UNIT 9 ANSWERS

UNIT 9: NUMBER 9

EXERCISE 1

1 ▶ X $4.50/litre, Y $4.20/litre. Y better value
2 ▶ P $3.25/kg, Q $3.30/kg. P better value
3 ▶ A $6.50/m, B $6.25/m. B better value
4 ▶ I $0.36/ml, II $0.32/ml. II better value
5 ▶ Tin A 167 g/$, tin B 176 g/$. Tin B gives better value.

EXERCISE 1*

1 ▶ 1.5 m tree is $30/m, 2 m is $27.50/m, 3.5 m is $31.43/m. The order, from worst value to best, is 3.5 m, 1.5 m, 2 m.
2 ▶ Marble $75/m^2, slate $72/m^2, limestone $72.22/m^2. Slate is cheapest.
3 ▶ Mega-Movie $1 per day per DVD; Films R Us $1.20 per day per DVD. Films R Us gives better value.
4 ▶ Small: $1.25/ml, Medium: $1.13/ml, Large: $1.12/ml. Large is best value.
5 ▶ Yellow: $195, Lime: $200, Rainbow: $207. Yellow is best value over 10 hours.

EXERCISE 2

1 ▶ $1380 2 ▶ £368
3 ▶ €28 200 4 ▶ €126
5 ▶ £862 500

EXERCISE 2*

1 ▶ £4800 2 ▶ €400
3 ▶ Overcharged by $3.75
4 ▶ 8 hrs
5 ▶ Students' own answers

EXERCISE 3

1 ▶ $8000 2 ▶ $10 000
3 ▶ $3166.67 4 ▶ $13 333.33
5 ▶ $5333.33

EXERCISE 3*

1 ▶ 38.7% 2 ▶ 23.3%
3 ▶ $50 000 4 ▶ $2 525 000
5 ▶ Students' own answers

EXERCISE 4

1 ▶ **a** £175 **b** €230 **c** ₽19 437.50
2 ▶ **a** R1258.50 **b** ₦14 943.75 **c** ₹492.75
3 ▶ **a** $2142.86 **b** $22.13 **c** $1630.43
4 ▶ $684 931.51
5 ▶ UK: $274 285.71; France: $271 739.13. France house cheaper by $2546.58

EXERCISE 4*

1 ▶ **a** $357.14 **b** €328.57 **c** ₹24 210.71
2 ▶ **a** R2554.03 **b** ₽11 834.09 **c** $222.22
3 ▶ £9003.22
4 ▶ Nigerian : South Africa = £800 : £1600 = ₦227 714.29 : R38 354.29
5 ▶ China : Australia : America = €600 : €1200 : €1800 =
= ₹4284.78 : $(AUS) 1904.35 : $(USA) 1956.52

EXERCISE 5 REVISION

1 ▶ Green: $3.80/kg
Mint: $3.80/kg
Same value for both!
2 ▶ $315
3 ▶ $666.67
4 ▶ **a** $171.43 **b** £91.30
5 ▶ $42.05

EXERCISE 5* REVISION

1 ▶ 1st: Wessex: 4.3 litres/100 km
2nd: Fizz: 4.4 litres/100 km
3rd: Tyrol: 4.5 litres/100 km
2 ▶ 48
3 ▶ $5671.23 per day
4 ▶ UK: $64 285.71 Spain: $65 217.39
UK is cheaper by $931.68 so buy in UK.
5 ▶ Students' own answers

EXAM PRACTICE: NUMBER 9

1 ▶ Pluto: 5.5 litres/100 km
Jupiter: 4.7 litres/100 km
Jupiter more economical.
2 ▶ $380
3 ▶ $1692.31
4 ▶ **a** £17.50 **b** $35.71 **c** $27.17
5 ▶ January 1st: £132 000
April 1st: £100 000
£32 000 less in April!

