternate angles)  
Bearing of $A$ from $B = 103°$ (3 s.f.)

**5**

Plan view  
\[
\text{speed} = \frac{\text{distance}}{\text{time}}  
\text{balloon: } 30 = \frac{28 + 62}{t}, \text{ so } t = \frac{90}{30} = 3\,\text{h}  
\text{truck: speed} = \frac{42.2363}{3} = 14.0787\,\text{km/h}...  
\text{Speed} = 14.1\,\text{km/h (3 s.f.)}
\]
Triangle ITJ:
\[ \tan(65\degree) = \frac{TJ}{25} \text{ (} \tan \theta = \frac{\text{opposite side}}{\text{adjacent side}} \text{)} \]

\[ TJ = 25 \times \tan(65\degree) = 53.6127\ldots \text{m} \]

Triangle ITK:
\[ \tan(75\degree) = \frac{TK}{25} \]

\[ TK = 25 \times \tan(75\degree) = 93.3013\ldots \text{m} \]

Plan view on triangle KTJ:
\[ KJ^2 = KT^2 + TJ^2 \text{ (Pythagoras’ theorem)} \]
\[ = 53.6127^2 + 93.3013^2 \]
\[ = 11579.45\ldots \]
\[ KJ = 107.608\ldots \text{m} \]
\[ KJ = 107.6 \text{ m (1 d.p.)} \]

**Handling Data 7 – Basic Skills Exercise**

1. \( \frac{5}{14} \)

2. (a) \( \frac{4}{11} \)
   
   (b) \( \frac{2}{9} \)

3. (a) \( \frac{2}{7} \)
   
   (b) \( \frac{1}{3} \)

4. (a) (i) \( \frac{5}{11} \)
   
   (ii) \( \frac{6}{11} \)
   
   (b) (i) \( \frac{16}{25} \)
   
   (ii) \( \frac{9}{25} \)
   
   (c) 0.798 (3 s.f.)

5. (a) 0.2
   
   (b) 0.1625
   
   (c) 0.097 25

6. (a) (i) \( \frac{5}{12} \)
   
   (ii) \( \frac{11}{60} \)

   (b) Let \( X \) be Emma’s score.
   
   \[ P(X \geq 25) = 1 - P(X < 25) = 1 - P(X \leq 20) \]
   
   \[ P(E) + P(E') = 1 = 1 - [P(X = 10) + P(X = 15) + P(X = 20)] \]
\[
= 1 - \left( \frac{4}{20} \times \frac{3}{19} \right) + \left( \frac{6}{20} \times \frac{4}{19} + \frac{4}{20} \times \frac{6}{19} \right) + \left( \frac{6}{20} \times \frac{7}{19} \right) = \frac{29}{38}
\]

\( \text{(P(A and B) = P(A) \times P(B))} \)

\( \text{(P(A or B) = P(A) + P(B) if A and B are mutually exclusive)} \)

9 (a) (i) 0.6

(ii) 0.2

(b) (i) 0.04

(ii) 0.055

10 (a) \( k = 0.1 \)

(b) (i) 0.8

(ii) 0.5

(e) 0.9

11 (a) (i) \( \frac{13}{30} \)

(ii) \( \frac{2}{7} \)

(b) (i) \( \frac{1}{15} \)

(ii) \( \frac{34}{35} \)

**HANDLING DATA 7 – EXAM PRACTICE EXERCISE**

1 (a) (i) \( P(\text{success}) = \frac{12750 + 14400 + 14800 + 12400}{80000} = \frac{54350}{80000} = \frac{1087}{1600} \)

(ii) \( P(\text{60-year old no change}) = \frac{7250 + 5600}{80000} = \frac{257}{1600} \)

(iii) \( P(\text{30-year old success}) = \frac{14800 + 12400}{80000} = \frac{17}{50} \)

(b) \( P(\text{Success/60 yr old}) = \frac{12750}{20000} = \frac{51}{80} \)

2 (a) As the herd has a very large number of cows, the proportions will be approximately the same when one, two or three cows are removed.

(i) \( P(F_1) \times P(F_2) = \frac{1}{5} \times \frac{1}{5} = \frac{1}{25} \)

\( \text{(P(A and B) = P(A) \times P(B))} \)

(ii) \( P(F_1 J_2 \ or \ J_1 F_2) = P(F_1 J_2) + P(J_1 F_2) \)

\[ = \frac{1}{5} \times \frac{4}{5} + \frac{4}{5} \times \frac{1}{5} = \frac{8}{25} \]

\( \text{(P(A or B) = P(A) + P(B) if A and B are mutually exclusive)} \)

(b) \( P(F \geq 2) = P(F = 2) + P(F = 3) \)

\[ = 3 \times p(F_1 F_2 J_3) + P(F_1 F_2 J_3) \]

\[ = 3 \times \frac{1}{5} \times \frac{1}{5} \times \frac{4}{5} + \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} = \frac{13}{125} \]

3 (a) Bag A Red (R) : Green (G) : Gold (g) = 1 : 2 : 3

Bag B Red (R) : Green (G) : Gold (g) = 1 : 1 : 2

(b) (i) \( P(g_1) \times P(g_2) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \)

\( \text{(P(A and B) = P(A) \times P(B))} \)

(ii) \( P(G g) = P(G_1 g_2) + P(g_1 G_2) \)

\[ = \frac{1}{3} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{4} = \frac{7}{24} \]

\( \text{(P(A or B) = P(A) + P(B) if A and B are mutually exclusive)} \)
(iii) \( P(R \geq 1) + P(R = 0) = 1 \)
\[ P(R \geq 1) = 1 - P(R = 0) = 1 - P(R') P(R') = \] 
\[ (P(E) + P(E') = 1) \]
\[ = 1 - \frac{5}{6} \times \frac{2}{4} \]
\[ = 1 - \frac{15}{24} \]
\[ = \frac{9}{24} = \frac{3}{8} \]

(ii) \( P(BW \text{ from Box Y}) = P(W) \times P(W_1) \times P(W_2) + P(W) \times P(B_1) \times P(W_2) + P(B \times P(W_1) \times P(B) \times P(W_2) \]
\[ = \frac{2}{5} \times \frac{4}{6} \times \frac{2}{5} \times \frac{3}{5} + \frac{2}{6} \times \frac{3}{5} \times \frac{3}{6} \times \frac{3}{5} \]
\[ + \frac{3}{5} \times \frac{3}{6} \times \frac{3}{5} \]
\[ = \frac{43}{75} \]

(P(A or B) = P(A) + P(B) if A and B are mutually exclusive)

(iii) \( P(B \geq 1) + P(B = 0) = 1 \)
\[ P(B \geq 1) = 1 - P(B = 0) = 1 - P(B') B' \]
\[ (P(E) + P(E') = 1) \]
\[ P(B' \cdot B') = P(WW \text{ from Box Y}) = \]
\[ \frac{7}{25} \]

5 (a) 1st Operation 2nd Operation 3rd Operation

(b) (i) \( P(\text{Cured 1st operation}) = P(W) \times P(S) = 0.90 \times 0.8 = 0.72 \)
\( (P(A \text{ and } B) = P(A) \times P(B)) \)

(ii) \( P(\text{Cured 3rd operation}) = P(W) \times P(F) \times P(F) \times P(S) = 0.9 \times 0.2 \times 0.4 = 0.0288 \)

(iii) \( P(\text{Cured}) = P(\text{Cured 1st operation}) + P(\text{Cured 2nd operation}) + P(\text{Cured 3rd operation}) = 0.72 + P(W) \times P(F) \times P(S) + 0.0288 \)
\[ = 0.72 + 0.90 \times 0.2 \times 0.6 + 0.0288 \]
\[ = 0.8568 \)

(P(A or B) = P(A) + P(B) if A and B are mutually exclusive)
1 (a) \( \frac{17}{100} \times 100 = 17 \) \( \text{as \% \ of \ } b = \frac{a}{b} \times 100 \)
(b) \( 175 \times 1.063 = 186.025 \text{ million} \)
Increase \( a \) by \( b\% \) = \( a \times \left(1 + \frac{b}{100}\right) \)
(c) Let population in 1990 be \( p \) million
\( p \times 1.174 = 175 \)
\( p = \frac{175}{1.174} = 149.06 \ldots \text{ million} \)
= 149 (nearest million)
2 (a) (i) \( u \times u \times u \times u \times u = u^5 \) \( \left( a^m \times a^n = a^{m+n}\right) \)
(ii) \( 3u + 2v - 7u + 4v + 11 = 3u - 7u + 2v + 4v + 11 = -4u + 6v + 11 \)
(iii) \( \frac{u^7 \times u^8}{u^3} = \frac{u^{7+8}}{u^3} = u^{15-3} \) \( \left( a^m \div a^n = a^{m-n}\right) \)
(b) \( u(5u - 1) - u(3u - 2) = 5u^2 - u - 3u^2 + 2u = 2u^2 + u \)
(c) \( (5u - 1)(3u - 2) = 5u(3u - 2) - 1(3u - 2) = 15u^2 - 10u - 3u + 2 = 15u^2 - 13u + 2 \)
3 (a) \[ \text{Area} = \frac{1}{2} \times \pi \times 1.8^2 = 1.62\pi \text{ cm}^2, \]
so \( k = 1.62 \)
(area of circle = \( \pi r^2 \))
(b) \[ \text{Area} = \frac{1}{2} \times (3 \times 2) = 3 \text{ cm}^2 \]
7 cm
6 cm
2 cm
3 cm
4 (a) \( P(B) = 1 - 0.55 - 0.25 - 0.12 = 0.08, \ x = 0.08 \)
(sum of all probabilities of an event = 1)
(b) \( P(R \text{ or } B) = P(R) + P(B) = 0.55 + 0.08 = 0.63 \)
\( (P(A \text{ and } B) = P(A) + P(B) \text{ if } A \text{ and } B \text{ are mutually exclusive}) \)
(c) \( P(GY) = P(GY \text{ or } YB) = P(GY) + P(YB) = 0.25 \times 0.12 + 0.12 \times 0.25 = 0.06 \)
\( (P(A \text{ and } B) = P(A) \times P(B) \text{ if } A \text{ and } B \text{ are independent}) \)
(d) Let \( E(YR) \) be expected number of times spinner lands on a \( Y \) or \( R \).
\( E(YR) = 200 \times 0.67 = 134 \text{ times} \)
( Expected number of outcomes = probability of event \times \text{number of trials} )
5 (a) Mean = \( \frac{a + b + c + d}{4} = 15 \)
\( \left( \text{mean} = \frac{\text{sum of numbers}}{\text{number of numbers}} \right) \)
\( 60 = a + b + c + d = 33 + d \)
d = 27
(b) Range = 23 = \( d - a = 27 - a \)
a = 4
(range = largest score - smallest score)
Sum = 60 = \( 4 + b + c + 27, b + c = 29, \)
so median = \( b + c \quad \frac{29}{2} = 14.5 \)
(Median is mean of the central pair in an even group of numbers arranged in ascending or descending order)
6 (a) Let \( M \) be midpoint of \( AB \)
\[ M = \left( \frac{0 + (-5)}{2}, \frac{4 + (-2)}{2} \right) \]
\[ M = \left( -\frac{21}{2}, 1 \right) \]
(midpoint of \( A(x_1, y_1) \) and \( B(x_2, y_2) \)
\[ = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \]
(b) Let \( m \) be the gradient of \( AB \)

