UNIT 6 ANSWERS

UNIT 6: NUMBER 6

EXERCISE 1

1 ▶ a Yes, each hotdog costs £1.80

b No, 5 apples are 32p each, 9 apples are 33p each

c No, Tom's speed is 13.33 km/hr, Ric's speed is 13.55 km/hr (4 s.f.)

2 ▶ a Yes, distance is $\frac{3}{4}$ of the time

b $s = \frac{3t}{4}$ **c** 18 km

3 ▶ 4 min 30 s (NOT in direct proportion)

4 ▶ Yes, speed ÷ time is a constant

5 ▶ a

t seconds	60	100	140	200	260
V cm³	300	500	700	1000	1300

b $V = 5t$ **c** 5 minutes

6 ▶ a 55 600 (3 s.f.) **b** 38.6 (3 s.f.)

7 ▶ a $468 **b** 7

8 ▶ a $97.50 **b** 2.3 m

EXERCISE 1*

1 ▶ 3 min 30 s (NOT in direct proportion)

2 ▶ a Yes, miles are $\frac{5}{8}$ of km or km is 1.6 of mile

b 1 : 1.6 **c** 62.5 miles

3 ▶ Extension at 4.5 N should be 20.25 mm

4 ▶ £300

5 ▶ a

Area, A cm²	25	30	40	70	100
Cost, $C	1125	1350	1800	3150	4500

b $C = 45A$ **c** 85 cm²

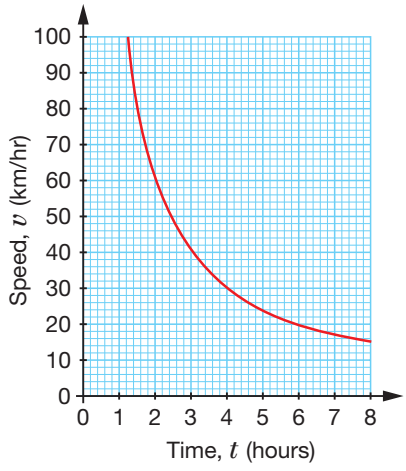
6 ▶ a 2.4 m **b** 13.3 m

7 ▶ a 360 g **b** 10

8 ▶ a 5.25 ohms **b** $R = 0.0035l$ **c** 2.2 m

ACTIVITY 1

Time (t hours)	2	3	4	5	6	8
Speed (v km/hr)	60	40	30	24	20	15
$t \times v$	120	120	120	120	120	120



Speed required is 48 km/hr
Yes, calculate 120 ÷ 2.5

EXERCISE 2

1 ▶ a Yes, product of time and speed is 600.

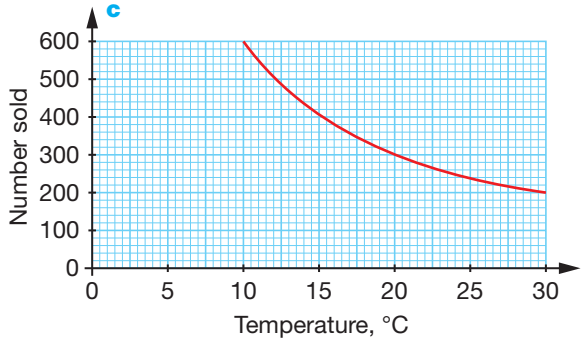
b 50 m/s

2 ▶ a

Temperature °C	10	16	20	25	30
Number sold	600	375	300	240	200

b $T \times N = 6000$

c



d The model predicts sales will increase without limit, which is unrealistic

3 ▶ a 20 years **b** 12 000 men

4 ▶ a 1.25 s **b** 40 Mb/s

c $s \times t = 20$

5 ▶ a 16 years **b** 500

6 ▶ a 4 days

b Yes, because 3 × 200 is the same as 300 × 2

EXERCISE 2*

1 ▶ a

A	3	4.8	5	6	32
B	16	10	9.6	8	1.5

b $A \times B = 48$

2 ▶ a 600 parts per million **b** 1 trillion

3 ▶ a 3 minutes

b No, if inversely proportional it should take 6 minutes

4 ▶ a 12 amps **b** $A \times R = 144$

c 24 ohms

5 ▶ a 24 days **b** 8

6 ▶ a 20 km/hr **b** 1 hour 58 mins

ACTIVITY 2

Will depend on the measured reaction time. An average reaction time is around 0.3 seconds ⇒ a speed of 208 km/hr

ACTIVITY 3

a i 2 **ii** 3 **iii** 4 **iv** 5 **v** 6

$a^{\frac{1}{2}}$ means the square root of a

b i 2 **ii** 3 **iii** 4 $a^{\frac{1}{3}}$ means the cube root of a

EXERCISE 3

1 ▶ 5 **2** ▶ 3 **3** ▶ 2 **4** ▶ $\frac{1}{2}$
5 ▶ 27 **6** ▶ 32 **7** ▶ $\frac{4}{5}$ **8** ▶ $\frac{8}{27}$
9 ▶ $\frac{9}{4}$ **10** ▶ 128 **11** ▶ 3 **12** ▶ 2
13 ▶ $x = \frac{1}{3}$ **14** ▶ $x = \frac{5}{2}$

EXERCISE 3*

1 ▶ 12 **2** ▶ 30 **3** ▶ $\frac{1}{2}$ **4** ▶ $\frac{1}{2}$
5 ▶ 27 **6** ▶ 25 **7** ▶ $\frac{8}{7}$ **8** ▶ $\frac{27}{64}$
9 ▶ $\frac{81}{16}$ **10** ▶ 4 **11** ▶ 5 **12** ▶ $\frac{125}{2}$
13 ▶ $x = \frac{3}{4}$ **14** ▶ $x = \frac{1}{10}$

EXERCISE 4

1 ▶ $\frac{1}{9}$ **2** ▶ $\frac{1}{8}$ **3** ▶ $\frac{1}{4}$ **4** ▶ 9
5 ▶ $\frac{1}{3}$ **6** ▶ 1 **7** ▶ 2 **8** ▶ $\frac{4}{3}$
9 ▶ 81 **10** ▶ $\frac{27}{8}$ **11** ▶ $\frac{9}{25}$ **12** ▶ $\frac{1}{2}$
13 ▶ $\frac{1}{5}$ **14** ▶ $\frac{1}{8}$ **15** ▶ 1 **16** ▶ 2
17 ▶ 4 **18** ▶ $\frac{3}{2}$ **19** ▶ 1 **20** ▶ 9
21 ▶ $x = -1$ **22** ▶ $x = -1$ **23** ▶ $x = -1$
24 ▶ $x = -2$ **25** ▶ $-\frac{1}{2}$

EXERCISE 4*

1 ▶ $\frac{1}{25}$ **2** ▶ $\frac{1}{64}$ **3** ▶ $\frac{1}{12}$ **4** ▶ 4
5 ▶ $\frac{49}{4}$ **6** ▶ $\frac{125}{64}$ **7** ▶ $\frac{9}{25}$ **8** ▶ $\frac{1}{8}$
9 ▶ $\frac{1}{2}$ **10** ▶ $\frac{1}{32}$ **11** ▶ 1 **12** ▶ 9
13 ▶ 25 **14** ▶ $\frac{16}{81}$ **15** ▶ $\frac{8}{27}$ **16** ▶ 4
17 ▶ $\frac{25}{49}$ **18** ▶ 1 **19** ▶ $\frac{5}{3}$ **20** ▶ $\frac{25}{8}$
21 ▶ $x = -2$ **22** ▶ $x = -\frac{1}{2}$ **23** ▶ $x = \frac{2}{3}$
24 ▶ $x = -9$ **25** ▶ $x = 4$

EXERCISE 5 REVISION

1 ▶ No, $B = 4A$ except in the last column

2 ▶ **a**

Depth, d metres	5	8	12	25	40
Pressure, P bars	0.5	0.8	1.2	2.5	4

b $P = 0.1d$
c Yes, pressure is 7.5 bars < 8.5 bars

3 ▶ **a** 152.5 kg
b 8 m

4 ▶ **a**

Number of workers, w	4	8	6	2
Number of days, d	12	6	8	24

b $dw = 48$ **c** 48

5 ▶ **a** 32 hours **b** 60 °C

6 ▶ **a** 6 days **b** 5 harvesters

7 ▶ 6 **8** ▶ 2 **9** ▶ 3 **10** ▶ $\frac{1}{3}$
11 ▶ 100 **12** ▶ 64 **13** ▶ 1 **14** ▶ $\frac{4}{5}$
15 ▶ $\frac{1}{64}$ **16** ▶ 8 **17** ▶ $\frac{9}{4}$ **18** ▶ $\frac{1}{5}$
19 ▶ $\frac{1}{9}$ **20** ▶ 8 **21** ▶ 8 **22** ▶ $\frac{1}{4}$
23 ▶ 2 **24** ▶ 3 **25** ▶ 1 **26** ▶ $x = 1$
27 ▶ $x = -1$ **28** ▶ $x = -\frac{1}{2}$

EXERCISE 5* REVISION

1 ▶ **a**

Force, F N	1.8	2.52	4.32	10.8	18
Acceleration, a m/s²	0.5	0.7	1.2	3	5

b $F = 3.6a$ **c** 25 m/s²

2 ▶ **a** 6 s **b** 162 MB

3 ▶ Yes, $XY = 144$ in all cases.

