EXAMINATION PRACTICE PAPER ANSWERS

Paper 1

1 a Both 10^6 factors cancel out.

$\frac{1.8}{5.1} \times 100 = 35.29\ldots = 35.3\%$ (3 s.f.)

b Divide both sides by 2 when ratio is 2 : 5
or divide both sides by 1.4 when ratio is 1.4 : 3.5

$1.4:3.5 = 14:35 = 2:5 = 1:2.5$

2 a Multiply each term in the bracket by 5.

$5(2y - 3) = 10y - 15$

b 'FOIL' expansion and simplification.

$(2x - 1)(x + 5) = 2x^2 + 10x - x - 5$
$= 2x^2 + 9x - 5$

c Common factor $4y$ produces the given product of $4y(1 + 6z)$.

$4y + 24yz = 4y(1 + 6z)$

d Common factor $2x$ produces the given product of $2x(x - 11)$.

$2x^2 - 22x = 2x(x - 11)$

e Divide both sides by 3.

$3x - 2 = 7$

Add 2 to both sides.

$3x = 9$

Divide both sides by 3.

$x = 3$

3 a The mid-point is the mean of the x-values and the y-values.

$\mathrm{M} = \left(\frac{2+8}{2}, \frac{1+5}{2}\right) = (5, 3)$

b Pythagoras' theorem.

$\mathrm{AB} = \sqrt{(8-2)^2 + (5-1)^2} = \sqrt{52} = 7.211\ldots$
$= 7.21$ cm (3 s.f.)

c Equation of line perpendicular to AB through M:

gradient $= \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$

Gradient of AB $= m_1 = \frac{5-1}{8-2} = \frac{4}{6} = \frac{2}{3}$

The condition for perpendicular gradients m_1 and m_2 is given by $m_1 \times m_2 = -1$

Gradient of line perpendicular to AB $= -\frac{3}{2}$

Use the equation of a straight line.

$y = mx + c$

M is on the line so must satisfy the equation.

$3 = -\frac{3}{2} \times 5 + c$

$c = 10\frac{1}{2}$

Multiply through by 2 to express equation as desired.

$y = -\frac{3}{2}x + 10\frac{1}{2}$

$2y = -3x + 21$

$3x + 2y - 21 = 0$

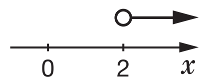
4 a $7x - 11 > 3$

Add 11 to both sides and then divide by 7.

$7x > 14$

$x > 2$

b



5 a Median is at $\frac{1}{2}(n + 1)$th position.

Median score is at the 8th position.

Median score = 2

b mean $= \frac{\Sigma fx}{\Sigma f}$

Mean $= \frac{5 \times 1 + 4 \times 2 + \cdots + 1 \times 5}{15}$

$= \frac{37}{15} = 2.47$ (3 s.f.)

6 a p(A or B) = p(A) + p(B) if A,B independent.
Note that the answer should be given in full as no rounding has been requested.

p(bb or gg) = p(bb) + p(gg) = $0.45 \times 0.45 + 0.12 \times 0.12$
$= 0.2169$

b Expected number of events = no. of trials × probability of the event

Number of green-eyed people $= 500 \times 0.12$
$= 60$

7 a area of trapezium $= \frac{1}{2}(a + b)h$

Area $= \frac{1}{2}(50 + 90)60 = 4.2 \times 10^3\,\text{mm}^2$

b volume of prism = cross-sectional area × length

Volume of prism $= 150 \times 4200 = 6.3 \times 10^5\,\text{mm}^3$

8 a i Elements in both A and B.

$A \cap B = \{1, 3\}$

ii Elements in A or B.

$A \cup B = \{1, 2, 3, 4, 5\}$

b A' is the complement of set A.

5 is a member of the set that is not A.

9 a

Age (years)	Frequency f	Mid-points x	fx	Cumulative frequency
$10 < t \leqslant 20$	8	15	$8 \times 15 = 120$	8
$20 < t \leqslant 30$	28	25	$28 \times 25 = 700$	36
$30 < t \leqslant 40$	30	35	$30 \times 35 = 1050$	66
$40 < t \leqslant 50$	10	45	$10 \times 45 = 450$	76
$50 < t \leqslant 60$	4	55	$4 \times 55 = 220$	80

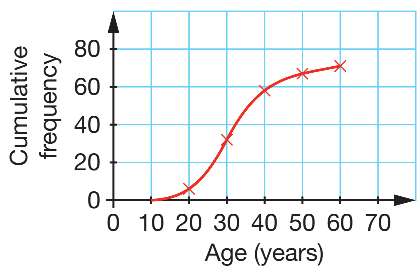
$\Sigma f = 80$ $\Sigma fx = 2540$

Mean estimate = $\frac{\Sigma fx}{\Sigma f}$, x are mid-points

Mean estimate = $\frac{\Sigma fx}{\Sigma f} = \frac{2540}{80} = 31.75$ years

b Check that final value is the total sum of 80.
Above.

c



d n 'large' so median at $\frac{1}{2}n = 40$
Median ≈ 32

Lower quartile at $\frac{1}{4}n = 20$
Lower quartile (LQ) ≈ 24.3 years

Upper quartile at $\frac{3}{4}n = 60$
Upper quartile (UQ) ≈ 38 years

IQR = UQ – LQ
Interquartile range (IQR) ≈ 13.7 years

10 a 32 capsules equally spaced around the wheel.
$\angle AOB = \frac{360}{32} = 11.25°$
$\angle BOC = 180° - 11.25° = 168.75°$

Triangle OBC is isosceles.
$\angle OCB = \frac{180° - 168.75°}{2} = 5.625°$

AC is a vertical line.
Angle required = $90° - 5.625° = 84.375°$

b $C = \pi d$
Distance = $\pi \times 135 = 424.115\ldots$ m = 424 m (3 s.f.)

c speed = $\frac{\text{distance}}{\text{time}}$

time = $\frac{\text{distance}}{\text{speed}} = \frac{424.115}{0.26} = 1631.2115\ldots$ s
= 27 mins 11 secs

11 a $3x - 4y = 15$
$4y = 3x - 15$
$y = \frac{3}{4}x - \frac{15}{4}$

$y = mx + c$; m is the gradient
Gradient = $\frac{3}{4}$

b Solve by eliminating y.