UNIT 9: ALGEBRA 9

ACTIVITY 1

$x = 0$, $y = 5$ or $x = 4$, $y = 3$
There is only one point of intersection, (4, –3); the line is a tangent to the circle.
No solutions

EXERCISE 1

1 ▶ (–2, 4), (3, 9)
2 ▶ $x = -1$, $y = 1$ or $x = 3$, $y = 9$
3 ▶ (–1, 1), (4, 16)
4 ▶ $x = -2$, $y = 4$ or $x = 4$, $y = 16$
5 ▶ $x = 1$, $y = 2$ or $x = 2$, $y = 3$
6 ▶ $x = 2$, $y = 1$ or $x = -3$, $y = -4$
7 ▶ (–2, –1), (1, 2)
8 ▶ (–1, –1), (2, 2)

EXERCISE 1*

1 ▶ $x = -3.45$, $y = -7.90$ or $x = 1.45$, $y = 1.90$
2 ▶ $x = 0.268$, $y = 1.80$ or $x = 3.73$, $y = 12.2$
3 ▶ $x = -1.54$, $y = -4.17$ or $x = 4.54$, $y = 20.2$
4 ▶ $x = 0.586$, $y = -0.757$ or $x = 3.41$, $y = -9.24$
5 ▶ $(-4, -2)$, $(2, 4)$
6 ▶ $(-3, \frac{1}{3})$, $(1, 3)$
7 ▶ $(6.85, 7.85)$, $(0.146, 1.15)$
8 ▶ $(1.73, 2.67)$, $(-1.73, 2.67)$, $(2.24, 1.6)$, $(-2.24, 1.6)$

EXERCISE 2

1 ▶ $x = -0.2$, $y = 1.4$ or $x = 1$, $y = -1$
2 ▶ $x = 0.8$, $y = 0.6$ or $x = 0$, $y = 1$
3 ▶ $x = -2$, $y = -1.5$ or $x = 3$, $y = 1$
4 ▶ $x = -\frac{2}{3}$, $y = 6$ or $x = 2$, $y = -2$
5 ▶ $x = -2.87$, $y = 4.87$ or $x = 0.87$, $y = 1.13$
6 ▶ $x = 9.74$, $y = -6.74$ or $x = 2.26$, $y = 0.74$
7 ▶ $x = 1$, $y = -1$
8 ▶ $x = \frac{1}{2}$, $y = 1$
9 ▶ $x = 2.17$, $y = 0.17$ or $x = 7.83$, $y = 5.83$
10 ▶ $x = 0.785$, $y = 3.22$ or $x = 2.55$, $y = 1.45$
11 ▶ **a** $y = 3$ **b** $x = 29.85$ and $y = 3$
c 29.85 cm
12 ▶ 4 cm

EXERCISE 2*

1 ▶ $x = 1$, $y = 0$ or $x = 7$, $y = -12$
2 ▶ $x = -2$, $y = 2$ or $x = -1$, $y = 3$
3 ▶ $x = \frac{2}{3}$, $y = \frac{1}{3}$ or $x = \frac{1}{3}$, $y = \frac{2}{3}$
4 ▶ (6, –6); tangent
5 ▶ **a** (6, 1), (–2, 7) **b** AB = 10
6 ▶ **a** $y = 1.5$
b A(–1.68, 1.5), B(1.68, 1.5), AB = 3.36 cm
7 ▶ (–2238, 5996), (2238, 5996)
8 ▶ **a** $y = -3$
b (–2.65, –3), (2.65, –3), diameter is 5.30 cm
9 ▶ (7.53, 0.88), No
10 ▶ (23.7, 21.8)
11 ▶ (2.67, –1.78)
12 ▶ (44, –22), length 49.2 m

EXERCISE 3

Note: other counter-examples exist for some of these.

1 ▶ $3 + 3 = 6$
2 ▶ Rhombus
3 ▶ 2
4 ▶ $2 - (-1) > 2 + (-1)$
5 ▶ $(1 + 2)^2 \neq 1^2 + 4$
6 ▶ $41^2 + 41 + 41 = 41(41 + 1 + 1)$
7 ▶ $2^2 + 3^2 = 13$
8 ▶ $0.5^2 < 0.5$

EXERCISE 3*

Note: other counter-examples exist for some of these.