4 ▶ **a**

Number of pipes, n	1	4	6	8	10
Time, t hrs	18	4.5	3	2.25	1.8

b $nt = 18$ **c** 12

5 ▶ **a** 126 160 **b** 50 cm²

6 ▶ **a** 40 sides **b** $\frac{1}{2}$

7 ▶ 11 **8** ▶ −5 **9** ▶ 4 **10** ▶ 1000
11 ▶ 16 **12** ▶ $\frac{3}{4}$ **13** ▶ 1 **14** ▶ 1
15 ▶ $\frac{25}{16}$ **16** ▶ $\frac{1}{32}$ **17** ▶ $\frac{1}{125}$ **18** ▶ 256
19 ▶ $\frac{1}{3}$ **20** ▶ $-\frac{1}{2}$ **21** ▶ $\frac{1}{81}$ **22** ▶ −3
23 ▶ 27 **24** ▶ $\frac{125}{8}$ **25** ▶ $\frac{3}{16}$ **26** ▶ $x = -2$
27 ▶ $x = -\frac{1}{3}$ **28** ▶ $x = -6$

EXAM PRACTICE: NUMBER 6

1 ▶ **a**

Volume, v litres	7.5	15	30	45
Cost, £C	8.5	17	34	51

b $C = \frac{17}{15}v$

2 ▶ **a** 3 pages **b** 8

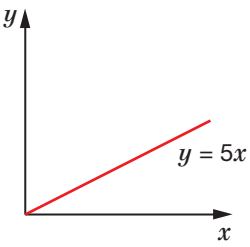
3 ▶ **a** 8 **b** 4 **c** 8 **d** $\frac{2}{3}$
e 27 **f** 8 **g** $\frac{3}{2}$ **h** $\frac{1}{81}$
i 3 **j** $\frac{8}{27}$ **k** $\frac{1}{3}$ **l** $\frac{1}{4}$
m 6 **n** $\frac{8}{27}$ **o** 2 **p** 4
q $x = -\frac{1}{3}$

UNIT 6: ALGEBRA 6

ACTIVITY 1 Y Y Y N Y Y N N

EXERCISE 1

1 ▶ **a** $y = 5x$ **b** 30 **c** 5

d 

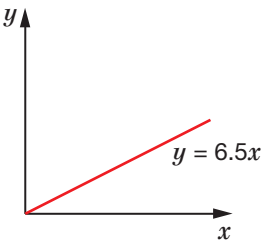
2 ▶ **a** $d = 4t$ **b** 60 **c** 45

3 ▶ **a** $y = 5x$ **b** $y = 50$

c $x = 13$

4 ▶ **a** $y = 6.5x$

b $y = 91$ **c** $x = 22$

d 

5 ▶ **a** $x = 5$ **b** $x = 10.1$ (1 d.p.)

c $x = 8.125$

6 ▶ **a** $y = \frac{x}{60}$ **b** $y = 9$

7 ▶ **a** $y = 2x$ **b** 10 cm **c** 7.5 kg

8 ▶ **a** $e = \frac{M}{20}$ **b** 5 m **c** 120 kg

9 ▶ **a** $l = 75t$ **b** 1950 sales

10 ▶ **a** $N = 7t$

b Yes, as 210 people would turn up to swim

EXERCISE 1*

1 ▶ **a** $v = 9.8t$ **b** 49 m/s **c** 2.5 s

2 ▶ **a** $c = \frac{m}{3}$ **b** \$2.50 **c** 600 g

3 ▶ **a** $d = 150$ m **b** 1500 m **c** 266.7 g

4 ▶ **a** $m = 6.5n$ **b** 975 g

c 1540 approx

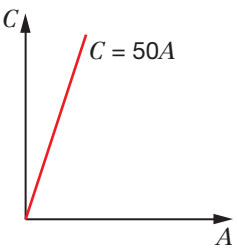
5 ▶ **a** $h = \frac{3y}{2}$ **b** 0.75 m **c** 4 months

6 ▶ **a** $d = 500t$ **b** $d = 2500$ km

c $t = 4.5$ hours

d **i** The distance doubles.

ii The distance halves.

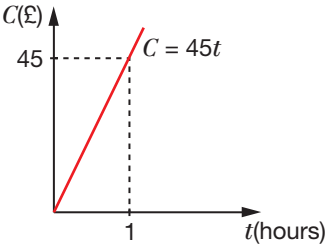
7 ▶ **a** 

b $C = 50A$ **c** £4250

8 ▶ **a** $y = \frac{23x}{3}$ **b** 184 **c** 21

9 ▶ **a** $d = 10.5t$ **b** 84 km **c** 0.73 hours

d **i** It is trebled. **ii** It is divided by 3.

10 ▶ **a** 

b $C = 45T$ **c** 23 hours

EXERCISE 2

1 ▶ **a** $y = 4x^2$ **b** 144 **c** 4

2 ▶ **a** $p = 2q^2$ **b** 18 **c** 7

3 ▶ **a** $v = 2w^3$ **b** 54 **c** 4

4 ▶ **a** $m = 10\sqrt{n}$ **b** 20 **c** 25

5 ▶ **a** $y = 5t^2$

b 45 m

c $\sqrt{20}$ s $\simeq 4.47$ s

6 ▶ **a** $P = \frac{h^3}{20}$ **b** \$86.40 **c** 8 cm

7 ▶ **a** $E = 5s^2$ **b** $E = 20$ J **c** $s = 6.2$ m/s

d The kinetic energy, E, is multiplied by 4.

8 ▶ **a** $C = 0.05s^3$ **b** $C =$ £6.25

EXERCISE 2*

1 ▶

g	2	4	6
f	12	48	108

2 ▶

n	1	2	5
m	4	32	500

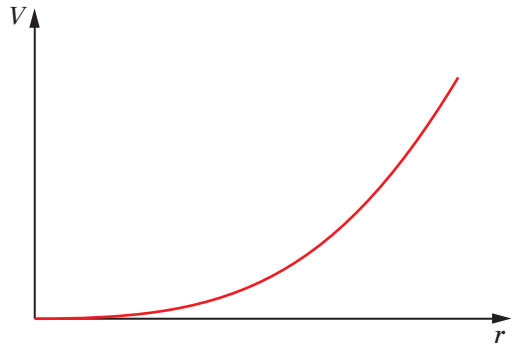
3 ▶ **a** $T = \frac{R^2}{450}$ **b** $T = 50$ minutes

4 ▶ **a** $d = 4.9t^2$ **b** 490 m **c** 15 s

d The distance moved is multiplied by 4.

5 ▶ **a** $V = 4.188r^3$ **b** 33 504 cm^3

c



6 ▶ **a** $R = \left(\frac{5}{256}\right)s^2$

b 113 km/h

7 ▶ **a** $H = 1.5\sqrt[3]{y}$

b 512 years old

8 ▶ $x = 10\sqrt{2}$

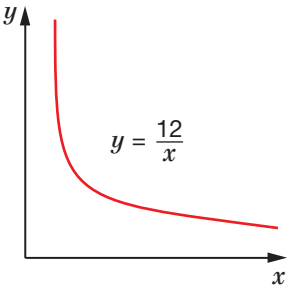
ACTIVITY 2 $t^2 \approx 3.95 \times 10^{-20} d^3$

Planet	d (million km)	t (Earth days) (2 s.f.)	t^2/d^3
Mercury	57.9	88	0.04
Jupiter	778	4300	0.04
Venus	108	220	0.04
Mars	228	680	0.04
Saturn	1430	11 000	0.04
Uranus	2870	31 000	0.04
Neptune	4500	60 000	0.04

EXERCISE 3

1 ▶ **a** $y = \frac{12}{x}$ **b** $y = 6$ **c** $x = 4$

d



2 ▶ **a** $d = \frac{250}{t}$ **b** $d = 125$ **c** $t = 5$

3 ▶ **a** $P = \frac{3000}{V}$ **b** $P = 2000$ N/m^2

c $V = 2.5$ m^3 **d** The volume is halved.

4 ▶ **a** $t = \frac{600\,000}{p}$

b No, it takes 4 minutes. When $p = 2500$ W, $t = 240$ seconds.

5 ▶ **a** $m = \frac{36}{n^2}$ **b** $m = 9$ **c** $n = 6$

6 ▶ **a** $V = \frac{100}{w^3}$ **b** $V = 100$ **c** $w = 5$

7 ▶ **a** $I = 4 \times \frac{10^5}{d^2}$ **b** 0.1 candle power

8 ▶ **a** $L = \frac{1}{4d^2}$ **b** 25 days

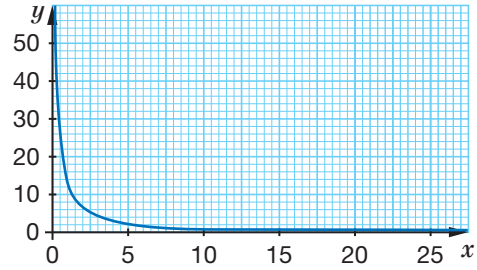
EXERCISE 3*

1 ▶ **a** $C = \frac{5000}{t}$ **b** \$277.78 **c** 12.5 °C

2 ▶ **a** $V = \frac{750}{t}$

b No; the balloon pops when $V = 25\,000$ cm^3

3 ▶ **a** Graph showing inverse proportion.



b 12 **c** It is always 12.

4 ▶

b	2	5	10
a	50	8	2

5 ▶ **a** $r \propto t$, in fact $r = \frac{20}{t}$

b

r	1	4	5	10
t	20	5	4	2

6 ▶ **a** $R = \frac{2}{r^2}$ **b** $\frac{2}{9}$ ohm

7 ▶ **a**

Day	N	T
Mon	400	25
Tues	447	20
Wed	500	16

b 407 approx.

8 ▶ 4.5 s

ACTIVITY 3

	Rabbit	Dog	Man	Horse
Pulse (beats/min)	165	135	83	65
Mass (kg)	5	12	70	200

Total heartbeats for a human life-span of 75 years $\simeq 3.27 \times 10^9$

According to theory:

	Rabbit	Dog	Man	Horse
Life-span (years)	37.7	46.1	75	95.8

Theory clearly not correct.