\[
m = \frac{-2 - 4}{5 - 0} = \frac{-6}{5}
\]

(Gradient of \( AB \) = \( \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} \))

(c) \( y = \frac{6}{5}x + 4 \)

(Equation of straight line: \( y = mx + c; \) \( m \) is gradient, \( c \) is \( y \)-axis intercept)

(d) Let perpendicular bisector of \( AB \) be line \( L \)

Gradient of line \( L = -\frac{5}{6} \)

(Products of the gradients of two perpendicular lines = \(-1\); \( m_1 \times m_2 = -1 \))

Equation of line \( L \):

\[
y = -\frac{5}{6}x + c
\]

(Line \( L \) passes through point \( M\left(-\frac{1}{2}, 1\right) \). \( M \) must satisfy the equation.)

\[
1 = -\frac{5}{6} \times \left(-\frac{1}{2}\right) + c, \text{ so } c = -\frac{1}{12}
\]

\[
y = -\frac{5}{6}x - \frac{13}{12} \text{ or } 10x + 12y + 13 = 0
\]

(multiply through by 12 to produce \( ax + by + c = 0 \))

7 (a) \( P \cap Q = \emptyset \) as there are no members of set \( P \) that are also in set \( Q \)

(Sets \( P \) and \( Q \) are mutually exclusive)

(b) \( x = 29 \)

(29 is NOT a member of \( P \cup Q \) but is in \( \in \) )

(c) \( R = \{2, 13, 19\} \)

8 (a)

<table>
<thead>
<tr>
<th>Time ( t ) (minutes)</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0 &lt; t \leq 40 )</td>
<td>20</td>
</tr>
<tr>
<td>( 0 &lt; t \leq 80 )</td>
<td>55</td>
</tr>
<tr>
<td>( 0 &lt; t \leq 120 )</td>
<td>115</td>
</tr>
<tr>
<td>( 0 &lt; t \leq 160 )</td>
<td>148</td>
</tr>
<tr>
<td>( 0 &lt; t \leq 200 )</td>
<td>155</td>
</tr>
<tr>
<td>( 0 &lt; t \leq 240 )</td>
<td>160</td>
</tr>
</tbody>
</table>
10 (a) (i) \( p^7 \times p^{11} = p^{18} \) (since \( a^m \times a^n = a^{m+n} \))
(ii) \( p^{11} \div p^5 = p^6 \) (since \( a^m \div a^n = a^{m-n} \))
(iii) \((2p + 1)^2 - (p - 1)^2 = (2p + 1)(2p + 1) - (p - 1)(p - 1)
= (4p^2 + 4p + 1) - (p^2 - 2p + 1)
= 3p^2 + 6p = 3p(p + 2) \)

(b) (i) \( 11p - 1 = 7p + 1 \)
\( 4p = 2 \)
(Do same operation to both sides to isolate \( p \))
\( p = \frac{2}{4} = \frac{1}{2} \)
(ii) \( 11p - 1 = 4p + 1 \)
\( 11(11p - 1) = 4(4p + 1) \)
\( 121p - 11 = 16p + 4 \)
\( 105p = 15 \)
\( p = \frac{15}{105} = \frac{1}{7} \)

11 \( 3m - 10n = 20 \) \( \text{(1)} \)
\( 5m + 2n = 6 \) \( \times 5 = \text{(3)} \)
\( 25m + 10n = 30 \)

(Decision made to eliminate \( n \), so make \( n \) values 'same'. Other variable \( m \) could also be eliminated by making \( m \) values 'same')

\( (3) + (1): 28m = 50, m = \frac{50}{28} = \frac{25}{14} = \frac{111}{14} \)

(Express \( m \) into any equation to find \( n \), say (2))

\( (2): 5 \times \frac{111}{14} + 2n = 6, 2n = \frac{41}{14}, n = \frac{41}{28} = -\frac{13}{28} \)

Point of intersection of the two lines is \( \left( \frac{111}{14} - \frac{13}{28} \right) \)

(Effective calculators allow checking of your answers, so questions of this type can be produced with more efficiency)

12 Let \( x = 0.492 = 0.492492492 \ldots \) \( \times \) by 1000 as there are 3 recurring decimals
\( 1000x = 492.492492492 \ldots \) \( \text{(2)} \)
\( 999x = 492 \)

((2) - (1) to eliminate recurring decimals)
\( x = \frac{492}{999} = \frac{164}{333} \) as required

(Perform numerator and denominator by 3)

13 (a) (b) \( \text{ } \)
\( \text{ } \)
\( \text{ } \)
\( \text{ } \)
\( \text{ } \)
\( \text{ } \)

(c) (Enlargements: Area of object \( \times k^2 \)
= Area of image, if \( k \) is the scale factor of enlargement)
(i) Area of \( Q_1 = 54 \times \left( \frac{1}{2} \right)^2 = 13 \frac{1}{2} \) units\(^2 \)
(ii) Area of \( R_1 = 6 \times 5^2 = 150 \) units\(^2 \)

14 Upper bound mass = 155 kg
Greatest number of cases lifted ‘safely’, \( n \), is worst case i.e. when the safe loading is minimised and the case weight is maximised.
\( n = \frac{1750}{155} = 11.290 \ldots \) say 11

15 \( 1 - \frac{2x - 7 + 3}{x^2 + 2x - 15} \)
\( = \frac{(x + 5)(x - 3) - (2x - 1)(x - 3)}{(x + 5)(x - 3)} \)
\( = \frac{(x + 5) - (2x - 1)}{x + 5} = \frac{6 - x}{x + 5} \)

(Express as a single fraction with a common denominator of \((x + 5)(x - 3)\))

16 (a) (i) \( 5p - 7 \geq p + 3, 4p \geq 10, p \geq 2.5 \)
(ii) \( 3(2p - 7) < 2(p + 3) \)
\( 6p - 21 < 2p + 6 \)
\( 6p - 2p < 6 + 21 \)
\( 4p < 27 \)
\( p < 6.25 \)

(b) The full solution set is \( 2.5 \leq p < 6.25 \), so the integer solution set = \{3, 4, 5, 6\}

17 (a) \( I \propto \frac{1}{d^2} \), so \( I = \frac{k}{d^2} \), where \( k \) is a constant of proportionality
\( 10^6 = \frac{k}{500} \), so \( k = 10^6 \), \( I = \frac{10^6}{d^2} \)

(b) At \( I = 4cd, 4 = \frac{10^6}{d^2} \), \( d = 500 \), so \( p = 0.5 \)
18 (a) (i) \( f(0) = 6 \)
(ii) \( f(g(0)) = f(2) = -2 \)
(\text{Find } g(0) \text{ first from the } g(x) \text{ graph and substitute into } f(x))

(b) \( f(x) = 2, x = -3, 1, 5 \) (Draw \( y = 2 \) on graph and find where it intercepts \( y = f(x) \)).

(c) On \( y = g(x) \) gradient at \( x = 7 \), \( m \approx -\frac{1}{2} \)

\[ = -2 \left( \frac{\text{rise}}{\text{run}} \right) \]

19 (a) Angle \( MLJ = 30^\circ \)
\text{(angles in the same segment)}

(b) Angle \( JLK = 73^\circ \)
\text{(alternate segment theorem)}

(c) Angle \( GJM = 30^\circ \)
\text{(alternate segment theorem)}

\text{Angle } \angle GMJ = 69^\circ \text{ (angle sum of a triangle = 180°)}

\text{Angle } \angle LMP = 38^\circ \text{ (angle sum in a straight line = 180°)}

\text{Angle } \angle MPL = 112^\circ \text{ (angle sum of a triangle = 180°)}

(d) Angle \( JKL = 69^\circ \) \text{(opposite angles in a cyclic quadrilateral add up to 80°)}

20 (a) \( 5x^2 - 20x + 12 = 5 \left[ x^2 - 4x + \frac{12}{5} \right] \)
\[ = 5 \left[ (x - 2)^2 - \frac{8}{5} \right] \]
\[ = 5(x - 2)^2 - 8 \]
so \( a = 5, b = -2 \) and \( c = -8 \)

(b) \( f(x)_{\text{min}} \) occurs at \( x = 2, y = -8 \)
\text{min point is } (2, -8)

21 (a) \( P(C \cap S \cap V) = \frac{7}{50} \)
\( \left( P(E) = \frac{n(E)}{n(\Omega)} \right) \)

(b) \( P(C \cap S') = \frac{6 + 9}{50} = \frac{15}{50} = \frac{3}{10} \)

(c) \( P(S \cup V') = \frac{12 + 4 + 7 + 6}{50} = \frac{29}{50} \)

(d) \( P(C \mid V) = \frac{n(C \cap V)}{n(V)} = \frac{9 + 7}{9 + 7 + 4 + 12} \)
\[ = \frac{16}{35} = \frac{4}{7} \]

\text{(Sample space is reduced from } n(\Omega) \text{ to } n(V))

22

\( y = \sin x \text{ graph and find where it intercepts } y = \sin x \).