1 ▶ $4^2 - 2^2 = 12$
2 ▶ $1^3 + 3^3 = 28$
3 ▶ $0.5 \times 1 < 0.5 + 1$
4 ▶ $0.5^3 < 0.5^2$
5 ▶ $1 < 2$ but $-1 \times 1 > -1 \times 2$
6 ▶ $-2 < -1$ but $(-2)^2 > (-1)^2$
7 ▶ $(1 + 2)^2 \neq 1^2 + 2^2$
8 ▶ $101^4 + 29 \times 101^2 + 101$
$= 101(101^3 + 29 \times 101 + 1)$

EXERCISE 4

1 ▶ $n + (n + 1) + (n + 2) = 3n + 3 = 3(n + 1)$
2 ▶ **a** $n - 1$, n, $n + 1$
b $\dfrac{(n - 1) + n + (n + 1)}{3} = \dfrac{3n}{3} = n$
3 ▶ $n^2 + (n + 1)^2 = 2n^2 + 2n + 1 = 2(n^2 + n) + 1$
4 ▶ $(3n + 1)^2 - (3n - 1)^2$
$= 9n^2 + 6n + 1 - (9n^2 - 6n + 1) = 12n$
5 ▶ Let the integers be $a = n - 1$, $b = n$ and $c = n + 1$ then $c^2 - a^2 = (n + 1)^2 - (n - 1)^2$
$= 4n = 4b$

EXERCISE 4*

1 ▶ $n + (n + 1) + (n + 2) + (n + 3) = 4n + 6$
$= 4(n + 1) + 2$
2 ▶ $8 \times \dfrac{n(n + 1)}{2} = 4n^2 + 4n = (2n + 1)^2 - 1$
3 ▶ Substituting c for n gives $ac^2 + bc + c$
$= c(ac + b + 1)$
4 ▶ $n(n + 1) + (n + 1)(n + 2) = 2n^2 + 4n + 2$
$= 2(n + 1)^2$
5 ▶ $(n - 1)n(n + 1) = n^3 - n$
a At least one of $(n - 1)$, n or $(n + 1)$ is even.
b At least one of $(n - 1)$, n or $(n + 1)$ is divisible by 3.
c Since $n^3 - n$ is divisible by 2 and 3 then it is divisible by 6.

ACTIVITY 2

1 ▶ **a** 8 **b** 8
c $\{(n + 4) + (n + 5)\} - \{n + (n + 1)\} = 8$
d $\{(n + 5) + (n + 6)\} - \{n + (n + 1)\} = 10$
e $\{(n + m) + (n + m + 1)\} - \{n + (n + 1)\} = 2m$
2 ▶ **a** $(n + 1)(n + 4) - n(n + 5) = 4$
b $(n + 1)(n + m) - n(n + m + 1) = m$
c Same as part **b**.

EXERCISE 5

1 ▶ $(2n + 1) + (2n + 3) = 4n + 4 = 4(n + 1)$
2 ▶ $(2n + 1) - (2m + 1) = 2n - 2m = 2(n - m)$
3 ▶ $(2n + 1)^2 = 4n^2 + 4n + 1 = 2(2n^2 + 2n) + 1$
4 ▶ $2m(2n + 1) = 2(2mn + m)$

5 ▶ $x = 2n + 5 = 2(n + 2) + 1$

6 ▶ $n^2 - 2n + 1 = (n - 1)^2$

7 ▶ $100a + 10b + 5 = 5(20a + 2b + 1)$

8 ▶ $(n - 1)^2 + n^2 = (n + 1)^2 \Rightarrow n^2 - 4n = 0 \Rightarrow$ $n(n - 4) = 0 \Rightarrow n = 4$ is only possible answer.

EXERCISE 5*

1 ▶ $2n + (2n + 2) + (2n + 4) = 6(n + 1)$

2 ▶ $(2n + 1)^3 = 8n^3 + 12n^2 + 6n + 1$
$= 2(4n^3 + 6n^2 + 3n) + 1$

3 ▶ $(2n + 3)^2 - (2n + 1)^2 = 4(2n + 2)$

4 ▶ $x = -4n + 10 = 2(-2n - 5)$

5 ▶ If n is even $= 2m$ then

$$\frac{n(n+1)}{2} = \frac{2m(2m+1)}{2} = m(2m + 1);$$

if n is odd $= 2m + 1$ then

$$\frac{n(n+1)}{2} = \frac{(2m+1)(2m+2)}{2} =$$

$(2m + 1)(m + 1)$. (Note it is easier to say one of n or $n + 1$ must be even so result follows.)