EXERCISE 4

1 ▶ a **2** ▶ b^2 **3** ▶ $\frac{1}{c^3}$

4 ▶ d^5 **5** ▶ e **6** ▶ f

7 ▶ $\frac{1}{a^2}$ **8** ▶ $\frac{1}{g^2}$ **9** ▶ a^2

10 ▶ $\frac{1}{b}$ **11** ▶ c^3 **12** ▶ d

13 ▶ e^2 **14** ▶ $\frac{1}{f}$

15 ▶ **a** $2^3 \div 2^3 = 2^0$ **b** 8

c $2^3 \div 2^3 = 8 \div 8 = 1$

d $2^3 \div 2^3 = 2^0 = 1$

e $7^5 \div 7^5 = 7^0 = 16\,807 \div 16\,807 = 1$

f $a^0 = 1$

16 ▶ **a** $\frac{1}{3}$ **b** $\frac{1}{16}$ **c** $\frac{1}{100\,000}$
d $\frac{4}{3}$ **e** $\frac{125}{64}$ **f** $\frac{4}{5}$
g $\frac{16}{121}$ **h** $\frac{10}{7}$ **i** 100 000
j $\frac{125}{8}$ **k** $5^0 = 1$ **l** $7^1 = 7$

EXERCISE 4*

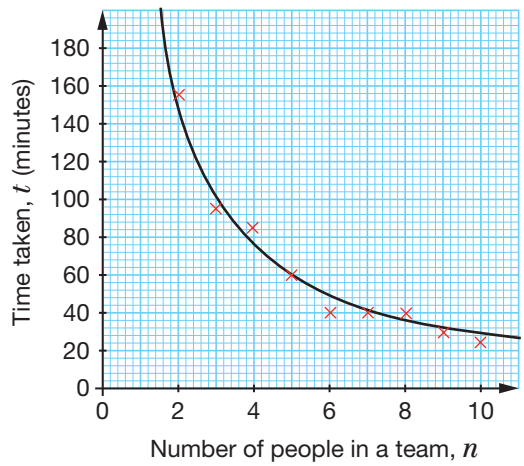
1 ▶ a^2 **2** ▶ b^3 **3** ▶ $\frac{2}{c^4}$
4 ▶ $\frac{4}{c^2}$ **5** ▶ $\frac{12}{a}$ **6** ▶ $\frac{8}{b}$
7 ▶ a **8** ▶ 1 **9** ▶ $\frac{1}{c}$
10 ▶ $\frac{1}{d}$ **11** ▶ $-9a^6$ **12** ▶ $\frac{1}{a^2}$
13 ▶ $\frac{1}{c^2}$ **14** ▶ e **15** ▶ $k = 2\frac{1}{3}$
16 ▶ $k = -6$ **17** ▶ $x = 3, y = -2$
18 ▶ $x = \frac{4}{5}, y = -\frac{3}{5}$
19 ▶ **a** 9 **b** $\frac{125}{12}$ **c** $\frac{125}{8}$
20 ▶ **a** 4 **b** $\frac{1}{3}$ **c** -2
d $-\frac{1}{2}$ **e** $\frac{7}{2}$ **f** $\frac{7}{4}$

EXERCISE 5 REVISION

1 ▶ **a** $y = 6x$ **b** $y = 42$ **c** $x = 11$
2 ▶ **a** $p = 5q^2$ **b** $p = 500$ **c** $q = 11$
3 ▶ **a** $c = \frac{3}{4}a^2$ **b** \$675 **c** 28.3 m²
4 ▶

n	1	2	4	8
t	80	40	20	10

5 ▶ $g \propto h^3$
6 ▶ $a = 4, b = 2$
7 ▶ **a**



b Answers close to $t = \frac{300}{n}$
c Answer using students' formulae from part **b**
$t = \frac{300}{n}$ gives t = 20 minutes.

8 ▶ **a** Graph B **b** Graph D
c Graph A **d** Graph C
9 ▶ a^2 **10** ▶ a^4 **11** ▶ $\frac{1}{d^2}$
12 ▶ $\frac{1}{b^{\frac{3}{2}}}$
13 ▶ c
14 ▶ **a** 10 **b** 2 **c** $\frac{2}{3}$ **d** $\frac{5}{7}$
e -3 **f** $-\frac{1}{3}$ **g** $-\frac{2}{3}$ **h** $\frac{1}{10}$
15 ▶ 1

EXERCISE 5* REVISION

1 ▶ **a** $y^2 = 50z^3$ **b** 56.6 **c** 5.85
2 ▶ 1500 m
3 ▶ **a** $m = \frac{8839}{\sqrt{n}}$ **b** 3.23×10^5
c 7.81×10^{-5}
4 ▶

x	0.25	1	4	25
y	20	10	5	2

$y = \frac{10}{\sqrt{x}}$

5 ▶ $w \propto \frac{1}{t^2}$ (rule B)
6 ▶ **a** $D = \frac{6390}{r^2}$
b D = 10.2 cm (1 d.p.)
c r = 10.0 cm (1 d.p.) **d** $\frac{1}{4}d$ cm
7 ▶ **a** $F = \frac{1.0584 \times 10^{17}}{d^2}$ **b** 4 N
8 ▶

p	2	4	6
w	7	$1\frac{3}{4}$	$\frac{7}{9}$

9 ▶ a^2 **10** ▶ $\frac{3}{c^2}$ **11** ▶ $\frac{2}{a}$
12 ▶ d **13** ▶ $-9a^6$ **14** ▶ 2
15 ▶ 1 **16** ▶ $5x^2y^3$

EXAM PRACTICE: ALGEBRA 6

1 ▶ **a** $y = \frac{x}{2}$ **b** 2.5 **c** 1
2 ▶ **a** $p = \frac{1}{5}q^2$ **b** 80 **c** 30
3 ▶ **a** 9000 **b** $N = \frac{9000}{d^2}$
c 2250 **d** 3 mm
4 ▶ **a** $D = \frac{9540}{r^2}$ **b** 42.4 cm
c 12.6 cm **d** $\frac{x}{\sqrt{3}}$
5 ▶ **a** $2a^2$ **b** b^{-2} **c** c^4 **d** $x = -\frac{1}{2}$

UNIT 6: SEQUENCES

ACTIVITY 1 Seema; 13 rows with 9 balloons left over

EXERCISE 1

1 ▶ 2, 4, 6, 8
2 ▶ −9, −6, −3, 0
3 ▶ 15, 10, 5, 0
4 ▶ 2, 4, 8, 16
5 ▶ 12, 6, 3, 1.5
6 ▶ Add 4; 19, 23, 27
7 ▶ Subtract 5; −7, −12, −17
8 ▶ Double; 48, 96, 192
9 ▶ Halve; 4, 2, 1
10 ▶ Add 0.3; 1.4, 1.7, 2

EXERCISE 1*

1 ▶ −1, 0.5, 2, 3.5
2 ▶ 3, 1.75, 0.5, −0.75
3 ▶ 1, 2.5, 6.25, 15.625
4 ▶ 3, −1, $\frac{1}{3}$, $-\frac{1}{9}$
5 ▶ 2, 3, 5, 8
6 ▶ Add $2\frac{1}{2}$; 13, $15\frac{1}{2}$, 18
7 ▶ Divide by 3; 3, 1, $\frac{1}{3}$
8 ▶ Square; 65 536, 4.3×10^9, 1.8×10^{19}
9 ▶ Multiply by $-\frac{1}{2}$; $\frac{1}{16}$, $-\frac{1}{32}$, $\frac{1}{64}$
10 ▶ Double then add 1; 63, 127, 255

EXERCISE 2

1 ▶ 3, 5, 7, 9
2 ▶ 4, 9, 14, 19
3 ▶ 30, 27, 24, 21
4 ▶ 2, 5, 10, 17
5 ▶ 3, 6, 9, 12
6 ▶ 2, $\frac{3}{2}$, $\frac{4}{3}$, $\frac{5}{4}$
7 ▶ 8
8 ▶ 7
9 ▶ 7
10 ▶ 22

EXERCISE 2*

1 ▶ −1, 4, 9, 14
2 ▶ 97, 94, 91, 88
3 ▶ 1, $\frac{3}{2}$, 2, $\frac{5}{2}$
4 ▶ 3, 7, 13, 21
5 ▶ 3, 6, 11, 18
6 ▶ 3, $\frac{5}{3}$, $\frac{7}{5}$, $\frac{9}{7}$
7 ▶ 8
8 ▶ 12
9 ▶ 10
10 ▶ 51

ACTIVITY 2

a gives the difference between the terms of the sequence

n	1	2	3	4	10
$3n + 2$	5	8	11	14	32

3 = difference between the terms of the sequence

3 + 2 gives the first term of the sequence

a gives the difference between the terms of the sequence. $a + b$ gives the first term of the sequence

EXERCISE 3

1 ▶ 17, 20, 23
2 ▶ −7, −10, −13
3 ▶ 11.5, 13, 14.5
4 ▶ 56, 76, 99
5 ▶ 0, 8, 19
6 ▶ −4, −11, −20

EXERCISE 3*

1 ▶ 58, 78, 101
2 ▶ 1, −6, −15
3 ▶ 1, 8, 17
4 ▶ 35.5, 46.5, 59
5 ▶ 71, 101, 139
6 ▶ −6, −19, −38

ACTIVITY 3

74, 100, 130; 150, 215, 297; −54, −90, −139

ACTIVITY 4

3, 5, 7, 9, 11, 13

$b = 2t + 1$; 201 balloons

$b = 3s + 1$ (s = no. of squares); $b = 5h + 1$ (h = no. of hexagons); $b = 7h + 5$ (h = no. of hexagons)