\( \text{(If } g(x) = 2f(x), g(x) \text{ is a stretch of } f(x) \text{ parallel to the } y\text{-axis of scale factor 2 and } 2f(x) + 1 \text{ is } g(x) + 1 \text{ which is a translation of } g(x) \)

\text{along vector } \begin{pmatrix} 0 \\ 1 \end{pmatrix} \)

\( A \) is the maximum point of \( g(x) \) in the domain which is \((90^\circ, 3)\)

\( B \) is the minimum point of \( g(x) \) in the domain which is \((270^\circ, -1)\)

23

Area of triangle = area of circle of circumference \( 10\pi \)

Circumference of circle = \( 10\pi \)

\[ 10\pi = \pi d \]
\[ r = 5 \]

Area of circle, \( A = \pi \times r^2 = 25\pi \) \( (A = \pi r^2) \)

Area of triangle, \( A = \frac{1}{2} \times a \times h \)

\text{(Area of triangle = } \frac{1}{2} \times \text{base } \times \text{perpendicular height)}

\[ a^2 = h^2 + \left( \frac{a}{2} \right)^2, h^2 = a^2 - \left( \frac{a}{2} \right)^2 = \frac{3a^2}{4}, h = \frac{a\sqrt{3}}{2} \]

\text{(Pythagoras’ theorem)}

\[ A = \frac{1}{2} \times a \times \frac{a\sqrt{3}}{2} = \frac{a^3}{4} \]

Both areas are equal so, \( 25\pi = \frac{a^3}{4} \)

\[ a^3 = \frac{100\pi}{\sqrt{3}} \]

24 (a) \( 240 = \pi r^2 \nu, \) so \( \nu = \frac{240}{\pi r^2} \)

(b) \( A = 2\pi r^2 + 2\pi r\nu = 2\pi r^2 + 2\pi r \times \frac{240}{\pi r^2} \)
\[ A = 2\pi r^2 + \frac{480}{r} \text{ as required} \]
(cylinder: Volume = \(\pi r^2 h\), Curved surface area = \(2\pi rh\))

(c) \[ A = 2\pi r^2 + 480r^{-1} \text{, so } \frac{dA}{dr} = 4\pi r - 480r^{-2} \]
\[ = 0 \text{ at stationary point} \]
(Stationary points occur when the gradient is 0, i.e. \(\frac{dy}{dx} = 0\))

\[ 4\pi r = 480r^{-2} = \frac{480}{r^2}, r^3 = \frac{480}{4\pi} = \frac{120\pi}{4} = 30\pi \]
\[ r = \sqrt[3]{\frac{120\pi}{4}} \text{ cm} = 3.36 \ldots \]

(d) Take a small step to the left of \(r = \sqrt[3]{\frac{120\pi}{4}}\), say \(r = 3.3\)

\[ \frac{dA}{dr} \text{ at } r = 3.3 \text{ is equal to } -2.6\ldots \]

Take a small step to the right of \(r = \sqrt[3]{\frac{120\pi}{4}}\), say \(r = 3.4\)

\[ \frac{dA}{dr} \text{ at } r = 3.4 \text{ is equal to } +1.2\ldots \]

The curve shape around the stationary point is a U shape, so \(A\) will be a minimum value when \(r = \sqrt[3]{\frac{120\pi}{4}}\)

\[ A_{\text{min}} = 214 \text{ cm}^2 \text{ (3 s.f.)} \]
(Efficient use of the ‘Ans’ button on the calculator is helpful for part d)

**EXAMINATION PRACTICE PAPERS 1B SOLUTIONS**

1. \[ \frac{3.1^2 + 1.3^2}{\sqrt{3.1^2 - 1.3^2}} = 1.1944\ldots = 1.19 \text{ (3 s.f.)} \]

2. 24 mg = 60%  
   (Find 1%, so that 100% can be calculated)

   \[ \frac{24}{60} = 1\% \]

   Daily recommended daily dose:

   \[ 100\% = 100 \times \frac{24}{60} = 40\text{ g} \]

   so recommended weekly vitamin C dose

   \[ = 7 \times 40\text{ g} = 280\text{ g} \]

3. (a) speed = \(\frac{\text{distance}}{\text{time}}\)

   \[ d = 168 \times 2.95 = 495.6 \text{ km} = 496 \text{ km} \]
   (nearest km)

   (Time from Nimes to Paris = 22 h 45 min = 19 h 48 min = 2.95 h)

   \[ \text{Efficient use of the ‘Ans’ button on the calculator is helpful for part d} \]

   (b) speed = \(\frac{\text{distance}}{\text{time}}\)

   \[ \frac{831}{168} = 4.9464\ldots = 4 \text{ h 56 min 47 s} \]

   Timetable time for journey

   \[ = 22:45 - 16:20 = 6 \text{ h 25 min} \]

   Total stoppage time

   \[ = 6 \text{ h 25 min} - 4 \text{ h 56 min 47 s} = 1 \text{ h 28 min 13 s} \]

   \[ = 1 \text{ h 28 min} \]

   (cylinder: Volume = \(\pi r^2 h\), Curved surface area = \(2\pi rh\))

4. \[ 1\% \times \left( \frac{7}{3} + 3\pi \right) = \frac{3}{2} \times \left( \frac{22}{3} \times \frac{7}{3} \right) \]

   \[ = \frac{3}{2} \times \frac{49}{9} = \frac{49}{6} = \frac{81}{6} \]

5. (a) \( x = 2464 = 2^5 \times 7 \times 11, y = 1372 = 2^2 \times 7^3 \)

   (Long division by prime factors should be shown)

   (b) (i) HCF = \(2^2 \times 7\), so \(m = 2, n = 1\)

   (ii) LCM = \(2^3 \times 7^3 \times 11\), so \(p = 5, q = 3, r = 1\)

   \[ (p + q + r)^{(m + n)} = 9^3 = 729 \]

6. (a) Yeast grams = \(\frac{20}{5} \times 35 = 140\text{ g} \]

   (b) \( L = \frac{90}{7.5} = 60, L = 60 \)

   (c) Brown flour: White flour: Yeast

   \[ = 2000: 500: 35 = 400: 100: 7 \]

   \[ p = 400, q = 100, r = 7 \]

7. Total time of all six runners is

   \[ = 6 \times (2 \text{ min 15.5 s}) = 12 \text{ min} + 6 \times 15.5 \text{ s} = 12 \text{ min} + 93 \text{ s} = 13 \text{ min 33 s} \]

   Total time of the other 5 runners is

   \[ 5 \times 2 \text{ min} + (13.6 + 15.9 + 15.8 + 18.3 + 14.1) \text{ s} = 10 \text{ min} + 77.7 \text{ s} = 11 \text{ min 17.7 s} \]

   (Mean = \(\frac{\text{sum of all scores}}{\text{number of scores}}\))

   Lucia’s time = 13 min 33 s – 11 min 17.7 s

   \[ = 2 \text{ min} 15.3 \text{ s} \]

   All the times in order are

   1st 2 min 13.6 s

   2nd 2 min 14.1 s

   3rd 2 min 15.3 s

   4th 2 min 15.8 s

   5th 2 min 15.9 s

   6th 2 min 18.3 s

   So Lucia came 3rd and was awarded the bronze medal.
Let perimeter of a semi-circle be 
\[ p = \text{diameter} + \text{half of circumference} \]
\[ p = 2r + \pi r = r(2 + \pi) \]
\[ p_{\text{max}} = 105 \text{ cm}, \quad p_{\text{min}} = 95 \text{ cm} \]
Let area of semi-circle be \( A \)
\[ A = \frac{1}{2} \pi r^2 \]
(a) \( A_{\text{max}} \) occurs when radius is \( r_{\text{max}} \)
\[ 105 = r_{\text{max}} (2 + \pi), \quad r_{\text{max}} = \frac{105}{2 + \pi}, \quad \text{so} \quad A_{\text{max}} = \frac{1}{2} \pi \left( \frac{105}{2 + \pi} \right)^2 \]
\[ = 655.09327\ldots \text{ cm}^2 \quad = 655 \text{ cm}^2 \quad (3 \text{ s.f.}) \]
(b) \( A_{\text{min}} \) occurs when radius is \( r_{\text{min}} \)
\[ 95 = r_{\text{min}} (2 + \pi), \quad r_{\text{min}} = \frac{95}{2 + \pi}, \quad \text{so} \quad A_{\text{min}} = \frac{1}{2} \pi \left( \frac{95}{2 + \pi} \right)^2 \]
\[ = 536.2554\ldots \text{ cm}^2 \]
\[ = 536 \text{ cm}^2 \quad (3 \text{ s.f.}) \]