6 ▶ $a + b + c = 3n \Rightarrow c = 3n - a - b$ so $100a + 10b + c = 99a + 9b + 3n =$ $3(33a + 3b + n)$

7 ▶ Let $b = 2n + 1$ then $a * b = 2a + (2n + 1) =$ $2(a + n) + 1$

8 ▶ $100a + 10b + c = 100a + 10(a + c) + c =$ $110a + 11c = 11(10a + c)$; for $abcd$ rule is $a + c = b + d$; $1000a + 100b + 10c + d$ $= 1000a + 99b + 10c + b + d =$ $1001a + 99b + 11c = 11(91a + 9b + c)$

ACTIVITY 3

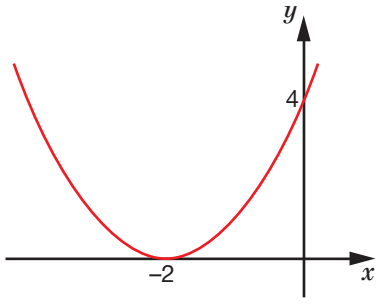
1 ▶ $(a - b) = 0$ and division by zero is not allowed.

2 ▶ $(b - a) < 0$ and when dividing an inequality by a negative number the inequality must be reversed.

EXERCISE 6

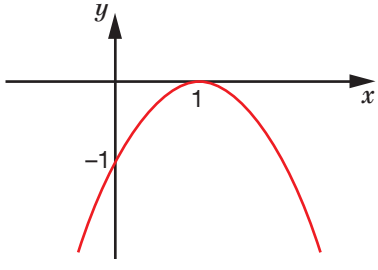
1 ▶ **a** $x^2 + 4x + 4 = (x + 2)^2 \geqslant 0$

b



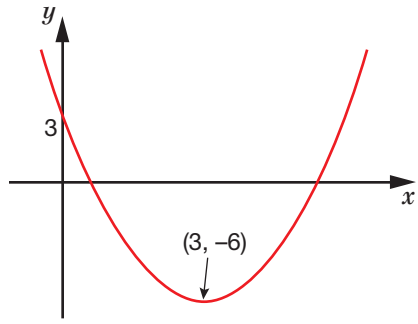
2 ▶ **a** $2x - x^2 - 1 = -(x - 1)^2 \leqslant 0$

b



3 ▶ **a** $(x - 3)^2 - 6$ **b** -6

c



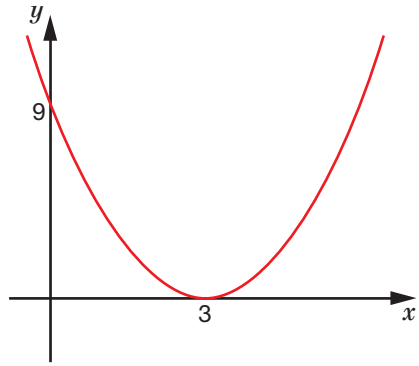
4 ▶ **a** $2(x + 1)^2 - 5$ **b** $(-1, -5)$

5 ▶ **a** $c \geqslant 25$ **b** $b \leqslant 8$

EXERCISE 6*

1 ▶ **a** $x^2 - 6x + 9 = (x - 3)^2 \geqslant 0$

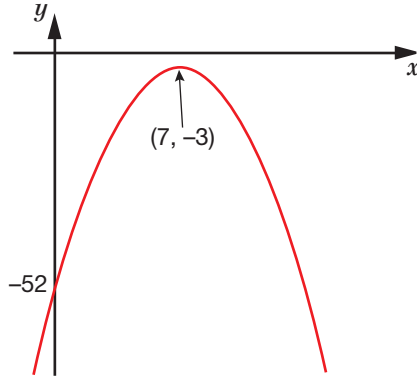
b



2 ▶ **a** $14x - x^2 - 52 = -(x - 7)^2 - 3 < 0$

b -3

c



3 ▶ $2x^2 - 32x + 129 = 2(x - 8)^2 - 128 + 129 =$ $2(x - 8)^2 + 1 \geqslant 0 \Rightarrow 2x^2 + 129 > 32x$

4 ▶ $x^2 + 2bx + 4 = x + b^2 + 4 - b^2 \Rightarrow$ smallest value is $4 - b^2$

5 ▶ **a** $(x - y)^2 \geqslant 0$
$(x - y)^2 = x^2 - 2xy + y^2$
$\Rightarrow x^2 - 2xy + y^2 \geqslant 0 \Rightarrow x^2 + y^2 \geqslant 2xy$

b $x^2 + y^2 = 2xy \Rightarrow (x - y^2) = 0 \Rightarrow x = y$

6 ▶ **a** $t^2 + 2 - t^2 = 2t^2 - 4t + 4$

b $2t^2 - 4t + 4 = 2t - 1^2 + 2$ which has a minimum value of 2 so minimum distance is $\sqrt{2}$

c When $t = 1$ so 13:00

EXERCISE 7 REVISION

1 ▶ **a** (–3, 9), (4, 16) **b** (–1, –7), (5, 5)

