ANSWERS

NUMBER 1 – BASIC SKILLS EXERCISE

1 **(a)** 27
(b) 56
(c) 26
(d) 12

2 $4\frac{1}{3} = \frac{13}{3}, \frac{52}{12} = \frac{4 \times 13}{4 \times 3} = \frac{13}{3},$

$\frac{6.5}{1.5} = \frac{65}{15} = \frac{5 \times 13}{5 \times 3} = \frac{13}{3}$

3 **(a)** $\frac{1}{3}$

(b) $\frac{10}{21}$

(c) $3\frac{5}{12}$

(d) $3\frac{5}{7}$

(e) $\frac{1}{35}$

(f) $\frac{5}{9}$

4 $3\frac{1}{2} \div 2\frac{1}{3} = \frac{7}{2} \div \frac{7}{3}$

$= \frac{7}{2} \times \frac{3}{7}$

$= \frac{3}{2}$

$= 1\frac{1}{2}$

5 $4\frac{2}{3} - 2\frac{1}{2} + 1\frac{3}{4} = \frac{14}{3} - \frac{5}{2} + \frac{7}{4}$

$= \frac{56}{12} - \frac{30}{12} + \frac{21}{12}$

$= \frac{47}{12}$

$= 3\frac{11}{12}$

6 $\frac{0.12}{32} \div \frac{0.024}{7.2} = \frac{12}{3200} \div \frac{24}{7200}$

$= \frac{12}{3200} \times \frac{7200}{24}$

$= \frac{9}{8}$

$= 1\frac{1}{8}$

7 $\frac{1}{4} - \left(\frac{1}{4} \times \frac{1}{4}\right) + \left(\frac{1}{4} \div \frac{1}{4}\right) = \frac{1}{4} - \left(\frac{1}{4} \times \frac{1}{4}\right)$

$+ \left(\frac{1}{4} \div \frac{1}{4}\right)$

$= \frac{1}{4} - \left(\frac{1}{4} \times \frac{1}{4}\right)$

$+ \left(\frac{1}{4} \times \frac{4}{1}\right)$

$= \frac{1}{4} - \frac{1}{16} + 1$

$= \frac{4}{16} - \frac{1}{16} + \frac{16}{16}$

$= \frac{19}{16}$

$= 1\frac{3}{16}$

8 $\frac{4}{2 + \frac{2}{3+4}} = \frac{4}{2 + \frac{2}{3+4}}$

$= \frac{4}{2 + \frac{2}{7}}$

$= \frac{4}{\frac{14}{7} + \frac{2}{7}}$

$= \frac{4}{\frac{16}{7}}$

$= \frac{28}{16}$

$= 1\frac{3}{4}$

9 **(a)** 8
(b) –16
(c) – 48
(d) $-\frac{1}{3}$
(e) 48

10 **(a)** 3
(b) 16
(c) 8
(d) 38
(e) 4
(f) 2

11 **(a)** 12 300
(b) 12 400
(c) 12 300
(d) 439 000
(e) 550 000
(f) 0.0130
(g) 1.01
(h) 0.01000

12 **(a)** 1.294
(b) 1.295
(c) 1.295
(d) 1.200
(e) 0.100
(f) 340.005
(g) 1.000
(h) 0.000499

NUMBER 1 – EXAM PRACTICE EXERCISE

1 **(a)** $4\frac{2}{3} \div 3\frac{5}{9} - 1\frac{3}{8} = \frac{14}{3} \div \frac{32}{9} - \frac{11}{8}$

$= \frac{14}{3} \times \frac{9}{32} - \frac{11}{8} = \frac{21}{16} - \frac{22}{16} = -\frac{1}{16}$

(b) $\frac{1}{4} = \frac{7}{28}$; $\frac{2}{7} = \frac{8}{28}$; $\frac{3}{14} = \frac{6}{28}$; so Karim eats the most.

(c) $\frac{1}{4} + \frac{2}{7} + \frac{3}{14} = \frac{7}{28} + \frac{8}{28} + \frac{6}{28}$

$= \frac{7+8+6}{28} = \frac{21}{28} = \frac{3}{4}$ so $\frac{3}{4}$ is eaten,

leaving $1 - \frac{3}{4} = \frac{1}{4}$ uneaten.

2 **(a)** **(i)** 0.001⁝8548 = 0.002 to 3 d.p.
the 8 rounds the 1 up to 2
(ii) 0.00185⁝48 = 0.00185 to 3 s.f.
the 4 does not round anything up
(iii) 0.00 ⁝18548 = 0.00 to 2 d.p.
the 1 does not round anything up
(iv) 0.0018⁝548 = 0.0019 to 2 s.f.
the 5 rounds the 8 up to 9

(b) **(i)** One of the following:
Lowest common multiple of 2 and 10 is 10, not 2 − 10
The numerator (top) of the first fraction has not been multiplied by anything to keep the value of the fraction correct.

(ii) $\frac{9}{2} - \frac{25}{10} = \frac{45}{10} - \frac{25}{10} = \frac{45-25}{10} = \frac{20}{10} = 2$

3 **(a)** $(5^2 \div 4 - 6 \times 3^2 \div 2^3) = \frac{25}{4} - \frac{6 \times 9}{8}$

$= \frac{25}{4} - \frac{27}{4} = \frac{-2}{4} = -\frac{1}{2}$

so $1 \div 2 \times (5^2 \div 4 - 6 \times 3^2 \div 2^3)$

$= 1 \div 2 \times \frac{-1}{2} = \frac{1}{2} \times \frac{-1}{2} = -\frac{1}{4}$

(b) $u = \frac{8}{3} \Rightarrow \frac{1}{u} = \frac{3}{8}$; $v = \frac{6}{5} \Rightarrow \frac{1}{v} = \frac{5}{6}$

so $\frac{1}{u} + \frac{1}{v} = \frac{3}{8} + \frac{5}{6} = \frac{3 \times 3 + 5 \times 4}{24} = \frac{29}{24}$

$\Rightarrow f = \frac{24}{29}$

4 **(a)** $187\frac{1}{2} = \frac{375}{2}$; $3\frac{1}{8} = \frac{25}{2}$

number of presses $= \frac{375}{2} \div \frac{25}{8}$

$= \frac{375}{2} \times \frac{8}{25} = \frac{15}{1} \times \frac{4}{1} = 60$

(b) Time from Granada to Antequera is 72 minutes
Time from Granada to Sevilla is 168 minutes
So fraction is $\frac{72}{168} = \frac{6}{14} = \frac{3}{7}$

5 (a) The fraction of the lower school that play football is

$\frac{5}{11} \times \frac{3}{10} = \frac{3}{22}$

$\frac{6}{11}$ of the school are in the upper school so the fraction of the upper school that play football is $\frac{6}{11} \times \frac{7}{12} = \frac{7}{22}$

$\Rightarrow$ fraction of school that play football is $\frac{3}{22} + \frac{7}{22} = \frac{10}{22} = \frac{5}{11}$

(b) The smallest number of students must be the smallest whole number divisible by both 45 and 12, i.e. the Lowest Common Multiple of 45 and 12.
$45 = 3^2 \times 5$, $12 = 2^2 \times 3$ therefore the LCM is $2^2 \times 3^2 \times 5 = 180$ students

ALGEBRA 1 – BASIC SKILLS EXERCISE

1 $2xy + 2xz$
2 $2xy$
3 $10a + 5$
4 $7b$
5 $9ab$
6 $7a^5$
7 $5a^6$
8 $72a^5$
9 $12a - 6b$
10 $12a + 16b$
11 $-a - b$
12 $6b - 2a$
13 5
14 −2
15 −3
16 −9
17 49
18 1
19 −2
20 −1
21 $\frac{3}{2}$
22 1.8
23 −1
24 2
25 3
26 −1
27 2
28 −1
29 3
30 2

ALGEBRA 1 – EXAM PRACTICE EXERCISE

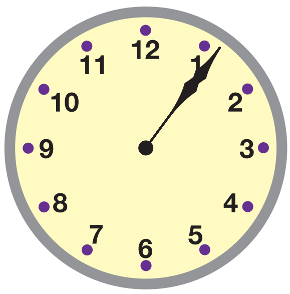
1 The numbers are x, $x + 2$ and $x + 4$,
so $x + (x + 2) + (x + 4) = 648$
$3x + 6 = 648$, $3x = 642$
$x = 214$ so the numbers are 214, 216 and 218

2 Interior angle at B is
$180 - (145 - 6x) = 35 + 6x$
Angle $C = 35 + 6x$ as it is an isosceles triangle
Angle sum of a triangle is 180°, so
$35 + 6x + 35 + 6x + 70 - 4x = 180$
$8x + 140 = 180$, $8x = 40$
$x = 5$
so angles are 65°, 65° and 50°

3 The width of the screen is $x - 0.5$
The length of the phone is $2x$ so the length of the screen is $2x - 3$
The perimeter of the screen is 32
so $2(x - 0.5) + 2(2x - 3) = 32$
$2x - 1 + 4x - 6 = 32$
$6x = 39$
$x = 6.5$
So the screen measures 6 cm by 10 cm and the area is $6 \times 10 = 60$ cm²

4 Let x be the diameter of the circle.
The side of the square is x so the perimeter of the square is $4x$
The circumference of the circle is πx
So $4x + \pi x = 30$, $x(4 + \pi) = 30$
$x = \frac{30}{4 + \pi}$
$x = 4.20$ (3 s.f.)
So lengths are $4 \times 4.20 = 16.8$ cm and $\pi \times 4.20 = 13.2$ cm (3 s.f.)

5 (a) Let x be the number of minutes after 12:00
In 60 minutes the minute hand moves 360° or 6° per minute



In x minutes the minute hand moves $6x$° from the vertical.
The hour hand moves at $\frac{1}{12}$ of the speed of the minute hand
In x minutes the hour hand moves $\frac{6x}{12} = \frac{x}{2}$ degrees from the vertical.
Angle between hands must equal
$90° \Rightarrow 6x - \frac{x}{2} = 90 \Rightarrow \frac{11x}{2} = 90 \Rightarrow x = 16.36$

$x = 16$ minutes 22 secs so time is 12:16:22 to the nearest second.

(b) Let x be the number of minutes after 12:00

In x minutes the minute hand moves $6x°$ from the vertical.

The time will be after 01:00. At 01:00 the minute hand has moved 360°, so the angle of the minute hand will be $6x - 360$ degrees from the vertical.

The hour hand will have moved $\frac{x}{2}$ degrees from the vertical.

$\Rightarrow 6x - 360 = \frac{x}{2} \Rightarrow \frac{11x}{2} = 360 \Rightarrow x = 65.45..$

$x = 1$ hour 5 minutes and 27 seconds so time is 01:05:27 to the nearest second.

OR

The time will be after 01:00

Let y be the number of minutes after 01:00.

In y minutes the minute hand moves $6y°$ from the vertical.

At 01:00 the hour hand has moved 30° from the vertical so y minutes later it has moved $30 + \frac{y}{2}$ degrees

$\Rightarrow 6y = 30 + \frac{y}{2} \Rightarrow \frac{11y}{2} = 30 \Rightarrow y = 5.45..$ or 5 mins 27 secs to nearest second

So time is 01:05:27 to the nearest second.

GRAPHS 1 – BASIC SKILLS EXERCISE

1 **(a)** 2 **(b)** $-\frac{1}{2}$

2 45 m

3 $\frac{2}{3}$ m

4 28.6

5 $p = -4$

6 No. Gradient of AB is 2, gradient of BC ≠ 2, it is 2.02 to 3 s.f.

7 −63

8 3

9 A and D

10 The second point in the table should be (0, 3).

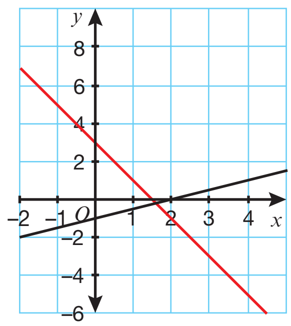
11

x	−3	$a = -2$	0	1	3	$c = 4$
y	11	8	2	−1	$b = -7$	−10

12 **(a)**

x	−2	0	2	4
$y = 3 - 2x$	7	3	−1	−5
$2y - x + 2 = 0$	−2	−1	0	1

(b)



(c) $y = 3 - 2x$: gradient $= -2$, intercept = (0, 3)

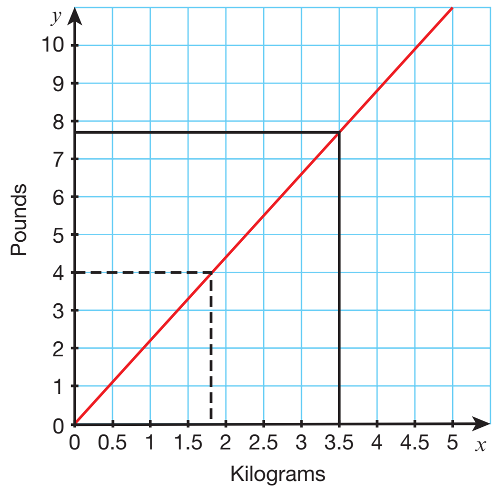
$2y - x + 2 = 0$: gradient $= \frac{1}{2}$, intercept = (0, −1)

(d) (1.6, −0.2)

13 **(a)** 5 kg = 11 lbs

Draw straight line from (0, 0) to (5, 11)

(b) **(i)** 7.7 lbs **(ii)** 1.82 kg



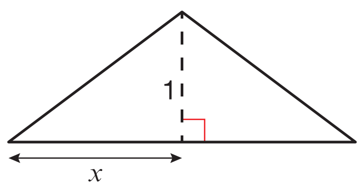
14 **(a)** approximately \$61

(b) approximately 7.1 km

(c) approximately 1.7 km

GRAPHS 1 – EXAM PRACTICE EXERCISE

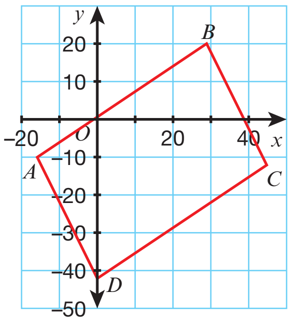
1 **(a)** $\frac{1}{x} = \frac{1}{3} \Rightarrow x = 3$ so width is 6 m



(b) Gradient $= \frac{(5p - 9) - (p - 9)}{(p + 7) - (p - 1)} = \frac{5p - p}{7 + 1}$

$= \frac{4p}{8} = \frac{p}{2} \Rightarrow p = 1$

2 Sketching the position of the points shows that if it is a parallelogram $AB \parallel DC$ and $AD \parallel BC$



(a) Gradient of $AB = \frac{20 - -10}{29 - -16} = \frac{30}{45} = \frac{2}{3}$

gradient of $DC = \frac{-12 - -42}{45 - 0} = \frac{30}{45} = \frac{2}{3}$

$\Rightarrow$ AB is parallel to DC as the gradients are the same.

gradient of $AD = \frac{-42 - -10}{0 - -16} = \frac{-32}{16} = -2$

gradient of $BC = \frac{-12 - 20}{45 - 29} = \frac{-32}{16} = -2$

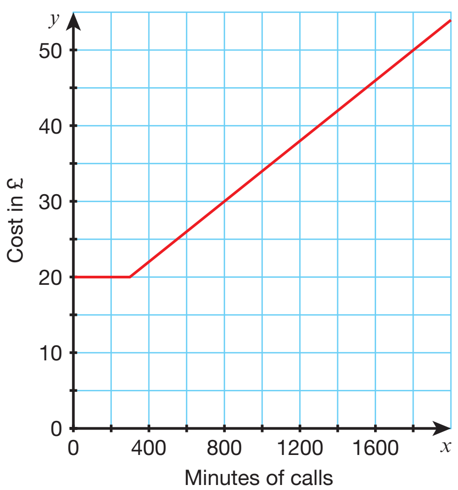
$\Rightarrow$ AD is parallel to BC as the gradients are the same.
$\Rightarrow$ $ABCD$ is a parallelogram as it has two pairs of opposite parallel sides.

(b) Gradient of $AO = \frac{0 - -10}{0 - -16} = \frac{10}{16} = \frac{5}{8}$

As the gradient of $AO \neq$ gradient of AB, O does not lie on AB.

3 **(a)**

***t* (min)**	0	300	1000	1800
***C* (£)**	20	20	34	50



(b) Gradient of graph after 300 minutes is

$\frac{50 - 20}{1800 - 300} = \frac{30}{1500} = 0.02$

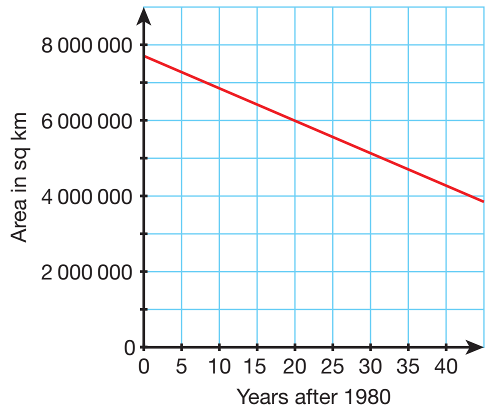
This is 0.02 £/min or 2p per minute

(c) 16 h 40 mins = 1000 mins. From graph bill should be £34, so she should complain.

4 **(a)** The area is decreasing by 86 000 km² per year, so the formula must be of the form $A = -86\,000y + c$
When $y = 0$, $A = 7.7 \times 10^6$, so the formula is $A = 7.7 \times 10^6 - 86\,000y$

(b)

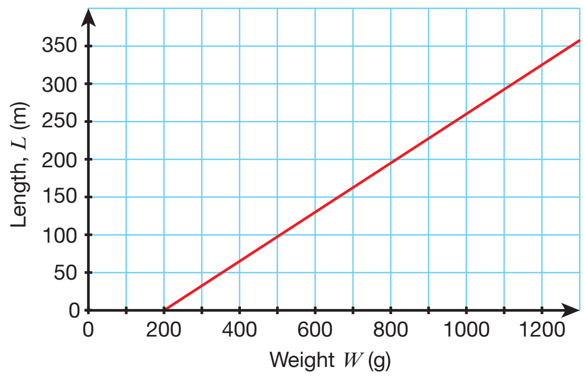
***x* (year after 1980)**	0	20	40
***y* (area in km²)**	7.7×10^6	5.98×10^6	4.26×10^6



(c) **(i)** 600 000 km²
(ii) 2011

5 **(a)**

Weight, *W* (g)	200	1200
Length, *L* (m)	0	325



(b) Reading from graph gives
(i) 231 m
(ii) 503 g
(Note as you are reading from a graph, your answers might not be the same, but they should be within ±5 units.)

(c) Gradient is $\frac{330}{1000} = 0.33$ so equation is of the form $L = 0.33W + c$

Substituting in one of the known points, say (200, 0), gives $c = -66$
$\Rightarrow$ equation is $L = 0.33W - 66$

SHAPE AND SPACE 1 – BASIC SKILLS EXERCISE

1 $x = 36°$, $y = 106°$, $z = 38°$
2 $x = 60°$, $y = 30°$
3 $x = 33°$, $y = 33°$, $z = 83°$
4 $x = 75°$

5 **(a)** Exterior angles sum to 360
$22x - 80 = 360$, $x = 20$
(b) Angle $A = 70$, angle $B = 70$,
angle $C = 40$ so isosceles with $AC = BC$

6 **(a)** Interior angles sum to 360°

$$x + \frac{7}{8}x + \frac{29}{24}x + \frac{2}{3}x = 360$$

$$\frac{15}{4}x = 360, x = 96$$

(b) Substituting $x = 96$ gives interior angle
$A = 64°$, $B = 96°$, $C = 84°$ and $D = 116°$
$A + D = 180°$ so AB is parallel to DC
(corresponding angles)
or
$B + C = 180°$ so AB is parallel to DC
(corresponding angles)

7 Acute angle between 12 and 8 is 120°

$\frac{1}{3} \times 360°$

Angle between minute hand and 12 is

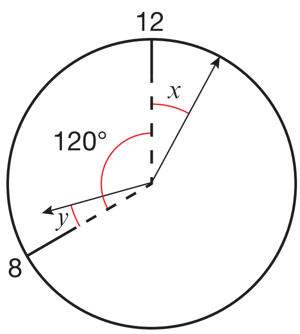
$$x = \frac{6}{60} \times 360 = 36°$$

Whole turn is 60 mins so 6 mins is $\frac{6}{60}$ of 360°

Hour hand travels at $\frac{1}{12}$ speed of minute hand.

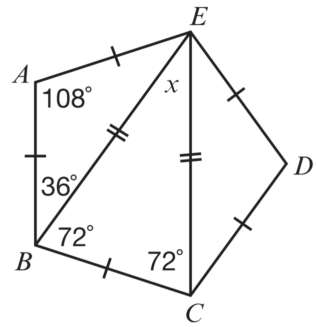
$$\Rightarrow y = \frac{1}{12} \times 36° = 3°$$

$\Rightarrow$ acute angle between hands is
$120 + x - y = 153°$



8 $x = 100°$, $y = 75°$, $z = 135°$

9 Mark all the equal sides.



(a) Exterior angle of a regular pentagon is
$360 \div 5 = 72$
angle $A = 180 - 72 = 108°$
ΔABE is isosceles,
so $A\hat{B}E = (180 - 108) \div 2 = 36°$
Angle $B = 108$ so $E\hat{B}C = 108 - 36 = 72°$
ΔBCE is isosceles so $B\hat{C}E = 72°$
By angle sum of ΔBCE, $x = 36°$
(b) $A\hat{B}E = B\hat{E}C$ so AB is parallel to CE
(alternate angles)
or
$A\hat{B}C + B\hat{C}E = 180$ so AB is parallel to CE (corresponding angles)
or
$B\hat{A}E + A\hat{E}C = 180$ so AB is parallel to CE (corresponding angles)

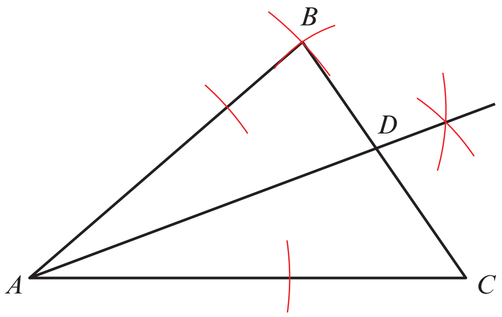
10 Exterior angle is $360 \div 20 = 18$ so interior angle is $180 - 18 = 162°$

11 Sum of interior angles is $180(n - 2) = 3060$ so $n = 19$

12 **(a)** Triangle shown is isosceles so interior angle is 156°, exterior angle is 24° and number of sides is $360 \div 24 = 15$
(b) $15 \times 156 = 2340°$

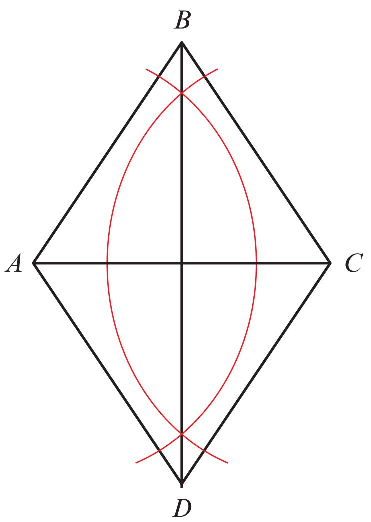
13 $a = 4.5$
14 $a = 6$, $b = 4.5$
15 $a = 4.5$, $b = 2.5$

16 (a) and (b)



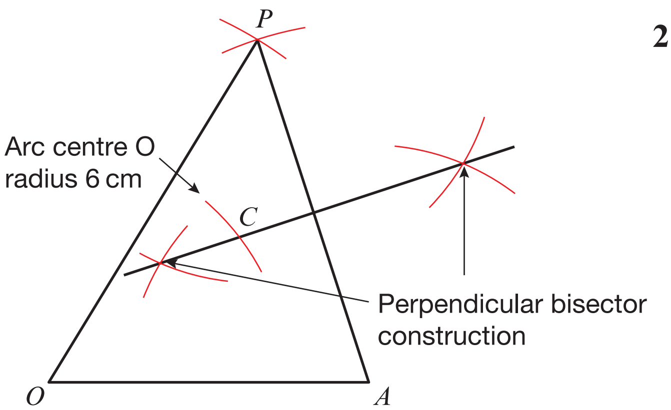
(c) 4.36 cm

17 (a) and (b)



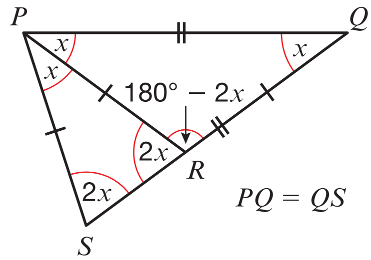
(c) 7.21 cm

18 (a) and (b) CP = 4.9 so CP = 9.8 m



SHAPE AND SPACE 1 – EXAM PRACTICE EXERCISE

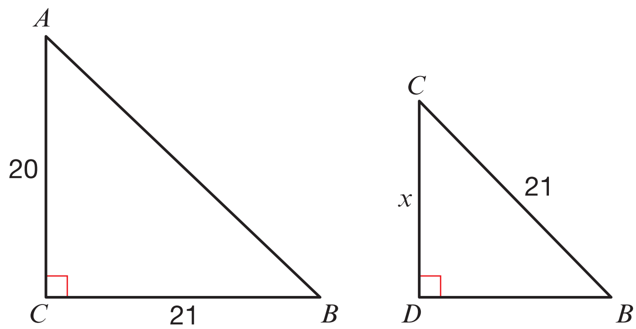
1 (a) Using isosceles triangle properties and angle sum on a straight line gives the angles shown in the diagram.



$5x = 180$ (angle sum of PRS)

$x = 36°$

(b) Draw out the similar triangles as shown.



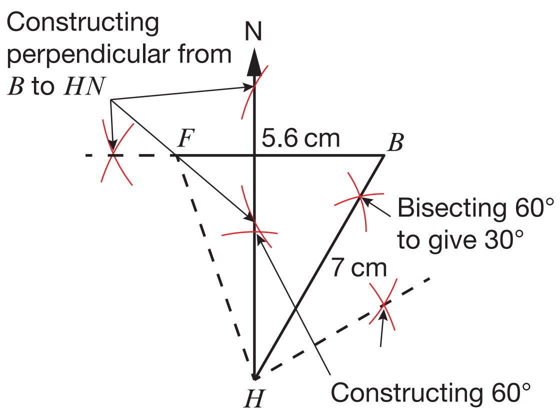
$AB^2 = 20^2 + 21^2 \Rightarrow AB = 29$ Pythagoras

$\frac{x}{21} = \frac{20}{29}$

$x = \frac{420}{29}$

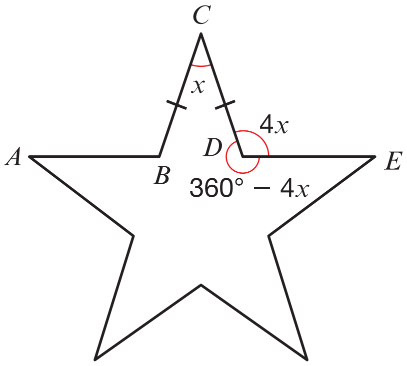
2 (a) 35 km ≡ 7 cm, 28 km ≡ 5.6 cm.

Draw HN, then construct a 60° angle at H. Bisect this to give 30° and measure 7 cm to find B. From B construct the perpendicular to HN and measure 5.6 cm to find F.

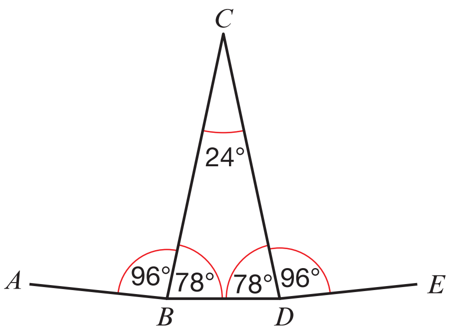


(b) *FH* measures 6.4 cm and angle *FHN* measures 14°
So bearing is 180 – 14 = 166°
FH represents 6.4 × 5 = 32 km
So It took 32 ÷ 4 = 8 h

3 **(a)** The interior angle at *D* is $360 - 4x$
Due to the rotational symmetry, all the interior angles are the same.
The sum of the interior angles of a 10-sided polygon is $(10 - 2) \times 180 = 1440°$
$5x + 5(360 - 4x) = 1440$
$5x + 1800 - 20x = 1440$
$15x = 360$
$x = 24°$

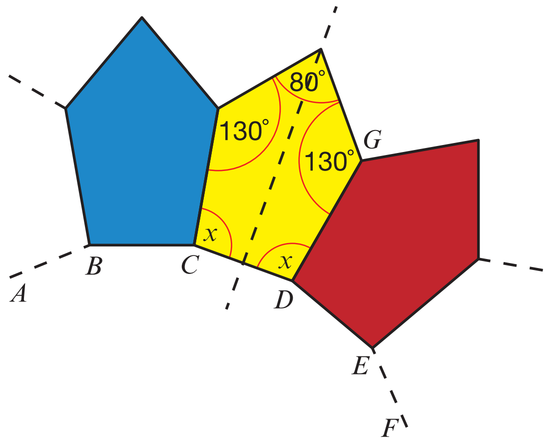


(b) Calculate the angles as shown.
Triangle *BCD* is isosceles.



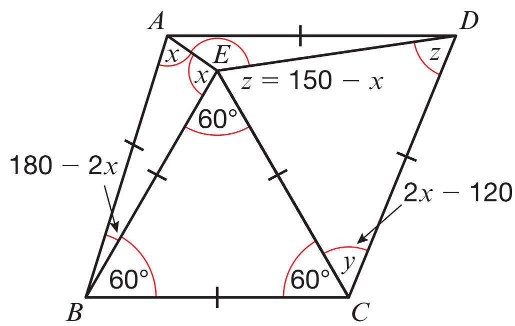
Angle sum at *B* is 174° so *ABD* is not a straight line.
Angle sum on a straight line is 180°.
OR Extend *AB* and *DE* to intersect at *F*.
Show the angle at *F* is not 180°.

4 The interior angles of a pentagon sum to $3 \times 180 = 540$
By symmetry the angle in the yellow pentagon at *G* is 130



Let the angle in the yellow pentagon at *C* be x
The angle in the yellow pentagon at *D* is x by symmetry
$2x + 130 + 130 + 80 = 540$
$x = 100°$
The angle in the blue pentagon at *C* is also 100°
The interior angle of the polygon is
$360 - 100 - 100 = 160°$
exterior angle of polygon = 20°
number of sides = 360 ÷ 20 = 18°

5 Add marks showing equal sides
Mark in 60° angles in equilateral triangle
ABE is isosceles, so angle at *A* is x and angle at *B* is $180 - 2x$ (angle sum of triangle)
Angles at *B* and *C* sum to 180° (*AB* parallel to *CD*, complementary angles)
$180 - 2x + 60 + 60 + y = 180$
$y = 2x - 120$
Triangle *CDE* is isosceles
$2z + 2x - 120 = 180$
$z = 150 - x$
Angles at *E* sum to 360 so
$AED + x + 60 + 150 - x = 360$
$AED = 150°$



SETS 1 – BASIC SKILLS EXERCISE

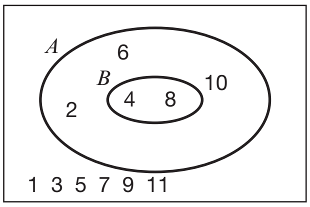
1 **(a)** Any multiples of 3
(b) Any negative integers
(c) Any sport
(d) Any make of car

2 **(a)** {multiples of 3}
(b) {negative integers}
(c) {sports}
(d) {makes of car}

3 **(a)** {2, 4, 6, 8}
(b) {4, 9, 16}
(c) {January, June, July}
(d) {Red, Amber (or Orange), Green}

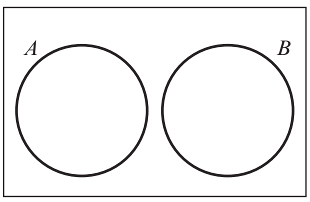
4 **(a)** True
(b) False
(c) False
(d) True

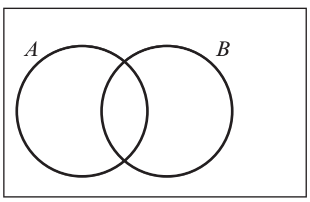
5 a, b and d

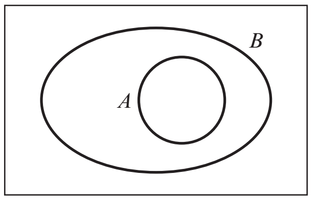
6 **(a)** ℰ


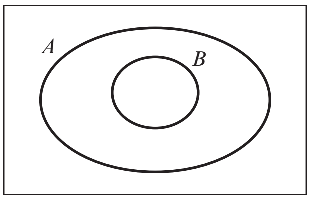
(b) {1, 3, 5, 7, 9, 11},
odd numbers between 1 and 11 inclusive
OR odd numbers between 0 and 12
(c) 8
(d) Yes. All multiples of 4 are also multiples of 2

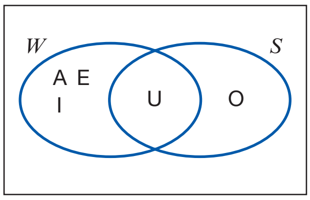
7 **(a)** Because 10 is not a member of ξ
(b) {5, 15}
(c) 2 Factors are 1 and 5
(d) {5}

8 **(a)** ℰ


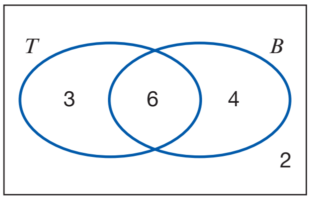
(b) ℰ


(c) ℰ


(d) ℰ


9 **(a)** $W = \{\text{A, E, I, U}\}$, $W' = \{\text{O}\}$,
$S = \{\text{O, U}\}$, $S' = \{\text{A, E, I}\}$
(b) ℰ


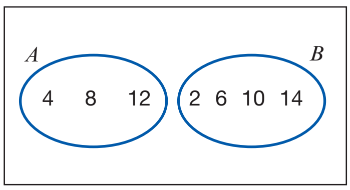
(c) **(i)** {A, E, I, O, U} or ℰ
(ii) {U}

10 **(a)** ℰ


(b) 15

SETS 1 – EXAM PRACTICE EXERCISE

1 **(a)** **(i)** False
(ii) True
(iii) False
(iv) False
(b) **(i)** $A \cap C = \varnothing$
(ii) $C \cup D = C$
(iii) $A \cap B \neq \varnothing$

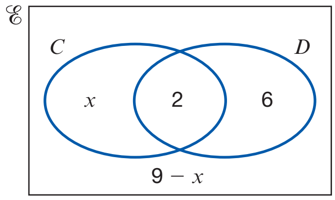
2 **(a)** ℰ


$A \cup B = \mathscr{E}$

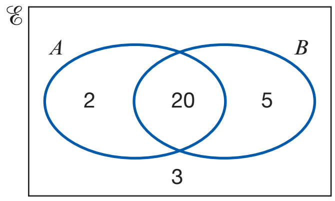
(b) **(i)** 8

(ii) 2

(iii) In the Venn diagram, x can be any number between 0 and 9

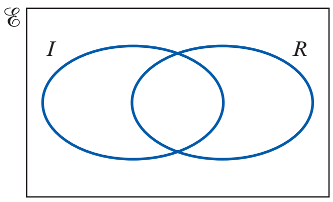


3 **(a)**



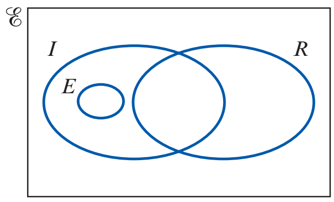
(b) 20

4 **(a)**



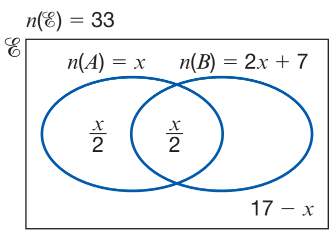
(b) $I \cap R$ is the set of isosceles right-angled triangles, so angles are 90°, 45° and 45°

(c)



All equilateral triangles are isosceles, so E is a subset of I

5 **(a)**



As $n(A) = x$ and $n(A \cap B) = \frac{x}{2}$ then $n(A \cap B') = \frac{x}{2}$

$\frac{x}{2} + 2x + 7 + 17 - x = 33$

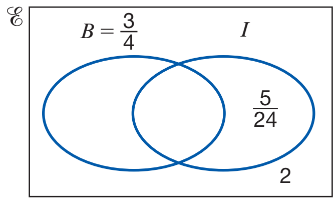
$\frac{3x}{2} = 9$

$x = 6$

$n(B) = 19$, $n(A \cap B) = 3$

$n(A' \cap B) = 19 - 3 = 16$

(b) **(i)**



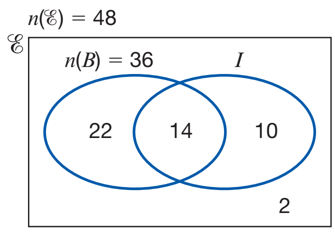
Fraction using B and I is

$\frac{3}{4} + \frac{5}{24} = \frac{23}{24}$

2 students are $\frac{1}{24}$ of the group

number of students is $24 \times 2 = 48$

(ii) Turn the fractions into numbers and fill in the Venn diagram



From the Venn diagram $n(B \cap I) = 14$

NUMBER 2 – BASIC SKILLS EXERCISE

1 **(a)** 1.45×10^5
(b) 1.23×10^8
(c) 1×10^6
(d) 1×10^9

2 **(a)** 1.38×10^5
(b) 9.74×10^8
(c) 3.13×10^3
(d) 3.16×10^4

3 **(a)** 350
(b) 5750
(c) 1 250 000
(d) 93 210

4 6×10^{199}
5 4×10^1
6 2.8×10^{100}
7 3.2×10^{100}
8 2×10^{50}
9 4×10^{200}
10 2.89×10^{50}

11 **(a)** 1×10^{-1}
(b) 5×10^{-3}
(c) 2.5×10^{-1}
(d) 7×10^{-6}

12 (a) 1.23×10^{-2}
(b) 1.24×10^{-2}
(c) 1.60×10^{-4}
(d) 8.89×10^{-3}

13 (a) 0.035
(b) 0.005 75
(c) 0.000 001 25
(d) 0.000 932 1

14 6×10^{-199}

15 4×10^{-1}

16 -1.7×10^{-99}

17 2.3×10^{-99}

18 2×10^{-50}

19 4×10^{-200}

20 1.85×10^{-6}

21 2.66×10^{-22}

22 2.41×10^{-6}

23 1.62×10^{-3}

24 1.38×10^{-2}

25 (a) 4.08×10^{-7}
(b) 1.76×10^{-9}
(c) 3.87×10^{-11}
(d) 4.83×10^{-4}

26 2.90×10^{-5} km

27 (a) 3.02×10^{30} mm^2
(b) 1.69×10^{-8} %

28 6.32×10^{-13} km/s

29 9.46×10^{12} km/year

30 (a) $\frac{1}{4}$
(b) $\frac{1}{10}$
(c) $\frac{3}{4}$
(d) $\frac{3}{5}$
(e) $\frac{7}{20}$

31 150 m

32 \$360

33 42 g

34 8%

35 0.01 %

36 loss of 37.5% or −37.5% increase

37 12%

38 1650 m

39 528 kg

40 \$912

41 21 250 cm^2

42 \$90

43 €897

44 £56.25

45 (a) 56.7 s
(b) 19% improvement

46 (a) 1.39 m
(b) 14%

47 (a) €11 880
(b) €120 000 $\times \left(1 + \frac{x}{100}\right)\left(1 - \frac{x}{100}\right)$

48 $x \times \left(1 + \frac{y}{100}\right)$

49 $x \times \left(1 - \frac{y}{100}\right)$

50 +44%

NUMBER 2 – EXAM PRACTICE EXERCISE

1 (a) (i) Let V be volume of all five planets:
$V = 1.43 \times 10^{24} + 8.27 \times 10^{23} + 1.08 \times 10^{21} + 1.63 \times 10^{20} + 7.15 \times 10^{18}$
$= 2.2582\ldots \times 10^{24}$ m^3
$= 2.26 \times 10^{24}$ m^3 (3 s.f.)
(ii) Volume of Jupiter − Volume of Pluto $= 1.43 \times 10^{24} - 7.15 \times 10^{18}$
$= 1.4299\ldots \times 10^{24}$ m^3
$= 1.43 \times 10^{24}$ m^3
(b) Volume Mars × k = Volume Earth
$1.63 \times 10^{20} \times k = 1.08 \times 10^{21}$
$k = \frac{1.08 \times 10^{21}}{1.63 \times 10^{20}} = 7$
(nearest integer)

2 (a) (i) DNA molecule width − Water molecule width $= 2.15 \times 10^{-9} - 2.70 \times 10^{-10}$
$= 0.000\,000\,001\,88$ m
$= 1.88 \times 10^{-9}$ m
(ii) Grain of sand width − Human hair width $= 5.25 \times 10^{-4} - 7.50 \times 10^{-5}$
$= 0.000\,45$
$= 4.50 \times 10^{-4}$ m
(b) Width Covid-19 virus : Human hair
$= 1.60 \times 10^{-7} : 7.50 \times 10^{-5} = 1 : \frac{7.50 \times 10^{-5}}{1.60 \times 10^{-7}}$
$= 468.75$
$= 1 : n$ where $n = 469$ to the nearest integer

3 1 January 2022:
Maira account = \$15 000 × 1.08 − (0.08 × \$15 000 × 0.40) = \$15 720

(Multiply by 1.08 to increase by 8%.
Multiply by 0.08 to find 8% of \$15 000.
Multiply by 0.40 to find 40% of the profit.)
Jurgen account = \$18 000 × 0.88 = \$15 840
Jurgen has \$15 840 − \$ 15 720 = \$ 120 more in his account than Maira on 1 Jan 2022.
(Multiply by 0.88 to decrease by 12%)

4 **(a)** London's population in 1900 as a percentage of its population in 2000 $= \frac{5}{7.27} \times 100 = 68.8\%$
(x as a % of $y = \frac{x}{y} \times 100$)

(b) **(i)** Percentage change in London's population from 1950 to 2000 $= \frac{7.27 - 8.2}{8.2} \times 100 = -11.3\%$

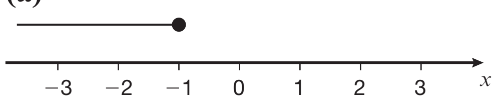
(ii) Percentage change in London's population from 1900 to 2020 $= \frac{9.3 - 5}{5} \times 100 = +86.0\%$
(% change $= \frac{\text{change}}{\text{original}} \times 100$)

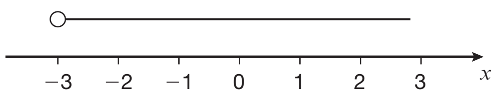
5 **(a)** Percentage of female doctors by country:
England: $\frac{98\,974}{98\,974 + 107\,221} \times 100 = 48\%$
Scotland: $\frac{11\,012}{11\,012 + 9766} \times 100 = 53\%$
Wales: $\frac{4711}{4711 + 5531} \times 100 = 46\%$
Northern Ireland: $\frac{3337}{3337 + 3207} \times 100 = 51\%$
Scotland has the highest percentage of female doctors in 2019.

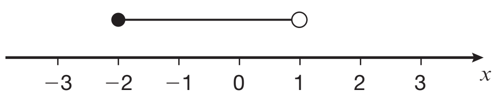
(b) Number of male doctors = 125 725
New number of male doctors = 130 754
$125\,725 \times p = 130\,754$,
so $p = \frac{130\,754}{125\,725} = 1.04$, so $k = 4$

ALGEBRA 2 – BASIC SKILLS EXERCISE

1 $\frac{4}{x}$
2 $2a$
3 $\frac{2x}{y}$
4 $4x^2$
5 $\frac{5ad}{7b}$
6 4
7 z
8 1
9 $4bc$
10 b
11 $\frac{17x}{12}$
12 $\frac{7}{6x}$
13 $\frac{14z}{15}$
14 $\frac{3}{5x}$
15 $\frac{(4x - x^2)}{6}$
16 $\frac{x^2}{6}$
17 $a^2 + b^2$
18 $\frac{a + b^3}{b}$
19 ± 4
20 ± 4
21 ± 3
22 3
23 ± 2
24 4
25 9
26 4
27 9
28 4
29 3^{10}
30 a^6
31 5^7
32 x^2
33 4^6
34 y^{18}
35 7^4
36 z^3
37 **(a)** $2 > -2$
(b) $-2 > -5$
(c) $20\% < \frac{1}{4}$
(d) $-0.3 > -\frac{1}{3}$

38 **(a)**


(b)


(c)


39 **(a)** $-2 \leq x < 2$
(b) $x \leq -1$ or $x > 2$

40 $x \le 2$
41 $x > 4$
42 $x < -2$
43 $-4 < x \le 0$

ALGEBRA 2 – EXAM PRACTICE EXERCISE

1 **(a)** $\frac{12x^3y^2z}{5x^2y^4} \div \frac{8xz}{15y^3} \times \frac{yz}{9x^2}$

$= \frac{12x^3y^2z}{5x^2y^4} \times \frac{15y^3}{8xz} \times \frac{yz}{9x^2}$ To divide, 'turn upside down and multiply'

$= \frac{12y^2z}{5y^4} \times \frac{15y^3}{8z} \times \frac{yz}{9x^2}$ 'Cancelling' x

$= \frac{12y^2z}{5} \times \frac{15}{8z} \times \frac{z}{9x^2}$ 'Cancelling' y

$= \frac{12y^2}{5} \times \frac{15}{8} \times \frac{z}{9x^2}$ 'Cancelling' z

$= \frac{y^2z}{2x^2}$ 'Cancelling' the numbers

(b) $\left(\frac{1}{x} - \frac{3x}{x^2}\right) = \frac{1}{x} - \frac{3}{x} = \frac{-2}{x}$

Deal with $\left(\frac{1}{x} - \frac{3x}{x^2}\right)$ first (BIDMAS)

$\Rightarrow \frac{1}{x^2} \div \left(\frac{1}{x} - \frac{3x}{x^2}\right) = \frac{1}{x^2} \times \frac{x}{-2} = \frac{-1}{2x}$

To divide, turn fraction upside down and multiply

$\Rightarrow 1 - \left[\frac{1}{x^2} \div \left(\frac{1}{x} - \frac{3x}{x^2}\right)\right] = 1 - \frac{-1}{2x}$

$= \frac{2x}{2x} + \frac{1}{2x} = \frac{2x+1}{2x} \Rightarrow a = 2.$

2 **(a)** Ava's age = y, Ben's age = $y - 4$, Charlie's age = $2(y - 4)$

Sum of ages = $y + y - 4 + 2(y - 4) = 4y - 12$

$4y - 12 > 27$ and $4y - 12 < 41$

(or $27 < 4y - 12 < 41$)

(b) $4y - 12 < 41$

$4y < 53 \Rightarrow y < 13.25$ y is an integer

Ava is 13, Ben is 9 and Charlie is 18.

(c) $4y - 12 > 27$

$4y > 39$

$y > 9.75$ y is an integer

Ava is 10, Ben is 6 and Charlie is 12

3 **(a)** Method 1: $a^4 \div a^3 = a^1$ Subtracting indices rule

$\sqrt{x+1} = 4$

$x + 1 = 16$

$x = 15$

Method 2: $\frac{a^{\sqrt{x+1}}}{a^3} = a$ Multiplying both sides by a^3

$a^{\sqrt{x+1}} = a^4$

$\sqrt{x+1} = 4$

$x + 1 = 16$

$x = 15$

(b) **(i)** $1 + \frac{1}{a} = \frac{a}{a} + \frac{1}{a} = \frac{a+1}{a}$

Simplify denominator first

$\frac{1}{1+\frac{1}{a}} = 1 \div \frac{a+1}{a}$

$= 1 \times \frac{a}{a+1} = \frac{a}{a+1}$

(ii) $\frac{1}{1+\frac{1}{1+\frac{1}{a}}} = \frac{1}{1+\frac{a}{a+1}}$ Using result from part i

$1 + \frac{a}{a+1} = \frac{a+1+a}{a+1} = \frac{2a+1}{a+1}$

Simplify denominator first

$\frac{1}{1+\frac{a}{a+1}} = 1 \div \frac{2a+1}{a+1}$

$= 1 \times \frac{a+1}{2a+1} = \frac{a+1}{2a+1}$

4 First third of journey is x km at a speed of 60 km/h

Time taken for the first third of the journey is $\frac{x}{60}$ hours. Time = $\frac{\text{distance}}{\text{speed}}$

Remaining two-thirds of journey is $2x$ km at 40 km/h

Time taken for remaining two-thirds is $\frac{2x}{40}$ hours. Time = $\frac{\text{distance}}{\text{speed}}$

$\frac{x}{60} + \frac{2x}{40} = \frac{3}{2}$ total journey time is 1.5 hours or $\frac{3}{2}$ hours

$\frac{x}{60} + \frac{2x}{40} = \frac{3}{2}$

$\frac{2x}{120} + \frac{6x}{120} = \frac{180}{120}$ multiplying both sides by 120

$2x + 6x = 180$

$x = \frac{180}{8}$

$3x = \frac{3 \times 180}{8}$ $3x$ is total length of journey

$= 67.5$ km

5 **(a)** $\frac{1}{R} = \frac{b}{ab} + \frac{a}{ab} = \frac{b+a}{ab} \Rightarrow \mathrm{R} = \frac{ab}{a+b}$

(b) a becomes $a + 1$, b becomes $b - 1$

Substitute these values into $R = \frac{ab}{a+b}$

$R_{new} = \frac{(a+1)(b-1)}{a+1+b-1} = \frac{(a+1)(b-1)}{a+b}$

Change in R is $R_{new} - R = \frac{(a+1)(b-1)}{a+b}$

$-\frac{ab}{a+b} = \frac{ab+b-a-1-ab}{a+b} = \frac{b-a-1}{a+b}$

% change $= \frac{b-a-1}{a+b} \div \frac{ab}{a+b} \times 100$

% change is $\frac{\text{Change in } R}{\text{Original } R} \times 100$

$= \frac{b-a-1}{a+b} \times \frac{a+b}{ab} \times 100$

$= \frac{b-a-1}{ab} \times 100$

GRAPHS 2 – BASIC SKILLS EXERCISE

1 $y = 3x - 1$

2 $y = -\frac{1}{4}x + 2$

3 $y = x$

4 $y = 2x + 1$

5 $y = -\frac{1}{3}x + 4$

6 $y = 4x - 2$

7 $y = -0.4x + 1$

8 $y = 0.2x$

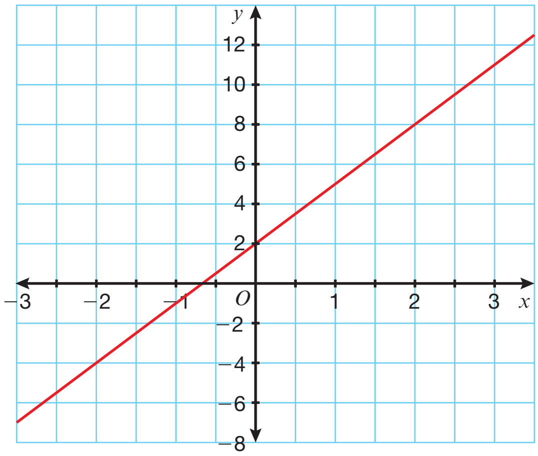
9 $y = -2x + 12$

10 $y = \frac{x}{3} + \frac{5}{3}$

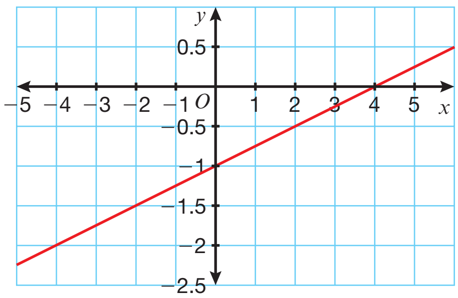
11 $y = -\frac{x}{2} - 1$

12 $y = 3x - 5$

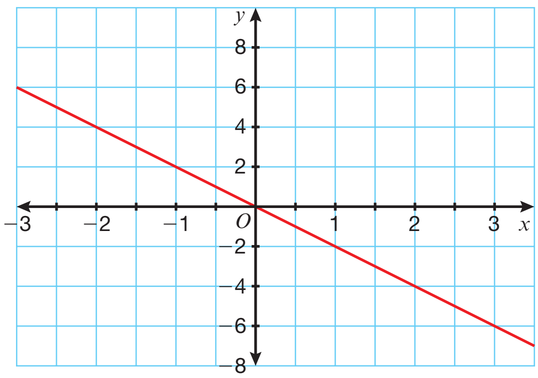
13 3, 2



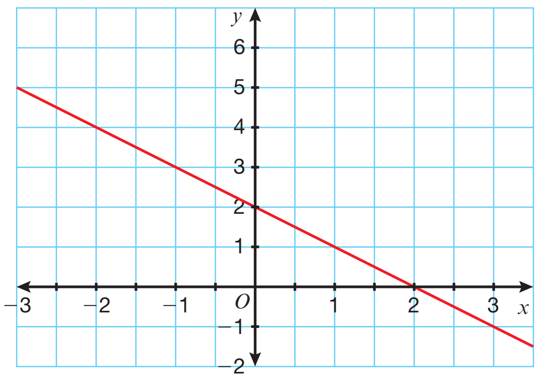
14 $\frac{1}{4}$, -1



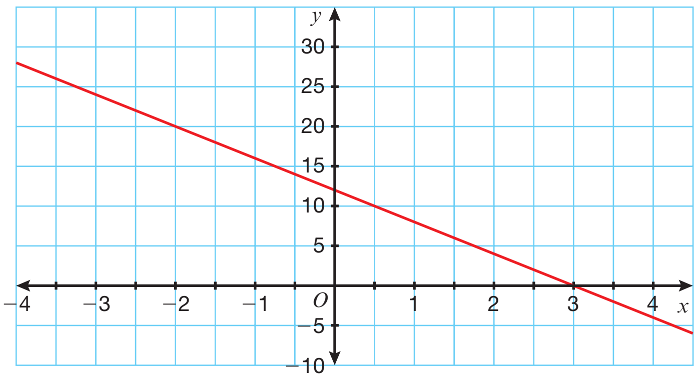
15 –2, 0



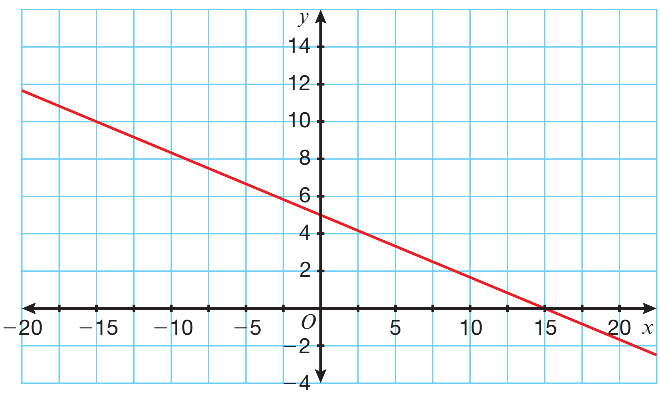
16 –1, 2



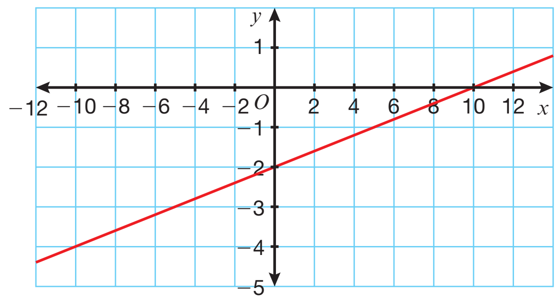
17 (3, 0), (0, 12)



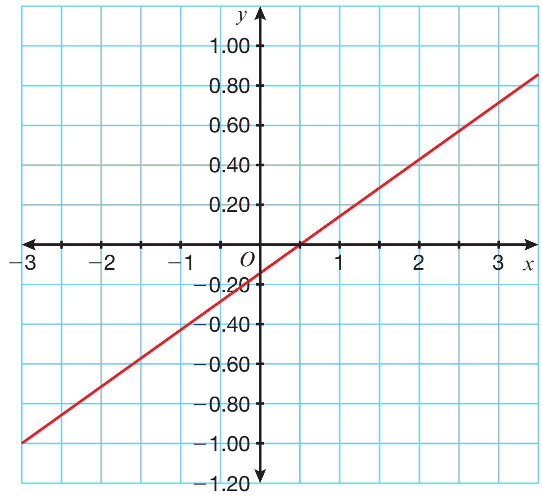
18 (15, 0), (0, 5)



19 (10, 0), 0, –2)



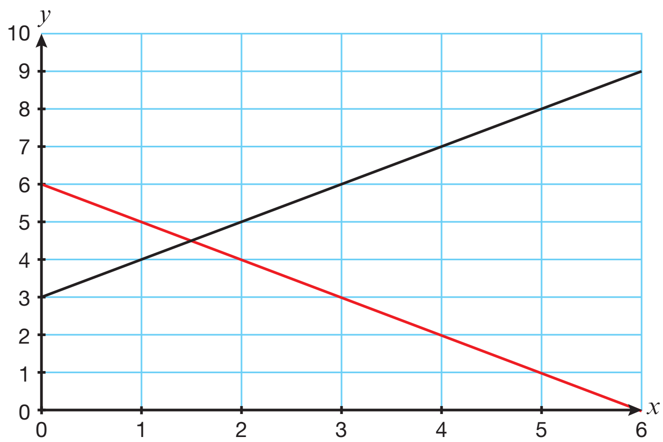
20 $\left(\frac{1}{2}, 0\right), \left(0, -\frac{1}{7}\right)$



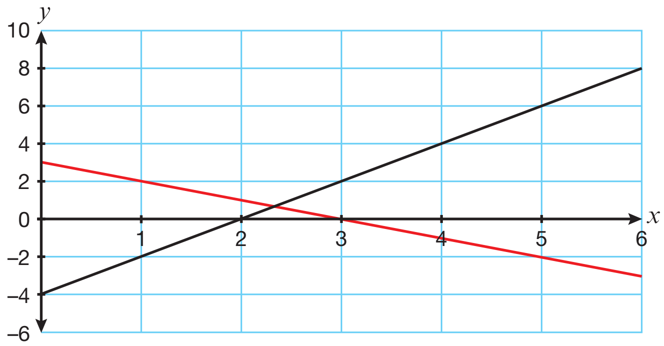
21 (1.5, 4.5)

x	0	3	6
***y* = *x* +3**	3	6	9

x	0	3	6
y* = 6 − *x	6	3	0



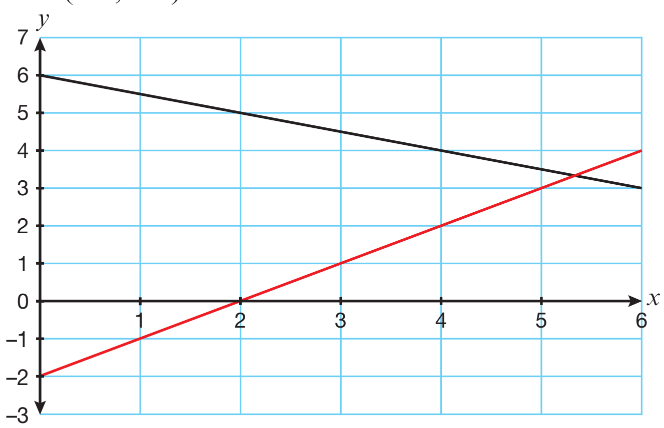
22 $\left(\frac{7}{3}, \frac{2}{3}\right)$



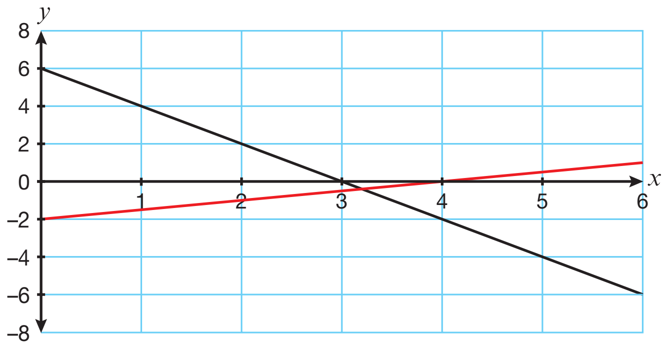
23 (2, 1)

24 (4, 2)

25 (5.3, 3.3)

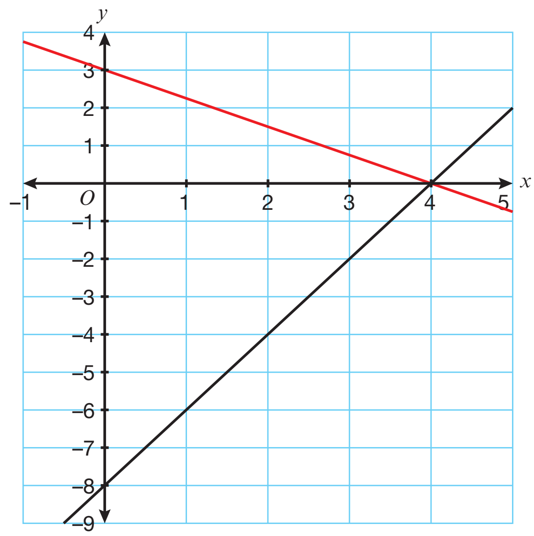


26 (3.2, –0.4)



GRAPHS 2 – EXAM PRACTICE EXERCISE

1



(a) $y = 2x - 4$ has gradient 2 so L_2 must be $y = 2x + c$
L intersects the x-axis at (4, 0)
Substituting $x = 4$, $y = 0$ into $y = 2x + c$ gives
$c = -8$ so M is $y = 2x - 8$

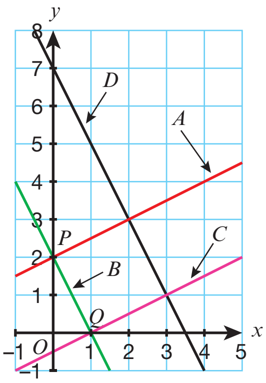
(b) L intersects the y-axis at (0,3), M intersects the y-axis at (0,−8) and they both intersect the x-axis at (4, 0)
Area $= \frac{1}{2} \times 11 \times 4 = 22$ square units

2 **(a)** Rearrange the equations as

A: $y = \frac{1}{2}x + 2$, B: $y = -2x + 2$

C: $y = \frac{1}{2}x - \frac{1}{2}$, D: $y = -2x + 7$

A sketch helps you to understand and answer the question.



A and C have the same gradient of $\frac{1}{2}$ so they are parallel and one pair of opposite sides.

B and D have the same gradient of -2 so they are parallel and the other pair of opposite sides.

(b) A and B have a common point, P, (0, 2) so this is a vertex

B and C have a common point, Q, (1, 0) so this is a vertex.

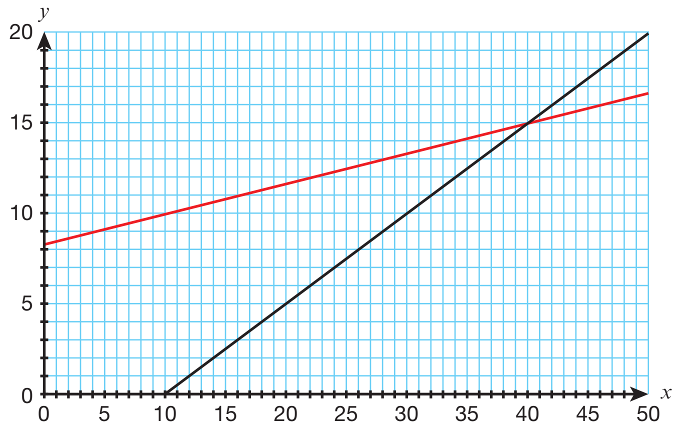
$PQ^2 = 1^2 + 2^2 = 5$ Pythagoras' Theorem
$PQ = \sqrt{5}$ so perimeter $= 4\sqrt{5}$

3 **(a)**

Age (years)	Mia	Priya
Now	x	y
10 years ago	$x - 10$	$y - 10$
10 years' time	$x + 10$	$y + 10$

Ages 10 years ago were $x - 10$, $y - 10$
$x - 10 = 6(y - 10)$
$x - 10 = 6y - 60$
$x + 50 = 6y$
Ages in 10 years' time will be $x + 10$, $y + 10$
$x + 10 = 2(y + 10)$
$x + 10 = 2y + 20$
$x - 10 = 2y$

(b) Some points for $x + 50 = 6y$ are (10, 10), (22, 12) and (46, 16)
Some points for $x - 10 = 2y$ are (10, 0), (30, 10) and (50, 20)
or rearrange equations as $y = \frac{x}{6} + \frac{50}{6}$ and $y = \frac{x}{2} - 5$ and make a table of values using 3 widely spaced values of x



The graphs intersect at (40, 15) so Mia is 40 years old and Priya is 15 years old.

4 **(a)** 200 minutes on the phone costs $200p$ cents and 200 texts cost $200t$ cents.
$200p + 200t = 2800$
Cost of \$28 must be expressed in cents
$p + t = 14$
100 minutes on the phone costs $100p$ cents and 300 texts cost $300t$ cents.
$100p + 300t = 2200$
Cost of \$22 must be expressed in cents
$p + 3t = 22$

(b) Table of values for $p + t = 14$

p	0	7	14
t	14	7	0

Table of values for $p + 3t = 22$

p	1	7	22
t	7	5	0

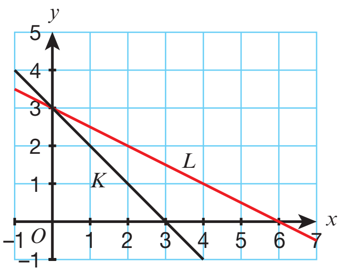
(c) The graphs intersect at $p = 10$, $t = 4$, so 150 minutes on the phone will cost 150×10 cents, 250 texts will cost 250×4 cents.
Total cost is 2500 cents or \$25.