ACTIVITY 5

4, 8, 12, 16, 20, 24; $4n$

EXERCISE 4

1 ▶ $\frac{1}{n}$
2 ▶ $\frac{1}{2n - 2}$
3 ▶ $3n + 1$
4 ▶ $34 - 4n$
5 ▶ **a** 1, 3, 5, 7, 9, 11 **b** $c = 2l - 1$
c c is always odd, 50 layers.
6 ▶ **a** 8, 10, 12, 14, 16, 18
b $s = 2n + 6$ **c** 47th

EXERCISE 4*

1 ▶ $\frac{n + 1}{n}$
2 ▶ $\frac{2n - 1}{2n + 1}$
3 ▶ $4n - 1$
4 ▶ $9 - 3n$
5 ▶ **a** 6, 10, 14, 18, 22, 26
b $s = 4n + 2$ **c** 202
6 ▶ **a** 10, 16, 22, 28, 34, 40
b $s = 6n + 4$ **c** 32nd

EXERCISE 5

1 ▶ **a** 3, 6, 9, 12; 300
b 8, 14, 20, 26; 602
c 3, 10, 17, 24; 696
d 18, 15, 12, 9; −279
e 4, 0, −4, −8; −392
2 ▶ **a** $5n + 2$ **b** $4n - 2$ **c** $21 - 2n$
3 ▶ **a** $4n - 3 = 101 \Rightarrow n = 26$, yes
b $5n - 1 = 168 \Rightarrow n = 33.8$, no
c $45 - 5n = -20 \Rightarrow n = 13$, yes
4 ▶ **a** 108 **b** 103 **c** 103 **d** 106
5 ▶ 119 **6** ▶ 6 **7** ▶ 12 **8** ▶ 24
9 ▶ $a = 3$, $d = 5$ **10** ▶ $a = 20$, $d = -2$
11 ▶ 11 weeks **12** ▶ $10.10

EXERCISE 5*

1 ▶ **a** 2, 11, 20; 893
b −2, −11, −20; −893
c −27, −24, −21; 270
d 2.5, 3, 3.5; 52
e 0.25, 0, −0.25; −24.5
2 ▶ **a** $13 - 3n = -230 \Rightarrow n = 81$, yes
b $2.5 + 1.5n = 99.5 \Rightarrow n = 64\frac{2}{3}$, no
c $5n - 36 = 285 \Rightarrow n = 64.2$, no
3 ▶ **a** 1004 **b** 1009 **c** $1000\frac{1}{3}$
4 ▶ $a = 5$, $d = 3$

5 ▶ –92 **6 ▶** $8 + 7n$

7 ▶ 123 **8 ▶** $-10 - 4n$

9 ▶ 4 **10 ▶** 6700 days or 18 years

11 ▶ 1001 days **12 ▶** 18 weeks

ACTIVITY 6

$\frac{n(n+1)}{2}$

EXERCISE 6

1 ▶ 500 500 **2 ▶** 3320 **3 ▶** 10 100

4 ▶ 40 000 **5 ▶** 920 **6 ▶** 9960

7 ▶ 2385 **8 ▶** 750 **9 ▶** 770

10 ▶ 78 **11 ▶** $2550

12 ▶ a $6n$ **b** 1261

EXERCISE 6*

1 ▶ 4425 **2 ▶** –16 600 **3 ▶** 2420

4 ▶ 6375 **5 ▶** 140 500 **6 ▶** 0

7 ▶ $n^2 + 4n$ **8 ▶** 6640 **9 ▶** 1504

10 ▶ 25 **11 ▶** $\frac{7}{16}$, $1\frac{9}{16}$ hekats

12 ▶ a 1150 **b** $33 120

ACTIVITY 7

Number of square	1	2	3	4	5
Number of grains	$1 = 2^0$	$2 = 2^1$	$4 = 2^2$	$8 = 2^3$	$16 = 2^4$
Total number	1	3	7	15	$31 = 2^5 - 1$

Number of square	6	7	64
Number of grains	$32 = 2^5$	$64 = 2^6$	2^{63}
Total number	$63 = 2^6 - 1$	$127 = 2^7 - 1$	$2^{64} - 1$

$2^{64} - 1 \approx 1.84 \times 10^{19}$

4.43×10^{20} mm^3 or 443 km^3

2.77×10^{17} mm or 2.77×10^{11} km

1800:1

EXERCISE 7 REVISION

1 ▶ a 32, 44, 58 **b** 40, 40, 38

2 ▶ a 1800 m **b** $800 + 200n$

c 36 days

3 ▶ a Sequence 1; 3072 **b** $4n - 1$

4 ▶ a Odd numbers **b** 4, 9, 16, 25

c 121 **d** n^2 **e** 29

5 ▶ $a = -2$, $d = 3$ **6 ▶** $a = 27$, $d = -4$

7 ▶ 203 **8 ▶** 101

9 ▶ 45 days **10 ▶** 1072

11 ▶ 1704 **12 ▶ a** $4n - 3$ **b** 9730

EXERCISE 7* REVISION

1 ▶ a 20, 28, 30 **b** 10, 3, –7

2 ▶ a £2.90 **b** $50 + 20n$ **c** 12 years

3 ▶ a 17, 21, 25, 29

b 7, 13 **c** 1, 3, 7, 13, …; 21st term

4 ▶ a 1, 8, 21, 40, 65

b $s = 3n^2 - 2n$ **c** 15th

5 ▶ $a = -8$, $d = 4$ **6 ▶** $a = 10$, $d = -2.5$

7 ▶ $35 + 17n$ **8 ▶** 1232

9 ▶ $72.50, $87.50, … $177.50

10 ▶ $n(2n + 7)$ **11 ▶** 8 days **12 ▶** 270 m

EXAM PRACTICE: SEQUENCES

1 ▶ a 80, 76, 72

b 3, 10, 21 **c** $0, \frac{1}{3}, \frac{1}{2}$

2 ▶ a –9, –13, –17

b 52, 68, 86 **c** 45, 60, 77

3 ▶ $a = 2$, $d = 6$ **4 ▶** 650

5 ▶ 2 **6 ▶** 741

UNIT 6: SHAPE AND SPACE 6

EXERCISE 1

1 ▶ 100° (Angle at centre of circle is twice angle at circumference.)

2 ▶ 30° (Angle at centre of circle is twice angle at circumference.)

3 ▶ ∠CAB = 105° (Angle at centre of circle is twice angle at circumference.)

∠ABO = 45° (Angles in a quadrilateral sum to 360°.)

4 ▶ ∠BAO + ∠CAO = 40° (Angle at centre of circle is twice angle at circumference.)

∠OAC = 30° (Base angles in an isosceles triangle are equal.)

∠BAO = 40 – 30 = 10°

∠ABO = 10° (Base angles in an isosceles triangle are equal.)

5 ▶ ∠ACB = 40° (Angles on a straight line sum to 180°.)

∠AOB = 80° (Angle at centre of circle is twice angle at circumference.)

Reflex ∠AOB = 280° (Angles at a point sum to 360°.)

6 ▶ ∠ABC = 110° (Angles on a straight line sum to 180°.)

Reflex ∠AOC = 220° (Angle at centre of circle is twice angle at circumference.)

∠AOC = 140° (Angles at a point sum to 360°.)

7 ▶ ∠CBD = 30° (Angles in the same segment are equal.)

∠ACB = 60° (Angles in a triangle sum to 180°.)

8 ▶ ∠ACB = 70° (Angles on a straight line sum to 180°.)
∠AEB = 70° (Angles in the same segment are equal.)
∠BED = 110° (Angles on a straight line sum to 180°.)

9 ▶ ∠WYX = 40° (Angles in the same segment are equal.)
∠XYZ = 100° (Opposite angles of a cyclic quadrilateral sum to 180°.)
∠WYZ = 100 – 40 = 60°

10 ▶ ∠ZWY = 24° (Angles in the same segment are equal.)
∠ZWX = 44 + 24 = 68°
∠ZYX = 112° (Opposite angles of a cyclic quadrilateral sum to 180°.)

11 ▶ ∠OAB = ∠OBA = 35° (Base angles of an isosceles triangle are equal.)
∠AOB = 110° (Angles in a triangle sum to 180°.)
∠COB = 180 – 110 = 70° (Angles on a straight line sum to 180°.)
∠OBC = 90° (Angle between the tangent and a radius is 90°.)
x = 180 – 90 – 70 = 20° (Angles in a triangle sum to 180°.)

12 ▶ ∠CBO = ∠CDO = 90° (Angle between the tangent and a radius is 90°.)
∠BOD = 112° (Angle at the centre is twice the angle at the circumference.)
g = 360 – (112 + 90 + 90) = 68° (Angles in a quadrilateral sum to 360°.)

13 ▶ **a** Students' explanations may vary, e.g.
∠GHO = x, ∠HGO = x (Base angles in an isosceles triangle are equal)
∠HOG = 180 – 2x (Angles in a triangle total 180°)
∠FGH = 90° (Angle in a semicircle is 90°)
∠FGO = ∠GFO = 90 – x (Base angles in an isosceles triangle are equal)
∠GOF = 180 – (180 – 2x) = 2x (Angle at the centre is twice the angle at the circumference)

b ∠GFO = 49°, ∠HGO = 41°

14 ▶ a = 180 – 108 = 72° (Angles on a straight line sum to 180°)
b = 180 – 72 = 108° (Opposite angles in a cyclic quadrilateral sum to 180°)
c = 180 – 87 = 93° (Opposite angles in a cyclic quadrilateral sum to 180°)

15 ▶ ∠ADB and ∠BCA are angles in the same segment

16 ▶ ∠ADC + ∠ABC sum to 180°

EXERCISE 1*

1 ▶ ∠AOC = 360 – 220 = 140° (Angle at centre of circle is twice angle at circumference.)