2 ▶ **a** $x = 1.37$, $y = -0.63$ or $x = -4.37$, $y = -6.37$

b $x = 1.82$, $y = -0.82$; $x = -0.82$, $y = 1.82$ (symmetry)

3 ▶ **a** $x = 1.7$

b A(1.7, 1.05), B(1.7, –1.05), 2.11 m

4 ▶ e.g. $2^2 + 2^2 = 8$ (even)

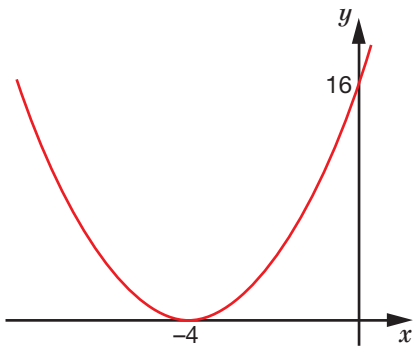
5 ▶ $(2n + 1) + (2n + 3) + (2n + 5) + (2n + 7) = 8(n + 2)$

6 ▶ $2m \times 2n = 4mn$

7 ▶ **a** $x^2 + 8x + 16 = (x + 4)^2 \geqslant 0$

b (–4, 0)

c



EXERCISE 7* REVISION

1 ▶ **a** $\left(-\frac{1}{2}, \frac{1}{2}\right)$, (3, 18) **b** (–2, –2), (1, 4)

2 ▶ **a** $x = 0.64$, $y = 0.27$ or $x = -3.14$, $y = -7.28$

b $x = 1.85$, $y = 0.15$; $x = -0.18$, $y = 2.18$

3 ▶ (15, –1.658), (15, 1.658)

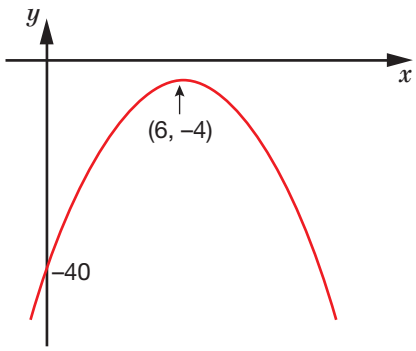
4 ▶ e.g. $4^3 - 2^3 = 56$ (odd)

5 ▶ Let the numbers be $a = n - 1$, $b = n$ and $c = n + 1$ then $ac = (n - 1)(n + 1) = n^2 - 1 = b^2 - 1$

6 ▶ $(2n + 1)(2n + 3) = 4n^2 + 8n + 3 = 4(n^2 + 2n + 1) - 1$

7 ▶ **a** $12x - x^2 - 40 = -(x - 6)^2 - 4 < 0$

b



EXAM PRACTICE: ALGEBRA 9

1 ▶ (4.73, 6.46), (1.27, –0.464)

2 ▶ (0, –4), (3.2, 2.4)

3 ▶ e.g. $(1 + 4)^2 \neq 1^2 + 16$

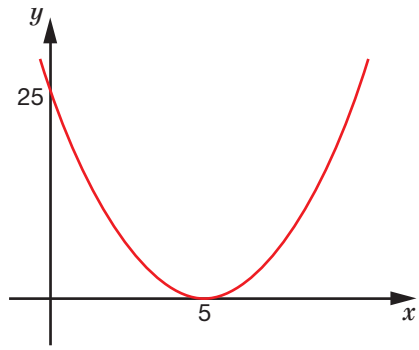
4 ▶ Let the numbers be $n + 4$ and n, then $(n + 4)^2 - n^2 = n^2 + 8n + 16 - n^2 = 8n + 16 = 8(n + 2)$