5 **(a)** Let equation of L be $y = mx + c$, then equation of K is $y = 2mx + c$
Gradient of K is twice gradient of L and they both have the same y intercept.

L: Substituting $(-2, 4)$ gives $4 = -2m + c$
K: Substituting $(4, -1)$ gives
$-1 = 2m \times 4 + c$ so $-1 = 8m + c$

(b) Subtract the two equations:
$4 - (-1) = -2m - 8m$
$5 = -10m$
$m = -\frac{1}{2}$
and $c = 3$

(c) Equation of L is $y = -\frac{x}{2} + 3$
L intersects the x-axis at (6, 0) and the y-axis at (0, 3)
Equation of K is $y = -x + 3$
K intersects the x-axis at (3, 0) and the y-axis at (0, 3)



$A = \frac{1}{2} \times 3 \times 3 = 4.5$ square units

SHAPE AND SPACE 2 – BASIC SKILLS EXERCISE

1 $a = 6.40$
2 $b = 4.47$
3 $c = 15.0$
4 $AC = 36.6$
5 $a = 5.39$
6 $a = 5.20$
7 **(a)** $r = 11.7$
(b) $a = 18.7$
8 **(a)** $XC = 2$ cm
(b) $AC = 4.47$ cm

9 $k = 3$ Using Pythagoras', horizontal distance is 3 and vertical distance is 15 gives hypotenuse of $3\sqrt{26}$

10 Using Pythagoras', $(\text{length of square})^2 = (a-c)^2 + (b-d)^2$. But area is $(\text{length of square})^2$

So $A = (a-c)^2 + (b-d)^2$

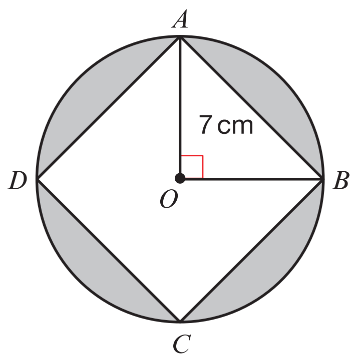
11 $a = 5$, $b = 17$ or vice-versa. Use Pythagoras' with horizontal distance 20 and vertical distance 90
$20^2 + 90^2 = 8500$ so $OP = 10\sqrt{85}$. 5 and 17 are prime numbers which multiply to give 85.

12 $k = 3$ Use Pythagoras' $(\text{length of base diagonal})^2 = (10x)^2 + (10x)^2 = 200x^2$
$(\text{internal diagonal})^2 = 200x^2 + 100x^2 = 300x^2$
so internal diagonal $= 10\sqrt{3}\,x$ so $k = 3$

13 $a = 90°$, $b = 30°$
14 $a = 70°$, $b = 20°$
15 $a = 55°$, $b = 70°$
16 $a = 90°$, $b = 45°$
17 $2a = 36°$, $3a = 54°$
18 $x = 70°$, $y = 55°$, $z = 35°$
19 $a = 60°$
20 $a = 140°$
21 $a = 50°$
22 $a = 140°$
23 $a = 100°$
24 $a = 80°$
25 $a = 290°$
26 $a = 102°$
27 $x = 130°$, $y = 25°$, $z = 65°$
28 $a = 40°$, $b = 20°$
29 $a = 120°$, $b = 30°$
30 $a = 40°$, $b = 60°$
31 $a = 35°$, $b = 25°$
32 $a = 65°$, $b = 115°$
33 $a = 50°$, $y = 130°$
34 $a = 110°$, $b = 70°$
35 $a = 60°$, $b = 60°$
36 $x = 130°$, $y = 65°$, $z = 115°$

SHAPE AND SPACE 2 – EXAM PRACTICE EXERCISE

1



(a) Let O be the centre of the circle, so triangle AOB is a right-angled triangle.
If $AO = BO = r$, then from Pythagoras' Theorem: $7^2 = r^2 + r^2 = 2r^2$, so $r^2 = \frac{49}{2}$
area of the shaded segments = circle area − square area $= \pi\frac{49}{2} - 7^2 = 49\left(\frac{\pi}{2} - 1\right)$
$= 49\left(\frac{\pi - 2}{2}\right)$

percentage of shaded area of area of

circle $= \dfrac{49\left(\frac{\pi-2}{2}\right)}{\pi\frac{49}{2}} \times 100 = \frac{100}{\pi}(\pi-2)$

$= m(\pi-2),\ m = \frac{100}{\pi}$

(b) Area of one of the original circle

shaded segments $= \dfrac{49\left(\frac{\pi-2}{2}\right)}{4}$

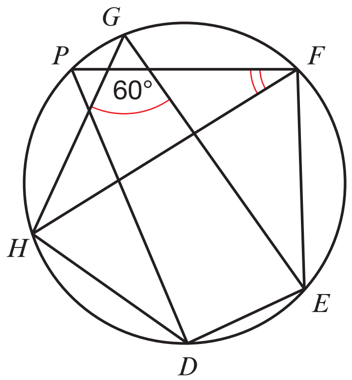
$= 49\left(\frac{\pi-2}{8}\right) = \frac{49}{8}(\pi-2) = a$

Let segment of the enlarged circle be A, so if the scale factor of enlargement = 4

$A = 4^2 \times a = 16 \times \frac{49}{8}(\pi-2) = 98(\pi-2)$

$= n(\pi-2),\ n = 98$

2



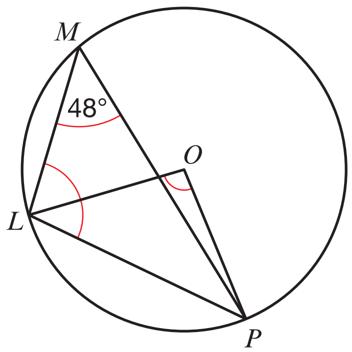
Angle HDE = 180° − 0° = 120°
($GHDE$ is a cyclic quadrilateral: Opposite angles in a cyclic quadrilateral sum to 180°)

Angle HDP $= \frac{1}{3} \times 120° = 40°$

(Angle HDE: Angle EDP =l : 2)

Angle HDP = Angle HFP = 40°
(Both angle HDP and angle HFP are formed in the same segment off chord HP: Angles in the same segment are equal).

3



Angle LOP = 2 × 48° = 96°
(Angle at centre = 2 × angle at circumference off the same chord in the same segment)

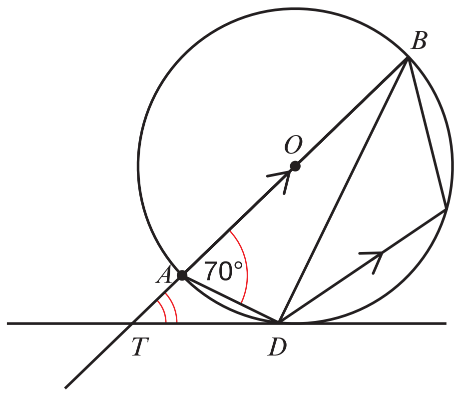
Angle OLP = Angle OPL $= \frac{180° - 96°}{2} = 42°$

(Triangle OLP is isosceles, so the base angles are equal)

Angle MPL $= \frac{2}{3} \times 42° = 28°$

Angle MLP = 180° − 48° − 28° = 104°
(Angle sum of a triangle = 180°)

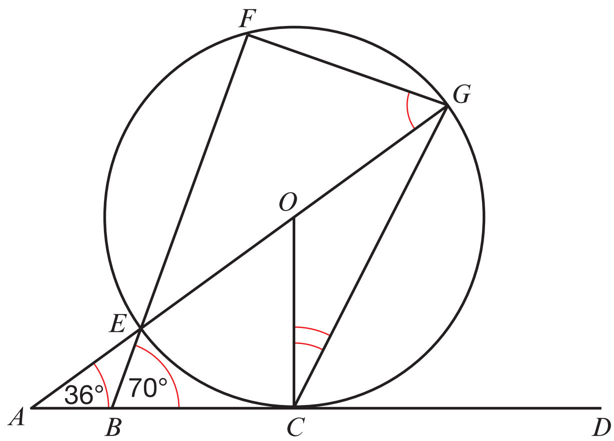
4



(a) Angle ADB = 90°
(Angle in a semicircle is 90°. AB is the diameter of the circle)
Angle ABD = 20°
(Angle sum of a triangle = 180°)
Angle BDC = 20°
(Angle ABD and Angle BDC are alternate angles)
Angle ADC = 110°
(Angle BAD and Angle ADC are also co-interior angles that sum to 180°)

(b) Angle ADT = Angle ABD = 20°
(Alternate segment theorem)
Angle ATD = 180° − 110° − 20° = 50°
(Angle sum of a triangle = 180°)

5



(a) Angle $OCA = 90°$ (Radius OC is perpendicular to tangent AD.)
Angle $AOC = 180° - 90° - 36° = 54°$
(Angle sum of a triangle = 180°)
Angle $COG = 180° - 54° = 126°$
(Sum of angles in a straight line = 180°)
Angle $OCG = \frac{180° - 126°}{2} = 27°$
(Triangle OCG is isosceles, so the base angles are equal.)

(b) Angle $BEO = 360° - 70° - 90° - 54° = 146°$
(Angle sum of a quadrilateral = 360°)
Angle $FEO = 180° - 146° = 34°$
(Sum of angles in a straight line = 180°)
Angle $FGO = 180° - 90° - 34° = 56°$
(Angle sum of a triangle = 180°. Angle in a semi-circle is 90°. AB is the diameter of the circle.)

(c) Triangle ECG is a right-angled triangle as EG is a diameter of the circle.
Using Pythagoras': $EG^2 = EC^2 + CG^2$,
$(2r)^2 = EC^2 + (4s)^2$
$EC^2 = 4r^2 - 16s^2 = 4(r^2 - 4s^2) =$
$4(r + 2s)(r - 2s)$ – using a difference of squares
$EC = \sqrt{4(r + 2s)(r - 2s)}$
$= 2\sqrt{(r + 2s)(r - 2s)}$ as required.

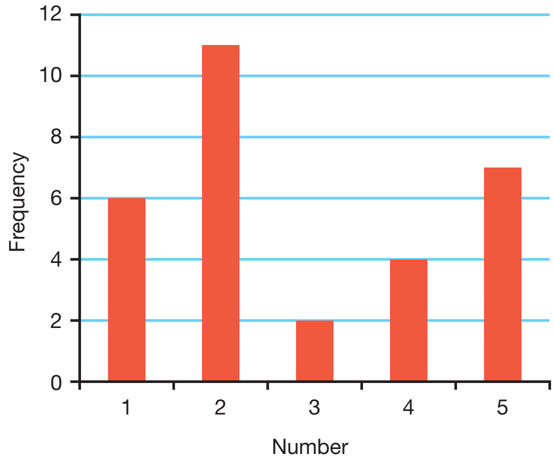
HANDLING DATA 1 – BASIC SKILLS EXERCISE

1 **(a)** Phone make – categorical
(b) Number of goals scored – discrete
(c) Height of a horse – continuous
(d) Number of coins – discrete
(e) Time to eat a pizza – continuous
(f) Hair colour – categorical

2 **(a)**

Number	Tally	Frequency
1	卌 |	6
2	卌 卌 |	11
3	||	2
4	||||	4
5	卌 ||	7

(b)

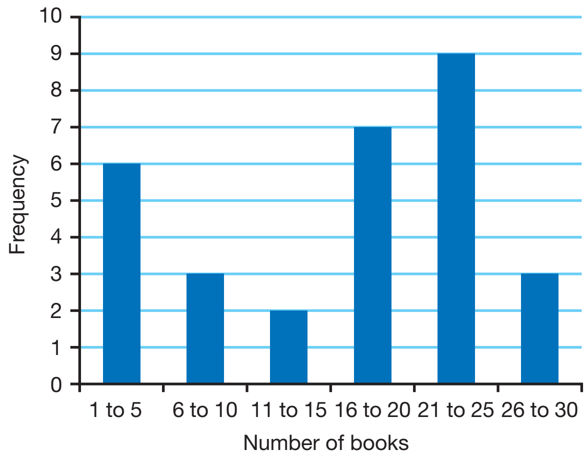


(c) Does not appear very random, but the sample is too small to draw any definite conclusions.

3 **(a)**

Number of guides	Tally	Frequency
1–5	卌 |	6
6–10	|||	3
11–15	||	2
16–20	卌 ||	7
21–25	卌 ||||	9
26–30	|||	3

(b)

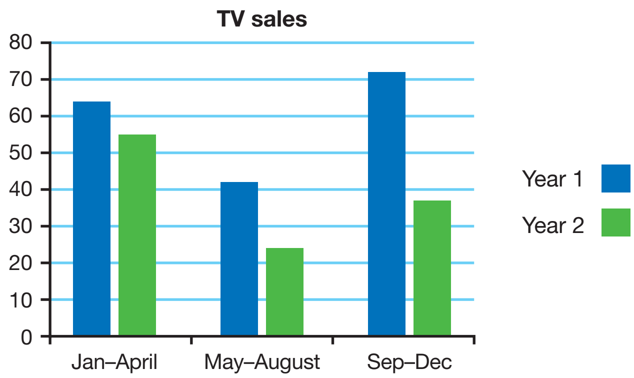


(c) There are many students with a lot of revision guides, quite a few with not many guides, but not many students with a middling number of guides.

4 **(a)**

	January–April	May–August	September–December	Total
Year 1	64	42	72	178
Year 2	55	24	37	116
Total	119	66	109	294

(b)



(c) Year 2 sales were worse overall, especially from April–December

5 **(a)** 7.5% had a rabbit
Percentages must sum to 100'
or '% must sum to 100

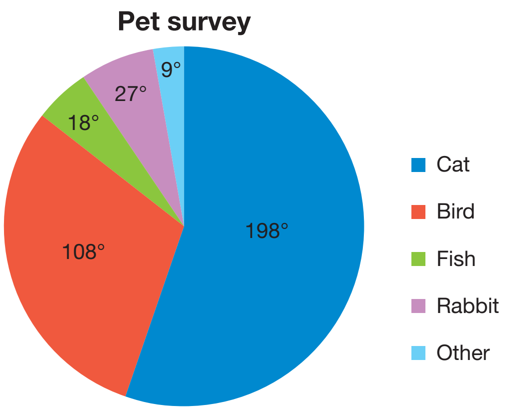
(b) Angle for cat $= \frac{55}{100} \times 360° = 198°$

Angle for bird $= \frac{30}{100} \times 360° = 108°$

Angle for rabbit $= \frac{7.5}{100} \times 360° = 27°$

Angle for fish $= \frac{5}{100} \times 360° = 18°$

Angle for other $= \frac{2.5}{100} \times 360° = 9°$



6 Orange juice is 56°

$\frac{56}{14} = 4$ so 4° corresponds to 1 person

Coffee $\frac{88}{4} = 22$ people

Tea $\frac{104}{4} = 26$ people

Milk $\frac{72}{4} = 18$ people

Other $\frac{40}{4} = 10$ people

7 Data in order is 0, 1, 2, 3, 4, 5, 6, 7, 7, 7, 7, 9
mean = 4.83, median = 4.5 and mode = 7

8 Data in order:

0	0	0	0	0	0	0
1	1	1	1	1	2	2
2	2	3	3	3	3	4
4	4	4	4	5	5	6
6	6					

(a) Sum of data = 73
mean is 73 ÷ 30 = 2.43 (3 s.f.)
median is 2 and mode is 0

(b) She would probably use the mode as this is the lowest average.

9 Total sent over the week = 7 × 32 = 224
Total sent over the first 6 days = 176
Number sent on seventh day = 224 – 176 = 48

10 Data in order: 0, 0, 0, 0, 0, 1, 1, 1, 2, 3

(a) Mean = 0.8, median is 0.5 and mode is 0

(b) Goals scored this season = 12 × 1.25 = 15, goals last season = 8
Total goals = 23, total matches = 22,
mean = 23 ÷ 22 = 1.05 (3 s.f.)

11 Total points needed: 857 × 7 = 5999
Total so far: 6 × 840 = 5040
Points needed: 5999 – 5040 = 959

12

State	A	B	C	Total
Population in millions	21	26	18	**65**
Mean	0.1	0.09	0.15	
Number infected in millions	2.1	2.34	2.7	**7.14**

Mean for whole country $= \frac{7.14}{65} = 0.11$ (2 d.p.)

13 $x = 56$, $y = 73$

14 $x = 7$, $y = 42$

15 Sum of the first 19 prime numbers squared
= 19 × 1314 = 24 966

mean is $\frac{24\,966 + 71^2}{20} = 1500.35$

HANDLING DATA 1 – EXAM PRACTICE EXERCISE

1 **(a)** Data in order are 29, 32, 32, 35, 83, 86, 95

Mean = $\frac{392}{7}$ = 56, median is 35 and mode = 32

(b) Mode cannot be calculated as frequencies are not known.

Median cannot be calculated as the distribution of sales is not known.

Mean can be calculated. Total sales for the first week are 392.

Total sales for the next three weeks (21 days) are 60 × 21 = 1260.

Total sales for the four-week period of 28 days are 392 + 1260 = 1652

mean is $\frac{1652}{28}$ = 59

2 **(a)** Two of the following:

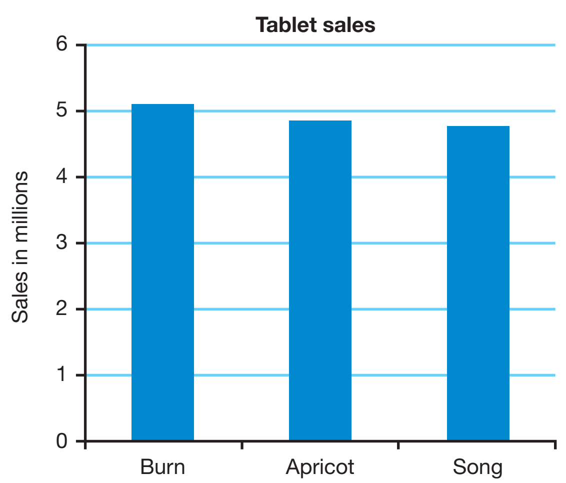
The vertical axis does not start at zero.

The vertical axis has no zig-zag line to show it does not start at zero.

The bars are not of equal widths.

The size of the type is not the same.

(b)



Note: Bars must not touch each other and the vertical scale must be linear.

3 $t = 50°$ (Angles must sum to 360)

50° corresponds to 40 pupils so 10° corresponds to 40 ÷ 5 = 8 pupils

100° corresponds to 8 × 10 = 80 pupils = p

60° corresponds to 8 × 6 = 48 pupils = q

70° corresponds to 8 × 7 = 56 pupils = r

80° corresponds to 8 × 8 = 64 pupils = s

4 **(a)** New mean is $\frac{12a + 16a + 21a + 27a}{4}$

$= \frac{a(12 + 16 + 21 + 27)}{4} = a \times 19$

The mean has been multiplied by a.

(b) Let s be the total of the numbers in set A, then $x = \frac{s}{n}$ and $s = xn$

Let t be the total of the numbers in set B, then $y = \frac{t}{m}$ and $t = my$

The total of all $n + m$ numbers in set C is $s + t = nx + my$ so the mean is $\frac{nx + my}{n + m}$

5 **(a)** Range is $d - 6 = 15$ so $d = 25$

Median is the mean of b and 14 so $b = 12$

Mode is 8 so $a = 8$

Mean = 14 so the sum of the numbers is 8 × 14 = 112

Sum of the numbers is 94 + c so $c = 18$

$a = 8$, $b = 12$, $c = 18$ and $d = 25$

(b) Mean =17 so

$\frac{w + x + y + z}{4} = 17$

$w + x + y + z = 17 \times 4 = 68$ (equation 1)

Range = 8 so $z - w = 8$ (equation 2)

Median = 16 so $\frac{x + y}{2} = 16$ so

$x + y = 16 \times 2 = 32$ (equation 3)

Substitute equation 3 into equation 1

$w + 32 + z = 68$ so

$w + z = 68 - 32 = 36$ (equation 4)

Add equations 2 and 4 to get

$2z = 44$

$z = 22$ and $w = 14$

$x + y = 32$ so $x = 15$ and $y = 17$ since the median is 16

Integers are 14, 15, 17 and 22.

NUMBER 3 – BASIC SKILLS EXERCISE

1 **(a)** 7, 14, 21, 28

(b) 11, 22, 33, 44

(c) 17, 34, 51, 68

2 **(a)** 1, 3, 5, 15

(b) 1, 2, 4, 5, 10, 20

(c) 1, 2, 7, 14, 49, 98

3 **(a)** $3^3 \times 5 \times 7$

(b) $2^3 \times 3^4 \times 5 \times 7$

(c) $3^9 \times 5^3 \times 7^3$

(d) $2^2 \times 3^3 \times 5^3 \times 7$

4 **(a)** $2^7 \times 3^4 \times 5^2 \times 7^4$

(b) $2^8 \times 3^4 \times 5^2 \times 7^4$

5 $N = 2^9 \times 3^2 \times 5^6$ therefore the largest odd factor is $3^2 \times 5^6 = 140\,625$

6 **(a)** $60 = 2^2 \times 3 \times 5$, $70 = 2 \times 5 \times 7$ therefore
HCF $= 2 \times 5 = 10$
LCM $= 2^2 \times 3 \times 5 \times 7 = 420$
(b) $140 = 2^2 \times 5 \times 7$, $84 = 2^2 \times 3 \times 7$
therefore HCF $= 2^2 \times 7 = 28$
LCM $= 2^2 \times 3 \times 5 \times 7 = 420$
(c) $525 = 3 \times 5^2 \times 7$, $40 = 2^3 \times 5$ $441 = 3^2 \times 7^2$
therefore HCF = 1
LCM $= 2^3 \times 3^2 \times 5^2 \times 7^2 = 88\,200$

7 HCF = 3, LCM $= 2^4 \times 3^2 \times 5^2 \times 7^2 \times 11^2$

8 15, 21

9 **(a)** HCF $= 5pq$, LCM $= 140\,pq$
(b) HCF $= 2xyz$, LCM $= 12x^2y^2z^2$
(c) HCF $= 6a^2b^3c^2$, LCM $= 36a^4b^3c^4$

10 **(a)** 1, 2, 3, 4, 6, 12
(b) Factors of 12

11 $48 = 2^4 \times 3$, $45 = 3^2 \times 5$ therefore
LCM $= 2^4 \times 3^2 \times 5 = 720$ s or 12 minutes

12 $88 = 2^3 \times 11$, $110 = 2 \times 5 \times 11$ therefore
LCM $= 2^3 \times 5 \times 11 = 440$ so 4.4 seconds

13 **(a)** 5 : 11
(b) 1 : 5 : 17
(c) 3 : 4 : 70
(d) $x : 4y : 20$

14 **(a)** 1 : 24
(b) 3 : 1440 = 1 : 480
(c) 4 : 1000 = 1 : 250
(d) $10 : \frac{900\,000}{60 \times 60} = 1 : 25$

15 $\frac{3}{x} = \frac{x}{27}$ so $x^2 = 81$ so $x = 9$ or $x = -9$

16 **(a)** $\frac{360}{9} = 40$ so 160 : 200
(b) $\frac{133}{7} = 19$ so 19 : 38 : 76
(c) $\frac{1000}{10} = 100$ so 100 : 200 : 300 : 400
(d) $\frac{352}{11} = 32$ so 64 : 96 : 192

17 $\frac{3450}{15} = 230$ so shares are £460, £1380 and £1610 so difference is £1150

18 $\frac{9}{4} \times 12 = 27$ therefore Petra is 27 years old

19 $x : y = 175 : 100 = 7 : 4$

20 $\frac{9}{7} \times 3.5 = 4.5$ therefore longest charging time is 4 h 30 mins

21 $\frac{3}{103} \times 51.5 = 1.5$ therefore the distance swum is 1.5 km

22 $\frac{5}{8} - \frac{3}{8} = \frac{1}{4}$ therefore $\frac{1}{4}$ of the weight of both sloths is 6 kg and the total weight is 24 kg hence 9 kg and 15 kg

23 $\frac{5}{10} - \frac{2}{10} = \frac{3}{10}$ so $\frac{3}{10}$ of total length is 120 cm and the total length is 400 cm.

24 Total no of parts is 16.
Sugar : milk : flour is 4 : 5 : 7
$\frac{5}{16} - \frac{4}{16} = \frac{1}{16} = 60$ g so the total weight
$= 60 \times 16 = 960$ g
therefore the weight of flour $= \frac{7}{16} \times 960 = 420$ g

25 LCM of 2 and 9 is 18, 1 : 2 = 9 : 18 and
9 : 5 = 18 : 10 so $a : b : c = 9 : 18 : 10$

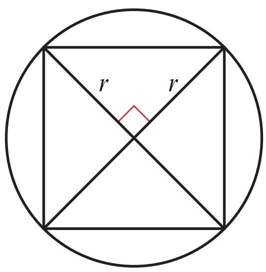
26 $a : b = 2 : 3 = 8 : 12$, $b : c = 12 : 15$
so $a : c = 8 : 15$

27 Angles A and C are $\frac{2}{3}$ of the angle sum of the triangle
$\frac{8}{8 + x} = \frac{2}{3}$
$24 = 16 + 2x$
$x = 4$

28 LCM of 3 and 7 is 21, 2 : 3 = 14 : 21 and
7 : 5 = 21 : 15 so $P : W : R = 14 : 21 : 15$.
14 + 21 + 15 = 50 = therefore the fraction of roses that are red is $\frac{15}{50} = \frac{3}{10}$ and hence the number is $\frac{3}{10}\,r$

29 Area of the circle is πr^2
Area of square is $4 \times \frac{1}{2}\, r^2 = 2r^2$

The square is made up of 4 right-angled triangles with base and height of r.
Alternatively calculate the length of the side of the square using Pythagoras' Theorem, length $\sqrt{r^2 + r^2} = \sqrt{2r^2}$
Area = length × length = $2r^2$.



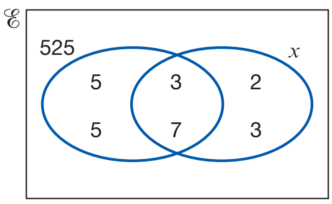
Ratio is $\pi r^2 : 2r^2 = \pi : 2$

30 Let F be father's age and S be son's age
$F = 3S$ and $F + 12 = 2(S + 12)$
$3S + 12 = 2S + 24$
$S = 12$, $F = 36$
In 36 years, $F = 72$, $S = 48$
So 3 : 2

NUMBER 3 – EXAM PRACTICE EXERCISE

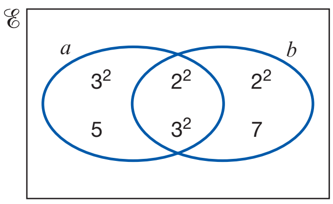
1 **(a)** $10^6 = 2^6 \times 5^6$ therefore the greatest is 5^6 = 15 625 An odd factor cannot contain any power of 2

(b) $525 = 3 \times 5^2 \times 7$, $21 = 3 \times 7$,
$3150 = 2 \times 3^2 \times 5^2 \times 7$
$x = 2 \times 3^2 \times 7 = 126$

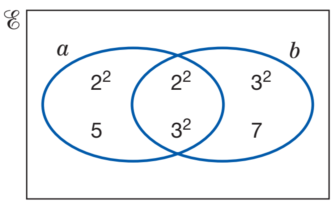


2 **(a)** $60 = 2^2 \times 3 \times 5$ $x = 2^2 \times 3 = 12$,
$y = 3 \times 5 = 15$
x and y must share the factor 3, but not the factors 2 or 5

(b) $45\,360 = 2^4 \times 3^4 \times 5 \times 7$, $36 = 2^2 \times 3^2$
There are two possible cases, shown in the Venn diagrams.



Case 1

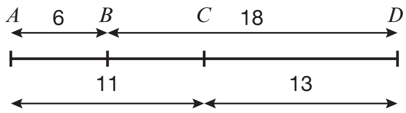


Case 2

Case 1 $a = 2^2 \times 3^4 \times 5 = 1620$,
$b = 2^4 \times 3^2 \times 7 = 1008$

Case 2 $a = 2^4 \times 3^2 \times 5 = 720$,
$b = 2^2 \times 3^4 \times 7 = 2268$
As $a < b$, Case 2 applies so $x = 4$ and $y = 2$

(3) **(a)** $1 + 3 = 4$ and $11 + 13 = 24$, so multiply the first ratio by 6 so they can be directly compared, i.e. 6 : 18 and 11 : 13.



The diagram shows that $BC = 5$, so
$AB : BC : CD = 6 : 5 : 13$

(b) **(i)** 11 : 3 = 22 : 6 and 5 : 2 = 15 : 6 so men : women : children = 22 : 15 : 6

(ii) $22 + 15 + 6 = 43$ so the fractional difference of men and women is
$\frac{22}{43} - \frac{15}{43} = \frac{7}{43}$

Total population is $42 \times \frac{43}{7} = 258$ therefore

the number of children is $\frac{6}{43} \times 258 = 36$

4 Number of V, C and M is $0.6 \times 140 = 84$
60% are not Mushroom
Ratio of 2 : 5 : 7 is a total of 14 parts
number of Meat is $\frac{7}{14} \times 84 = 42$

number of Veggie is $\frac{2}{14} \times 84 = 12$

difference is $42 - 12 = 30$ pizzas

5 $720 = 2^4 \times 3^2 \times 5$, $1260 = 2^2 \times 3^2 \times 5 \times 7$,
$1800 = 2^3 \times 3^2 \times 5^2$

HCF of 720, 1260 and 1800 is $2^2 \times 3^2 \times 5 =$ 180 so 180 parcels

Number of pints of milk is $\frac{720}{180} = 4$

Number of loaves of bread is $\frac{1260}{180} = 7$

Number of cans of beans is $\frac{1800}{180} = 10$

180 parcels each with 4 pints of milk,
7 loaves of bread and 10 cans of beans

ALGEBRA 3 – BASIC SKILLS EXERCISE

1 $a(7 - a)$
2 $3x(1 - 4x)$
3 $ab(a + b)$
4 $4xy(1 - 2xy)$
5 $\frac{1}{3}pqr(qr^2 + 2p^2q)$
6 $(x + 1)(2x - 5)$

7 $x + 4$
8 $1 + a$
9 $2x$
10 $\frac{5}{xy}$
11 $\frac{2(x+2)}{7}$
12 x^2y
13 6
14 $\frac{3}{5}$
15 11
16 4
17 14
18 15
19 $\frac{1}{5}$
20 $\frac{2}{3}$
21 –49
22 $\frac{3}{4}$
23 ± 4
24 ± 8
25 $1\frac{1}{5}$
26 6
27 Let £x be Zazoo's winnings $\Rightarrow$ Yi's winnings are £$3x$ and Xavier's are £$\frac{3x}{2}$

$\Rightarrow x + 3x + \frac{3x}{2} = 11000 \Rightarrow \frac{11x}{2} = 11000$
$\Rightarrow \frac{x}{2} = 1000 \Rightarrow x = 2000$

$\Rightarrow$ Zazoo gets £2000, Yi gets £6000 and Xavier gets £3000.
28 (4, 1)
29 (3, 2)
30 (1, 2)
31 (1, 1)
32 (1, 4)
33 (1, 8)
34 (3, 1)
35 (6, 2)
36 (2, 1)
37 (2, 1)
38 (–1, 5)
39 (−0.4, −9.2)
40 $x = 24$, $y = 15$
41 6 stools

ALGEBRA 3 – EXAM PRACTICE EXERCISE

1 **(a)** **(i)** $\frac{2}{5}p^2r^2(r - 2p)$

(ii) $(x - 1)(3 - x)$

(b) **(i)** $\frac{10x^2 + 5x}{5x} - 1 = \frac{5x(2x+1)}{5x} - 1$

$= (2x + 1) - 1 = 2x$

(ii) $\frac{3x^3y + 9x^2y^2}{x^2 + 3xy} = \frac{3x^2y(x+3y)}{x(x+3y)}$

$= 3xy$

(c) $\frac{x^2 - xy}{xy^2 + y^3} \div \frac{xy - y^2}{x^3 + x^2y}$

$= \frac{x^2 - xy}{xy^2 + y^3} \times \frac{x^3 + x^2y}{xy - y^2}$

$= \frac{x(x-y)}{y^2(x+y)} \times \frac{x^2(x+y)}{y(x-y)} = \frac{x^3}{y^3}$

$= \left(\frac{x}{y}\right)^3$ so $n = 3$

2 **(a)** **(i)** $\frac{2(x+1)}{5} - \frac{3(x+1)}{10} = x$

$\frac{4x + 4 - 3x - 3}{10} = x$

$x + 1 = 10x$

$x = \frac{1}{9}$

(ii) $\frac{36}{x} - 4x = 0$

$36 - 4x^2 = 0$
$36 = 4x^2$
$x^2 = 9$
$x = \pm 3$

(b) Let C be the amount Carla gets.
Then Bobbie gets $1.4C$
To increase by 40%, multiply by 1.4
And Anna gets $0.7C + 500$
$0.7C + 500 + 1.4C + C = 16000$
$3.1C = 15500$
$C = 5000$
Anna gets \$4000, Bobbie gets \$7000 and Carla gets \$5000.

3 $4y - 2x - 15 = 2x + y$ simplifies to $3y - 4x = 15$
$x : y = 3 : 5$ so $5x = 3y$
Substituting gives
$5x - 4x = 15$
$x = 15$
$y = 25$
angle B = angle C = 55° so angle A = 70°
Ratio angle A : angle B = 70 : 55 = 14 : 11

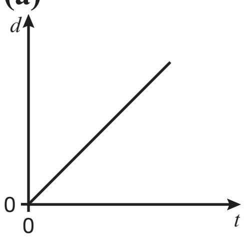
4 **(a)** $12 = 2^2 + 2a + b$
$-6 = (-1)^2 - a + b$
$2a + b = 8$
$b - a = -7$
$3a = 15$
$a = 5$
$b = -2$
equation of curve is $y = x^2 + 5x - 2$

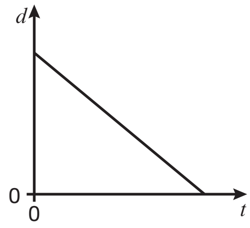
(b) Let a be number of ash trees and b be the number of beech trees.
$20a + 30b = 3100$
$30a + 20b = 2900$
$2a + 3b = 310$
$3a + 2b = 290$
$6a + 9b = 930$
$6a + 4b = 580$
$5b = 350$
$b = 70$
$a = 50$
Ash costs \$50, beech costs \$70

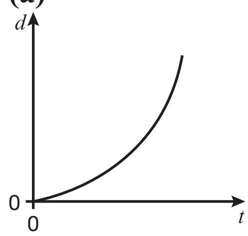
5 $(x + 100) : (y + 100) = 4 : 3$
$\frac{x + 100}{y + 100} = \frac{4}{3}$
$3(x + 100) = 4(y + 100)$
$3x - 4y = 100$ equation 1
$(x - 100) : (y - 100) = 11 : 7$
$\frac{x - 100}{y - 100} = \frac{11}{7}$
$7(x - 100) = 11(y - 100)$
$7x - 11y = -400$ equation 2
$21x - 28y = 700$ equation 1 × 7
$21x - 33y = -1200$ equation 2 × 3
$5y = 1900 \Rightarrow y = 380$
$x = 540$
Bike costs \$540, laptop costs \$380

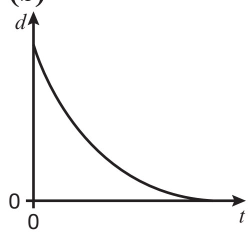
GRAPHS 3 – BASIC SKILLS EXERCISE

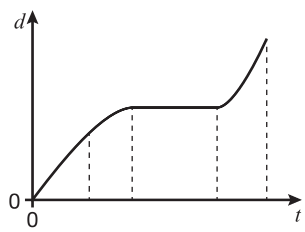
1 8 m/s
2 –4 m/s
3 **(a)** 15 m/s
(b) 4 s
(c) –9 m/s

4 **(a)**


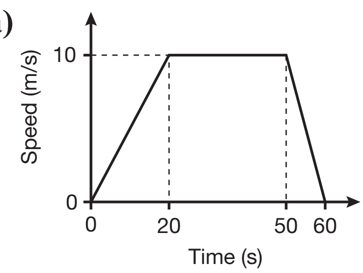
(b)


5 **(a)**


(b)


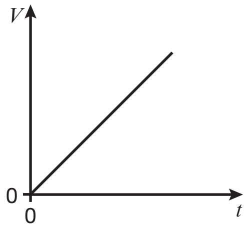
6


7 5 m/s²
8 −0.5 m/s²
9 **(a)** 3 m/s²
(b) 0 m/s²
(c) 2 m/s²
(d) 43.3 m/s

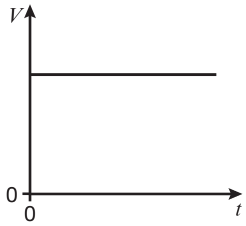
10 **(a)**


(b) **(i)** 0.5 m/s²
(ii) –1 m/s²
(iii) 7.5 m/s

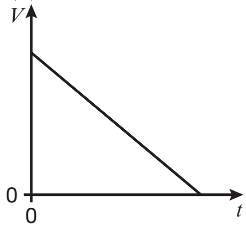
11 **(a)**



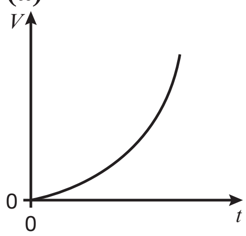
(b)



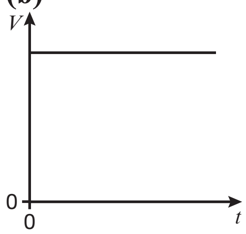
(c)



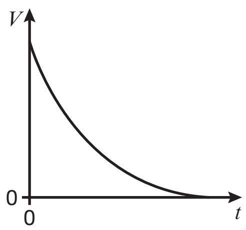
12 **(a)**



(b)



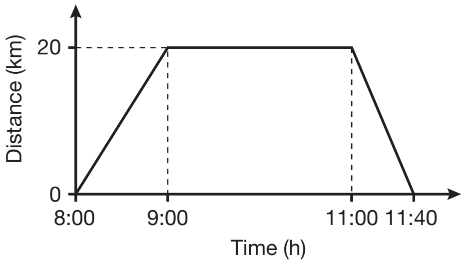
(c)



GRAPHS 3 – EXAM PRACTICE EXERCISE

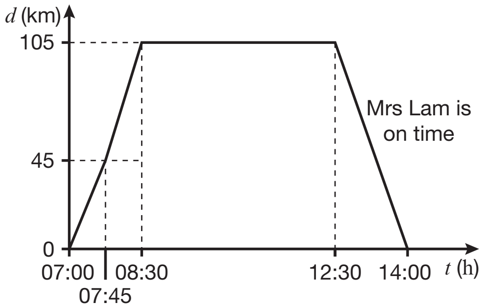
1 Gradient of a distance–time graph = velocity

(a)



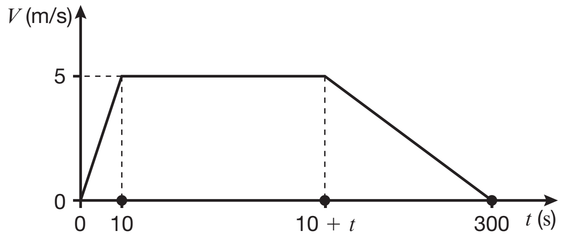
(b) 11: 40 (Return speed = 30 km/h, $30 = \frac{20}{t}$, $t = \frac{2}{3}$ hr = 40 min)

2



3 **(a)** Area under a speed–time graph = distance travelled

Gradient of a speed–time graph = acceleration



(b) **(i)** 1 km = 1000 m

Area under graph = Distance travelled

$1000 = \frac{1}{2}(t + 300)5$

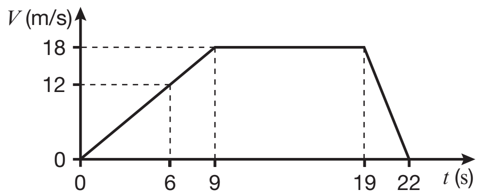
$400 = t + 300$

$t = 100$ s

(ii) Acceleration, $a = -\frac{5}{300 - 110}$

$a = -0.0263$ m/s² (3 s.f.)

4 **(a)**



(b) Initial acceleration = 2 m/s², so final retardation = 6 m/s²

(c) Mean speed of the hawk = $\frac{\text{area under the speed – time graph}}{\text{time of travel}}$

Area under graph = 0.5 × 6 × 12 + 0.5 × (12 + 18) × 3 + 0.5 × (10 + 13) × 18 = 288 m

Mean speed of the hawk = 288 ÷ 22 = 13.1 m/s

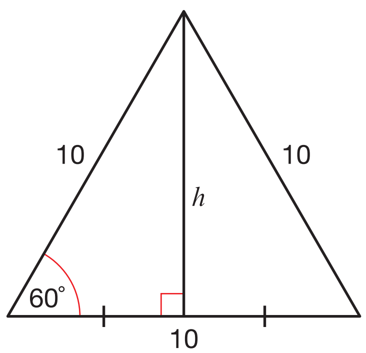
5 **(a)** Be careful to work in consistent units.
1km = 1000 m
1 hour = 3600 s
(i) 400 ÷ 62.5 = 6.4 m/s
(ii) $\frac{0.4 \times 60 \times 60}{62.5} = 23.04 \approx 23.0$ km/h
(b) Area under speed-time graph = 400 m
$400 = 0.5 \times (50.5 + 62.5) \times S_{max}$,
so $S_{max} = 7.08$ m/s
(c) Initial acceleration = gradient of first phase = (7.0796…) ÷ 12 = 0.58997… = 0.590 m/s^2

SHAPE AND SPACE 3 – BASIC SKILLS EXERCISE

1 $x = 8.39$ m, $y = 3.53$ m
2 $x = 12.1$ cm, $y = 5.12$ cm
3 $x = 6.53$ cm, $y = 1.55$ cm
4 $x = 34.6$ cm, $y = 29.1$ cm
5 $\theta = 43.6°$
6 $\theta = 5.20°$
7 $\theta = 59.6°$
8 $\theta = 16.1°$
9 71.6°
10 144 cm
11 $x = 7.71$, $y = 7.96$
12 $x = 14.3$, $y = 24.9$
13 $x = 17.3$, $y = 34.6$
14 $x = 17.3$, $y = 6$
15 $\theta = 16.8°$
16 $\theta = 59.5°$
17 $\theta = 50.2°$
18 $\theta = 11.0°$
19 3.95 m
20 **(a)** 4.5 km
(b) 2.25 km
(c) 3.90 km

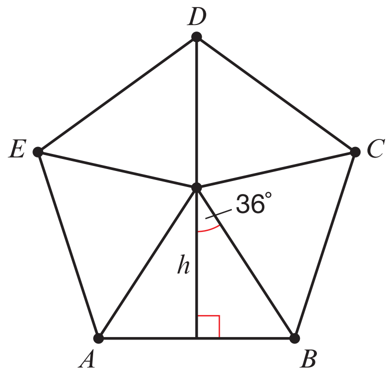
SHAPE AND SPACE 3 – EXAM PRACTICE EXERCISE

1 **(a)**



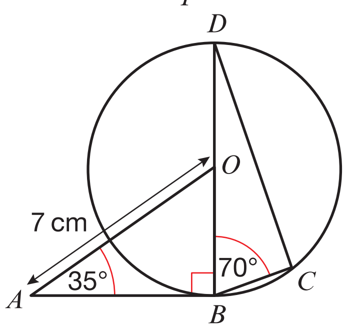
Area of triangle = $\frac{1}{2} \times 10 \times h$
$= \frac{1}{2} \times 10 \times (10 \times \sin 60°) = 25\sqrt{3}$
Area of circle = $\pi r^2 = 25\sqrt{3}$, $r^2 = \frac{25\sqrt{3}}{\pi}$
so $r = \frac{5\sqrt[4]{3}}{\sqrt{\pi}}$
Circumference of circle = $2\pi r = 2\pi \times \frac{5\sqrt[4]{3}}{\sqrt{\pi}} =$
$10\sqrt{\pi} \times \sqrt[4]{3} = 10 \times \pi^{\frac{1}{2}} \times 3^{\frac{1}{4}} = 10 \times \pi^a \times 3^b$
$a = \frac{1}{2}$, $b = \frac{1}{4}$

2 **(a)**



Angle at centre of pentagon = 360°, so angle of each triangle at centre of pentagon
$= \frac{360}{5} = 72°$
$AB = 2 \times 6.8 \times \sin(36°) = 7.9939…$cm
Perimeter = 5 × 7.9939 = 39.969…cm = 40.0 cm (3 s.f.)
(b) Let shaded area be A
$A = \frac{\text{area of circle} - \text{area of pentagon}}{5}$
$= \frac{\pi \times 6.8^2 - 5 \times 6.8 \times \sin(36°) \times 6.8 \times \cos(36°)}{5}$
$= 7.0650…\text{cm}^2$
$p = \frac{7.0650}{\pi \times 6.8^2} \times 100 = 4.8635…\% = 4.86\%$
The value of p is 4.86.

3



Triangle OAB:
$\sin(35°) = \frac{OB}{7}$, so $OB = 7 \times \sin(35°)$
= 4.0150…cm
$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$
Area of circle = $\pi \times 4.0150^2 = 50.6440…\text{cm}^2$
area of circle = πr^2

Area of triangle $BCD = \frac{1}{2} \times BC \times DC$

(Angle BCD = 90° as angles in a semi-circle are right-angles)

Triangle BCD:

$\sin(70°) = \frac{CD}{2 \times 4.01503}$,

so $CD = 2 \times 4.01503 \times \sin(70°) = 7.5458\ldots$cm

$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$

$\cos(70°) = \frac{BC}{2 \times 4.01503}$,

so $BC = 2 \times 4.01503 \times \cos(70°) = 2.7464\ldots$cm

$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$

Area of triangle BCD

$= \frac{1}{2} \times 2.7464 \times 7.5458 = 10.362\ldots \text{cm}^2$ or

Area $= \frac{1}{2} \times BD \times BC \times \sin 70°$

Area in circle and outside of triangle BCD

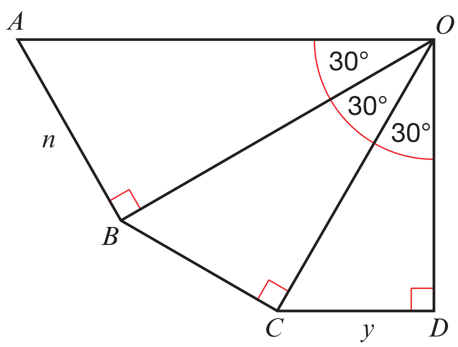
$= 50.644 - 10.362 = 40.282 \text{ cm}^2$

% of whole circle not occupied by triangle

$BCD = \frac{40.282}{50.644} \times 100 = 79.540\ldots\%$

So value of $p = 79.5$ (3 s.f.)

4



(a) Triangle OAB

$\tan 30° = \frac{n}{OB}$, so $OB = \frac{n}{\tan 30°} = \sqrt{3}n$

$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

$\tan 30° = \frac{1}{\sqrt{3}}$

Triangle OBC

$\cos(30°) = \frac{OC}{\sqrt{3}\,n}$,

so $OC = \cos 30° \times \sqrt{3}\,n = \frac{3}{2}n$

$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$

$\cos 30° = \frac{\sqrt{3}}{2}$

Triangle OCD

$\sin 30° = \frac{y}{\frac{3}{2}n}$, so $y = \frac{3}{2}n \times \sin 30° = \frac{3}{4}n$

as required.

$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$

$\sin 30° = \frac{1}{2}$

(b) Let area of the pentagon be A.

A = area of triangle OAB + area of triangle OBC + area of triangle OCD

Area of triangle OAB

$= \frac{1}{2} \times n \times 3\sqrt{n}$

$= \frac{\sqrt{3}}{2}n^2$

Area of triangle $OBC = \frac{1}{2} \times BC \times OC$

$= \frac{1}{2} \times (\sqrt{3}n \times \sin(30°)) \times \frac{3}{2}n = \frac{3\sqrt{3}}{8}n^2$

Area of triangle $OCD = \frac{1}{2} \times CD \times OD$

$= \frac{1}{2} \times \frac{3}{4}n \times \left(\frac{3}{2}n \times \cos(30°)\right) = \frac{9\sqrt{3}}{32}n^2$

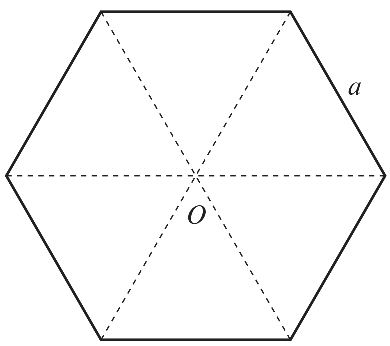
So $A = \frac{\sqrt{3}}{2}n^2 + \frac{3\sqrt{3}}{8}n^2 + \frac{9\sqrt{3}}{32}n^2$

$= \frac{16\sqrt{3}}{32}n^2 + \frac{12\sqrt{3}}{32}n^2 + \frac{9\sqrt{3}}{32}n^2$

$= \frac{37\sqrt{3}}{32}n^2$

If $A = \frac{37\sqrt{3}}{2} = \frac{37\sqrt{3}}{32}n^2$, $n^2 = 16$, $n = 4$

5



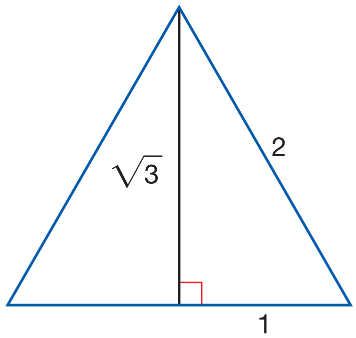
Pentagon $PQRSTU$ is regular so each triangle is equilateral.

Area of triangle $OST = \frac{1}{2} \times a \times (a \times \sin 60°)$

$= \frac{\sqrt{3}}{4}a^2$

Area of pentagon $= 6 \times \frac{\sqrt{3}}{4}a^2 = \frac{3\sqrt{3}}{2}a^2$ as required.

6



Pythagoras' theorem: Let height of equilateral triangle = h

$2^2 = h^2 + 1^2$, $h = \sqrt{3}$

If area of equilateral triangle = area of regular pentagon

$\frac{1}{2} \times 2 \times \sqrt{3} = \frac{3\sqrt{3}}{2} a^2$, $1 = \frac{3}{2} a^2$, $a^2 = \frac{2}{3}$,

$a = \sqrt{\frac{2}{3}}$

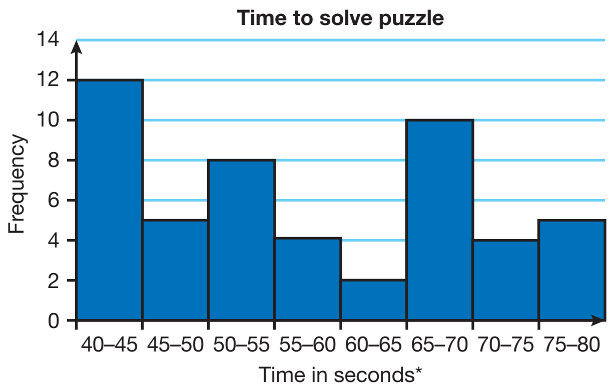
$P = 6a = 6 \times \sqrt{\frac{2}{3}} = 3 \times 2 \times \frac{\sqrt{2}}{\sqrt{3}} = \sqrt{3} \times 2 \times 2^{\frac{1}{2}}$

$= \sqrt{3} \times 2^{\frac{3}{2}} = \sqrt{3} \times 2^k$, so $k = \frac{3}{2}$

HANDLING DATA 2 – BASIC SKILLS EXERCISE

1 (a)

Time	Frequency
$40 \le t < 45$	12
$45 \le t < 50$	5
$50 \le t < 55$	8
$55 \le t < 60$	4
$60 \le t < 65$	2
$65 \le t < 70$	10
$70 \le t < 75$	4
$75 \le t < 80$	5

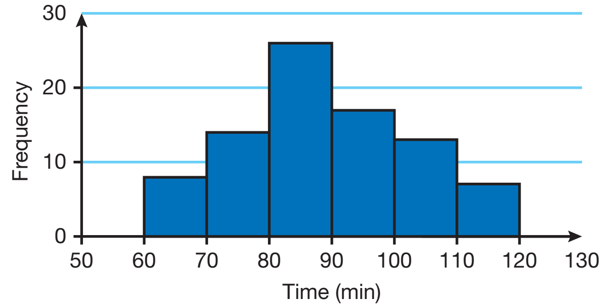


* Upper bounds are not included in groups

(b) 58%

2 (a) 85

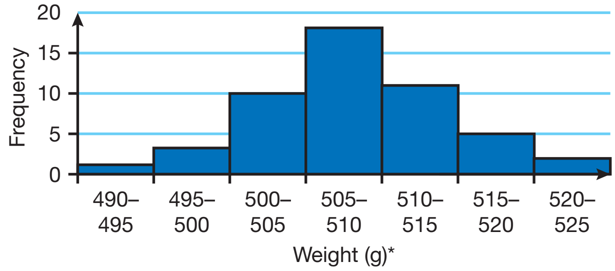
(b)



* Upper bounds are not included in groups

(c) 23.5%

3 (a)

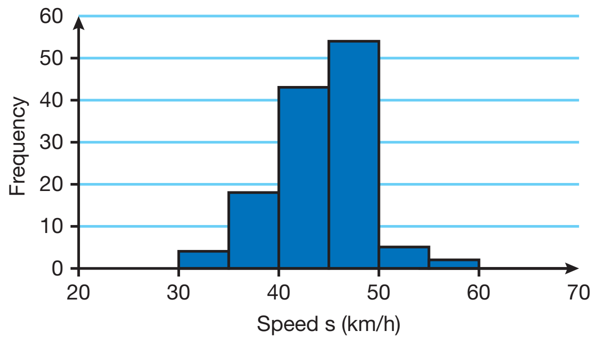


* Upper bounds are not included in groups

(b) Evidence suggests the mean is between 505 and 510 so 500 is probably minimum weight.

4 (a) 126

(b)

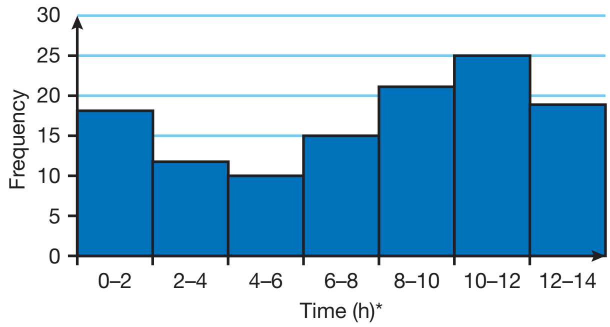


* Upper bounds are not included in groups

(c) Speed limit is probably 50 km/h as there is a sharp cut off at that speed.

5 (a) 15

(b)



* Upper bounds are not included in groups

(c) 38.3 – assuming that the 4 is included and not the 10

6 **(a)** 50 lessons
(b) 16–20
(c) 15.3

7 **(a)** 24
(b) $4 < m \le 6$
(c) 4.75 kg

8 **(a)** 25
(b) $17.5 < w \le 20.5$
(c) 17.9 minutes
(d) $17.5 < w \le 20.5$

9 **(a)** 32
(b) $14 \le w < 15$
(c) 14.3
(d) Decrease as 13 is below the mean

10 **(a)** mean = 1.69, median = 1.72
(b) new mean = 1.76

11 **(a)** 14.2 m
(b) 1.40 m

12 **(a)** mean = 7.45, median = 7.45
(b) new mean = 8.2

13 1.74 m

14 **(a)** 83 cm
(b) 9.37 cm

15 21 m 56 s

16 **(a)** 126 s
(b) 129 s

HANDLING DATA 2 – EXAM PRACTICE EXERCISE

1 Treat the number of calls as continuous data as we only have the data in class intervals.
(a) February is the only month with 28 days.
(b) 16–20 calls
(Modal class in the one with the highest frequency)
(c)

Number of Calls	Frequency (f)	Midpoint (x)	$f \times x$
1–5	2	3	$2 \times 3 = 6$
6–10	4	8	$4 \times 8 = 32$
11–15	7	13	$7 \times 13 = 91$
16–20	9	18	$9 \times 18 = 162$
21–25	6	23	$6 \times 23 = 138$
	$\Sigma f = 28$		$\Sigma fx = 429$

Estimated mean = $\frac{429}{28}$ = 15.3 calls
(Estimated mean = , where x is the mid-point of each class interval.)

2 Treat the length of words as continuous data as we only have the data in class-intervals
(a) 1000 students
(b) 601–800 words
(Modal class in the one with the highest frequency)
(c)

Number of Words	Frequency (f)	Midpoint (x)	$f \times x$
401–600	150	500.5	$150 \times 500.5 = 75075$
601–800	425	700.5	$425 \times 700.5 = 297712.5$
801–1000	350	900.5	$350 \times 900.5 = 315175$
1001–1200	75	1100.5	$75 \times 1100.5 = 82537.7$
	$\Sigma f = 1000$		$\Sigma fx = 770500.5$

Estimated mean = $\frac{770\,500.5}{1000}$ = 770.5 words
(Estimated mean = $\frac{\Sigma fx}{\Sigma f}$, where x is the mid-point of each class interval.)

3 **(a)** 540 Munros
(b) 3000–3300 ft
(Modal class in the one with the highest frequency)
(c)

Height (h feet)	Frequency (f)	Midpoint (x)	$f \times x$
$3000 < h \le 3300$	300	3150	$300 \times 3150 = 945000$
$3300 < h \le 3600$	135	3450	$135 \times 3450 = 465750$
$3600 < h \le 3900$	80	3750	$80 \times 3750 = 300000$
$3900 < h \le 4200$	20	4050	$20 \times 4050 = 81000$
$4200 < h \le 4500$	5	4350	$5 \times 4350 = 21750$
	$\Sigma f = 540$		$\Sigma fx = 1813500$

Estimated mean = $\frac{1\,813\,500}{540}$ = 3358.3. . .

= 3360 ft (nearest 10 ft)

(Estimated mean = $\frac{\Sigma fx}{\Sigma f}$, where x is the mid-point of each class interval.)

4 **(a)** $\sum f = 50 = x + 23 + p + 5 = 28 + x + p$, so $p = 22 - x$

(b)

Speed (s mph)	Frequency (f)	Midpoint (x)	$f \times x$
$90 < s \le 100$	x	95	$x \times 95 = 95x$
$100 < s \le 110$	23	105	$23 \times 105 = 2415$
$110 < s \le 120$	$22 - x$	115	$(22 - x) \times 115$
$120 < s \le 130$	5	125	$5 \times 125 = 625$
	$\sum f = 50$		$\sum fx = 5570 - 20x$

Estimated mean $= 107.8 = \frac{5570 - 20x}{50}$

$107.8 \times 50 = 5570 - 20x$, so $x = 9$

(Estimated mean $= \frac{\sum fx}{\sum f}$, where x is the mid-point of each class interval.)

5 **(a)**

(b)

Delay (d mins)	Midpoint (x)	Frequency (f)	$f \times x$
$0 \le d < 30$	15	10	$10 \times 15 = 150$
$30 \le h < 60$	45	14	$14 \times 45 = 630$
$60 \le h < 90$	75	16	$16 \times 75 = 1200$
$90 \le h < 120$	105	11	$11 \times 105 = 1155$
$120 \le h < 150$	135	8	$8 \times 135 = 1080$
$150 \le h < 180$	165	1	$1 \times 165 = 165$
		$\sum f = 60$	$\sum fx = 4380$

Estimated mean $= \frac{4380}{60} = 73$ mins

(Estimated mean $= \frac{\sum fx}{\sum f}$, where x is the mid-point of each class interval.)

(c) Estimate because midpoints used as there are no exact values.

(d) Median class interval is $60 \le d < 90$ as this is where the 30th value must be placed.

NUMBER 4 – BASIC SKILLS EXERCISE

1 **(a)** \$535
(b) \$572.45
(c) \$612.52
(d) \$655.40

2 **(a)** £104
(b) £108.16
(c) £121.67
(d) £148.02

3 **(a)** ฿26 250
(b) ฿27 562.50
(c) ฿31 907.04
(d) ฿66 332.44

4 **(a)** ¥41 000
(b) ¥38 088
(c) ¥29 658.67
(d) ¥19 547.48

5 **(a)** €3345.56
(b) 12 years

6 **(a)** €1343.92
(b) 14 years

7 20.2 years

8 ₹10 675

9 **(a)** $60 \times 0.94^{10} = 32.3$ km
(b) $60 \times 0.94^{30} = 9.38$ km

10 **(a)** $100 \times 0.98 = 98$ g
(b) $100 \times .98^5 = 90.39 \approx 90.4$ g
(c) $100 \times .98^{10} = 81.7$ g

11 $2 \times 0.9975^{60} = 1.72$ litres

12 0.5% monthly increase hence $1.005^{120} = 1.819$ so 81.9% increase which is not enough.
0.52% monthly increase gives $1.0052^{120} = 1.86$ so 86% increase which is enough.

13 €7366.96

14 **(a)** \$4255.70
(b) \$5978.95

15 €652.70

16 **(a)** £16 769.97
(b) £13 887.21

17 $\frac{150}{125} \times 100 = \$120$

18 $\frac{24}{120} \times 100 = \$20$

19 $\frac{2125}{85} \times 100 = €2500$

20 80 s

21 \$600

22 12

23 \$4329

24 €1811.59

25 £409.36

26 €2573.53

27 \$1283.76

28 \$888 889

29 €6863.56

30 £2326.24

31 Let Q be the factor of depreciation in the first year, then $Q - 0.1$ is the factor for the second year.
$60\,000(Q)(Q + 0.1) = 30\,000$
$60\,000Q(Q - 0.1) = 30\,000$
$Q^2 + 0.1Q - 0.5 = 0$
$Q^2 - 0.1Q - 0.5 = 0$
$Q = 0.75987...$
$Q \approx 0.76$
R = $(1 - 0.759...) \times 100 = 24.1\%$ (3 s.f.)

32 Final radius is $\sqrt{\frac{730}{\pi}} = 15.24356...$

Original area was $\frac{730}{1.15} = 634.7826...$

therefore original radius is $\sqrt{\frac{634.78...}{\pi}} = 14.2146959...$

Fractional increase in radius is $\frac{15.24356...}{14.21469...} = 1.07238...$ therefore

percentage increase is 7.24%

33 **(a)** $\frac{120\,000}{0.9875^{48}} = 219\,482$ so the lost area is $219\,482 - 120\,000 = 99\,500$ hectares

(b) $120\,000 \times 0.9875^{48} = 65\,608$ so the lost area is $120\,000 - 65\,608 = 54\,400$ hectares

34 **(a)** $\frac{412}{1.005^{20}} = 373$ ppm

(b) $412 \times 1.005^{20} = 455$ ppm

NUMBER 4 – EXAM PRACTICE EXERCISE

1 Total amount in account after 5 years $= \$50\,000 \times 1.035^5 = \$59\,384.32$
Interest after 35% deduction of interest $= (\$59\,384.32 - \$50\,000) \times 0.65 = \$6099.81$
Percentage gained from original investment $= \frac{6099.81}{50\,000} \times 100 = 12.2\%$ (3 s.f.)

2 Total amount in account after first 3 years $= €12\,000 \times 1.0325^3 = €13\,208.44$
Total amount in account after final 7 years $= €13\,208.44 \times 1.0225^7 = €15\,434.57$
Total interest gained after 10 years $= €15\,434.57 - €12\,000 = €3\,434.57$
Percentage gained from original investment $= \frac{3434.57}{12\,000} \times 100 = 28.6\%$ (3 s.f.)

3 Let Q be the factor of depreciation each year.
After three years' depreciation
$£50\,000 \times Q^3 = £25\,000$
So, $Q^3 = 0.5$, $Q = \sqrt[3]{0.5} = 0.7937...$
Therefore, % depreciation each year $= (1 - 0.793\,7) \times 100 = 20.6\%$ (3 s.f.)

4 **(a)** Let H be the height of the tree.
$H \times (1.075)^3 = 12$, so $H = \frac{12}{1.075^3}$
$= 9.66$ m (3 s.f.)
(b) $x \times 1.075 \times 1.05 \times 1.025 = 1.8$
$x = 1.56$ metres (3 s.f.)

5 **(a)** Let required price be £P
$P = £46\,800 \times (1 - 0.152) = £39\,686.40$
$P = £39\,686$ (nearest £)
(b) Let required price be €Q
18% = €3848, so 1% = €213.78,
so 100% = €21 377.78
$Q = €21\,378$ (nearest €)

ALGEBRA 4 – BASIC SKILLS EXERCISE

Note that answers can be correct but look different to the answer given. For example, the two answers given for Q1 and Q6 are just different rearrangements of the same expression.

1 $b - \frac{c^2}{a}$ or $\frac{ab - c^2}{a}$

2 $\frac{cd - b}{a}$

3 $\frac{a + c}{bd}$

4 $\sqrt{a(b+c)}$

5 $\sqrt{\frac{a}{c-b}}$

6 $\left(\frac{c}{a}+b\right)^2$ or $\left(\frac{ab+c}{a}\right)^2$

7 $\left(\frac{c}{a}\right)^2+b$

8 $\frac{c-f}{d+e}$

9 $\frac{a-bc}{c-1}$ or $\frac{bc-a}{1-c}$

10 $\frac{ab+cd}{a+c}$

11 $t(p^2-s)$

12 $\frac{rs}{r+s}$

13 $h=\frac{3V}{\pi r^2}$

14 $r=\sqrt[3]{\frac{3V}{4\pi}}$

15 $s=\frac{v^2-u^2}{2a}$

16 $h=\frac{A}{2\pi r}-r$

17 $a=\frac{s}{n}-\frac{d(n-1)}{2}$

18 $a=\frac{2(s-ut)}{t^2}$

19 $a=\frac{S(1-r)}{1-r^n}$

20 $x=a\sqrt{1-\frac{y^2}{b^2}}$ or $\frac{a}{b}\sqrt{b^2-y^2}$

21 $t=g\left(\frac{T}{2\pi}\right)^2$

22 $r=\sqrt{\frac{GmM}{F}}$

23 $d=\left(\frac{F}{k}\right)^3$

24 $r=\frac{6a}{5m^2-1}$

25 $b=\frac{2A}{a\sin C}$, $b=4$

26 $a=\frac{2A-bh}{h}$, $a=8$

27 $\sin A=\frac{a\sin B}{b}$, $A=48.6°$ (3 s.f.)