2 ▶ Let ∠BAO = x, extend AB to P and OC to Q
∠PBC = x (Corresponding angles are equal.)
∠BCO = x (Alternate angles are equal.)
∠ABC = 180 – x (Angles on a straight line sum to 180°.)
Reflex ∠AOC = 360 – 2x (Opposite angles of a cyclic quadrilateral sum to 180°.)
∠AOC = 2x (Angles at a point sum to 360°.)
$x + x$ + 180 – x + 2x = 360 (Angles in a quadrilateral sum to 360°.)
x = 60°

3 ▶ Reflex ∠AOC = 230° (Angles at a point sum to 360°.)
∠ABC = 115° (Angle at centre of circle is twice angle at circumference.)

4 ▶ ∠AOC = 360 – reflex ∠AOC (Angles at a point sum to 360°.)
Reflex ∠AOC = 2 × ∠ABC (Angle at centre of circle is twice angle at circumference.)
70 + 40 + ∠ABC + ∠AOC = 360 (Angles in a quadrilateral sum to 360°.)
110 + ∠ABC + (360 – 2 × ∠ABC) = 360
∠ABC = 110°

5 ▶ ∠VRS = ∠RVS = 54° (Base angles in an isosceles triangle are equal.)
∠TUV = 54° (Angles in the same segment are equal.)

6 ▶ ∠TRS = 52° (Alternate angles are equal.)
∠TUS = 52° (Angles in the same segment are equal.)
∠UVT = 76° (Angles in a triangle sum to 180°.)

7 ▶ ∠PZW = 119° (Angles in a triangle sum to 180°.)
∠WZY = 61° (Angles on a straight line sum to 180°.)
∠WXY = 61° (Angles in the same segment are equal.)
∠PXY = 119° (Angles on a straight line sum to 180°.)

8 ▶ ∠XWY = 90° (Angle in a semicircle is 90°.)
∠XWZ = 90 – 68 = 22°
∠XYZ = 22° (Angles in the same segment are equal.)

9 ▶ ∠OCB = x (Base angles in an isosceles triangle are equal.)
∠OBC = 180 – 2x (Angles in a triangle sum to 180°.)

$\angle OBA = 2x$ (Angles on a straight line sum to 180°.)

$\angle OAB = 2x$ (Base angles in an isosceles triangle are equal.)

$\angle BOA = 180 - 4x$ (Angles in a triangle sum to 180°.)

$\angle XOA = 180 - x - (180 - 4x) = 3x$ (Angles on a straight line sum to 180°.)

10 ▶ $\angle ABC = x$ (Angle at centre of circle is twice angle at circumference.)

$\angle BCD = 180 - 90 - x = 90° - x$ (Angles in a triangle sum to 180°.)

11 ▶ $\angle ADB = x°$ (Base angles in an isosceles triangle are equal.)

$\angle BDC = (180 - 4x)°$ (Base angles in an isosceles triangle are equal. Angles in a triangle total 180°).

Therefore $\angle ADC = (180 - 3x)°$

Therefore $\angle ADC + \angle ABC = 180°$ so quadrilateral is cyclic.

12 ▶ $\angle AFE + \angle EDA = 180°$ (opposite angles of a cyclic quadrilateral total 180°)

Similarly, $\angle ABC + \angle CDA = 180°$

But $\angle CDA + \angle EDA = \angle CDE$.

Therefore $\angle ABC + \angle CDE + \angle EFA = 360°$

13 ▶ $\angle BEC = \angle CDB$ (angles in the same segment are equal)
Therefore $\angle CEA = \angle BDA$

14 ▶ Join AO. $\angle OAX = 90°$ (Angle in a semicircle is 90°)

Therefore $\angle OAY = 90°$ (Angles on a straight line sum to 180°)

OX = OY (radii) and OA is common

Therefore triangles OYA and OXA are congruent (RHS) and AX = AY

15 ▶ Let $\angle QYZ = y°$ and $\angle YZQ = x°$.

So $\angle ZWX = y°$ and $\angle PWZ = 180 - y°$

In $\triangle YQZ$: $x + y + 20 = 180$

Therefore $x + y = 160$ (1)

In $\triangle PWZ$: $180 - y + x + 30 = 180$

Therefore $y - x = 30$ (2)

From (1) and (2) $x = 65$ and $y = 95$

Therefore angles of the quadrilateral are 65°, 85°, 95°, 115°

16 ▶ Draw OP and OQ. Let $\angle POQ = 2x°$.

Therefore $\angle PRQ = x°$ and $\angle XOQ = (180 - x)°$

Therefore $\angle XRQ + \angle XOQ = 180°$ and RXOQ is a cyclic quadrilteral

ACTIVITY 1

Circle	∠ECB	∠OCB	∠OBC	∠BOC	∠BAC
C_1	60°	30°	30°	120°	60°
C_2	$x°$	$(90 - x)°$	$(90 - x)°$	$2x°$	$x°$

$\angle ECB = \angle BAC$

EXERCISE 2

1 ▶ $\angle TRS = 40°$ (Alternate segment theorem)

$\angle RTS = 70°$ (Angles in a triangle sum to 180°.)

2 ▶ $\angle TPQ = 30°$ (Angles in a triangle sum to 180°.)

$\angle QTB = 30°$ (Alternate segment theorem)

3 ▶ $\angle CBT = 40°$ (Base angles in an isosceles triangle are equal.)

$\angle BCT = 100°$ (Angles in a triangle sum to 180°.)

$\angle DCT = 80°$ (Angles on a straight line sum to 180°.)

$\angle DTA = 80°$ (Alternate segment theorem)

4 ▶ $\angle YZT = 50°$ (Base angles in an isosceles triangle are equal.)

$\angle YTX = 50°$ (Alternate segment theorem)

5 ▶ $\angle T_1T_2M = 75°$ (Alternate segment theorem)

$\angle T_1T_2M = 75°$ (Base angles in an isosceles triangle are equal.)

$\angle T_2MT_1 = 30°$ (Angles in a triangle sum to 180°.)

6 ▶ $\angle T_1T_2A = 105°$ (Alternate segment theorem)

$\angle T_1T_2B = 75°$ (Angles on a straight line sum to 180°.)

$\angle T_2T_1C = 105°$ (Alternate segment theorem)

$\angle T_2T_1B = 75°$ (Angles on a straight line sum to 180°.)

$\angle T_2BT_1 = 30°$ (Angles in a triangle sum to 180°.)

7 ▶ $\angle TBC = 25°$ (Base angles in an isosceles triangle are equal.)

$\angle ABT = 75°$ (Alternate segment theorem)

$\angle ABC = 75 + 25 = 100°$

8 ▶ $\angle TDC = 60°$ (Alternate segment theorem)

$\angle EDT = 50°$ (Alternate segment theorem)

$\angle EDC = 60 + 50 = 110°$

9 ▶ **a** 90° (Angle between tangent and radius is 90°)

b (Angles in a triangle sum to 180°.)

c 60° (Base angles in an isosceles triangle are equal.)

d 60° (Alternate segment theorem)

10 ▶ **a** 90° (Angle between tangent and radius is 90°)

b (Angles in a triangle sum to 180°.)

c 20° (Base angles in an isosceles triangle are equal.)

d 20° (Alternate segment theorem)

11 ▶ **a** $\angle NTM = \angle NPT$ (Alternate segment theroem)

b $\angle PLT = \angle NTM$ (Corresponding angles are equal)

12 ▶ ∠ATF = ∠FDT (Alternate segment theorem)

∠FDT = ∠BAF (Alternate angles, AC parallel to DT)

13 ▶ **a** ∠ATC = ∠ABT (Alternate segment theorem)

b ∠ABT = ∠BTD (Alternate angles, AB parallel to CD)

14 ▶ ∠CTB = ∠CDT (Alternate segment theorem)

∠CBT is common

Therefore all three angles are equal and the triangles are similar

EXERCISE 2*

1 ▶ $\angle BT_1T_2 = 65°$ (Base angles in an isosceles triangle are equal.)

$\angle T_1DT_2 = 65°$ (Alternate segment theorem)

2 ▶ ∠TED =110° (Opposite angles of a cyclic quadrilateral sum to 180°.)

∠EDT = 35° (Base angles in an isosceles triangle are equal.)

∠ATE = 35° (Alternate segment theorem)

3 ▶ $\angle T_1T_2C = 60°$ (Alternate segment theorem)

$\angle ET_2T_1 = 80°$ (Alternate segment theorem)

$\angle ET_2C = 60 + 80 = 140°$

4 ▶ $\angle TDC = 180 - 2(40 + x) = 100 - 2x$ (Base angles in an isosceles triangle are equal.)

$\angle TOC = 200 - 4x$ (Angle at centre of circle is twice angle at circumference.)

$200 - 4x + 2x = 180$ (Angles in a triangle sum to 180°.)

$x = 10°$

5 ▶ ∠ATE = 55° (alternate segment theorem)

∠TBC = 125° (angles on straight line sum to 180°)

∠BTC = 35° (angle sum of triangle is 180°)

∠ATB = 90° (angles on straight line sum to 180°)

Therefore AB is a diameter

6 ▶ $\angle ATD = x°$ (alternate segment theorem), ∠DTC = 90° (CD is a diameter), therefore $\angle BTC = (90 - x)°$ (angles on straight line sum to 180°)

Therefore ∠TBC is a right angle (angle sum of triangle is 180°)

7 ▶ $\angle ACT = x + 9°$ (Alternate segment theorem)

$\angle CAT = x + 9°$ (Base angles in an isosceles triangle are equal.)

$3x + 18 = 180$ (Angles in a triangle sum to 180°.)

$x = 54°$

8 ▶ **a** ∠BAE = 90° (Angle in a semicircle is 90°.)

∠DAE = 90 – 35 = 55°

b ∠BAE = ∠BDE = 90° (Angle in a semicircle is 90°.)

∠DAE = 90 – 35 = 55°

∠DBE = 55° (Angles in the same segment are equal.)