$3x - 4y = 15$ [1] × 3
$5x + 6y = 6$ [2] × 2
$9x - 12y = 45$ [3]
$10x + 12y = 12$ [4]

[3] + [4] $19x = 57$
$x = \frac{57}{19} = 3$ sub in [2]
[2] $15 + 6y = 6$, so $y = -1\frac{1}{2} \Rightarrow (3, -1\frac{1}{2})$

12 $f(x) = 3x - 2$, $g(x) = \frac{1}{x}$

a i Find g(3) to use as the input into f(x).
$fg(3) = f(\frac{1}{3}) = 1 - 2 = -1$

ii Find f(3) to use as the input into g(x).
$gf(3) = g(7) = \frac{1}{7}$

b i g(x) is the input into f(x).
$fg(x) = f(\frac{1}{x}) = 3(\frac{1}{x}) - 2 = \frac{3 - 2x}{x}$

ii if $x = 0$ there is no output for fg(x).

c $f(x) = fg(x)$
$3x - 2 = \frac{3 - 2x}{x}$

Multiply both sides by x.
$(3x - 2)x = 3 - 2x$
$3x^2 - 2x = 3 - 2x$
$x^2 = 1 \rightarrow x = \pm 1$

13 a i

Number of calories (n)	Frequency	Frequency density = frequency ÷ class width
$0 < n \leqslant 1000$	90	0.09
$1000 < n \leqslant 2000$	130	0.13
$2000 < n \leqslant 2500$	140	0.28
$2500 < n \leqslant 4000$	120	0.08

ii Histogram can now be completed.

b UQ is at $\frac{3}{4}$ of area of histogram.

UQ is $\frac{3}{4} \times 480 = 360$th person = 2500 calories.

14 a i Inversely implies that r^2 is on the denominator.

$R \alpha \frac{1}{r^2}$, so $R = \frac{k}{r^2}$, where k is a constant.

Substitute $R = 0.9$ and $r = 2$ into the formula.

$0.9 = \frac{k}{2^2}$, so $k = 3.6$

ii

b Substitute values into formula then solve for r.

$R = \frac{3.6}{r^2}$, $0.1 = \frac{3.6}{r^2}$, $r^2 = 36$, $r = 6$ mm

15 shaded area = large rectangle − small rectangle = 35

$(x + 4)(x + 1) - 15 = 35$

$(x^2 + 5x + 4) - 15 = 35$

$x^2 + 5x - 46 = 0$

Use the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

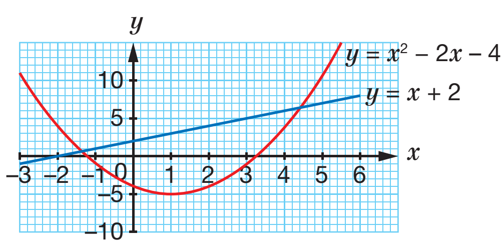
$a = 1$, $b = 5$, $c = -46$

$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(-46)}}{2(1)}$, $x = 4.73$ cm (3 s.f.)

16 a $x = -1.2,\ 3.2$ (1 d.p.)

b $y = x^2 - 2x - 4 = x + 2$ so draw $y = x + 2$

Solutions are $x = 4.4, -1.4$ (1 d.p.)



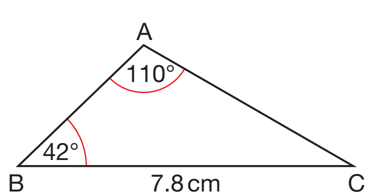
c i $\frac{dy}{dx} = 2x - 2$

ii Gradient = 0 at minimum point.

$\frac{dy}{dx} = 0 = 2x - 2$, $x = 1$, $y = -5$

Min point (1, −5)

17



a sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\angle ACB = 28°$, sine rule $\frac{AB}{\sin 28°} = \frac{7.8}{\sin 110°}$

$AB = \frac{7.8}{\sin 110°} \times \sin 28° = 3.89688\ldots$ cm

$AB = 3.90$ cm (3 s.f.)

b Note 3.90 is not used!

Area $= \frac{1}{2} \times 7.8 \times 3.89688 \times \sin 42° = 10.1693\ldots \text{cm}^2$

area $= \frac{1}{2}ab \sin C$

Area = 10.2 cm^2 (3 s.f.)

18 a p(X and Y) = p(X) × p(Y)

p(AN) $= \frac{3}{6} \times \frac{2}{5} = \frac{6}{30} = \frac{1}{5}$

b p(X) + p(X′) = 1

p(not same) = 1 − p(same)

p(same) = p(AA) + p(NN)

p(X or Y) = p(X) + p(Y)

$= \frac{3}{6} \times \frac{2}{5} + \frac{2}{6} \times \frac{1}{5} = \frac{8}{30} = \frac{4}{15}$

p(not same) $= 1 - \frac{4}{15} = \frac{11}{15}$

19 Volume of the thin cylinder of water increase after the sphere is submerged = volume of the sphere.

$V = \pi r^2 h$ (volume of cylinder)

$V = \frac{4}{3}\pi r^3$ (volume of sphere)

$\pi \times 30^2 \times 5 = \frac{4}{3} \times \pi \times r^3$ where r is the radius of the sphere.

Rearrange to make r the subject.

$r^3 = \frac{3 \times 30^2 \times 5}{4}$

$r = \sqrt[3]{\frac{3 \times 30^2 \times 5}{4}} = 15$ cm

$A = 4\pi r^2$ (surface area of sphere)

$A = 4\pi \times 15^2 = 2827.43\ldots \text{cm}^2 = 2830 \text{ cm}^2$ (3 s.f.)

20 $\frac{x}{x + 2} + \frac{x + 17}{x^2 + x - 6} = 1$

Factorise the denominator.

$\frac{3}{(x + 2)} + \frac{x + 17}{(x + 2)(x - 3)} = 1$

Multiply first fraction by $\frac{(x - 3)}{(x - 3)}$ which is 1.