28 $\cos A=\frac{b^2+c^2-a^2}{2bc}$ or $\frac{a^2-b^2-c^2}{-2bc}$, $\cos A=0.7$

29 $v=\frac{fu}{u-f}$, $v=6\frac{2}{3}$

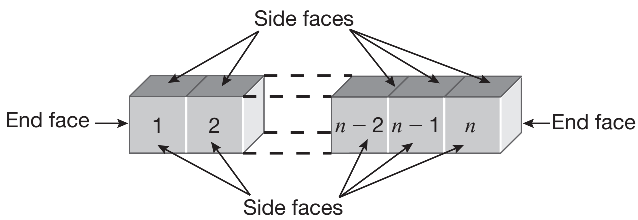
30 $c=\frac{b^2-(2ax+b)^2}{4a}$, $c=-3$

ALGEBRA 4 – EXAM PRACTICE EXERCISE

1 **(a)** Perimeter of room is 18 m so the wall area is $18 \times 2.6 = 46.8$ m²
$N = 2 + 0.4 \times 46.8 = 20.72$ so she needs to buy 21 rolls.

(b) Making A the subject of the formula:
$A=\frac{N-2}{0.4}$
Substituting $N = 15$ gives $A = 32.5$ m²
If A is just less than 32.5 m², say 32 m², Juan will still need 15 rolls as $A \le 32.5$ m²
Substituting $N = 14$ gives $A = 30$ m² and if A is greater than 30 then 15 rolls are needed.
$30\text{ m}^2 < A \le 32.5\text{ m}^2$

2 **(a)** The diagram shows n cubes joined together.



There are two end faces each with an area of 1 cm² so the end face area = 2 cm²
Each cube has 4 side faces exposed, each of area 1 cm² so each cube's side face area = 4 cm²
n cubes have a total side face area of $4n$ cm²
$A = 4n + 2$

(b) Making n the subject of the formula gives $n=\frac{A-2}{4}$
Substituting $A = 214$ gives $n = 53$

(c) n is an integer. In the formula $n=\frac{A-2}{4}$, if A is odd then $A - 2$ is also odd and an odd number divided by 4 (an even number) can never be an integer.

3 (a) Substitute $C = \frac{100N}{33}$ into $F = \frac{9C + 160}{5}$ gives

$$9C = 9 \times \frac{100N}{33} = \frac{300N}{11}$$

$$F = \frac{\frac{300N}{11} + 160}{5}$$

$$= \frac{60N}{11} + 32$$

$$= \frac{60N + 11 \times 32}{11}$$

$$= \frac{60N + 352}{11}$$

(b) Let the temperature where they read the same be T

then $T = \frac{60T + 352}{11}$

Substituting T for F and N

in $F = \frac{60N + 352}{11}$ gives

$11T = 60T + 352$
$49\ T = -352$
$T = -7.2$ (1 d.p.)

4 (a) $t = 2\pi\sqrt{\frac{l}{9.8}}$

$$\frac{t}{2\pi} = \sqrt{\frac{l}{9.8}}$$

$$\left(\frac{t}{2\pi}\right)^2 = \frac{l}{9.8}$$

$$l = 9.8\left(\frac{t}{2\pi}\right)^2$$

($l = \frac{9.8t^2}{4\pi^2}$ is also correct)

(b) Method 1:
Actual length $= 1.05 \times 9.8\left(\frac{1}{2\pi}\right)^2$
$= 0.26065$ m (to 5 s.f.)

$t = 2\pi\sqrt{\frac{0.26065}{9.8}} = 1.0247$ s (5 s.f.)

Increase is 0.0247 s $\Rightarrow$ % increase

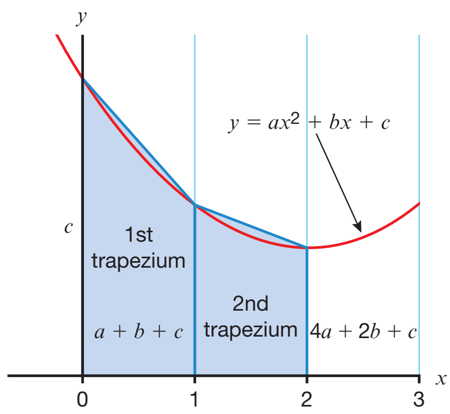
$= \frac{0.0247}{1} \times 100 = 2.47 \approx 2.5\%$ (2.s.f.)

Method 2: Let l be the length of the second pendulum; then the length of the manufactured pendulum is $1.05l$. Let t_1 and t_2 be the times of swings respectively (note $t_1 = 1$)

$$\frac{t_2}{t_1} = \left(2\pi\sqrt{\frac{1.05l}{9.8}}\right) \div \left(2\pi\sqrt{\frac{l}{9.8}}\right)$$

$= \sqrt{1.05}$
$= 1.0247$
So the % increase is 2.5% (2 s.f.)
1.025 is multiplier for 2.5% increase

5 (a) The heights of each trapezium are calculated by substituting $x = 0$, 1 and 2 respectively into $y = ax^2 + bx + c$



When $x = 0$, $y = c$
When $x = 1$, $y = a + b + c$
When $x = 2$, $y = 4a + 2b + c$

The area of a trapezium $= \frac{1}{2}(a + b)h$

In this case $h = 1$
Area of first trapezium
$= \frac{1}{2}(c + a + b + c)$

$= \frac{1}{2}(a + b + 2c)$

Area of second trapezium
$= \frac{1}{2}(a + b + c + 4a + 2b + c)$

$= \frac{1}{2}(5a + 3b + 2c)$

Total area
$= \frac{1}{2}(a + b + 2c + 5a + 3b + 2c)$

$= \frac{1}{2}(6a + 4b + 4c) = 3a + 2b + 2c$

(b) Difference in areas

$$= (3a + 2b + 2c) - \left(\frac{8a}{3} + 2b + 2c\right) = \frac{a}{3}$$

$$\text{Percentage error} = \frac{\frac{a}{3}}{\frac{8a}{3} + 2b + 2c} \times 100$$

$$= \frac{a}{8a + 6b + 6c} \times 100 = \frac{100a}{8a + 6b + 6c}\%$$

GRAPHS 4 – BASIC SKILLS EXERCISE

1

x	−2	−1	0	1	2	3
y	5	3	3	5	9	15

2

x	−2	−1	0	1	2	3
y	−3	−5	−5	−3	1	7

3

x	−3	−2	−1	0	1	2	3	4
y	10	7	6	7	10	15	22	31

4

x	−3	−2	−1	0	1	2	3	4
y	−4	−6	−6	−4	0	6	14	24

5

x	−3	−2	−1	0	1	2	3
y	10	4	0	−2	−2	0	4

$x = -1$ or 2

6

x	−4	−3	−2	−1	0	1	2	3	4
y	6	0	−4	−6	−6	−4	0	6	14

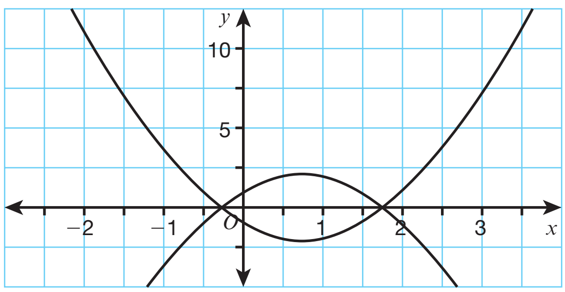
$x = -3$ or 2

7

x	−3	−2	−1	0	1	2	3	4	5
y	7	0	−5	−8	−9	−8	−5	0	7

$x = -2$ or 4

8 **a, b** $x \approx -0.3$ or 1.8 for both equations.



GRAPHS 4 – EXAM PRACTICE EXERCISE

1 (a)

t	0	1	2	3	4	5
y	0	5	20	45	80	125

(b) (i) 61 m

(ii) 2.2 s

2 Substitute $x = 0$ into formula for y to produce $y = 5$ and hence solve for $p = 5$.

(a) $p = 5$

(b)

x	0	1	2	3	4	5
y	5	0	−1	2	9	20

(c) $x = 1$ or 2.5

3 Substitute $x = 0$ and $y = -3$, $x = 4$ and $y = 13$ to produce simultaneous equations:

$-3 = q$

$13 = 32 + 4p + q$

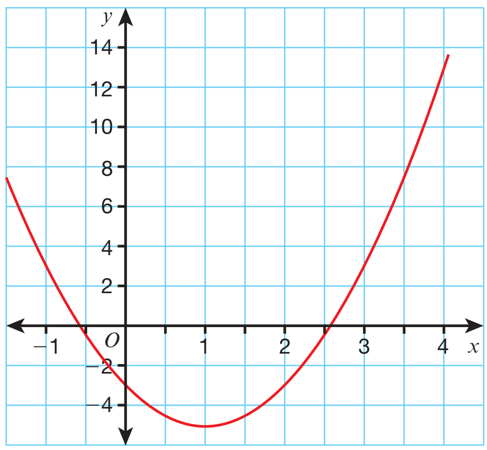
Solve them to find p and q.

(a) $p = -4$, $q = -3$

(b)

x	−2	−1	0	1	2	3	4
y	13	3	−3	−5	−3	3	13

(c) $x \approx -0.6$ or 2.6



4 Substitute $t = 1$ into formula for p to produce $p = 7$ to solve for $k = 3$.

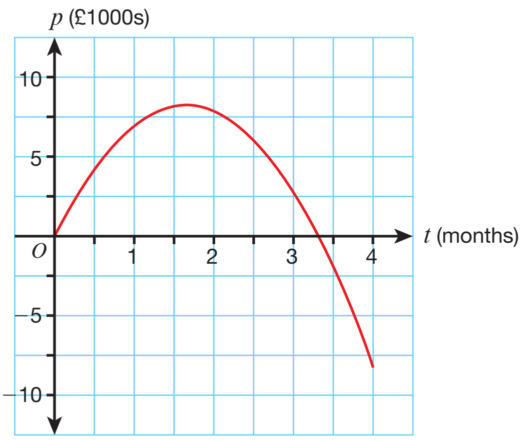
(a) $k = 3$

(b)

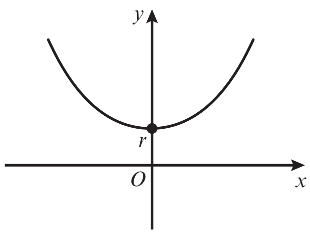
t	0	1	2	3	4
p	0	7	8	3	−8

(c) (i) £8333 @ $t = 1.7$ months

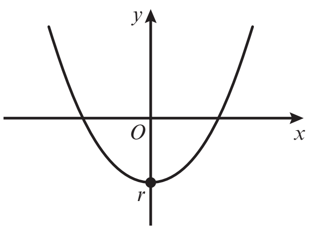
(ii) $t > 3.3$ months



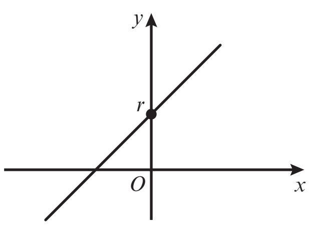
5 (a)



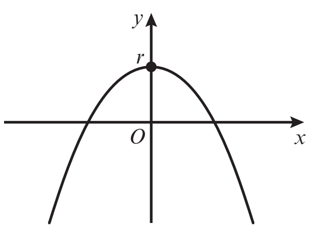
(b)



(c)



(d)



SHAPE AND SPACE 4 – BASIC SKILLS EXERCISE

1 (a) 22.4 cm
(b) 26.4 cm
(c) 32.1°

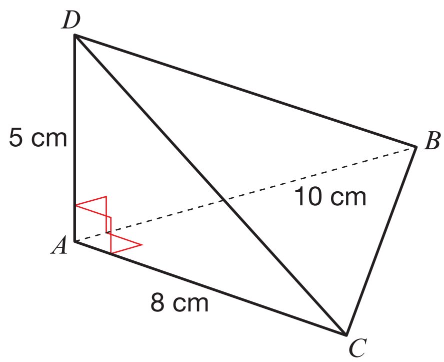
2 (a) 70.7 m
(b) 60.4 m
(c) 41.8°
(d) 9038 m^2

3 (a) 16.7°
(b) $DF = 94.3$ m
(c) 9.03°
(d) 2.19 m/s

4 79.2 m

5 Show by clear working that area = 600 cm^2, by Pythagoras' theorem.

6



The diagram shows a tetrahedron.
AD is perpendicular to both AB and AC.
$AB = 10$ cm. $AC = 8$ cm. $AD = 5$ cm.
Angle $BAC = 90°$
Let angle $BDC = \theta$
Area of triangle $BDC = \frac{1}{2} \times BD \times CD \times \sin(\theta)$

(Area of a triangle = $\frac{1}{2} ab\sin C$)

Triangle ACD:
$CD^2 = 5^2 + 8^2 = 89$, $CD = 9.4340\ldots$cm
Triangle ABD:
$BD^2 = 5^2 + 10^2 = 125$, $BD = 11.180\ldots$cm
Triangle ABC:
$BC^2 = 8^2 + 10^2 = 164$, $BD = 12.806\ldots$cm
Triangle BCD:
(Cosine Rule : $a^2 = b^2 + c^2 - 2bc \cos A$)
$164 = 125 + 89 - 2 \times 11.180 \times 9.4340 \times \cos\theta$
(Rearrange to make $\cos\theta$ the subject and find θ)
$\cos\theta = 0.23703...$, so $\theta = 76.289...°$

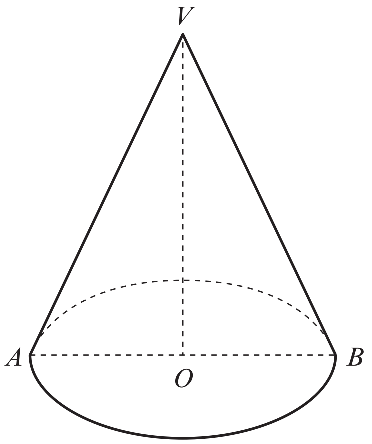
Area of triangle BDC

$= \frac{1}{2} \times 11.180 \times 9.4340 \times \sin(76.289°)$

$= 51.2 \text{ cm}^2$ (3 s.f.)

SHAPE AND SPACE 4 – EXAM PRACTICE EXERCISE

1 **(a)**



(Total surface area of solid cone = Area of circular base + curved surface area) $= \pi r^2 + \pi rl$

$75\pi = \pi \times 5^2 + \pi \times 5 \times l$

$50\pi = 5\pi l$, $l = 10$ cm

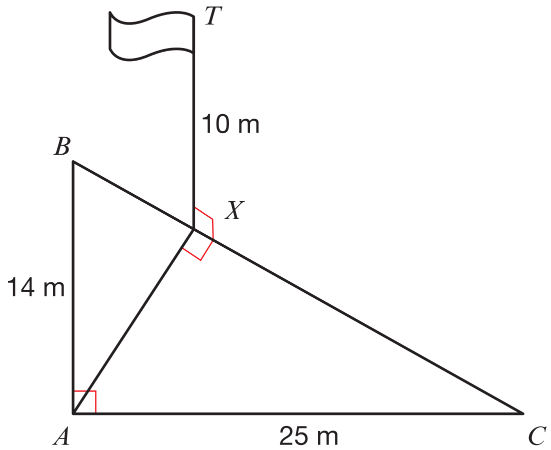
(b) Let angle $AVB = 2\theta$

Triangle OVB:

$\sin(\theta) = \frac{OB}{VB} = \frac{5}{10}$ $\left(\cos\theta = \frac{\text{opposite side}}{\text{hypotenuse}}\right)$

$\theta = 30°$, so angle $AVB = 60°$

2



Triangle ABC:

Let angle $ACX = \theta$

$\tan(\theta) = \frac{AB}{AC} = \frac{14}{25}$ $\left(\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}\right)$

$\theta = 29.249...°$

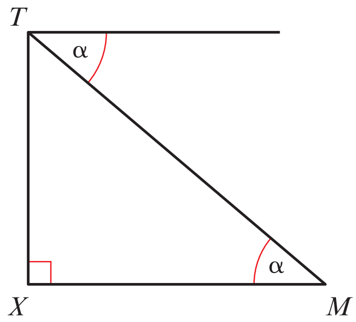
Triangle AXC:

$\cos(\theta) = \frac{XC}{AC} = \frac{XC}{25}$ $\left(\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}\right)$

$XC = 21.813...$m

$XM : MC = 3 : 1 = 16.360 : 5.453$

Triangle TXM:



Calculate angle $XMT = \alpha$

(Alternate angles)

$\tan(\alpha) = \frac{TX}{XM} = \frac{10}{16.360}$

$\left(\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}\right)$

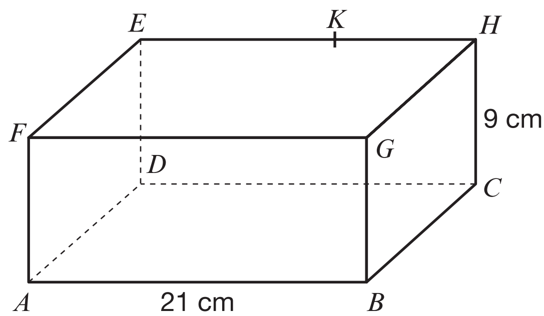
$\alpha = 31.435...° = 31.4°$ (3 s.f.)

But angle of depression is angle

$XTM = 45° - 31.4° = 13.6°$

This could have been calculated immediately using tan ( ) $= \frac{XM}{TX}$

3 **(a)**



$EK : KH = 2 : 1 = 14 : 7$, so $EK = 14$ cm, $KH = 7$ cm

$BC : CH = 4 : 3 = 12 : 9$, so $BC = 12$ cm

Drop a perpendicular line from K to DC to point L.

Triangle BCL:

$LB^2 = 7^2 + 12^2$, $LB = 13.892\ldots$cm

Triangle KLB:

$KB^2 = LB^2 + KB^2$, $KB^2 = 13.892^2 + 9^2$,

$KB = 16.553\ldots$cm

Triangle ADL:

$LA^2 = 14^2 + 12^2$, $LA = 18.439\ldots$cm
Triangle KLA:
$KA^2 = LA^2 + KL^2$, $KA^2 = 18.439^2 + 9^2$,
$KA = 20.518\ldots$cm
Now consider triangle AKB:
Let angle $AKB = \theta$
(Cosine rule : $a^2 = b^2 + c^2 - 2bc\cos A$)
$21^2 = 20.518^2 + 16.553^2 - 2 \times 20.518 \times 16.553 \times \cos\theta$
(Re-arrange to make $\cos\theta$ the subject and find θ)
$\cos\theta = 0.37392...$, so $\theta = 68.043°$, so angle $AKB = 68.0°$ (3 s.f.)

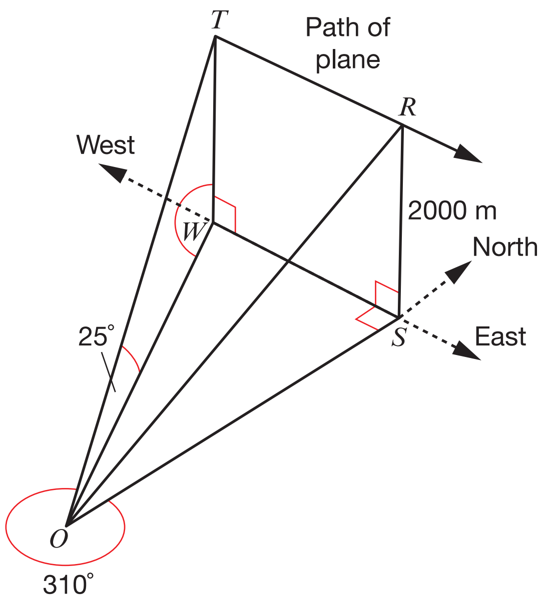
(b) Let required angle $KAL = \alpha$
Triangle KAL:

$\tan(\alpha) = \frac{KL}{AL} = \frac{9}{18.439}$

$\left(\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}\right)$

$\alpha = 26.017...°$, so angle $KAL = 26.0°$ (3 s.f.)

4



(a) Triangle OWT:

$\tan(25°) = \frac{TW}{OW} = \frac{2000}{OW}$

$\left(\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}\right)$

$OW = \frac{2000}{\tan(25°)} = 4289.0...$m

$= 4290$ m (3 s.f.)
Triangle OWS:
Angle $WOS = 360 - 310 = 50°$
(Plan view on base triangle OWS)

$\cos(50°) = \frac{OS}{OW} = \frac{OS}{4289.0...}$

$\left(\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}\right)$

$OS = 4289.0 \times 4\cos(50°) = 2757.0...$ m
$= 2760$ m (3 s.f.)

(b) Triangle ROS:

Let required angle $ROS = \theta$

$\tan(\theta) = \frac{RS}{OS} = \frac{2000}{2757.0}$

$\left(\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}\right)$

$\theta = 35.958... = 36.0°$ (3 s.f.)

(c) Speed $= \frac{\text{distance}}{\text{time}} = \frac{WS}{1} = ...$m/min

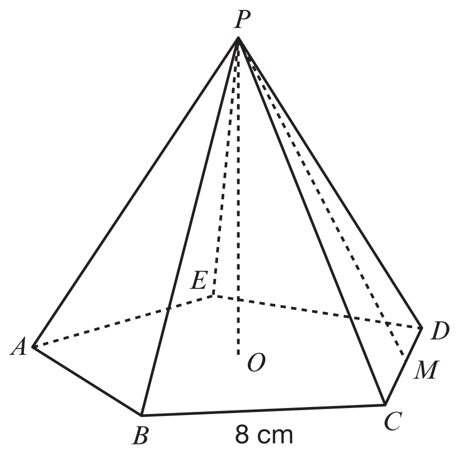
(Units required to be in km and hours to give an answer of speed in km/h)
Triangle SOW:
$OW^2 = OS^2 + SW^2$
(Pythagoras' theorem)
$4289.0^2 = 2757.0^2 + SW^2$

$SW = \sqrt{4289.0^2 - 2757.0^2} = 3285.5...$m

Speed $= \frac{3.2855}{\frac{1}{60}} = 197.13... \frac{\text{km}}{\text{hr}}$

$= 197$ km/h (3 s.f.)

5 **(a)**



The base of the pyramid is a regular pentagon, so angle $DOC = \frac{360°}{5} = 72°$

Triangle OCM:

$\tan(36°) = \frac{CM}{OM} = \frac{4}{OM}$

$\left(\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}\right)$

$OM = \frac{4}{\tan(36°)} = 5.5055...\text{cm}$

Triangle *POM*:

Required angle is $PMO = \alpha$

$\tan(\alpha) = \frac{PO}{OM} = \frac{10}{5.5055} = 1.8164...,$

so angle $PMO = 61.2°$ (3 s.f.)

(b) Weight of pyramid + weight of water = 1000 g

Area of pyramid base

= 5 × area of triangle *ODC*

$= 5 \times \frac{1}{2} \times 8 \times 5.5055$

$= 110.11...\text{cm}^2$

(Volume of pyramid

$= \frac{1}{3} \times$base area × perpendicular height)

Volume of pyramid $= \frac{1}{3} \times 110.11 \times 10$

$= 367.03...\text{cm}^3$

$\left(\text{Density} = \frac{\text{Mass}}{\text{Volume}}\right)$

$1000\,\text{kg/m}^3 = 1\ \text{g/cm}^3$

Weight of water = density × volume

= 1 × 367.03 g.

So $w + 367.03 = 1000$

(working in units of g)

$w = 633$ g (3 s.f.)

HANDLING DATA 3 – BASIC SKILLS EXERCISE

1

Median	Q_1	Q_3	Range	IQR
6	1	7	8	6

2

Median	Q_1	Q_3	Range	IQR
5	1	6	9	5

3

Median	Q_1	Q_3	Range	IQR
4	1.5	7	11	5.5

4

Median	Q_1	Q_3	Range	IQR
62	48	83	69	35

5

Median	Q_1	Q_3	Range	IQR
6	5	8	9	3

6

Median	Q_1	Q_3	Range	IQR
3.9	2.1	9.0	9	6.0

7

Median	Q_1	Q_3	Range	IQR
67	42	77	84	35

8

Median	Q_1	Q_3	Range	IQR
0.56	0.46	0.68	0.7	0.22

9

Median	Q_1	Q_3	Range	IQR
2	1	3	5	2

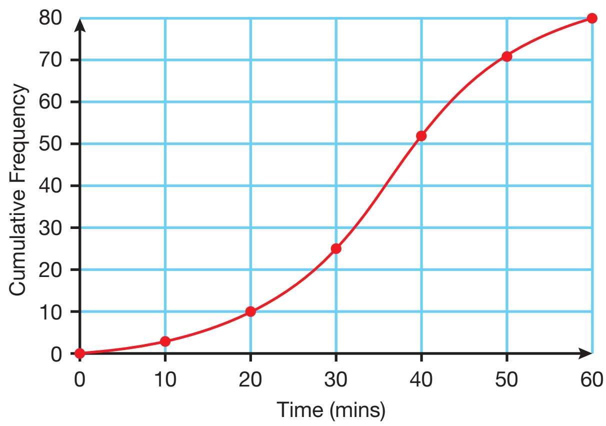
10

Median	Q_1	Q_3	Range	IQR
2	1	2.5	5	1.5

11 (a)

Time (t mins)	C.F.
$0 < t \leq 10$	3
$10 < t \leq 20$	10
$20 < t \leq 30$	25
$30 < t \leq 40$	52
$40 < t \leq 50$	71
$50 < t \leq 60$	80

(b)



(c) $Q_2 = 36$, $Q_1 = 27$, $Q_3 = 43$, IQR = 16

(d) 50 students

12 (a)

Speed s (m.p.h)	C.F.
$s \leq 55$	0
$55 < s \leq 60$	6
$60 < s \leq 65$	25
$65 < s \leq 70$	71
$70 < s \leq 75$	85
$75 < s \leq 80$	90

(c) $Q_2 = 67$, $Q_1 = 64.5$, $Q_3 = 69.5$, IQR = 5

(d) 18%

13 (a)

Weight w (kilograms)	Country A Cum. freq.	Country B Cum. freq.
$w \le 2.0$	0	0
$2.0 < w \le 2.5$	14	0
$2.5 < w \le 3.0$	43	3
$3.0 < w \le 3.5$	66	23
$3.5 < w \le 4.0$	80	74
$4.0 < w \le 4.5$	80	80

(c) Country A: $Q_2 = 2.95$, $Q_1 = 2.62$, $Q_3 = 3.37$, IQR = 0.75
Country B: $Q_2 = 3.65$, $Q_1 = 3.45$, $Q_3 = 3.78$, IQR = 0.33

(d) Babies heavier in country B, more variation in weight in country A

HANDLING DATA 3 – EXAM PRACTICE EXERCISE

1 Median = 23.5, $Q_1 = 17.5$, $Q_3 = 31$, range = 28, IQR = 13.5

2 (a)

Weight w (g)	Cum. freq.
$66 < w \le 68$	5
$68 < w \le 70$	18
$70 < w \le 72$	36
$72 < w \le 74$	46
$74 < w \le 76$	54
$76 < w \le 78$	60

(c) $Q_2 = 71.4$, IQR = 4.1
(d) 15%

3 (a)

Time t (min)	Cum. freq.
$0 < t \le 20$	3
$20 < t \le 40$	10
$40 < t \le 60$	21
$60 < t \le 80$	31
$80 < t \le 100$	52
$100 < t \le 120$	77
$120 < t \le 140$	100

(c) $Q_2 = 98$, $Q_1 = 68$, $Q_3 = 119$, IQR = 51
(d) 35%

4 (a)

Time t (milliseconds)	Cum. freq. before drink	Cum. freq. after drink
$t \le 160$	0	0
$160 < t \le 180$	10	0
$180 < t \le 200$	45	0
$200 < t \le 220$	76	8
$220 < t \le 240$	80	49
$240 < t \le 260$	80	74
$260 < t \le 280$	80	80

(c) Before drink $Q_2 = 197$, $Q_1 = 189$, $Q_3 = 206$, IQR = 17

After drink $Q_2 = 237$, $Q_1 = 229$, $Q_3 = 245$, IQR = 16

(d) Drink lengthens reaction times by approximately 40 milliseconds, but doesn't after the spread.
This possibly means that everybody is equally affected.

5 (a)

Diameter d (cm)	Frequency Short Wood	Frequency Waley Wood
$0 < d \le 10$	6	2
$10 < d \le 20$	15	5
$20 < d \le 30$	27	12
$30 < d \le 40$	50	50
$40 < d \le 50$	71	92
$50 < d \le 60$	85	98
$60 < d \le 70$	95	100
$70 < d \le 80$	100	100

(c) Short Wood $Q_2 = 40$, $Q_1 = 29$, $Q_3 = 53$, IQR = 24

Waley Wood $Q_2 = 40$, $Q_1 = 36$, $Q_3 = 46$, IQR = 10

(d) Waley Wood is much more uniform in size. This possibly means it is a plantation with all the trees having been planted at the same time. Short Wood is more diverse in size, possibly a wild wood.

NUMBER 5 – BASIC SKILLS EXERCISE

1 1×10^8

2 4×10^{13}

3 5×10^{15}

4 1×10^{11}

5 2×10^2

6 2×10^3

7 5×10^7

8 5×10^8

9 3×10^{14}

10 3×10^{15}

11 1×10^3

12 2×10^{10}

13 2×10^6

14 4×10^{-3}

15 2×10^{-12}

16 5×10^7

17 2×10

18 8×10^{-3}

19 upper bound = 3.52, lower bound = 2.93

20 upper bound = 4.08, lower bound = 3.15

21 upper bound = 2.33, lower bound = 1.73

22 upper bound = 1.71, lower bound = 1.50

23 upper bound = 2.29, lower bound = 2.15

24 upper bound = 0.159, lower bound = 0.09832

25 upper bound = 3.38, lower bound = 2.59

26 **(a)** upper bound = 15.9 m^2, lower bound = 9.62 m^2

(b) upper bound = 14.1 m, lower bound = 11.0 m

27 upper bound = 12.1, lower bound = 11.7

28 upper bound = 104, lower bound = 78

29 upper bound = 17 N/cm^2, lower bound = 15 N/cm^2

30 upper bound = 75 000 mm^2, lower bound = 64 000 mm^2

NUMBER 5 – EXAM PRACTICE EXERCISE

1 $a_{max} = 2.5$ m, $b_{max} = 100.5$ m, $\alpha_{max} = 40.5°$
$a_{min} = 1.5$ m, $b_{min} = 99.5$ m, $\alpha_{min} = 39.5°$

(a) upper bound of h

$= a_{max} + (b_{max}\ x \tan(\alpha_{max}))$

$\left(\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}\right)$

$= 2.5 + 100.5 \times \tan(40.5°) = 88.335\ldots$m
$= 88.3$ m (3 s.f.)

(b) lower bound of h

$= a_{min} + (b_{min} \times \tan(\alpha_{max}))$

$\left(\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}\right)$

$= 1.5 + 99.5 \times \tan(39.5°) = 83.521\ldots$m
$= 83.5$ m (3 s.f.)

2 Let area of the base be A cm^2 and force be F N.
$A_{max} = 12.5 \times 12.5 = 156.25$ cm^2, $F_{max} = 55$ N
$A_{min} = 11.5 \times 11.5 = 132.25$ cm^2, $F_{min} = 45$ N

(a) upper bound of $P = \frac{F_{max}}{A_{min}} = \frac{55}{132.25}$

$= 0.41587\ldots$N/cm^2

$= 0.416$ N/cm^2

(b) lower bound of $P = \frac{F_{min}}{A_{max}} = \frac{45}{156.25}$

$= 0.288$ N/cm^2

3 $d_{max} = 2.55$ m, $t_{max} = 2.75$ s
$d_{min} = 2.45$ m, $t_{min} = 2.25$ s

(speed $= \frac{\text{distance}}{\text{time}}$, Circumference of a circle $= 2\pi r$)

(a) upper bound of V $= \frac{d_{max}}{t_{min}} = \frac{2 \times \pi \times 0.5 \times 2.55}{2.25}$

$= 3.5604\ldots$m/s $= 3.6$ m/s (2 s.f.)

(b) lower bound of V $= \frac{d_{min}}{t_{max}} = \frac{2 \times \pi \times 0.5 \times 2.45}{2.75}$

$= 2.7988\ldots$m/s $= 2.8$ m/s (2 s.f.)

4 $R_{max} = 12.85$ cm, $r_{max} = 10.35$ cm
$R_{min} = 12.75$ cm, $r_{min} = 10.25$ cm
(Area of a circle $= \pi r^2$)

$A_{max} = \pi R_{max}^2 - \pi r_{min}^2 = \pi(R_{max}^2 - r_{min}^2)$
$= \pi(12.85^2 - 10.25^2) = 60.06\pi$ cm^2

$A_{min} = \pi R_{min}^2 - \pi r_{max}^2 = \pi(R_{min}^2 - r_{max}^2)$
$= \pi(12.75^2 - 10.35^2) = 55.44\pi$ cm^2

So, if $p\pi \leq A < q\pi$
$55.44\,\pi \leq A < 60.06\pi$
$p = 55$, $q = 60$ (2 s.f.)

5 $a_{max} = 12.55$, $b_{max} = 3.255$, $c_{max} = 1.755$ and $d_{max} = 3.855$
$a_{min} = 12.45$, $b_{min} = 3.245$, $c_{min} = 1.745$ and $d_{min} = 3.845$

(a) $Z_{max} = \frac{a_{max} - b_{min}}{c_{min} + d_{min}} = \frac{12.55 - 3.245}{1.745 + 3.845}$
$= 1.6645$, upper bound of $T = 10^{Z_{max}}$
$= 46$ (2 s.f.)

(b) $Z_{min} = \frac{a_{min} - b_{max}}{c_{max} + d_{max}} = \frac{12.45 - 3.255}{1.755 + 3.855}$
$= 1.6390\ldots$, lower bound of $T = 10^{Z_{min}}$
$= 44$ (2 s.f.)

ALGEBRA 5 – BASIC SKILLS EXERCISE

1 $x^2 - 3x - 10$

2 $x^2 + 16x + 64$

3 $x^4 - 10x^2 + 25$

4 $8x^2 + 2x - 6$

5 $10x^3 + x^2 - 2x$

6 $x^3 - 4x^2 - 3x + 18$

7 $9x^3 - 18x^2 - 25x + 50$

8 $8x^3 - 36x^2 + 54x - 27$

9 $x^2 - 5$

10 -9

For questions 11–14, there is no need to multiply out the brackets.

11 $(x^2 + 3)$ is a common factor.
$(x^2 + 3)(2x + 1) + (x^2 + 3)(1 - 2x)$
$= (x^2 + 3)\,[2x + 1 + 1 - 2x] = 2(x^2 + 3)$

12 $(4x + 1)$ is a common factor.
$(4x + 1)^2 - (4x + 1)(x + 1)$
$= (4x + 1)\,[(4x + 1) - (x + 1)]$
$= (4x + 1)\,[4x + 1 - x - 1] = 3x(4x + 1)$

13 $(4x + 3)$ is a common factor.
$\pi(4x + 3) - 3(4x + 3) = (4x + 3)(\pi - 3)$

14 $(1 - \cos x)$ is a common factor.
$2x(1 - \cos x) - 3\,(1 - \cos x)$
$= (1 - \cos x)\,(2x - 3)$

15 $(x - 3)(x - 4) = 0$, $x = 3$ or 4

16 $(x - 1)(x + 9) = 0$, $x = 1$ or -9

17 $x(x - 11) = 0$, $x = 0$ or 11

18 $(x - 4)(x + 5) = 0$, $x = 4$ or -5

19 $(3x - 5)(x + 2) = 0$, $x = \frac{5}{3}$ or -2

20 $(4x + 3)(x - 2) = 0$, $x = -\frac{3}{4}$ or 2

21 $(2x - 3)(3x - 2) = 0$, $x = \frac{3}{2}$ or $-\frac{2}{3}$

22 $(5x - 8)(2x + 5) = 0$, $x = \frac{8}{5}$ or $-\frac{5}{2}$

23 $(x - 4)(x + 6) = 0$, $x = 4$ or -6

24 $2(x - 1)(x + 3) = 0$, $x = 1$ or -3

25 $2(2x + 1)(x - 5) = 0$, $x = \frac{1}{2}$ or 5

26 $2x(2x + 3)(x + 2) = 0$, $x = 0, -\frac{3}{2}$ or -2

27 $(x^2 - 9)(x^2 - 4) = 0$, $x = \pm 3$ or ± 2

28 $7(x + 2) = 0$ so $x = -2$

29 $(x - 4)(x - 1)$ so $x = 4$ or $x = 1$

30 $(3x + 1)(2x - 1) = 0$ so $x = \frac{1}{2}$ or $-\frac{1}{3}$

31 $\frac{1}{2} \times 2x \times x = 3x(x - 3)$

The areas are the same.
$x^2 = 3x^2 - 9x$
$2x^2 - 9x = 0$
$x(2x - 9) = 0$
$x = 0$ or 4.5
$x = 0$ is not a real-life solution so $x = 4.5$

32 $x(x + 2) = 15$
$x^2 + 2x - 15 = 0$
$(x + 5)(x - 3)$
$x = 3$
$x = -5$ is not a real-life solution

33 The coconut hits the ground when $x = 14$ so
$5t^2 + 3t = 14$
$5t^2 + 3t - 14 = 0$
$(5t - 7)(t + 2) = 0$ so $t = \frac{7}{5}$
$t = -2$ is not a real-life solution

34 Let one integer be x, then the other integer is $x + 4$ ($x - 4$ is also correct – see later)
$x^2 + (x + 4)^2 = 208$
$x^2 + x^2 + 8x + 16 = 208$
$2x^2 + 8x - 192 = 0$
$x^2 + 4x - 96 = 0$
$(x + 12)(x - 8) = 0$
$x = -12$ or $8 \Rightarrow$ numbers are -12 and -8 or 8 and 12
If using $x - 4$:
$x^2 + (x - 4)^2 = 208$
$x^2 + x^2 - 8x + 16 = 208$
$2x^2 - 8x - 192 = 0$
$x^2 - 4x - 96 = 0$
$(x - 12)(x + 8) = 0$
$x = 12$ or $-8 \Rightarrow$ numbers are 12 and 8 or -8 and -12

35 $(4x - 3)^2\ (x + 1)^2 + (2x + 4)^2$ Pythagoras' theorem
$16x^2 - 24x + 9 = x^2 + 2x + 1 + 4x^2 + 16x + 16$
$11x^2 - 42x - 8 = 0$
$(11x + 2)(x - 4) = 0$
$x = 4$ as $x = -\frac{2}{11}$ is not a real-life solution
triangle sides are 5, 12 and 13 so area is $\frac{1}{2} \times 5 \times 12 = 30\text{ cm}^2$

36 **(a)** $(a + b)(a - b)$
(b) $2^{24} - 1^2 = (2^{12} + 1)(2^{12} - 1)$
so suitable integers are $2^{12} + 1 = 4097$ and $2^{12} - 1 = 4095$

ALGEBRA 5 – EXAM PRACTICE EXERCISE

1 Internal angle sum of a quadrilateral is 360°
$8x^2 - 32 + 22x - 16 + 20x + 4 + 6x^2 + 12 = 360$
$14x^2 + 42x - 392 = 0$
$x^2 + 3x - 28 = 0$ Dividing by 14
$(x - 4)(x + 7) = 0 \Rightarrow x = 4$
$x = -7$ is not a real-life solution
angles are $A = 96°$, $B = 72°$, $C = 84°$ and $D = 108°$
it is cyclic as $A + C = 180°$ or $B + D = 180°$

2 **(a)** $\frac{1}{2}(3x + 5 + x + 1) \times (x + 3) = 35$
See formula sheet for area of a trapezium.
$(2x + 3)(x + 3) = 35$
$2x^2 + 9x - 26 = 0$
$(x - 2)(2x + 13) = 0$
$x = 2$ $x = -\frac{13}{2}$ is not a real-life solution.
(b) $(x - 2)^2 + (2x + 6)^2 = (3x - 2)^2$ Pythagoras' theorem.
$x^2 - 4x + 4 + 4x^2 + 24x + 36 = 9x^2 - 12x + 4$
$4x^2 - 32x - 36 = 0$
$x^2 - 8x - 9 = 0$ Dividing equation by 4
$(x - 9)(x + 1) = 0$
$x = 9$ $x = -1$ is not a real-life solution
Sides are 7, 24 and 25 so perimeter is 56 cm

3 **(a)** $(x + 2)(x + a) = x^2 + px + 6$
$\Rightarrow x^2 + ax + 2x + 2a = x^2 + px + 6$
$2a = 6$
$a = 3$
factors are $(x + 2)$ and $(x + 3)$
$(x + 2)(x + 3) = x^2 + 5x + 6$
$p = 5$
(b) **(i)** $(2x + 3)(x - 7)$
(ii) $2\left(x + \frac{1}{2}\right)^2 - 11\left(x + \frac{1}{2}\right) = 21$
$2\left(x + \frac{1}{2}\right)^2 - 11\left(x + \frac{1}{2}\right) - 21 = 0$
Compare this with $2x^2 - 11x - 21 = 0$
so replace x by $\left(x + \frac{1}{2}\right)$ in b.
This gives $\left(2\left(x + \frac{1}{2}\right) + 3\right)\left(\left(x + \frac{1}{2}\right) - 7\right) = 0$
$(2x + 4)\left(x - 6\frac{1}{2}\right) = 0$
$x = -2$ or $x = 6\frac{1}{2}$

4 **(a)** $(x - 5)\left(x + \frac{2}{3}\right) = 0$
$(x - 5)(3x + 2) = 0$ Multiplying by 3
$3x^2 - 13x - 10 = 0$ Any multiple of this equation is correct

(b) To have one solution, when factorised the equation must be $(x - a)(x - a) = 0$
$(x - a)^2 = 0$
$x^2 - 2ax + a^2 = 0$
$2a = 6$
$a = 3$
$p = 9$
equation is $(x - 3)^2 = 0$ so the solution is $x = 3$

5 **(a)** Let x be the number of jars she bought so each jar costs $\frac{2000}{x}$ cents.

In the other shop, each jar costs $\frac{2000}{x} - 20$ and she could have bought $x + 5$ jars.

$(x + 5)\left(\frac{2000}{x} - 20\right) = 2000$

$2000 - 20x + \frac{10000}{x} - 100 = 2000$

$\frac{10000}{x} - 20x - 100 = 0$

$10000 - 20x^2 - 100x = 0$ Multiplying equation by x

$20x^2 + 100x - 10000 = 0$ Multiplying by -1 and re-arranging

$x^2 + 5x - 500 = 0$ Dividing by 20

(b) $x^2 + 5x - 500 = 0$
$(x + 25)(x - 20) = 0$
$x = 20$ $x = -25$ is not a real-life solution

Priti bought 20 jars.

(c) Each jar costs $\frac{2000}{x} = 100$ cents or \$1

GRAPHS 5 – BASIC SKILLS EXERCISE

1 $y > -3$

2 $x < 2$ or $x \geq 4$

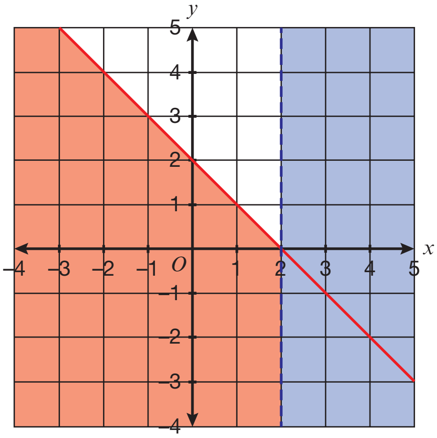
3 $x + y \geq 5$

4 $y < 2x + 2$

5 $x \geq 0$, $y \geq 0$ and $x + 2y < 6$

6 $x + y \geq -4$, $y - 3x > -4$ and $3y - x \leq 4$

7



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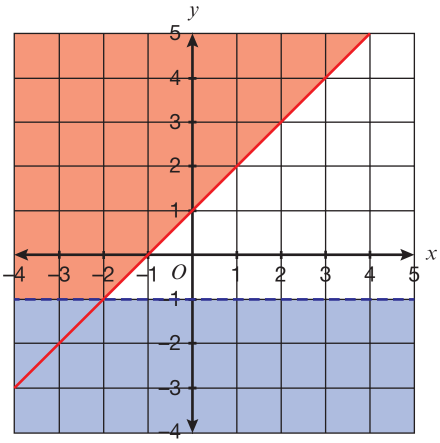

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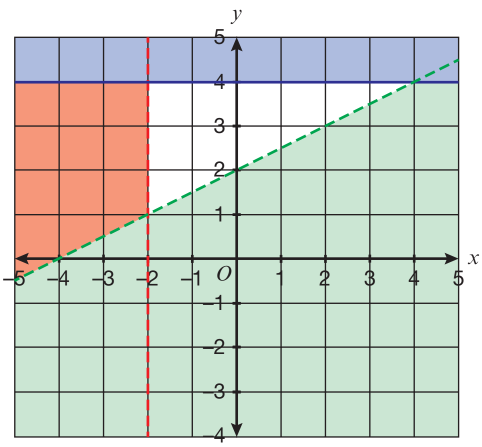

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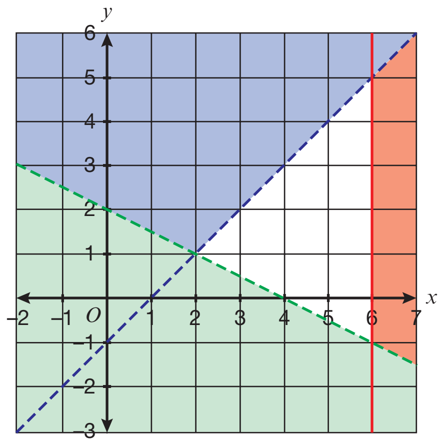


A sketch will help you to answer Q11−Q25

11 **(a)** $-\frac{3}{4}, \frac{4}{3}$

(b) $\frac{6}{7}, \frac{-7}{6}$

12 a and d, b and c

13 Sketch shows right angle is at A.

Gradient of $AB = \frac{1}{3}$, gradient of $AC = -3$,

$\frac{1}{3} \times -3 = -1$ hence AB is perpendicular to AC

14 Gradient of L is $-\frac{3}{7}$, gradient of M is

$\frac{12 - -9}{5 - -4} = \frac{21}{9} = \frac{7}{3}$

$-\frac{3}{7} \times \frac{7}{3} = -1$ so L is perpendicular to M

15 $9y + 5x = 18$

16 -5

17 (a) $\left(-2, -1\frac{1}{2}\right)$

(b) $\left(11\frac{1}{2}, -13\right)$

18 $(-2, 3)$

19 Sketch shows diagonals are AC and BD.

Midpoint of AC is $\left(\frac{7-4}{2}, \frac{1+1}{2}\right) = \left(1\frac{1}{2}, 1\right)$,

Midpoint of BD is $\left(\frac{4-1}{2}, \frac{3-1}{2}\right) = \left(1\frac{1}{2}, 1\right)$

As midpoint is the same, diagonals bisect each other.

20 Gradient of AB is $\frac{1}{3}$ so a perpendicular gradient is -3
The midpoint of AB is (1,0)
Equation is $y = -3x + 3$

21 A lies on $2y = x + 2$ so the median passes through A and midpoint of BC.

The midpoint of BC is $\left(\frac{5-1}{2}, \frac{1+3}{2}\right) = (2, 2)$

which lies on $2y = x + 2$ hence it is a median.

22 **(a)** 13
(b) 15

23 Centre of circle is $C\,(3, 0)$. $AC = 5$ so the radius is 5. $CP = \sqrt{2^2 + 21} = 5$ and P lies on circle.

24 $AB^2 = (2\sqrt{3})^2 + 2^2 = 12 + 4 = 16$
$AB = 4$. $BC = 1 - {}^-3 = 4$
$AC^2 = (2\sqrt{3})^2 + (-2)^2 = 12 + 4 = 16$
$AC = 4$
As all sides are equal, triangle is equilateral.

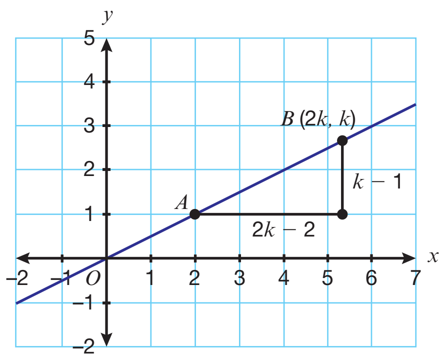
25 $CT^2 = 3^2 + 2^2$ so the radius is $\sqrt{13}$

The gradient of CT is $-\frac{2}{3}$ and the gradient of the tangent is $\frac{3}{2}$

The equation of tangent is $y = \frac{3x}{2} + c$

Substitute $(2, -1)$ hence $c = -4$ and the tangent is $2y = 3x - 8$

26 Let the coordinates of B be $(2k, k)$
B lies on $2y = x$



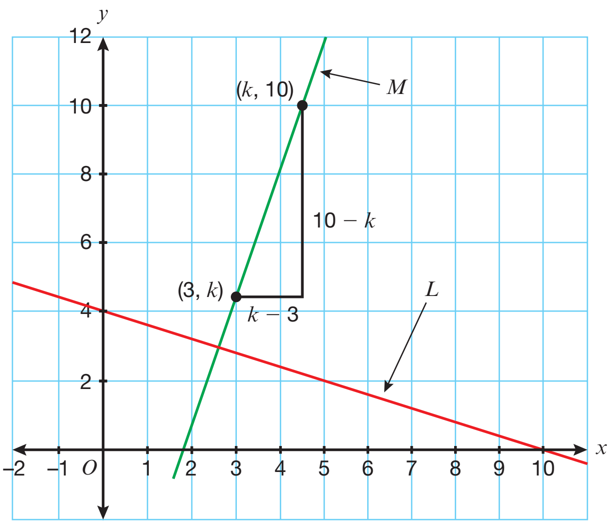
$AB^2 = (k-1)^2 + (2k-2)^2$
$20 = k^2 - 2k + 1 + 4k^2 - 8k + 4$
$5k^2 - 10k - 15 = 0$
$k^2 - 2k - 3 = 0$
$(k+1)(k-3) = 0$
$k = -1$ or 3

Coordinates of B are $(-2, -1)$ or $(6, 3)$

GRAPHS 5 – EXAM PRACTICE EXERCISE

1 The gradient of the line passing through $(-2, 0)$ and $(0, 4)$ is 2
The equation of the line is $y = 2x + 4$
One required inequality is $y \le 2x + 4$
$\le$ as the line is solid
The gradient of the line passing through $(0, -1)$ and $(2, 0)$ is $\frac{1}{2}$
The equation of the line is $y = \frac{x}{2} - 1$ or $2y + 2 = x$
One required inequality is $2y + 2 \ge x$
$\ge$ as the line is solid
The gradient of the line passing through $(0, 6)$ and $(8, 0)$ is $-\frac{3}{4}$
The equation of the line is $y = -\frac{3x}{4} + 6$ or $4y + 3x = 24$
One required inequality is $4y + 3x < 24$
$<$ as the line is dotted
The three inequalities are:
$y \le 2x + 4$, $2y + 2 \ge x$, $4y + 3x < 24$

2 A sketch with a guess for k will help you understand the problem. Axes need to be equal aspect so that the two lines look as though they are at right angles



The gradient of L is $-\frac{2}{5}$

Rearranging L gives $y = -\frac{2}{5}x + \frac{23}{5}$

The gradient of M is $\frac{5}{2}$ $\quad -\frac{2}{5} \times \frac{5}{2} = -1$

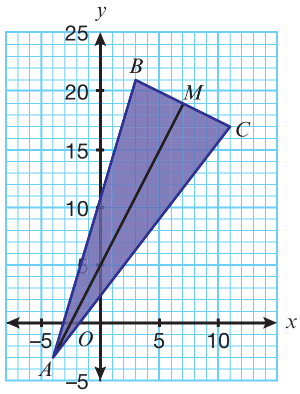
The gradient of line joining the two points is $\frac{10-k}{k-3}$

$\frac{k-10}{3-k}$ is also correct and will give the same answer.

$\frac{10-k}{k-3} = \frac{5}{2}$

$2(10 - k) = 5(k - 3)$
$20 - 2k = 5k - 15$
$7k = 35$
$k = 5$

3 A sketch shows that AB and AC will be the equal sides.



(a) $AB^2 = (21 + 3)^2 + (3 + 4)^2 = 625$
$AB = 25$
$AC^2 = (17 + 3)^2 + (11 + 4)^2 = 625$
$AC = 25$
So ABC is an isosceles triangle.

(b) M is $\left(\frac{3+11}{2}, \frac{21+17}{2}\right) = (7,1\ 9)$

Gradient of AM is $\frac{19+3}{7+4} = 2$

Gradient of BC is $\frac{17-21}{11-3} = -\frac{1}{2}$

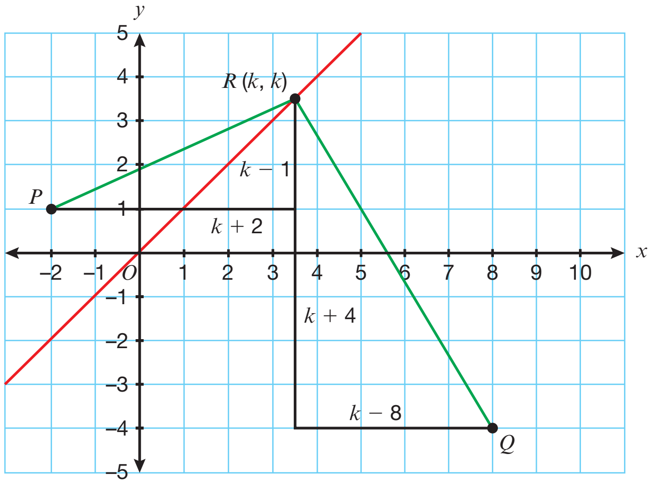
Product of gradients is $2 \times \frac{-1}{2} = -1$

So AM is perpendicular to BC

4 Let the point R be (k, k)

R lies on $y = x$ so coordinates are equal.

A rough sketch helps.



Gradient of PR is $\frac{k-1}{k--2} = \frac{k-1}{k+2}$

$\frac{1-k}{-2-k}$ is also correct

Gradient of QR is $\frac{k--4}{k-8} = \frac{k+4}{k-8}$

$\frac{-4-k}{8-k}$ is also correct

As PR is perpendicular to QR

$\frac{k-1}{k+2} \times \frac{k+4}{k-8} = -1$

$(k-1)(k+4) = -(k+2)(k-8)$

$k^2 - 3k - 4 = -k^2 + 6k + 16$

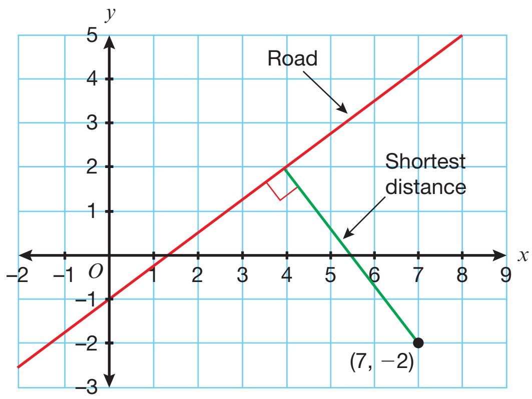
$2k^2 - 3k - 20 = 0$

$(2k+5)(k-4) = 0$

$k = 4$ or $k = -\frac{5}{2}$

R is (4, 4) or (–2.5, –2.5)

5 Shortest distance is along the perpendicular to the road passing through (7, −2).



Gradient of the line (road) is $\frac{3}{4}$

Rearranging $4y + 4 = 3x$ gives $y = \frac{3}{4}x - 1$

The gradient of perpendicular is

$-\frac{4}{3}$ $-\frac{4}{3} \times \frac{3}{4} = -1$

The equation of perpendicular is

$y = -\frac{4}{3}x + c$ or $3y + 4x = d$

Substituting $x = 7$, $y = -2$ gives $d = 22$ so equation of perpendicular is $3y + 4x = 22$

Intersection is given by solving $3y + 4x = 22$ and $4y + 4 = 3x$ simultaneously.

$3y + 4x = 22$ (1)

$4y + 4 = 3x$ (2)

$9y + 12x = 66$ (3)

(3) is (1) multiplied by 3

$16y - 12x = -16$ (4)

(4) is (2) rearranged and multiplied by 4

$25y = 50$ Add (3) and (4)

$y = 2$, $x = 4$ so the lines intersect at (4, 2)

Distance from (7, −2) to (4, 2) is

$\sqrt{(7-4)^2 + (-2-2)^2} = \sqrt{25} = 5$

Kyle must walk 500 m.

SHAPE AND SPACE 5 – BASIC SKILLS EXERCISE

1 (6, 10)

2 (2, 11)

3 $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$

4 13 units

5 (1, −2)

6 (−3, 4)

7 (6, 5)

8 (1, 8)

9 (2, −1)

10 (−4, 3)

11 Rotation of 180° about 0

12 Rotation 90° clockwise about centre (6, 8)

13 (2, 4)

14 (9, 12)

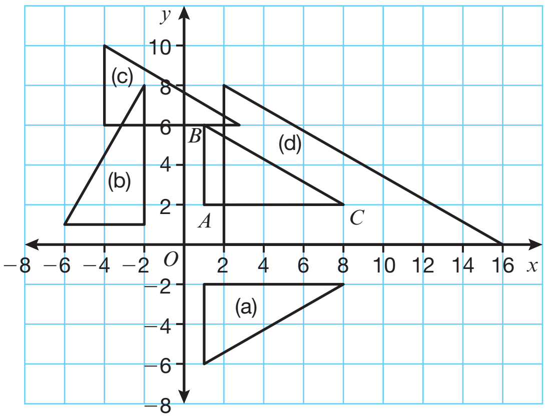
15 100

16 40

17 **(a)** (3, −4)
(b) (−3, 4)
(c) (4, −3)
(d) (10, −2)

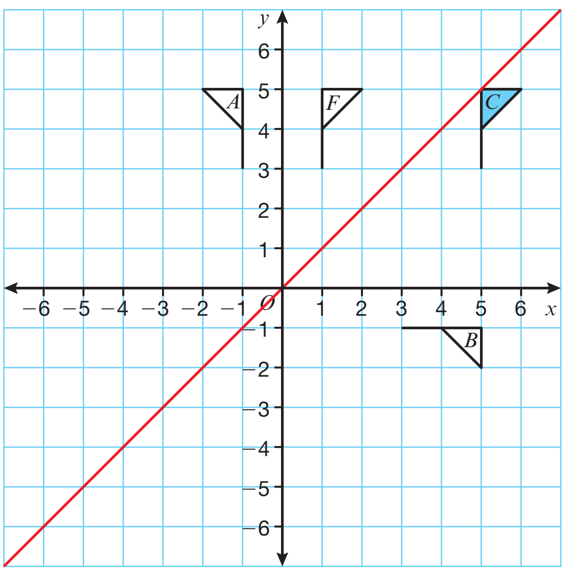
18 **(a)** (−3, −5)
(b) (3, 5)
(c) (−5, −3)
(d) (−8, 8)

19



(a) $A'(1, -2)$, $B'(1, -6)$, $C'(8, -2)$
(b) $A'(-2, 1)$, $B'(-6, 1)$, $C'(-2, 8)$
(c) $A'(-4, 6)$, $B'(-4, 10)$, $C'(3, 6)$
(d) $A'(2, 0)$, $B'(2, 8)$, $C'(16, 0)$

20



(c) Rotation of 90° clockwise around 0

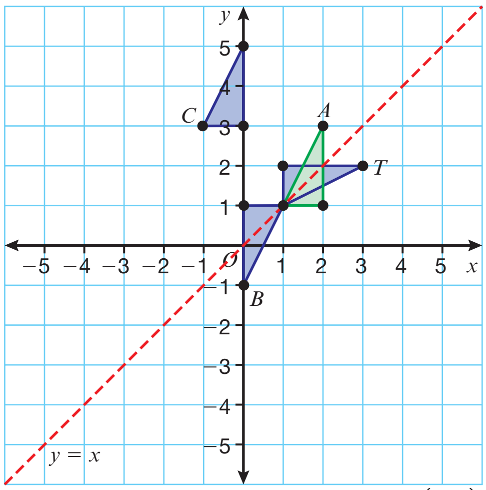
(e) Translation along $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$

SHAPE AND SPACE 5 – EXAM PRACTICE EXERCISE

1 $x = -11$, $y = -1$
Perform the inverse operation of each translation in reverse order on point (4, 8).
1. Translate along vector $\begin{pmatrix} -3 \\ 3 \end{pmatrix}$
2. Rotation of 90° in an anticlockwise direction about 0
3. Reflection in x-axis

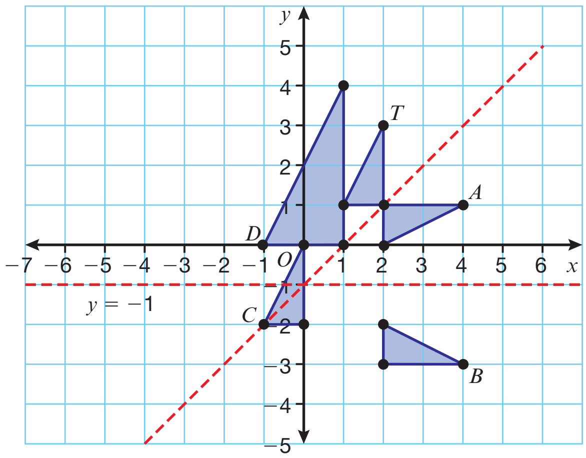
2 $A(1, -5)$, $B(-1, -5)$, $C(1, -9)$
Perform the inverse operation of each translation in reverse order on triangle JKL.
1. Translate along vector $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$
2. Rotation of 90° in a clockwise direction about 0
3. Reflection in y-axis

3



A translates to C along vector $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$

4



(*e*) Rotation of 90° clockwise about centre (0, −1)

(*f*) Enlargement of scale factor +2 about centre (−1, −4)

5 **(a)** The image of A after the translation along vector $\begin{pmatrix} 6 \\ 6 \end{pmatrix}$ is at $(7, 6 + \sqrt{3})$

(b) The image hexagon would have a new perimeter of $6 \times 12 = 72$.
Each triangle within the original hexagon is an equilateral triangle of side 2.
Area of whole hexagon image
= 6 × area of an equilateral triangle side 12 = $6 \times A_1$

$A_1 = \frac{1}{2} \times 12 \times 12 \times \sin(60°)$

(Area of triangle $= \frac{1}{2}ab\sin C$)

$= \frac{1}{2} \times 12 \times 12 \frac{\sqrt{3}}{2}$

$= 36\sqrt{3}$

$= 2^2 \times 3^2 \times 3^{\frac{1}{2}} = 2^2 \times 3^{\frac{5}{2}}$

So the total hexagon area $= 6 \times A_1$

$= (3 \times 2) \times 2^2 \times 3^{\frac{5}{2}} = 2^3 \times 3^{\frac{7}{2}}$

$a = 3, b = \frac{7}{2}$

HANDLING DATA 4 – BASIC SKILLS EXERCISE

1 0.8

2 **(a)** $\frac{1}{2}$

(b) $\frac{5}{6}$

(c) 1

(d) 0

3 **(a)** $\frac{3}{29}$

(b) $\frac{9}{58}$

(c) $\frac{17}{29}$

(d) 0

4 **(a)** 0

(b) $\frac{9}{25}$

(c) $\frac{16}{25}$

(d) $\frac{9}{25}$

5 **(a)** $\frac{1}{13}$

(b) $\frac{2}{13}$

(c) $\frac{3}{4}$

(d) $\frac{3}{26}$

6 **(a)** $\frac{1}{8}$

(b) $\frac{1}{2}$

(c) $\frac{3}{4}$

(d) $\frac{5}{8}$

7 **(a)** RG GR GG

(b) $\frac{2}{3}$

8 **(a)**

×	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

(b) **(i)** $\frac{1}{36}$

(ii) 0

(iii) $\frac{11}{36}$

(iv) $\frac{2}{9}$

9 **(a)**

	10	30	50
9	10	30	50
16	16	30	50
25	25	30	50
36	36	36	50

(b) **(i)** $\frac{1}{12}$

(ii) 0

(iii) $\frac{11}{12}$

(iv) $\frac{1}{3}$

10 **(a)** $\frac{25}{28}$

(b) $\frac{1}{6}$

(c) Number not cured =

$\frac{1}{9} \times 90 + \frac{3}{14} \times 84 + \frac{1}{10} \times 99 = 10 + 18 + 9.9$

$\approx$ 38 horses

11 **(a)**

		Pink Box		
		△	○	☆
Blue Box	□	□△	□○	□☆
	▭	▭△	▭○	▭☆
	☆	☆△	☆○	☆☆

(b) **(i)** $\frac{5}{9}$

(ii) $\frac{4}{9}$

12 10

13 $\frac{3}{5}$

14 40

15 5

16 $\frac{1}{245}$

17 **(a)**

	2	**3**	**5**	**7**	**11**
13	15	16	18	20	24
17	19	20	22	24	28
19	21	22	24	26	30
23	25	26	28	30	34
29	31	32	34	36	40

(b) P(number with a zero) = $\frac{5}{25}$, expected

numbers with a zero = $50 \times \frac{5}{25} = 10$

18 72 letters with a line of symmetry

HANDLING DATA 4 – EXAM PRACTICE EXERCISE

1 **(a)** $8x = 1$

$x = \frac{1}{8}$, so P(black) = $\frac{3}{8}$

(Sum of all probabilities = 1)

(b) P(not red or green) = $6x = \frac{6}{8}$

P(not red or green) = $\frac{3}{4}$

(c) If the spinner lands 25 times on one colour its probability must be $\frac{1}{4}$ which leads to the conclusion that it is most likely to have been blue as P(blue) = $\frac{1}{4}$.