∠ABE = 180 – 90 – 20 = 70° (Angles in a triangle sum to 180°.)

∠ABD = 70 + 55 = 125°

∠AED = 180 – 125 = 55° (Opposite angles of a cyclic quadrilateral sum to 180°.)

∠BED = 55 – 20 = 35°

c ∠ADE = 70° (angle sum of triangle is 180°)

∠ACD = 35° (angle sum of triangle is 180°)

Therefore ∠ACD is isosceles

9 ▶ **a** ∠BTD = 110° (Angles in a triangle sum to 180°.)

∠DTA = 70° (Angles on a straight line sum to 180°.)

b ∠TCD = 70° (Alternate segment theorem)

∠BCT = 110° (Angles on a straight line sum to 180°.)

c ∠CBT is common and ∠CTB = 40° (alternate segment theorem) therefore all three angles are equal and triangles BCT and BTD are similar

10 ▶ ∠TEB = 80° (Alternate segment theorem)

∠ATD = 80° (Vertically opposite angles are equal.)

∠TFD = 80° (Alternate segment theorem)

∠EFT = 20° (Angles in a triangle sum to 180°.)

11 ▶ ∠TEB = 115° (Alternate segment theorem)

∠ATD = 115° (Vertically opposite angles are equal.)

∠TED = 115° (Alternate segment theorem)

∠DEB = 360 – 115 – 115 = 130° (Angles at a point sum to 180°.)

12 ▶ **a** Triangles ACG and ABF are right-angled

b Angles ACG and ABF are equal and in the same segment of the chord FG

13 ▶ **a** **i** $\angle T_2CT_1$, T_2T_1B

ii $\angle AT_1C$, $\angle BT_1D$, $\angle T_1DB$, $\angle CT_2T_1$

b Triangles BT_1T_2 and T_1BD are isosceles, therefore $BT_2 = BD$

14 ▶ **a** $\angle EOC = 2x°$ (angle at centre is twice angle at circumference)

$\angle CAE = (180 - 2x)°$ (opposite angles of a cyclic quadrilateral sum to 180°)

b $\angle AEB = x°$ (angle sum of triangle is 180°)

Therefore triangle ABE is isosceles

c ∠ECB = $x°$ (base angles of isosceles triangle are equal)

Therefore ∠BEC = $(180 - 2x)°$ = ∠CAE

Since ∠BAE and ∠BEC are equal and angles in alternate segments, BE must be the tangent to the larger circle at E

ACTIVITY 2

OM is common; OA = OB (radii of same circle);

∠OMA = 90° (angles on a straight line sum to 180°)

Therefore the triangles are congruent (RHS).

OAM and OBM are congruent, so AM = AB.

Therefore M is the mid-point of AB.

EXERCISE 3

1 ▶ OM = 15 cm

2 ▶ **a** 90° **b** 65° **c** 130°

3 ▶ 12 **4 ▶** 3 **5 ▶** 8

6 ▶ 8 **7 ▶** 4 **8 ▶** 3

ACTIVITY 3

∠APD = ∠BPC (Vertically opposite angles are equal)

∠CDA = ∠CBA (Angles in same segment off chord AC are equal)

∠BAD = ∠BCD (Angles in same segment off chord BC are equal)

⇒ Triangles APD and CPB are similar

$\Rightarrow \frac{CP}{AP} = \frac{BP}{DP}$

⇒ AP × BP = CP × DP

ACTIVITY 4

∠BAD = ∠BCD (Angles in same segment off chord BD are equal)

∠P is common to triangle APD and triangle BPC

∠ADP = ∠CBP

⇒ Triangles APD and CPB are similar

$\Rightarrow \frac{CP}{AP} = \frac{BP}{DP}$

⇒ AP × BP = CP × DP

EXERCISE 3*

1 ▶ **a** AM = 6 cm (The perpendicular from the centre of a circle to a chord bisects the chord.)

b AO = 10 cm

2 ▶ AB = 20 cm **3 ▶** 6 **4 ▶** 3

5 ▶ 8 **6 ▶** 5 **7 ▶** 4 **8 ▶** 4

EXERCISE 4

1 ▶ 16 cm **2 ▶** x = 2 or 14

3 ▶ **a** a = 38° (The angle between the tangent and the chord is equal to the angle in the alternate segment.)

b b = 35° (The angle between the tangent and the chord is equal to the angle in the alternate segment.)

c = 93° (Angles in a triangle sum to 180° or the angle between the tangent and the chord is equal to the angle in the alternate segment.)

d = 93° (Angles on a straight line sum to 180°.)

c e = 62° (The angle between the tangent and the chord is equal to the angle in the alternate segment.)

g = 79° (The angle between the tangent and the chord is equal to the angle in the alternate segment.)

f = 39° (Angles on a straight line sum to 180°.)

4 ▶ **a** ∠TAB = 58° (Angles in a triangle sum to 180° and tangents drawn to a circle from a point outside the circle are equal in length.)

a = 58° (The angle between the tangent and the chord is equal to the angle in the alternate segment.)

b b = 55° (The angle in a semicircle is a right angle. Angles in a triangle sum to 180°. The angle between the tangent and the chord is equal to the angle in the alternate segment.)

c c = 66° (Alternate angles are equal.)

d = 66° (The angle between the tangent and the chord is equal to the angle in the alternate segment.)

e = 48° (Angles in a triangle sum to 180°.)

5 ▶ **a** 28° (Angle between the tangent and a radius is 90°.)

b 160° (Angle between the tangent and a radius is 90° and angles in a quadrilateral sum to 360°.)

c 124° (Angles in a triangle sum to 180° and the base angles in an isosceles triangle are equal.)

d 76° (Angles round a point sum to 360°.)

e 52° (Angles in a triangle sum to 180° and the base angles in an isosceles triangle are equal.)

6 ▶ ∠DOB = 114° (Angle between the tangent and a radius is 90° and angles in a quadrilateral sum to 360°.)

Reflex ∠DOB = 246° (Angles round a point sum to 360°)

∠OBC = 20° (Angle between the tangent and a radius is 90°.)

∠DCB = 57° (Angle at the circumference is twice the angle at the centre.)

∠ODC = 37° (Angles in a quadrilateral sum to 360°.)

EXERCISE 4*

1 ▶ **a** 90° **b** 45°
c 90° **d** All of them

2 ▶ $x = 3$

3 ▶ Angle ABC = 180° – x (angles on a straight line)

Also angle ABC = 180° – angle ADC (opposite angles of a cyclic quadrilateral sum to 180°)

So angle ADC = x

Angle ADC + angle CDT = 180° (angles on a straight line)

So $x + y = 180°$

4 ▶ **a** Angle on straight line adjacent to 79° and 53° is 48°.

$a = 48°$ (alternate segment theorem)

b $b = 42°$ (angle sum of triangle)

$c = b = 42°$ (alternate segment theorem)

$d = 80°$ (alternate segment theorem, or angles on a straight line)

c $e = 65°$ (alternate segment theorem)

Angle between radius and chord = 90 – 65 = 25° (angle between the tangent and a radius is 90°)

$f = 25°$ (angles subtended by same arc)

d Angles between tangents and chord opposite 62° angle are both 62° (alternate segment theorem)

$g = 180 – 62 – 62 = 56°$ (angle sum of triangle)

e $h = 83 – 42 = 41°$ (exterior angle property)

Angle adjacent to $i = 42°$ (alternate segment theorem)

$i = 55°$ (angles on a straight line)

f $j = 70°$ (alternate segment theorem)

$k = 58°$ (alternate segment theorem)

$l = 110°$ (opposite angles of a cyclic quadrilateral sum to 180°)

$m = 180 – 58 – 58 = 64°$ (isosceles triangle formed by tangents of equal length)

5 ▶ Let angle BAC = x

Angle BAD = 2 × angle BAC = $2x$ (CA bisects angle BAD)

Angle BOD = 2 × angle BAD = $4x$ (angle at centre = 2 × angle at circumference)

Angle DCT = angle DAC = x (alternate segment theorem)

Therefore angle DCT = $\frac{1}{4}$ angle BOD

6 ▶ $x = 360 – (90 + 90 + 132) = 48°$ (Angle between the tangent and a radius is 90°. Angles in a quadrilateral sum to 360°.)

$y = 90 – 58 = 32°$ (Angle between the tangent and a radius is 90°.)

$z = 360 – (228 + 32 + 66) = 34°$ (Angles at a point sum to 360°. Angles in a quadrilateral sum to 360°. Angle at the centre is twice the angle at the circumference.)

EXAM PRACTICE: SHAPE AND SPACE 6

1 ▶ **a** OA = 6.5 cm

b AM = 6 cm

c OM = 2.5 cm

2 ▶ **a** $x = 16$

b $x = 10$

3 ▶ **a** $x = 60°$ (Angles in a triangle sum to 180°), $y = 60°$ (Alternate segment theorem), $z = 55°$ (Alternate segment theorem)

b $x = 40°$ (Isosceles triangles), $y = 70°$ (Radius perpendicular to tangent), $z = 40°$ (Alternate segment theorem)

4 ▶ Angle OBA = 90° – $3x$ (angle between tangent and radius = 90°)

Angle OAB = 90° – $3x$ (base angle of isosceles triangle OAB, equal radii)

Angle OAC = x (base angle of isosceles triangle OAC, equal radii)

Therefore angle BAC = $90° – 3x + x = 90° – 2x$

Angle BOC = 2 × angle BAC = $180° – 4x$ (angle at centre = 2 × angle at circumference)

Angle TBO = angle TCO = 90° (angles between tangents and radii)

In quadrilateral TBOC, $y + 90 + 180 – 4x + 90 = 360°$

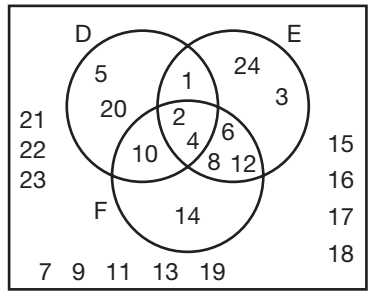
Therefore $y = 4x$

UNIT 6: SETS 2

EXERCISE 1

1 ▶ **a** 35 **b** 3 **c** 11 **d** 2 **e** 64

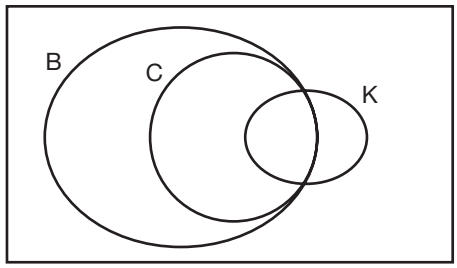
2 ▶ **a**



b {1, 2, 4, 5, 6, 8, 10, 12, 14, 20}

c 16

3 ▶ a Diagram not unique.



b All cards that are black or a king or both.