Let \( AB = d \)
\[ d^2 = 40^2 + 50^2 = 4100 \]
(Pythagoras' theorem)
\[ d = \sqrt{4100} = \sqrt{41 \times 100} = 10\sqrt{41}, \quad \text{so} \quad k = 10 \]
(b) Bearing of ship \( A \) from ship \( B \) is \( 180^\circ + \theta \) (Bearing is clockwise from North)
\[ \tan \theta = \frac{40}{50}, \quad (\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}) \]
\[ \text{so} \quad \theta = 38.65\ldots^\circ \quad \text{so, bearing of ship} \quad A \text{ from ship } B = 219^\circ \text{ (nearest degree)} \]

Either \( x^2 = 0, \quad x = 0 \)
Or \( 2x - 3 = 0, \quad x = \frac{3}{2} = 1\frac{1}{2} \)

If \( n \) is an integer.
A general term for an odd number = \( 2n + 1 \)
The next odd number = \( (2n + 1) + 2 = 2n + 3 \)
The difference between the squares of these numbers
\[ = (2n + 3)^2 - (2n + 1)^2 \]
\[ = (2n + 3)(2n + 3) - (2n + 1)(2n + 1) \]
\[ = (4n^2 + 12n + 9) - (4n^2 + 4n + 1) \]
\[ = 8n + 8 = 8(n + 1) \text{ which is a multiple of } 8 \]

192\pi = \pi \times 4^2 \times BC 
(Volume of cylinder = \pi r^2 h)
\[ BC = \frac{192\pi}{16\pi} = 12 \text{ cm} \]
Required angle = \( BAC \), triangle \( BAC \) is right-angled
Let angle \( BAC = \theta \)
\[ \tan(\theta) = \frac{BC}{AC} = \frac{12}{8}, \quad \text{so} \quad \theta = 56.3^\circ (3 \text{ s.f.}) \]

(a) The exterior angle of the polygon = \( 180^\circ - 144^\circ = 36^\circ \) 
\( (n\text{-sided regular polygon: exterior angle} = \frac{360^\circ}{n}) \)
So \( 36^\circ = \frac{360^\circ}{n}, \quad n = 10 \)
The shape has 10 sides so is a decagon.
(b) Perimeter = \( 10 \times 12 = 120 \text{ cm} \)
(c) The polygons are similar figures
Scale factor of length = \( \frac{960}{120} = 8 \) 
(Similar figures: small area \times k^2 = larger area, where \( k \) is length scale factor)
Enlarged area = \( 8^2 \times A = 64A \)
14 The toys are not replaced.
Let \( X \) be the number of toys that are the same from 3 random picks.
T: Teddy bears, R: Robots, D: Dolls

\[
P(X = 2) = 3 \times P(TTT') + 3 \times P(RRR') + 3 \times P(DDD')
\]

(\( \times 3 \) as the ‘not’ option can occur in 3 places)

\[
(P(A \text{ or } B) = P(A) + P(B) \text{ if } A \text{ and } B \text{ are mutually exclusive})
\]

\[
P(A \text{ and } B) = P(A) \times P(B) \text{ if } A \text{ and } B \text{ are independent}
\]

\[
P(X = 2) = 3 \times \frac{5}{15} \times \frac{4}{14} \times \frac{10}{13} + \frac{3}{15} \times \frac{4}{14} \times \frac{10}{13} + \frac{3}{15} \times \frac{4}{14} \times \frac{10}{13}
\]

\[
= 9 \times \frac{5}{15} \times \frac{4}{14} \times \frac{10}{13} = \frac{60}{91}
\]

15 (a) If \( g(x) = \frac{10}{\sqrt{x} - 3} \):

\[
y = \frac{10}{\sqrt{x} - 3}
\]

(Re-write in terms of \( y = \ldots \))

\[
x = \frac{10}{\sqrt{y} - 3}
\]

(Switch \( x \) and \( y \) variables)

\[
y - 3 = \frac{10}{x}, \quad y = \frac{10}{x} + 3 = \frac{10 + 3x}{x}
\]

(Re-arrange to make \( y \) the subject)

\[
g^{-1}(x) = \frac{10 + 3x}{x}
\]

(Replace \( y \) with \( g^{-1}(x) \))

(b) If \( gh(p) = -1, \ g\left(\frac{p - 5}{p}\right) = -1, \frac{10}{p - 5} - 3 = -1, \frac{10}{(p - 5) - 3p} = -1, \frac{-10p}{p - 5} = -1, \)

(Find \( h(p) \) first and input this into \( g(x) \))

\[
\frac{10}{p - 5} - 3p = 2p + 5, \quad 8p = 5, \quad p = \frac{5}{8}
\]

16 (a) Esther’s investment = €12 000 \times 1.025 \times 1.035^n = €16 763.64

(Increase \( a \) by \( b\% \) for \( n \) years compound interest = \( a \times \left(1 + \frac{b}{100}\right)^n\))

(b) Let \( EP \) be the amount Ivan invests

\[
p \times 1.015^2 = €1236.27,
\]

so \( p = \frac{1236.27}{1.015^2} = 1200 \)

Ivan invests €1200 into his Savings Bond.

17 \( (x + 1) : (x + 2) = (y + 1) : (y + 3) \)

\[
\frac{x + 1}{x + 2} = \frac{y + 1}{y + 3}
\]

\[
(x + 1)(y + 3) = (x + 2)(y + 1)
\]

\[
x + 3y + x + 3 = xy + x + 2y + 2
\]

\[
y = 2x + 1
\]

18 (a) (Density = \frac{\text{Mass}}{\text{Volume}})

Volume of cylinder = \( \pi r^2 h \)

Mass = Density \times Volume

\[
= 2710 \times \pi \times 0.5^2 \times 1.20 = 813\pi \text{ kg}
\]

\( k = 813 \)

(b) (Pressure = \frac{\text{Force}}{\text{Area}})

\[
\text{Pressure} = \frac{2.5 \times 10^3}{\pi \times 0.5^2} = 31830.98... \text{ N/m}^2
\]

\[
= 3.18 \times 10^4 \text{ N/m}^2 \text{ (3 s.f.)}
\]

19 \( x^2 + y^2 = 26 \) \[1\]

\[
y = 3 - 2x \] \[2\]

(subs into \[1\])

\[
x^2 + (3 - 2x)^2 = 26
\]

(expand out brackets)

\[
x^2 + 9 - 12x + 4x^2 = 26
\]

\[
5x^2 - 12x - 17 = 0
\]

\[
(5x - 17)(x + 1) = 0, \text{ so } 5x - 17 = 0 \text{ or } x + 1 = 0
\]

\[
x = \frac{17}{5} = \frac{3^2}{5}, \quad x = -1
\]

(substitute into \[2\])

\[
y = 3 - 2\left(\frac{17}{5}\right) = -\frac{19}{5} = -\frac{34}{5}, \quad y = 3 - 2(-1) = 5
\]

\[
P\left(\frac{3^2}{5}, \frac{-34}{5}\right), Q(-1, 5)
\]

(Write answers as coordinate pairs)

\( P \) & \( Q \) are interchangeable!

20 After 3 hours Rover \( B \) has travelled

\[
\frac{3 \times 4 \times 60 \times 60}{100} = 432 \text{ m}
\]

(Draw a sketch of the situation. There are two possible positions for Rover \( A \), shown as \( A \) and \( A' \) on the sketch.)

(Using the sine rule in triangle \( ABC \) with the usual notation.)
\[
\sin(A) = \frac{250}{432} = 0.579
\]
or 120.23° (2 d.p.)

The smallest possible value of \( x \) is given by Rover \( A \) being at position \( A' \) on the sketch, corresponding to \( x = 120.23° \)

When \( x = 120.23° \), \( B = 180 - 30 - 120.23 \)

\[
= 29.77°
\]

\[
\sin(29.77°) = \frac{250}{\sin(30°)}
\]

\[
= 248.26\text{ m} (2\text{ d.p.})
\]

speed is

\[
\frac{248.26}{3} \text{ m/h} = \frac{248.26 \times 100}{3 \times 60 \times 60}
\]

\[
= 2.30\text{ cm/s} (3\text{ s.f.})
\]

\[
\vec{PS} = 2\vec{a}
\]

\[
\vec{PR} = 2\vec{a} + \vec{b}
\]

\[
\vec{MQ} = -\vec{a} + \vec{b} = \vec{b} - \vec{a}
\]

If \( PNR \) is a straight line \( \vec{PR} = k\vec{PN} \)

\[
\vec{PN} = \vec{PM} + \frac{1}{3}\vec{MQ} = \vec{a} + \frac{1}{3}(\vec{b} - \vec{a}) = \frac{1}{3}(2\vec{a} + \vec{b})
\]

\[
= \frac{1}{3}\vec{PR}, \text{ so } \vec{PR} = 3\vec{PN}
\]

\[
PNR \text{ in a straight line as } k = 3
\]

\[
t_n = 3n + 1
\]

\[
t_1 = 4, t_2 = 7, t_3 = 10 \ldots \text{ so sequence is an arithmetic progression with } a = 4, d = 3, n = 20
\]

\[
(S_n = \frac{n}{2}(2a + (n - 1)d) \text{ Sum to } n \text{ terms for an arithmetic progression})
\]

\[
S_{20} = \frac{20}{2}(2 \times 4 + (20 - 1) \times 3) = 650
\]

23 (a) (i) \( s = (t - 1)^2 + 3t = (t - 1)(t - 1)^2 + 3t = (t - 1)(t^2 - 2t + 1) + 3t = t(t^2 - 2t + 1) - 1(t^2 - 2t + 1) + 3t = t^3 - 3t^2 + t^2 - t + 2t - 1 + 3t = t^3 - 3t^2 + 6t - 1
\]

\[
v = \frac{ds}{dt} = 3t^2 - 6t + 6 \text{ m/s}
\]