2 **(a)** P(4) = 0.35

P(prime) = p (2 or 3 or 5)

= 0.1 + 0.05 + 0.2

P(prime) = 0.35

(b) Li throws two numbers a total number of 40 times from 100. The probability of this event must = 0.4

The only two numbers which have a probability sum of 0.4 is 3 and 4. (P(3) = 0.05, P(4) = 0.35)

Most likely numbers are 3 and 4.

3 **(a)**

	Milkshake	Orange juice	Tea	Coffee	Total
Year 10	2	10	7	5	24
Year 11	3	12	4	7	26
Total	5	22	11	12	50

(b) **(i)** P(year 10, milkshake) = $\frac{2}{50} = \frac{1}{25}$

(ii) P(year 11, not tea or coffee) =

$\frac{3+12}{50} = \frac{15}{50} = \frac{3}{10}$

(c) P(not milkshake/year 10) = $\frac{22}{24} = \frac{11}{12}$

4 **(a)**

	35	42	120
20	5	2	20
63	7	21	3
96	1	6	24

(b) **(i)** P(even) = $\frac{4}{9}$

(ii) P(prime or triangular) = $\frac{2}{3}$

(Triangle numbers are 1, 3, 6, 10, 15, 21…)

(c) P(odd) = $\frac{5}{9}$, therefore the expected number of odds from 90 choices

$= \frac{5}{9} \times 90 = 50$

5 Let the number of purple jellyfish be x

P(purple) = $\frac{x}{60}$

P(white) = $\frac{60-x}{60}$

If 40 white jellyfish are added:

P(purple) = $\frac{x}{100}$, P(white) = $\frac{100-x}{100}$

$\frac{100-x}{100} = 3 \times \frac{60-x}{60}$

$20(100 - x) = 100(60 - x)$
$100 - x = 300 - 5x$
$4x = 200, x = 50$

P(purple) = $\frac{50}{100} = \frac{1}{2}$

NUMBER 6 – BASIC SKILLS EXERCISE

1 **(a)** $\frac{126}{3} = \frac{294}{7}$ so yes

(b) $\frac{275}{50} \neq \frac{500}{80}$ so yes

(c) $\frac{254}{12} = \frac{317.5}{15}$ so yes

2 $\frac{1227 \times 1.9}{1.2} = 1942.75$ seconds or 32 minutes

22.75 seconds

3 Brand A $\frac{60}{25}$ = €2.4/ml, Brand B $\frac{70}{30}$

= €2.$\dot{3}$/ml so Brand B is cheaper

4 **(a)** $\frac{104.5 \times 60}{9.5} = 660$ tonnes in one minute

(b) $\frac{297 \times 9.5}{104.5} = 27$ seconds

5 **(a)** $\frac{12\,000}{11 \times 24 \times 60} = 758$ m (3 s.f.)

(b) $\frac{11 \times 534}{12000} = 0.4895$ days or 11 hours 45 mins

6 **(a)**

Number of people	50	80	160	200
Cost in £	125	200	400	500

(b) $\frac{50}{125} \times 875 = 350$ people

7 Between 0–5 mins temperature drop is 1° C/min, between 5–15 mins it is 0.9° C/min and between 15–30 mins it is 0.867° C. Temperature drop is not constant so Kit is wrong.

8 1 square contains $\frac{30}{100} \times \frac{1}{20} \times 85 = 1.275$ g of solids.
15 g of solids needs $\frac{15}{1.275} = 11.76$ squares so at least 12 squares are needed.

9 $x \times y$ = constant if x and y are in inverse proportion

x	2	3	4	6
y	18	12	10	6
xy	36	36	40	36

(4, 10) is not in inverse proportion.

10 Number of scarves × temperature = constant as they are in inverse proportion. Constant = 32 × 15 = 480

Number of scarves	120	60	40	32	24
Temperature (°C)	4	8	12	15	20

11 **(a)** $\frac{24}{64}$ = 0.375 seconds

(b) $\frac{15}{0.25}$ = 60 Mbs

12 Number of cleaners × days = constant as they are in inverse proportion
Constant = 3 × 8 = 24
(a) 2 × 12 = 24 so 12 days
(b) 4 × 6 = 24 so 4 window cleaners

13 Number of desks × time = constant as they are in inverse proportion. $\frac{3}{4}$ h = 45 min, constant = 3 × 45 = 135. Easier to work in minutes.
(a) 5 × 27 = 135 so 27 minutes
(b) n × 15 = 135 so n = 9 desks

14 Time × temperature = constant as they are in inverse proportion
Constant = 20 × 9 = 180
(a) t × 15 = 180 so t = 12 minutes
(b) 24 × T = 180 so T = 7.5°C

15 Number of waiters × time = constant as they are in inverse proportion
Constant = 12 × 3 = 36 (for 60 people)
(a) 18 × 2 = 36 therefore 18 waiters
(b) 24 × 1.5 = 36 therefore 24 waiters are needed to serve 60 people in 1.5 minutes
90 secs = 1.5 mins
24 × $\frac{100}{60}$ = 40 waiters needed to serve 100 people in 90 seconds

16 **(a)** 150 km of flight produces 1 g so 150 × 1000 = 150 000 km produces 1 kg
1000 g = 1 kg mean distance per bee is 150 000 ÷ 10 000 = 15 km
(b) 12 × 150 000 km will produce 12 kg
number of bees = 12 × 150 000 ÷ 45
= 40 000

17 $\frac{1}{25}$

18 $\frac{1}{27}$

19 $\frac{1}{16}$

20 $\frac{1}{49}$

21 $\frac{1}{5}$

22 $\frac{1}{108}$

23 $\frac{9}{8}$ or $1\frac{1}{8}$

24 1

25 9

26 $\frac{2}{3}$

27 $\frac{1}{7}$

28 $\frac{1}{16}$

29 $\frac{8}{125}$

30 $\frac{8}{27}$

31 $\frac{9}{4}$

32 $\frac{5}{3}$

33 1

34 $\frac{1}{3}$

35 16

36 9

37 4

38 0

39 $\frac{1}{2}$

40 $-\frac{2}{3}$

41 0

42 $\frac{3}{4}$

43 $\frac{1}{2}$

44 1 or −2

NUMBER 6 – EXAM PRACTICE EXERCISE

1 One machine produces $\frac{3000}{5} = 6000$ shoes in 20 days

One machine produces $\frac{6000}{20} = 300$ shoes every day

Five machines working for 6 days produce $5 \times 6 \times 300 = 9000$ shoes

To complete the order, 27 000 shoes must be produced in 10 days. 36 000 − 9 000 = 27 000

1 machine will produce $300 \times 10 = 3000$ shoes in 10 days

To produce 27 000 shoes in 10 days will take

$\frac{27000}{3000} = 9$ machines

Kiko must order an extra 9 – 5 = 4 machines

2 Temperature decreases by 10 °C in 500 m hence 2 °C in 100 m and 14 °C in 700 m

so the temperature at 700 m is 20 – 14 = 6 °C

Below 700 m, let T be temperature in °C and d be depth in metres

$T \times d =$ constant as T and d are in inverse proportion

$T \times d = k$

$k = 6 \times 700 = 4200$

$T \times d = 4200$

If $T = 4$, then $d = \frac{4200}{4} = 1050$ m

3 **(a)** $(5^4)^{\frac{3}{8}} \times (5^{10})^{-\frac{1}{4}} \div 100^{-\frac{1}{2}} = 5^{\frac{3}{2}}\ 5^{\frac{5}{2}} \div 10^{-1}$

$= 5^{-1} \times 10$

$= \frac{1}{5} \times 10$

$= 2$

(b) $27\sqrt{27} = 3^3 \times (3^3)^{\frac{1}{2}}$

$= 3^3 \times 3^1 \times 3^{\frac{1}{2}}$

$= 3^4 \times 3^{\frac{1}{2}}$

$= 81\sqrt{3}$

(c) $27\sqrt{27} = (3^3)^k$

$3^3 \times 3^{\frac{3}{2}} = 3^{2k}$

$3^{\frac{9}{2}} = 3^{2k}$

$k = \frac{9}{4}$

4 **(a)** $2^3 = (2^2)^k$ so $2k = 3$ and $k = \frac{3}{2}$

(b) $2\sqrt{32} = 4^k$

$2^1 \times (2^5)^{\frac{1}{2}} = (2^2)^k$

$2^{\frac{2}{7}} = 2^{2k}$

$k = \frac{7}{4}$

(c) $\frac{1}{32} = 8^k$

$32^{-1} = (2^3)^k$

$(2^5)^{-1} = 2^{3k}$

$2^{-5} = 2^{3k}$

$k = \frac{-5}{3}$

5 **(a)** **(i)** $6 = 2 \times 3$

$= (2^3)^{\frac{1}{3}} \times (3^2)^{\frac{1}{2}}$

$= x^{\frac{1}{3}} \times y^{\frac{1}{2}}$

(ii) $4\sqrt{3} = 2^2 \times 3^{\frac{1}{2}}$

$= (2^3)^{\frac{2}{3}} \times (3^2)^{\frac{1}{4}}$

$= x^{\frac{2}{3}} \times y^{\frac{1}{4}}$

(iii) $\frac{1}{3\sqrt{4}} = 3^{-1} \times 4^{-\frac{1}{2}}$

$= 3^{-1} \times (2^2)^{-\frac{1}{2}}$

$= 3^{-1} \times 2^{-1}$

$= (3^2)^{-\frac{1}{2}} \times (2^3)^{-\frac{1}{3}}$

$= x^{-\frac{1}{3}} \times y^{-\frac{1}{2}}$

(b) $\frac{3\sqrt{6}}{8} = 3 \times \sqrt{2} \times \sqrt{3} \times 2^{-3}$

$= 3 \times 2^{\frac{1}{2}} \times 3^{\frac{1}{2}} \times 2^{-3}$

$= 2^{-\frac{5}{2}} \times 3^{\frac{3}{2}}$

$2^a \times 3^b = 2^{-\frac{5}{2}} \times 3^{\frac{3}{2}}$

$a = \frac{-5}{2}$, $b = \frac{3}{2}$

ALGEBRA 6 – BASIC SKILLS EXERCISE

1 **(a)** $y = 9x$
(b) $y = 90$
(c) $x = 5$

2 $a = 20b$

b	10	5	30
a	200	300	600

3 **(a)** $y = 8x$
(b) $y = 80$
(c) $x = 5$

4 **(a)** $y = \left(\frac{x}{4}\right)^3$
(b) $y = 512$

5 **(a)** $y = 0.4x^2$
(b) $y = 90$
(c) $x \approx 6.12$

6 **(a)** $p = 20\sqrt{q}$
(b) $p = 160$
(c) $q = 6.25$

7 **(a)** $d = 5t^2$
(b) $d = 20$ m
(c) $t \approx 4.24$ s

8 **(a)** $p = 1.5n^2$
(b) $p = €216$
(c) $n = 20$

9 **(a)** $y = 4x^3$
(b) $y = 256$
(c) $x = 6$

10 **(a)** $e = 0.5v^2$
(b) $e = 1250$ kJ
(c) $v = 1414$ m/s

11 **(a)** $A = 15h^2$
(b) $A = 135$ m^2
(c) $h = 6$ m

12 **(a)** $y = \frac{48}{x}$
(b) $y = 6$
(c) $x = 4$

13 **(a)** $p = \frac{50}{q}$
(b) $p = 2.5$
(c) $q = 2.5$

14 **(a)** $y = \frac{80}{x^2}$
(b) $y = 5$
(c) $x = 12.6$

15 **(a)** $p = \frac{2500}{\sqrt{q}}$
(b) $p = 250$
(c) $q = 2500$

16 **(a)** $p^2 = \frac{800}{q^3}$
(b) $p = 3.54$
(c) $q = 3.17$

17

b	125	8	1
a	2	5	10

18 **(a)** $N = \frac{9000}{d^2}$
(b) $N = 2250$
(c) $d = 3$

19 $\frac{1}{2}$

20 -1

21 -2

22 -3

23 $\frac{1}{3}$

24 $-\frac{1}{2}$

25 $-\frac{1}{3}$

26 $-\frac{1}{4}$

27 $\frac{1}{2}$

28 2

29 3

30 3

31 -5

32 -4

33 -4

34 -3

35 1

36 $\frac{1}{2}$

37 1

38 $\frac{1}{4}$

39 2

40 2

41 –1

42 10^4

43 $a^{-2} = \frac{1}{a^2}$

44 $b^{-1} = \frac{1}{b}$

45 c^2

46 d^2

47 $e^{-1} = \frac{1}{e}$

48 $f^{-2} = \frac{1}{f^2}$

49 $g^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{g}}$

50 h^2

51 $x = \frac{3}{10}, y = -\frac{9}{10}$

52 $x = \frac{3}{4}, y = 0$

53 $x = 2, y = -\frac{3}{5}$

54 $x = \frac{3}{5}, y = \frac{7}{3}$

55 $x = \frac{1}{2}$

56 $x = 3$ or 4

ALGEBRA 6 – EXAM PRACTICE EXERCISE

1 **(a)** $C \propto d, C = kd$
If $d = 49$ cm, $C = \$60$
$60 = k \times 49, k = 1.2$
$C = 1.2d$
(b) If $d = 65$ cm
$C = 1.2 \times 65 = \$\,78$
(c) If $C = \$80$
$80 = 1.2 \times d, d = 66.\dot{6}$ cm

2 **(a)** $n \propto \frac{1}{t^2}, n = \frac{k}{t^2}$
If $n = 10^3, t = 0.5$
$10^3 = \frac{k}{0.5^2}$
$k = 2.5 \times 10^2$
$n = \frac{2.5 \times 10^2}{t^2}$
(b) If $t = 2$
$n = \frac{2.5 \times 10^2}{2^2} = 62.5$
(c) If $n = 1$
$1 = \frac{2.5 \times 10^2}{t^2}, t = \sqrt{2.5 \times 10^2} = 15.8$ yrs

3 **(a)** $v \propto \sqrt{d}, v = k\sqrt{d}$
If $d = 10$ m, $v = 9.8$ m/s
$9.8 = k \times \sqrt{10}, k = \frac{9.8}{\sqrt{10}}$
$v = \frac{9.8\sqrt{d}}{\sqrt{10}} = 9.8\sqrt{\frac{d}{10}}$
(b) **(i)** If $d = 50$ m, $v = 9.8\sqrt{\frac{50}{10}}$
$= 21.9$ m/s (3 s.f.)
(ii) If $d = 1000$ m, $v = 9.8\sqrt{\frac{1000}{10}}$
$= 98$ m/s
(c) If $v = 1$ m/s
$1 = 9.8\sqrt{\frac{d}{10}}, \frac{1}{9.8} = \sqrt{\frac{d}{10}}, \left(\frac{1}{9.8}\right)^2 = \frac{d}{10}$
$d = 10 \times \left(\frac{1}{9.8}\right)^2 = 0.104$ m (3 s.f.)
(d) $790 \text{ km/h} = \frac{790 \times 1000}{60 \times 60} = 219.4$ m/s
(Convert 790 km/h into m/s)
$219.4 = 9.8\sqrt{\frac{d}{10}}, \frac{219.4}{9.8} = \sqrt{\frac{d}{10}},$
$\left(\frac{219.4}{9.8}\right)^2 = \frac{d}{10}$
$d = 10 \times \left(\frac{219.4}{9.8}\right)^2 = 5010$ m (3 s.f.)

4 $x = k_1z^3$ and $x = k_2y^2$, where k_1 and k_2 are constants
So $x = k_1z^3 = k_2y^2$ therefore $k_1z^3 = k_2y^2$,
$z^3 = \frac{k_2}{k_1} \times y^2, 50^3 = \frac{k_2}{k_1} \times 25^2, \frac{k_2}{k_1} = 200$
$z^3 = 200 \times 10^2$, so $z^3 = 20\,000, z = 27.1$ (3 s.f.)

5 $ab = 125$ implies that $5^m \times 5^n = 5^3$
so $m + n = 3$ (1)
$a^4b^{-2} = 5^{-9}$ implies that $5^{4m} \times 5^{-2n} = 5^{-9}$
so $4m - 2n = -9$ (2)
Solving equations (1) and (2):
(1): $m = 3 - n$
substituting into (2) gives
$4(3 - n) - 2n = -9$
$12 - 4n - 2n = -9$
$21 = 6n$
$n = 3.5$
substituting into (1) gives
(1): $m + 3.5 = 3$
$m = -0.5$

SEQUENCES 1 – BASIC SKILLS EXERCISE

1 16, 19.5, 23 (add 3.5)

2 0.2, 0, −0.2 (subtract 0.2)

3 $\frac{1}{16}, \frac{1}{32}, \frac{1}{64}$ $\left(\frac{1}{2^{n-1}}\right)$

4 0.32, 0.064, 0.0128 (divide by 5)

5 −9, 27, −81 (multiply by −3)

6 35, 48, 63 $n^2 - 1$

7 −2, 4, 10, 16, …

8 80, 76, 72, 68

9 $-\frac{1}{3}, 0, \frac{1}{3}, \frac{2}{3}$

10 $2, \frac{5}{2}, \frac{10}{3}, \frac{17}{4}$

11 3, 10, 21, 36

12 $\frac{1}{3}, -\frac{1}{4}, -\frac{3}{5}, -\frac{5}{6}$

13 56, 76, 99

14 2, −8, −21

15 4, 10, 18

16 74, 100, 130

17 8, 5, 1

18 −6, −4, −1

19 $2n + 3$

20 $29 - 3n$

21 $\frac{n-1}{n+1}$

22 $3n - 2$

23 $13 - 4n$

24 2^{n-1}

25 No, $78 - 3n = -218 \Rightarrow n = 98.67$ …

26 $-23 + 5n > 1000 \Rightarrow n > 204.6$ 205th term is 1002

27 First sequence is $4n - 7$, second sequence is $3n + 9$. $4n - 7 = 3n + 9$ so $n = 16$

28 2nd sequence is $11n + 8$ so $n^2 - 4 = 11n + 8$ and $n^2 - 11n - 12 = 0$
$(n + 1)(n - 12) = 0$
$n = 12$
The term is 140

29 17th term = $\frac{1}{52}$ or 0.0192…

30 $n = 1$ gives $1 + a + b = 3$
$n = 2$ gives $4 + 2a + b = 3$ so $a = -3$, $b = 5$

31 $a = 6$, $d = 5$, $n = 20$ so $S_{20} = 1070$

32 $a = -2$, $d = -4$, $a + (n - 1)d = -46$
$n = 12$
$S_{12} = -288$

33 $S_{38}\ \frac{38}{2}(1 + 297) = 5662$

34 $a = 4$, $d = 4$, $n = 100$
$S_{100} = 20\,200$

35 First term = $461 - 7 \times 67 = -8$ $S_{68} = \frac{68}{2}(-16 + 67 \times 7) = 15\,402$

36 **(a)** $120 = \frac{10}{2}[6 + (10 - 1)d]$ so $d = 2$

(b) $120 = \frac{10}{2}[2a + (10 - 1)3]$ so $a = -\frac{3}{2}$

37 $a + 6d = 37$, $a + 17d = 92$ so $a = 7$, $d = 5$

$S_{20} - S_9 = \frac{20}{2}(14 + 19 \times 5) - \frac{9}{2}$

$(14 + 8 \times 5) = 847$

38 $270 = \frac{n}{2}[12 + (n - 1)3]$
$n^2 + 3n - 180 = 0$
$(n + 15)(n - 12) = 0$
$n = 12$

39 Sum of first 80 even numbers = $\frac{80}{2}$ $(4 + 79 \times 2) = 6480$

Even multiples of 3 are multiples of 6. Need to subtract $6 + 12 + 18 + \ldots + 156$
Number of terms given by $6 + (n - 1)$
$6 = 156$ so $n = 26$
Sum of $6 + 12 + 18 + \ldots + 156 = \frac{26}{2}$ $(12 + 25 \times 6) = 2106$

answer is $6480 - 2106 = 4374$

40 $935 = \frac{17}{2}(a + 103)$ so $a = 7$ and

$d = \frac{103 - 7}{17 - 1} = 6$

41 $a = -11$, $d = 2$ gives

$540 = \frac{n}{2}[-22 + (n - 1)2]$

$n^2 - 12n - 540 = 0$
$(n - 30)(n + 18) = 0$
$n = 30$
last term is $-11 + 29 \times 2 = 47$

42 $145 = \frac{10}{2}(2a + 9d)$ (1)

Sum of first 20 terms is $145 + 645 = 790$

$790 = \frac{20}{2}(2a + 19d)$ (2)

Solving (1) and (2) simultaneously gives
$a = -8$, $d = 5$

43 $64 = a + 11d$ and $504 = \frac{12}{2}(2a + 11d)$
so $a = 20$, $d = 4$

$S_{24} = \frac{24}{2}(40 + 23 \times 4) = 1584$

44 First term ($k = 1$) is 3, common difference is 4

$S_n = \frac{n}{2}[6 + (n - 1)4] = n(2n + 1)$

45 **(a)** $0 = 48 + (k - 1)(-3)$ so $k = 17$
(b) After the 17th term, terms are negative and thus reducing the sum of the series.

largest sum is $S_{17} = \frac{17}{2}[96 + 16 \times (-3)]$
$= 408$
Note S_{16} is also correct as the 17th term is zero.

SEQUENCES 1 – EXAM PRACTICE EXERCISE

1 **(a)** The common difference is 4 so the sequence continues as 21, 25, 29, nth term is $4n - 3$
(b) 25 is a square number and is 7th in the sequence.
Next square number is 36. $4n - 3$ is always odd so 36 is not a member of the sequence.
or $4n - 3 = 36$ so $n = 9.75$ and hence 36 is not a member of the sequence.
Next square number is 49 so $4n - 3 = 49$ and $n = 13$
(c) T is the sequence 1, 3, 7, 13, …
Table of differences shows 2nd difference is constant and equal to 2

1		3		7		13
	2		4		6	
		2		2		

Extending the table gives (shown in red)

1		3		7		13		21		31
	2		4		6		8		10	
		2		2		2		2		

T is 1, 3, 7, 13, 21, 31, …
the sixth square number in S is in the 31st position.
Substituting $n = 31$ into $4n - 3$ gives the value as 121 (or 11^2)

2 **(a)** $\frac{n + 1}{2n + 1}$

(b) $n + 1 = 99$ so $n = 98$
$2n + 1 = 195$
$n = 97$
$\frac{99}{195}$ is not a member of the sequence

$\frac{n + 1}{2n + 1}$

(c) $\frac{n + 1}{2n + 1} = 0.52 \Rightarrow n + 1 = (2n + 1)(0.52)$

$= 1.04n + 0.52$

$0.04n = 0.48$
$n = 12$
so 13th term is the first with a value less than 0.52.
12th term equals 0.52 so is not less than 0.52

3 Subtracting two consecutive terms gives d
$(10x - 9) - (4x + 10) = d$
$6x - 19 = d$
$(12x - 10) - (10x - 9) = d$
$2x - 1 = d$
$6x - 19 = 2x - 1$
$4x = 18$
$x = 4.5$
$d = 8$ and $a = 28$
Sum from the 20th to the 30th terms
$= S_{30} - S_{19}$ Sum includes the 20th term
$S_{30} = \frac{30}{2}(56 + 29 \times 8) = 4320$
$S_{19} = \frac{19}{2}(56 + 18 \times 8) = 1900$
$S_{30} - S_{19} = 4320 - 1900 = 2420$

4 $a + 4d = 56$ (1) 5th term is $a + 4d$
$\frac{5}{2}(2a + 4d) = 300$ $S_5 = \frac{5}{2}[2a + (5 - 1)d]$
$a + 2d = 60$ (2)
Solving (1) and (2) simultaneously gives
$a = 64, d = -2$
$S_5 : S_n = 6 : 11$
$S_n = \frac{11}{6} \times 300 = 550$
$\frac{n}{2}[128 - 2(n - 1)] = 550$
$64n - n^2 + n = 550$
$n^2 - 65n + 550 = 0$
$(n - 55)(n - 10) = 0$
$n = 55$ or $n = 10$

5 **(a)** $1 + 2 + 3 + \ldots + 99 + 100 = 5050$
smallest share is $\frac{1}{5050} \times 10100 = £2$
largest share is $\frac{100}{5050} \times 10100 = £200$
(b) $1 + 2 + 3 + 4 + \ldots + n =$
$\frac{n}{2}[2 + (n - 1) \times 1] = \frac{n}{2}(n + 1) = \frac{n(n+1)}{2}$
Smallest share is $1 \div \left[\frac{n(n+1)}{2}\right]$ of the
amount $= A \times \frac{2}{n(n+1)} = \frac{2A}{n(n+1)}$
Largest share is $n \div \left(\frac{n(n+1)}{2}\right)$ of the
amount $= A \times n \times \frac{2}{n(n+1)} = \frac{2A}{n+1}$

SHAPE AND SPACE 6 – BASIC SKILLS EXERCISE

1 12
2 3
3 8
4 8
5 4
6 3
7 5
8 4
9 6
10 3
11 8
12 5
13 4
14 4
15 **(a)** $x = 16$
(b) $x = 10$
16 62°
17 55°
18 124°
19 132°
20 54°
21 66°
22 30°
23 222°
24 57°
25 112°
26 68°
27 42°
28 44°

29 42°

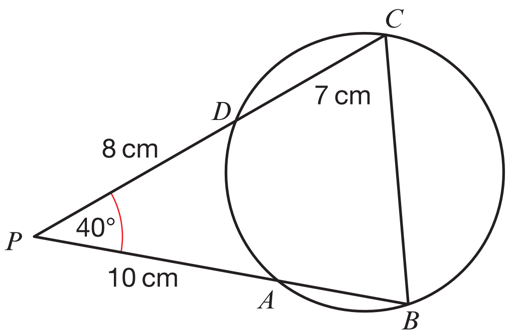
30 228°

31 44°

32 **(a)** $x = 60°$, $y = 60°$, $z = 55°$
(b) $x = 40°$, $y = 70°$, $z = 40°$

SHAPE AND SPACE 6 – EXAM PRACTICE EXERCISE

1



(a) $PB \times PA = PC \times PD$ (intersecting chords theorem)
$PB \times 10 = 15 \times 8$

$PB = \frac{15 \times 8}{10} = 12$ cm, so AB

$= 12 - 10 = 2$ cm

(b) If angle $DPA = 40°$, BC can be found from the cosine rule.

(cosine rule: $a^2 = b^2 + c^2 - 2bc\cos(A)$)

$BC^2 = PC^2 + PB^2 - 2 \times PC \times PB \times \cos(40°)$
$= 15^2 + 12^2 - 2 \times 15 \times 12 \times \cos(40°)$
$BC = 9.6553\ldots$cm $= 9.66$ cm (3 s.f.)

(c) Angle BCD can be found from the sine rule.

(sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$)

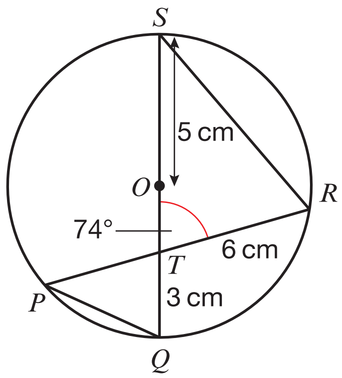
$\frac{BC}{\sin(DPA)} = \frac{PB}{\sin(BCD)}$,

$\frac{9.6553}{\sin(40°)} = \frac{12}{\sin(BCD)}$

$\sin(BCD) = \frac{12 \times \sin(40°)}{9.6553} = 0.79888\ldots$,

$BCD = 53.0°$ (2 s.f.)

2



(a) $PT \times RT = ST \times QT$ (intersecting chords theorem)
$PT \times 6 = 7 \times 3$

$PT = \frac{7 \times 3}{6} = 3.5$ cm, so $PR = 3.5 + 6$

$= 9.5$ cm

(b) If the angle $OTR = 74°$, consider triangle RST and use the cosine rule

(cosine rule: $a^2 = b^2 + c^2 - 2bc\cos(A)$)

$RS^2 = RT^2 + ST^2 - 2 \times RT \times ST \times \cos(74°)$
$= 6^2 + 7^2 - 2 \times 6 \times 7 \times \cos(74°)$
$RS = 7.8643\ldots$cm $= 7.86$ cm (3 s.f.)

(c) angle QPT = angle TSR

(Angles in the same segment are equal.)

So consider triangle RST to find angle TSR hence finding angle QPT.

(cosine rule: $a^2 = b^2 + c^2 - 2bc\cos(A)$)

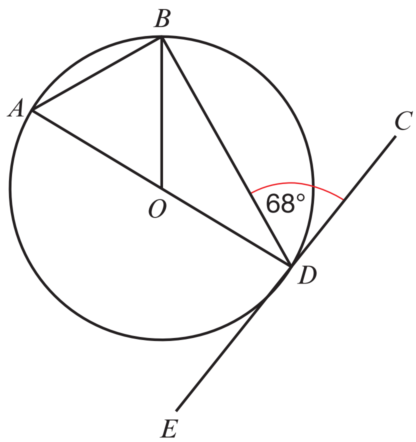
$RT^2 = RS^2 + ST^2 - 2 \times RS \times ST \times \cos(RST)$
$6^2 = 61.846 + 7^2 - 2 \times 61.846 \times 7 \times \cos(RST)$

$\cos(RST) = \frac{61.846 + 7^2 - 6^2}{2 \times 61.846 \times 7}$, so angle

$RST = 85.041°\ldots$,

So angle RST = angle $QPT = 85.0°$ (3 s.f.)

3



(a) angle $BAO = 68°$ (alternate segment theorem)

angle BAO = angle $ABO = 68°$ (triangle ABO is isosceles)

angle $AOB = 180° - 2 \times 68° = 44°$ (angle sum of triangle = 180°)

angle $BOD = 180° - 44° = 136°$ (angle sum of a straight line = 180°)

(b) angle $ABD = 90°$ (angles in a semi-circle = 90° at the circumference)

So triangle ABD is a right-angled triangle.

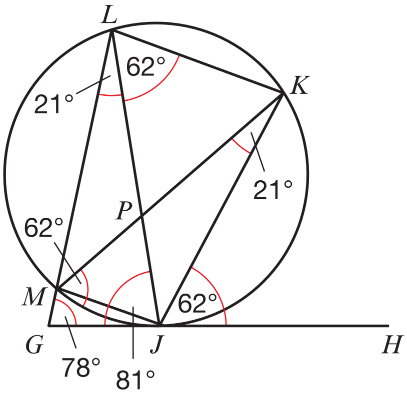
$AD^2 = AB^2 + BD^2$, $(2r)^2 = (2s)^2 + BD^2$

$BD^2 = 4r^2 - 4s^2 = 4(r^2 - s^2) = 4(r + s)(r - s)$

$BD = \sqrt{4(r + s)(r - s)} = 2\sqrt{(r + s)(r - s)}$

as required

4



(a) angle $PLK = 62°$ (alternate segment theorem)

angle PLK = angle $PMJ = 62°$ (angles in the same segment are equal)

angle MLP = angle $PKJ = 21°$ (angles in the same segment are equal)

Triangle GLJ:

angle $LJG = 180° - (21° + 78°) = 81°$ (angle sum of a triangle = 180°)

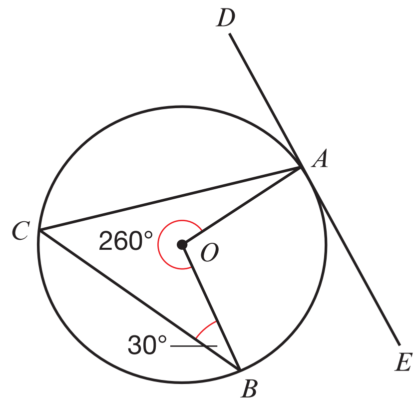
angle $LJK = 180° - (81° + 62°) = 37°$ (angle sum of a straight line = 180°)

(b) angle LMK = angle $KJL = 37°$ (angles in the same segment are equal)

angle $GMJ = 180° - (37° + 62°) = 81°$ (angle sum of a straight line = 180°)

angle GMK = angle GMJ + angle PMJ = $81° + 62° = 143°$

5



(a) **(i)** Draw line AB on diagram as shown.

angle $BOA = 100°$ (angle sum in a circle = 360°)

angle $ACB = 50°$ (angle at centre of circle = 2 × angle at circumference off the same chord)

angle $CAO = 360° - (50° + 260° + 30°) = 20°$ (angle sum of a quadrilateral = 360°)

(ii) angle $ABO = \frac{180° - 100°}{2} = 40°$ (triangle ABO is isosceles so base angles are equal.)

angle ABC = angle ABO + angle OBC
$= 40° + 30° = 70°$

(b) angle EAB = angle $ACB = 50°$ (alternate segment theorem)

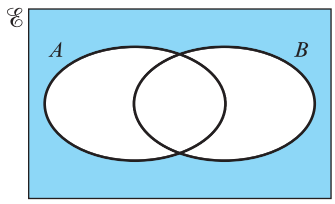
angle $FAB = 25°$ (line FA bisects angle BAE)

angle $AFB = 180° - 50° = 130°$ (opposite angles in a cyclic quadrilateral = 180°)

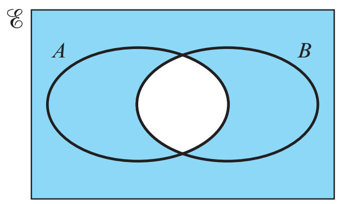
angle $FBA = 180° - 130° - 25° = 25°$ (angle sum of a triangle = 180°)

SETS 2 – BASIC SKILLS EXERCISE

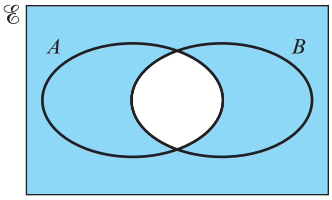
1 **(a)**



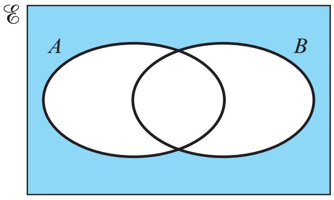
(b)



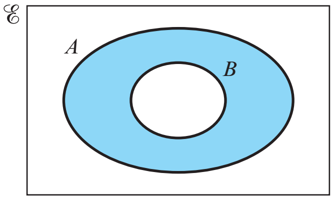
(c)



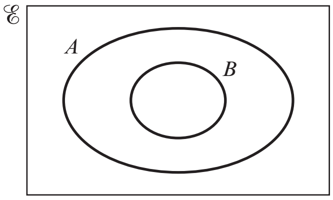
(d)



2 **(a)**

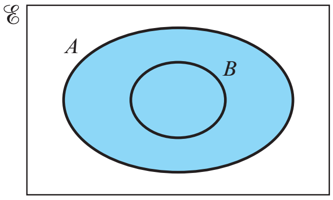


(b)

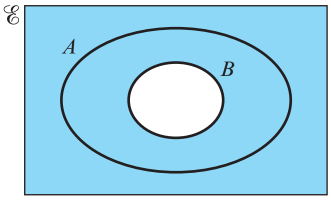


or ∅

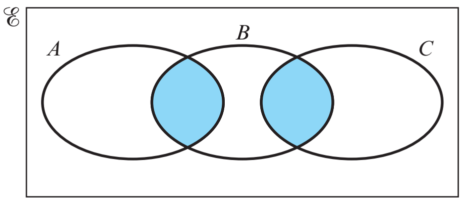
(c)



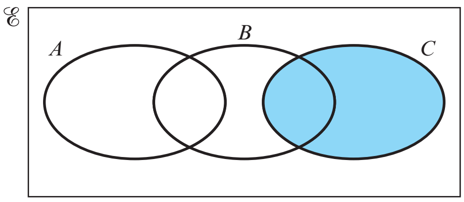
(d)



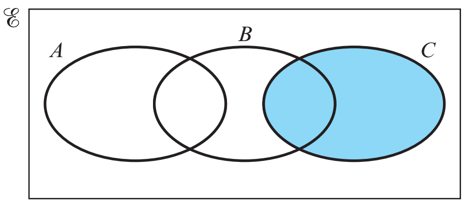
3 **(a)**



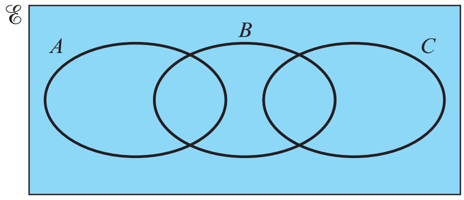
(b)



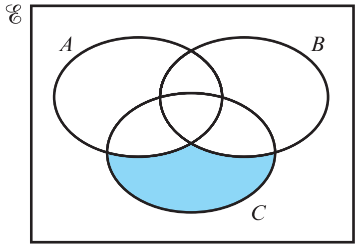
(c)



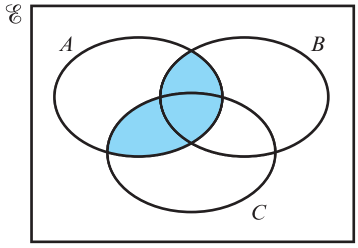
(d)



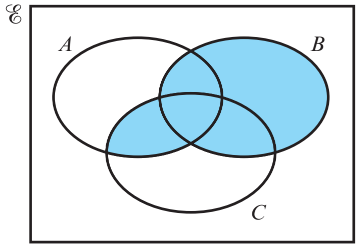
4 **(a)**



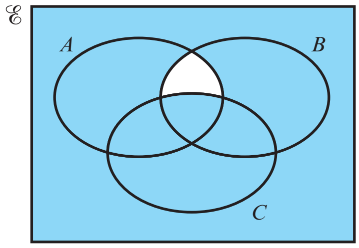
(b)



(c)



(d)



5 **(a)** $A' \cap B$

(b) $(A' \cap B) \cup (A \cap B')$

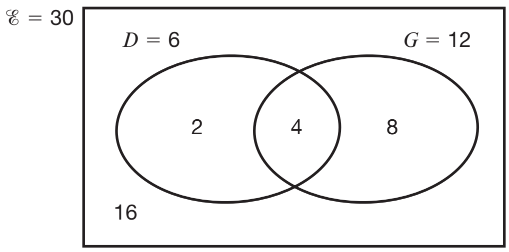
6 **(a)** $A' \cup B$

(b) $(A \cap B') \cup C$

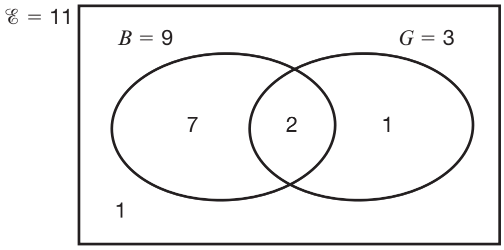
7 **(a)** There are no tabby cats over 10 years old.

(b) There are some non-tabby cats under 10 years old.

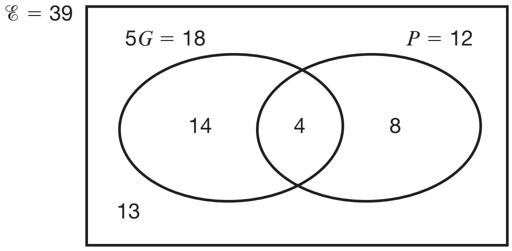
8



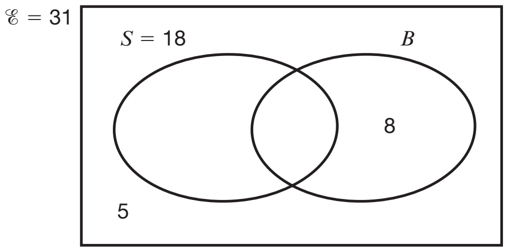
16 play neither.

9 

2 have both.

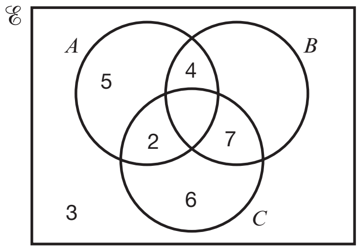
10 

8 cannot.

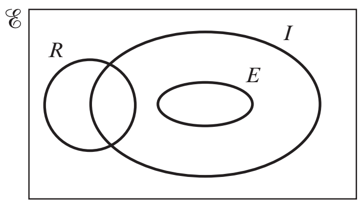
11 

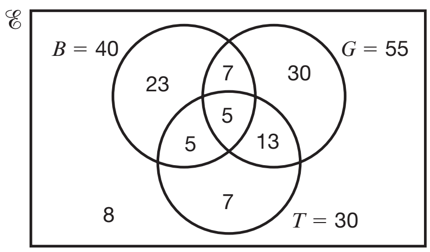
31 animals are in the field.

12 **(a)** 21
(b) 13



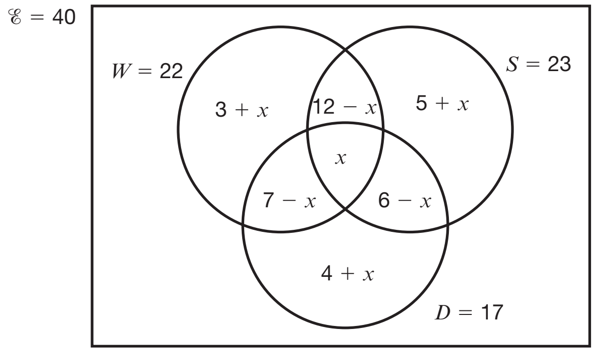
13 I = Isosceles triangles
E = Equilateral triangles
R = Right-angled triangles



14 

Number of pensioners =
$23 + 7 + 30 + 5 + 5 + 13 + 7 = 90$

15



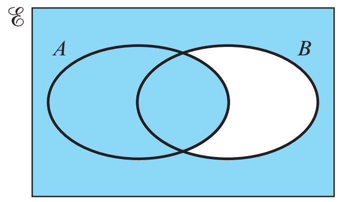
$(3 + x) + (12 – x) + (5 + x) + (7 – x) +$
$(6 – x) + (4 + x) + x = 40$
$x = 3$ so 3 teenagers enjoy all three sports.

16 **(a)** $A = \{–1, 0, 1\}$
(b) $B = \{3, 4, 5\}$
(c) $C = \{–1, 0, 1\}$
(d) $D = \{1, 2, 3, 4\}$
(e) $E = \{–2, 2\}$
(f) $F = \{0, 1, 2, 3, 4, 5, 6\}$

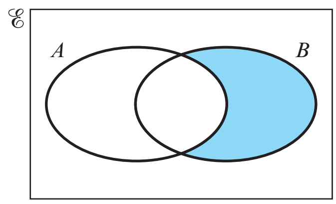
17 **(a)** $A = \{x: x < 5\}$
(b) $B = \{x: x \geq –8\}$
(c) $C = \{x: –2 < x < 4\}$
(d) $D = \{x: 3 \leq x \geq 8\}$
(e) $E = \{x: –2 \leq x \leq 2\}$ or $C = \{x: –3 < x < 3\}$
(f) $F = \{x: x = 2y \text{ and } 2 \leq y \leq 4\}$ or
$D = \{x: x = 2y \text{ and } 1 < y < 5\}$

SETS 2 – EXAM PRACTICE EXERCISE

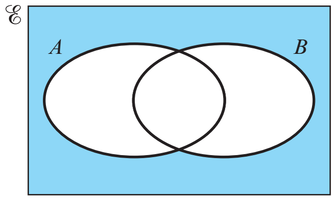
1 **(a)** **(i)** $(A \cup B') =$



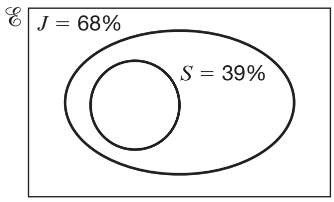
so $(A \cup B')' =$



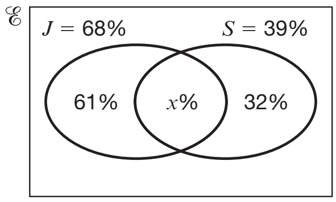
(ii) $A' \cap B' =$



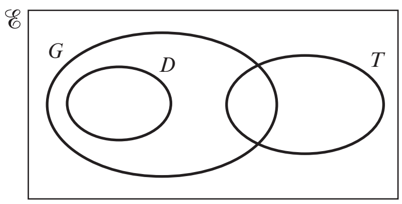
(b) (i) Largest intersection is 39% when swimming is a subset of jogging.



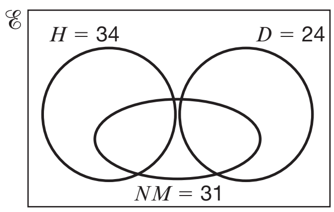
(b) (ii) Smallest intersection is when the percentage not in J or S is 0%.
Let the % in $J \cap S$ be x, then $(68 - x) + x + (39 - x) = 100$
$\Rightarrow x = 7\% \Rightarrow$ smallest percentage is 7%.



2 **(a)** Suzie has some green T shirts.
(b) All Suzie's dresses are green.
(c)

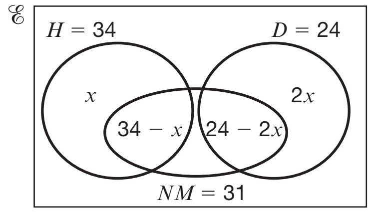


3 The Venn diagram shows H representing horses, D representing donkeys and NM representing non-microchipped
H and *D* do not intersect.



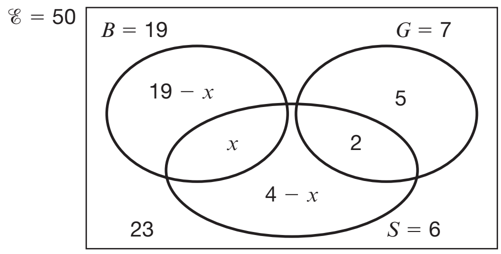
Let x be the number of horses with microchips so $2x$ is the number donkeys with microchips
$\Rightarrow 34 - x$ is the number of horses without microchips, $24 - 2x$ is the number of donkeys without microchips

Putting these into the Venn diagram gives:



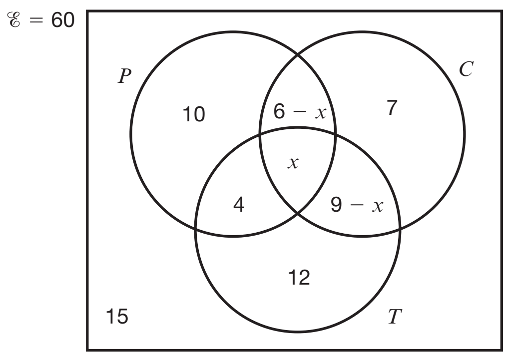
$\Rightarrow 34 - x + 24 - 2x = 31 \Rightarrow 3x = 27 \Rightarrow x = 9$
$\Rightarrow$ horses without microchips $= 34 - x = 25$

4 Let B represent blue cars, G green cars and S soft tops.
The numbers represent the numbers of cars.
B and *G* do not intersect.



The total number of cars is 50
$19 - x + x + 4 - x + 2 + 5 + 23 = 50$
$x = 3$

5 **(a)** Of the 6 that like peppermint and chocolate, some might like toffee.
Of the 9 that like chocolate and toffee, some might like peppermint.
Let P represent peppermint lovers, C chocolate lovers and T toffee lovers.
The numbers represent the numbers of teenagers.



(b) $6 - x$ from the Venn diagram
(c) All the numbers must sum to 60.
$10 + (6 - x) + x + 4 + 7 + (9 - x) + 12 + 15 = 60$
$x = 3$ so 3 teenagers like all three.

NUMBER 7 – BASIC SKILLS EXERCISE

1 2.64

2 4.37

3 0.245

4 6.75

5 5.94

6 0.314

7 27.3

8 2 400 000

9 26 200

10 15.6

11 755 000

12 25.5

13 862 000

14 2.79

15 2.31

16 5.68

17 14.7

18 0.104

19 $\frac{1}{9}$

20 $\frac{2}{9}$

21 $\frac{1}{3}$

22 $\frac{4}{11}$

23 $\frac{7}{11}$

24 $\frac{17}{99}$

25 $\frac{71}{99}$

26 1

27 $\frac{4}{33}$

28 $\frac{34}{99}$

29 $\frac{1}{45}$

30 $\frac{7}{90}$

31 $\frac{25}{333}$

32 $\frac{41}{333}$

33 $\frac{1234}{9999}$

34 $\frac{2468}{9999}$

35 $\frac{11}{15}$

36 $\frac{17}{45}$

37 $\frac{2}{1125}$

38 $\frac{211}{9000}$

NUMBER – 7 EXAM PRACTICE EXERCISE

1 **(a)** 6.99×10^4
(b) 7.85×10^7
(c) 2.90×10^3

2 **(a)** 3.16×10^0
(b) 1.11×10^1
(c) 2.87×10^7

3 **(a)** 0.773
(b) 0.992
(c) 0.129

4 Let $x = 0.0\dot{2}\dot{3}$
$10x = 0.2323\ldots$
$1000x = 23.2323\ldots$

$990x = 23$, so $x = \frac{23}{990}$

Let $y = 0.1\dot{7}$
$10y = 1.777\ldots$
$100y = 17.777\ldots$

$90y = 16$, so $y = \frac{16}{90}$

$x + y = \frac{23}{990} + \frac{16}{90} = \frac{199}{990}$, so

$p = 199$, $q = 990$

5 **(a)** Let $p = 0.xyxy\ldots$

$100p = xy.xyxy\ldots$

$99p = xy = 10x + y$, so $p = \frac{10x + y}{99}$, as required.

(The number xy means there are 10 x's and 1y)

(b) Let $p = 5.xyzxyz\ldots$

$1000p = 5xyz.xyzxyz\ldots$

(The number xyz means there are 100 x's, 10 y's and 1z)

$999p = 5xyz - 5 = 5000 + 100x + 10y + z - 5 = 4995 + 100x + 10y + z$

so $p = \frac{4995 + 100x + 10y + z}{999}$, as required.

ALGEBRA 7 – BASIC SKILLS EXERCISE

1 5, 6

2 −1, 5

3 −1, 4

4 −4, −3

5 2

6 2, 0

7 −3, 5

8 −4, 8

9 $-3 \pm \sqrt{3}$

10 $\frac{-3 \pm \sqrt{3}}{2}$

11 $3 \pm \sqrt{\frac{23}{2}}$

12 $2 \pm \sqrt{5}$

13 $-1 \pm \sqrt{\frac{8}{3}}$

14 $3 \pm \sqrt{\frac{53}{5}}$

15 1.59, 4.41

16 −0.257, 2.59

17 3.11, −1.61

18 −57.9, −4.29

19 −3.30, 0.379

20 −2.45, −0.147

21 $x < -5$ or $x > 3$

22 $-2 \le x \le 7$

23 $-\frac{1}{3} < x < \frac{1}{2}$

24 $x < -3.45$ or $x > 1.45$

25 $2 - \sqrt{2} < x < 2 + \sqrt{2}$

26 $-4 \le x \le -2$ or $3 \le x \le 6$ $x \ge 6$ or $x \le 4$ or $-1.65 \le x \le 3.64$

27 $b = 6$, $c = -7$

28 $(x + 3\pi)(x - \pi) = 0$, $x = -3\pi$ or $x = \pi$

29 $(x + p)(x + q) = 0$, $x = -p$ or $x = -q$

30 $9 - (x - 3)^2$

31 $b = -2$, $c = -4$

32 $x^2 + (x + 2)^2 = 202$, 9 and 11 or −11 and −9

33 $x(x + 3) = 9 \times 5$, $x = 5.37$

34 **(a)** $(x - 2)^2 + 5$
(b) $x = 2$
(c) 5

35 $b = -5$, $c = 6$

36 8 m by 5 m

37 $x^2 + (5 - x)^2 = 4^2$ so the sides are 1.18 cm and 3.82 cm (3 s.f.)

38 $\pi(w + 2)^2 - \pi 2^2 = \pi 2^2$
The width of the path is 0.828 m (3 s.f.)

39 $A + B + C + D = 360°$
$3x^2 + 24x - 315 = 0$
$x^2 + 8x - 105 = 0$
$(x - 7)(x + 15) = 0$
$x = 7$
The angles are $A = 60°$, $B = 120°$, $C = 135°$ and $D = 45°$
$A + B = 180°$ therefore a trapezium (or $C + D = 180°$)

40 $600 < 3x(x + 5) < 700$, $11.9 < x < 13.0$

$(x - 3)(2x + 3) > \frac{1}{2}(x + 2)(2x - 6)$

$x^2 - 2x - 3 > 0$

$x > 3$

41 Area of Rectangle $= (2x + 3)(x - 3)$

$= 2x^2 - 6x + 3x - 9 = 2x^2 - 3x - 9$

Area of Triangle = half × base × height

$= 0.5(2x - 6)(x + 2)$

$= 0.5(2x^2 + 4x - 6x - 12)$

$= 0.5(2x^2 - 2x - 12) = x^2 - x - 6$

For Area of Rectangle > Area of Triangle, then:

$2x^2 - 3x - 9 > x^2 - x - 6$

$2x^2 - x^2 - 3x + x - 9 + 6 > 0$

$x^2 - 2x - 3 > 0$

$(x + 1)(x - 3) > 0$

Solving, $x < -1$ or $x > 3$. As x must be greater than zero then $x > 3$ for the area of the rectangle to be greater than the area of the triangle.

ALGEBRA 7 – EXAM PRACTICE EXERCISE

1 **(a)** $f(x) = -3[x^2 + 4x - 1] = -3[(x + 2)^2 - 5]$

$= -3(x + 2)^2 + 15$

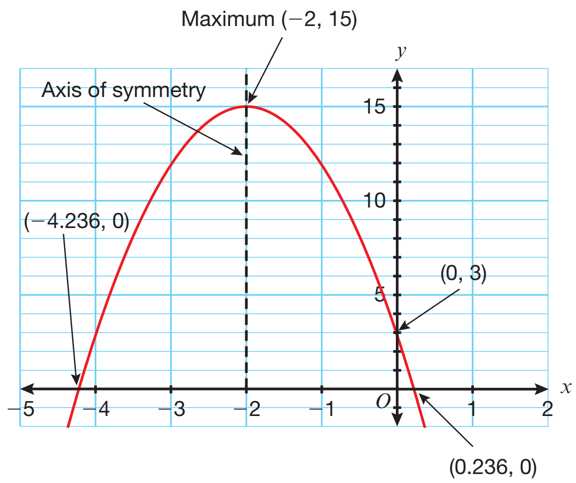
(b) $-3(x + 2)^2 + 15 = 0$

$3(x + 2)^2 = 15$

$(x + 2)^2 = 5$

$x = -2 \pm \sqrt{5}$

(c)



2 **(a)** Formula sheet: Curved surface area of cylinder $= 2\pi rh$, surface area of sphere $= 4\pi r^2$

Surface area of cylinder $= 2\pi r \times 20 = 40\pi r$

The two hemispherical ends have a surface area equal to the surface area of a sphere.

Surface area of ends $= 4\pi r^2$

$4\pi r^2 + 40\pi r \leq 800\pi$

$r^2 + 10r - 200 \leq 0$

Dividing both sides by 4π and rearranging.

$(r + 20)(r - 10) \leq 0$

$0 < r \leq 10$ as $r > 0$

(b) Maximum volume is when $r = 10$

Volume of cylinder $= \pi \times 10^2 \times 20$

$= 2000\pi$

Volume of two hemispheres

$= \frac{4}{3} \times \pi \times 10^3 = \frac{4000}{3}\pi$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Total volume $\frac{10\,000}{3}\pi = 10\,500$ cm^3 to 3 s.f.

3 **(a)** The sum of the internal angles = 360°

$2x^2 + 50 + 3x^2 - 18 + 3x + 40 + 12x + 18 = 360$

$5x^2 + 15x - 270 = 0$

$x^2 + 3x - 54 = 0$

$(x + 9)(x - 6) = 0$

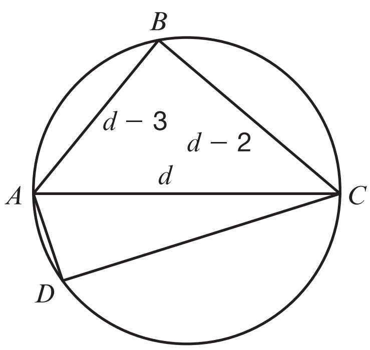
$x = 6$ or $x = -9$

When $x = -9$ the angle $D = -90$ so $x \neq -9$

Substituting $x = 6$ gives $A = 122°$, $B = 90°$, $C = 58°$ and $D = 90°$

Since $B + D = 180°$, $ABCD$ is cyclic (or $A + C = 180°$)

(b) Since $B = 90°$, AC is a diameter.



By Pythagoras' theorem in triangle ABC

$(d - 3)^2 + (d - 2)^2 = d^2$

$d^2 - 6d + 9 + d^2 - 4d + 4 = d^2$

$d^2 - 10d + 13 = 0$

Using the quadratic formula gives

$d = \frac{10 \pm \sqrt{100 - 4 \times 13}}{2}$

$d = \frac{10 \pm \sqrt{48}}{2}$

$d = \frac{10 \pm \sqrt{16 \times 3}}{2}$

$d = \frac{10 \pm 4\sqrt{3}}{2}$

$d = 5 \pm 2\sqrt{3}$

$d = 5 - 2\sqrt{3} \cong 1.5$ means AB and BC are negative so $d = 5 + 2\sqrt{3}$ cm

4 **(a)** Area of garden = $6 \times 8 = 48$ m²
Total area of the flower beds
$= (6 - x)(8 - x) = 48 - 14x + x^2$
$\Rightarrow$ area of path $= 48 - (48 - 14x - x^2)$
$= 14x - x^2$
Or: area of one path is $6x$, area of the other path is $8x$. When added together the overlap of area x^2 is counted twice, so area is $6x + 8x - x^2 = 14x - x^2$.

(b) $(14x - x^2) : 48 = 1 : 4 \Rightarrow \frac{14x - x^2}{48} = \frac{1}{4}$
$\Rightarrow \frac{14x - x^2}{12} = \frac{1}{1} \Rightarrow 14x - x^2 = 12$ or
$x^2 - 14x + 12 = 0$

Using the formula to solve $x^2 - 14x + 12 = 0$,
$a = 1,\ b = -14,\ c = 12$

$\Rightarrow x = \frac{14 \pm \sqrt{196 - 48}}{2} \Rightarrow x = 7 \pm \sqrt{37}$

$\Rightarrow x = 13.1$ or 0.917 (3 s.f.)
$x < 6 \Rightarrow x = 0.917$ m
If $x > 6$ m path takes up more than the width of the garden.

5 **(a)** Volume of sweet is $[\pi(r + 5)^2 - \pi r^2] \times 3$
Volume of hole is $\pi r^2 \times 3$
Ratio of volumes is 4 : 1
so $[\pi(r + 5)^2 - \pi r^2] \times 3 = 4\pi r^2 \times 3$
$r^2 + 10r + 25 - r^2 = 4r^2$
Dividing both sides by π and 3 and simplifying
$4r^2 - 10r - 25 = 0$

(b) $r = \frac{10 \pm \sqrt{100 + 4 \times 4 \times 25}}{2 \times 4}$

$= \frac{10 \pm \sqrt{500}}{8}$

$= \frac{5 \pm 5\sqrt{5}}{4}$

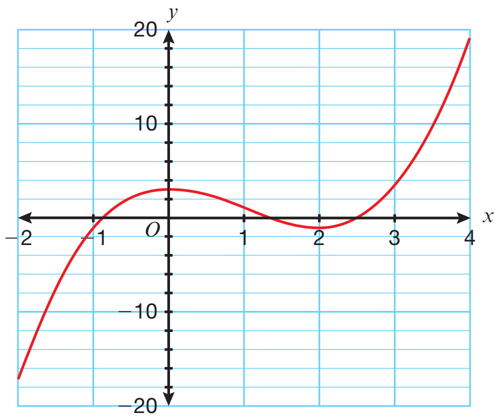
The positive value of $r = 4.0451\ldots$
Volume of sweet is $= 4 \times \pi \times 4.0451^2 \times 3$
Volume of sweet is 4 times the volume of the hole
Volume of sugar is $0.6 \times 4 \times \pi \times 4.0451^2 \times 3$
= 370 mm³ (3 s.f.)
370 mm³ = 0.370 cm³ (3.s.f.)

GRAPHS 6 – BASIC SKILLS EXERCISE

1 **(a)**

x	−2	−1	0	1	2	3	4
y	−17	−1	3	1	−1	3	19

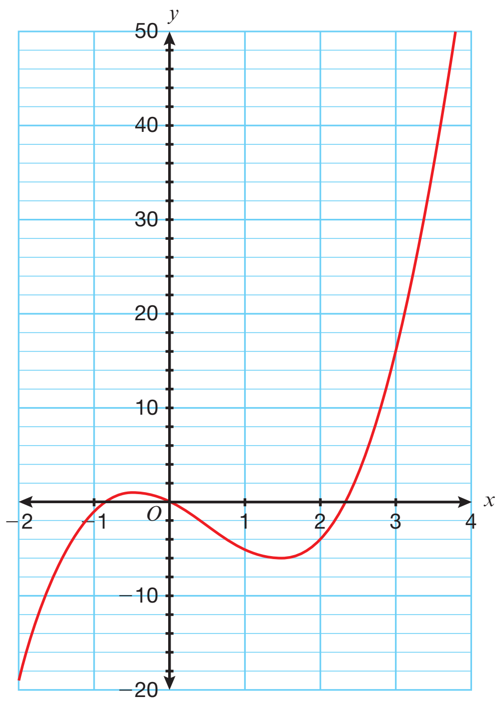
(b)



2 **(a)**

x	−2	−1	0	1	2	3	4
y	−19	0	1	−4	−3	16	65

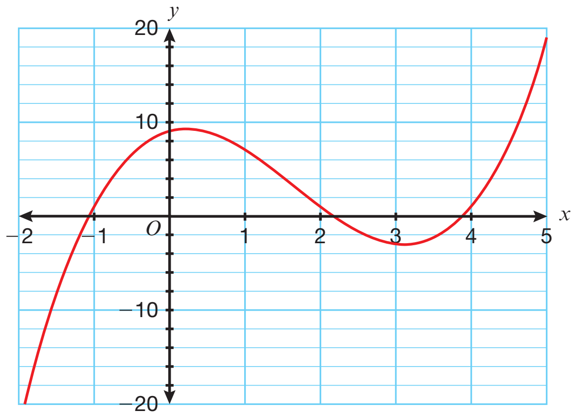
(b)



3 **(a)** $a = -5$

x	−2	−1	0	1	2	3	4	5
y	−23	1	9	7	1	−3	1	18

(b)

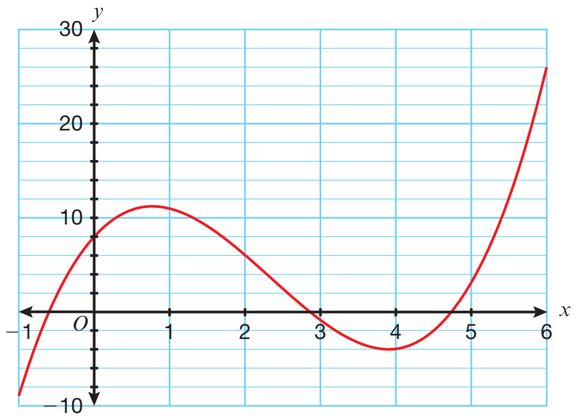


(c) $x \approx -1.1, 2.2$ or 3.8

4 **(a)** $a = -7, b = 8$

x	−1	0	1	2	3	4	5	6
y	−9	8	11	6	−1	−4	3	26

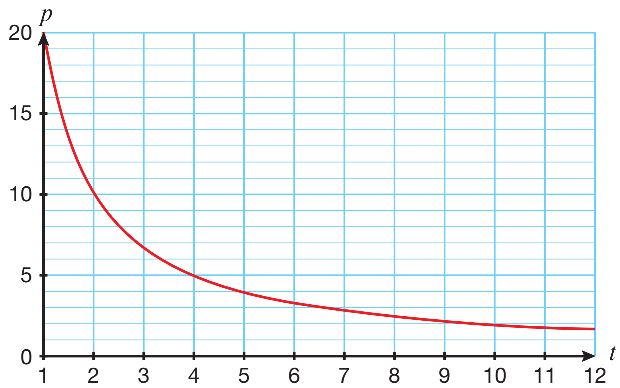
(b)



(c) $x \approx -0.6, 2.8$ or 4.7

5 **(a)**

t	1	2	4	6	8	10	12
p	20	10	5	3.3	2.5	2	1.7



(b) **(i)** $1\frac{2}{3}$ months

(ii) $2222

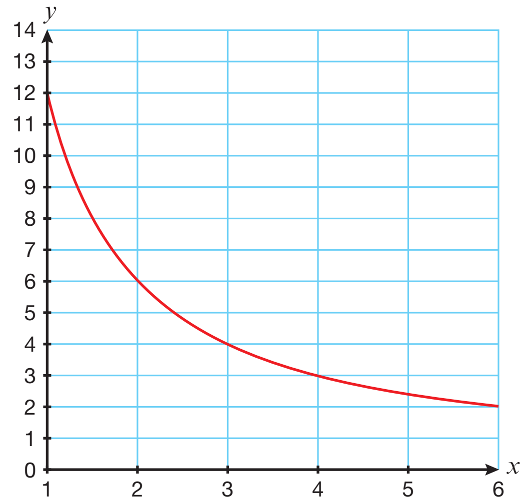
(c) $x = 80$

6 Substitute $x = 6$ into formula for y to find value of $k = 12$

(a) $k = 12$

x	1	2	3	4	5	6
y	12	6	4	3	2.4	2

(b)

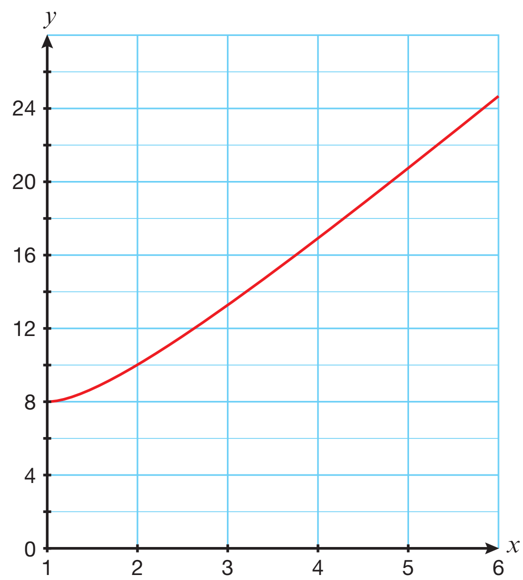


7 Substitute $x = 1$ into formula for y to find value of $a = 4$

(a) $a = 4$

x	1	2	3	4	5	6
y	8	10	13.3	17	20.8	24.7

(b)

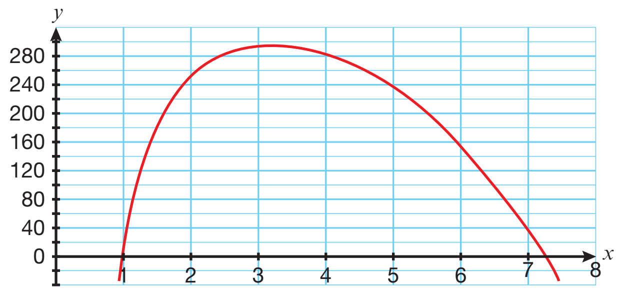


(c) $x = 1, y = 8$

8 **(a)**

x	1	2	3	4	5	6	7
y	9	252	294	284	237	155	34.3

(b)



(c) **(i)** By inspection $y_{max} \approx 295$

(ii) $x^2 + \frac{200}{x} + \frac{200}{x^2} = 210,$

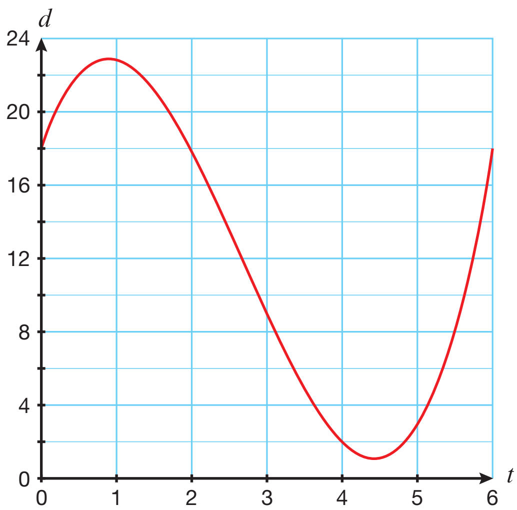
$-x^3 - \frac{200}{x} - \frac{200}{x^2} + 410 = 200$

Draw $y = 200$ and x values in the domain are $x \approx 1.6$ or 5.5

GRAPHS 6 – EXAM PRACTICE EXERCISE

1 **(a)**

t	0	1	2	3	4	5	6
d	18	23	18	9	2	3	18



(b) 23.1 m depth at 00:54

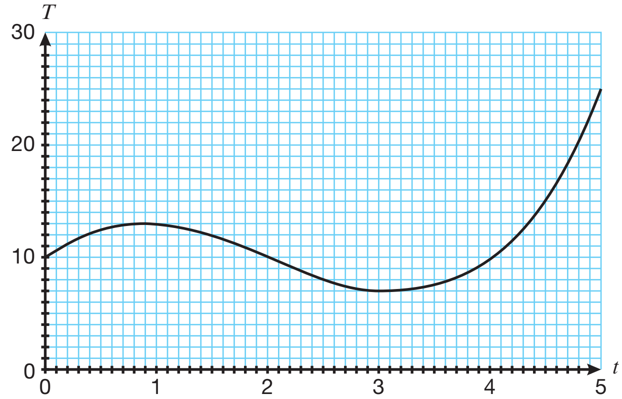
(c) 02:54 ≤ t ≤ 05:36

2 **(a)** Substitute $t = 0$ into formula for T to find value of b, then substitute $t = 5$ into formula for T to find value of a.