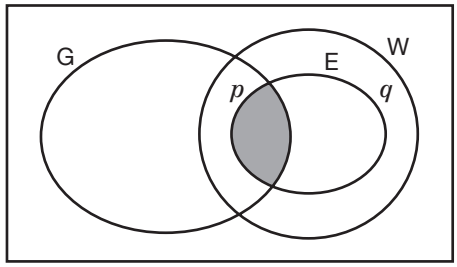
c All cards that are black or a king or both.

d All cards that are red or a king or both.

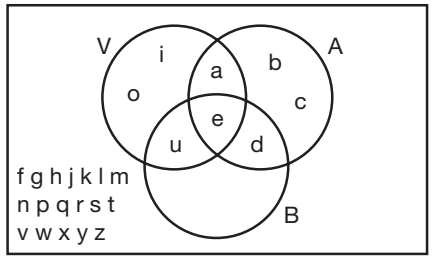
4 ▶ a All houses with electricity have mains water. E ⊂ W

b House p has mains water and gas but no electricity.

c and **d**



5 ▶ a

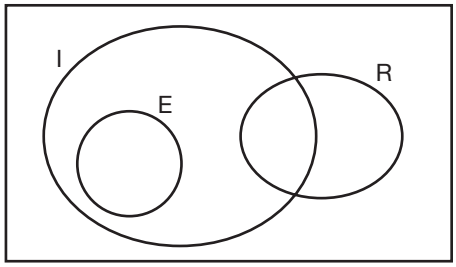


b {a, e, i, o, u, b, c, d}

c Consonants

d Yes

6 ▶ a



b An isosceles right-angled triangle.

c I ∪ E = isosceles triangles, I ∪ R = triangles that are isosceles or right-angled or both.

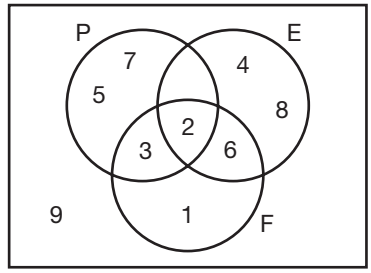
d Equilateral triangles; ∅

EXERCISE 1*

1 ▶ a 39 **b** 22 **c** 8 **d** 12

e 7 different types of ice-cream

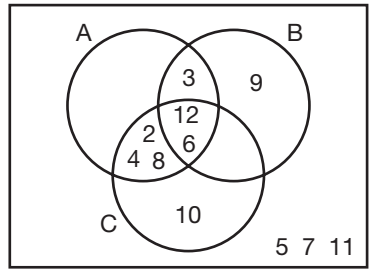
2 ▶ a



b P′ ∩ E = {4, 6, 8}, E ∩ F = {2, 6}, P ∩ F′ = {5, 7}

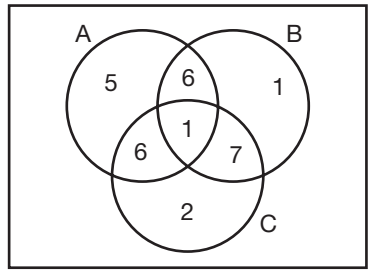
c The even prime factor of 6

3 ▶



4 ▶ 17

5 ▶



$n(A \cup B \cup C) = 28$

6 ▶ 2^n

EXERCISE 2

1 ▶ 6 **2 ▶** 93 **3 ▶** 22 **4 ▶** 41

5 ▶ a 10 **b** 8 **c** 5

6 ▶ 18

EXERCISE 2*

1 ▶ a 4 **b** 14

2 ▶ 11 **3 ▶** 10 **4 ▶** 100 **5 ▶** 8

6 ▶ a 4 **b** 3

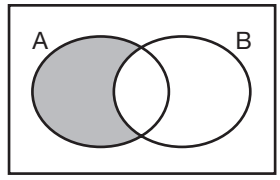
7 ▶ $8 \leqslant x \leqslant 14$, $0 \leqslant y \leqslant 6$

8 ▶ 40% ⩽ percentage who do both ⩽ 65%

EXERCISE 3

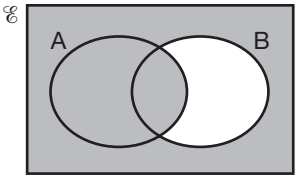
1 ▶

a

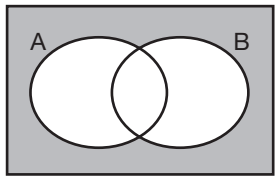


A ∩ B′

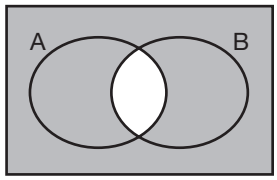
b



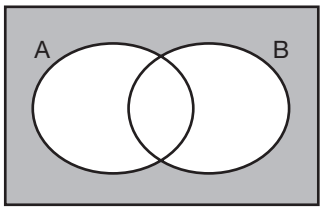
A ∪ B′

c 

$A' \cap B'$

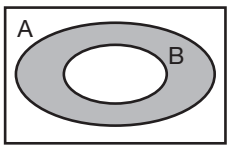
d 

$A' \cup B'$

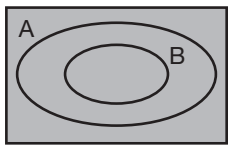
e 

$(A \cup B)'$

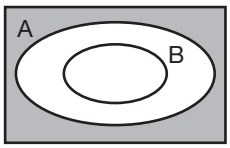
2 ▶

a 

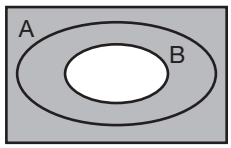
$A \cap B'$

b 

$A \cup B'$

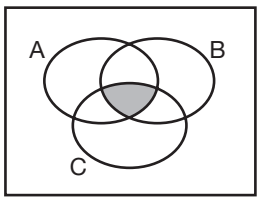
c 

$A' \cap B'$

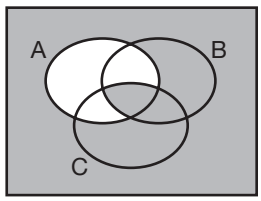
d 

$A' \cup B'$

3 ▶

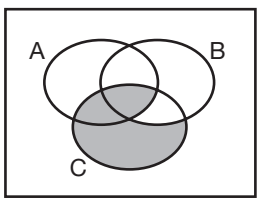
a 

$A \cap B \cap C$

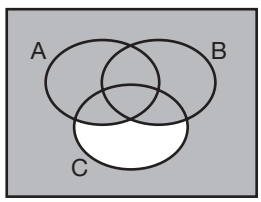
b 

$A' \cup B \cap C$

4 ▶

a 

$(A \cup B') \cap C$

b 

$A \cup B \cup C'$

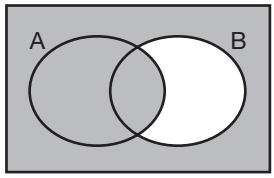
5 ▶ **a** $A \cap B'$ **b** $A' \cap B'$ **c** $A' \cup B'$