5 ▶ $(2n + 1)(2n + 3) = 4n^2 + 8n + 3 = 2(2n^2 + 4n + 1) + 1$

6 ▶ **a** $x^2 - 10x + 25 = (x - 5)^2 \geqslant 0$

b (5, 0)

c



UNIT 9: GRAPHS 8

EXERCISE 1

1 ▶ **a** 2 **b** 3 **c** –1 **d** –2 **e** $x = 1$

2 ▶ **a** 2 **b** 2 **c** 0.1 **d** 5.8 **e** $x = 0.18$ or 1.8

3 ▶ **a** –0.25 **b** –1 **c** –0.44 **d** –4 **e** $x = \pm 0.71$

4 ▶ **a** 4 **b** 2 **c** –4 **d** $x = 3$

EXERCISE 1*

1 ▶ **a** 2 **b** 4 **c** –1 **d** –3 **e** $x = 1.5$

2 ▶ **a** –2.75 **b** 1.25 **c** –0.75 **d** –4 **e** $x = -1$ or 0.33

3 ▶ **a** 1 **b** –0.37 or 1.37 **c** –1.3, 0.17 or 1.13 **d** –1.13, –0.17, 1.3

4 ▶ **b**

x co-ordinate	–4	–3	–2	–1	0	1	2	3	4
Gradient	–8	–6	–4	–2	0	2	4	6	8

c The value of the gradient is twice the x-co-ordinate.

5 ▶ **a**

x	0	1	2	3	4	5
2^x	1	2	4	8	16	32

b 1.4, 5.5

c $x = 4.1$, $y = 17$ (2 s.f.)

EXERCISE 2

1 ▶ a i 1 m/s ii 0 m/s iii 2 m/s

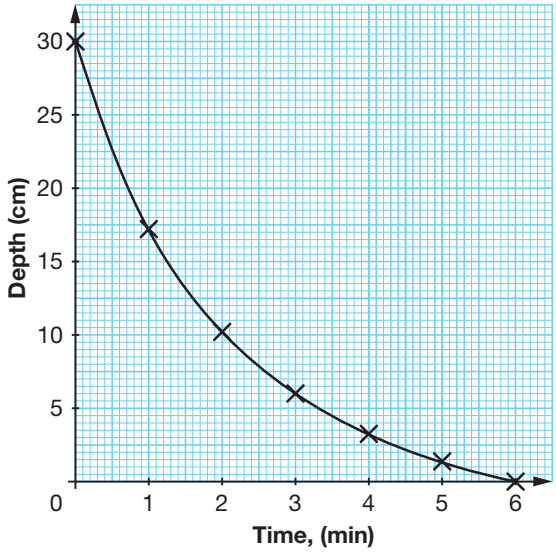
b 0–20 s gradually increased speed then slowed down to a stop
20–30 s stationary
30–40 s speed increasing
40–50 s travelling at a constant speed of 2 m/s
50–60 s slowing down to a stop

2 ▶ a i $\frac{1}{4}$ m/s² ii 0 m/s² iii $-\frac{1}{4}$ m/s²

b 0–20 s accelerating up to a speed of 5 m/s
20–30 s running at a constant speed of 5 m/s
30–50 s decelerating to a speed of 2.5 m/s
50–60 s running at a constant speed of 2.5 m/s
60–80 s decelerating to a stop