(ii) \( a = \frac{dv}{dt} = 6t - 6 \text{ m/s}^2
\]

(b) \( a = 6t - 6, \ t = 1
\)

At \( t = 1, v = 3 \times (1)^2 - 6 \times 1 + 6 = 3 \text{ m/s}

24 (a) Volume of space \( V = \text{Volume of cylinder } V_c - \text{Volume of spheres } V_s
\)

\[
V_c = \pi r^2 \times 4r = 4\pi r^3
\]

(radius of cylinder = \( r \), height of cylinder = \( 4r \))

\[
V_s = 2 \times \frac{4}{3} \times \pi \times r^3 = \frac{8\pi r^3}{3}
\]

\[
V = 4\pi r^3 - \frac{8\pi r^3}{3} = \frac{12\pi r^3}{3} - \frac{8\pi r^3}{3} = \frac{4\pi r^3}{3}
\]

as required

(b) \( \frac{4\pi r^3}{3} = \frac{9\pi}{2}
\)

So, \( \frac{4r^3}{3} = \frac{9}{2}, 8r^3 = 27, r^3 = \frac{27}{8}
\)

\[
r = \sqrt[3]{\frac{27}{8}} = \frac{3}{2} \equiv 1.5 \text{ cm}
\]

(e) Required fraction = \( \frac{4\pi r^3}{3} ÷ 4\pi r^3
\)

\[
= \frac{4\pi r^3}{12\pi r^3} = \frac{1}{3}
\]

EXAMINATION PRACTICE PAPERS 2A SOLUTIONS

1

Let required bearing of Moritz to Kielder be \( \theta \).

So \( \theta = 180° + 70° = 250°
\)

(Bearings are measured clockwise from North)
2 (a) Sequence 10, 9.5, 9, 8.5... is an arithmetic progression with:
\[
a = 10, \ d = -0.5
\]
\[
t_n = 10 + (n - 1)\times(-0.5) = 10 - 0.5n + 0.5 = 10.5 - 0.5n = 0.5(21 - n)
\]
\[(t_n = a + (n - 1)\ d \text{ is the } n\text{th term of an arithmetic progression})\]

(b) If \(S_n = 0\)
\[
(S_n = \frac{n}{2} [2a + (n - 1)d] \text{ is the sum to } n\text{ terms of an arithmetic progression})
\]
\[
0 = \frac{n}{2} [2 \times 10 + (n - 1) \times (-0.5)]
\]
(Divide both sides by \(\frac{n}{2}\))
\[
0.5(n - 1) = 20
\]
\[
40 = n - 1
\]
\[
n = 41
\]

3
\[
A
\]
\[
\begin{array}{c}
2^2 \\
3^2 \\
7 \\
x^2 \\
y
\end{array}
\]
\[
\begin{array}{c}
2^3 \\
3^2 \\
9 \\
x^2 \\
y
\end{array}
\]

Let \(A: 12x^3y^2z^5\)
\(B: 21x^3y^2z^2\)

HCF = \(3x^3y^2z^2\) (Intersection: \(A \cap B\))

LCM = \(84x^3y^2z^5\) (Union: \(A \cup B\))

4
\[
\frac{2v - w}{3} = \frac{2v + w}{5} + u
\]
(multiply both sides of the equation by 15)
\[
5(2v - w) = 3(2v + w) + 15u
\]
\[
10v - 5w = 6v + 3w + 15u
\]
(Expand out brackets both sides)
\[
4v = 8w + 15u
\]
(Isolate \(v\) on the LHS of the equation)
\[
v = \frac{8w + 15u}{4}
\]
(Divide both sides of equation by 4)

5

Let price of shoes before sales tax be \(\$p\).
\[p \times 1.15 = 92\]
(Increase \(a\) by \(b\% = a \times (1 + \frac{b}{100})\))
\[p = \frac{92}{1.15}, \ p = \$80\]

6 (a) Equation of a straight line: \(y = mx + c\)
\([m: \text{gradient, } c: \text{y intercept}]\)
\[
m = \frac{8 - 4}{3 - 1} = \frac{4}{2} = -1
\]
\[
\text{(Gradient of } AB = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1})
\]

so \(y = -x + c\), so \((1, 4)\) satisfies the equation as it is on the line.
\(4 = -1 + c\), so \(c = 5\), equation of \(L: y = -x + 5\),
\(x + y - 5 = 0; a = 1, b = 1, c = -5\)

(b) Midpoint of \(AB = \left(\frac{-3 + 4}{2}, \frac{4 + 8}{2}\right) = (-1, 6)\)
(Midpoint of \(A(x_1, y_1)\) and \(B(x_2, y_2)\))
\[
\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)
\]
Let gradient of \(L\) be \(m_1\) and gradient of \(M\) be \(m_2\)
\(m_1 \times m_2 = -1, -1 \times m_2 = -1, m_2 = 1\)
(Product of the gradients of two perpendicular lines = -1; \(m \times m_2 = -1\))

Equation of \(M: y = x + c\), so \((-1, 6)\) satisfies the equation as it is on the line.
Therefore \(6 = -1 + c, c = 7\), \(y = x + 7, -x + y - 7 = 0, a = -1, b = 1, c = -7\)

7
Let \(y = 2x^2 - x - 6 = (2x + 3)(x - 2)\)
(Factorising helps sketch the curve)
At the \(x\)-axis \(y = 0\), so \(0 = (2x + 3)(x - 2)\),
\(x = 2\) or \(-\frac{3}{2}\)

So \(2x^2 - x - 6 \geq 0\) when \(x \leq -\frac{3}{2}, x \geq 2\)
(y-values above or on the \(x\)-axis satisfy the inequality)

8

So the inequality is satisfied when \(x \leq -\frac{3}{2}\) or \(x \geq 2\).
Angle \(OFA = 3x\) (opposite angles)
Angle \(AOF = 90 - x\)
\((\Delta AOD \text{ is a right angled } \Delta \text{ as } AD \text{ is a tangent and } OA \text{ is a radius})\)
Angle \(ACB = 2x\)
(alternate segment theorem)
Angle \(BAC = \frac{1}{2} (180 - 2x) = 90 - x\)
\((\Delta ACB \text{ is isosceles})\)
Angle \(AOB = 4x\)
\((2 \times \text{angle } BCA, \text{angle subtended at centre is twice angle at circumference})\)
Angle \(OAB = \frac{1}{2} (180 - 4x) = 90 - 2x\)
\((\Delta OAB \text{ is isosceles})\)

**9 (a)** Percentage > 90 mins = \(\frac{3}{24} \times 100 = 12.5\%\)

(b) Modal class is \(30 < x \leq 60\)
(Most popular group)

(c) Mean = \(\frac{\sum f_i x_i}{\sum f_i}\), where is the midpoint of each class

\[
\text{Mean} = \frac{4 \times 15 + 10 \times 45 + 7 \times 75 + 3 \times 105}{24} = \frac{1350}{24} = 56.25 \text{ mins} = 56 \text{ mins 15 s}
\]

(d) New mean = \(\frac{1350 + 56.25}{25} = 56.25 \text{ mins},\)
so remains unchanged

**10 (a)** \(\text{speed}_{\text{max}} = \frac{\text{distance}_{\text{max}}}{\text{time}_{\text{max}}} = \frac{805}{137.5} = 5.85454...\)

\(= 5.85 \text{ m/s (3 s.f.)}\)

(b) \(\text{speed}_{\text{min}} = \frac{\text{distance}_{\text{min}}}{\text{time}_{\text{min}}} = \frac{795}{142.5} = 5.5789...\)

\(= 5.58 \text{ m/s (3 s.f.)}\)

**11 (a)**

\[
\begin{array}{c}
\text{First passenger} \\
\text{Overweight} \\
\text{Not Overweight} \\
\hline
\text{Second passenger} \\
\text{Overweight} \\
\text{Not Overweight}
\end{array}
\]

\(p\)

\(1 - p\)

\(1 - p\)

\(p\)

\(p\)

\begin{align*}
\text{(b) (i)} & \text{ Branches required are: } \\
& \text{‘Overweight – Not overweight’ and} \\
& \text{‘Not overweight – overweight’} \\
& \text{Probability } = p \times (1 - p) + (1 - p) \times p \\
& = 2p (1 - p) \text{ or } 2p - 2p^2 \\
\end{align*}

\(\text{(ii)}\)

\(2p - 2p^2 = 0.05\)

\[2p^2 - 2p + 0.025 = 0\]

(Solving this quadratic using the quadratic formula.)

\[p = \frac{1 \pm \sqrt{1 - 4 \times 1 \times 0.025}}{2 \times 1} = \frac{1 \pm \sqrt{0.9}}{2}\]

\[p = 0.974 \text{ or } p = 0.0257\]

12 Let \(a = 2n + 1, b = 2n + 3, c = 2n + 5 \text{ and } d = 2n + 7\)
(If \(n\) is an integer, \(2n\) is always even, so \(2n + 1\) is odd)

\[
d^2 - a^2 = (2n + 7)^2 - (2n + 1)^2 \\
= 4n^2 + 28n + 49 - (4n^2 + 4n + 1) \\
= 4n^2 + 28n + 49 - 4n^2 - 4n - 1 \\
= 24n + 48 = 24(n + 2)
\]

\(24(n + 2)\) is divisible by 24 so, \(d^2 - a^2\) is divisible by 24.