(a) $a = -6$, $b = 10$

t	0	1	2	3	4	5
T	10	13	10	7	10	25

(b)

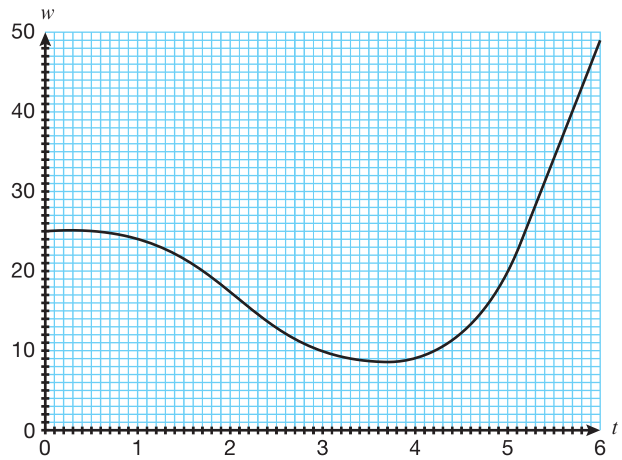


(c) $T = 6.9$°C, $t = 10{:}12$

3 **(a)**

t	0	1	2	3	4	5	6
w	25	24	17	10	9	20	49

(b)



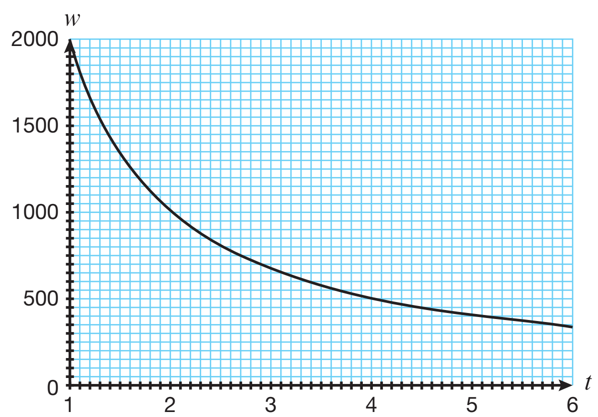
(i) 12.6 mph

(ii) 01:40, 05:00

4 **(a)**

t	1	2	3	4	5	6
w	2000	1000	667	500	400	333

(b)



(i) 571 approx
(ii) 15 March approx

5 Substitute $t = 1$ and $t = 10$ into formula for V to form two simultaneous equations:

$25 = a + b$ [1]

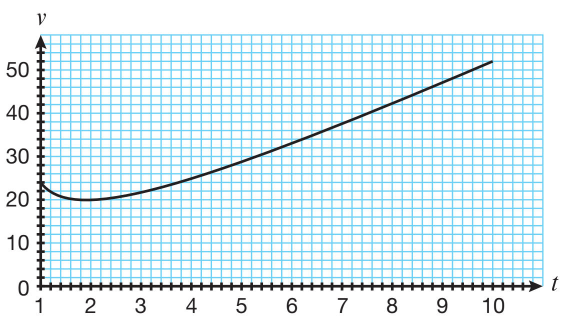
$52 = 10a + \frac{b}{10}$ [2]

Solve these equations to find values of a and b.

(a) $a = 5$, $b = 20$

t	1	2	4	6	8	10
v	25	20	25	33.3	42.5	52

(b)



(c) **(i)** $v = 20$ m/s, $t = 2$. Time = 12:02
(ii) $t \geq 4$. Time for speed to be at least 25 m/s is: 12:04 ≤ time ≤ 12:10

SHAPE AND SPACE 7 – BASIC SKILLS EXERCISE

1 **(a)** $A = 9.72$ cm², $P = 14.3$ cm
(b) $A = 49.1$ cm², $P = 28.6$ cm
(c) $A = 13.1$ cm², $P = 28.2$ cm

2 $r = 4.67$ cm, $A = 34.2$ cm²

3 $x = 100°$, $P = 18$ cm

4 $A = 92.47$ cm²

5 18.5 cm²

6 $P = 153$ mm $A = 625$ mm²

7 $r = 6$ cm, $V = 144\pi$ or 452 cm³

8 4 : 5

9 **(a)** $A = 2369$ mm²
(b) $V = 8247$ mm³

10 **(a)** $V = 320$ cm³
(b) $A = 341$ cm²

11 **(a)** $x = 25.5°$
(b) $V = 126$ cm³
(c) $A = 190$ cm²

12 $V = 24$ cm³

13 **(a)** $P = 20$ cm
(b) $A = 31.4$ cm²

14 $x = 4.5$ cm

15 $\frac{4a}{21}$ cm²

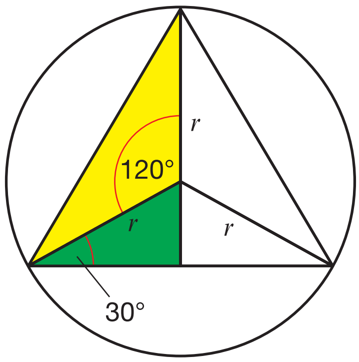
16 **(a)** $V = 2048$ cm³, **(b)** $A = 1875$ cm²

17 $n = 3$

18 **(a)** Diameter of Moon = 3480 km
(b) surface area of Earth = 5.09×10^8 km²

SHAPE AND SPACE 7 – EXAM PRACTICE EXERCISE

1 Let r be the radius of the circle so $\pi r^2 = k\pi$ hence $r = \sqrt{k}$



Area of red triangle is $\frac{1}{2} \times r \times r \times \sin 120°$

$= \frac{r^2}{2} \sin 60° = \frac{\sqrt{3}}{4} r^2$

Area of a triangle $= \frac{1}{2} ab\sin C$, $\sin 60° = \frac{\sqrt{3}}{2}$

The area of the equilateral triangle is $\frac{3\sqrt{3}}{4} r^2 = \frac{3\sqrt{3}}{4} k$

OR

The base of the green triangle is

$r\cos 30° = \frac{r\sqrt{3}}{2}$

Height of the the green triangle is

$r\sin 30° = \frac{r}{2}$

The area of the green triangle is

$\frac{1}{2} \times \frac{r\sqrt{3}}{2} \times \frac{r}{2} = \frac{\sqrt{3}}{8} r^2$

The area of the equilateral triangle is

$6 \times \frac{\sqrt{3}}{8} r^2 = \frac{3\sqrt{3}}{4} r^2 = \frac{3\sqrt{3}}{4} k$

Six green triangles make up the equilateral triangle.

There are other equally valid ways of calculating the area of the triangle.

The blue area is $k\pi - \frac{3\sqrt{3}}{4}k$

$= k\left(\pi - \frac{3\sqrt{3}}{4}\right)$ cm^2

2 **(a)** Curved surface area of a cylinder $= 2\pi rh$

Inside height is $d = 2r \Rightarrow$

Curved surface area $= 2\pi r \times 2r = 4\pi r^2$

Total inside surface area

$= \pi r^2 + 4\pi r^2 = 250$ Base is πr^2

$5\pi r^2 = 250$

$r = \sqrt{\frac{50}{\pi}} = 3.989...$ cm

Volume of a cylinder $= \pi r^2 h$

Volume $= \pi r^2 \times 2r = 2\pi r^3 = 400$ cm^3 (2 s.f.)

(b) Area scale factor $= \frac{360}{250} = \frac{36}{25}$

Length scale factor $= \sqrt{\frac{36}{25}} = \frac{6}{5}$

Volume scale factor $= \left(\frac{6}{5}\right)^3 = \frac{216}{125}$

or 216 : 215

3 The maximum number of pieces is when the cube has maximum volume, the cylinder has minimum outside diameter and maximum inside diameter and the pieces have minimum length.

Therefore, the cube side length is 3.05 cm, the cylinder has outside diameter 4.95 mm and inside diameter 3.05 mm and piece length of 5.95 mm.

Working in mm

$30.5^3 = \frac{\pi}{4}(4.95^2 - 3.05^2) \times l$

$l = 2376.65...$

number of pieces $= 2376.65.... \div 5.95 = 399.4...$

Number of pieces must be an integer so number is 399.

4 **(a)** Flat end area is

$\pi(kr)^2 - \pi r^2 = \pi r^2(k^2 - 1)$

$A = 2\pi r^2(k^2 - 1)$

Remember there are two flat end faces.

Curved outer area is

$2\pi kr \times r = 2\pi krz^2$

Curved area = circumference × height

Curved inner area is

$2\pi r \times r = 2\pi r^2$

$B = 2\pi kr^2 + 2\pi r^2$

$= 2\pi r^2(k + 1)$

$A : B = \frac{2\pi r^2(k^2 - 1)}{2\pi r^2(k + 1)}$

$= \frac{(k + 1)(k - 1)}{(k + 1)}$

$= k - 1$

$k^2 - 1 = (k + 1)(k - 1)$

'difference of two squares'

(b) Length scale factor = 2 so the area scale factor = 4.

Both areas will be 4 times larger so the ratio will not change.

5 **(a)** The cone that is removed is similar to the original cone.

The area scale factor is $= \frac{a}{4} \div a = \frac{1}{4}$

The length scale factor $= \sqrt{\frac{1}{4}} = \frac{1}{2}$

i.e. removed cone is half the height of the original cone.

Volume scale factor $= \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

So the removed cone has a volume $= \frac{1}{8}V$

Therefore, the volume of truncated cone $= \frac{7}{8}V$ cm^3

(b) The total surface area of the removed cone $= \frac{A}{4}$ Area scale factor is $\frac{1}{4}$

Curved surface area of removed cone

$= \frac{A}{4} - \frac{a}{4}$

Base area of removed cone $= \frac{a}{4}$

Surface area of truncated cone is surface area of original cone less curved surface of removed cone plus surface area of top of truncated cone.

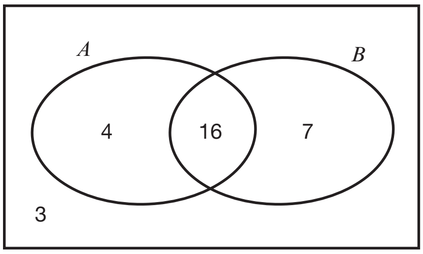
Surface area of truncated cone

$A - \left(\frac{A}{4} - \frac{a}{4}\right) + \frac{a}{4} = A - \frac{A}{4} + \frac{a}{4} + \frac{a}{4}$

$= \frac{3A}{4} + \frac{a}{2}$

SETS 3 – BASIC SKILLS EXERCISE

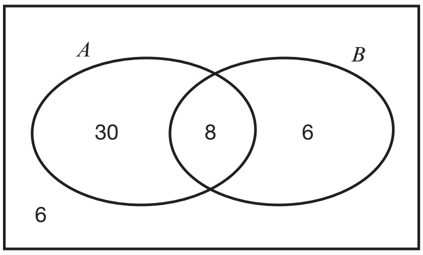
1 **(a)** $n(\mathscr{E}) = 30$



(b) **(i)** $\frac{16}{30} = \frac{8}{15}$

(ii) $\frac{11}{30}$

2 **(a)** $n(\mathscr{E}) = 50$

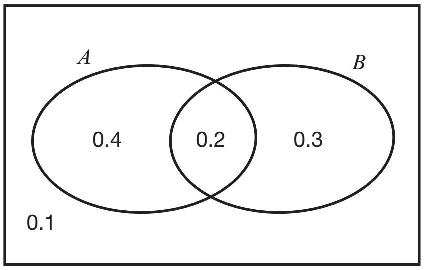


(b) **(i)** $\text{P}(B) = \frac{14}{50} = 0.28$

(ii) $\text{P}(A \cup B) = \frac{44}{50} = 0.88$

(iii) $\text{P}(A \cap B) = \frac{8}{50} = 0.16$

3 $\mathscr{E}$



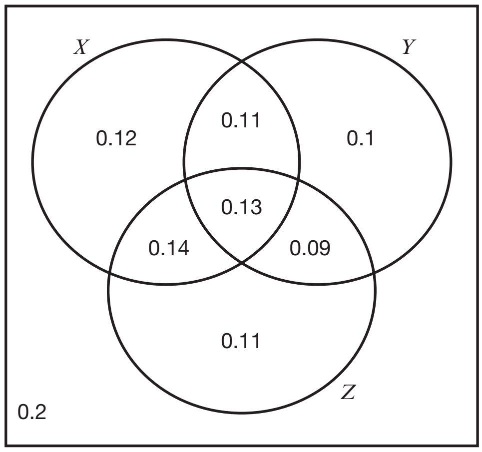
(a) $\text{P}(A \cap B) = 0.2$
(b) $\text{P}(A \cup B') = 0.7$

4 **(a)** 45
(b) **(i)** 29
(ii) 11
(c) **(i)** $\frac{20}{45} = \frac{4}{9}$

(ii) $\frac{7}{45}$

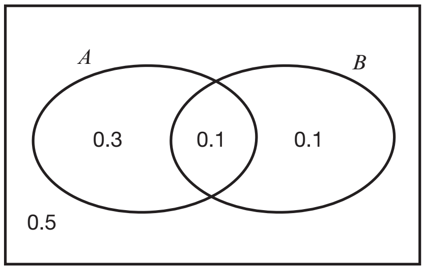
(iii) $\frac{16}{45}$

5 $\mathscr{E}$



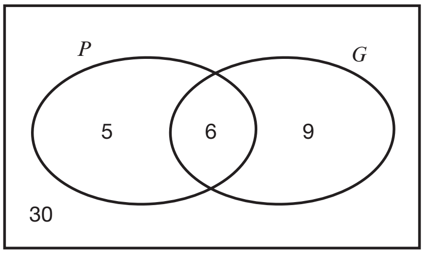
(a) $\text{P}(X \cup Y) = 0.69$
(b) $\text{P}(Y \cap Z') = 0.21$
(c) $\text{P}(X \cap (Y \cup Z')) = 0.36$

6 $\mathscr{E}$



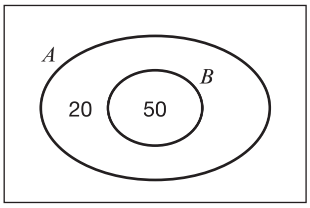
(a) $P(B) = 0.2$
(b) $P(B|A) = \frac{0.1}{0.4} = \frac{1}{4} = 0.25$

7 $n(\mathscr{E}) = 50$



$\frac{6}{11}$

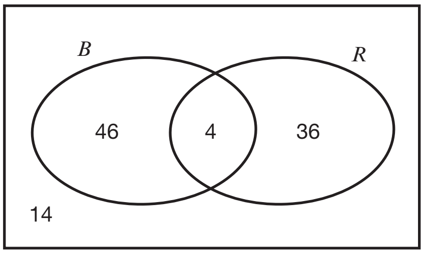
8 $n(\mathscr{E}) = 100$



(a) $\text{P}(A|B) = \frac{50}{50} = 1$

(b) $\text{P}(B|A) = \frac{50}{70} = \frac{5}{7}$

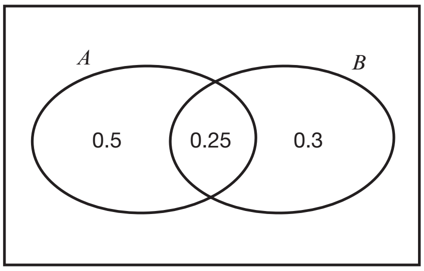
9 **(a)** $n(\mathscr{E}) = 100$



(b) $\text{P}((B \cup R)') = \frac{14}{100} = 0.14$

(c) $\text{P}(R|B) = \frac{4}{50} = 0.08$

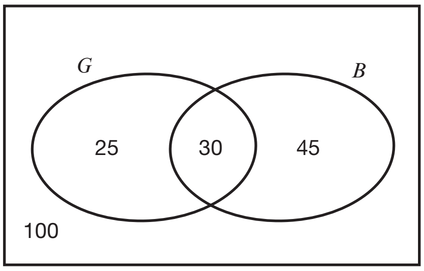
10 $\mathscr{E}$



(a) $\text{P}(B) = 0.55$

(b) $\text{P}(A'|B) = \frac{0.3}{0.55} = \frac{6}{11}$

11 $n(\mathscr{E}) = 200$



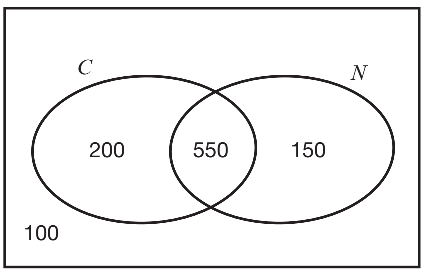
(a) $(2x + 1) + (3x - 6) + (4x - 3) + 100 = 200$
$x = 12$

(b) $\text{P}(B) = \frac{75}{200} = \frac{3}{8} = 0.375$

(c) $\text{P}(B|G) = \frac{30}{55} = \frac{6}{11}$

(d) $\text{P}(G|B) = \frac{30}{75} = \frac{2}{5} = 0.4$

12 $n(\mathscr{E}) = 1000$

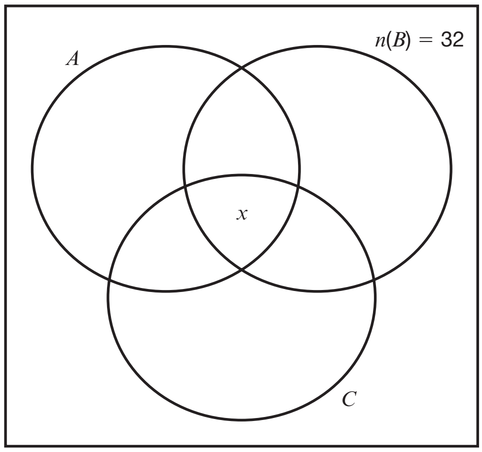


(a) $\frac{370}{1000} = \frac{37}{100}$

(b) $\frac{370}{570} = \frac{37}{57}$

(c) $\frac{200}{530} = \frac{20}{53}$

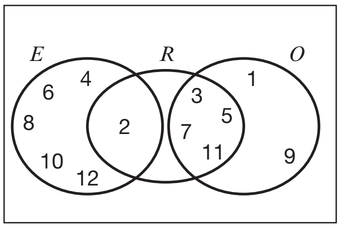
13 $n(\mathscr{E}) = 100$



$\text{P}((A \cap C)|B) = \frac{1}{8} = \frac{x}{32}$ so $x = 4$

$\text{P}(A \cap B \cap C) = \frac{4}{100} = \frac{1}{25} = 0.04$

14 **(a)** $n(\mathscr{E}) = 12$



(b) **(i)** $\text{P}(R \cap E) = \frac{1}{12}$

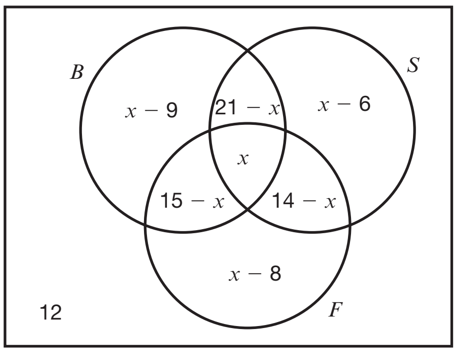
(ii) $\text{P}(R \cap E \cap O) = 0$

(iii) $\text{P}((R \cup E)') = \frac{2}{12} = \frac{1}{6}$

(c) $\text{P}(R|O) = \frac{4}{6} = \frac{2}{3}$

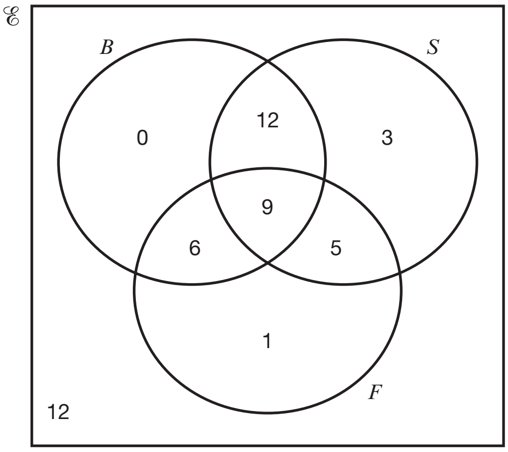
15 **(a)** Let $x = n(B \cap S \cap F)$

$\mathscr{E}$



Summing, putting expression = 48 and solving gives $x = 9$

(b)



(i) $\frac{9}{48} = \frac{3}{16}$

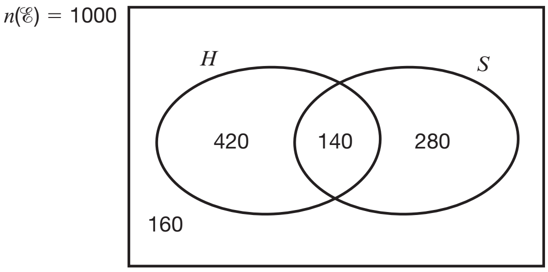
(ii) $\frac{23}{48}$

(c) $\frac{27}{36} = \frac{3}{4}$

SETS 3 – EXAM PRACTICE EXERCISE

1 **(a)** $2x(x + 1) + 10x + x^2 + 6x + 160 = 1000$
Number of spectators sums to 1000
$3x^2 + 18x - 840 = 0$
$x^2 + 6x - 240 = 0$
$(x + 20)(x - 14) = 0$
$x = 14$ or use quadratic formula
$x = -20$ is not possible

Venn diagram becomes (numbers represent number of spectators)



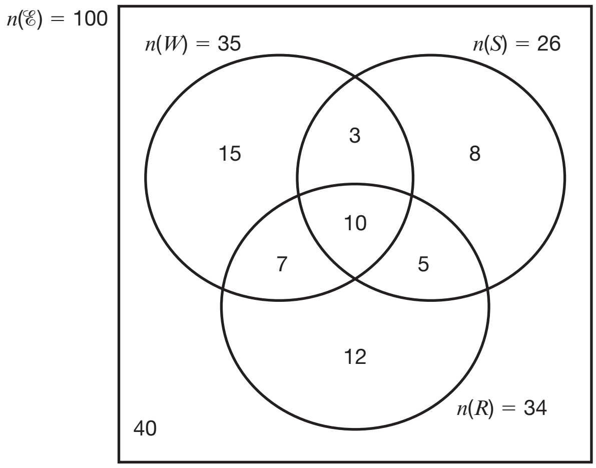
(b) **(i)** $\frac{420 + 140 + 280}{1000} = \frac{840}{1000} = 0.84$

or $\frac{1000 - 160}{1000} = \frac{840}{1000} = 0.84$

(ii) $\frac{420 + 280}{1000} = \frac{700}{1000} = 0.7$

(c) $\frac{140}{140 + 280} = \frac{140}{420} = \frac{1}{3}$

2 **(a)**



(b) **(i)** $\frac{34}{100} = \frac{17}{50} = 0.34$

(ii) $\frac{12}{100} = \frac{3}{25} = 0.12$

(iii) $\frac{7}{100} = 0.07$

(c) $P(S|R) = \frac{15}{34}$

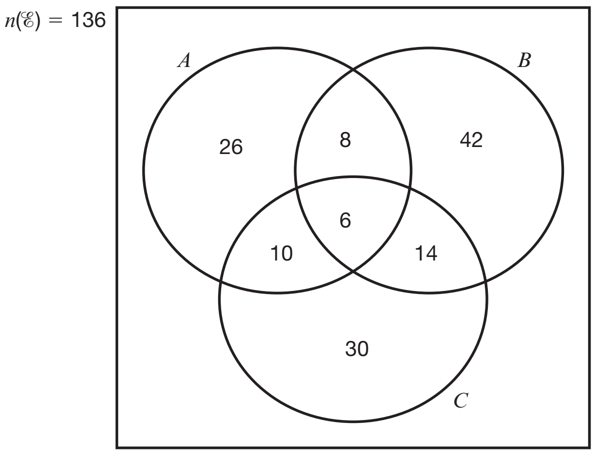
3 **(a)** $P(A|B) = 0.2$

$\frac{x + 3 + y}{70} = 0.2$

$P(C|A) = 0.32$

$\frac{2x + y}{50} = 0.32$

Simplifying gives:
$x + y = 11$
$2x + y = 16$
Solving simultaneously gives
$x = 5$ and $y = 6$
The Venn diagram can now be filled in.



Total number of elements = 136

(b) $P(A \cap C) = \frac{16}{136} = \frac{2}{17}$

(c) $P(B|C) = \frac{20}{60} = \frac{1}{3}$

4 **(a)** $P((M \cap E)|S) = \frac{4}{21}$

$\frac{x}{42} = \frac{4}{21}$

$x = 8$

$P((E \cap S)|E) = \frac{3}{8}$

$\frac{y+8}{64} = \frac{3}{8}$

$y = 16$

$n(E \cap M) = 40$

$z + 8 = 40$

$z = 32$

$n(M \cap S) = 10$

$w + 8 = 10$

$w = 2$

$n(E) = 64$

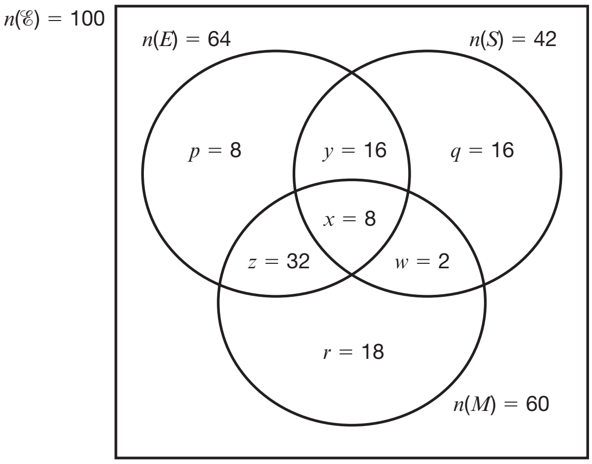
$p = 8$

$n(S) = 42$

$q = 16$

$n(M) = 60$

$r = 18$



(b) **(i)** $\frac{18}{60} = \frac{3}{10} = 0.3$

(ii) $\frac{w+x+y+z}{100} = \frac{16+8+32+2}{100}$

$= \frac{58}{100} = \frac{29}{50} = 0.58$

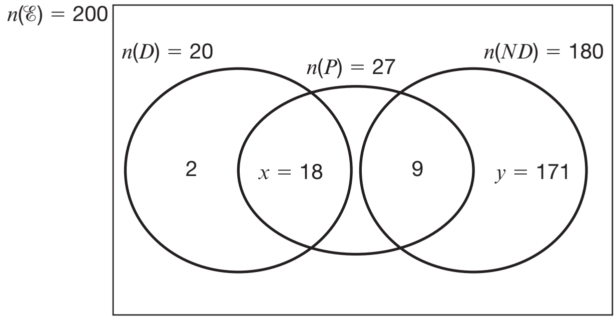
(c) $\frac{y+z}{64} = \frac{48}{64} = \frac{3}{4} = 0.75$

5 **(a)** $n(D) = 0.1 \times 200 = 20$

$n(ND) = 200 - 20 = 180$ or 0.9×200

$\frac{x}{20} = 0.9 \Rightarrow x = 18$ $\frac{y}{180} = 0.95 =$

$h \Rightarrow y = 171$



(b) $\frac{9}{9+18} = \frac{1}{3}$

(c) $\frac{2}{2+171} = \frac{2}{173}$

NUMBER 8 – BASIC SKILLS EXERCISE

1 **(a)** 5×10^5 cm
(b) 8×10^{-5} km
(c) 200 km
(d) 0.6 mm

2 9×10^{-6} km

3 2×10^5 micrometres

4 Number of thicknesses of paper $= 2^{50}$
height $= 2^{50} \times 0.004 \times 25.4 \times 10^{-6}$
$= 1.14 \times 10^8$ km

5 **(a)** 5×10^2 m²
(b) 4×10^7 cm²
(c) 4×10^6
(d) 2 km²

6 1.96×10^{-4} km²

7 1.75×10^{25} cm²

8 5.40×10^{-13} km²

9 **(a)** 2×10^9ml
(b) 4×10^9 m³
(c) 80 m³
(d) 7×10^9 mm³

10 **(a)** 980 ml
(b) 0.98 litres
(c) 9.8×10^{-13} km³

11 Volume of rain drop $= \frac{4}{3} \times \pi \times 0.75^3$ mm³,

volume of reservoir $= 1.24 \times 10^8 \times 10^9$ mm³.
The number of raindrops
$= (1.24 \times 10^8 \times 10^9)$
$\div \left(\frac{4}{3} \times \pi \times 0.75^3\right) = 7.02 \times 10^{16}$

12 1 litre of a chemical contains $3 \times 10^{22} \times 10^3 = 3 \times 10^{25}$ molecules.
Volume of ocean is $1.15 \times 10^{10} \times 10^9$ m^3 = $1.5 \times 10^9 \times 10^9 \times 10^3$ litres = 1.5×10^{21} litres
Number of molecules per litre is
$(3 \times 10^{25}) \div (1.5 \times 10^{21}) = 2 \times 10^4$ or 20 000

13 1230 km/h

14 133 mm

15 0904

16 4.5×10^{-3} seconds

17 2.92 mm

18 Time taken is 19 h 19 mins,
average speed = 880 km/h = 244 m/s

19 50 m^3

20 £361.18

21 1.87 g/cm^3

22 4.45 g/cm^3

23 Density of A is 0.911 (floats), of B is 1.1 (sinks), of C is 0.802 (floats)

24 $\frac{a + 0.001b}{2}$

25 3×10^{-4} N

26 20 400 N/m^2

27 1 000 000 cm^2

28 4500 cm^3

29 102 N

30 1.76×10^5 N/m^2

31 Cylinder volume = $\pi \times 6^2 \times 10$
= 360π cm^3
Dimensions changed to cm.

12 tonnes/m^3 = $\frac{12 \times 1000}{10^6}$

= 1.2×10^{-2} kg/cm^3
Mass of cylinder = $360\pi \times 1.2 \times 10^{-2}$

= $\frac{108}{25}\pi$ kg.

This exerts a force of $10 \times \frac{108}{25}\pi = \frac{216}{5}\pi$ N on the table.

Area in contact with the table = $\pi \times 6^2$
= 36π cm^2
The pressure is $\frac{216}{5}\pi \div 36\pi = \frac{6}{5}$ N/cm^2
or 1.2 N/cm^2

NUMBER 8 – EXAM PRACTICE EXERCISE

1 **(a)** mass = density × volume
3 cm^3 of gold has a mass of 3×19.3
= 57.9 g
1 cm^3 of silver has a mass of 10.5 g.
4 cm^3 of the alloy has a mass of
57.9 + 10.5 = 68.4 g
density of alloy is $\frac{68.4}{4}$ = 17.1 g/cm^3

(b) x cm^3 of gold has a mass of $x \times 19.3$ g
1 cm^3 of silver has a mass of 10.5 g
$1 + x$ cm^3 of the alloy has a mass of
$19.3x + 10.5$ g
density of alloy is $\frac{19.3x + 10.5}{1 + x}$
= 18.1 g/cm^3
solving for x gives
$19.3x + 10.5 = 18.1(1 + x)$
$19.3x + 10.5 = 18.1 + 18.1x$
$1.2x = 7.6$
$x = \frac{19}{3}$

2 **(a)** The candle uses $\frac{1}{15} \times 60 = 4$ g of wax every hour
The wax has a density of $900 \times \frac{1000}{10^9}$
= 9×10^{-4} g/mm^3
4 g of wax has a volume of $\frac{4}{9 \times 10^{-4}}$
= 4444.4... mm^3
Let the distance between marks be
x mm
Volume of cylinder is $\pi r^2 h$ and $r = 10$ mm
$\Rightarrow x \times \pi \times 10^2 = 4444.4.. \Rightarrow x = 14.147...$
or 14.1 mm to 3 s.f.

(b) Volume of cone = $\frac{1}{3}\pi r^2 h$, top half has
$r = 20$ mm and $h = 100$ mm
OR work out volume of whole cone,
top half has $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$ of volume by similarity
Volume of top half of cone
= $\frac{1}{3} \times \pi \times 20^2 \times 100 = \frac{40\,000\pi}{3}$ mm^3

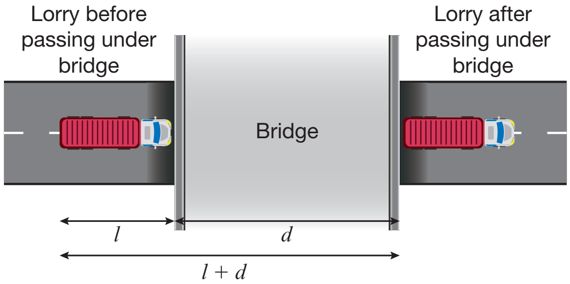
Weight of top half of cone

$= \frac{40\,000\pi}{3} \times 9 \times 10^{-4} = 12\pi$ g

Time taken $12\pi \div 4 = 3\pi$ hours

Uses 4 g of wax every hour (see part a)

3 **(a)** The diagram shows the lorry just as the front passes under the bridge and just when the back leaves the bridge.



The back of the lorry travels a distance l + d metres in a time of t seconds

OR the front of the lorry travels a distance $l + d$ metres in a time of t seconds

$\Rightarrow t = \frac{l + d}{\text{speed}}$

Speed must be in consistent units, i.e. m/s

v km/h = 1000v m/h = $\frac{1000v}{3600} = \frac{5v}{18}$ m/s

$t = \frac{18(l + d)}{5v}$

(b) $t = \frac{18(l + d)}{5v}$

$vt = \frac{18(l + d)}{5}$

$v = \frac{18(l + d)}{5t}$

Substituting values gives

$v = \frac{18(16.5 + 60)}{5 \times 2.7}$

v = 102 km/h

Units in the expression are lengths in metres, time in seconds and speed in km/h

102 km/h = $\frac{102}{1.6}$ = 63.75 mph,

so it is breaking the speed limit

4 **(a)** Area of hose = $\pi \times 6^2$ mm²

16 m/s = 16 000 mm

volume of water that comes out of hose in one second

$= \pi \times 6^2 \times 16\,000$ mm³

$= (\pi \times 6^2 \times 16\,000) \div 1000$ cm³

$= 576\pi$ cm³

1000 cm³ = 1 litre so 576π cm³

$= 0.576\pi$ litres/s

Time taken to fill pond $= \frac{20\,000}{0.576\pi}$ seconds

$= \frac{20\,000}{0.576 \times \pi \times 60}$ minutes

= 184.207… minutes = 3 hours

4 minutes (and 12.4 seconds)

(b) Area of the hose $= \pi \times \frac{d^2}{4}$ mm²

16 m/s = 16 000 mm/s

Volume of water that comes out of the hose every second is $= \pi \times \frac{d^2}{4} \times 16\,000$

$= 4000\pi d^2$ mm³ $= 4000\pi d^2 \div 1000$ cm³

$= 4\pi d^2$ cm³ $= 4\pi d^2 \div 1000$ litres

$= 0.004\pi d^2$ litres

2 hours is 2 × 60 × 60 seconds

= 7200 seconds

$7200 = 15\,000 \div 0.004\pi d^2 \Rightarrow d^2$

$= \frac{15\,000}{7200 \times 0.004\pi} = 165.78\ldots$

d = 12.9 (3 s.f.)

5 Volume of A $= \pi \times r^2 \times h$ cm³ so the mass of A $= \pi r^2 hd$ kg

Force A exerts on table $= 10\pi r^2 hd$ N

Each kg exerts a force of 10N

Area of A in contact with table $= \pi r^2$ cm²

pressure exerted by A on table =

$\frac{10\pi r^2 hd}{\pi r^2} = 10hd$ N/cm²

Volume of B $= \pi \times R^2 \times hc$m³ so the mass of A $= \pi R^2 hd$ kg

Force B exerts on table $= 10\pi R^2 hd$ N

Each kg exerts a force of 10N

Area of B in contact with table $= \pi R^2$ cm²

The pressure exerted by B on table =

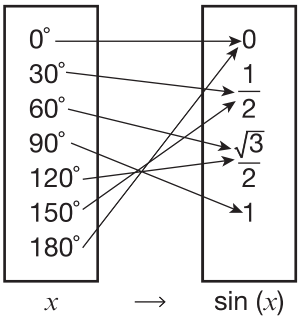
$\frac{10\pi R^2 hd}{\pi R^2} = 10hd$ N/cm²

The pressure exerted by each cylinder is the same and equal to $10hd$ N/cm²

ALGEBRA 8 – BASIC SKILLS EXERCISE

1 **(a)** Function as any vertical line only cuts the graph once
(b) Not a function as a vertical line can cut the graph more than once
(c) Not a function as there is no value when $x = 0$
(d) Not a function as a vertical line can cut the graph more than once

2 **(a)**



(b) It is a function as it is a many-to-one mapping.

3 $f : x \rightarrow 4x^2 + 12x + 9$

4 **(a)** 1
(b) 11
(c) 7
(d) $7 - 2y^2$

5 **(a)** **(i)** 1.5
(ii) 2.5
(b) 0 as $1 \div 0$ is undefined

6 **(a)** 2
(b) 0

7 **(a)** 2, 3
(b) −1, 6

8 **(a)** $2x + 2$
(b) $2x + 1$

9 **(a)** $8 + 6x$
(b) $4 + 6x$

10 $f(x) = x^2 + 6x$

11 $f(x) = 4x^2$

12 −5

13 1 or $-\frac{2}{3}$

14 $a = -2$, $b = 4$

15 $f(a) = a^2 + b$, $f(b) = ab + b$
$f(a) = f(b)$
$a^2 + b = ab + b$
$a^2 - ab = 0$
$a(a - b) = 0$
$a = 0$ or $a = b$

16 **(a)** $x = \frac{1}{2}$
(b) $x = 0$
(c) $x < -3$
(d) $x \geq 3$

17 **(a)** $-3 < x < 3$
(b) None
(c) $x = 180n$, n is an integer
(d) $x = 90 + 180n$, n is an integer

18 **(a)** All real numbers
(b) $g(x) \geq -1$
(c) $h(x) \geq 0$

19 **(a)** range = {−1, 0, 3}
(b) Range is all real numbers ≥ 0
(c) Range is all real numbers ≥ −4

20 **(a)** 12
(b) 3
(c) 16
(d) −1

21 $x = \frac{3}{4}$

22 $k = 1$ or $k = -\frac{1}{3}$

23 $-\frac{1}{2}$

24 $k = 2$ or $k = -1$

25 $2 - 3x^2$

26 **(a)** $2 - \frac{x}{7}$
(b) $x^2 - 5$
(c) $\frac{4}{x + 2}$

27 $f : x \rightarrow \frac{x + 2}{1 - x}$

28 7

29 $f(x) = x$

30 **(a)** $f^{-1}(x) = \frac{x}{x - 1}$
(b) Self inverse

31 $f(x) = \frac{x+3}{4}$

32 $f(x) = \frac{1}{1-x}$

33 **(a)** **(i)** x

(ii) x

(b) Inverse of each other

(c) π

34 **(a)** **(i)** $\sqrt{(3x+3)}$

(ii) $3\sqrt{(x+1)} + 2$ or $3\sqrt{x+5}$

(b) **(i)** $x < -1$

(ii) $x < 0$

35 $g(x) = \frac{5x+4}{3}$ $g(x)$ is the inverse of $f(x)$

36 **(a)** **(i)** $1 + 3x$

(ii) $5 + 3x$

(b) $(fg)^{-1}(x) = g^{-1}f^{-1}(x) = \frac{x-1}{3}$ because

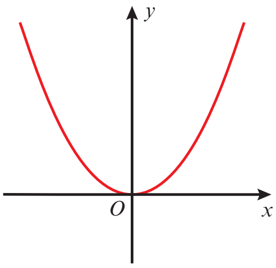
$g^{-1}(x) = \frac{x+1}{3}$ and $f^{-1}(x) = x - 2$

37 $a = -2,\ b = 4$

38 $a = \frac{1}{10}$

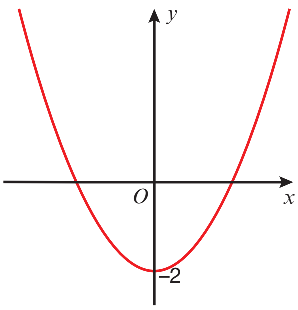
39 $g(x) = 3x + 1$ $\quad g(x) = [(6x - 5) + 7] \div 2$

40 **(a)** Range is $\{y : y \geq 0\}$



(b) $y = \sqrt{x}$, Restriction on domain $x \geq 0$

41 **(a)** Range is $\{y : y \geq -2\}$



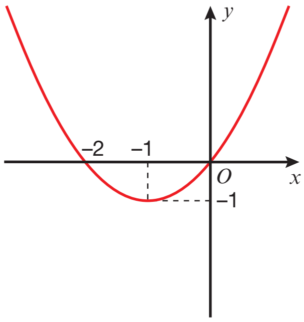
(b) $y = \sqrt{x+2}$, Restriction on domain $x \geq -2$

42 **(a)** $f(x) = (x+1)^2 - 1$

(b) $\{y : y \geq -1\}$

(c) $y = (x+1)^2 - 1 \Rightarrow (x+1)^2$
$= y + 1 \Rightarrow x = \sqrt{y+1} - 1 \Rightarrow f^{-1}(x)$
$= \sqrt{x+1} - 1$

Restriction on domain $x \geq -1$



43 **(a)** $f(x) = (x-1)^2 - 1 + 4 = (x-1)^2 + 3$

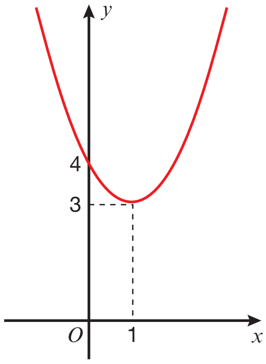
(b) Function is a positive quadratic, minimum value at (1, 3)

Cuts the y axis at (0, 4)

Range is $\{y : y \geq 3\}$

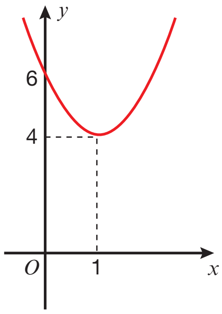
(c) $y = (x-1)^2 + 3 \Rightarrow y - 3 = (x-1)^2$
$\Rightarrow x - 1 = \sqrt{y-3} \Rightarrow x = \sqrt{y-3} + 1$
$\Rightarrow f^{-1}(x) = \sqrt{x-3} + 1$

Restriction on domain $x \geq 3$



44 **(a)** $f(x) = 2[x^2 - 2x + 3]$
$= 2[(x-1)^2 + 2] = 2(x-1)^2 + 4$

(b) $\{y : y \geq 4\}$



(c) $y = 2(x-1)^2 + 4 \Rightarrow \frac{y-4}{2} = (x-1)^2$
$\Rightarrow x = \sqrt{\frac{y-4}{2}} + 1$
$\Rightarrow f^{-1}(x) = \sqrt{\frac{x-4}{2}} + 1$

Restriction on domain $x \geq -4$

ALGEBRA 8 – EXAM PRACTICE EXERCISE

1 **(a)** **(i)** $y = 7 - x$
$x = 7 - y$
$f^{-1}(x) = 7 - x$
(ii) $g(x)$ and $g^{-1}(x)$ are self inverse
(b) $(x - 1)^2 < 7 - x$
$x^2 - x - 6 < 0$
$(x - 3)(x + 2) < 0$
$-2 < x < 3$
A sketch of $y = (x + 2)(x - 3)$ is a positive quadratic passing through $(-2, 0)$ and $(3, 0)$ showing $x < 0$ for $-2 < x < 3$

2 **(a)** $fg(x) = f(2x + 1)$
$= (2x + 1 - 1)^2$
$= 4x^2$
$gf(x) = g[(x - 1)^2]$
$= 2(x - 1)^2 + 1$
$= 2(x^2 - 2x + 1) + 1$
$= 2x^2 - 4x + 3$
$2fg(x) = gf(x)$
$8x^2 = 2x^2 - 4x + 3$
$6x^2 + 4x - 3 = 0$
(b) $2fg(k) = gf(k)$
$6k^2 + 4k - 3 = 0$
Solving for k using the quadratic formula

$$k = \frac{-4 \pm \sqrt{4^2 - 4 \times 6 \times -3}}{2 \times 6}$$

$$= \frac{-4 \pm \sqrt{88}}{12}$$

$$= \frac{-4 \pm \sqrt{4 \times 22}}{12}$$

$$= \frac{-4 \pm 2\sqrt{22}}{12}$$

$$= \frac{-2 \pm \sqrt{22}}{6}$$

So $p = -2$, $q = 22$ and $r = 6$

3 $f(3) = 5$
$5 = 3a + b$ Equation 1
$g(4) = f(2)$ so $g(4) = 2a + b$
$g^{-1}(x) = 3x - 5$
$y = 3x - 5$
$3x = y + 5$
$x = \frac{y + 5}{3}$
$g(x) = \frac{x + 5}{3}$
$g(4) = 3$
$3 = 2a + b$ Equation 2

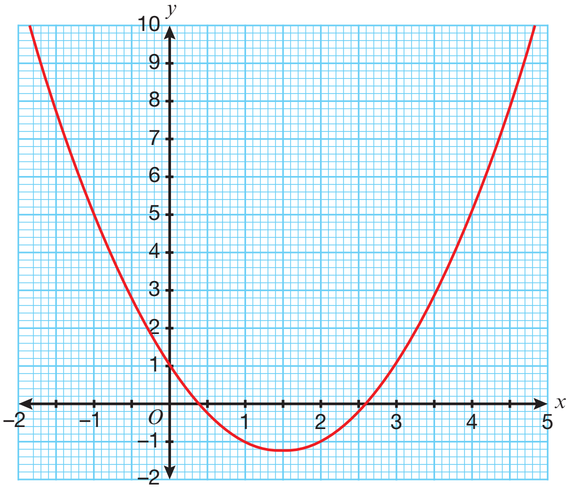
Solving Equation 1 and Equation 2 simultaneously
$5 = 3a + b$ (1)
$3 = 2a + b$ (2)
$2 = a$ Subtracting (2) from (1)
$b = -1$ Subtracting $a = 2$ into either (1) or (2)
$f(x) = 2x - 1$
$7 = 2x - 1$
$x = 4$
$f^{-1}(7) = 4$
Or
$f^{-1}(x) = \frac{x + 1}{2}$
$f^{-1}(7) = 4$

4 Answers will differ as read from a graph.
(a) **(i)** $fg(2) = 3.8$
(ii) $gf(4) = 6.2$
(iii) $gf^{-1}(3) = 0.57$
(b) $k > 0.4$
(c) -0.6 or 2.5

5 **(a)** Let $f(x)$ be the radius and x the area.
$f(x)$ is the output of a function, x the input
$\pi[f(x)]^2 = x$ $\pi r^2 = A$
$[f(x)]^2 = \frac{x}{\pi}$
$f(x) = \sqrt{\frac{x}{\pi}}$
(b) Let $g(x)$ be the circumference and x the radius
$f(x)$ is the output of a function, x the input
$g(x) = 2\pi x$ $C = 2\pi r$
(c) $gf(x) = g[f(x)] = g\left[\sqrt{\frac{x}{\pi}}\right] = 2\pi\sqrt{\frac{x}{\pi}}$
$= 2\sqrt{\frac{\pi^2 x}{\pi}} = 2\sqrt{\pi x}$
$gf(x)$ gives the circumference of a circle when the area is the input
(d) $a = 2\sqrt{\pi a}$
$a^2 = 4\pi a$
$a^2 - 4\pi a = 0$
$a(a - 4\pi) = 0$
$a = 0$ or $a = 4\pi$
$a = 0$ is not a real-world solution, so $a = 4\pi$
4π is the only value where the area and circumference are numerically the same

GRAPHS 7 – BASIC SKILLS EXERCISE

1 **(a)**



(b) **(i)** –0.79, 3.8, $y = 4$

(ii) –0.24, 4.2, $y = x + 2$

(iii) –1.3, 2.3, $y = 4 - 2x$

2 **(a)** $y = 2$

(b) $y = 1$

(c) $y = x$

(d) $y = 4 - x$

3 **(a)** $5x^2 - 3x + 17 = 0$

(b) $4x^2 - 4x - 7 = 0$

(c) $x^2 - 7x + 3 = 0$

(d) $3x^2 - 3x + 5 = 0$

4 **(a)** No solutions if $x^2 - 4x + 3 < -1$

$\Rightarrow x^2 - 4x < -4 \Rightarrow k < -4$

(b) $x^2 - 3x = p$

$x^2 - 4x + 3 = 3 + p - x$

There is one solution if $y = -x + 3 + p$ is a tangent to the curve.

$y = -x + 3 + p$ is a straight line with gradient -1 and intercept $3 + p$.

Using a ruler, the intercept is approximately 0.75 so $p = -2.25$

5 **(a)** **(i)** –1.7, 0, 1.7, $y = -x$

(ii) –1.325, $y = x + 1$

(iii) –1.8, –0.45, 1.2, $y = x^2 - 1$

(b) $k < -1.1$ or $k > 1.1$

6 **(a)** **(i)** –1, 1, $y = 2x$

(ii) –3.4, –0.59, $y = \frac{x}{2} - 2$

(iii) –2.7, 0.27, 1.4, $y = 4 - x^2$

(b) $-2 < k < 2$

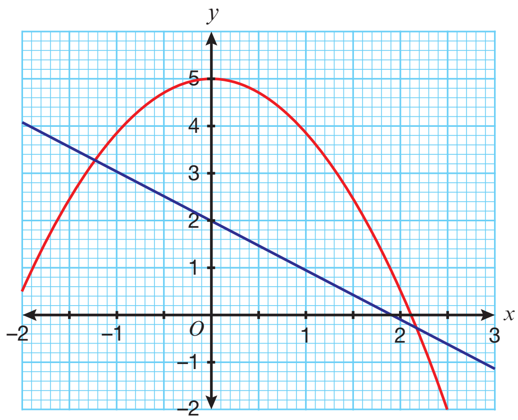
7 **(a)** $-0.6 < k < 2.1$

(b) $3x^3 - 15x^2 + 19x - 3 = 0$

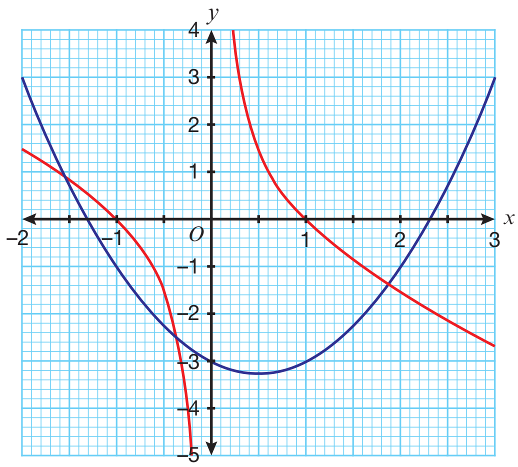
$3x^3 - 15x^2 + 18x = 3 - x$

$x^3 - 5x^2 + 6x = 1 - \frac{x}{3}$, find intersections with $y = 1 - \frac{x}{3}$, $x = 0.2, 1.8, 3$

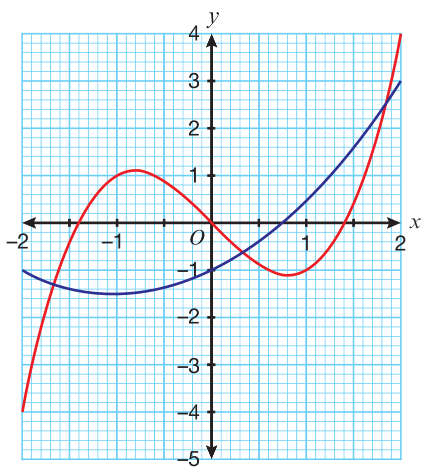
8 $(-1.3, 3.3)$, $(2.3, -0.3)$



9 $(-1.5, 0.9)$, $(-0.3, -2.5)$, $(1.9, -1.3)$



10 $(-1.7, -1.3)$, $(0.3, -0.6)$, $(1.8, 2.5)$

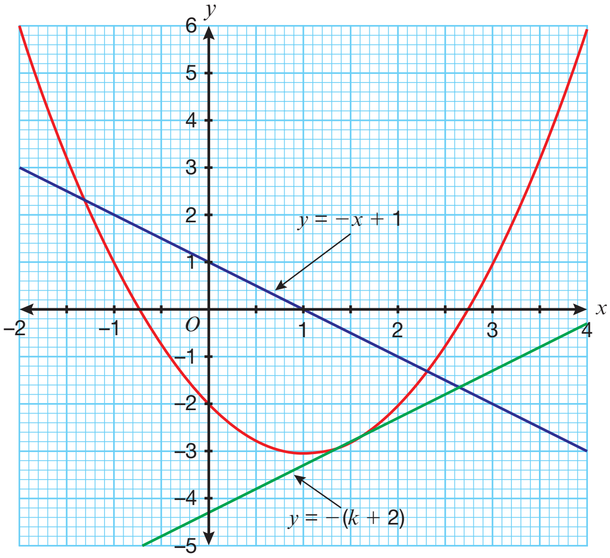


GRAPHS 7 – EXAM PRACTICE EXERCISE

1 **(a)**

x	−2	−1	0	1	2	3	4
y	6	1	−2	−3	−2	1	6

(b)



(c) $x^2 - x = 3$

$x^2 - 2x + x - 2 + 2 = 3$

$x^2 - 2x - 2 = -x + 1$

The line to plot is $y = -x + 1$

Solutions are $x = -1.3$ or 2.3

x values of the intersection points are the solutions

(d) $x^2 - 3x + k = 0$

$x^2 - 2x - x + k - 2 + 2 = 0$

$x^2 - 2x - 2 = x - k - 2$

$x^2 - 2x - 2 = x - (k + 2)$

The line to plot is $y = x - (k + 2)$

This is a straight line with gradient 1 and y intercept of $-(k + 2)$

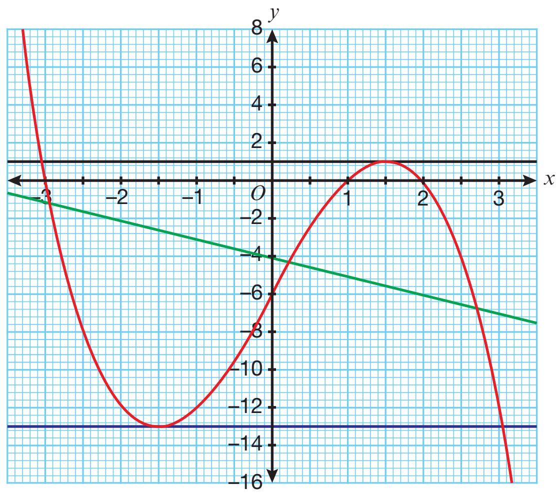
This line must be a tangent to the graph of $y = x^2 - 2x - 2$

Place a ruler on the graph at a gradient of 1, move it until it is a tangent to the curve

The intercept is $-4.25 = -(k + 2)$ so $k = 2.25$

$2 \le k \le 2.5$ is acceptable providing a tangent is drawn on the graph

2 **(a)** Draw two horizontal lines on the graph as shown, one touching the maximum point, the other the minimum point. Any horizontal line drawn between these two lines will intersect the graph at 3 points giving three solutions.



$x^3 - 7x = h$

$-x^3 + 7x = -h$

$-x^3 + 7x - 6 = -h - 6$

$-x^3 + 7x - 6 = -h - 6$ has three solutions if $-13.13 < -h - 6 < 1.13$

$-13.13 < -h - 6 < 1.13$

$\Rightarrow -1.13 < h + 6 < 13.13$

$\Rightarrow -7.13 < h < 6.13$

Values within 0.3 are acceptable

(b) $x^3 - 8x + 2 = 0$

$-x^3 + 8x - 2 = 0$

$-x^3 + 7x + x - 6 + 4 = 0$

$-x^3 + 7x - 6 = -x - 4$

Solutions are the x values of the intersections of $y = -x^3 + 7x - 6$ and $y = -x - 4$

$x = -2.9$ or $x = 0.3$ or $x = 2.7$

3 **(a)** The x values of the intersection of the two graphs are given by

$x^3 - 3x + \frac{1}{x} = \frac{x}{2} - 1$

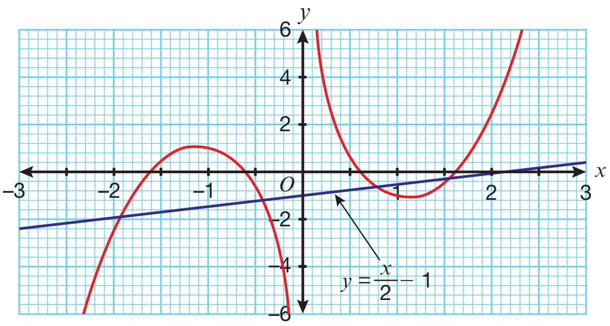
$x^4 - 3x^2 + 1 = \frac{x^2}{2} - x$

Multiplying both sides by x

$2x^4 - 6x^2 + 2 = x^2 - 2x$

Multiplying both sides by 2

$2x^4 - 7x^2 + 2x + 2 = 0$



(b) Adding the line $y = \frac{x}{2} - 1$ to the graph shows there are 4 solutions.
Reading from the graph,
$x = -1.9, -0.4, 0.8, 1.6$ to 1 d.p.