$\frac{(x - 3)}{(x - 3)} \times \frac{3}{(x + 2)} + \frac{x + 17}{(x + 2)(x - 3)} = 1$

Express LHS as a single fraction.

$\frac{3(x - 3) + x + 17}{(x - 3)(x + 2)} = 1$

$\frac{3x - 9 + x + 17}{(x - 3)(x + 2)} = 1$

Factorise numerator and cancel common factor $(x + 2)$.

$\frac{4x + 8}{(x - 3)(x + 2)} = \frac{4(x + 2)}{(x - 3)(x + 2)} = \frac{4}{(x - 3)} = 1$

Add 3 to both sides.

$4 = (x - 3)$, $x = 7$

21 Consider the sum of the first odd numbers:

$1 + 3 + 5 + 7 + \cdots$

This is an A.P. with $a = 1$ and $d = 2$

$t_n = a + (n - 1)d = 1 + (n - 1) \times 2$

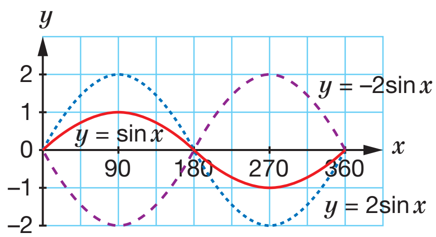
$t_n = 2n - 1$

Sum of the A.P. $1 + 3 + 5 + 7 + \cdots + (2n - 1)$

Sum of an A.P. $S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_n = \frac{n}{2}[2 \times 1 + (n - 1) \times 2] = \frac{n}{2}[2n] = n^2$

22



$f(x) = \sin x$

Stretch of $f(x)$ SF = 2 parallel to y-axis.

$2f(x) = 2\sin x$

$-2f(x) = -2\sin x$

Reflection of $2f(x)$ in x-axis.

Min point (90, –2) Max point (270, 2)

Paper 2

1

180 units at £0.0825 per unit	£14.85
440 units at £0.0705 per unit	£31.02
Total amount	£45.87
Tax at 15% of the total amount	£6.88
Amount to pay	£52.75

620 – 180 = 440 units

15% of £45.87 = 0.15 × £45.87 = £6.88

2 (triangle ABD is isosceles)

$\angle ABD = \frac{180 - 38}{2} = 71°$

(corresponding angle to $\angle ABD$)

$p = 71°$

(angle sum of a straight line = 180°)

$\angle BDE = 180 - 71 = 109°$

3 Sweets are the same in number.

Arul Nikos

$x + 6 = 4x - 6$

$12 = 3x$

$x = 4$

4 mean $= \frac{\text{sum of scores}}{\text{number of scores}}$

$158 = \frac{S_4}{4}$, $S_4 = 156 \times 4 = 632$ cm

$156 = \frac{632 + y}{5}$, where y is Sienna's height in cm.

$156 \times 5 = 632 + y$, $y = 148$ cm

5 **a** tin : lead = 1 : 2

240 g ÷ sum of parts = 1 part

240 = 3 parts, 1 part = 80

tin : lead = 80 g : 160 g

b tin : lead = 75 : m

$m = 75 \times 2 = 150$ g

mass of solder = 75 + 150 = 225 g

6 $48 = 2^4 \times 3$

First find prime factors of both numbers.

$180 = 2^2 \times 3^2 \times 5$

a Product of common factors of both numbers.

HCF $= 2^2 \times 3 = 12$

b LCM $= 2^4 \times 3^2 \times 5 = 720$

7 **a** Round one number up and one down as this is a product.

$g = 10$, $h = 0.8$

b Square both sides.

$v = \sqrt{2gh}$

Divide both sides by $2h$.

$v^2 = 2gh$

$g = \frac{v^2}{2h}$

8 **a** Kitchen chairs

b **i** P ∪ Q = {1, 2, 3, 4, 5, 6, 7, 8, 9}

ii Yes, an integer cannot be both odd and even.

ϕ means that the set is empty.

9 **a** Let angle PQR = θ

$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$

$\sin(\theta) = \frac{4.7}{7.6}$, $\theta = 38.2009\ldots$

Use the inverse sin 'button' used to find angle.

$\theta = 38.2°$ (1 d.p.)

b **i** $\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$

$QR_{max} = 7.65 \times \cos(38.2009\ldots)$

Upper bound of 7.6 = 7.65

= 6.01 cm (3 s.f.)

ii $QR_{min} = 7.55 \times \cos(38.2009\ldots)$

Lower bound of 7.6 = 7.55

= 5.93 cm (3 s.f.)

10 **a** 60 × 1.20 = 72 s

b cost per sec $= \frac{12}{72}$, cost per min $= \frac{12}{72} \times 60 = 10$ c/min

c old rate: 12 c/min; new rate: 10 c/min

percentage change $= \frac{\text{change}}{\text{original}} \times 100$

% decrease $= \frac{2}{12} \times 100 = 16\frac{2}{3}\%$

11 $2x^2 = 8x - 7$

Rearrange in form $ax^2 + bx = c$.

$2x^2 - 8x = -7$

Divide both sides by 2.

$x^2 - 4x = -\frac{7}{2}$

Complete the square.

$(x-2)^2 = -\frac{7}{2} + 4 = \frac{1}{2}$

Take square root of both sides.

$x - 2 = \pm\sqrt{\frac{1}{2}}$

$x = 2 \pm \frac{1}{\sqrt{2}} = \frac{2\sqrt{2} \pm 1}{\sqrt{2}}$

12 a Ratios of lengths are $8:12 = 2:3$

$5:x = 2:3$, $\frac{5}{x} = \frac{2}{3}$, $5 \times 3 = 2 \times x$, $x = \frac{15}{2}$, $x = 7.5$

b $y:15 = 2:3$, $\frac{y}{15} = \frac{2}{3}$, $3 \times y = 15 \times 2$, $y = \frac{30}{3}$, $y = 10$

c Length are in ratio $x:y$; areas are in ratio $x^2:y^2$.