13 \((AM \text{ is a line of symmetry, so } \Delta ABM \text{ is a right-angled triangle. } M \text{ is midpoint of } BC \text{ so } BM = 1)\)
\(AM^2 + BM^2 = AB^2\) (Pythagoras’ theorem)

\(AM^2 + 1 = 3^2\)

\(AM^2 = 8\)

\(AM = \sqrt{8}\)

\(= 2\sqrt{2}\)

\(AX = \frac{3}{4} AM\)

\(AX = \frac{3}{4} \times 2\sqrt{2}\)

\(= \frac{3\sqrt{2}}{2} \text{ cm}\)
14 (a) \[ 1 - \frac{1}{x+a} - \frac{x-1}{x} = \frac{x(x+a) - x - (x-1)(x+a)}{x(x+a)} \]

(Lowest common denominator is \(x(x+a)\))

\[ = \frac{x^2 + ax - x - (x^2 - x + ax - a)}{x(x+a)} \]

\[ = \frac{x^2 + ax - x^2 + x - ax + a}{x(x+a)} \]

\[ = \frac{a}{x(x+a)} \]

(b) By inspection \(a = 2\), \(\frac{2}{x(x+2)} = 2 \times \frac{1}{3} = \frac{2}{3}\),

\[ 6 = 2x(x + 2) = 2x^2 + 4x \]

\[ 0 = x^2 + 2x - 3 = (x + 3)(x - 1), \]

\(x = 1\) or \(x = -3\)

15 Using the sine rule gives \(\frac{AC}{\sin(60^\circ)} = \frac{38.7}{\sin(52^\circ)}\)

\[ AC = \frac{38.7 \times \sin(60^\circ)}{\sin(52^\circ)} \]

The minimum value of \(AC\) will be given when 38.7 is a minimum, \(\sin(60^\circ)\) is a minimum and \(\sin(52^\circ)\) is a maximum.

38.7 is correct to 3 s.f. so the minimum value is 38.65

\(60^\circ\) is correct to 2 s.f. so the minimum value is 68.5

\(52^\circ\) is correct to 2 s.f. so the maximum value is 52.5

Minimum value of \(AC\) is \(\frac{38.65 \times \sin(68.5^\circ)}{\sin(52.5^\circ)}\)

\[ = 45.327 \ldots = 45.3\text{ m to 3 s.f.} \]

16 (a) The perimeter \(= r + 4r + r + 4r + \pi r = 10r + \pi r \)

(Curved length of a semicircle = \(\pi r\))

\(10r + \pi r = 50\), \(r(10 + \pi) = 50\),

\(r = \frac{50}{10 + \pi} = 3.8047\ldots\)

Area of shape \(= \frac{\pi r^2}{2} + 4r \times r = r^2\left(\frac{\pi}{2} + 4\right)\)

\[ = (3.8047\ldots)^2\left(\frac{\pi}{2} + 4\right) = 80.64\text{ cm}^2(4\text{ s.f.}) \]

(b) (i) Volume \(= 80.64\ldots \times 25 = 2016\text{ cm}^3(4\text{ s.f.})\)

(ii) (Surface area excluding the flat ends = perimeter \(\times 25\))

Surface area \(= 50 \times 25 + 2 \times 80.64\ldots = 1411\text{ cm}^2(4\text{ s.f.})\)

(c) (i) Length scale factor = \(30 \div 50 = 0.6\text{ volume scale factor} = 0.6^3\)

Volume of new shape is \(80.64\ldots \times 25 = 2016\text{ cm}^3(4\text{ s.f.})\)

(ii) Area scale factor = \(0.6^2\)

Area of new shape is \(1411 \times 0.6^2 = 508\text{ cm}^2(3\text{ s.f.})\)

17 (a) (i) \(3^p = x \Rightarrow (3^\frac{p}{3})^q = (3^\frac{p}{3})^q \Rightarrow x^q = x^p \Rightarrow q = 3p - 1 \Rightarrow x^q = x^3p - 1 \Rightarrow x^q = x^3p \)

(ii) \(3^p = x \Rightarrow (3^\frac{p}{3})^q = (3^\frac{p}{3})^q \Rightarrow y^p = y^q \Rightarrow y^q = y^p \)

(b) \(\frac{-y}{3^p} = y \Rightarrow \left(\frac{-y}{3^p}\right)^{3^q} = \left(\frac{-y}{3^p}\right)^{3^q} \Rightarrow y = \left(\frac{3^q}{3^p}\right) = \left(\frac{3^q}{3^p}\right) \Rightarrow y^q = y^p \)

18 (a) Shot hits the ground when \(h = 0\), so \(5t^2 - 6t - 2 = 0\) (Rearrange formula to make squared part positive)

\(A = 5, B = -6, C = -2\)

(Using quadratic formula: \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\))

\[ t = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(5)(-2)}}{2(5)} = 1.47\text{ s (3 s.f.)} \]
(b) \( h = -5 \left[ t^2 - \frac{6t}{5} + \frac{2}{5} \right] = -5 \left[ \left( \frac{3}{5} \right)^2 - \frac{9}{25} - \frac{10}{25} \right] \)
\[ = -5 \left[ \left( \frac{3}{5} \right)^2 - \frac{19}{25} \right] = -5 \left[ \left( \frac{3}{5} \right)^2 - \frac{9}{25} \right] \]
\[ = -5 \left( \frac{3}{5} \right)^2 + \frac{19}{5} \]
Therefore \( a = -5, b = -\frac{3}{5}, c = 3 \frac{4}{5} \)

(c) \( h_{max} = 3.8\ m \) at \( t = \frac{3}{2} \ s \)

19 Total surface area is the sum of the areas both ends plus the curved surface area
Curved surface area = circumference \times height
Total surface area of \( A = 2 \times \pi r^2 + 2\pi r \times 3.5 = 9\pi r^2 \)
Total surface area of \( B = 2 \times \pi R^2 + 2\pi R \times R = 4\pi R^2 \)
\( 9\pi r^2 = 4\pi R^2 \)

Surface areas are equal
\( r^2 = \frac{4\pi R^2}{9\pi} \quad r = \frac{2R}{3} \quad (1) \)

Square rooting both sides
Volume of \( A = \pi r^2 \times 3.5r = 3.5\pi r^3 \)
Volume of \( B = \pi R^2 \times R = \pi R^3 \)
Volume of \( A = \frac{3.5\pi r^3}{\pi R^3} \)
From (1) \( r^3 = \left( \frac{2R}{3} \right)^3 = \frac{8R^3}{27} \)
Volume of \( A = \frac{3.5\pi r^3}{\pi R^3} = 3.5 \times \frac{8R^3}{27} = \frac{28}{27} \)
ratio 28 : 27
so \( m = 28, n = 27 \)

20 (a) \( \overline{PA} = \frac{1}{3} \overline{a}, \overline{AB} = \overline{b} - \overline{a}, \overline{AQ} = \frac{2}{3} (\overline{b} - \overline{a}) \)
\( \overline{PQ} = \overline{PA} + \overline{AQ} = \frac{1}{3} \overline{a} + \frac{2}{3} (\overline{b} - \overline{a}) \)
\[ = \frac{2}{3} \overline{b} - \frac{1}{3} \overline{a} \]
\( \overline{Q\hat{B}} = \frac{1}{3} \overline{AB} = \frac{1}{3} (\overline{b} - \overline{a}) \quad \overline{BR} = \frac{1}{3} \overline{b} \)
\( \overline{QR} = \overline{Q\hat{B}} + \overline{BR} = \frac{1}{3} (\overline{b} - \overline{a}) + \frac{1}{3} \overline{b} \)
\[ = \frac{2}{3} \overline{b} - \frac{1}{3} \overline{a} \]
\( \overline{PQ} = \overline{QR}, \overline{PQ} \) and \( \overline{QR} \) are parallel with a common point \( Q \), so \( \overline{PQR} \) is a straight line and therefore \( P, Q \) and \( R \) are collinear points.

21 (a) Let reflection in \( x \)-axis be \( R \) and translation be \( T \), so \( TR(P) = Q \)
(undo the translation first and then the reflection by using their inverses)
\( \left( \frac{3}{7} \right) + \left( \frac{1}{-2} \right) = \left( \frac{4}{5} \right) \)
(This point is then reflected in \( x \)-axis)
\( P \left( \frac{4}{-5} \right) \)

(b) \( OP = \sqrt{4^2 + (-5)^2} = \sqrt{16 + 25} = \sqrt{41} \)
(Pythagoras’ theorem)
\( k = 41 \)

22 (a) \[ y = \frac{4}{3} \]
(i) \( f(x) + 1 = \cos(x) + 1 \)
(Translation along vector \( \left( \begin{array}{c} 0 \\ 1 \end{array} \right) \))
(ii) \( -f(x) = -\cos(x) \) (Reflection in \( x \)-axis)
(iii) \( 2f(x) = 2\cos(x) \)
(Stretch parallel to \( y \)-axis scale factor 2)

(b) If \( g(x) = x^2 \)
\[ = g(\cos(\pi x) + \pi) \]
\[ = g(\cos(\pi x)) + \pi \]
\[ = (\cos(\pi x))^2 + \pi \]
\[ = \cos^2(\pi x) + \pi \]

23 \( u = k_1v^2, u = k_2\sqrt{w}, \) so \( k_150^2 = k_2\sqrt{144}, \)
\[ \frac{k_1}{k_2} = \frac{\sqrt{144}}{50^2} = \frac{\sqrt{144}}{2500} = \frac{\sqrt{144}}{2500} \]
\[ = \frac{12}{500} = \frac{3}{625} = \frac{25}{v^2} \]
\[ \sqrt{v} = \frac{25}{\sqrt{5}} = 72.168... \]
\[ = 7.22 \times 10^1 \) (3 s.f.)
24 (a) Total surface area
\[ = 2x^2 + 2x^2 + 2xd + 2xd + xd + xd = 4x^2 + 6xd \]
\[ 4x^2 + 6xd = 1400, \quad 2x^2 + 3xd = 700 \]
\[ d = \frac{700 - 2x^2}{3x} \quad (1) \]

Length of tape used
\[ L = 2 \times 3x + 2(d + x) + 2(2x + d) \]
\[ = 12x + 4d \quad (2) \]

(Substituting \( d \) from (1) into (2))
\[ L = 12x + 4 \times \frac{700 - 2x^2}{3x} \]
\[ = 12x + \frac{2800}{3x} - \frac{8x^2}{3x} \]
\[ = \frac{28x}{3} + \frac{2800}{3x} \]

(b) To minimise \( L \), differentiate with respect to \( x \)
\[ \frac{dL}{dx} = \frac{28}{3} - \frac{2800}{3x^3} \]

(Derivative of \( \frac{1}{x} = x^{-1} = -x^2 = -\frac{1}{x^2} \))

For a maximum or minimum, \( \frac{dL}{dx} = 0 \)
\[ \frac{28}{3} - \frac{2800}{3x^3} = 0 \]
\[ \frac{28}{3} = \frac{2800}{3x^3} \]
\[ x^3 = 100 \]
\[ x = 10 \]
and \( L = 186 \frac{2}{3} \)

When \( x = 9 \), \( L = 187.7 \ldots \), when \( x = 11 \), \( L = 187.5 \ldots \)

It is a minimum as the graph of \( L \) against \( x \) is continuous for \( x > 0 \) and the shape of the graph of \( L \) against \( x \) is a U shape.