6 ▶ **a** $A' \cup B$ **b** $(A' \cap B') \cup (A \cap B)$

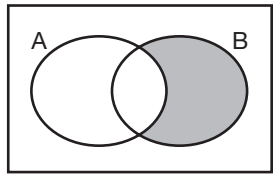
c $A \cap B'$

EXERCISE 3*

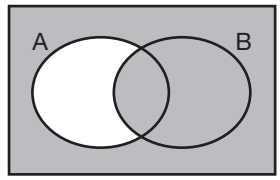
1 ▶

a 

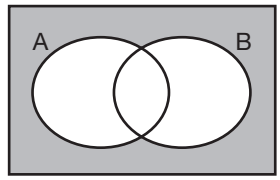
$(A' \cap B')$

b 

$(A \cup B')'$

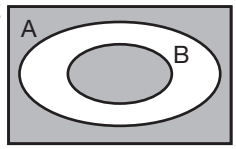
c 

$(A \cap B')'$

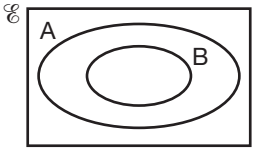
d 

$(A \cup B)'$

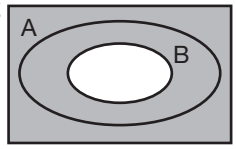
2 ▶

a 

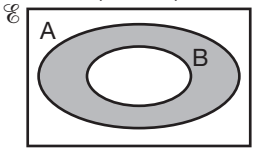
$(A \cap B')'$

b 

$(A \cup B')'$

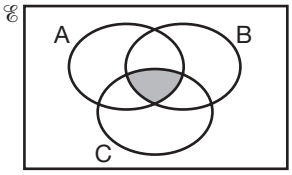
c 

$(A \cap B)'$

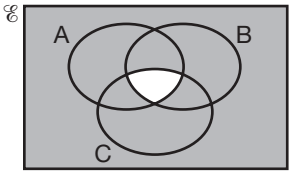
d 

$(A' \cup B)'$

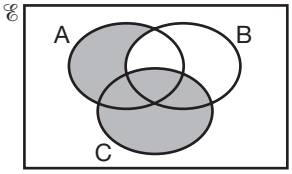
3 ▶

a 

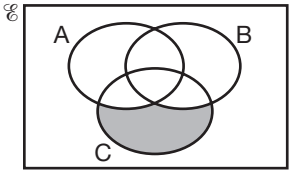
$A \cap B \cap C$

b 

$(A \cap B \cap C)'$

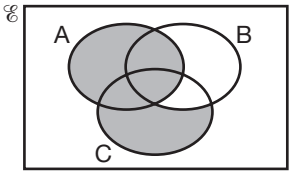
c 

$(A \cap B') \cup C$

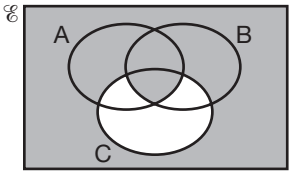
d 

$(A \cup B)' \cap C$

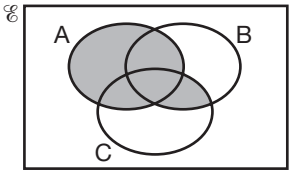
4 ▶

a 

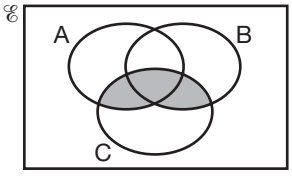
$A \cup (B' \cap C)$

b 

$(A \cap B) \cup C'$

c 

$A \cup (B \cap C)$

d 

$(A \cup B) \cap C$

5 ▶ **a** $B \cap (A \cup C)'$

b $(B' \cap C)$

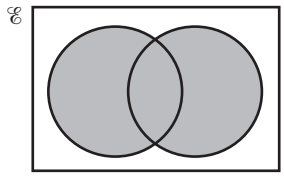
c $(B' \cap C) \cup (A \cap B \cap C')$

6 ▶ **a** $A \cap C \cap B'$ **b** $A' \cap (B \cup C)$

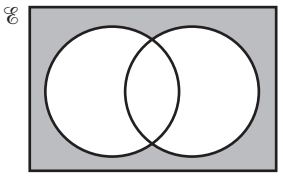
c $[A' \cap (B \cup C)] \cup [A \cap (B \cup C)']$

ACTIVITY 1

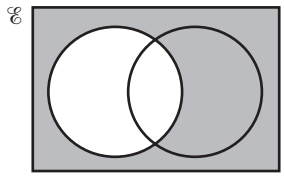
$A \cup B$



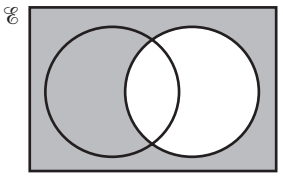
$(A \cup B)'$



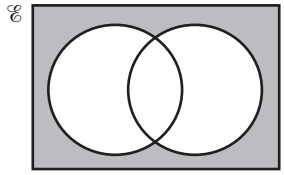
A'



B'



$A' \cap B'$



$(A \cup B)' = A' \cap B'$

EXERCISE 4

1 ▶ **a** {Tuesday, Thursday}
b {Red, Amber, Green}
c {1, 2, 3, 4, 5, 6}
d {–1, 0, 1, 2, 3, 4, 5, 6}

2 ▶ **a** {Africa, Antarctica, Asia, Australia, Europe, North America, South America}
b {all Mathematics teachers in the school}
c {1, 2, 3, 4, 5} **d** {–3, –2, –1, 0, 1, 2}

3 ▶ **a** $\{x: x < 7, x \in \mathbb{N}\}$
b $\{x: x > 4, x \in \mathbb{N}\}$
c $\{x: 2 \leqslant x \leqslant 11, x \in \mathbb{N}\}$
d $\{x: -3 < x < 3, x \in \mathbb{N}\}$
e $\{x: x \text{ is odd}, x \in \mathbb{N}\}$
f $\{x: x \text{ is prime}, x \in \mathbb{N}\}$

4 ▶ **a** $\{x: x > 3, x \in \mathbb{N}\}$
b $\{x: x \leqslant 9, x \in \mathbb{N}\}$
c $\{x: 5 < x < 19, x \in \mathbb{N}\}$
d $\{x: -4 \leqslant x \leqslant 31, x \in \mathbb{N}\}$
e $\{x: x \text{ is a multiple of } 5, x \in \mathbb{N}\}$ or $\{x: x = 5y, y \in \mathbb{N}\}$
f $\{x: x \text{ is a factor of } 48, x \in \mathbb{N}\}$

EXERCISE 4*

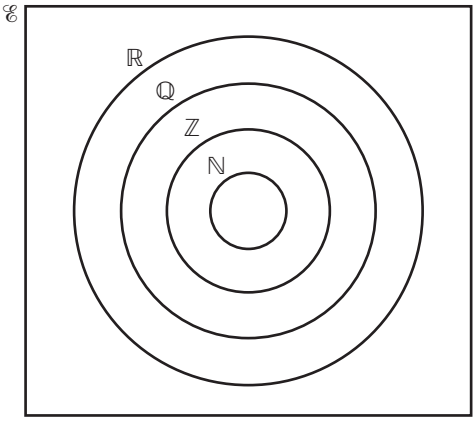
1 ▶ **a** {2, 4, 6, 8, 10, 12}
b {3, 7, 11, 15, 19, 23} **c** {2, 4, 6}
d {integers between 1 and 12 inclusive}

2 ▶ **a** {0, 1, 4} **b** $\{\frac{1}{4}, \frac{1}{2}, 1, 2, 4\}$
c {1} **d** {(1, 1), (2, 2)}

3 ▶ **a** ∅ **b** $(1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32})$
c {2} **d** {–3, 2}

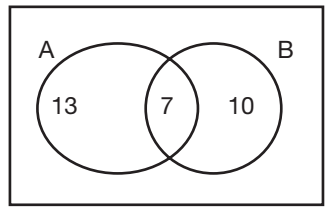
4 ▶ **a** ∅ **b** {1, 2, 4, 8, 16}
c ∅ **d** $\{-1 + \sqrt{7}, -1 - \sqrt{7}\}$

5 ▶



EXERCISE 5 REVISION

1 ▶ **a**



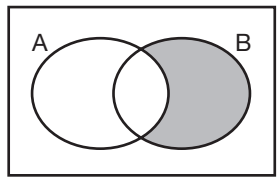
b 17 **c** 30

2 ▶ **a** 6 **b** 2 **c** 10

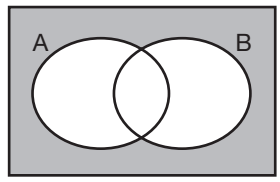
3 ▶ **a** 17% **b** 52% **c** 31%

4 ▶

a



b



5 ▶ $A' \cup B'$

6 ▶ **a** {–2, –1, 0, 1, 2, 3}
b {1, 2, 3, 4} **c** ∅

7 ▶ **a** $\{x: x \text{ is even}, x \in \mathbb{N}\}$
b $\{x: x \text{ is a factor of } 24, x \in \mathbb{N}\}$
c $\{x: -1 \leqslant x \leqslant 4, x \in \mathbb{N}\}$

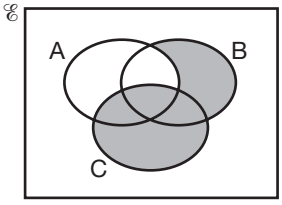
8 ▶ $\{x: x \geqslant -4, x \in \mathbb{Z}\}$

EXERCISE 5* REVISION

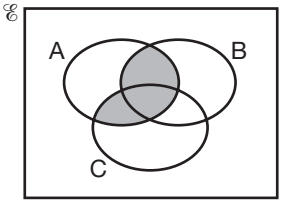
1 ▶ 34 **2** ▶ 10 **3** ▶ 2

4 ▶

a



b



5 ▶ **a** $(A \cup B') \cap C$
b $A \cup B \cup C'$

6 ▶ **a** {–1, 1}

b {0, –4}

c ∅

7 ▶ **a** $\{x: x > 5, x \in \mathbb{N}\}$

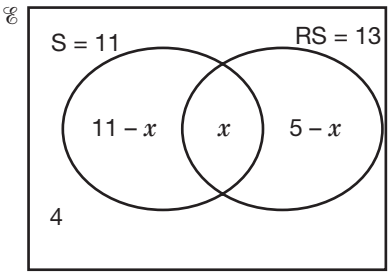
b $\{x: 4 < x < 12, x \in \mathbb{N}\}$

c $\{x: x$ is a multiple of $3, x \in \mathbb{N}\}$ or $\{x: x = 3y, y \in \mathbb{N}\}$

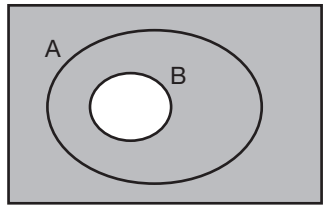
8 ▶ $\{x: -\frac{1}{4} < x \leqslant 4, x \in \mathbb{R}\}$

EXAM PRACTICE: SETS 2

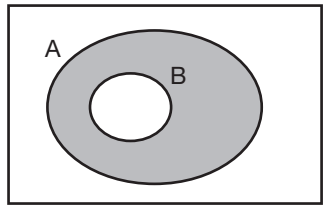
1 ▶ 8



2 ▶ **a**



b



3 ▶ A′ ∩ B

4 ▶ {–1, 0, 1, 2}

5 ▶ $\{x: x = 4y, y \in \mathbb{N}\}$

6 ▶ **a** 7 **b** 9