Ratios of areas are $4:9$

Let area of Q be A

$80:A = 4:9$, $\frac{80}{A} = \frac{4}{9}$, $80 \times 9 = 4 \times A$,

$A = 180\,\text{cm}^2$

13 Intersecting chords theorem.

$AX \times XB = CX \times DX$

$2.8 \times 1.6 = 1.2 \times DX$

$DX = 3.7333\ldots\text{cm}$

Pythagoras' theorem

$AD^2 = AX^2 + DX^2$

$= 2.8^2 + (3.7333\ldots)^2$

$AD = 4.67\,\text{cm}$ (3 s.f.)

Let angle $DAX = \theta$

$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

$\tan\theta = \frac{DX}{AX} = \frac{3.7333\ldots}{2.8}$

$\theta = 53.1°$ (3 s.f.)

14 a i $\mathbf{OX} = k\mathbf{OB} = k\begin{pmatrix}5\\2\end{pmatrix}$

ii $\mathbf{AX} = \mathbf{AO} + \mathbf{OX} = k\begin{pmatrix}5\\2\end{pmatrix} - \begin{pmatrix}1\\2\end{pmatrix}$

iii $\mathbf{XC} = \mathbf{XO} + \mathbf{OC} = \begin{pmatrix}4\\0\end{pmatrix} - k\begin{pmatrix}5\\2\end{pmatrix}$

b If $\mathbf{AX} = \mathbf{XC}$, $5k - 1 = 4 - 5k$, $k = \frac{1}{2}$

c X is halfway along OB as $k = \frac{1}{2}$.

$\mathbf{XC} = \frac{1}{2}\mathbf{AX}$ implies $\mathbf{AX} = \mathbf{XC}$ and vectors are parallel, so AXC is a straight line with X as its mid-point.

15 a Multiplying a quantity by 1.03 increases it by 3%...

Investment value = $1200 \times (1.03)^5$ = $1391.13

b Let original cost be $$y$,

$y \times (0.70) = \$98$

Multiplying a quantity by 0.70 decreases it by 30%.

$y = \frac{98}{0.70} = \$140$

16 a Area of rectangle = base × width

$y = x(28 - 2x) = 28x - 2x^2$

b Graph of y against x will be an inverted parabola which has a maximum point where $\frac{dy}{dx} = 0$

If $y = ax^n$, $\frac{dy}{dx} = nax^{n-1}$

$\frac{dy}{dx} = 28 - 4x = 0$ at max point

Solve for x.

$x = 7$

Substitute x into width $= 28 - 2x$

$y = 28 - 2 \times 7 = 14$

$y_{max} = 7 \times 14 = 98\,\text{m}^2$

17 a area of circle $= \pi r^2$

Area $= \frac{110}{360} \times \pi \times 12^2 = 138\,\text{cm}^2$ (3 s.f.)

b Total perimeter $= 2r + \frac{120}{360} \times 2\pi r = 2r + \frac{2}{3}\pi r = \frac{2}{3}r(3 + \pi)$

Perimeter $= 2 \times 12 + \left(\frac{110}{360}\right) \times 2 \times \pi \times 12$

$= 47.0$ (3 s.f.)

circumference of circle $= 2\pi r$

$200 = \frac{2}{3}r(3 + \pi)$

Rearrange to make r the subject.

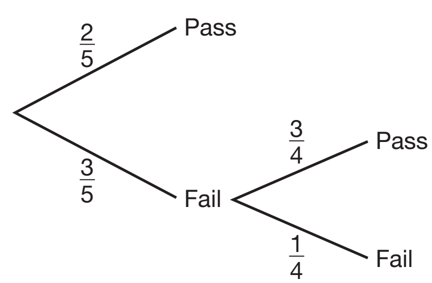
$600 = 2r(3 + \pi)$

$300 = r(3 + \pi)$

$r = \frac{300}{3 + \pi}$ cm

18 a $p(A) + p(A') = 1$

First attempt Second attempt



b p(pass course) = p(pass 1st time) + p(pass 2nd time)

p(pass 2nd time) = fail 1st test and pass 2nd test

$= \frac{2}{5} + \frac{3}{5} \times \frac{3}{4} = \frac{17}{20}$

19 Solve by substitution.

$y = x - 1$ [1]

$2x^2 + y^2 = 2$ [2]

subs [1] into [2]

[2] $2x^2 + (x - 1)^2 = 2$

$2x^2 + x^2 - 2x + 1 = 2$

Quadratic factorisation.

$3x^2 - 2x - 1 = 0$

$(3x + 1)(x - 1) = 0$

$x = -\frac{1}{3}$ or 1 sub into [1]

$y = -\frac{4}{3}$ or 0 $(1, 0), \left(-\frac{1}{3}, -\frac{4}{3}\right)$

20 a $x = 3t - t^2 + \frac{8}{t} = 3t - t^2 + 8t^{-1}$

If $x = f(t)$, $v = \frac{dx}{dt}$, $a = \frac{dv}{dt}$

$v = 3 - 2t - 8t^{-2} = 3 - 2t - \frac{8}{t^2}$ m/s

b $a^{-m} = \frac{1}{a^m}$

$a = \frac{dv}{dt} = -2 + 16t^{-3} = -2 + \frac{16}{t^3}$ m/s²

c $a = 0 = -2 + \frac{16}{t^3}$, $t^3 = 8$, $t = 2$ s

21 $(2x - 1)(x + 1)(x + 2) - (x + 1)(x + 2)(x + 3) = 0$

$(x + 1)(x + 2)[(2x - 1) - (x + 3)] = 0$

$(x + 1)(x + 2)(x - 4) = 0$

$x = -1, -2$ or 4

Be careful not to divide by $(x + 1)(x + 2)$ first as solutions can be lost!

22 area of triangle $= \frac{1}{2} \times$ base $\times$ perpendicular height

Let height be y. Consider equilateral triangle of side $4n$.

$A = \frac{1}{2} \times 4n \times y = 2ny$

Pythagoras' theorem

$(4n)^2 = y^2 + (2n)^2$

$16n^2 = y^2 + 4n^2$

$12n^2 = y^2$

$\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$

$2\sqrt{3}\,n = y$

$A = 2n \times 2\sqrt{3}\,n = 4\sqrt{3}\,n^2$

Paper 3

1 $\frac{3.23 \times 10^4}{1.8 \times 10^6} = 24.0749\ldots = 2.41 \times 10^1$ (3 s.f.)