When \( x = 10 \), \( d = \frac{700 - 2 \times 10^2}{3 \times 10} = \frac{50}{3} \)

Using (1)
Dimensions of box are 10 cm \( \times \) 20 cm \( \times \) \( \frac{50}{3} \) cm
Volume of box = \( \frac{10000}{3} \) cm\(^3\)

EXAMINATION PRACTICE PAPERS 2B SOLUTIONS

1. Bag A: kg per $ = \frac{2.5}{2} = 1.25$ kg/$
   Bag B: kg per $ = \frac{4}{3.20} = 1.25$ kg/$
   Same value for money for both bags.
   Alternatively:
   Bag A: $ per kg = \frac{2}{2.5} = 0.80$ $/kg
   Bag B: $ per kg = \frac{3.20}{4} = 0.80$ $/kg

2. Area of whole circle = 240 cm\(^2\)
   \[ A = \pi r^2 = 240 = \pi r^2 \frac{240}{\pi} = r^2, \quad r = \sqrt{\frac{240}{\pi}} \]
   \[ = 8.74039 \ldots \text{cm} \]
   Perimeter of semi-circle = \( \frac{1}{2} \times 2\pi r + 2r \)
   \[ = \pi r + 2r = r(\pi + 2) = 44.9 \text{ cm} \quad (3 \text{ s.f.}) \]

3. Let expression be \( E \),
   \[ E = 0.347 \ 695 \ldots = 3.48 \times 10^{-1} \ (3 \text{ s.f.}) \]

4. 40% of the girls and 70% of the boys did not choose the fish option
   \[ \frac{40}{100} \times \frac{45}{100} + \frac{70}{100} \times \frac{55}{100} = \frac{56.5}{100} = 56.5\% \]
   (55% are boys)
   OR percentage who chose fish is
   \[ \frac{60}{100} \times \frac{45}{100} + \frac{30}{100} \times \frac{55}{100} = \frac{43.5}{100} = 43.5\% \]
   (55% are boys)
   percentage who did not choose fish is
   \[ 100\% - 43.5\% = 56.5\% \]

5. \[ T_n = \frac{t}{\nu} = \frac{2\pi - 1}{2\pi + 1} \]
   \[ T_1 = \frac{1}{3}, \ T_2 = \frac{1}{3}, \ T_3 = \frac{5}{7}, \ T_4 = \frac{7}{9} \]
   \[ T_1 \times T_2 \times T_3 \times T_4 = \frac{1}{9} \]

6. Expected number of non-white
   \[ = p(\text{non-white}) \times \text{number of trials} \]
   \[ = \frac{n - 3}{n} \times n = n - 3 \]
7 (a) \[ p \times (1.05)^3 = £1389.15 \]
   (Divide both sides by 1.05^3)
   \[ p = \frac{1389.15}{1.05^3} = 1200 \]

(b) \[ \% \text{ profit} = \frac{1389.15 - 1200}{1200} \times 100 \]
   = 15.7625 = 15.8% (3 s.f.)
   (% profit = \frac{\text{change}}{\text{original}} \times 100)

8 Pressure = \frac{\text{Force}}{\text{Area}} = \frac{15}{\pi} \text{N/m}^2

(Area of circle = \pi r^2)

Circumference = 2\pi r, so 30 = 2\pi r

(Divide both sides by 2\pi)

\[ r = \frac{30}{2\pi} = 4.7746... \text{cm} = 0.047746... \text{m} \]

(100 cm = 1 m)

pressure = \frac{15}{\pi \times 0.047746^2} = 2094.4... \text{N/m}^2
   = 2090 \text{ N/m}^2 (3 \text{ s.f.})

9 \[ m = \frac{t + 1}{\sqrt{t - 1}} \]

(Square both sides)

\[ m^2 = \frac{t + 1}{t - 1} \]

(multiply both sides by \( (t - 1) \))

\[ m^2(t - 1) = t + 1 \]

\[ m^2t - m^2 = t + 1 \]

(add \( m^2 \) and subtract \( t \) from both sides)

\[ m^2t - t = m^2 + 1 \]

(factorise LHS for \( t \))

\[ t(m^2 - 1) = m^2 + 1 \]

(divide both sides by \( (m^2 - 1) \))

\[ t = \frac{m^2 + 1}{m^2 - 1} = \frac{m^2 + 1}{(m + 1)(m - 1)} \]

10 (a) \[ p(A') = \frac{28}{44} = \frac{7}{11} \]
   (Set \( A' \) are all elements not in \( A \))

(b) \[ p(B \cap C') = \frac{10}{44} = \frac{5}{22} \]
   (Elements in \( B \) and also not in \( C \))

(c) \[ p(A \cup B \cup C') = \frac{4}{44} = \frac{1}{11} \]
   (Elements not in \( A \) or \( B \) or \( C \))

(d) \[ p((A \cap B \cap C)/(A \cup B)) = \frac{2}{32} = \frac{1}{16} \]
   (Sample space is reduced to \( A \cup B \))

11 (a)

Regular pentagon exterior angle = \( \frac{360^\circ}{5} = 72^\circ \)

(Exterior angle of a regular \( n \)-sided pentagon = \( \frac{360^\circ}{5} \))

Regular pentagon interior angle = \( 180^\circ - 72^\circ = 108^\circ \)

Angle at point \( P \):

Interior angle of \( A \) = \( 360^\circ - 2 \times 108^\circ = 144^\circ \)

Exterior angle of \( A \) = \( 180^\circ - 144^\circ = 36^\circ \)

36° = \( \frac{360°}{n} \), \( n = 12 \) so 12 pentagons will surround polygon \( A \).

(b) The 12-sided polygon is a regular dodecagon.

12 (a) \[ 100 = \frac{1}{2} (8.5 + 12.5) \times V_{\text{max}} \]
   (Area under speed–time graph = distance travelled)

\[ V_{\text{max}} = \frac{200}{21} = 9.5238... \text{m/s} = 9.52 \text{ m/s} \]

(3 s.f.)

(b) Acceleration = \( \frac{9.5238}{4} = 2.3809... \text{m/s}^2 \)
   = 2.38 \text{ m/s}^2 (3 \text{ s.f.})
   (Gradient of speed–time graph = acceleration)
PC × PD = PB × PA
(intersecting chords theorem)
Let \( AB = p \)
\[ 15 \times 8 = (10 + p)10 \]
\[ 120 = 100 + 10p \]
(Subtract 100 from both sides)
\[ 20 = 10p \]
(Divide both sides by 10)
\[ p = 2 \]
Now angle \( APD = 30^\circ \)
Triangle \( BPC \):
(Cosine Rule: \( a^2 = b^2 + c^2 - 2bc \cos A \))
\[ BC^2 = 15^2 + 12^2 - 2 \times 15 \times 12 \times \cos(30^\circ) \]
\[ BC = 7.5651... \text{ cm} = 7.57 \text{ cm (3 s.f.)} \]

14
\[ (11\sqrt{3} - a)(3\sqrt{3} + a) = 95 + 32b\sqrt{3} \]
(Expand out the LHS and compare irrational and rational parts)
LHS = \[ 99 + 11\sqrt{3}a - 3\sqrt{3}a - a^2 \]
\[ = 99 - a^2 + 8a\sqrt{3} \]
(Rational) \[ 99 - a^2 = 95, 4 = a^2, \ a = 2 \]
(Irrational) \[ 8 \times 2\sqrt{3} = 32b\sqrt{3} \]
\[ b = \frac{1}{2} \]
\[ a^2 + b^2 = 4 + \frac{1}{4} = \frac{17}{4} \] as required.