4 **(a)** The y values of the points of intersection of the graphs $x^2 + y^2 = 16$ and $y = x^2 - 5$ are given by eliminating x between the two equations.
$x^2 = y + 5$
$y^2 + y + 5 = 16$
$y^2 + y = 11$

(b) Reading the y values from the points of intersection gives $y = -3.8$ (3.9) or $y = 2.8$ (2.9)

(c) $y = x^2 + k$ is a parabola with vertex on the y-axis at $(0, k)$. If the vertex is on the y-axis and within the circle, then there will be exactly two intersections with the circle.
$-4 < k < 4$

5 **(a)** $\sin x - 2\cos x = 1$ so $\sin x - 1 = 2\cos x$ and the x values of the points of intersection of the two graphs are the solutions.
$x = -270°, -145°, 90°, 215°$
Solutions within 5° are acceptable

(b) $\cos x + \frac{x}{180} = \frac{1}{2}$

$2\cos x + \frac{x}{90} = 1$

$2\cos x = -\frac{x}{90} + 1$

L is $y = -\frac{x}{90} + 1$

SHAPE AND SPACE 8 – BASIC SKILLS EXERCISE

1 **(a)** $\mathbf{p} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$, $\sqrt{26}$

(b) $\mathbf{q} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$, $\sqrt{10}$

(c) $\mathbf{r} = \begin{pmatrix} -4 \\ 13 \end{pmatrix}$, $\sqrt{185}$

(d) $\mathbf{s} = \begin{pmatrix} 7 \\ 0 \end{pmatrix}$, 7

2 **(a)** $\mathbf{p} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$, $\sqrt{29}$

(b) $\mathbf{q} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, 1

(c) $\mathbf{r} = \begin{pmatrix} 12 \\ 1 \end{pmatrix}$, $\sqrt{145}$

(d) $\mathbf{s} = \begin{pmatrix} -2 \\ -42 \end{pmatrix}$, $\sqrt{1768}$

3 **(a)** **(i)** $\mathbf{p} + \mathbf{q} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$

(ii) $2\mathbf{p} - \mathbf{q} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 3 \\ -5 \end{pmatrix} = \begin{pmatrix} 3 \\ 9 \end{pmatrix}$

(b) $m\mathbf{p} + n\mathbf{q} = \begin{pmatrix} 12 \\ -13 \end{pmatrix} = m\begin{pmatrix} 3 \\ 2 \end{pmatrix} + n\begin{pmatrix} 3 \\ -5 \end{pmatrix}$
$= \begin{pmatrix} 12 \\ -13 \end{pmatrix}$
$3m + 3n = 12$
$m + n = 4$ (1)
$2m - 5n = -13$ (2)
(1): $m = 4 - n \rightarrow$ (2)
(2): $2(4 - n) - 5n = -13$
$8 - 2n - 5n = -13$
$-7n = -21$
$n = 3, m = 1$

(c) $\mathbf{p} + r\mathbf{q} = \begin{pmatrix} s \\ -8 \end{pmatrix} = s\begin{pmatrix} 3 \\ 2 \end{pmatrix} + r\begin{pmatrix} 3 \\ -5 \end{pmatrix}$
$= \begin{pmatrix} 5 \\ -8 \end{pmatrix}$
$3 + 3r = 5$ (1) × 5 → (3)
$2 - 5r = -8$ (2) × 3 → (4)
$15 + 15r = 5s$ (3)
$6 - 15r = -24$ (4)
(3) + (4) : $21 = 5s - 24$
$4s = 5s, s = 9, r = 2$

(d) $u(\mathbf{p} + \mathbf{q}) + r(2\mathbf{p} - \mathbf{q}) = r\begin{pmatrix} 0 \\ 21 \end{pmatrix}$
$u\begin{pmatrix} 6 \\ -3 \end{pmatrix} + v\begin{pmatrix} 3 \\ 9 \end{pmatrix} = \begin{pmatrix} 0 \\ 21 \end{pmatrix}$
$= 6u + 3v = 0$ (1)
$-3u + 9v = 21$ (2) × 2 → (3)
$-6u + 18v = 42$ (3)
(1) + (3): $21v = 42$,
$v = 2, u = -1$

4 $m = 2, n = -1$

5 **(a)** $\overrightarrow{AB} = -\mathbf{a} + \mathbf{b}$
(b) $\overrightarrow{AM} = -\frac{1}{2}(\mathbf{b} - \mathbf{a})$
(c) $\overrightarrow{OM} = -\frac{1}{2}(\mathbf{a} + \mathbf{b})$

6 **(a)** $\overrightarrow{AB} = -2\mathbf{x} + 2\mathbf{y}$
(b) $\overrightarrow{AM} = \mathbf{y} - \mathbf{x}$
(c) $\overrightarrow{OM} = \mathbf{x} + \mathbf{y}$

7 **(a)** $\overrightarrow{ED} = \mathbf{p}$
(b) $\overrightarrow{DE} = -\mathbf{p}$
(c) $\overrightarrow{AC} = \mathbf{p} + \mathbf{q}$
(d) $\overrightarrow{AE} = 2\mathbf{q} - \mathbf{p}$

8 **(a)** $\overrightarrow{RS} = -\mathbf{r} + \mathbf{s}$
(b) $\overrightarrow{OP} = \frac{3}{2}\mathbf{r}$
(c) $\overrightarrow{PQ} = -\frac{3}{2}\mathbf{r} + 2\mathbf{s}$
(d) $\overrightarrow{OM} = \mathbf{s} + \frac{3}{4}\mathbf{r}$

9 **(a)** **(i)** $\overrightarrow{PQ} = \mathbf{q} - \mathbf{p}$
(ii) $\overrightarrow{PR} = \frac{1}{3}(\mathbf{q} - \mathbf{p})$
(iii) $\overrightarrow{OR} = \mathbf{p} + \frac{1}{3}(\mathbf{q} - \mathbf{p})$
$= \mathbf{p} + \frac{1}{3}\mathbf{q} - \frac{1}{3}\mathbf{p}$
$= \frac{2}{3}\mathbf{p} + \frac{1}{3}\mathbf{q} = \frac{1}{3}(2\mathbf{p} + \mathbf{q})$
(b) **(i)** $\overrightarrow{OS} = k\,\overrightarrow{OR}$
$= \frac{3}{5}\overrightarrow{OR} \quad = -k = \frac{3}{5}$
(ii) $\overrightarrow{OS} = \frac{3}{5} \times \frac{1}{3}(2\mathbf{p} + \mathbf{q}) = \frac{1}{5}(2\mathbf{p} + \mathbf{q})$

10 **(a)** **(i)** $\overrightarrow{MP} = \frac{2}{5}\mathbf{p}$
(ii) $\overrightarrow{PQ} = \mathbf{q} - \mathbf{p}$
(iii) $\overrightarrow{PN} = \frac{2}{5}(\mathbf{q} - \mathbf{p})$
(iv) $\overrightarrow{MN} = \overrightarrow{MP} + \overrightarrow{PN}$
$= \frac{2}{5}\mathbf{p} + \frac{2}{5}(\mathbf{q} - \mathbf{p})$
$= \frac{2}{5}\mathbf{q}$
(b) $\overrightarrow{OQ} = \mathbf{q}$, $\overrightarrow{MN} = \frac{2}{5}\mathbf{q}$ } OQ is parallel to MN
MN $= \frac{2}{5}$ OQ **or**
OQ $= \frac{5}{2}$ MN

11 **(a)** **(i)** $\mathbf{a}$
(ii) $\frac{1}{2}(\mathbf{a} + \mathbf{b})$
(iii) $\mathbf{a} - \mathbf{b}$
(iv) $\frac{1}{2}(\mathbf{a} + \mathbf{b})$
(b) Parallel and equal lengths
(c) *PQRS* is a trapezium

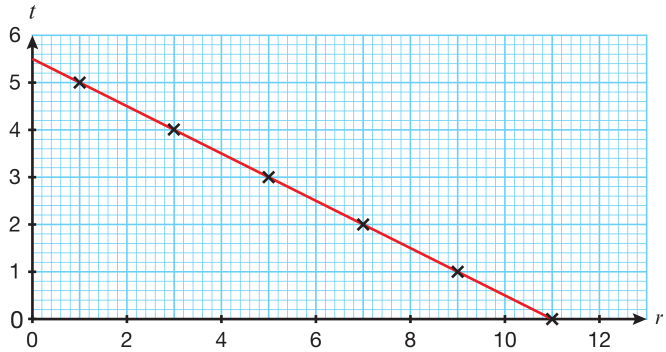
12 **(a)** $\mathbf{w} = \begin{pmatrix} -7 \\ 19 \end{pmatrix}$
(b) $\sqrt{410}$, 339.8°
(c) 5.06 km/h

13 **(a)** $\begin{pmatrix} 1 \\ 5 \end{pmatrix}$
(b) $\begin{pmatrix} 13 \\ 10 \end{pmatrix}$
(c) 5.10
(d) 052.4°

14 **(a)**

t	0	1	2	3	4	5
r	$\begin{pmatrix} 1 \\ 5 \end{pmatrix}$	$\begin{pmatrix} 3 \\ 4 \end{pmatrix}$	$\begin{pmatrix} 5 \\ 3 \end{pmatrix}$	$\begin{pmatrix} 7 \\ 2 \end{pmatrix}$	$\begin{pmatrix} 9 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 11 \\ 0 \end{pmatrix}$

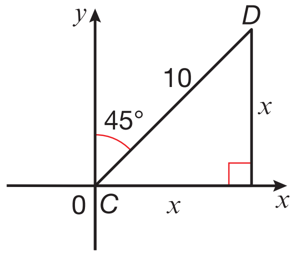
(b)



(c) 8050 km/h, 117°

SHAPE AND SPACE 8 – EXAM PRACTICE EXERCISE

1 **(a)** $m\mathbf{a} - n\mathbf{b} = \begin{pmatrix} 8 \\ -11 \end{pmatrix} = \mathbf{c}$, so
$2m + n = 8$ (1)
$-m - 4n = -11$ (2)
(2) × 2: $-2m - 8n = -22$ (3)
(1) + (3): $-7n = -14$, so $n = 2$ and $m = 3$
(b) If $m = 1$, $n = 1$ and $c = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$.
(c)



$10^2 = 2x^2$
$x^2 = 50$
$x = \sqrt{50}$
$\quad = 5\sqrt{2}$

So vector $\mathbf{d} = \begin{pmatrix} 3 \\ -5 \end{pmatrix} + \begin{pmatrix} 5\sqrt{2} \\ 5\sqrt{2} \end{pmatrix}$

$= \begin{pmatrix} 3 + 5\sqrt{2} \\ 5\sqrt{2} - 5 \end{pmatrix}$

2 **(a)** $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$

$= \overrightarrow{OA} + \frac{m}{m+n}\overrightarrow{AB}$

$= \boldsymbol{a} + \frac{m}{m+n}(\mathbf{b} - \mathbf{a}) = \frac{(m+n)\boldsymbol{a} + m(\mathbf{b} - \mathbf{a})}{m+n}$

$= \frac{n\mathbf{a} + m\mathbf{b}}{m+n}$

(b) $\overrightarrow{OP} = \frac{2\begin{pmatrix} -4 \\ 3 \end{pmatrix} + 3\begin{pmatrix} 2 \\ 1 \end{pmatrix}}{3+2}$

$= \frac{\begin{pmatrix} -2 \\ 9 \end{pmatrix}}{5}$

$\overrightarrow{OP} = \frac{1}{5}\sqrt{(-2)^2 + 9^2}$

$= \frac{1}{5}\sqrt{85}$

$= \frac{1}{5}\sqrt{5 \times 17}$

$= \sqrt{\frac{17}{5}}$

as required

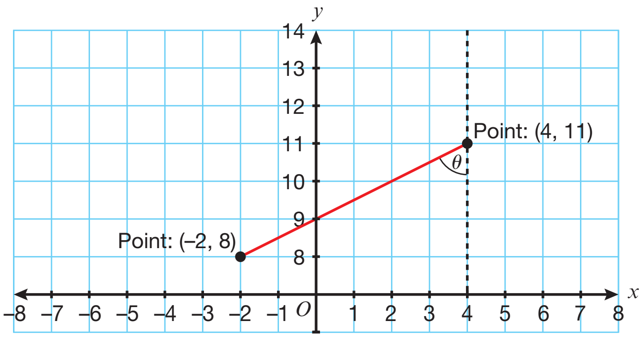
3 **(a)** **(i)** $t = 0, \mathbf{r} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

(ii) $t = 4, \mathbf{r} = \begin{pmatrix} -3 \\ 10 \end{pmatrix}$

(b) (Each hour that passes, the boat travels along vector $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ km, so the length of this vector is the distance travelled in 1 h)

Speed $= \sqrt{(-1)^2 + 2^2} = \sqrt{5}$ km/h

(c) At 15:00, $t = 3$, $\mathbf{r} = \begin{pmatrix} -2 \\ 8 \end{pmatrix}$ km



Bearing of boat from $L = 180° + \theta°$

$\tan(\theta) = \frac{6}{3} = 2$, so $\theta = 63.43°\ldots$

Bearing of boat from
$L = 180° + 63.43°\ldots = 243°$ (3 s.f.)

4 $\overrightarrow{AQ} = \mathrm{k}\,\overrightarrow{AB} = \mathrm{k}(\mathbf{b} - \mathbf{a})$

$\overrightarrow{OP} = \overrightarrow{OA} + \frac{3}{4}\overrightarrow{AM} = \mathbf{a} + \frac{3}{4}\left(\frac{1}{2}\boldsymbol{b} - \mathbf{a}\right)$

$= \frac{1}{4}\boldsymbol{a} + \frac{3}{8}\boldsymbol{b}$

$\overrightarrow{OQ} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}) = \mathbf{a} + \lambda\mathbf{b} - \lambda\mathbf{a}$
$= (1 - \lambda)\mathbf{a} + \lambda\mathbf{b}$

(If vector $m\mathbf{p}$ is collinear with vector $n\mathbf{q}$, then $\frac{m}{n}$ = a constant)

Now OPQ are collinear so:

$\frac{\lambda}{1-\lambda} = \frac{\frac{3}{8}}{\frac{1}{4}} = \frac{3}{2}$

$2\lambda = 3 - 3\lambda$
$5\lambda = 3$
$\lambda = \frac{3}{5}$

$\overrightarrow{AQ} = \overrightarrow{AO} + \overrightarrow{OQ} = -\mathbf{a} + \left(1 - \frac{3}{5}\right)\mathbf{a} + \frac{3}{5}\mathbf{b}$

$= -\frac{3}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$

$= \frac{3}{5}(\mathbf{b} - \mathbf{a})$

$= \frac{3}{5}\overrightarrow{\mathrm{AB}}$

$AQ : QB = 3 : 2$

5 **(a)** $\overrightarrow{CD} = \overrightarrow{CB} + \overrightarrow{BA} + \overrightarrow{AD}$
$= -\mathbf{c} - \mathbf{b} + 3\mathbf{c} = 2\mathbf{c} - \mathbf{b}$

(b) If BPD are collinear $\overrightarrow{BP} = k\,\overrightarrow{BD}$
$\overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{CD} = \mathbf{c} + 2\mathbf{c} - \mathbf{b} = 3\mathbf{c} - \mathbf{b}$
Now $\overrightarrow{BP} = \overrightarrow{BA} + \lambda\overrightarrow{AC} = -\mathbf{b} + \lambda(\mathbf{b} + \mathbf{c})$
$= (\lambda - 1)\mathbf{b} + \lambda\mathbf{c}$ (1)
(If vector $m\mathbf{p}$ is collinear with vector $n\mathbf{q}$, then $\frac{m}{n}$ = a constant)
$\frac{\lambda}{\lambda - 1} = \frac{3}{-1}$, $\lambda = \frac{3}{4}$, so from (1)
$\overrightarrow{BP} = \frac{1}{4}(3\mathbf{c} - \mathbf{b})$ so $\overrightarrow{BP} = \frac{1}{4}\overrightarrow{BD}$
$\overrightarrow{AP} = \overrightarrow{AB} + \overrightarrow{BP} = \mathbf{b} + \frac{1}{4}(3\mathbf{c} - \mathbf{b})$
$= \frac{3}{4}(\mathbf{b} + \mathbf{c}) = \frac{3}{4}\overrightarrow{AC}$
$AP : PC = 3 : 1$

(c) $\overrightarrow{CD} = 2\mathbf{c} - \mathbf{b} = 2\begin{pmatrix}3\\0\end{pmatrix} - \begin{pmatrix}4\\1\end{pmatrix} = \begin{pmatrix}2\\-1\end{pmatrix}$
$|\overrightarrow{CD}| = \sqrt{(2)^2 + (-1)^2} = \sqrt{5}$

HANDLING DATA 5 – BASIC SKILLS EXERCISE

1 **(a)** $\frac{1}{12}$
(b) $\frac{5}{24}$
(c) $\frac{7}{24}$

2 $P(O_1E_2 \text{ or } E_1O_2) = P(O_1E_2) + P(E_1O_2)$
$= \frac{4}{7} \times \frac{3}{6} + \frac{3}{7} \times \frac{4}{6} = \frac{24}{42} = \frac{4}{7}$
P(A or B) = P(A) + P(B) if A and B are mutually exclusive

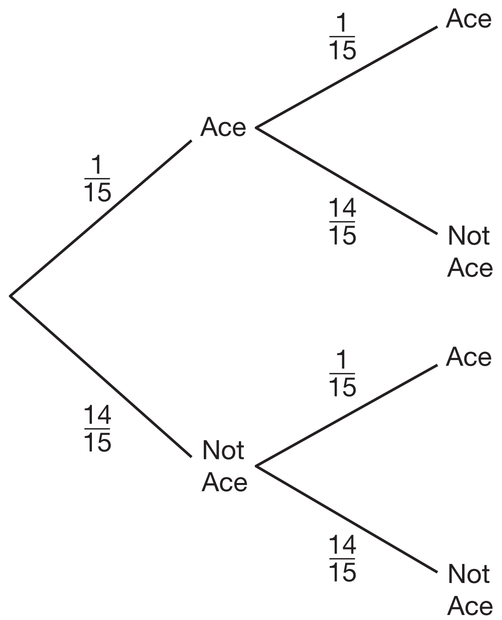
3 Either all 3 discs are even or 1 is even and two are odd to sum an even number.
$P(E_1E_2E_3 \text{ or } E_1O_2O_3 \times 3)$
$= P(E_1E_2E_3) + P(E_1O_2O_3) \times 3$
$= \frac{4}{9} \times \frac{3}{8} \times \frac{2}{7} + \frac{4}{9} \times \frac{5}{8} \times \frac{4}{7} \times 3$
$= \frac{264}{504}$
$= \frac{11}{21}$
P(A or B) = P(A) + P(B) if A and B are mutually exclusive

4 Let Z be the number of Z's that are revealed
$P(Z \geq 1) + P(Z = 0) = 1$
$P(Z \geq 1) = 1 - P(Z = 0)$
$= 1 - P(Z_1' Z_2' Z_3')$
$= 1 - \frac{6}{9} \times \frac{5}{8} \times \frac{4}{7}$
$= 1 - \frac{5}{21}$
$= \frac{16}{21}$

5 $P(W < 2) + P(W \geq 2) = 1$
$P(W \geq 2) = 1 - P(W < 2)$
$= 1 - [P(W = 0) + P(W = 1)]$
$= 1 - \left[\left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) + 3 \times \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right)\right]$
$= \frac{1}{2}$
Scarlett is correct.

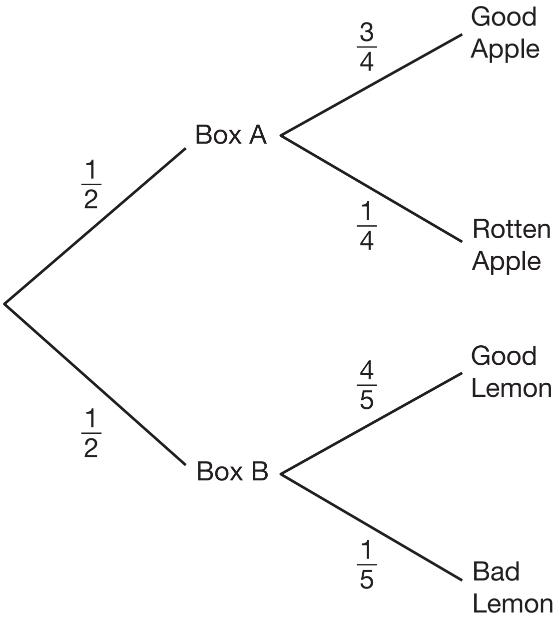
6 **(a)** $\frac{1}{4}$
(b) $\frac{1}{2}$
(c) $\frac{1}{4}$

7 **(a)**



(b) (i) $\frac{1}{225}$
(ii) $\frac{196}{225}$
(iii) $\frac{28}{225}$

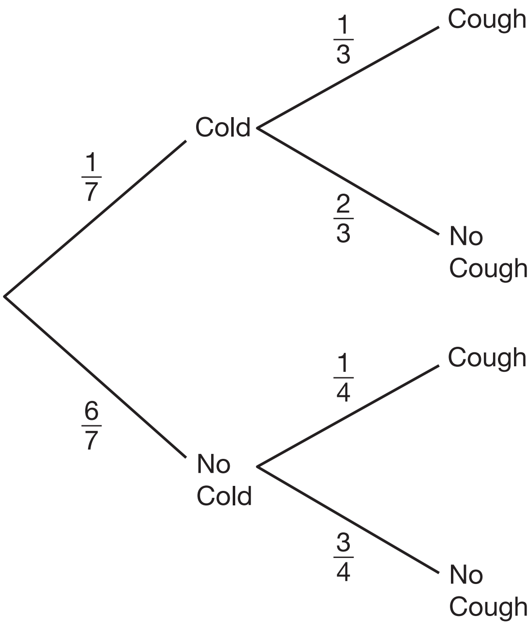
8 **(a)**



(b) $\frac{3}{8}$

(c) $\frac{1}{10}$

9 **(a)**



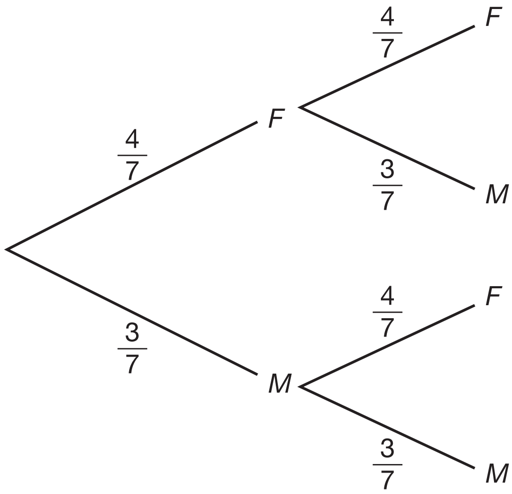
(b) **(i)** $\frac{1}{21}$

(ii) $\frac{3}{14}$

(iii) $\frac{13}{42}$

HANDLING DATA 5 – EXAM PRACTICE EXERCISE

1 **(a)** Let event F be a female baby giant panda.
Let event M be a male baby giant panda.



(b) $P(M_1M_2) = P(M_1) \times P(M_2)$

$= \frac{3}{7} \times \frac{3}{7} = \frac{9}{49}$

$P(A \text{ and } B) = P(A) \times P(B)$

(c) $P(M_1F_2 \text{ or } F_1M_2)$

$= P(M_1F_2) + P(F_1M_2)$

$= \frac{3}{7} \times \frac{4}{7} + \frac{4}{7} \times \frac{3}{7} = \frac{24}{49}$

$P(A \text{ or } B) = P(A) + P(B)$ if A and B are mutually exclusive

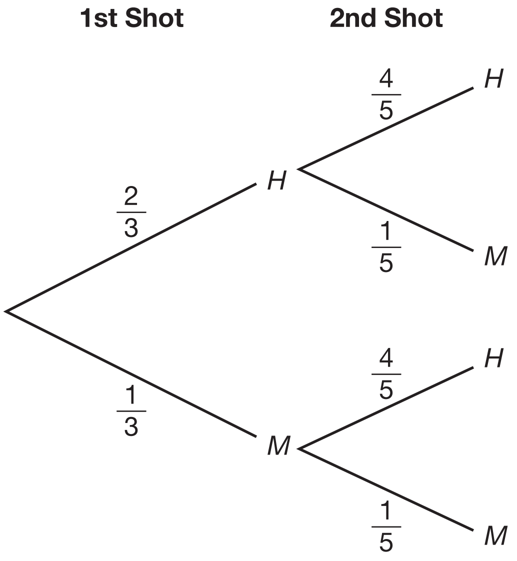
2 **(a)**

	1st Shot	2nd Shot
Hits	$\frac{2}{3}$	$\frac{4}{5}$
Misses	$\frac{1}{3}$	$\frac{1}{5}$

Table completed as $P(E) + P(E') = 1$

Let event H be a hit of the bullseye.

Let event M be a miss of the bullseye.



(b) (i) $P(H_1H_2) = P(H_1) \times P(H_2)$

$= \frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$

P(A and B) = P(A) × P(B)

(ii) $P(H = 1) = P(H_1M_2 \text{ or } M_1H_2)$

$= P(H_1M_2) + P(M_1H_2)$

$= \frac{2}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{4}{5} = \frac{6}{15} = \frac{2}{5}$

P(A or B) = P(A) + P(B) if A and B are mutually exclusive

iii) $P(H \geq 1) + P(H = 0) = 1$

$P(H \geq 1) = 1 - P(H = 0)$

$= 1 - P(M_1M_2)$

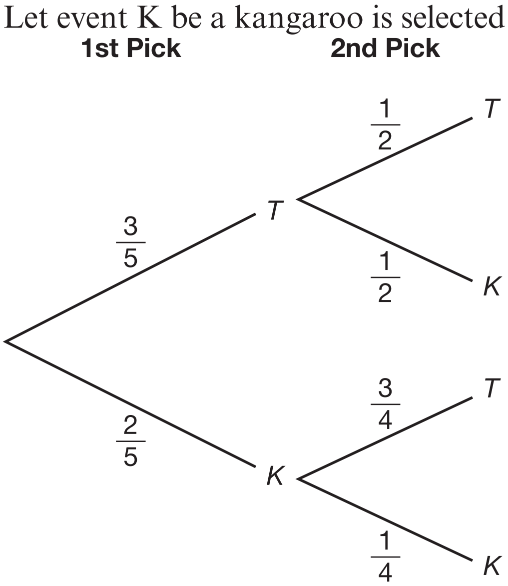
$P(E) + P(E') = 1$

$= 1 - \frac{1}{3} \times \frac{1}{5}$

$= 1 - \frac{1}{15}$

$= \frac{14}{15}$

3 **(a)** Let event T be a teddy bear is selected.
Let event K be a kangaroo is selected.



(b) (i) $P(T_1T_2) = P(T_1) \times P(T_2)$

$= \frac{3}{5} \times \frac{1}{2} = \frac{3}{10}$

P(A and B) = P(A) × P(B)

(ii) $P(T_1K_2 \text{ or } K_1T_2)$

$= P(T_1K_2) + P(K_1T_2)$

$= \frac{3}{5} \times \frac{1}{2} + \frac{2}{5} \times \frac{3}{4}$

$= \frac{12}{20} = \frac{3}{5}$

P(A or B) = P(A) + P(B) if A and B are mutually exclusive

(iii) $P(K \geq 1) + P(K = 0) = 1$

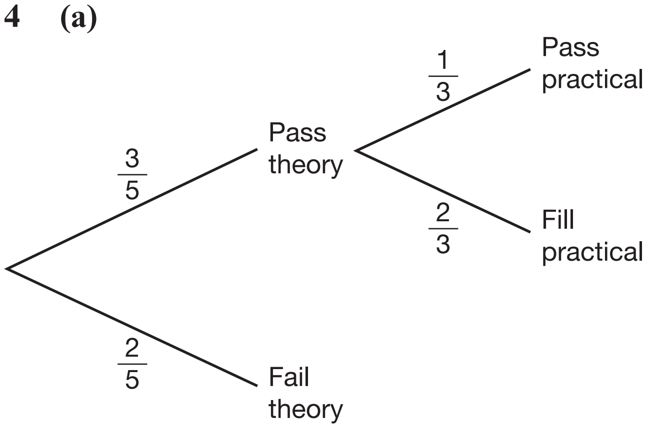
$P(K \geq 1) = 1 - P(K = 0)$

$= 1 - P(T_1T_2)$

$P(E) + P(E') = 1$

$= 1 - \frac{3}{10} = \frac{7}{10}$

4 **(a)**



(b) $P(FT \text{ or } PT,FP) = P(FT) + P(PT,FP)$

$= \frac{2}{5} + \frac{3}{5} \times \frac{2}{3} = \frac{4}{5}$

P(A or B) = P(A) + P(B) if A and B are mutually exclusive

(c) $P(PT,PP,PA) = \frac{3}{5} \times \frac{1}{3} \times \frac{1}{4} = \frac{1}{20}$

P(A and B) = P(A) × P(B)

5 **(a)** Let d be the number of diamonds in the box
Let r be the number of rubies in the box

$P(D_1D_2) = P(D_1) \times P(D_2)$

$= \frac{d}{20} \times \frac{d-1}{19} = \frac{21}{38}$

P(A and B) = P(A) × P(B)

$d(d - 1) = 210$

$d^2 - d - 210 = 0$

$(d - 15)(d + 14) = 0$

$d = 15$ and $r = 5$

(b) $P(D_1R_2 \text{ or } R_1D_2) = P(D_1R_2) + P(R_1D_2)$

$= \frac{15}{20} \times \frac{5}{19} + \frac{5}{20} \times \frac{15}{19} = \frac{30}{76} = \frac{15}{38}$

P(A or B) = P(A) + P(B) if A and B are mutually exclusive

NUMBER 9 – BASIC SKILLS EXERCISE

1 $660

2 $625

3 Yes. Saskia has been over charged by £4.

4 €4140

5 15 hours

6 $700

7 €1233.75

8 2 h 30 mins

9 $885 (Aus)

10 £892.86

11 44.32 yuan

12 5200 reais

13 804.74 yuan

14 £10 050.25

15 Australia: £2127.12
Brazil: £2000
Spain: £2200
Cheapest purchase is in Brazil.

16 Malaysia : India : China = $200 : $200 : $600
811.85 ringitts, 14 740.74 rupees, 3920 yuan

17 A: 33.3 g/£ , B: 33.3 g/£ so same value!

18 Square: €25/m^2, Octagonal: €24/m^2,
Octagonal are better value.

19 Everamp: 20 h/£, Dynamo: 24 h/£,
Dynamo is better value.

20 kg bag $1.90 per kg, 21 kg bag $1.75 per kg so 12 kg bag better value per kg.

NUMBER 9 – EXAM PRACTICE EXERCISE

1 Let €x be the amount that Aria receives.

Aria	Blake	Chloe
x	$0.25x$	$1.25x$

So, $1750 = x + 0.25x + 1.25x = 2.5x$
$x = 700$
So Aria: €700 Blake: €175 Chloe: €875

2 Total amount paid by Kofi
= 24 × €36 = €864
Money received by Kofi
= $\frac{2}{3}$ × 24 ×30 × €1.80 = €864
Kofi's overall profit = 0.25 × €864 = €216
Number of bottles left
= $\frac{1}{3}$ × 24 × 30 = 240
So price of each bottle to secure 25% profit
= $\frac{€216}{240}$ = €0.90 per bottle

3 **(a)** Total parts = 6, so 1 part of £7200 = £1200
Malaysia: £1200 = 5.48 × £1200 = 6576 ringitts
India: £2400 = 99.50 × £2400 = 238 800 rupees
China: £3600 = 8.82 × £3600 = 31 752 yuan

(b) Return journey: 25% of 31752 yuan = 7938 yuan
If 1€ = 8 yuan, 1 yuan = € $\frac{1}{8}$, so
7938 yuan = 7938 × $\frac{1}{8}$ euros
= €992.25

4 Tangerine cost = $25 + $0.35 × 60 × 24
= $529 × 1.175 = $621.58
Aardvark cost = $90 + $0.30 × 60 × 24
= $522 × 1.175 = $613.35
So Aardvark is cheaper by $8.23 and is therefore better value assuming the quality of service is the same.
$p = 8.23$

5 **(a)** g = 1.025 × £44 940, so g = £46 063.50
(b) s × 0.96 = £600, so s = £625
(c) Price of 1 kg of gold on 1 Jan 2022
= 0.975 × £44 940 = £43 816.50
Price of 1 kg of silver on 1 Jan 2022
= 1.04 × £625 = £650
500 g of gold = 0.5 × £43 816.50
= £21 908.25
500 g of silver = 0.5 × £650 = £325
Total price = £22 233.25

ALGEBRA 9 – BASIC SKILLS EXERCISE

1 $(-4, 12), (3, 5)$

2 $(3, 2), (-3, -2)$

3 $(3, 1), (9, 4)$

4 $(2, 3), (3, 2)$

5 $(1, 1)$

6 $(-4.16, -6.16), (2.16, 0.162)$

7 $(3.92, 1.08), (-1.92, 6.92)$

8 $(3.24, 1.24), (-1.24, -3.24)$

9 $(91.65, 18.33)$

10 Equation of top of egg cup is $y = 4$
Intersection at $(-2, 4)$ and $(2, 4)$,
radius = 2 cm

11 Eliminating y gives $x^2 - 4x + 5 = 0$. Using quadratic formula $b^2 - 4ac = 16 - 20 = -4$

12 Substituting $y = k - x$ into $x^2 + y^2 = 25$ gives $2x^2 - 2kx + (k^2 - 25) = 0$. This has one solution if, using quadratic formula, $b^2 - 4ac = 0$. Substituting values gives
$4k^2 = 4 \times 2 \times (k^2 - 25)$
$k^2 = 50$
$k = \pm 5\sqrt{2}$

13 $p = 4$, $2^4 - 1 = 15$

14 $x = 0$

15 $n = 5$ gives 99 which is not prime.

16 False if a and b are of opposite signs; for example, $(-2)^2 = (2)^2$ but $-2 \neq 2$

17 $(2m + 1) - (2n + 1) = 2(m - n)$ which is even.

18 $(2n - 1) + (2n + 1) + (2n + 3) = 6n + 3$ this leaves remainder 3 when divided by 6
or $(2n + 1) + (2n + 3) + (2n + 5)$
$= 6(n + 1) + 3$

19 $S = \frac{n}{2}[2 + (n - 1)2] = \frac{n}{2}[2n] = n^2$

20 Sum $= \frac{n}{2}[2a + (n - 1)]$. Let $n = 2p + 1$

$S = (2p + 1)\left[a + \frac{2p + 1 - 1}{2}\right] = (2p + 1)$

$(a + p)$ which is divisible by $n = (2p + 1)$

21 $(2n - 1)(2n + 1)(2n + 3)$
$= 8n^3 + 12n^2 - 2n - 3$
$= 2(4n^3 + 6n^2 - n - 1) - 1$

22 $n(n + 1) + (n + 1) = n^2 + 2n + 1 = (n + 1)^2$

23 $(2n + 1)^2 = 4n^2 + 4n + 1 = 4(n^2 + n) + 1$

24 $(2m + 1)^2 - (2n + 1)^2 = 4(m^2 + m - n^2 - n)$
$= 4[m(m + 1) - n(n + 1)]$
Either m or $m + 1$ is even with a factor of 2, likewise n and $n + 1$ hence 8 is a factor.

25 $(n + 1)^2 - n^2 = 2n + 1 = (n + 1) + n$

26 $(x - 4)^2 + 1$

27 $-(x - 3)^2$

28 $2x^2 - 24x + 73 = 2(x - 6)^2 + 1 > 0$

29 $(2x + 1)^2 \geq 0$

30 $x^2 + 14x + c = (x + 7)^2 - 49 + c$
$c - 49 \geq 0$
$c \geq 49$

31 $x^2 + bx + 4 = \left(x + \frac{b}{2}\right)^2 - \frac{b^2}{4} + 4$

$4 - \frac{b^2}{4} \geq 0$

$b^2 \leq 16$
$-4 \leq b \leq 4$

32 $\sqrt{3a}(\sqrt{18a} + \sqrt{2a}) = \sqrt{36a^2}$
$+ 2a = 6a + 2a = 8a$

33 $2^{127} - 2 = 2(2^{126} - 1)$
$2^{127} - 2$ has a factor of 2.
$2^{127} - 2$, $2^{127} - 1$, 2^{127} are three conecutive integers. 2^{127} is even, $2^{127} - 1$ is prime,
$2^{127} - 2$ has a factor of 3 as one of any three consecutive integers has a factor of 3.
$2^{127} - 2$ has a factor of 6.

34 Using the quadratic formula gives $b^2 - 4ac = (2\sqrt{c})^2 - 4c = 0$ therefore there is only one solution.

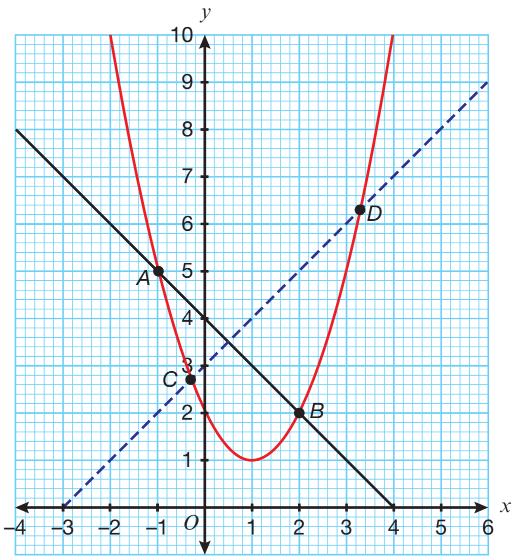
35 $abc = 100a + 10b + 5 = 5(20a + 2b + 1)$

36 $100a + 10b + c = 100a + 10(a + c) + c$
$= 110a + 11c = 11(10a + c)$

37 $[(n + 1)^2 - 6(n + 1) + 10] - [n^2 - 6n + 10]$
$= n^2 + 2n + 1 - 6n - 6 + 10 - n^2 + 6n - 10$
$= 2n - 5$ which is an odd number.

ALGEBRA 9 – EXAM PRACTICE EXERCISE

1



Midpoint of AB is $\left(\frac{-1+2}{2}, \frac{5+2}{2}\right) = \left(\frac{1}{2}, \frac{7}{2}\right)$

Midpoint is mean of the coordinates.

Gradient of AB is -1

gradient of perpendicular is 1

equation of CD is $y = x + C$

For perpendicular lines, product of gradients = -1

Substituting midpoint gives $\frac{7}{2} = \frac{1}{2} + C$

$C = 3$ so equation of CD is $y = x + 3$

Therefore, the coordinates of C and D are given by the solutions to the simultaneous equations

$y = x^2 - 2x + 2$ and $y = x + 3$

$x^2 - 2x + 2 = x + 3$

$x^2 - 3x - 1 = 0$

Using the quadratic formula with $a = 1$, $b = -3$ and $c = -1$ gives

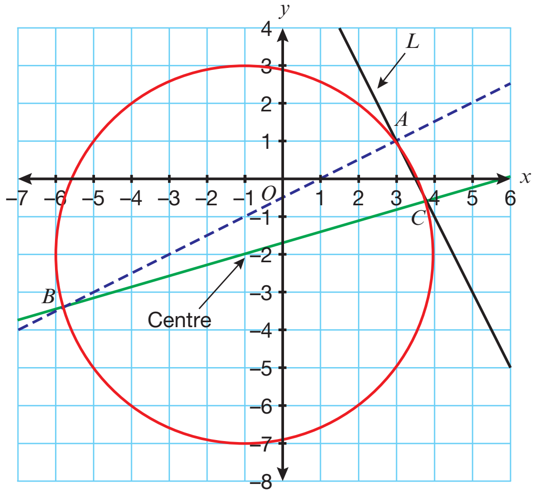
$x = \frac{3 \pm \sqrt{9+4}}{2} = \frac{3 \pm \sqrt{13}}{2}$

Substituting into $y = x + 3$ gives

$C = \left(\frac{3-\sqrt{13}}{2}, \frac{9-\sqrt{13}}{2}\right)$ and

$D = \left(\frac{3+\sqrt{13}}{2}, \frac{9+\sqrt{13}}{2}\right)$

2



(a) Gradient of AB is $\frac{1+3.4}{3+5.8} = \frac{1}{2}$

gradient of L is -2

For perpendicular lines, product of gradients = -1

Equation of L is $y = -2x + C$

Substituting the coordinates of A gives

$1 = -2 \times 3 + C$

$C = 7$ hence L is $y = -2x + 7$

(b) Coordinates of where L intersects the circle are given by the solutions to the simultaneous equations

$x^2 + y^2 + 2x + 4y = 20$ and

$y = -2x + 7$

$y = -2x + 7 \Rightarrow 4y = -8x + 28$ and

$y^2 = 4x^2 - 28x + 49$

Substituting into

$x^2 + y^2 + 2x + 4y = 20$ gives

$x^2 + 4x^2 - 28x + 49 + 2x - 8x + 28 = 20$

$5x^2 - 34x + 57 = 0$

$(5x - 19)(x - 3) = 0$

$x = 3$ or $\frac{19}{5}$

Or use quadratic formula to solve.

Substituting into $y = -2x + 7$ gives

$C = \left(\frac{19}{5}, \frac{-3}{5}\right)$ or $(3.8, -0.6)$

(c) AB and AC are chords intersecting at right angles therefore BC is a diameter of the circle.

The midpoint of BC is the centre of the circle $= \left(\frac{3.8-5.8}{2}, \frac{-0.6-3.4}{2}\right) = (-1, -2)$

B is $(-5.8, -3.4)$ given in question.

3 **(a)** Coordinates of A and B are given by the solutions to the simultaneous equations $4x^2 + y^2 - 4y = 0$ and $2x + y = 3$

$2x + y = 3$

$y = 3 - 2x$

$y^2 = 9 - 12x + 4x^2$

Substituting into $4x^2 + y^2 - 4y = 0$ gives

$4x^2 + 9 - 12x + 4x^2 - 12 + 8x = 0$

$8x^2 - 4x - 3 = 0$

Solving using the quadratic formula gives $x = \frac{1 \pm \sqrt{7}}{4}$

When $x = \frac{1 + \sqrt{7}}{4}$,

$y = 3 - 2 \times \frac{1 + \sqrt{7}}{4} = \frac{5 - \sqrt{7}}{2}$

so the coordinates of B are $\left(\frac{1 + \sqrt{7}}{4}, \frac{5 - \sqrt{7}}{2}\right)$

When $x = \frac{1 - \sqrt{7}}{4}$,

$y = 3 - 2 \times \frac{1 - \sqrt{7}}{4} = \frac{5 + \sqrt{7}}{2}$

so the coordinates of A are $\left(\frac{1 - \sqrt{7}}{4}, \frac{5 + \sqrt{7}}{2}\right)$

Distance in the x direction between A and B is $\frac{1 + \sqrt{7}}{4} - \frac{1 - \sqrt{7}}{4} = \frac{\sqrt{7}}{2}$

Distance in the y direction between A and B is $\frac{5 + \sqrt{7}}{2} - \frac{5 - \sqrt{7}}{2} = \sqrt{7}$

$AB^2 = \left(\frac{\sqrt{7}}{2}\right)^2 + (\sqrt{7})^2 = \frac{7}{4} + 7 = \frac{35}{4}$

$AB = \frac{\sqrt{35}}{2}$ cm

(b) Volume of wire $= \pi \times 0.05^2 \times \frac{\sqrt{35}}{2}$

$= 0.03285...\text{cm}^3$

1 mm = 0.1 cm ⇒ radius is 0.05 cm

Weight of gold $= 0.03285... \times 19.3$

$= 0.6341...$ g

Cost of gold $= 0.6341 \times 60 = 38.04...$

$= £38.0$ (3 s.f.)

4 **(a)** Any odd number is given by $2n + 1 \Rightarrow$

$(2n + 1)^2 = 4n^2 + 4n + 1 = 4n(n + 1) + 1$

one of n or $n + 1$ must be even so $n(n + 1)$ has a factor of 2 hence $4n(n + 1)$ has a factor of 8 and $4n(n + 1) + 1$ is 1 more than a multiple of 8

(b) $a^2 - b^2 = (a + b)(a - b)$

(i) As $a^2 - b^2$ is a prime number then one of the factors $(a + b)$ or $(a - b)$ must equal 1.

$a + b = 1$ so $a = 1 - b$ implies that a is negative as $b > 0$. But $a > 0$ so $a + b \neq 1$

$a - b = 1$ so $a = 1 + b$ implies that a and b are consecutive integers.

(ii) $a^2 - b^2 = (a + b) \times 1 \Rightarrow a + b = a^2 - b^2$ which is prime, so $a + b$ is also prime.

5 **(a)** From the formula sheet, the sum to n terms is given by $S = \frac{n}{2}[2a + (n - 1)d]$ where a is the first term and d the common difference.

$S = \frac{n}{2}[2 \times 2 + (n - 1)4] = \frac{n}{2}[4 + 4(n - 1)]$

$= \frac{4n}{2}[1 + n - 1] = 2n^2$ which is double a square number.

(b) The nth term of S is $a + (n - 1)d$

$= 2 + 4(n - 1) = 4n - 2$

nth term squared $= (4n - 2)^2$

$= 16n^2 - 16n + 4$

nth term squared $+ 12 = 16n^2 - 16n + 4 + 12 = 16(n^2 - n + 1)$ which is divisible by 16

GRAPHS 8 – BASIC SKILLS EXERCISE

1 **(a)** **(i)** −1.2

(ii) 1.8

(b) **(i)** (3.2, −1.6) and (0.8, 1.6)

(ii) (5.1, −1.0) and (−1.1, 1.0)

(iii) (4.0, −2.1) and (0, 2.1)

2 **(a)** −1.2

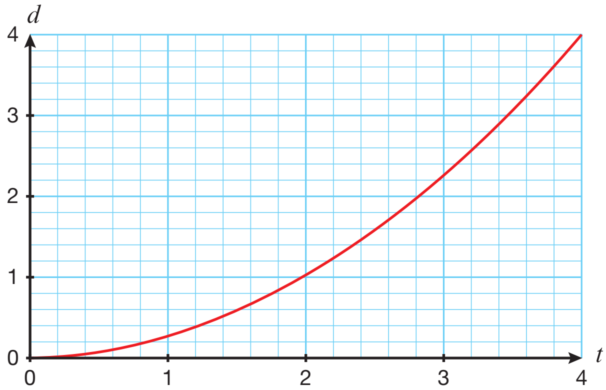
(b) $y = -1.2x + 2.1$

3 1.76 and −1.5

4 2.4 and 1.5

5 (a)

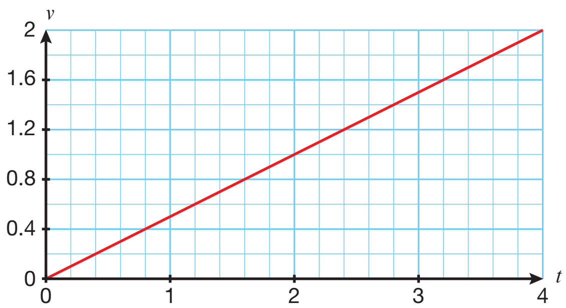
t	0	1	2	3	4
d	0	0.25	1	2.25	4



(b) Gradient gives velocity

(i) 0 m/s
(ii) 0.5 m/s
(iii) 1 m/s
(iv) 1.5 m/s
(v) 2 m/s

(c)

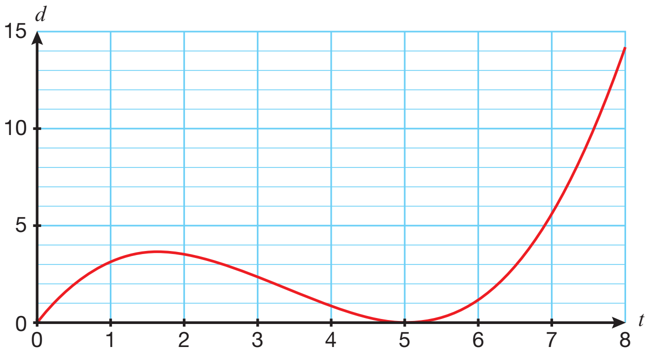


(d) Gradient gives acceleration
Acceleration is constant and equal to $0.5\ \text{m/s}^2$

6 (a) (i)

t	0	1	2	3	4	5	6	7	8
d	0	3.2	3.6	2.4	0.8	0	1.2	5.6	14.4

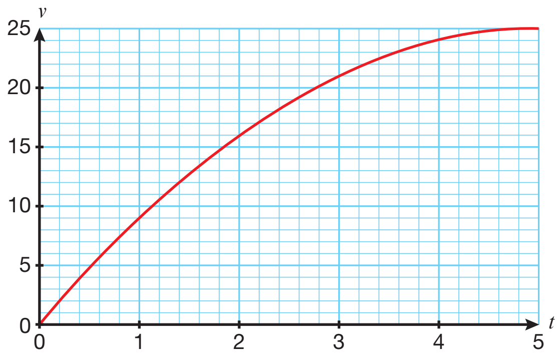
(ii)



(b) Starts off, slows down and stops after approximately 1.5 s, returns to start after 5 s then sets off at increasing speed

(c) Gradient gives velocity
(i) 1.6 m/s
(ii) −1.6 m/s
(d) 6.9 s

7 (a)



(b) Gradient gives acceleration
(i) $8\ \text{m/s}^2$
(ii) $4\ \text{m/s}^2$
(c) 2.5 s

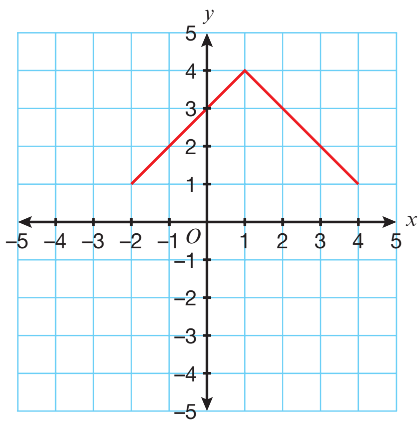
8 (a) (i)

t (days)	0	1	2	3	4	5	6	7	8
d (cm)	20	17	14.5	12.3	10.4	8.87	7.54	6.41	5.45

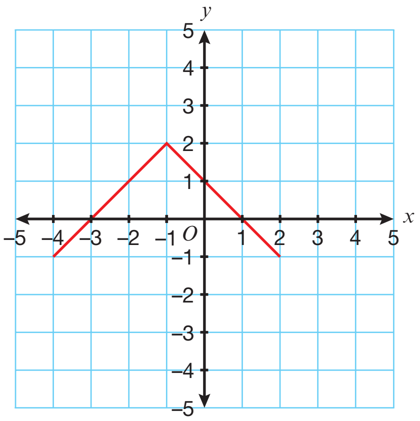
(ii)

(b) Approximately 4.3 days
(c) 2.3 cm/day
(d) 61.2 cm/day = 0.05 cm/h

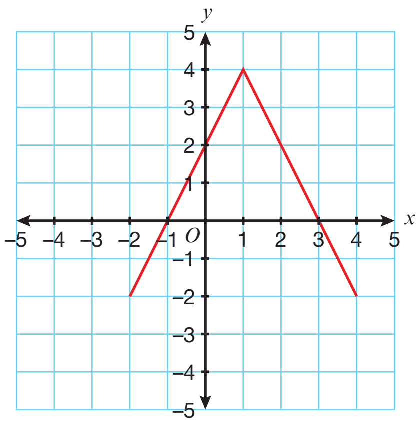
9 (a) (1, 4)



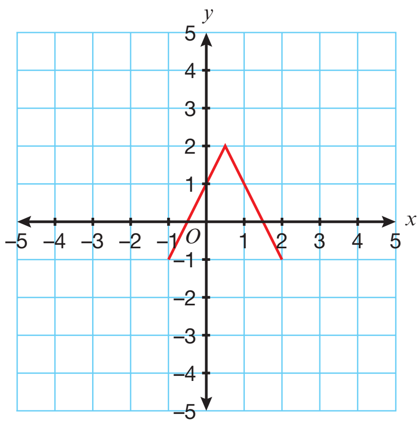
(b) $(-1, 2)$



(c) $(1, 4)$



(d) $(0.5, 2)$



10 **(a)** $(-2, 2)$
(b) $(2, -2)$
(c) $(-2, -1)$
(d) $(-4, -2)$

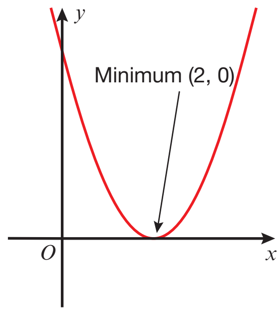
11 $y = f(x + 2)$ and $y = f(-x)$

12 $y = -x^2 + 2x + 3$

13 $y = -[\sin(x) + 2] = -\sin(x) - 2$

14 **(a)** $y = x^2 + 4x + 4$
(b) $y = x^2 - 4x + 4$

(c)



15 **(a)** $(0, k)$
(b) $(0, -2k)$
(c) $(0, k - 2)$

16 **(a)** k
(b) $2k$
(c) $2k$
(d) k

17 **(a)** $-m$
(b) $-m$
(c) $\frac{m}{2}$
(d) m

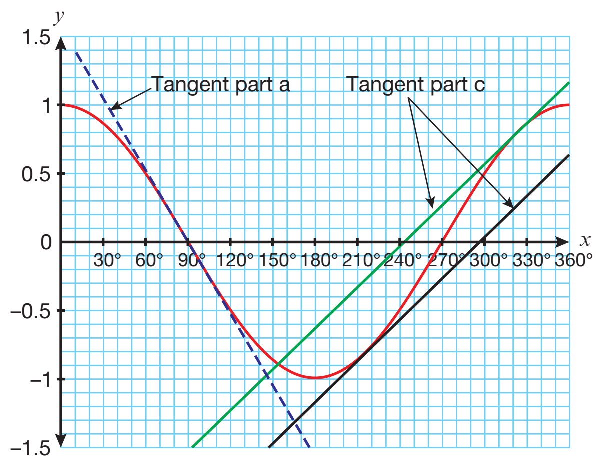
18 **(a)** $(0, p - 3)$
(b) $(0, p)$
(c) $(0, 3p)$
(d) $(0, p)$

19 **(a)** 3
(b) 2
(c) −1

20 **(a)** $(-6, 4)$
(b) $(-1, -3)$

GRAPHS 8 – EXAM PRACTICE EXERCISE

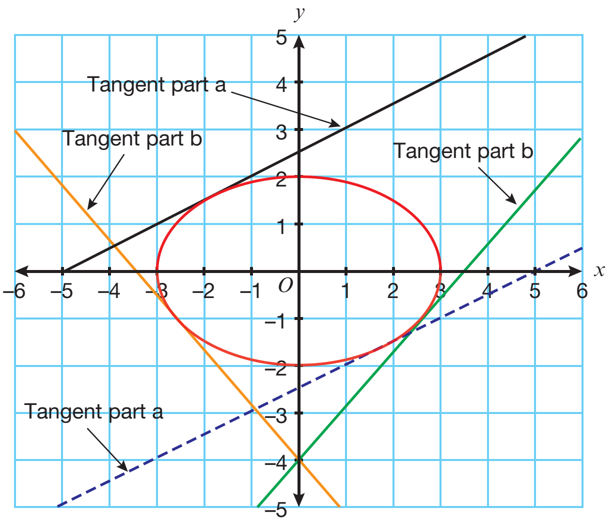
1



(a) Gradient is approximately −0.017 5
(b) Gradient $-m$ at (330°, 0.5)
(c) See graph. $x = 215°$ or $325°$

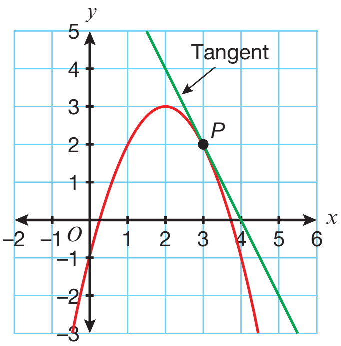
(d) The cosine curve is periodic with period 360°
$x = 215 + 360 = 575°$ or
$x = 325 + 360 = 685°$

2



(a) C = approximately 2.5 or
C = approximately −2.5 (see graph)
(b) m = approximately 1.1 or
m = approximately −1.1 (see graph)

3



(a) See graph. Gradient is −2
(b) **(i)** Stretch with scale factor $\frac{1}{a}$ parallel to the x-axis so P becomes $\left(\frac{3}{a}, 2\right)$, gradient is $a \times -2 = -2a$
(ii) Translation of $\begin{pmatrix} -a \\ 0 \end{pmatrix}$ so P becomes $(3 - a, 2)$, gradient is unchanged $= -2$
(iii) Stretch scale factor a parallel to the y axis so P becomes $(3, 2a)$, gradient is $a \times -2 = -2a$
(iv) Reflection in the y-axis so P becomes $(-3, 2)$, gradient becomes 2.

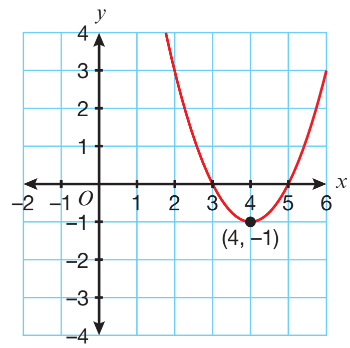
4 **(a)**

t (s)	1	2	3	4	5
v (m/s)	3	6	9	12	15

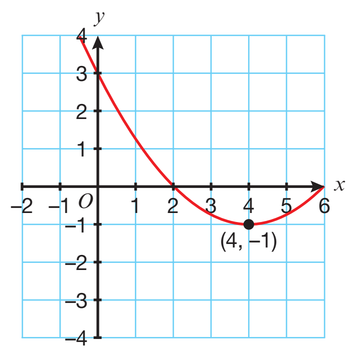
(b)

(c) Gradient is equal to 3 therefore the acceleration is 3 m/s²

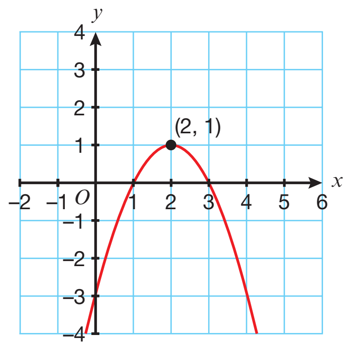
5 **(a)** **(i)** Translation of $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$



(ii) Stretch scale factor 2 parallel to x-axis



(iii) Reflection in the x-axis



(b) $g(x)$ has been reflected in the y-axis then translated by $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$.

The reverse of this is translate by $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ then reflect in the y-axis.

$(5, 3)$ translated by $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ gives $(5, 1)$.

$(5,1)$ reflected in the y-axis gives $(-5, 1)$

$\Rightarrow$ turning point is $(-5, 1)$

SHAPE AND SPACE 9 – BASIC SKILLS EXERCISE

1 $A'(90°, 10)$

2 $A'(180°, 9)$

3 $A'(180°, 10)$

4 $x = 60°, 120°$

5 $x = 60°, 300°$

6 $x = 45°, 225°$

7 9.22 cm

8 18.0 cm

9 15.6 cm

10 13.5 cm

11 11.9 cm

12 19.4 cm

13 46.5°

14 38.2°

15 55.1°

16 36.2°

17 20.7°

18 59.0°

19 **(a)** $C = 42.2°$, $a = 6.96$ m
(b) $C = 44.7°$, $a = 5.84$ m

20 **(a)** 68.9 m^2
(b) 120 m^2

21 92.1 m

22 Circle area = triangle area

Circumference of circle $= 6\pi = 2\pi r$, $r = 3$

Area of circle $= \pi r^2 = \pi \times 3^2 = 9\pi$

Let equilateral triangle have side x, so perimeter $p = 3x$

(Area of triangle $= \frac{1}{2}ab\sin C$)

$9\pi = \frac{1}{2} \times x \times x \times \sin(60°) = \frac{\sqrt{3}x^2}{4}$

$x^2 = \frac{36\pi}{\sqrt{3}} = \frac{3^2 \times 4 \times \pi}{\sqrt{3}} = 3^{\frac{3}{2}} \times 4 \times \pi$

$x = \sqrt{3^{\frac{3}{2}} \times 4 \times \pi} = 3^{\frac{3}{4}} \times 2 \times \pi^{\frac{1}{2}} = 2 \times \sqrt{\pi} \times 3^{\frac{3}{4}}$

$((a^m)^n = a^{mn})$

$p = 3x = 3 \times \left(2 \times \sqrt{\pi} \times 3^{\frac{3}{4}}\right) = 2 \times 3^{\frac{7}{4}} \times \sqrt{\pi}$

$= 2 \times 3^m \times n$

So $m = \frac{7}{4}$, $n = \sqrt{\pi}$

SHAPE AND SPACE 9 – EXAM PRACTICE EXERCISE

1 **(a)** **(i)** $2f(x) = 2\sin(x°)$

($2f(x)$ stretches the function by scale factor 2 parallel to y-axis)

P is transformed to $(90°, 2)$, Q is transformed to $(180°, 0)$

(ii) $-f(x) = -\sin(x°)$

($-f(x)$ reflects the function in the x-axis)

P is transformed to $(90°, -1)$, Q is transformed to $(180°, 0)$

(iii) $-2f(2x) + 2 = -2\sin(2x°) + 2$

The function $-2f(2x) + 2$

(i) stretches the function by scale factor $\frac{1}{2}$ parallel to x-axis.

(ii) stretches the function by scale factor 2 parallel to the y-axis.

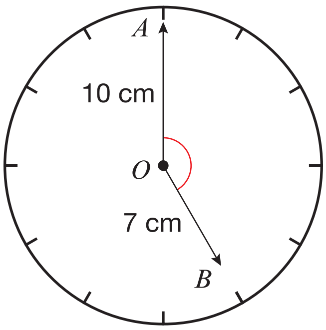
(iii) reflects the function in the x-axis.

(iv) translates the function along vector $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$

P is transformed to $(45°, 0)$, Q is transformed to $(90°, 2)$

(b) R is at $(30°, 0.5)$

2



(a) Triangle OAB at 05:00 angle AOB

$= 5 \times 30° = 150°$

$AB^2 = 10^2 + 7^2 - 2 \times 10 \times 7 \times \cos(150°)$

(cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$)

$AB = 16.4$ cm (3 s.f.)

(b) Triangle OAB at 17:50 angle AOB

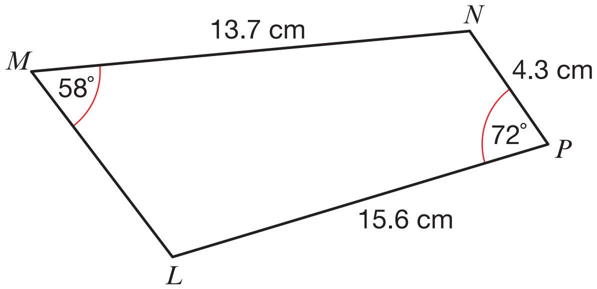
$= 4 \times 30° + (30° - \frac{50}{60} \times 30) = 125°$

$AB^2 = 10^2 + 7^2 - 2 \times 10 \times 7 \times \cos(125°)$

(cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$)

$AB = 15.1$ cm (3 s.f.)

3



Let required area $LMNP$ =

Area of triangle LNP + Area of triangle LMN

Area of triangle LNP

$= \frac{1}{2} \times 15.6 \times 4.3 \times \sin(72°) = 31.898\ldots \text{cm}^2$

(area of triangle $= \frac{1}{2}ab\sin C$)

Triangle LNP:

$LN^2 = 15.6^2 + 4.3^2 - 2 \times 15.6 \times 4.3 \times \cos(72°)$

(cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$)

$LN = 14.846\ldots$cm

Triangle LMN:

$\frac{\sin(MLN)}{13.7} = \frac{\sin(58°)}{14.846}$

(sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$)

$\sin(MLN) = 0.78259\ldots$,

angle $MLN = 51.498°\ldots$

angle $MNL = 180 - 58 - 51.498 = 70.502°\ldots$

(angle sum of a triangle =180°)

Area triangle LMN

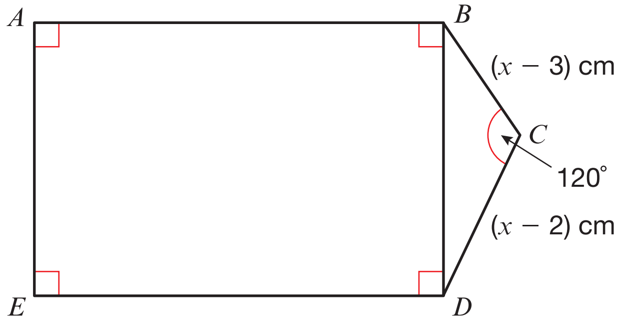
$= \frac{1}{2} \times 13.7 \times 14.846 \times \sin(70.502°)$

$= 95.863 \text{ cm}^2\ldots$

Area $LMNP = 31.898 + 95.863$

$= 127.761 \text{ cm}^2\ldots = 128 \text{ cm}^2$ (3 s.f.)

4



$ABDE$ is a rectangle in which $AB = 2BD$

Triangle BCD:

$BD^2 = (x - 3)^2 + (x - 2)^2 - 2(x - 3)(x - 2)$

$\cos(120°)$

(cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$)

$= (x^2 - 6x + 9) + (x^2 - 4x + 4) + (x^2 - 5x + 6)$

$= 3x^2 - 15x + 19$

Alternative: $AB \times BD = 14$ so $2(BD)^2 = 14$ and $BD^2 = 7$

$3x^2 - 15x + 19 = 7$ etc

Area $ABDE = 14 = AB \times BD = 2BD \times BD =$

$2(BD)^2 = 2(3x^2 - 15x + 19) = 6x^2 - 30x + 38$

$0 = 6x^2 - 30x + 24$

$0 = x^2 - 5x + 4$

$0 = (x - 4)(x - 1)$

$x = 4$ or $x = 1$

Discard $x = 1$ as the sides lengths are > 0.

If $x = 4$, $BC = 1$, $DC = 2$, $BD^2 = 7$,

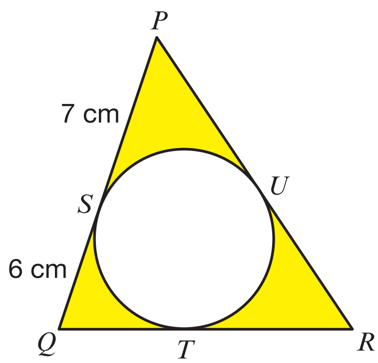
$BD = \sqrt{7}$, $AB = 2\sqrt{7}$

Required perimeter of pentagon

$ABCDE = 2AB + AE + BC + CD$

$= 4\sqrt{7} + \sqrt{7} + 1 + 2 = 5\sqrt{7} + 3$ cm

5



Perimeter of triangle PQR = 42 cm

$PS = PU = 7$ cm

$QS = QT = 6$ cm

$RT = RU = 8$ cm

Triangle PQR

$15^2 = 13^2 + 14^2 - 2 \times 13 \times 14 \times \cos(SQT)$

(Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$)

$\cos(SQT) = \frac{13^2 + 14^2 - 15^2}{2 \times 13 \times 14} = \frac{5}{13}$,

angle $SQT = 67.380°\ldots$

Let required area be A cm².

A = Area of triangle PQR − Area of circle

Area of triangle PQR

$= \frac{1}{2} \times 13 \times 14 \times \sin(67.380°) = 84{,}000\ldots \text{cm}^2$

(Area of triangle $= \frac{1}{2}ab\sin C$)

Consider circle centre O and triangle QSO.
Angle $OSQ = 90°$, so triangle QSO is a right-angled triangle.

OS = radius of circle r

$\tan\left(\frac{1}{2} \times 67.380°\right) = \frac{r}{6}$

$r = 4$ cm

Area of circle $= \pi \times 4^2 = 50.265\ldots \text{cm}^2$

$A = 84.000 - 50.265 = 33.7 \text{ cm}^2$ (3 s.f.)