2 width = height

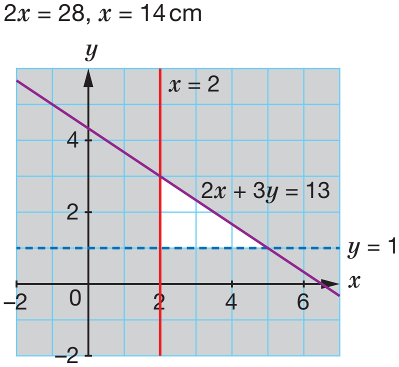
$6x = 4(x + 7)$

Solve for x.

$6x = 4x + 28$

$2x = 28$, $x = 14$ cm

3



Trial point (0, 0) could be used to check region $2x + 3y \leqslant 13$

4 mean $= \frac{\Sigma fx}{\Sigma f}$, where x are mid-points.

$5.5 = \frac{4 \times 1 + 15 \times 4 + 18 \times 7 + p \times 10}{37 + p}$

Solve for p.

$5.5(37 + p) = 190 + 10p$

$203.5 + 5.5p = 190 + 10p$

$13.5 = 4.5p$, $p = 3$

5 (ABCD is a cyclic quadrilateral, so opposite angles sum to 180°)

$x + (2x + 12) = 180°$

$3x = 168°$

$x = 56°$

(angle at centre of circle = 2 × angle at circumference)

$y = 2 \times 56°$

$y = 112°$

6 a i 'FOIL'

$(x - 3)(x + 7) = x^2 + 7x - 3x - 21 = x^2 + 4x - 21$

ii 'FOIL'

$(5x - 3)^2 = (5x - 3)(5x - 3) = 25x^2 - 30x + 9$

iii Expand last two brackets by 'FOIL' then expand this by $(3x - 1)$

$(3x - 1)(2x - 1)(x - 1) = (3x - 1)(2x^2 - 3x + 1)$

$= 6x^3 - 9x^2 + 3x - 2x^2 + 3x - 1$

$= 6x^3 - 11x^2 + 6x - 1$

iv $(\sqrt{7} - 1)(2\sqrt{7} + 1) = 2 \times 7 + \sqrt{7} - 2\sqrt{7} - 1 = 13 - \sqrt{7}$

b i $\frac{\sqrt{7}}{\sqrt{7}} = 1$

$\frac{\sqrt{7} + 1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{7 + \sqrt{7}}{7}$

ii $\frac{\sqrt{7} - 1}{\sqrt{7} - 1} = 1$, $a^2 - b^2 = (a + b)(a - b)$

$\frac{\sqrt{7}}{\sqrt{7} + 1} \times \frac{\sqrt{7} - 1}{\sqrt{7} - 1} = \frac{7 - \sqrt{7}}{\sqrt{7}^2 - 1^2} = \frac{7 - \sqrt{7}}{6}$

7 Let A_1 = area of ΔABC.

Let A_2 = total surface area of pyramid.

Base area + four triangular faces.

$A_2 = 10 \times 10 + 4A_1$

area $= \frac{1}{2} \times$ base $\times$ height

$A_1 = \frac{1}{2} \times 10 \times \text{AM}$

Pythagoras' theorem

$\text{AC}^2 = \text{AM}^2 + \text{MC}^2$

$\text{AM}^2 = 13^2 - 5^2 = 144$, AM = 12

$A_1 = 5 \times 12 = 60$, $A_2 = 100 + 4 \times 60$

$A_2 = 340$ cm²

8 n-sided regular polygon
exterior angle, $e = 180° - 120° = 60°$

$e = \dfrac{360}{n}$ for a regular n-sided polygon

$60 = \dfrac{360}{n}$, $n = 6$

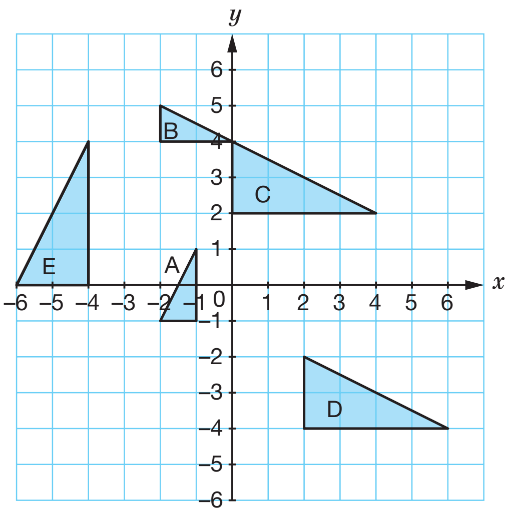
$4n$-sided polygon

$e = \dfrac{360}{4 \times 6} = 15°$

$e + i = 180°$ for a regular n-sided polygon

interior angle, $i = 180° - 15° = 165°$

9



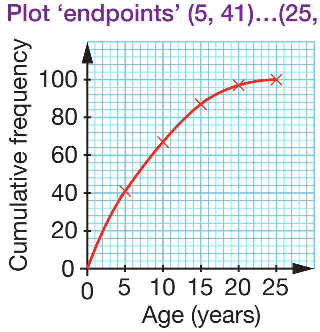
e Draw construction lines from vertices from E to A shows centre of enlargement is where they meet.

E mapped onto A by an enlargement SF = $\dfrac{1}{2}$ about centre (2, –2)

10 a

Age (years)	Cumulative frequency
$0 < t \leqslant 5$	41
$0 < t \leqslant 10$	67
$0 < t \leqslant 15$	87
$0 < t \leqslant 20$	97
$0 < t \leqslant 25$	100

b Plot 'endpoints' (5, 41)…(25, 100)



c Lower quartile = 3 years; upper quartile = 12 years.

LQ found at 25th value, UQ found at 75th value

IQR = UQ – LQ

Interquartile range = 12 – 3 = 9 years

11 a $y = mx + c$; m is gradient, c is y-intercept

gradient = $\dfrac{\text{rise}}{\text{run}}$

$m = 2$

$y = 2x - 1$

b Gradient of line perpendicular to line L is m_1.