15 (a) \( p = 2r + \frac{80}{360} \times 2\pi(2r) + \frac{80}{360} \times 2\pi r \)
\[ = 2r + \frac{4\pi}{9} = (2r + r) = 2r + \frac{4\pi}{3} \]
\[ = \frac{2r(3 + 2\pi)}{3} \]

(b) \( p = 18 = \frac{2r(3 + 2\pi)}{3}, \ r = \frac{27}{3 + 2\pi} \)
\[ = 2.9084... \text{ m} \]
Let area of lawn be \( A = \frac{80}{360} \times \pi(4r^2 - r^2) \)
\[ = \frac{2}{3} \pi r^2 \]
At \( r = 2.9084... \text{ m}, A = \frac{2}{3}(2.9084)^2 \)
\[ = 17.16... \text{ m}^2 \]
Cost = \$18 \times 17.16 = $318.89
\[ = $319 (\text{nearest } $) \]

16 (a) \[ 6x - 5y = 7 \]
\[ [1] \times 3 = [3] \]
\[ 4x + 3y = 11 \]
\[ 18x - 15y = 21 \]
\[ [3] \]
\[ 20x + 15y = 55 \]
\[ [4] \]
\[ [3] + [4]: 38x = 76, \text{ so } x = 2 \text{ substitutes into } [2] \]
\[ [2]: 8 + 3y = 11, \text{ so } 3y = 3, y = 1 \]
Lines intersect at point (2, 1)

(b) Let \( x = p^{-2} = \frac{1}{p^2} \) and \( y = q^{-1} = \frac{1}{q} \)
So \[ 2 = \frac{1}{p^2}, p^2 = \frac{1}{2}, p = \pm \frac{1}{\sqrt{2}} \]
Also \[ 1 = \frac{1}{q}, q = 1 \]

17
\[ AB = 8 + BD \]
Triangle \( BCD \): \( \tan(59^\circ) = \frac{BD}{BC} \) \[ [1] \]
Triangle \( ABC \): \( \tan(69^\circ) = \frac{BD + 8}{BC} \) \[ [2] \]
Let \([1] = [2] = BC \):
\[ BC = \frac{BD}{\tan(59^\circ)} = \frac{BD + 8}{\tan(69^\circ)} \]
$BD \times \tan(69) = (BD + 8) \times \tan(59)$

$BD \times \tan(69) - BD \times \tan(59) = 8 \times \tan(59)$

(Expand and factorise for $BD$)

$BD \times (\tan(69) - \tan(59)) = 8 \times \tan(59)$

$BD = \frac{8 \times \tan(59)}{(\tan(69) - \tan(59))} = 14.1518... \text{ m}$

so $AB = 8 + 14.1518... = 22.151... \text{ m} = 22.2 \text{ m (3 s.f.)}$

18 (a) If $g(x) = \frac{x + 60}{2}$

$y = \frac{x + 60}{2}$

(Re-write in terms of $y = \ldots$)

$x = \frac{y + 60}{2}$

(Switch $x$ and $y$ variables)

$2x = y + 60, y = 2x - 60$

(Re-arrange to make $y$ the subject)

$g^{-1}(x) = 2x - 60$

(b) $fgh(x) = fg(2x) = f\left(\frac{2x + 60}{2}\right)$

$= \sin\left(\frac{2x + 60}{2}\right) = 1$

If the domain for $f(x)$ is $0 \leq x \leq 90^\circ$

$\sin(x) = 1$ gives one solution which is

$x = 90^\circ$

$\frac{2x + 60}{2} = 90^\circ, 2x + 60 = 180^\circ$

$2x = 120^\circ, x = 60^\circ$

19 Complete the table:

<table>
<thead>
<tr>
<th>Minutes late $t$ (min)</th>
<th>0 &lt; $t$ ≤ 1</th>
<th>1 &lt; $t$ ≤ 3</th>
<th>3 &lt; $t$ ≤ 5</th>
<th>5 &lt; $t$ ≤ 6</th>
<th>6 &lt; $t$ ≤ 7</th>
<th>7 &lt; $t$ ≤ 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of pupils</td>
<td>6</td>
<td>14</td>
<td>8</td>
<td>8</td>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>Frequency density</td>
<td>6</td>
<td>7</td>
<td>4</td>
<td>8</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

(b) (The second class has a frequency density of 7 so it is now possible to calibrate the vertical scale.)

(c) Modal class is the most popular group with the highest frequency density: $5 < t \leq 6$

(d) Area of the histogram = total frequency

Area representing $t \geq 2.5$

$= 0.5 \times 7 + 2 \times 4 + 1 \times 8 + 1 \times 2 + 3 \times 4$

$= 33.5$

$P(t \geq 2.5) = \frac{33.5}{50} = \frac{67}{100}$

20 Let $m$ be the original number of male llamas

Let $f$ be the original number of female llamas

$m : f = 3 : 10, \quad \frac{m}{f} = \frac{3}{10}, \quad m = \frac{3f}{10}, \quad f = \frac{10m}{3}$ (1)

After the birth, number of male llamas is $m + 3$, the number of female llamas is $f + 2$

$(m + 3) : (f + 2) = 1 : 3, \quad \frac{m + 3}{f + 2} = \frac{1}{3}, \quad 3(m + 3) = f + 2$ (2)

Substituting (1) into (2)

$3(m + 3) = \frac{10m}{3} + 2$

$9(m + 3) = 10m + 6$

$9m + 27 = 10m + 6$

$m = 21, \quad f = \frac{10 \times 21}{3} = 70$ there were 91 llamas before the birth.

There are now 91 + 5 = 96 llamas in the herd.
21 (a) | x | 0.5 1 2 3 4 5 6 | y | 2.25 1.25 1.25 1.5 1.85 2.25 |

(b) 

(c) $2x + \frac{12}{x} = 13$

\[
2x + \frac{12}{x} - 5 = 8 \\
\frac{1}{4} \left(2x + \frac{12}{x} - 5\right) = 2 	ext{ so draw line } y = 2
\]

Solutions are where line intersects the curve at $\approx 1.1$ or $x \approx 5.4$

22 (AM is a line of symmetry, so $\Delta ABM$ is a right-angled triangle. M is midpoint of BC so $BM = 1$)

\[\text{AM} \text{ is a line of symmetry, so } \Delta \text{ABM} \text{ is a right-angled triangle. M is midpoint of } BC \text{ so } BM = 1\]

\[\text{AM} = \sqrt{3}\]

\[AX = \frac{2}{3} \text{ AM}\]

\[AX = \frac{2\sqrt{3}}{3} \text{ mm}\]

23 (a) Area of base (triangle ABC) is

\[\frac{1}{2} \times 2 \times \sqrt{3} = \sqrt{3}\]

\[h^2 + AX^2 = 2^2 \quad AD = 2 \text{ mm}\]

\[h^2 = 4 - \left(\frac{2x}{3}\right)^2 = 4 - \frac{4 \times 3}{9} = \frac{8}{3}\]

\[h = \frac{8}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \text{ or } \frac{2\sqrt{3}}{3} \text{ or } \frac{2\sqrt{3}}{3}\]

Volume $= \frac{1}{3} \times \sqrt{3} \times \frac{2\sqrt{3}}{3} = \frac{2\sqrt{3}}{3} \text{ mm}^3$

(b) Angle required is $\theta$ (see sketch)

\[\tan(\theta) = \frac{h}{AM} = \frac{2\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}\]

\[= \frac{2\sqrt{3}}{\sqrt{3}} \times \frac{3}{\sqrt{3}} = 2\sqrt{3}\theta = 70.5^\circ\]

\[MX = \frac{1}{2} \times AM\]

or $\sin(\theta) = \frac{h}{DM} = \frac{2\sqrt{3}}{\sqrt{3}} + \frac{\sqrt{3}}{3}\]

\[= 2\sqrt{3} \times \frac{3}{2\sqrt{3}} = \frac{2\sqrt{3}}{3}\theta\]

\[= 70.5^\circ\]

\[DM = AM\]

or $\cos(\theta) = \frac{MX}{DM} = \frac{\sqrt{3}}{\sqrt{3}} + \frac{\sqrt{3}}{3}\]

\[= \frac{\sqrt{3}}{3} \times \frac{1}{\sqrt{3}} = \frac{1}{3}\]

\[\theta = 70.5^\circ\]

\[MX = \frac{1}{2} \times AM, DM = AM\]

23 (a) \[
\frac{4x^2 - 9}{x + 2} \div \frac{2x^2 - 5x - 12}{x - 4}
\]

\[= \frac{4x^2 - 9}{x + 2} \times \frac{x - 4}{2x^2 - 5x - 12}
\]

\[= \frac{(2x - 3)(2x + 3)}{x + 2} \times \frac{x - 4}{(2x - 3)(x + 4)}
\]

\[= \frac{2x - 3}{x + 2}\]

(b) \[
\frac{4x^2 - 9}{x + 2} \div \frac{2x^2 - 5x - 12}{x - 4}
\]

\[= \frac{2x - 3}{x + 2} \div \frac{2x - 3}{x + 2} = \frac{x + 1}{2(x - 2)}
\]

(Using previous result)

\[\frac{2x - 3}{x + 2} = \frac{x + 1}{2(x - 2)}\]
2(2x - 3)(x - 2) = (x + 1)(x + 2)
(Multiplying both sides by 2(x + 2)(x - 2))
4x^2 - 14x + 12 = x^2 + 3x + 2
3x^2 - 17x + 10 = 0
(x - 5)(3x - 2) = 0
x = 5 or x = \frac{2}{3}

24 (a) Stone hits the sea when
s = -24 = 20t - 4t^2
(Stone is 24m below the cliff top)
so 4t^2 - 20t - 24 = 0
(Divide both sides by 4)
t^2 - 5t - 6 = 0, (t - 6)(t + 1) = 0, t = 6
(Note t ≠ -1)

(b) \frac{du}{dt} = 20 - 8t, at t = 6,
v = 20 - 8 \times 6 = -28 \text{ m/s}

(c) Mean speed = \frac{\text{distance}}{\text{time}}
distance = \text{distance to top} \times 2 + 24
speed at top is when v = 0 = 20 - 8t,
t = \frac{5}{2} \text{ s}
at t = \frac{5}{2}, s = 20 \times \frac{5}{2} - 4 \times \left(\frac{5}{2}\right)^2 = 25 \text{ m}
Mean speed = \frac{2 \times 25 + 24}{6}
= 12.3 \text{ m/s (3 s.f.)}