HANDLING DATA 6 – BASIC SKILLS EXERCISE

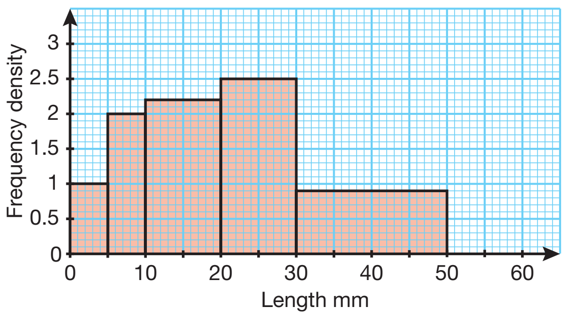
frequency density $= \frac{\text{frequency}}{\text{class width}}$

Be careful with the class widths when the data is continuous (i.e. time, weight, length, volume…)

1

Time, *t*	10 < *t* ≤ 20	20 < *t* ≤ 25	25 < *t* ≤ 30	30 < *t* ≤ 40	40 < *t* ≤50	50 < *t* ≤70	70 < *t* ≤95
Frequency	7	15	25	18	12	8	5
Frequency density	0.7	3	5	1.8	1.2	0.4	0.2

2 **(a)**

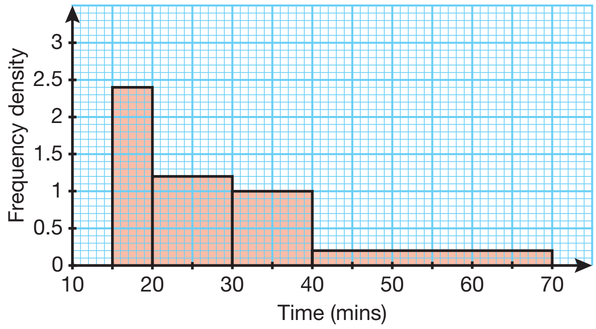


Length, *l* (mm)	Frequency density
0 < *l* ≤ 5	1
5 < *l* ≤ 10	2
10 < *l* ≤ 20	2.2
20 < *l* ≤ 30	2.5
30 < *l* ≤ 50	0.9

(b) 7

(c) 22.0 mm

3 **(a)**

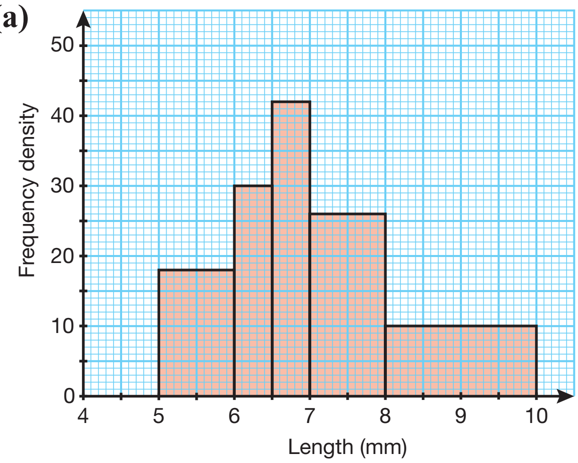


Time, *t* (min)	Frequency density
15 < *t* ≤ 20	2.4
20 < *t* ≤ 30	1.2
30 < *t* ≤ 40	1
40 < *t* ≤ 70	0.2

(b) 2

(c) 17 (16.8)

4 **(a)**

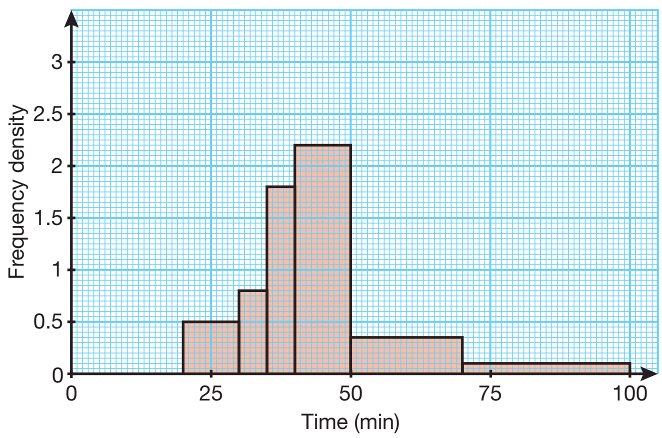


Length, *l* (mm)	Frequency density
5 < *l* ≤ 6	18
6 < *l* ≤ 6.5	30
6.5 < *l* ≤ 7	42
7 < *l* ≤ 8	26
8 < *l* ≤ 10	10

(b) 76

(c) 7.10 mm

5 **(a)**



Time, t (min)	Frequency density
$20 < t \le 30$	0.5
$30 < t \le 35$	0.8
$35 < t \le 40$	1.8
$40 < t \le 50$	2.2
$50 < t \le 70$	0.35
$70 < t \le 100$	0.1

(b) 71.2%
(c) 43.2 mins

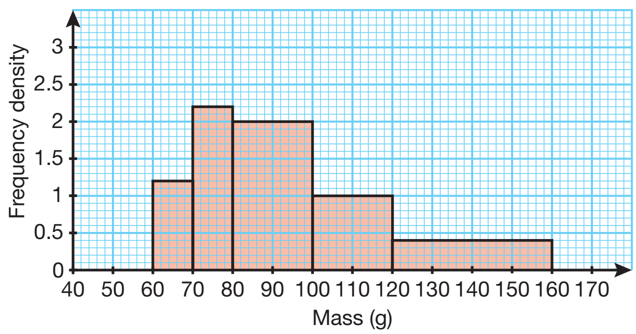
HANDLING DATA 6 – EXAM PRACTICE EXERCISE

Frequency density = $\frac{\text{frequency}}{\text{class-width}}$

Be careful with the class widths when the data is continuous (i.e. time, weight, length, volume…)

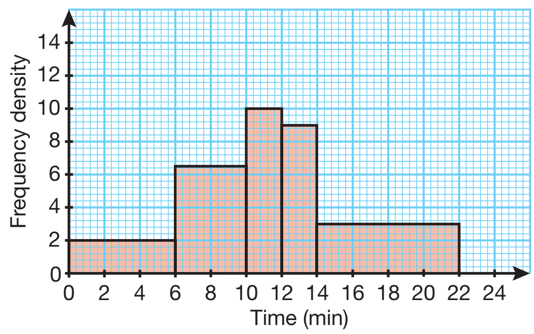
1 **(a) (b)**

Mass, m (g)	Frequency	Frequency density
$60 < m \le 70$	12	1.2
$70 < m \le 80$	22	2.2
$80 < m \le 100$	40	2
$100 < m \le 120$	20	1
$120 < m \le 160$	16	0.4



(c) 41

2 **(a)**



(b)

Time, t (min)	Frequency	Frequency density
$0 < t \le 6$	12	2
$6 < t \le 10$	26	6.5
$10 < t \le 12$	20	10
$12 < t \le 14$	18	9
$14 < t \le 22$	24	3

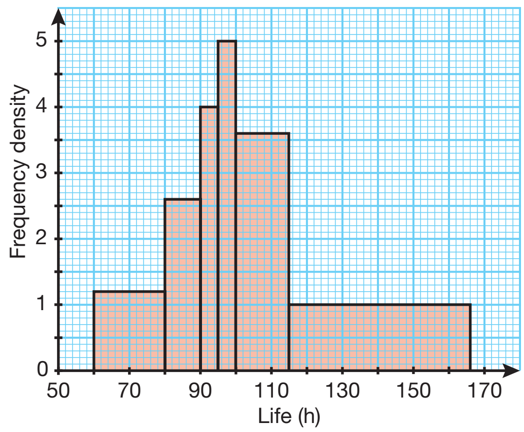
(c) 57%
(d) 11.2 min

3 **(a)**

Life, t (h)	Frequency	Frequency density
$60 < t \le 80$	24	1.2
$80 < t \le 90$	26	2.6
$90 < t \le 95$	20	4
$95 < t \le 100$	25	5
$100 < t \le 115$	54	3.6
$115 < t \le x$	51	1

(b) $x = 166$

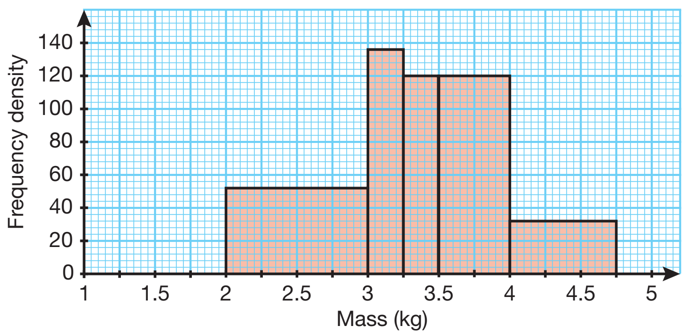
(c)



4 **(a)**

Mass, m kg	Frequency	Frequency density
$2 < m \le 3$	52	52
$3 < m \le 3.25$	34	136
$3.25 < m \le 3.5$	30	120
$3.5 < m \le 4$	60	120
$4 < m \le 4.75$	24	32

(b)

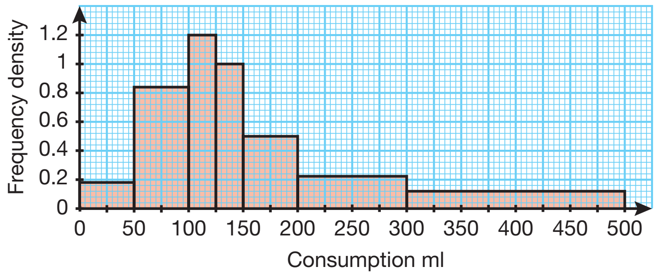


(c) 0.83
(d) 3.37 kg

5 **(a)**

Consumption, m (ml)	Frequency
$0 < m \le 50$	9
$50 < m \le 100$	42
$100 < m \le 125$	30
$125 < m \le 150$	25
$150 < m \le 200$	25
$200 < m \le 300$	22
$300 < m \le 500$	27

(b)



(c) 134 ml
(d) 50.1%

NUMBER 10 – BASIC SKILLS EXERCISE

1 a, b and d

2 **(a)** e.g. 4.2
(b) e.g. 5.4

3 **(a)** e.g. $\sqrt{137}$
(b) e.g. $\sqrt{9.9}$

4 **(a)** e.g. $k = 3$
(b) e.g. $k = 1$

5 $n = \frac{28}{3k}$ therefore rational

6 $\sqrt{63k}$

7 $8k\sqrt{3}$

8 **(a)** $a^2 b$

(b) $\frac{1}{4a}$

(c) $\frac{8a}{b}$

9 4.5

10 $\frac{1}{2}$

11 **(a)** $17\sqrt{3}$
(b) $\sqrt{7}$
(c) $5\sqrt{10}$

12 $a = 12$

13 **(a)** $2\sqrt{2} - 1$
(b) $11 - 6\sqrt{2}$
(c) 24

14 **(a)** $a\sqrt{b}$
(b) $a + b + 2\sqrt{a}\sqrt{b}$
(c) $1 - a$
(d) $4a$

15 $4\sqrt{ab}$

16 $a = 4,\ b = 12$

17 **(a)** $\sqrt{3}$
(b) 2
(c) 2
(d) $9\sqrt{3}$

18 **(a)** $\frac{22(7 - \sqrt{5})}{7}$

(b) $3 + 3\sqrt{3}$
(c) $\sqrt{7} + \sqrt{5}$

19 **(a)** $2a + 1 + \sqrt{2}(a + 1)$

(b) $\sqrt{a}$

(c) $4\sqrt{a} + 3$

20 $\frac{3\sqrt{2}}{8}$

21 $\frac{\sqrt{a}(a^2 + a + 1)}{a^3}$

22 All are equal to $\frac{3\sqrt{35}}{14}$

23 Radius = $\sqrt{12} = 2\sqrt{3} \Rightarrow$
Perimeter = $2 \times 2\sqrt{3} + \pi 2\sqrt{3}$

24 $x\sqrt{12} + x\sqrt{75} = 21$

$x\left(2\sqrt{3} + 5\sqrt{3}\right) = 21$

$x = \frac{21 \times \sqrt{3}}{(7\sqrt{3}) \times \sqrt{3}} = \sqrt{3}$

25 $x2\sqrt{2} - \frac{3x}{\sqrt{2}} = 5$

$4x - 3x = 5\sqrt{2}$
$x = 5\sqrt{2}$

26 $4\sqrt{2}$

27 $n = 4$

28 $x = \frac{2 \pm \sqrt{10}}{2}$

29 $x = 2\sqrt{3}$ or $x = \frac{\sqrt{3}}{3}$

30 $7\left(4a + 2\sqrt{2}a^2\right)$

31 $\sqrt{a}$

32 Let h be the third side of the triangle

$(\sqrt{2} + \sqrt{14})^2 = h^2 + (2 + \sqrt{7})^2$

Pythagoras' theorem

$16 + 2\sqrt{2}\sqrt{14} = h^2 + 11 + 4\sqrt{7}$

$5 + 4\sqrt{7} = h^2 + 4\sqrt{7}$ $\quad \sqrt{14} = \sqrt{2}\sqrt{7}$

$h^2 = 5$

$h = \sqrt{5}$

$\text{area} = \frac{1}{2}(2 + \sqrt{7}) \times \sqrt{5}$

Area of a triangle $= \frac{1}{2}$ base $\times$ height

$\text{area} = \left(\frac{2 + \sqrt{7}}{2}\right)\sqrt{5}$, so $p = \sqrt{5}$

NUMBER 10 – EXAM PRACTICE EXERCISE

1 **(a)** **(i)**

$\frac{a - \sqrt{a}}{a + \sqrt{a}} = \frac{(a - \sqrt{a})(a - \sqrt{a})}{(a + \sqrt{a})(a - \sqrt{a})}$

$= \frac{a^2 - 2a\sqrt{a} + a}{a^2 - a}$

$= \frac{a(a - 2\sqrt{a} + 1)}{a(a - 1)}$

$= \frac{a + 1 - 2\sqrt{a}}{a - 1}$

(ii)

$\frac{1}{\sqrt{a}} + \frac{1}{a} + \frac{1}{\sqrt{a^3}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{a}\sqrt{a}} + \frac{1}{\sqrt{a}\sqrt{a}\sqrt{a}}$

$= \frac{a + \sqrt{a} + 1}{\sqrt{a^3}}$

$= \frac{\sqrt{a^3}(a + \sqrt{a} + 1)}{\sqrt{a^3}\sqrt{a^3}}$

$= \frac{a^2\sqrt{a} + a^2 + a\sqrt{a}}{a^3}$

$= \frac{a\sqrt{a} + a + \sqrt{a}}{a^2}$

$= \frac{a + \sqrt{a}\,(a + 1)}{a^2}$

(b) $r = 25s$

$\frac{p}{q} = \frac{\sqrt{125} + \sqrt{25s}}{\sqrt{5} + \sqrt{s}}$

$= \frac{5\sqrt{5} + 5\sqrt{s}}{\sqrt{5} + \sqrt{s}}$

$= \frac{5(\sqrt{5} + \sqrt{s})}{\sqrt{5} + \sqrt{s}}$

$= 5$

$p : q = 5 : 1$

2 Total surface area is two ends plus curved surface area $= 2\pi r^2 + 2\pi rh$

$2\pi(3\sqrt{2})^2 + 2\pi(3\sqrt{2})(a\sqrt{2} + b\sqrt{3}) = 48\sqrt{6}\pi$

Divide both sides by π

$36 + 2(6a + 3b\sqrt{6}) = 48\sqrt{6}$

$36 + 12a + 6b\sqrt{6} = 48\sqrt{6}$

equating rational numbers and irrational numbers separately

$36 + 12a = 0$ and $6b = 48$

$a = -3$ and $b = 8$

3 **(a)** Using the quadratic formula with

$a = 1$, $b = -(1 + 2\sqrt{p})$ and $c = p + \sqrt{p}$

$b^2 - 4ac = (1 + 2\sqrt{p})^2 - 4(p + \sqrt{p})$

$= 1 + 4p + 4\sqrt{p} - 4p - 4\sqrt{p} = 1$

$x = \frac{(1 + 2\sqrt{p}) \pm 1}{2}$

$x = \frac{2 + 2\sqrt{p}}{2}$ or $x = \frac{2\sqrt{p}}{2}$

$x = 1 + \sqrt{p}$ or $x = \sqrt{p}$

(b) $(a + \sqrt{b})(a + \sqrt{b}) = a^2 + b + 2a\sqrt{b}$

$a^2 + b + 2a\sqrt{b} = 7 + \sqrt{48}$

$a^2 + b = 7(1)$ and $2a\sqrt{b} = \sqrt{48}(2)$

equating rational values and irrational values separately

(1) $\quad a^2 = 7 - b$, (2) $a^2b = 12$

Substituting (1) into (2) gives

$(7 - b)b = 12$

$b^2 - 7b + 12 = 0$

$(b - 4)(b - 3) = 0$

$b = 4$ or $b = 3$

$a^2 = 3$ or $a^2 = 4$

$a = \pm\sqrt{3}$ or $a = \pm 2$

$\sqrt{7 + \sqrt{48}} = \pm 2 \pm \sqrt{3}$

The positive possible values are $2 + \sqrt{3}$ or $2 - \sqrt{3}$

$(2 + \sqrt{3})^2 = 7 + 4\sqrt{48} = 7 + \sqrt{48}$ Correct

$(2 - \sqrt{3})^2 = 7 - 4\sqrt{3} = 7 - \sqrt{48}$ Incorrect

Positive value of $\sqrt{7 + \sqrt{48}} = 2 + \sqrt{3}$

4 Let w be the width

$w(a + \sqrt{2}) = (1 + a)\sqrt{2} + (a + 2)$

$w = \dfrac{(1 + a)\sqrt{2} + (a + 2)}{a + \sqrt{2}}$

$= \dfrac{[(1 + a)\sqrt{2} + (a + 2)](a - \sqrt{2})}{(a + \sqrt{2})](a - \sqrt{2})}$

$= \dfrac{a\sqrt{2} + a^2\sqrt{2} + a^2 + 2a - 2 - 2a - a\sqrt{2} - 2\sqrt{2}}{a^2 - 2}$

$= \dfrac{a^2(1 + \sqrt{2}) - 2(1 + \sqrt{2})}{a^2 - 2}$

$= \dfrac{(a^2 - 2)(1 + \sqrt{2})}{a^2 - 2}$

$= 1 + \sqrt{2}$ cm

5 Let h be the height of the triangle

$\frac{1}{2}(3 - \sqrt{2}) \times h = \sqrt{6} + \frac{\sqrt{3}}{2}$

$\sqrt{6} + \dfrac{\sqrt{3}}{2} = \dfrac{2\sqrt{6} + \sqrt{3}}{2}$

$h = \dfrac{2\sqrt{6} + \sqrt{3}}{3 - \sqrt{2}}$

$= \dfrac{(2\sqrt{6} + \sqrt{3})(3 + \sqrt{2})}{(3 - \sqrt{2})(3 + \sqrt{2})}$

$= \dfrac{6\sqrt{6} + 3\sqrt{3} + 2\sqrt{2}\sqrt{6} + \sqrt{6}}{9 - 2}$

$2\sqrt{2}\sqrt{6} = 2\sqrt{2}\sqrt{2}\sqrt{3}$

$= 4\sqrt{3}$

$h = \dfrac{7\sqrt{6} + 7\sqrt{3}}{7}$

$= \sqrt{6} + \sqrt{3}$ cm

Let H be the hypotenuse of triangle.

$H^2 = (3 - \sqrt{2})^2 + (\sqrt{6} + \sqrt{3})^2$

$= 11 - 6\sqrt{2} + 9 + 2\sqrt{3}\sqrt{6}$

$2\sqrt{3}\sqrt{6} = 2\sqrt{3}\sqrt{2}\sqrt{3} = 6\sqrt{2}$

$= 20$

$H = \sqrt{20}$

Perimeter $= (3 - \sqrt{2}) + (\sqrt{6} + \sqrt{3}) + \sqrt{20}$

$= \sqrt{3}\sqrt{3} - \sqrt{2} + \sqrt{2}\sqrt{3} + \sqrt{3} + \sqrt{4}\sqrt{5}$

$= \sqrt{2}(\sqrt{3} - 1) + \sqrt{3}(\sqrt{3} + 1) + 2\sqrt{5}$ cm

ALGEBRA 10 – BASIC SKILLS EXERCISE

1 $2 + 3x$

2 $3x$

3 $-5x$

4 $\dfrac{x + 6}{4}$

5 $\dfrac{x - 2}{3}$

6 $\dfrac{2(x - 5)}{x - 4}$

7 $\dfrac{x + 1}{x - 2}$

8 $\dfrac{2x - 3}{3x + 2}$

9 $\dfrac{93 - 11x}{35}$

10 $\dfrac{x^2 + x + 1}{x + 1}$

11 $\dfrac{2x + 5}{3(x - 2)}$

12 $\dfrac{3x}{(1 - x)(2 + x)}$

13 $\dfrac{x + 7}{(2x - 1)(3x + 1)}$

14 $\dfrac{-2}{(x + 4)(x + 6)}$

15 $\dfrac{1}{3(x - 6)}$

16 $\dfrac{-1}{2(x + 3)}$

17 $(x - 7)(x - 3)$

18 $3x$

19 x
$(x + 1)(x - 7)$

20 $\dfrac{x - 1}{x + 1}$

21 1

22 2

23 $\dfrac{3}{x + 2}$

24 $x + 3$

25 $x = 28$

26 $x = -2.427. \quad x = 2$

28 $x = -8$

29 $x = -\frac{1}{2}$ or -4

30 $x = -6$ or $x = 4$

31 $x = \pm 2$

32 $x = 2.4$

33 $x = -2$

34 $x = 2$ or $x = 3$

35 $\frac{1}{x-1}$

36 $\frac{2}{3}$

37 $3 - 2x$

38 $\frac{x^2 - 3x - 4}{x + 3} \div \frac{x + 1}{x^2 - x - 12} = \frac{(x-4)(x+1)}{x+3}$

$\times \frac{(x-4)(x+3)}{x+1} = (x-4)^2 \geq 0$

ALGEBRA 10 – EXAM PRACTICE EXERCISE

1 **(a)** $\frac{3y}{x} - \frac{y}{x+3} = y\left(\frac{3}{x} - \frac{1}{x+3}\right)$

$= y\left(\frac{3(x+3) - x}{x(x+3)}\right) = y\left(\frac{2x+9}{x(x+3)}\right)$

$\frac{3y}{x} - \frac{y}{x+3} = 2x + 9$

$y = \left(\frac{2x+9}{x(x+3)}\right) = 2x + 9$

$y = \frac{x(x+3)(2x+9)}{(2x+9)}$

$y = x(x+3)$

$y = x^2 + 3x$

$y = \left(x + \frac{3}{2}\right)^2 - \frac{9}{4}$

(b) **(i)** $\left(-\frac{3}{2}, -\frac{9}{4}\right)$, **(ii)** a minimum

y is a positive quadratic

2 **(a)** $x = \frac{t+3}{t-1}$

$x(t-1) = t + 3$

$xt - x = t + 3$

$t(x-1) = x + 3$

$t = \frac{x+3}{x-1}$

Substituting gives $y = \left(\frac{x+3}{x-1}\right)^2 - 4\left(\frac{x+3}{x-1}\right)$

$y = \frac{(x+3)^2 - 4(x-1)(x+3)}{(x-1)^2}$

$y = \frac{x^2 + 6x + 9 - 4x^2 - 8x + 12}{(x-1)^2}$

$y = \frac{-3x^2 - 2x + 21}{(x-1)^2}$

$y = \frac{-(3x-7)(x+3)}{(x-1)^2}$ or $\frac{(7-3x)(x+3)}{(x-1)^2}$

(b) $\left(\frac{x+3}{x-1}\right) \div y$

$= \left(\frac{x+3}{x-1}\right) \times \frac{(x-1)^2}{(7-3x)(x+3)}$

$= \frac{x-1}{7-3x}\left(\frac{x+3}{x-1}\right) : y = (x-1) : (7-3x)$

3 **(a)** $\frac{2x^2 + 5x - 12}{4x^2 - 9} \div \frac{x^2 + 2x - 8}{2x^2 + 5x + 3}$

$= \frac{2x^2 + 5x - 12}{4x^2 - 9} \times \frac{2x^2 + 5x + 3}{x^2 + 2x - 8}$

$= \frac{(2x-3)(x+4)}{(2x+3)(2x-3)} \times \frac{(2x+3)(x+1)}{(x+4)(x-2)}$

$= \frac{x+1}{x-2}$

(b) $\frac{2x^2 + 5x - 12}{4x^2 - 9} \div \frac{x^2 + 2x - 8}{2x^2 + 5x + 3}$

$= 1 + \frac{x}{x+2}$

$\frac{x+1}{x-2} = 1 + \frac{x}{x+2}$

Using the result from part a.

$\frac{x+1}{x-2} = 1 + \frac{x}{x+2}$

$(x+1)(x+2) = (x-2) + x(x+2) + x(x-2)$

Multiplying both sides by $(x-2)(x+2)$

$x^2 + 3x + 2 = x^2 - 4 + x^2 - 2x$

$x^2 - 5x - 6 = 0$

$(x+1)(x-6) = 0$

$x = -1$ or $x = 6$

4 Time taken to travel x km at $x + 10$ km/h is $\frac{x}{x+10}$ h

Time taken to travel $70 - x$ km at $x - 20$ km/h is $\frac{70-x}{x-20}$ h

$\frac{x}{x+10} + \frac{70-x}{x-20} = 1$

Multiply both sides by $(x+10)(x-20)$
$x(x-20) + (x+10)(70-x) = (x+10)(x-20)$
$x^2 - 20x - x^2 + 60x + 700 = x^2 - 10x - 200$
$x^2 - 50x - 900 = 0$
Solving using the quadratic formula gives $x = 25 + 5\sqrt{61}$ or 64.1 to 3 s.f.

5 **(a)** $y = \frac{8x^2+16x-10}{3x^2-3} \div \frac{2x+5}{3x-3}$

$= \frac{2(2x-1)(2x+5)}{3(x-1)(x+1)} \times \frac{3(x-1)}{(2x+5)}$

$= \frac{2(2x-1)}{x+1}$

(b) Half perimeter is $\frac{2x+5}{3x-3} + \frac{2(2x-1)}{x+1} = 5$

Multiply both sides by $3(x-1)(x+1)$
$(2x+5)(x+1) + 2(2x-1)3(x-1)$
$= 5 \times 3(x-1)(x+1)$
$2x^2 + 7x + 5 + 12x^2 - 18x + 6 = 15x^2 - 15$
$x^2 + 11x - 26 = 0$
$(x-2)(x+13) = 0$
$x = 2$
sides are $\frac{4+5}{6-3} = 3$ and $\frac{2(4-1)}{3} = 2$

area $= 3 \times 2 = 6$ cm^2

GRAPHS 9 – BASIC SKILLS EXERCISE

1 **(a)** $\frac{dy}{dx} = 3$

(b) $\frac{dy}{dx} = 0$

(c) $\frac{dy}{dx} = 3x^2$

(d) $\frac{dy}{dx} = 4x^3$

(e) $\frac{dy}{dx} = 5x^4$

(f) $\frac{dy}{dx} = 12x^5$

(g) $\frac{dy}{dx} = 15x^4$

(h) $\frac{dy}{dx} = 160x^7$

2 **(a)** $\frac{dy}{dx} = 6x^2 + 10x$

(b) $\frac{dy}{dx} = 14x - 3$

(c) $\frac{dy}{dx} = 15x^2$

(d) $\frac{dy}{dx} = 12x^3 - 10x$

(e) $\frac{dy}{dx} = 3x^2 + 10x$

(f) $\frac{dy}{dx} = 2x + 2$

(g) $\frac{dy}{dx} = 4x - 9$

(h) $\frac{dy}{dx} = 2x + 4$

3 **(a)** $\frac{dy}{dx} = -x^{-2} = -\frac{1}{x^2}$

(b) $\frac{dy}{dx} = -2x^{-3} = -\frac{2}{x^3}$

(c) $\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

(d) $\frac{dy}{dx} = -3x^{-4} = -\frac{3}{x^4}$

(e) $\frac{dy}{dx} = -16x^{-5} = -\frac{16}{x^5}$

(f) $\frac{dy}{dx} = -x^{-3} = -\frac{1}{x^3}$

(g) $\frac{dy}{dx} = 4x + 3 - 4x^{-2}$

(h) $\frac{dy}{dx} = 2 + 3x^{-2}$

4 **(a)** $\frac{dy}{dx}$ (at $x = 1$) $= 4$

(b) $\frac{dy}{dx}$ (at $x = 1$) $= -5$

(c) $\frac{dy}{dx}$ (at $x = 1$) $= 26$

(d) $\frac{dy}{dx}$ (at $x = 2$) $= 19$

5 **(a)** -2

(b) 2

(c) 0

(d) $\frac{-17}{8}$

6 $4y = 3x + 4$

7 $y = -3x$, $y = 3x - 9$

8 $p = 1$

9 $p = 3$, $q = -2$, so $p^3 + q^3 = 27 - 8 = 19$

10 **(a)** $\frac{dy}{dx} = 3x^2 - 3$

(b) (1, 0), (−1, 4)
(c) (−1, 4) is max, (1, 0) is min.

11 **(a)** $\frac{dy}{dx} = 3x^2 + 6x - 9$

(b) (−3, 28), (1, −4)

(c) (−3, 28) is max, (1, −4) is min.

12 **(a)** (0, 1), (4/3, −0.185185...)

(b) $x = 0$ is a max, $x = \frac{4}{3}$ is a min.

13 **(a)** $v = 24t$ m/s

(b) 48 m/s

(c) $a = 24$ m/s^2

(d) 24 m/s^2

14 **(a)** $v = 3t^2 + 8t - 3$ m/s

(b) 377 m/s

(c) $a = 6t + 8$ m/s^2

(d) 68 m/s^2

15 **(a)** $s = 125$ m

(b) $t = 5$

16 **(a)** $v = \frac{ds}{dt} = 3t^2 - 300$ km/s

(b) at $t = 5$, $v = -225$ km/s

(c) $a = \frac{dv}{dt} = 6t$ km/s^2

(d) at $t = 5$, $a = 30$ km/s^2

(e) $t = 10$ s

17 **(a)** $v = 24t$ m/s

(b) 72 m/s

(c) $a = 24$ m/s^2, (constant)

(d) 24 m/s^2

18 **(a)** $v = 3t^2 - 500$ km/s

(b) at $t = 10$ s, $v = -200$ km/s

(c) $a = 6t$ km/s^2

(d) at $t = 10$ s, $a = 60$ km/s^2

(e) $t = \left(\frac{\sqrt{500}}{3}\right)^{0.5}$ s, 12.9 s

19 **(a)** $\frac{dQ}{dt} = 3t^2 - 16t + 14$

(b) **(i)** 14 m^3/s^2
(ii) −6 m^3/s^2
(iii) 9 m^3/s^2

20 **(a)** $\frac{dC}{dt} = 4 - 16t^{-2} = 4 - \frac{16}{t^2}$

(b) $t = 2$, $C = 1$

(c) −12°C/month

GRAPHS 9 – EXAM PRACTICE EXERCISE

1 **(a)** $P = 5t^2 + \frac{10\,000}{t} + 10$

$P = 5t^2 + 10\,000t^{-1} + 10$

(Rewrite P in index form so differentiation is easier.)

$\frac{dP}{dt} = 10t - 10\,000t^{-2}$ flowers/year

(Now equate this to zero and solve for t.)

(b) $\frac{dP}{dt} = 0 = 10t - \frac{10\,000}{t^2}$

(Multiply by t^2.)
$0 = 10t^3 - 10\,000$
$10\,000 = 10t^3$
$1000 = t^3$
$t = 10$ yrs
(From the graph, the gradient of the curve is zero at $t = 10$ yrs.)
At $t = 10$, $P = 1510$
(Substitute $t = 10$ into equation for P.)

2 **(a)** $y = x^2 - 3x$ (Differentiate to find the gradient function.)

$\frac{dy}{dx} = 2x - 3$ (Find the gradient to the curve at $x = 4$.)

At $x = 4$, $\frac{dy}{dx} = 5$

Gradient of normal $= -\frac{1}{5}$

(If m_1 is gradient of tangent and m_2 is gradient of normal, $m_1 \times m_2 = -1$)

Equation of normal at $x = 4$ is

$y = -\frac{1}{5}x + c$

(Equation of a straight line is $y = mx + c$)

At $x = 4$, $y = 4$ (The y-value is found by substituting $x = 4$ into $y = x^2 - 3x$)

$4 = -\frac{1}{5} \times 4 + c$ (Point (4 ,4) is on the normal so it must satisfy the equation.)

$c = 4\frac{4}{5}$, so equation of normal is

$y = -\frac{1}{5}x + 4\frac{4}{5}$

$x + 5y = 24$ (Multiply the equation $y = -\frac{1}{5}x + 4\frac{4}{5}$ by 5 and re-arrange.)

$a = 1$, $b = 5$, $c = 24$

(b) At A, $y = 0$, so $x = 24$

At B, $x = 0$, so $y = \frac{24}{5}$

Area of triangle $OAB = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 24 \times \frac{24}{5} = \frac{288}{5}$ units^2

So $\frac{288}{5}$ = area of a square side x

$x = \sqrt{\frac{288}{5}} = \frac{12\sqrt{10}}{5}$, $p = 4x$

so $p = \frac{48\sqrt{10}}{5} = \frac{48\sqrt{2}}{\sqrt{5}}$ as required

$\left(\frac{48\sqrt{10}}{5} = \frac{48\sqrt{5} \times \sqrt{2}}{\sqrt{5} \times \sqrt{5}}\right)$

3 **(a)** At $A(1,5)$: $5 = 1 - 6 + p + q$

so $p + q = 10$ (1)

(Substitute coordinates of A into equation)

If A is a turning point $\frac{dy}{dx} = 0$
$= 3x^2 - 12x + p$
So at $x = 1$, $0 = 3 - 12 + p$,
so $p = 9$ and $q = 1$ (Using equation (1))

(b) $y = x^3 - 6x^2 + 9x + 1$

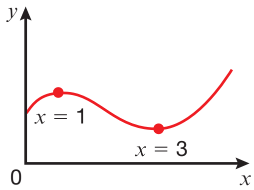
$\frac{dy}{dx} = 3x^2 - 12x + 9$

$0 = x^2 - 4x + 3$

(Gradient = 0 at turning points)

$0 = (x - 1)(x - 3)$
$x = 1$ or $x = 3$, so $y = 5$ or 1 so $A(1, 5)$, $B(3, 1)$

Due to the shape of the cubic curve, the smallest x value is the maximum point on the function.
So $A(1, 5)$ max point and $B(3, 1)$ min point.



4 **(a)** $v = t(k - 5t) = kt - 5t^2$

$a = \frac{dv}{dt} = k - 10t$

(Differentiate velocity function to get acceleration function)
$t = 0$, $a = 3$, so $3 = k - 10 \times 0$,
$k = 3$

(b) $v = 3t - 5t^2 = 0 = t(3 - 5t)$, so $t = 0$ or 0.6 s
$a = 3 - 10t = 3 - 10(0.6)$
(Take the value of $t = 0.6$ s)
$a = -3$ m/s^2

5 **(a)** **(i)** $V = \pi r^2 h$
$12 : 4 = h : (4 - r)$
(Similar triangles…)
$h = 3(4 - r)$
$V = \pi r^2 \times 3(4 - r) = 3\pi r^2(4 - r)$ as required

(ii) $V = 12\pi r^2 - 3\pi r^3$

$\frac{dV}{dr} = 24\pi r - 9\pi r^2 = 3\pi r(8 - 3r)$

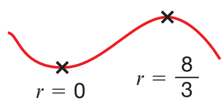
as required

(b) **(i)** $\frac{dV}{dr} = 0 = 3\pi r(8 - 3r)$

(Gradient of curve is flat at V_{max})

$r = 0$ or $\frac{8}{3}$, so $r = \frac{8}{3}$ cm

(Ignore $r = 0$ as it does not fit the model)



(Negative cubic curves will have this shape)

(ii) When $r = \frac{8}{3}$ cm , $V = 3\pi\left(\frac{8}{3}\right)^2\left(4 - \frac{8}{3}\right)$

$= \frac{256\pi}{9}$ cm^3, so $p = 9$

SHAPE AND SPACE 10 – BASIC SKILLS EXERCISE

1 36.4 m

2 21.4 m

3 18.4°

4 0.828 m

5 10.8°

6 10.39 m

7 1.79 m

8 82.7%

9 **(a)** 32.6°

(b) 38.5 units^2

10 **(a)** 243°

(b) 63.4°

11 125°

12 **(a)** 2250 m

(b) 3897 m

(c) 4500 m

13 **(a)** 13.9 km 279.7°

(b) He travels at 7.2 km/h so he does arrive by 18:00

14 2.5 km

15 **(a)** 25.5 km

(b) 022.7°

(c) 203°

SHAPE AND SPACE 10 – EXAM PRACTICE EXERCISE

1 Let cliff height WZ be h metres

Speed $= \frac{\text{Distance}}{\text{Time}}$

$0.75 = \frac{ZX - ZY}{60}$ [1]

Triangle WZX:
$\tan(60°) = \frac{ZX}{h}$, so $ZX = h\tan(60°)$ [2]

Triangle WZY:
$\tan(40°) = \frac{ZY}{h}$, so ZY $= h\tan(40°)$ [3]

Subs [2] and [3] into [1]

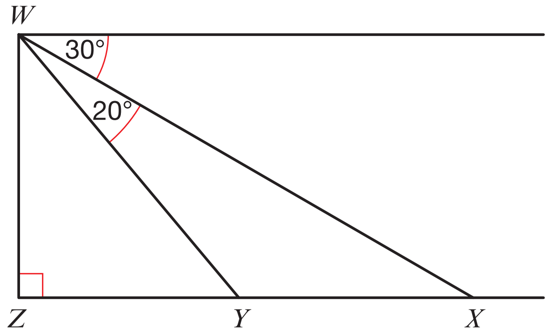
[1] $0.75 = \frac{h\tan(60°) - h\tan(40°)}{60}$

$= \frac{h[\tan(60°) - \tan(40°)]}{60}$

So $0.75 \times 60 = h[\tan(60°) - \tan(40°)]$

$h = \frac{45}{\tan(60°) - \tan(40°)}$

$h = 50.4$ m (3 s.f.)



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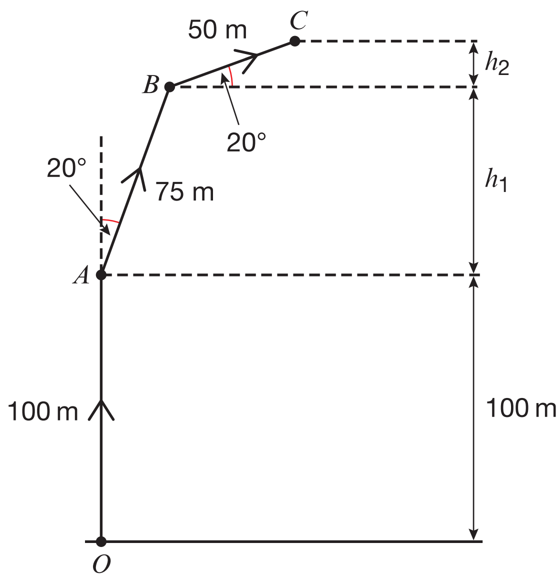


speed $= \frac{\text{distance}}{\text{time}}$

speed of descent $= \frac{100 + h_1 + h_2}{60}$ m/s

Motion from A to B:

$\sin(70°) = \frac{h_1}{75}$, $h_1 = 75 \times \sin(70°) = 70.4769\ldots$m

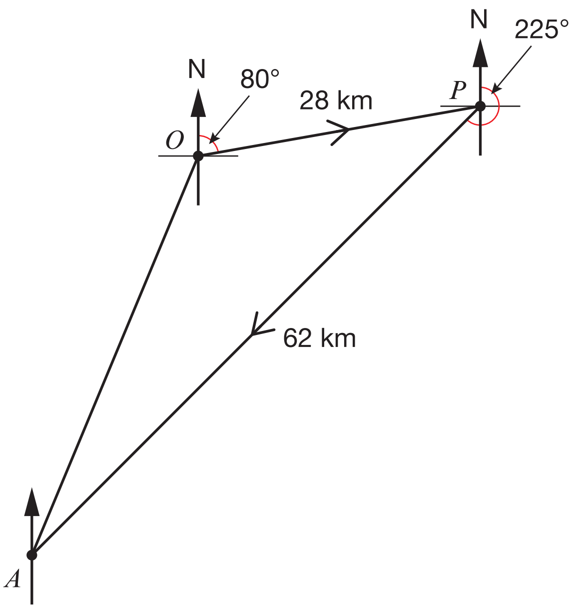
Motion from B to C:

$\sin(20°) = \frac{h_2}{50}$, $h_2 = 50 \times \sin(20°) = 17.1010\ldots$m

speed of descent $= \frac{100 + 70.4769 + 17.1010}{60}$

$= 3.1263\ldots$m/s $= 3.13$ m/s (3 s.f.)

3



(a) Angle $OPA = 80° - 45° = 35°$
(alternate angles, North to South = 180°)
cosine rule (SSSA condition so cosine rule)
$OA^2 = 28^2 + 62^2 - 2 \times 28 \times 62 \times \cos(35°)$
$= 1783.9041\ldots$
$OA = 42.2363\ldots$km
$OA = 42.2$ km (3 s.f.)

(b) Bearing of O from A:
sine rule (SASA condition so sine rule)
Let angle $OAP = \theta$

$$\frac{\sin\theta}{28} = \frac{\sin(35°)}{42.2363}$$

so $\sin\theta = \frac{\sin(35°)}{42.2363} \times 28$
$= 0.38024\ldots$
$\theta = 22.349°\ldots$
Angle $AOP = 180° - 35° - 22.349°$
$= 122.651°\ldots$
Bearing of O from $A = (80° + 122.651°) - 180°$ (alternate angles)
Bearing of O from $A = 022.7°$ (3 s.f.)

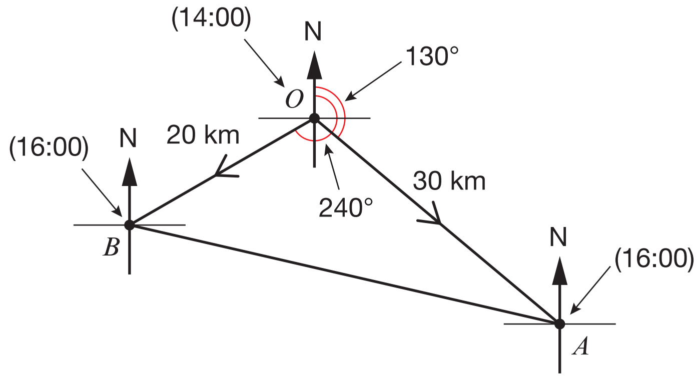
(c) speed $= \frac{\text{distance}}{\text{time}}$

balloon: $30 = \frac{28 + 62}{t}$, so $t = \frac{90}{30} = 3$ h

truck: speed $= \frac{42.2363}{3} = 14.0787$ km/h…

Speed = 14.1 km/h (3 s.f.)

4 (a)



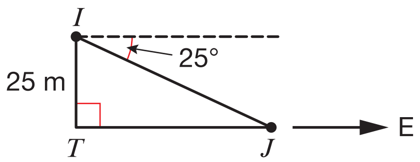
(b) Angle $BOA = 240° - 130° = 110°$
cosine rule
(SSSA condition so cosine rule)
$AB^2 = 20^2 + 30^2 - 2 \times 20 \times 30 \times \cos(110°) = 1710.4241\ldots$km
$AB = 41.3573\ldots$km
$AB = 41.4$ km (3 s.f.)

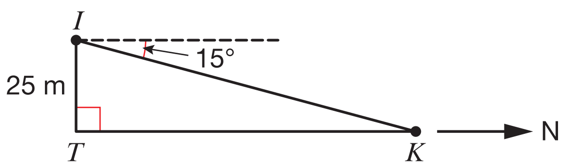
(c) Bearing of A from B:
sine rule (SASA condition so sine rule)
Let angle $OBA = \theta$

$$\frac{\sin\theta}{30} = \frac{\sin(110°)}{41.3573}$$

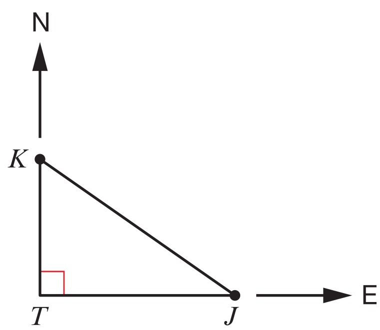
$\sin\theta = \frac{\sin(110°)}{41.3573} \times 30$
$= 0.68163\ldots$
$\theta = 42.9719°\ldots$
Bearing of A from $B = 060° + 042.9719°$
(alternate angles)
Bearing of A from $B = 103°$ (3 s.f.)

5





Plan view



Triangle ITJ:

$\tan(65°) = \frac{TJ}{25}$ $\left(\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}\right)$

$TJ = 25 \times \tan(65°)$
$= 53.6127\ldots$m
Triangle ITK:

$\tan(75°) = \frac{TK}{25}$

$TK = 25 \times \tan(75°)$
$= 93.3013\ldots$m
Plan view on triangle KTJ:
$KJ^2 = KT^2 + TJ^2$
(Pythagoras' theorem)
$= 53.6127^2 + 93.3013^2$
$= 11\ 579.45\ldots$
$KJ = 107.608\ldots$m
$KJ = 107.6$ m (1 d.p.)

HANDLING DATA 7 – BASIC SKILLS EXERCISE

1 $\frac{5}{14}$

2 **(a)** $\frac{4}{11}$

(b) $\frac{2}{9}$

3 **(a)** $\frac{2}{7}$

(b) $\frac{1}{3}$

4 **(a)** **(i)** $\frac{5}{11}$

(ii) $\frac{6}{11}$

(b) **(i)** $\frac{16}{25}$

(ii) $\frac{9}{25}$

(c) 0.798 (3 s.f.)

5 **(a)** 0.2

(b) 0.1625

(c) 0.097 25

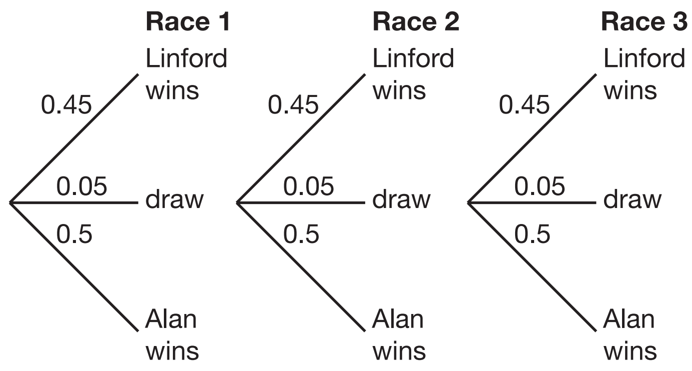
6 **(a)** **(i)** $\frac{5}{12}$

(ii) $\frac{11}{60}$

(b) **(i)** $\frac{7}{145}$

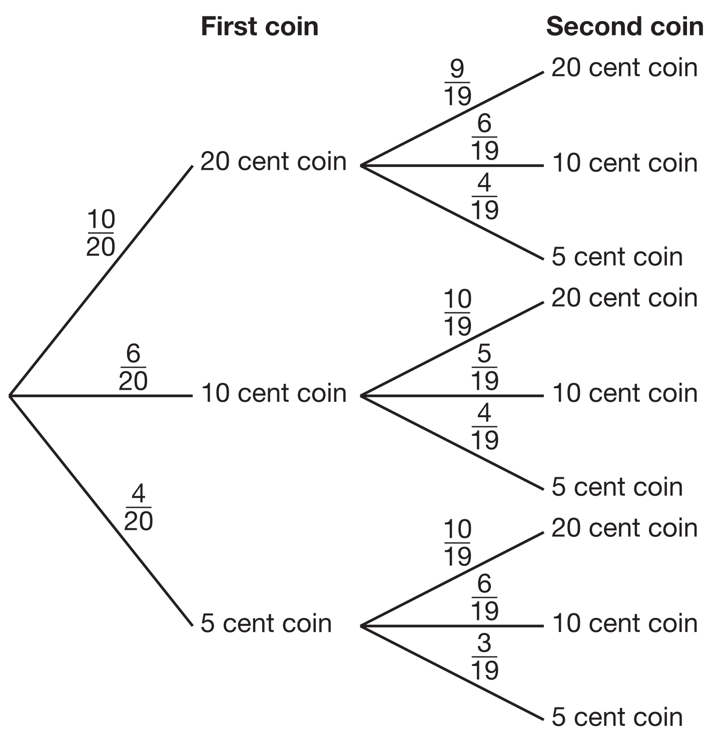
(ii) $\frac{7}{29}$

7 **(a)**



(b) P(Alan wins) = P(Alan wins 1st race) or P(draw then Alan wins 2nd race) or P(draw and Alan wins 3rd race)
$= 0.5 + 0.05 \times 0.5 + 0.05 \times 0.05 \times 0.5$
$= 0.526$

8 **(a)**



(b) Let X be Emma's score.
$P(X \geq 25) = 1 - P(X < 25)$
$= 1 - P(X \leq 20)$
$P(E) + P(E') = 1$
$= 1 - [P(X = 10) + P(X = 15)$
$+ P(X = 20)]$

$$= 1 - \left[\left(\frac{4}{20} \times \frac{3}{19}\right) + \left(\frac{6}{20} \times \frac{4}{19} + \frac{4}{20} \times \frac{6}{19}\right) + \left(\frac{6}{20} \times \frac{5}{19}\right)\right] = \frac{29}{38}$$

(P(A and B) = P(A) × P(B))
(P(A or B) = P(A) + P(B) if A and B are mutually exclusive)

9 **(a)** **(i)** 0.6

(ii) 0.2

(b) **(i)** 0.04

(ii) 0.055

10 **(a)** $k = 0.1$
(b) **(i)** 0.8
(ii) 0.5
(c) 0.9

11 **(a)** **(i)** $\frac{13}{30}$

(ii) $\frac{2}{7}$

(b) **(i)** $\frac{1}{15}$

(ii) $\frac{34}{35}$

HANDLING DATA 7 – EXAM PRACTICE EXERCISE

1 **(a)** **(i)** P(success)

$$= \frac{12\,750 + 14\,400 + 14\,800 + 12\,400}{80\,000}$$

$$= \frac{54\,350}{80\,000} = \frac{1087}{1600}$$

(ii) P(60-year old no change)

$$= \frac{7250 + 5600}{80\,000} = \frac{257}{1600}$$

(iii) P(30-year old success)

$$= \frac{14\,800 + 12\,400}{80\,000}$$

$$= \frac{17}{50}$$

(b) P(Success/60 yr old) $= \frac{12\,750}{20\,000} = \frac{51}{80}$

2 **(a)** As the herd has a very large number of cows, the proportions will be approximately the same when one, two or three cows are removed.

(i) $P(F_1) \times P(F_2) = \frac{1}{5} \times \frac{1}{5} = \frac{1}{25}$

(P(A and B) = P(A) × P(B))

(ii) $P(F_1J_2 \text{ or } J_1F_2) = P(F_1J_2) + P(J_1F_2)$

$$= \frac{1}{5} \times \frac{4}{5} + \frac{4}{5} \times \frac{1}{5} = \frac{8}{25}$$

(P(A or B) = P(A) + P(B) if A and B are mutually exclusive)

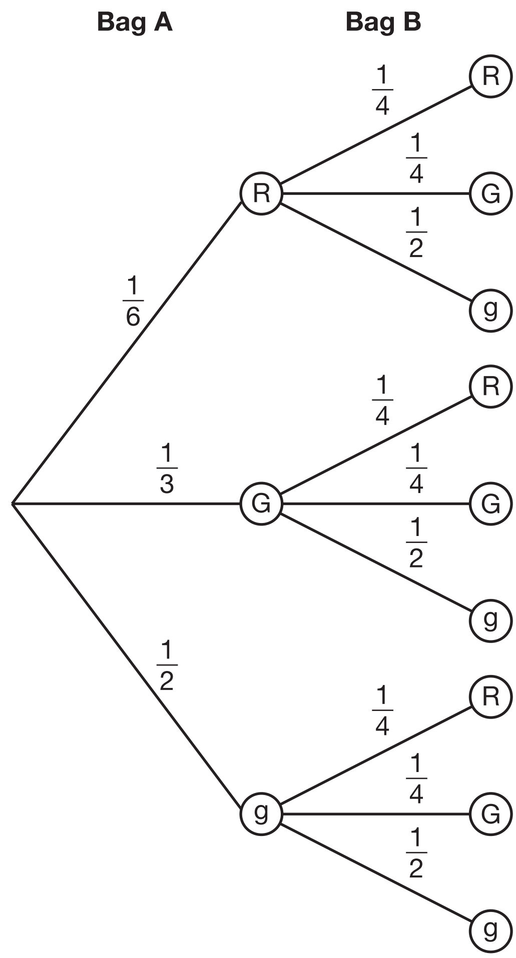
(b) $P(F \geq 2) = P(F = 2) + P(F = 3)$

$$= 3 \times p\,(F_1F_2J_3) + P(F_1F_2F_3)$$

$$= 3 \times \frac{1}{5} \times \frac{1}{5} \times \frac{4}{5} + \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} = \frac{13}{125}$$

3 **(a)** Bag A Red (R) : Green (G) : Gold (g) = 1 : 2 : 3

Bag B Red (R) : Green (G) : Gold (g) = 1 : 1 : 2



(b) **(i)** $P(g_1) \times P(g_2) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

(P(A and B) = P(A) × P(B))

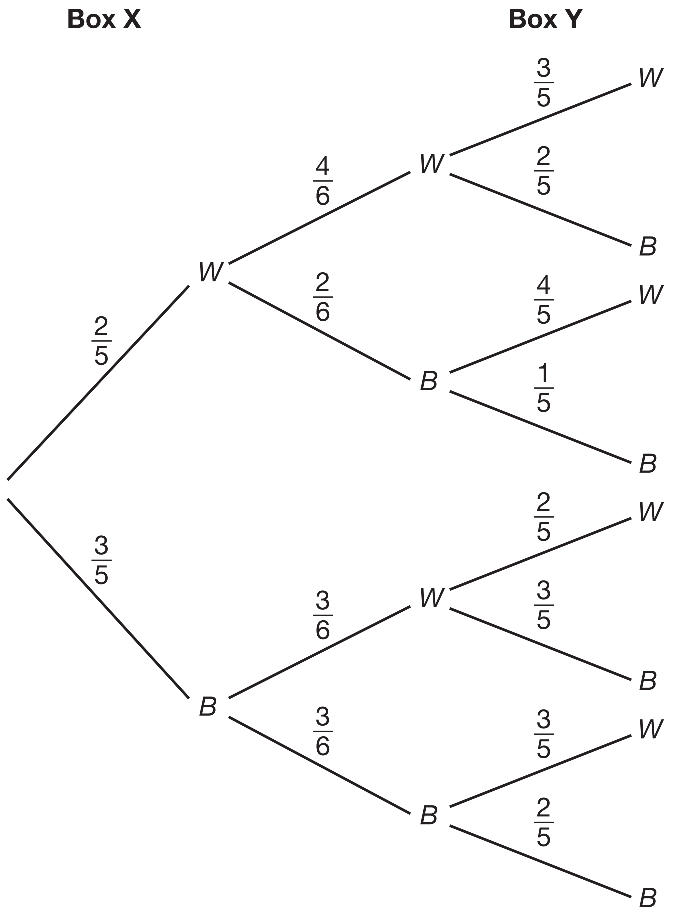
(ii) $P(Gg) = P(G_1g_2) + P(g_1G_2)$

$$= \frac{1}{3} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{4} = \frac{7}{24}$$

(P(A or B) = P(A) + P(B) if A and B are mutually exclusive)

(iii) P(R P($R \geq 1$) + P($R = 0$) =1
P($R \geq 1$) = 1 − P($R = 0$)
= 1 − P($R_1' \, R_2'$)
(P(E) + P(E') = 1)
$= 1 - \frac{5}{6} \times \frac{3}{4}$
$= 1 - \frac{15}{24}$
$= \frac{9}{24}$
$= \frac{3}{8}$

4 **(a)**

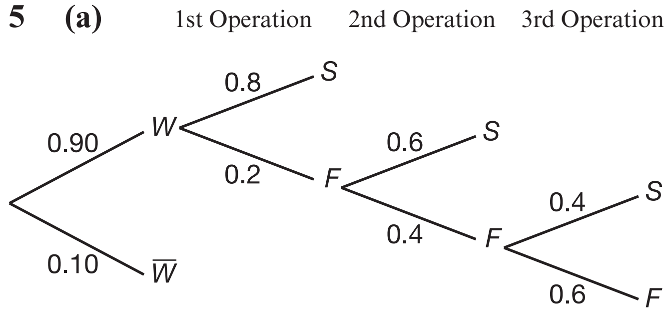


(b) **(i)** P(WW from Box Y) = P(W) × P(W_1)
× P(W_2) + P(B) × P(W_1) × P(W_2)
$= \frac{2}{5} \times \frac{4}{6} \times \frac{3}{5} + \frac{3}{5} \times \frac{3}{6} \times \frac{2}{5} = \frac{7}{25}$
(P(A and B) = P(A) × P(B))

(ii) P(BW from Box Y) = P(W) × P(W_1)
× P(B_2) + P(W) × P(B_1) × P(W_2)
+ P(B) × P(W_1) × P(B_2) + P(B) × P(B_1) × P(W_2)
$= \frac{2}{5} \times \frac{4}{6} \times \frac{2}{5} + \frac{2}{5} \times \frac{2}{6} \times \frac{4}{5} + \frac{3}{5} \times \frac{3}{6} \times \frac{3}{5}$
$+ \frac{3}{5} \times \frac{3}{6} \times \frac{3}{5}$
$= \frac{43}{75}$
(P(A or B) = P(A) + P(B) if A and B are mutually exclusive)

(iii) P($B \geq 1$) + P($B = 0$) = 1
P($B \geq 1$) = 1 − P($B = 0$)
= 1 − P($B_1' \, B_2'$) (P(E) + P(E') = 1
P($B_1' \, B_2'$) = P(WW from Box Y)
$= \frac{7}{25}$ P(B ≥ 1) $= 1 - \frac{7}{25} = \frac{18}{25}$

5 **(a)**



(b) **(i)** P(Cured 1st operation)
= P(W) × P(S) = 0.90 × 0.8 = 0.72
(P(A and B) = P(A) × P(B))

(ii) P(Cured 3[rd] operation)
= P(W) × P(F) × P(F) × P(S)
= 0.9 × 0.2 × 0.4 × 0.4 = 0.0288

(iii) P(Cured) = P(Cured 1st operation)
+ P(Cured 2nd operation) +
P(Cured 3rd operation)
= 0.72 + P(W) × P(F) × P(S) + 0.0288
= 0.72 + 0.90 × 0.2 × 0.6 + 0.0288
= 0.8568
(P(A or B) = P(A) + P(B) if A and B are mutually exclusive)

ANSWERS

EXAMINATION PRACTICE PAPERS 1A SOLUTIONS

1 **(a)** $\frac{17}{196} \times 100 = 8.6734... = 8.67\%$ (3 s.f.)

(a as % of $b = \frac{a}{b} \times 100$)

(b) $175 \times (1.063) = 186.025$ million
$= 186$ million (nearest million)
Increase a by $b\% = a \times \left(1 + \frac{b}{100}\right)$

(c) Let population in 1990 be p million
$p \times 1.174 = 175$
$p = \frac{175}{1.174} = 149.06\ldots$ million
$= 149$ (nearest million)

2 **(a)** **(i)** $u \times u \times u \times u \times u = u^5$ $(a^m \times a^n = a^{m+n})$

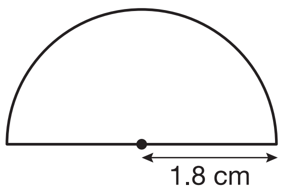
(ii) $3u + 2v - 7u + 4v + 11 = 3u - 7u + 2v + 4v + 11 = -4u + 6v + 11$

(iii) $\frac{u^7 \times u^8}{u^{11}} = \frac{u^{15}}{u^{11}} = u^4$ $(a^m \div a^n = a^{m-n})$

(b) $u(5u - 1) - u(3u - 2)$
$= 5u^2 - u - 3u^2 + 2u = 2u^2 + u$

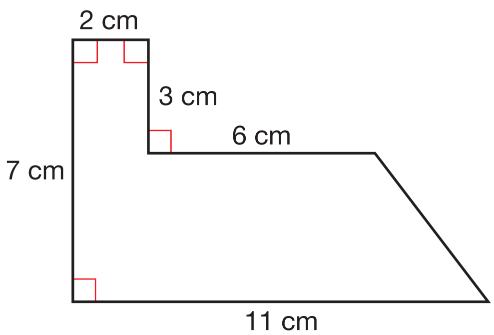
(c) $(5u - 1)(3u - 2) = 5u(3u - 2) - 1(3u - 2)$
$= 15u^2 - 10u - 3u + 2 = 15u^2 - 13u + 2$

3 **(a)**



Area $= \frac{1}{2} \times \pi \times 1.8^2 = 1.62\pi$ cm^2,
so $k = 1.62$
(area of circle $= \pi r^2$)

(b)



Let required area be A = area of trapezium + area of rectangle

$A = 2 \times 3 + \frac{1}{2}(8 + 11) \times 4 = 44$ cm^2

(trapezium area $= \frac{1}{2}(a + b)h$)

4 **(a)** $P(B) = 1 - 0.55 - 0.25 - 0.12$
$= 0.08$, $x = 0.08$
(sum of all probabilities of an event = 1)

(b) $P(R \text{ or } B) = P(R) + P(B) = 0.55 + 0.08$
$= 0.63$
($P(A$ and $B) = P(A) + P(B)$ if A and B are mutually exclusive)

(c) $P(GY) = P(GY \text{ or } YB)$
$= P(GY) + P(YB)$
$= 0.25 \times 0.12 + 0.12 \times 0.25 = 0.06$
($P(A$ and $B) = P(A) \times P(B)$ if A and B are independent)

(d) Let $E(YR)$ be expected number of times spinner lands on a Y or R.
$E(YR) = 200 \times 0.67 = 134$ times
(Expected number of outcomes = probability of event × number of trials)

5 **(a)** Mean $= \frac{a + b + c + d}{4} = 15$

$\left(\text{mean} = \frac{\text{sum of numbers}}{\text{number of numbers}}\right)$

$60 = a + b + c + d = 33 + d$
$d = 27$

(b) Range $= 23 = d - a = 27 - a$
$a = 4$
(range = largest score − smallest score)
Sum $= 60 = 4 + b + c + 27$, $b + c = 29$,
so median $= \frac{b + c}{2} = \frac{29}{2} = 14.5$
(Median is mean of the central pair in an even group of numbers arranged in ascending or descending order)

6 **(a)** Let M be midpoint of AB

$M = \left(\frac{0 + (-5)}{2}, \frac{4 + (-2)}{2}\right)$

$M\left(-2\frac{1}{2}, 1\right)$

(midpoint of $A(x_1, y_1)$ and $B(x_2, y_2)$ $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$)

(b) Let m be the gradient of AB

$m = \frac{-2-4}{-5-0} = \frac{-6}{-5} = \frac{6}{5}$

$\left(\text{Gradient of } AB = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}\right)$

(c) $y = \frac{6}{5}x + 4$

(Equation of straight line: $y = mx + c$; m is gradient, c is y-axis intercept)

(d) Let perpendicular bisector of AB be line L

Gradient of line $L = -\frac{5}{6}$

(Products of the gradients of two perpendicular lines = –1; $m_1 \times m_2 = -1$)

Equation of line L:

$y = -\frac{5}{6}x + c$

(Line L passes through point $M\left(-2\frac{1}{2}, 1\right)$. M must satisfy the equation.)

$1 = -\frac{5}{6} \times \left(-2\frac{1}{2}\right) + c$, so $c = -1\frac{1}{12}$

$= -\frac{13}{12}$

$y = -\frac{5}{6}x - \frac{13}{12}$ or $10x + 12y + 13 = 0$

(multiply through by 12 to produce $ax + by + c = 0$)

7 **(a)** $P \cap Q = \varnothing$ as there are no members of set P that are also in set Q

(Sets P and Q are mutually exclusive)

(b) $x = 29$

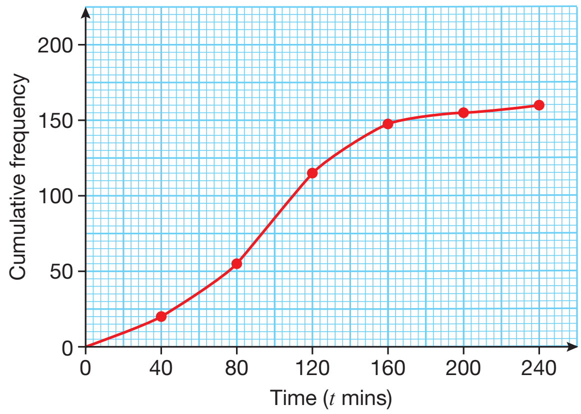
(29 is NOT a member of $P \cup Q$ but is in $\in$)

(c) $R = \{2, 13, 19\}$

8 **(a)**

Time t (minutes)	Cumulative frequency
$0 < t \leq 40$	20
$0 < t \leq 80$	55
$0 < t \leq 120$	115
$0 < t \leq 160$	148
$0 < t \leq 200$	155
$0 < t \leq 240$	160

(b)



(Plot end points: (40, 20), (80, 55)…)

(c) **(i)** Interquartile range (IQR)
= upper quartile – lower quartile
$= Q_3 - Q_1$
lower quartile $Q_1 \approx 65$ mins

(LQ at $\frac{1}{4} \times n = 40$th position)

upper quartile $Q_3 \approx 125$ mins

(UQ at $\frac{3}{4} \times n = 120$th position)

IQR ≈ 125 – 65 ≈ 60 mins

(ii) Bottom of the top 10th percentile (T_{10}) is at 144th position which is at 155 mins. Top 10th percentile is from 155 minutes $\leq t \leq$ 240 minutes.

9 **(a)** Let shaded segment area be A

A = Area of sector XYZ – Area of triangle XYZ

$= \frac{70}{360} \times \pi \times 7.5^2 - \frac{1}{2} \times 7.5^2 \times \sin(70°)$

(Area sector angle $\theta = \frac{\theta}{360°}\pi r^2$, area of triangle $= \frac{1}{2}ab\sin C$)

= 7.9323…cm²

= 7.93 cm² (3 s.f.)

(b) Let shaded segment perimeter be P

P = Chord XZ + Arc XZ

$= 2 \times 7.5 \times \sin(35°) + \frac{70}{360} \times 2\pi \times 7.5$

= 17.7666…cm

= 17.8 cm (3 s.f.)

(Arc length of sector angle $\theta = \frac{\theta}{360°} \times 2\pi r$)

10 **(a)** **(i)** $p^7 \times p^{11} = p^{18}$ $(a^m \times a^n = a^{m+n})$

(ii) $p^{11} \div p^7 = p^4$ $(a^m \div a^n = a^{m-n})$

(iii) $(2p + 1)^2 - (p - 1)^2$

$= (2p + 1)(2p + 1) - (p - 1)(p - 1)$

$= (4p^2 + 4p + 1) - (p^2 - 2p + 1)$

$= 3p^2 + 6p = 3p(p + 2)$

(b) **(i)** $11p - 1 = 7p + 1$

$4p = 2$

(Do same operation to both sides to isolate p)

$p = \frac{2}{4} = \frac{1}{2}$

(ii) $\frac{11p - 1}{4} = \frac{4p + 1}{11}$

(multiply both sides by 44)

$11(11p - 1) = 4(4p + 1)$

$121p - 11 = 16p + 4$

$105p = 15$

$p = \frac{15}{105} = \frac{1}{7}$

11 $3m - 10n = 20$ (1)

$5m + 2n = 6$ (2) × 5 = (3)

$25m + 10n = 30$ (3)

(Decision made to eliminate n, so make n values 'same'. Other variable m could also be eliminated by making m values 'same')

(3) + (1): $28m = 50$, $m = \frac{50}{28} = \frac{25}{14} = 1\frac{11}{14}$

(Substitute $m = 1\frac{11}{14}$ into any equation to find n, say (2))

(2): $5 \times 1\frac{11}{14} + 2n = 6$, $2n = -\frac{41}{14}$, $n = -\frac{41}{28} = -1\frac{13}{28}$

Point of intersection of the two lines is $\left(1\frac{11}{14}, -1\frac{13}{28}\right)$

(Modern calculators allow checking of your answers, so questions of this type can be produced with more efficiency)

12 Let $x = 0.\dot{4}9\dot{2} = 0.492\,492\,492\ldots$ (1)

(× by 1000 as there are 3 recurring decimals)

$1000x = 492.492492492\ldots$ (2)

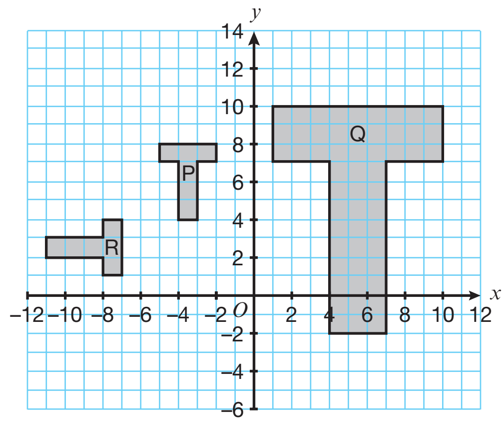
$999x = 492$

((2) – (1) to eliminate recurring decimals)

$x = \frac{492}{999} = \frac{164}{333}$ as required

(Divide numerator and denominator by 3)

13 **(a)**
(b)



(c) (Enlargements: Area of object × k^2 = Area of image, if k is the scale factor of enlargement)

(i) Area of $\mathbf{Q}_1 = 54 \times \left(\frac{1}{2}\right)^2 = 13\frac{1}{2}$ units^2

(ii) Area of $\mathbf{R}_1 = 6 \times 5^2 = 150$ units^2

14 Upper bound mass = 155 kg

Greatest number of cases lifted 'safely', n, is worst case i.e. when the safe loading is minimised and the case weight is maximised.