Product of the gradients of perpendicular lines = –1

$2 \times m_1 = -1,\ m_1 = -\dfrac{1}{2}$

$y = -\dfrac{1}{2}x + c$

(2, 3) $\quad 3 = -\dfrac{1}{2} \times 2 + c,\ c = 4$

$y = -\dfrac{1}{2}x + 4$

12 a $p^{\frac{1}{2}} = (3^8)^{\frac{1}{2}} = 3^4$, $k = 4$

b $(a^m)^n = a^{m \times n}$, $(ab)^m = a^m \times b^m$

$q^{-\frac{1}{3}} = (2^9 \times 5^{-6})^{-\frac{1}{3}} = (2^9)^{-\frac{1}{3}} \times (5^{-6})^{-\frac{1}{3}} = 2^{-3} \times 5^2$,

$m = -3, n = 2$

13 a $y = 5000x - 625x^2$

$\dfrac{dy}{dx} = 0$ at turning points

$\dfrac{dy}{dx} = 5000 - 1250x = 0$

$x = 4$, $y = 10\,000$

b (4, 10 000) is a maximum point as the curve is an inverted parabola ($y = -ax^2\ldots$)

c £4 as this produces the greatest profit of £10 000.

14 a $3600 = \pi r^2 + \dfrac{1}{2} \times 4\pi r^2 = 3\pi r^2$

area of circle = πr^2; surface area of sphere = $4\pi r^2$

$r^2 = \dfrac{3600}{3\pi}$, $r = \sqrt{\dfrac{1200}{\pi}} = 19.5441\ldots$, $d = 39.1$ cm (3 s.f.)

b volume of sphere = $\dfrac{4}{3}\pi r^3$

$V = \dfrac{1}{2} \times \dfrac{4}{3}\pi \times 19.5441 = 15\,635.2\ldots$,

$v = 15\,600$ cm^3 (3 s.f.)

15 a Similar figures of corresponding lengths l and L.

$l \times k = L$ where k is the length scale factor.

$l \times 3 = L$

$v \times k^3 = V$

6000×3^3 = volume of large drum, V

$V = 1.62 \times 10^5$ cm^3

b Number of large drums = $\dfrac{3240 \times 10^6}{1.62 \times 10^5} = 20\,000$

$a \times k^2 = A$

2000×3^2 = Area of a large drum = 18 000 cm^2

10 000 cm^2 = 1 m^2

Area of all large drums = $20\,000 \times \dfrac{18000}{10^4}$ m^2 = 36 000 m^2

Total cost = \$1.20 × 36 000 = \$43 200

16 a If $f(x) = \frac{x}{x-1}$, $f(3) = \frac{3}{3-1} = \frac{3}{2}$

b If $x = 1$, the denominator = 0

x cannot be 1

c i $ff(x) = f\left(\frac{x}{x-1}\right) = \frac{\frac{x}{x-1}}{\frac{x}{x-1} - 1} = \frac{\frac{x}{x-1}}{\frac{1}{x-1}} = \frac{x}{x-1} \times \frac{x-1}{1} = x$

ii The input is the output as $ff(x) = ff^{-1}(x) = x$

$f(x)$ is its own inverse

17 Triangle BAG has ∠A = 30°, ∠B = 20°, ∠G = 130°, AB = 1500 m

'SASA' so use the sine rule.

$\frac{BG}{\sin 30°} = \frac{1500}{\sin 130°}$, $BG = \frac{1500}{\sin 130°} \times \sin 30° = 979.055\ldots$

BG = 979 m (3 s.f.)

18 a $7 - x$

b All cells sum to 50.

Total must sum to 50:

$50 = x + 10 + 9 + 13 + (7 - x) + (8 - x) + 6$

$50 = 53 - x$, $x = 3$

19 a $3n$

b p(A and B) = p(A) × p(B) if A, B are independent.

$\frac{n}{3n} \times \frac{n-1}{3n-1} = \frac{1}{10}$

Solving for n by multiplying across by $30n(n-1)$

gives $n^2 - 7n = 0$

$n(n-7) = 0$, so $n = 7$, number of people in club = $3n$ = 21 people.

20 a $x^2 - 8x + 21 = (x-4)^2 - 16 + 21 = (x-4)^2 + 5$

$a = 4$, $b = 5$

b If $f(x) = x^2 - 8x + 21 = (x-4)^2 + 5$

$(x-4)^2$ will always be positive so, when $x = 4$, this value is 0.

$f(x)_{min} = 5$ when $x = 4$

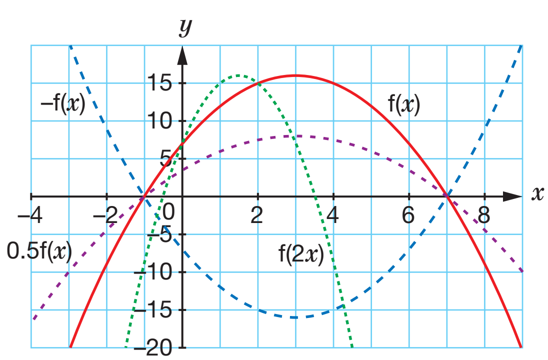
Also, the curve is a U-shaped parabola with minimum point (4, 5)

21 Let the first integer be n, the next integer is $n + 1$.

$(n+1)^2 - n^2 = (n^2 + 2n + 1) - n^2 = 2n + 1$

$n + (n+1) = 2n + 1$

22 a



b,c i $-f(x) = -(7 + 6x - x^2) = x^2 - 6x - 7$: Reflection of $f(x)$ in x-axis

ii $0.5f(x) = 0.5(7 + 6x - x^2) = 3.5 + 3x - 0.5x^2$: Stretch parallel to y-axis, SF = 0.5

iii $f(2x) = 7 + 6(2x) - (2x)^2 = 7 + 12x - 4x^2$: Stretch parallel to x-axis, SF = 0.5

Paper 4

1 Bearings are measured clockwise from North.

Bearing of Q from P = 250°.

2 a i $(3x-4)(3x+4) = 9x^2 + 12x - 12x - 16 = 9x^2 - 16$

ii 'FOIL' used to expand the brackets.

$(3x-4)^2 = (3x-4)(3x-4) = 9x^2 - 12x - 12x + 16$
$= 9x^2 - 24x + 16$

b i $2xy - 12x^2y^3 = 2xy(1 - 6xy^2)$

ii $3x^2y - 9x^3y^2 + 15x^4y^3 = 3x^2y(1 - 3xy + 5x^2y^2)$

c i $a^m \times a^n = a^{m+n}$

$xy \times x^3y^5 = x^4y^6$

ii $a^m \div a^n = a^{m-n}$

$\frac{x^2y^7z^5}{xy^2z^3} = xy^5z^2$

3 a $37\,000\,000\,000 = 3.7 \times 10^{10}$

b $7.5 \times 10^{-5} = 0.000\,075$

c Standard form is $a \times 10^n$ where $1 \leqslant a < 10$, n is an integer.

$\frac{2.5 \times 10^{-3}}{1.25 \times 10^7} = 2 \times 10^{-10}$

4 ∠CBA = 73° (alternate segment theorem)

∠OBC = 34° (△OBC is isosceles)

∠OBA = 73° − 34° = 39°

5 Increase by 2%: multiplying factor = 1.02

Big Bank: €1200 × $(1.02)^3$ = €1273.45

Increase by 3%: multiplying factor= 1.03

Small Bank: €1273.45 × $(1.03)^2$ = €1351.00

percentage profit = $\frac{\text{change}}{\text{original}} \times 100$

Percentage profit = $\frac{1351 - 1200}{1200} \times 100 = 12.6\%$ (3 s.f.)

6

$3x + 4y = 5$	[1] × 5 ⇒ [3]
$2x - 5y = 11$	[2] × 4 ⇒ [4]
$15x + 20y = 25$	[3]
$8x - 20y = 44$	[4]
$23x = 69$	[3]+[4]
$x = 3$	⇒ [1]

[1] $9 + 4y = 5$, $y = -1 \Rightarrow (3, -1)$

7 $\mathbf{OR} = \mathbf{OP} + \mathbf{PR} = \mathbf{OP} + \frac{5}{2}\mathbf{PQ} = \mathbf{OP} + \frac{5}{2}(\mathbf{OQ} - \mathbf{OP})$

$= \frac{5}{2}\mathbf{OQ} - \frac{3}{2}\mathbf{OP} = \frac{1}{2}(5\mathbf{OQ} - 3\mathbf{OP})$

Set up the vector path equation first, and then substitute in the vector elements.

$= \frac{1}{2}[5(4\mathbf{a} + \mathbf{b}) - 3(2\mathbf{a} + 5\mathbf{b})]$

$= \frac{1}{2}[14\mathbf{a} - 10\mathbf{b}] = 7\mathbf{a} - 5\mathbf{b}$

8 a $P(T \cap C \cap G') = \frac{6}{80} = \frac{3}{40}$

b $P((T \cap G \cap C)/T) = \frac{11}{42}$

c $P(T/(G \cup C)) = \frac{6 + 11 + 13}{58} = \frac{30}{58} = \frac{15}{29}$

d $P(A/B) = \frac{P(A \cap B)}{P(B)}$

$P(G/C') = \frac{13 + 15}{50} = \frac{28}{50} = \frac{14}{25}$

9 a,b $t_n = a + (n - 1)d$ for an A.P.

$t_{10} = 39 = a + 9d$ [1]

$t_5 = 19 = a + 4d$ [2]

$20 = 5d$ [1] – [2]

$d = 4 \Rightarrow$ [1], $a = 3$

c $Sn = \frac{n}{2}(a + l)$ for an A.P.

$S_{10} = \frac{10}{2}(3 + 39) = 210$

10 a $y_{max} = \frac{\text{max value}}{\text{min value}}$

$y_{max} = \frac{8.3}{6.6 - 2.6} = 2.1$ (2 s.f.)

b $y_{min} = \frac{\text{min value}}{\text{max value}}$

$y_{min} = \frac{7.3}{7.6 - 1.6} = 1.2$ (2 s.f.)

11 a $g(x) = \frac{2}{3x - 1}$, $g(2) = \frac{2}{6 - 1} = \frac{2}{5}$

b $f^2(x) = ff(x)$; note that $f^2(x) \neq [f(x)]^2$

$g^2(x) = gg(x) = g(\frac{2}{5}) = \frac{2}{\frac{6}{5} - 1} = \frac{2}{\frac{1}{5}} = 10$

c $x = \frac{2}{3x - 1}$

$x(3x - 1) = 2$

$3x^2 - x - 2 = 0$

$(3x + 2)(x - 1) = 0$

$x = -\frac{2}{3}, 1$

d Let $y = g(x)$

Switch y and x for the inverse of $g(x)$.

$y = \frac{2}{3x - 1}$

Make y the subject.

$x = \frac{2}{(3y - 1)}$, $x(3y - 1) = 2$, $3xy - x = 2$

Use proper notation for inverse function.

$3xy = x + 2$, $y = \frac{x + 2}{3x}$, $g^{-1}(x) = \frac{x + 2}{3x}$

12 RHS regular polygon, exterior angle $e = \frac{360}{10} = 36°$

$e = \frac{360}{n}$, for an n-sided regular polygon.

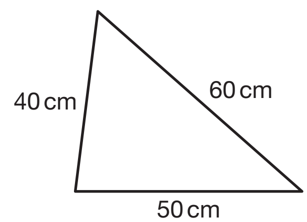
LHS regular polygon, $e = 60° - 36° = 24°$

$24° = \frac{360°}{n}$, $n = \frac{360}{24} = 15$ sides

13 Sum of parts = 4 + 5 + 6

150 cm $\Rightarrow$ 15 parts

1 part = 10 cm, so triangle sides are 40 cm, 50 cm and 60 cm.



Let angle between 40 and 50 sides be A:

cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

$\cos A = \frac{40^2 + 50^2 - 60^2}{2 \times 40 \times 50}$, $A = 82.819\ldots°$

Area of triangle $= \frac{1}{2} \times 40 \times 50 \times \sin(82.819°)$

area of triangle $= \frac{1}{2}ab\sin C$

$= 992.156\ldots\text{cm}^2 = 992\,\text{cm}^2$ (3 s.f.)