$n = \frac{1750}{155} = 11.290\ldots$ say 11

15 $1 - \frac{2x^2 - 7x + 3}{x^2 + 2x - 15}$

$= \frac{(x + 5)(x - 3) - (2x - 1)(x - 3)}{(x + 5)(x - 3)}$

$= \frac{(x + 5) - (2x - 1)}{(x + 5)} = \frac{6 - x}{x + 5}$

(Express as a single fraction with a common denominator of $(x + 5)(x - 3)$)

16 **(a)** **(i)** $5p - 7 \geq p + 3$, $4p \geq 10$, $p \geq 2.5$

(ii) $3(2p - 7) < 2(p + 3)$

$6p - 21 < 2p + 6$

$6p - 2p < 6 + 21$

$4p < 27$

$p < 6.25$

(b) The full solution set is $2.5 \leq p < 6.25$, so the integer solution set = {3, 4, 5, 6}

17 **(a)** $I \propto \frac{1}{d^2}$, so $I = \frac{k}{d^2}$, where k is a constant of proportionality

$10^6 = \frac{k}{1^2}$, so $k = 10^6$, $I = \frac{10^6}{d^2}$

(b) At $I = 4cd$, $4 = \frac{10^6}{d^2}$, $d = 500$, so $p = 0.5$

18 **(a)** **(i)** $f(0) = 6$

(ii) $fg(0) = f(2) = -2$

(Find $g(0)$ first from the $g(x)$ graph and substitute into $f(x)$)

(b) $f(x) = 2$, $x = -3, 1, 5$ (Draw $y = 2$ on graph and find where it intercepts $y = f(x)$).

(c) On $y = g(x)$ gradient at $x = 7$, $m \approx -\frac{4}{2}$

$= -2 \left(\text{gradient} = \frac{\text{rise}}{\text{run}}\right)$

19 **(a)** Angle $MLJ = 30°$
(angles in the same segment)

(b) Angle $JLK = 73°$
(alternate segment theorem)

(c) Angle $GJM = 30°$
(alternate segment theorem)
Angle $GMJ = 69°$
(angle sum of a triangle = 180°)
Angle $LMP = 38°$
(angle sum in a straight line = 180°)
Angle $MPL = 112°$
(angle sum of a triangle = 180°)
Angle $KPL = 68°$
(angle sum in a straight line = 180°)

(d) Angle $JKL = 69°$ (opposite angles in a cyclic quadrilateral add up to 80°)

20 **(a)** $5x^2 - 20x + 12 = 5\left[x^2 - 4x + \frac{12}{5}\right]$

$= 5\left[(x-2)^2 - 4 + \frac{12}{5}\right]$

$= 5\left[(x-2)^2 - \frac{8}{5}\right]$

$= 5(x-2)^2 - 8$

so $a = 5$, $b = -2$ and $c = -8$

(b) $f(x)_{min}$ occurs at $x = 2$, $y = -8$
min point is $(2, -8)$

21 **(a)** $P(C \cap S \cap V) = \frac{7}{50}$ $\left(P(E) = \frac{n(E)}{n(E)}\right)$

(b) $P(C \cap S') = \frac{6+9}{50} = \frac{15}{50} = \frac{3}{10}$

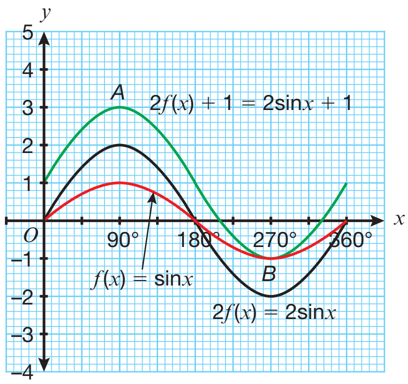
(c) $P(S \cup V') = \frac{12+4+7+6}{50} = \frac{29}{50}$

(d) $P(C \mid V) = \frac{n(C \cap V)}{n(V)} = \frac{9+7}{(9+7+4+12)}$

$= \frac{16}{32} = \frac{1}{2}$

(Sample space is reduced from $n(\mathscr{E})$ to $n(V)$)

22

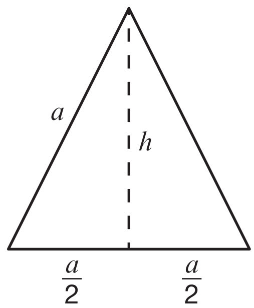


(If $g(x) = 2f(x)$, $g(x)$ is a stretch of $f(x)$ parallel to the y-axis of scale factor 2 and $2f(x) + 1$ is $g(x) + 1$ which is a translation of $g(x)$ along vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$)

A is the maximum point of $g(x)$ in the domain which is (90°, 3)

B is the minimum point of $g(x)$ in the domain which is (270°, −1)

23 Area of triangle = area of circle of circumference 10π
Circumference of circle = 10π



$10\pi = \pi d$ $(c = \pi d)$
$r = 5$
Area of circle, $A = \pi \times 5^2 = 25\pi$ $(A = \pi r^2)$
Area of triangle, $A = \frac{1}{2} \times a \times h$

(Area of triangle $= \frac{1}{2} \times$ base $\times$ perpendicular height)

$a^2 = h^2 + \left(\frac{a}{2}\right)^2, h^2 = a^2 - \left(\frac{a}{2}\right)^2 = \frac{3a^2}{4}, h = \frac{a\sqrt{3}}{2}$

(Pythagoras' theorem)

$A = \frac{1}{2} \times a \times \frac{a\sqrt{3}}{2} = \frac{a^2\sqrt{3}}{4}$

Both areas are equal so, $25\pi = \frac{a^2\sqrt{3}}{4}$

$a^2 = \frac{100\pi}{\sqrt{3}}$

24 **(a)** $240 = \pi r^2 y$, so $y = \frac{240}{\pi r^2}$

(b) $A = 2\pi r^2 + 2\pi r y = 2\pi r^2 + 2\pi r \times \frac{240}{\pi r^2}$

$A = 2\pi r^2 + \frac{480}{r}$ as required

(cylinder: Volume = $\pi r^2 h$, Curved surface area = $2\pi rh$)

(c) $A = 2\pi r^2 + 480r^{-1}$, so $\frac{dA}{dr} = 4\pi r - 480r^{-2}$

= 0 at stationary point

(Stationary points occur when the gradient is 0, i.e. $\frac{dy}{dx} = 0$)

$4\pi r = 480r^{-2} = \frac{480}{r^2}$, $r^3 = \frac{480}{4\pi} = \frac{120}{\pi}$, so

$r = \sqrt[3]{\frac{120}{\pi}}$ (= 3.36…)

(d) Take a small step to the left of

$r = \sqrt[3]{\frac{120}{\pi}}$, say $r = 3.3$

$\frac{dA}{dr}$ at $r = 3.3$ is equal to –2.6...

Take a small step to the right of

$r = \sqrt[3]{\frac{120}{\pi}}$, say $r = 3.4$ $\frac{dA}{dr}$

at $r = 3.4$ is equal to +1.2...

The curve shape around the stationary point is a U shape, so A will be a minimum value when $r = \sqrt[3]{\frac{120}{\pi}}$

$A = 2\pi r^2 + \frac{480}{r}$, $A_{min} = 213.79...\text{cm}^2$

$A_{min} = 214\ \text{cm}^2$ (3 s.f.)

(Efficient use of the 'Ans' button on the calculator is helpful for part d)

EXAMINATION PRACTICE PAPERS 1B SOLUTIONS

1 $\sqrt{\frac{3.1^2 + 1.3^2}{3.1^2 - 1.3^2}} = 1.1944... = 1.19$ (3 s.f.)

2 24 mg = 60%

(Find 1%, so that 100% can be calculated)

$\frac{24}{60} = 1\%$

Daily recommended daily dose:

$100\% = 100 \times \frac{24}{60} = 40$ g

so recommended weekly vitamin C dose = 7 × 40 g = 280 g

3 **(a)** speed = $\frac{\text{distance}}{\text{time}}$, $168 = \frac{d}{22.75 - 19.80}$,

$d = 168 \times 2.95 = 495.6$ km = 496 km (nearest km)

(Time from Nimes to Paris = 22 h 45 min − 19 h 48 min = 2.95 h)

(b) speed = $\frac{\text{distance}}{\text{time}}$, time = $\frac{831}{168}$

= 4.9464...h = 4 h 56 min 47 s

Timetable time for journey
= 22:45 − 16:20 = 6 h 25 min

Total stoppage time
= 6 h 25 min − 4 h 56 min 47 s
= 1 h 28 min 13 s
= 1 h 28 min

4 $1\frac{1}{2} \times \left(7\frac{1}{3} \div 3\frac{1}{7}\right)^2 = \frac{3}{2} \times \left(\frac{22}{3} \times \frac{7}{22}\right)^2$

$= \frac{3}{2} \times \frac{49}{9} = \frac{49}{6} = 8\frac{1}{6}$

5 **(a)** $x = 2464 = 2^5 \times 7 \times 11$, $y = 1372 = 2^2 \times 7^3$

(Long division by prime factors should be shown)

(b) **(i)** HCF = $2^2 \times 7$, so $m = 2$, $n = 1$

(ii) LCM = $2^5 \times 7^3 \times 11$, so $p = 5$, $q = 3$, $r = 1$

$(p + q + r)^{(m+n)} = 9^3 = 729$

6 **(a)** Yeast grams = $\frac{20}{5} \times 35 = 140$ g

(b) $L = \frac{90}{7.5} \times 5 = 60$, $L = 60$

(c) Brown flour : White flour : Yeast
= 2000 : 500 : 35 = 400 : 100 : 7
$p = 400$, $q = 100$, $r = 7$

7 Total time of all six runners is
= 6 × (2 min 15.5 s) = 12 min + 6 × 15.5 s = 12 min + 93 s = 13 min 33 s

Total time of the other 5 runners is
5 × 2 min + (13.6 + 15.9 + 15.8 + 18.3 + 14.1) s = 10 min + 77.7 s = 11 min 17.7 s

(Mean = $\frac{\text{sum of all scores}}{\text{number of scores}}$)

Lucia's time = 13 min 33 s – 11 min 17.7 s
= 2 min 15.3 s

All the times in order are
1st 2 min 13.6 s
2nd 2 min 14.1 s
3rd 2 min 15.3 s
4th 2 min 15.8 s
5th 2 min 15.9 s
6th 2 min 18.3 s

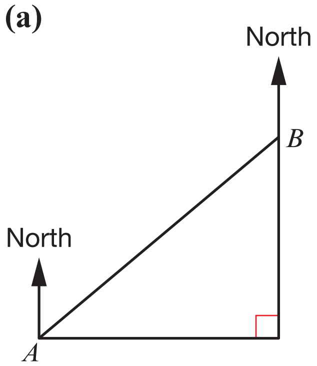
So Lucia came 3rd and was awarded the bronze medal.

8 Let perimeter of a semi-circle be
p = diameter + half of circumference
$p = 2r + \pi r = r(2 + \pi)$
$p_{max} = 105$ cm, $p_{min} = 95$ cm
Let area of semi-circle be A
$A = \frac{1}{2}\pi r^2$

(a) A_{max} occurs when radius is r_{max}
$105 = r_{max}(2 + \pi)$,
$r_{max} = \frac{105}{2+\pi}$, so $A_{max} = \frac{1}{2}\pi\left(\frac{105}{2+\pi}\right)^2$
$= 655.09327...\ \text{cm}^2 = 655\ \text{cm}^2$ (3 s.f.)

(b) A_{min} occurs when radius is r_{min}
$95 = r_{min}(2 + \pi)$,
$r_{min} = \frac{95}{2+\pi}$, so $A_{min} = \frac{1}{2}\pi\left(\frac{95}{2+\pi}\right)^2$
$= 536.2554..\text{cm}^2$
$= 536\ \text{cm}^2$ (3 s.f.)

9 **(a)**



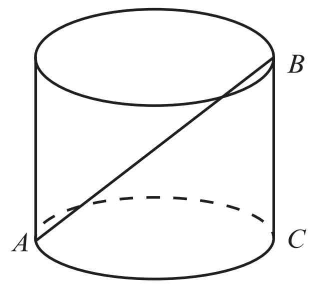
Let $AB = d$
$d^2 = 40^2 + 50^2 = 4100$
(Pythagoras' theorem)
$d = \sqrt{4100} = \sqrt{41} \times \sqrt{100} = 10\sqrt{41}$,
so $k = 10$

(b) Bearing of ship A from ship B is $180° + \theta$
(Bearing is clockwise from North)
$\tan\theta = \frac{40}{50}$, $\left(\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}\right)$
so $\theta = 38.65...°$ so, bearing of ship A from ship $B = 219°$ (nearest degree)

10 $(2x + 1)(x - 1)^2 = 1$
$(2x + 1)(x^2 - 2x + 1) = 1$
$2x(x^2 - 2x + 1) + 1(x^2 - 2x + 1) = 1$
$2x^3 - 4x^2 + 2x + x^2 - 2x + 1 = 1$
$2x^3 - 3x^2 = 0$
$x^2(2x - 3) = 0$
Either $x^2 = 0$, $x = 0$
Or $2x - 3 = 0$, $x = \frac{3}{2} = 1\frac{1}{2}$

11 If n is an integer.
A general term for an odd number $= 2n + 1$
The next odd number $= (2n + 1) + 2 = 2n + 3$
The difference between the squares of these numbers
$= (2n + 3)^2 - (2n + 1)^2$
$= (2n + 3)(2n + 3) - (2n + 1)(2n + 1)$
$= (4n^2 + 12n + 9) - (4n^2 + 4n + 1)$
$= 8n + 8 = 8(n + 1)$ which is a multiple of 8

12



$192\pi = \pi \times 4^2 \times BC$
(Volume of cylinder $= \pi r^2 h$)
$BC = \frac{192\pi}{16\pi} = 12$ cm
Required angle $= BAC$, triangle BAC is right-angled
Let angle $BAC = \theta$
$\tan(\theta) = \frac{BC}{AC} = \frac{12}{8}$, so $\theta = 56.3°$(3 s.f.)
$\left(\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}\right)$

13 **(a)** The exterior angle of the polygon $= 180° - 144° = 36°$
(n-sided regular polygon: exterior angle $= \frac{360°}{n}$)
So $36° = \frac{360°}{n}$, $n = 10$
The shape has 10 sides so is a decagon.

(b) Perimeter $= 10 \times 12 = 120$ cm

(c) The polygons are similar figures
Scale factor of length $= \frac{960}{120} = 8$
(Similar figures: small area $\times\, k^2$ = larger area, where k is length scale factor)
Enlarged area $= 8^2 \times A = 64A$

14 The toys are not replaced.
Let X be the number of toys that are the same from 3 random picks.
T: Teddy bears, R: Robots, D: Dolls
$P(X = 2) = 3 \times P(TTT') + 3 \times P(RRR') + 3 \times P(DDD')$
(× 3 as the 'not' option can occur in 3 places)
(P(A or B) = P(A) + P(B) if A and B are mutually exclusive)
(P(A and B) = P(A) × P(B) if A and B are independent)

$$P(X = 2) = 3 \times \frac{5}{15} \times \frac{4}{14} \times \frac{10}{13} + 3 \times \frac{5}{15} \times \frac{4}{14} \times \frac{10}{13} + 3 \times \frac{5}{15} \times \frac{4}{14} \times \frac{10}{13}$$
$$= 9 \times \frac{5}{15} \times \frac{4}{14} \times \frac{10}{13} = \frac{60}{91}$$

15 **(a)** If $g(x) = \frac{10}{x-3}$:
$y = \frac{10}{x-3}$
(Re-write in terms of $y = \ldots$)
$x = \frac{10}{y-3}$
(Switch x and y variables)
$y - 3 = \frac{10}{x}$, $y = \frac{10}{x} + 3 = \frac{10+3x}{x}$
(Re-arrange to make y the subject)
$g^{-1}(x) = \frac{10+3x}{x}$
(Replace y with $g^{-1}(x)$)

(b) If $gh(p) = -1$, $g\left(\frac{p-5}{p}\right) = -1$,
$$\frac{10}{\left(\frac{p-5}{p}\right) - 3} = -1$$
(Find $h(p)$ first and input this into $g(x)$)
$$\frac{10}{\frac{(p-5)-3p}{p}} = -1, \quad \frac{10p}{-2p-5} = -1,$$
so $10p = 2p + 5$, $8p = 5$,
$p = \frac{5}{8}$

16 **(a)** Esther's investment =
€12 000 × 1.025 × 1.035^9 = €16 763.64
(Increase a by b% for n years compound interest = $a \times \left(1 + \frac{b}{100}\right)^n$)

(b) Let €p be the amount Ivan invests
$p \times 1.015^2 =$ €1236.27,
so $p = \frac{1236.27}{1.015^2} = 1200$
Ivan invests €1200 into his Savings Bond.

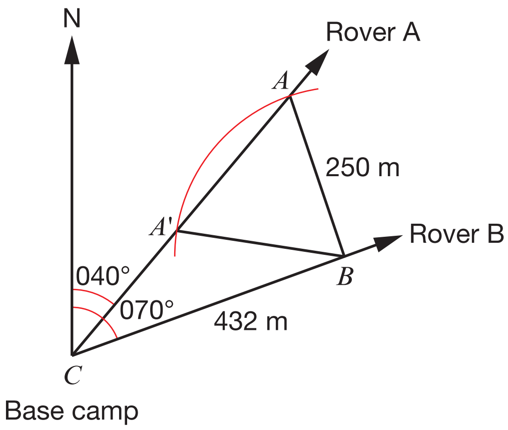
17 $(x + 1) : (x + 2) = (y + 1) : (y + 3)$
$\frac{x+1}{x+2} = \frac{y+1}{y+3}$
$(x + 1)(y + 3) = (x + 2)(y + 1)$
$xy + 3x + y + 3 = xy + x + 2y + 2$
$y = 2x + 1$

18 **(a)** (Density = $\frac{\text{Mass}}{\text{Volume}}$,
Volume of cylinder = $\pi r^2 h$)
Mass = Density × Volume
$= 2710 \times \pi \times 0.5^2 \times 1.20 = 813\pi$ kg
$k = 813$

(b) (Pressure = $\frac{\text{Force}}{\text{Area}}$)
Pressure = $\frac{2.5 \times 10^4}{\pi \times 0.5^2} = 31830.98\ldots$ N/m²
$= 3.18 \times 10^4$ N/m² (3 s.f.)

19 $x^2 + y^2 = 26$ [1]
$y = 3 - 2x$ [2]
(subs into [1])
[1] $x^2 + (3 - 2x)^2 = 26$
(expand out brackets)
$x^2 + (3 - 2x)(3 - 2x) = 26$
$x^2 + 9 - 12x + 4x^2 = 26$
$5x^2 - 12x - 17 = 0$
$(5x - 17)(x + 1) = 0$, so $5x - 17 = 0$ or $x + 1 = 0$
$x = \frac{17}{5} = 3\frac{2}{5}$, $x = -1$
(substitute into [2])
[2] $y = 3 - 2\left(\frac{17}{5}\right) = -\frac{19}{5} = -3\frac{4}{5}$, $y = 3 - 2(-1) = 5$
$P\left(3\frac{2}{5}, -3\frac{4}{5}\right)$, $Q(-1, 5)$
(Write answers as coordinate pairs)
P & Q are interchangeable!

20 After 3 hours Rover B has travelled
$\frac{3 \times 4 \times 60 \times 60}{100} = 432$ m
(Draw a sketch of the situation. There are two possible positions for Rover A, shown as A and A' on the sketch.)
(Using the sine rule in triangle ABC with the usual notation.)



$\frac{\sin(A)}{432} = \frac{\sin(30°)}{250} \sin(A) = 0.864$ $A = 59.77°$

or 120.23° (2 d.p.)

The smallest possible value of x is given by Rover A being at position A' on the sketch, corresponding to $A' = 120.23°$
When $A' = 120.23°$, $B = 180 - 30 - 120.23 = 29.77°$

$\frac{b}{\sin(29.77°)} = \frac{250}{\sin(30°)} b = 248.26$ m (2 d.p.)

speed is $\frac{248.26}{3}$ m/h $= \frac{248.26 \times 100}{3 \times 60 \times 60}$

$= 2.30$ cm/s (3 s.f.)

21 $\overrightarrow{PS} = 2\mathbf{a}$
$\overrightarrow{PR} = 2\mathbf{a} + \mathbf{b}$
$\overrightarrow{MQ} = -\mathbf{a} + \mathbf{b} = \mathbf{b} - \mathbf{a}$
If PNR is a straight line $\overrightarrow{PR} = k\overrightarrow{PN}$
$\overrightarrow{PN} = \overrightarrow{PM} + \frac{1}{3}\overrightarrow{MQ} = \mathbf{a} + \frac{1}{3}(\mathbf{b} - \mathbf{a}) = \frac{1}{3}(2\mathbf{a} + \mathbf{b})$

$= \frac{1}{3}\overrightarrow{\mathrm{PR}}$, so $\overrightarrow{PR} = 3\overrightarrow{PN}$

PNR in a straight line as $k = 3$

22 $t_n = 3n + 1$
$t_1 = 4$, $t_2 = 7$, $t_3 = 10$ … so sequence is an arithmetic progression with $a = 4$, $d = 3$, $n = 20$
($S_n = \frac{n}{2}\{2a + (n-1)d\}$ Sum to n terms for an arithmetic progression)
$S_{20} = \frac{20}{2}\{2 \times 4 + (20 - 1) \times 3\} = 650$

23 (a) (i) $s = (t - 1)^3 + 3t = (t - 1)(t - 1)^2 + 3t$
$= (t - 1)(t^2 - 2t + 1) + 3t$
$= t(t^2 - 2t + 1) - 1(t^2 - 2t + 1) + 3t$
$= (t^3 - 2t^2 + t - t^2 + 2t - 1) + 3t$
$= t^3 - 3t^2 + 6t - 1$

$v = \frac{ds}{dt} = 3t^2 - 6t + 6$ m/s

(ii) $a = \frac{dv}{dt} = 6t - 6$ m/s^2

(b) $a = 6t - 6$, $t = 1$
At $t = 1$, $v = 3 \times (1)^2 - 6 \times 1 + 6 = 3$ m/s

24 (a) Volume of space V = Volume of cylinder V_c − Volume of spheres V_s
$V_c = \pi(r)^2 \times 4r = 4\pi r^3$
(radius of cylinder = r, height of cylinder = $4r$)
$V_s = 2 \times \frac{4}{3} \times \pi \times r^3 = \frac{8\pi r^3}{3}$

$V = 4\pi r^3 - \frac{8\pi r^3}{3} = \frac{12\pi r^3}{3} - \frac{8\pi r^3}{3} = \frac{4\pi r^3}{3}$

as required

(b) $\frac{4\pi r^3}{3} = \frac{9\pi}{2}$

So, $\frac{4r^3}{3} = \frac{9}{2}$, $8r^3 = 27$, $r^3 = \frac{27}{8}$,

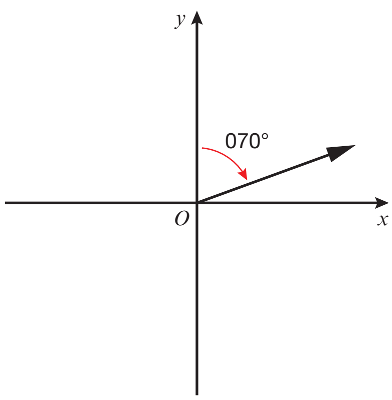
$r = \sqrt[3]{\frac{27}{8}} = \frac{\sqrt[3]{27}}{\sqrt[3]{8}} = \frac{3}{2} = 1.5$ cm

(c) Required fraction $= \frac{\frac{4\pi r^3}{3}}{4\pi r^3} = \frac{4\pi r^3}{3} \div 4\pi r^3$

$= \frac{4\pi r^3}{12\pi r^3} = \frac{1}{3}$

EXAMINATION PRACTICE PAPERS 2A SOLUTIONS

1

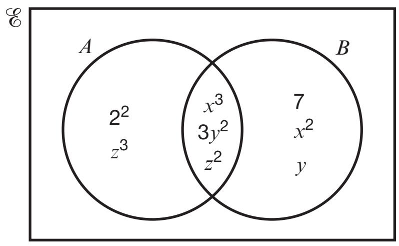


Let required bearing of Moritz to Kielder be θ.
So $\theta = 180° + 70° = 250°$
(Bearings are measured clockwise from North)

2 **(a)** Sequence 10, 9.5, 9, 8.5…is an arithmetic progression with:
$a = 10$, $d = -0.5$
$t_n = 10 + (n - 1) \times (-0.5) = 10 - 0.5n + 0.5 = 10.5 - 0.5n = 0.5(21 - n)$
($t_n = a + (n - 1)\,d$ is the nth term of an arithmetic progression)

(b) If $S_n = 0$
($S_n = \frac{n}{2}[2a + (n - 1)d]$ is the sum to n terms of an arithmetic progression)
$0 = \frac{n}{2}[2 \times 10 + (n - 1) \times (-0.5)]$
(Divide both sides by $\frac{n}{2}$)
$0.5(n - 1) = 20$
$40 = n - 1$
$n = 41$

3



Let A: $12x^3y^2z^5$
B: $21x^5y^3z^2$
HCF $= 3x^3y^2z^2$ (Intersection: A ∩ B)
LCM $= 84x^5y^3z^5$ (Union: A ∪ B)

4 $\frac{2v - w}{3} = \frac{2v + w}{5} + u$

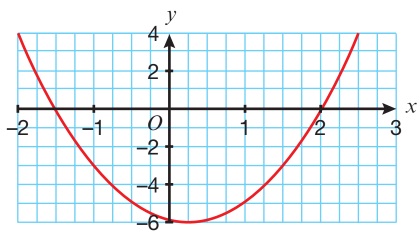
(multiply both sides of the equation by 15)
$5(2v - w) = 3(2v + w) + 15u$
$10v - 5w = 6v + 3w + 15u$
(Expand out brackets both sides)
$4v = 8w + 15u$
(Isolate v on the LHS of the equation)
$v = \frac{8w + 15u}{4}$
(Divide both sides of equation by 4)

5 Let price of shoes before sales tax be \$$p$.
$p \times 1.15 = 92$
(Increase a by $b\% = a \times (1 + \frac{b}{100})$)
$p = \frac{92}{1.15}$, $p = \$80$

6 **(a)** Equation of a straight line: $y = mx + c$
(m: gradient, c: y intercept)
$m = \frac{8 - 4}{-3 - 1} = \frac{4}{-4} = -1$
$\left(\text{Gradient of } AB = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}\right)$
so $y = -x + c$, so (1, 4) satisfies the equation as it is on the line.
$4 = -1 + c$, so $c = 5$, equation of L: $y = -x + 5$,
$x + y - 5 = 0$; $a = 1$, $b = 1$, $c = -5$

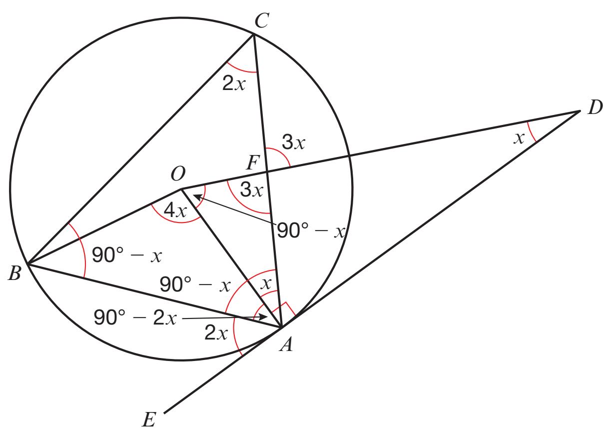
(b) Midpoint of $AB = \left(\frac{1 - 3}{2}, \frac{4 + 8}{2}\right) = (-1, 6)$
(Midpoint of $A(x_1, y_1)$ and $B(x_2, y_2)$ $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$)
Let gradient of L be m_1 and gradient of M be m_2
$m_1 \times m_2 = -1$, $-1 \times m_2 = -1$, $m_2 = 1$,
(Product of the gradients of two perpendicular lines $= -1$; $m \times m_2 = -1$)
Equation of M: $y = x + c$, so $(-1, 6)$ satisfies the equation as it is on the line.
Therefore $6 = -1 + c$, $c = 7$, $y = x + 7$,
$-x + y - 7 = 0$, $a = -1$, $b = 1$, $c = -7$

7 Let $y = 2x^2 - x - 6 = (2x + 3)(x - 2)$
(Factorising helps sketch the curve)
At the x-axis $y = 0$, so $0 = (2x + 3)(x - 2)$,
$x = 2$ or $-\frac{3}{2}$



So $2x^2 - x - 6 \geq 0$ when $x \leq -\frac{3}{2}$, $x \geq 2$
(y-values above or on the x-axis satisfy the inequality)

8



Angle $OFA = 3x$ (opposite angles)
Angle $AOF = 90 - x$
(ΔAOD is a right angled Δ as AD is a tangent and OA is a radius)
Angle $ACB = 2x$
(alternate segment theorem)
Angle $BAC = \frac{1}{2}(180 - 2x) = 90 - x$
(ΔACB is isosceles)
Angle $AOB = 4x$
(2 × angle BCA, angle subtended at centre is twice angle at circumference)
Angle $OAB = \frac{1}{2}(180 - 4x) = 90 - 2x$
(ΔOAB is isosceles)
Angle OAF $= (90 - x) - (90 - 2x) = x$
$x + (90 - x) + 3x = 180$
(Angle sum of ΔOAF)
$3x = 90x = 30$

9 **(a)** Percentage > 90 mins $= \frac{3}{24} \times 100 = 12.5\%$

(b) Modal class is $30 < x \leq 60$
(Most popular group)

(c) Mean $= \frac{\Sigma fx}{\Sigma f}$, where is the midpoint of each class

Mean =
$\frac{4 \times 15 + 10 \times 45 + 7 \times 75 + 3 \times 105}{24}$

$= \frac{1350}{24} = 56.25$ mins $= 56$ mins 15 s

(d) New mean $= \frac{1350 + 56.25}{25} = 56.25$ mins, so remains unchanged

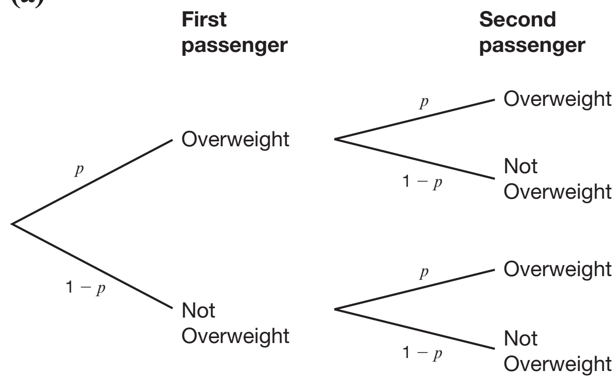
10 **(a)** $\text{speed}_{max} = \frac{\text{distance}_{max}}{\text{time}_{min}} = \frac{805}{137.5} = 5.85454...$

$= 5.85$ m/s (3 s.f.)

(b) $\text{speed}_{min} = \frac{\text{distance}_{min}}{\text{time}_{max}} = \frac{795}{142.5} = 5.5789...$

$= 5.58$ m/s (3 s.f.)

11 **(a)**



(b) **(i)** Branches required are:
'Overweight – Not overweight' and 'Not overweight – overweight'
Probability $= p \times (1 - p) + (1 - p) \times p$
$= 2p(1 - p)$ or $2p - 2p^2$

(ii) $2p - 2p^2 = 0.05$
$2p^2 - 2p + 0.05$
$= p^2 - p + 0.025 = 0$
(Solving this quadratic using the quadratic formula.)

$p = \frac{1 \pm \sqrt{12 - 4 \times 1 \times 0.025}}{2 \times 1} = \frac{1 \pm \sqrt{0.9}}{2}$

$p = 0.974$ or $p = 0.0257$

12 Let $a = 2n + 1$, $b = 2n + 3$, $c = 2n + 5$ and $d = 2n + 7$
(If n is an integer, $2n$ is always even, so $2n + 1$ is odd)
$d^2 - a^2 = (2n + 7)^2 - (2n + 1)^2$
$= 4n^2 + 28n + 49 - (4n^2 + 4n + 1)$
$= 4n^2 + 28n + 49 - 4n^2 - 4n - 1$
$= 24n + 48 = 24(n + 2)$
$24(n + 2)$ is divisible by 24
so, $d^2 - a^2$ is divisible by 24.

13 (AM is a line of symmetry, so ΔABM is a right-angled triangle. M is midpoint of BC so $BM = 1$)
$AM^2 + BM^2 = AB^2$ (Pythagoras' theorem)
$AM^2 + 1 = 3^2$
$AM^2 = 8$
$AM = \sqrt{8}$
$= 2\sqrt{2}$
$AX = \frac{3}{4} AM$

$AX = \frac{3}{4} \times 2\sqrt{2}$
$= \frac{3\sqrt{3}}{2}$ cm

14 (a) $1 - \frac{1}{x+a} - \frac{x-1}{x}$

$= \frac{x(x+a) - x - (x-1)(x+a)}{x(x+a)}$

(Lowest common denominator is $x(x + a)$)

$= \frac{x^2 + ax - x - (x^2 - x + ax - a)}{x(x+a)}$

$= \frac{x^2 + ax - x - x^2 + x - ax + a}{x(x+a)}$

$= \frac{a}{x(x+a)}$

(b) By inspection $a = 2$, $\frac{2}{x(x+2)} = \frac{2}{3}$,

$6 = 2x(x + 2) = 2x^2 + 4x$
$0 = x^2 + 2x - 3 = (x + 3)(x - 1)$,
$x = 1$ or $x = -3$

15 Using the sine rule gives $\frac{AC}{\sin(69°)} = \frac{38.7}{\sin(52°)}$

$AC = \frac{38.7 \times \sin(69°)}{\sin(52°)}$

The minimum value of AC will be given when 38.7 is a minimum, sin(69°) is a minimum and sin(52°) is a maximum.

38.7 is correct to 3 s.f. so the minimum value is 38.65

69° is correct to 2 s.f. so the minimum value is 68.5°

52° is correct to 2 s.f. so the maximum value is 52.5°

Minimum value of $AC = \frac{38.65 \times \sin(68.5°)}{\sin(52.5°)}$

$= 45.327 \ldots = 45.3$ m to 3 s.f.

16 (a) The perimeter $= r + 4r + r + 4r + \pi r$
$= 10r + \pi r$

(Curved length of a semicircle $= \pi r$)

$10r + \pi r = 50$, $r(10 + \pi) = 50$,

$r = \frac{50}{10+\pi} = 3.8047\ldots$

Area of shape $= \frac{\pi r^2}{2} + 4r \times r = r^2\left(\frac{\pi}{2} + 4\right)$

$= (3.8047\ldots)^2\left(\frac{\pi}{2} + 4\right) = 80.64\ \text{cm}^2$ (4 s.f.)

(b) (i) Volume $= 80.64\ldots \times 25$
$= 2016\ \text{cm}^3$ (4 s.f.)

(ii) (Surface area excluding the flat ends = perimeter × 25)

Surface area $= 50 \times 25 + 2 \times 80.64\ldots$
$= 1411\ \text{cm}^2$ (4 s.f.)

(c) (i) Length scale factor $= \frac{30}{50}$

$= 0.6$ volume scale factor $= 0.6^3$
Volume of new shape is 2016×0.6^3
$= 435\ \text{cm}^3$ (3 s.f.)

(ii) Area scale factor $= 0.6^2$
Area of new shape is 1411×0.6^2
$= 508\ \text{cm}^2$ (3 s.f.)

17 (a) (i) $3^{\frac{p}{q}} = x \Rightarrow \left(3^{\frac{p}{q}}\right)^q$

$= x^q \Rightarrow 3^p = x^q\, 3^{p-1}$

$= 3^p \times 3^{-1} \Rightarrow 3^{p-1} = \frac{x^q}{3}$

(ii) $3^{\frac{p}{q}} = x \Rightarrow \left(3^{\frac{p}{q}}\right)^{\frac{q^2}{p}} = x^{\frac{q^2}{p}} \Rightarrow 3^q = x^{\frac{q^2}{p}}$

(b) $3^{\frac{-q}{p}} = y \Rightarrow \left(3^{\frac{-q}{p}}\right)^{\frac{p^2}{-q}} = y^{\frac{p^2}{-q}} \Rightarrow 3^p = y^{\frac{-p^2}{q}}$

(c) $x^q = \left(3^{\frac{p}{q}}\right)^q = 3^p \quad y^p = \left(3^{\frac{-q}{p}}\right)^p = 3^{-q}$

$x^q : y^p = 3 : 1 \Rightarrow \frac{3^p}{3^{-q}} = 3 \Rightarrow 3^p \div 3^{-q}$
$= 3 \Rightarrow 3^{p+q} = 3^1 \Rightarrow p + q = 1$ (1)

$x^q \times y^p = 243 \Rightarrow 3^p \times 3^{-q} = 243 \Rightarrow$
$3^{p-q} = 3^5 = p - q = 5$ (2)
Add (1) and (2) gives $2p = 6$ $p = 3$, $q = -2$

$x = 3^{\frac{3}{-2}} = \frac{1}{3^{\frac{3}{2}}} = \frac{1}{\sqrt{3^3}} = \frac{1}{3\sqrt{3}} = \frac{\sqrt{3}}{9}$

$y = 3^{\frac{2}{3}} = 9^{\frac{1}{3}} = \sqrt[3]{9}$

18 (a) Shot hits the ground when $h = 0$, so $5t^2 - 6t - 2 = 0$ (Rearrange formula to make squared part positive)

$A = 5$, $B = -6$, $C = -2$

(Using quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$)

$t = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(5)(-2)}}{2(5)} = 1.47$ s (3 s.f.)

(b) $h = -5\left[t^2 - \frac{6t}{5} - \frac{2}{5}\right] = -5\left[\left(t - \frac{3}{5}\right)^2 - \frac{9}{25} - \frac{10}{25}\right]$

$= -5\left[\left(t - \frac{3}{5}\right)^2 - \frac{19}{25}\right] = -5\left[\left(t - \frac{3}{5}\right)^2 - \frac{9}{25}\right]$

$= -5\left(t - \frac{3}{5}\right)^2 + \frac{19}{5}$

Therefore $a = -5$, $b = -\frac{3}{5}$, $c = 3\frac{4}{5}$

(c) $h_{max} = 3.8$ m at $t = \frac{3}{5}$ s

19 Total surface area is the sum of the areas both ends plus the curved surface area
Curved surface area = circumference × height

Total surface area of $A = 2 \times \pi r^2 + 2\pi r \times 3.5r$
$= 9\pi r^2$

Total surface area of $B = 2 \times \pi R^2 + 2\pi R \times R$
$= 4\pi R^2$

$9\pi r^2 = 4\pi R^2$ Surface areas are equal

$r^2 = \frac{4\pi R^2}{9\pi}$ $r = \frac{2R}{3}$ (1)

Square rooting both sides

Volume of $A = \pi r^2 \times 3.5r = 3.5\pi r^3$

Volume of $B = \pi R^2 \times R = \pi R^3$

$\frac{\text{Volume of } A}{\text{Volume of } B} = \frac{3.5\pi r^3}{\pi R^3}$

From (1) $r^3 = \left(\frac{2R}{3}\right)^3 = \frac{8R^3}{27}$

$\frac{\text{Volume of } A}{\text{Volume of } B} = \frac{3.5\pi r^3}{\pi R^3} = \frac{3.5 \times 8R^3}{R^3 \times 27} = \frac{28}{27}$

ratio 28 : 27

so $m = 28$, $n = 27$

20 **(a)** $\overrightarrow{PA} = \frac{1}{3}\boldsymbol{a}$, $\overrightarrow{AB} = \boldsymbol{b} - \boldsymbol{a}$, $\overrightarrow{AQ} = \frac{2}{3}(\boldsymbol{b} - \boldsymbol{a})$

$\overrightarrow{PQ} = \overrightarrow{PA} + \overrightarrow{AQ} = \frac{1}{3}\boldsymbol{a} + \frac{2}{3}(\boldsymbol{b} - \boldsymbol{a})$

$= \frac{2}{3}\boldsymbol{b} - \frac{1}{3}\boldsymbol{a}$

(b) $\overrightarrow{QB} = \frac{1}{3}\overrightarrow{AB} = \frac{1}{3}(\boldsymbol{b} - \boldsymbol{a})$ $\overrightarrow{BR} = \frac{1}{3}\boldsymbol{b}$

$\overrightarrow{QR} = \overrightarrow{QB} + \overrightarrow{BR} = \frac{1}{3}(\boldsymbol{b} - \boldsymbol{a}) + \frac{1}{3}\boldsymbol{b}$

$= \frac{2}{3}\boldsymbol{b} - \frac{1}{3}\boldsymbol{a}$

$\overrightarrow{PQ} = \overrightarrow{QR}$, PQ and QR are parallel with a common point Q, so PQR is a straight line and therefore P, Q and R are collinear points.

21 **(a)** Let reflection in x-axis be R and translation be T, so $TR(P) = Q$

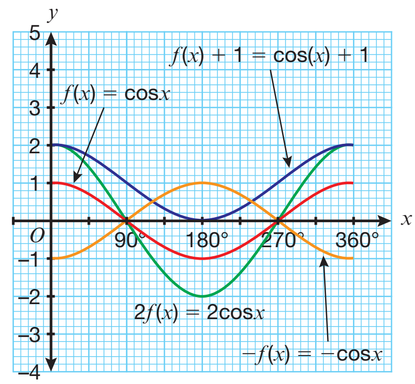
(Undo the translation first and then the reflection by using their inverses)

$\begin{pmatrix}3\\7\end{pmatrix} + \begin{pmatrix}1\\-2\end{pmatrix} = \begin{pmatrix}4\\5\end{pmatrix}$ (This point is then relected in x-axis)

$P\begin{pmatrix}4\\-5\end{pmatrix}$

(b) $OP = \sqrt{4^2 + (-5)^2} = \sqrt{16 + 25} = \sqrt{41}$
(Pythagoras' theorem)
$k = 41$

22 **(a)**



(i) $f(x) + 1 = \cos(x) + 1$

(Translation along vector $\begin{pmatrix}0\\1\end{pmatrix}$)

(ii) $-f(x) = -\cos(x)$ (Reflection in x-axis)

(iii) $2f(x) = 2\cos(x)$

(Stretch parallel to y-axis scale factor 2)

(b) If $g(x) = x^2$

$= gf(\pi x) + \pi$

$= g(\cos(\pi x)) + \pi$

$= (\cos(\pi x))^2 + \pi$

$= \cos^2(\pi x) + \pi$

23 $u = k_1 v^2$, $u = k_2\sqrt{w}$, so $k_1 50^2 = k_2\sqrt{144}$,

$\frac{k_1}{k_2} = \frac{\sqrt{144}}{50^2} = \frac{\sqrt{w}}{v^2} = \frac{\sqrt{625}}{v^2}$

$= \frac{12}{2500} = \frac{3}{625} = \frac{25}{v^2}$

$v^2 = \frac{25 \times 625}{3}$, $v = \sqrt{\frac{25^3}{5}} = 72.168...$

$= 7.22 \times 10^1$ (3 s.f.)

24 **(a)** Total surface area

$= 2x^2 + 2x^2 + 2xd + 2xd + xd + xd$

$= 4x^2 + 6xd$

$4x^2 + 6xd = 1400$, $2x^2 + 3xd = 700$

$$d = \frac{700 - 2x^2}{3x} \qquad (1)$$

Length of tape used

$L = 2 \times 3x + 2(d + x) + 2(2x + d)$

$= 12x + 4d \qquad (2)$

(Substituting d from (1) into (2))

$L = 12x + 4 \times \frac{700 - 2x^2}{3x}$

$= 12x + \frac{2800}{3x} - \frac{8x^2}{3x}$

$= 12x - \frac{2800}{3x} - \frac{8x}{3}$

$= \frac{28x}{3} + \frac{2800}{3x}$

(b) To minimise L, differentiate with respect to x

$\frac{dL}{dx} = \frac{28}{3} - \frac{2800}{3x^2}$

(Derivative of $\frac{1}{x} = x^{-1} = -x^{-2} = -\frac{1}{x^2}$)

For a maximum or minimum, $\frac{dL}{dx} = 0$

$\frac{dL}{dx} = 0$

$\frac{28}{3} - \frac{2800}{3x^2} = 0$

$\frac{28}{3} = \frac{2800}{3x^2}$

$x^2 = 100$

$x = 10$

and $L = 186\frac{2}{3}$

When $x = 9$, $L = 187.7$ ..., when $x = 11$, $L = 187.5$...

It is a minimum as the graph of L against x is continuous for $x > 0$ and the shape of the graph of L against x is a U shape.

When $x = 10$, $d = \frac{700 - 2 \times 10^2}{3 \times 10} = \frac{50}{3}$

Using (1)

Dimensions of box are 10 cm × 20 cm × $\frac{50}{3}$ cm

Volume of box = $\frac{10000}{3}$ cm^3

EXAMINATION PRACTICE PAPERS 2B SOLUTIONS

1 Bag A: kg per \$ = $\frac{2.5}{2}$ = 1.25 kg/\$

Bag B: kg per \$ = $\frac{4}{3.20}$ = 1.25 kg/\$

Same value for money for both bags.

Alternatively:

Bag A: \$ per kg = $\frac{2}{2.5}$ = 0.80 \$/kg

Bag B: \$ per kg = $\frac{3.20}{4}$ = 0.80 \$/kg

2 Area of whole circle = 240 cm^2

$A = \pi r^2 = 240 = \pi r^2, \frac{240}{\pi} = r^2, r = \sqrt{\frac{240}{\pi}}$

= 8.74039...cm

Perimeter of semi-circle = $\frac{1}{2} \times 2\pi r + 2r$

$= \pi r + 2r = r(\pi + 2) = 44.9$ cm (3 s.f.)

3 Let expression be E,

$E = 0.347\,695\ldots = 3.48 \times 10^{-1}$ (3 s.f.)

4 40% of the girls and 70% of the boys did not choose the fish option

$\frac{40}{100} \times \frac{45}{100} + \frac{70}{100} \times \frac{55}{100} = \frac{56.5}{100} = 56.5\%$

(55% are boys)

OR percentage who chose fish is

$\frac{60}{100} \times \frac{45}{100} + \frac{30}{100} \times \frac{55}{100} = \frac{43.5}{100} = 43.5\%$

(55% are boys)

percentage who did not choose fish is 100% − 43.5% = 56.5%

5 $T_n = \frac{t_n}{u_n} = \frac{2n - 1}{2n + 1}$

$T_1 = \frac{1}{3}, T_2 = \frac{3}{5}, T_3 = \frac{5}{7}, T_4 = \frac{7}{9}$

$T_1 \times T_2 \times T_3 \times T_4 = \frac{1}{9}$

6 Expected number of non-white

= p(non-white) × number of trials

$= \frac{n - 3}{n} \times n = n - 3$

7 **(a)** $p \times (1.05)^3 = £1389.15$

(Divide both sides by 1.05^3)

$p = \frac{1389.15}{1.05^3} = 1200$

(b) $\%\ \text{profit} = \frac{1389.15 - 1200}{1200} \times 100$

$= 15.7625 = 15.8\%$ (3 s.f.)

($\%\ \text{profit} = \frac{\text{change}}{\text{original}} \times 100$)

8 $\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{15}{\pi r^2}$

(Area of circle $= \pi r^2$)

Circumference $= 2\pi r$, so $30 = 2\pi r$

(Divide both sides by 2π)

$r = \frac{30}{2\pi} = 4.7746...\text{cm} = 0.047746...\text{m}$

(100 cm = 1 m)

$\text{pressure} = \frac{15}{\pi \times 0.047746^2} = 2094.4...N/m^2$

$= 2090\ \text{N/m}^2$ (3 s.f.)

9 $m = \sqrt{\frac{t+1}{t-1}}$

(Square both sides)

$m^2 = \frac{t+1}{t-1}$

(multiply both sides by $(t - 1)$)

$m^2(t - 1) = t + 1$

$m^2t - m^2 = t + 1$

(add m^2 and subtract t from both sides)

$m^2t - t = m^2 + 1$

(factorise LHS for t)

$t(m^2 - 1) = m^2 + 1$

(divide both sides by $(m^2 - 1)$)

$t = \frac{m^2 + 1}{m^2 - 1} = \frac{m^2 + 1}{(m + 1)(m - 1)}$

10 **(a)** $p(A') = \frac{28}{44} = \frac{7}{11}$

(Set A' are all elements not in A)

(b) $p(B \cap C') = \frac{10}{44} = \frac{5}{22}$

(Elements in B and also not in C)

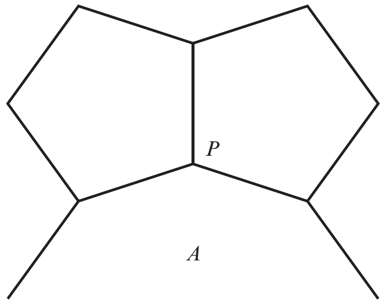
(c) $p(A \cup B \cup C)' = \frac{4}{44} = \frac{1}{11}$

(Elements not in A or B or C)

(d)) $= p((A \cap B \cap C)/(A \cup B)) = \frac{2}{32} = \frac{1}{16}$

(Sample space is reduced to $A \cup B$)

11 **(a)**



Regular pentagon exterior angle $= \frac{360°}{5} = 72°$

(Exterior angle of a regular n-sided pentagon $= \frac{360°}{5}$)

Regular pentagon interior angle
$= 180° - 72° = 108°$

Angle at point P:

Interior angle of $A = 360° - 2 \times 108°$
$= 144°$

Exterior angle of $A = 180° - 144° = 36°$

$36° = \frac{360°}{n}$, $n = 12$ so 12 pentagons will surround polygon A.

(b) The 12-sided polygon is a regular dodecagon.

12 **(a)** $100 = \frac{1}{2}(8.5 + 12.5) \times V_{max}$

(Area under speed–time graph = distance travelled)

$V_{max} = \frac{200}{21} = 9.5238...\text{m/s} = 9.52\ \text{m/s}$

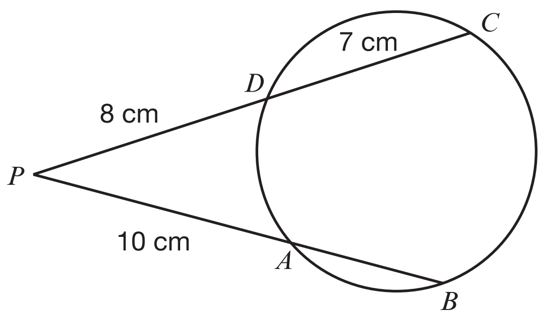
(3 s.f.)

(b) $\text{Acceleration} = \frac{9.5238}{4} = 2.3809...\text{m/s}^2$

$= 2.38\ \text{m/s}^2$ (3 s.f.)

(Gradient of speed–time graph = acceleration)

13



$PC \times PD = PB \times PA$
(intersecting chords theorem)
Let $AB = p$
$15 \times 8 = (10 + p)10$
$120 = 100 + 10p$
(Subtract 100 from both sides)
$20 = 10p$
(Divide both sides by 10)
$p = 2$, so $AB = 2$ cm
Now angle $APD = 30°$
Triangle BPC:
(Cosine Rule: $a^2 = b^2 + c^2 - 2bc\text{Cos}A$)
$BC^2 = 15^2 + 12^2 - 2 \times 15 \times 12 \times \cos(30°)$
$BC = 7.5651\ldots\text{cm} = 7.57$ cm (3 s.f.)

14 $(11\sqrt{3} - a)(3\sqrt{3} + a) = 95 + 32b\sqrt{3}$
(Expand out the LHS and compare irrational and rational parts)
LHS $= 99 + 11\sqrt{3}\,a - 3\sqrt{3}\,a - a^2$
$= 99 - a^2 + 8a\sqrt{3}$
$= 95 + 32b\sqrt{3}$
(Rational) $99 - a^2 = 95,\ 4 = a^2,\quad a = 2$
(Irrational) $8 \times 2\sqrt{3} = 32b\sqrt{3},\ b = \frac{1}{2}$
$a^2 + b^2 = 4 + \frac{1}{4} = \frac{17}{4}$ as required.

15 **(a)** $p = 2r + \frac{80}{360} \times 2\pi(2r) + \frac{80}{360} \times 2\pi r$

$= 2r + \frac{4\pi}{9} = (2r + r) = 2r + \frac{4\pi r}{3}$

$= \frac{2r(3 + 2\pi)}{3}$

(b) $p = 18 = \frac{2r(3 + 2\pi)}{3},\ r = \frac{27}{3 + 2\pi}$
$= 2.9084\ldots\text{m}$
Let area of lawn be $A = \frac{80}{360} \times \pi(4r^2 - r^2)$
$= \frac{2}{3}\pi r^2$
At $r = 2.9084\ldots\text{m}$, $A = \frac{2}{3}\pi(2.9084)^2$
$= 17.716\ldots\text{m}^2$
Cost $= \$18 \times 17.716 = \$318.89$
$= \$319$ (nearest \$)

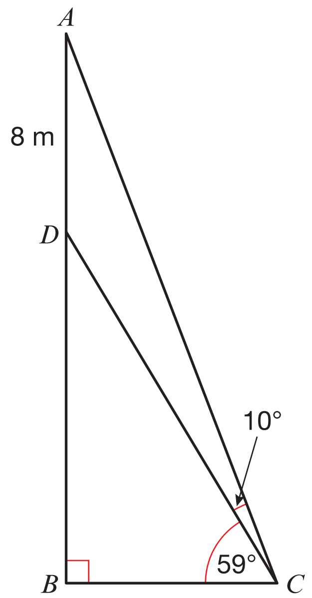
16 **(a)**
$6x - 5y = 7$ [1] × 3 = [3]
$4x + 3y = 11$ [2] × 5 = [4]
$18x - 15y = 21$ [3]
$20x + 15y = 55$ [4]
[3] + [4]: $38x = 76$, so $x = 2$ substitutes into [2]
[2]: $8 + 3y = 11$, so $3y = 3$, $y = 1$
Lines intersect at point (2, 1)

(b) Let $x = p^{-2} = \frac{1}{p^2}$ and $y = q^{-1} = \frac{1}{q}$

So $2 = \frac{1}{p^2},\ p^2 = \frac{1}{2},\ p = \pm\frac{1}{\sqrt{2}}$

Also $1 = \frac{1}{q} =,\ q = 1$

17



$AB = 8 + BD$
Triangle BCD: $\tan(59°) = \frac{BD}{BC}$ [1]

Triangle ABC: $\tan(69°) = \frac{BD + 8}{BC}$ [2]

Let [1] = [2] = BC:

$BC = \frac{BD}{\tan(59)} = \frac{BD + 8}{\tan(69)}$

$BD \times \tan(69) = (BD + 8) \times \tan(59)$
$BD \times \tan(69) - BD \times \tan(59) = 8 \times \tan(59)$
(Expand and factorise for BD)
$BD \times (\tan(69) - \tan(59)) = 8 \times \tan(59)$

$BD = \frac{8 \times \tan(59)}{(\tan(69) - \tan(59))} = 14.1518...\text{m}$

so $AB = 8 + 14.1518...$
$= 22.151...\text{m} = 22.2$ m (3 s.f.)

18 **(a)** If $g(x) = \frac{x + 60}{2}$

$y = \frac{x + 60}{2}$
(Re-write in terms of $y = ...$)
$x = \frac{y + 60}{2}$
(Switch x and y variables)
$2x = y + 60$, $y = 2x - 60$
(Re-arrange to make y the subject)
$g^{-1}(x) = 2x - 60$

(b) $fgh(x) = fg(2x) = f\left(\frac{2x + 60°}{2}\right)$

$= \sin\left(\frac{2x + 60°}{2}\right) = 1$

If the domain for $f(x)$ is $0 \leq x \leq 90°$ $\sin(x) = 1$ gives one solution which is $x = 90°$

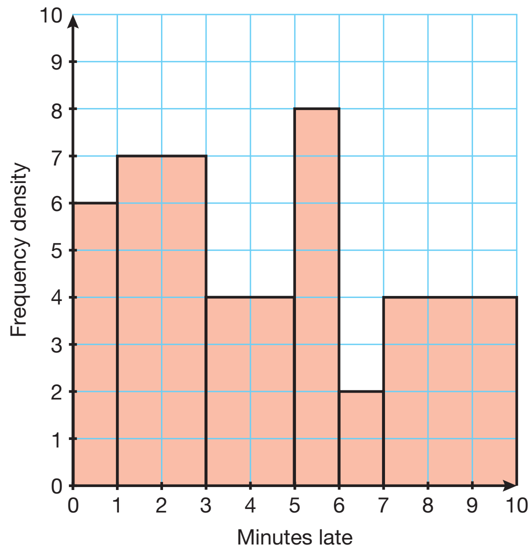
$\frac{2x + 60}{2} = 90°$, $2x + 60 = 180°$,

$2x = 120°$, $x = 60°$

19 Complete the table:

Minutes late t (min)	$0 < t \leq 1$	$1 < t \leq 3$	$3 < t \leq 5$	$5 < t \leq 6$	$6 < t \leq 7$	$7 < t \leq 10$
Number of pupils	6	14	8	8	2	12
Frequency density	6	7	4	8	2	4

(b) (The second class has a frequency density of 7 so it is now possible to calibrate the vertical scale.)



(c) Modal class is the most popular group with the highest frequency density: $5 < t \leq 6$

(d) Area of the histogram = total frequency
Area representing $t \geq 2.5$
$= 0.5 \times 7 + 2 \times 4 + 1 \times 8 + 1 \times 2 + 3 \times 4$
$= 33.5$
$P(t \geq 2.5) = \frac{33.5}{50} = \frac{67}{100}$

20 Let m be the original number of males llamas
Let f be the original number of female llamas

$m : f = 3 : 10$, $\frac{m}{f} = \frac{3}{10}$, $m = \frac{3f}{10}$, $f = \frac{10m}{3}$ (1)

After the birth, number of male llamas is $m + 3$, the number of female llamas is $f + 2$

$(m + 3) : (f + 2) = 1 : 3$, $\frac{m + 3}{f + 2} = \frac{1}{3}$,
$3(m + 3) = f + 2$ (2)
Substituting (1) into (2)

$3(m + 3) = \frac{10m}{3} + 2$

$9(m + 3) = 10m + 6$

$9m + 27 = 10m + 6$

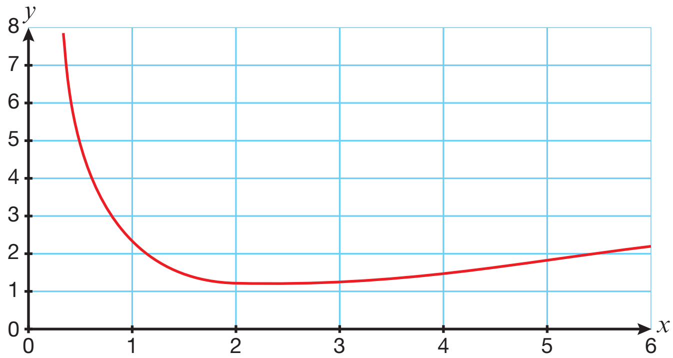
$m = 21$, $f = \frac{10 \times 21}{3} = 70$ there were 91 llamas before the birth.

There are now $91 + 5 = 96$ llamas in the herd.

21 (a)

x	0.5	1	2	3	4	5	6
y	5	2.25	1.25	1.25	1.5	1.85	2.25

(b)



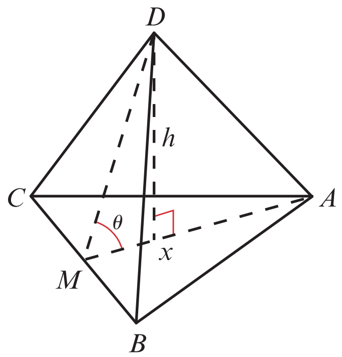
(c) $2x + \frac{12}{x} = 13$

$2x + \frac{12}{x} - 5 = 8$

$\frac{1}{4}\left(2x + \frac{12}{x} - 5\right) = 2$ so draw line $y = 2$

Solutions are where line intersects the curve at ≈ 1.1 or $x \approx 5.4$

22 (AM is a line of symmetry, so ΔABM is a right-angled triangle. M is midpoint of BC so $BM = 1$)



$AM^2 + BM^2 = AB^2$

(Pythagoras' theorem)

$AM^2 + 1 = 2^2$

$AM^2 = 3$

$AM = \sqrt{3}$

$AX = \frac{2}{3}AM$

$AX = \frac{2\sqrt{3}}{3}$ mm

(a) Area of base (triangle ABC) is

$\frac{1}{2} \times 2 \times \sqrt{3} = \sqrt{3}$

$h^2 + AX^2 = 2^2$ $\quad AD = 2$ mm

$h^2 = 4 - \left(\frac{2\sqrt{3}}{3}\right)^2 = 4 - \frac{4 \times 3}{9} = \frac{8}{3}$

$h = \sqrt{\frac{8}{3}} = \frac{2\sqrt{2}}{\sqrt{3}}$ or $\frac{2\sqrt{2}\sqrt{3}}{3}$ or $\frac{2\sqrt{6}}{3}$

Volume $= \frac{1}{3} \times \sqrt{3} \times \frac{2\sqrt{2}}{\sqrt{3}} = \frac{2\sqrt{2}}{3}$ mm³

(b) Angle required is θ (see sketch)

$\tan(\theta) = \frac{h}{MX} = \frac{2\sqrt{2}}{\sqrt{3}} \div \frac{\sqrt{3}}{3}$

$= \frac{2\sqrt{2}}{\sqrt{3}} \times \frac{3}{\sqrt{3}} = 2\sqrt{2}, \theta = 70.5°$

$MX = \frac{1}{3} \times AM$

or $\sin(\theta) = \frac{h}{DM} = \frac{2\sqrt{2}}{\sqrt{3}} \div \frac{2\sqrt{3}}{3}$

$= \frac{2\sqrt{2}}{\sqrt{3}} \times \frac{3}{2\sqrt{3}} = \frac{2\sqrt{2}}{3}\theta$

$= 70.5°$

$DM = AM$

or $\cos(\theta) = \frac{MX}{DM} = \frac{\sqrt{3}}{3} \div \sqrt{3}$

$= \frac{\sqrt{3}}{3} \times \frac{1}{\sqrt{3}} = \frac{1}{3}$

$\theta = 70.5°$

$MX = \frac{1}{3} \times AM, DM = AM$

23 (a) $\frac{4x^2 - 9}{x + 2} \div \frac{2x^2 - 5x - 12}{x - 4}$

$= \frac{4x^2 - 9}{x + 2} \times \frac{x - 4}{2x^2 - 5x - 12}$

$= \frac{(2x - 3)(2x + 3)}{x + 2} \times \frac{x - 4}{(2x + 3)(x - 4)}$

$= \frac{2x - 3}{x + 2}$

(b) $\frac{4x^2 - 9}{x + 2} \div \frac{2x^2 - 5x - 12}{x - 4}$

$= \frac{2x - 1}{x + 4}, \frac{2x - 3}{x + 2} = \frac{x + 1}{2(x - 2)}$

(Using previous result)

$\frac{2x - 3}{x + 2} = \frac{x + 1}{2(x - 2)}$

$2(2x - 3)(x - 2) = (x + 1)(x + 2)$
(Multiplying both sides by $2(x + 2)(x - 2)$)
$4x^2 - 14x + 12 = x^2 + 3x + 2$
$3x^2 - 17x + 10 = 0$
$(x - 5)(3x - 2) = 0$
$x = 5$ or $x = \frac{2}{3}$

24 **(a)** Stone hits the sea when
$s = -24 = 20t - 4t^2$
(Stone is 24m below the cliff top)
so $4t^2 - 20t - 24 = 0$
(Divide both sides by 4)
$t^2 - 5t - 6 = 0$, $(t - 6)(t + 1) = 0$, $t = 6$
(Note $t \neq -1$)

(b) $v = \frac{ds}{dt} = 20 - 8t$, at $t = 6$,
$v = 20 - 8 \times 6 = -28$ m/s

(c) Mean speed $= \frac{\text{distance}}{\text{time}}$
distance = distance to top × 2 + 24
speed at top is when $v = 0 = 20 - 8t$,
$t = \frac{5}{2} s$
at $t = \frac{5}{2}$, $s = 20 \times \frac{5}{2} - 4 \times \left(\frac{5}{2}\right)^2 = 25$m
Mean speed $= \frac{2 \times 25 + 24}{6}$
= 12.3 m/s (3 s.f.)