14 a arc length $= \frac{\theta}{360} \times 2\pi r$

Perimeter, $p = \frac{60 \times 3}{360} \times 2 \times \pi \times 4 = 12.6$ cm (1 d.p.)

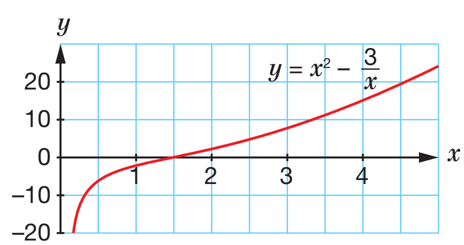
b area of triangle $= \frac{1}{2}ab\sin C$; area of sector $= \frac{\theta}{360} \times \pi r^2$

Area $= \frac{1}{2} \times 8 \times 8 \times \sin 60° - \frac{180}{360} \times \pi \times 4^2 = 2.6\,\text{cm}^2$ (1 d.p.)

15 a

x	0.5	1	1.5	2	3	4	5
y	–5.75	–2	0.25	2.5	8	15.25	24.4

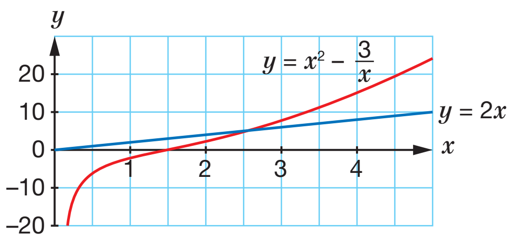
b



c Read off graph where curve cuts the x-axis.

$x \approx 1.4$

d $x^2 - 2x - \frac{3}{x} = 0$, $x^2 - \frac{3}{x} = 2x$, so draw $y = 2x$.



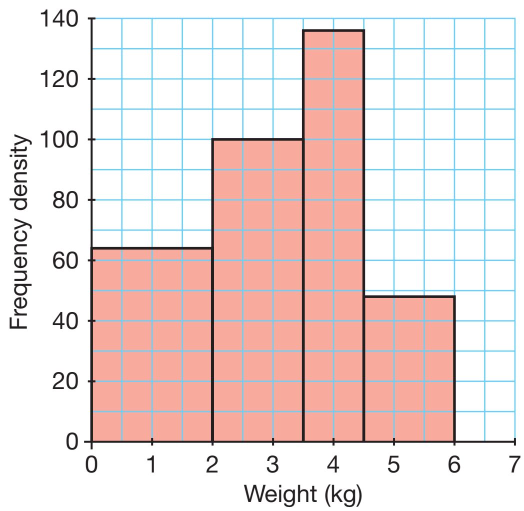
Read off x-axis where curve cuts $y = 2x$

$x \approx 2.5$

16 a frequency density = frequency ÷ class width

Weight (w kg)	Frequency	Frequency density
$0 < w \leq 2$	128	128 ÷ 2 = 64
$2 < w \leq 3.5$	150	150 ÷ 1.5 = 100
$3.5 < w \leq 4.5$	136	136 ÷ 1 = 136
$4.5 < w \leq 6$	72	72 ÷ 1.5 = 48

b



17 a Consider a right-angled triangle with hypotenuse AB = $4 + x$

Pythagoras' theorem

$$(x + 4)^2 = (5 - x)^2 + (6 - x)^2$$
$$x^2 + 8x + 16 = 25 - 10x + x^2 + 36 - 12x + x^2$$
$$0 = x^2 - 30x + 45, \text{ as required.}$$

b Use the quadratic formula

$a = 1$, $b = -30$, $c = 45$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-30) \pm \sqrt{(-30)^2 - 4(1)45}}{2(1)} = 1.58\ldots \text{ or } 28.4\ldots,$$

so $x = 1.58$ cm (3 s.f.)

18 a $v = \frac{dx}{dt} = 2t^2 - 9t + 4 = 0$ when stationary.

$(2t - 1)(t - 4) = 0$, $t = \frac{1}{2}$ or 4

b $a = \frac{dv}{dt} = 4t - 9$, at $t = 3$, $a = 12 - 9 = 3$

19 a $p(R) = \frac{3}{8} = \frac{x}{48}$, $x = 18$ red beads

b $p(R) = \frac{18 + r}{48 + r} = \frac{1}{2}$, $36 + 2r = 48 + r$, $r = 12$ red beads

20 a Let length of diagonal of square base be y

Pythagoras' theorem

$y^2 = x^2 + x^2 = 2x^2$, $y = \sqrt{2}\,x$, $\frac{1}{2}y = \frac{x}{\sqrt{2}}$

Consider right-angled triangle of height h and base $\frac{1}{\sqrt{2}}x$ and hypotenuse x.

$x^2 = h^2 + \frac{1}{2}x^2$, $h^2 = \frac{1}{2}x^2$, $h = \frac{x}{\sqrt{2}}$

b Let angle between an edge and the base be θ.

$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

$$\tan\theta = \frac{\frac{x}{\sqrt{2}}}{\frac{x}{\sqrt{2}}} = 1,\ \theta = 45°$$

21 Let A be the area of the field to be maximized.

Width of field $= 150 - 2x$

$A = x(150 - 2x) = 150x - 2x^2$

$\frac{dA}{dx} = 150 - 4x = 0$ at turning point,

solving $x = 37.5$, $y = 75$, $A_{max} = 2812.5$

Maximum value as shape of A vs x graph is an inverted parabola yielding a maximum point.

22 volume of a sphere $= \frac{4}{3}\pi r^3$; volume of cone $= \frac{1}{3}\pi r^2 h$

curved surface area of cone $= \pi r l$; slant height is l where $l^2 = r^2 + h^2$

volume of sphere = volume of cone

$\frac{4}{3}\pi r^3 = \frac{1}{3}\pi r^2 h$, so $4r = h$

Total surface area of cone, $A = \pi r^2 + \pi r l$

$l^2 = r^2 + h^2 = r^2 + 16r^2 = 17r^2$,

so $A = \pi r^2 + \pi r\sqrt{17}\,r = \pi r^2(1 + \sqrt{17})$