3 ▶ b i −9.6 °C/min ii −6.7 °C/min iii −5.5 °C/min

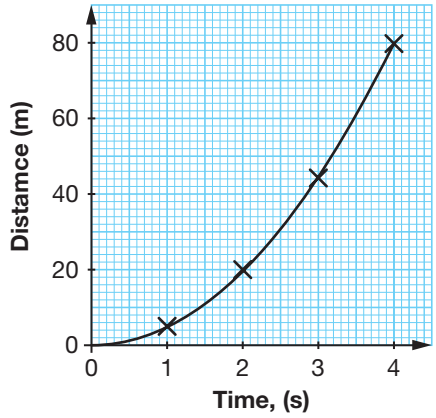
4 ▶ a



b i −12 cm/min ii −4 cm/min iii −1.4 cm/min

5 ▶ b i 0.5 m/s² ii 1.5 m/s² iii 2.5 m/s²

6 ▶ a



b i 10 m/s² ii 20 m/s² iii 30 m/s²

EXERCISE 2*

1 ▶ a

t (min)	0	20	40	60	80	100	120
N	10	40	160	640	2560	10 240	40 960

b i 0.7 ii 44 iii 710

2 ▶

t (months)	0	1	2	3	4
N (millions)	2	2.1	2.21	2.32	2.43

t (months)	5	6	7	8	9
N (millions)	2.55	2.68	2.81	2.95	3.1

b i 97 000 ii 119 000 iii 144 000

3 ▶ a

t (min)	0	10	20	30	40
V (cm³)	2000	1700	1445	1228	1044

t (min)	50	60	70	80	90
V (cm³)	887	754	641	545	463

b i −28 cm³/min ii −8.9 cm³/min

c t = 0, −32.5 cm³/min

4 ▶ a

t (s)	0	10	20	30	40
M (g)	120	96	76.8	61.4	49.2

t (s)	50	60	70	80	90
M (g)	39.3	31.5	25.2	20.1	16.1

b i −1.71 g/s ii −0.56 g/s

c At t = 0, −2.68 g/s

5 ▶ a i 1.67 m/h ii −1.67 m/h iii 0 m/h

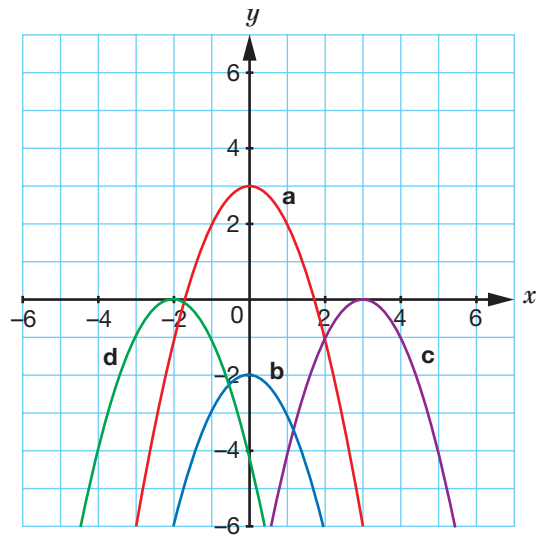
b Max at t = 0, 4, 8, 12, ±2.36 m/h

6 ▶ a i −9.44 m/s ii 0 m/s iii 1.56 m/s

b Max at t = 1.75, −11.7 m/s

EXERCISE 3

1 ▶

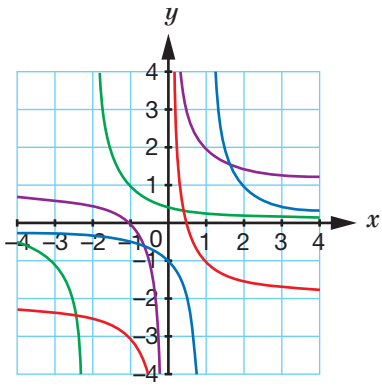


a (0, 3) **b** (0, −2) **c** (3, 0) **d** (−2, 0)

2 ▶ **a** $\begin{pmatrix}0\\-7\end{pmatrix}$ **b** $\begin{pmatrix}7\\0\end{pmatrix}$

c $\begin{pmatrix}0\\7\end{pmatrix}$ **d** $\begin{pmatrix}-7\\0\end{pmatrix}$

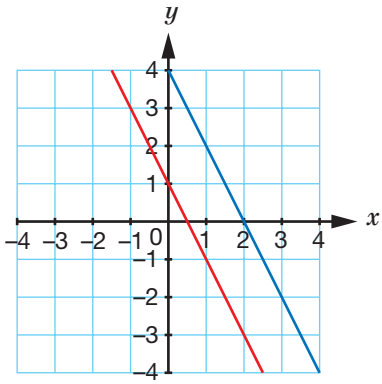
3 ▶



a Red curve **b** Blue curve

c Purple curve **d** Green curve

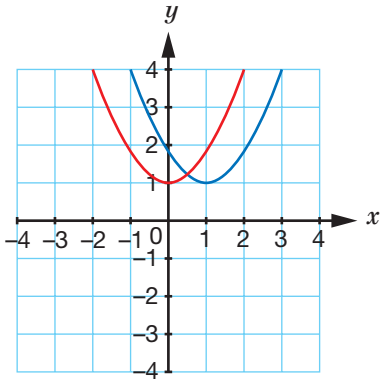
4 ▶



a Red line **b** Blue line

c $y = 4 - 2x$

5 ▶



a Red curve **b** Blue curve

c $y = x^2 - 2x + 2$

6 ▶ **a** $y = f(x) - 2$

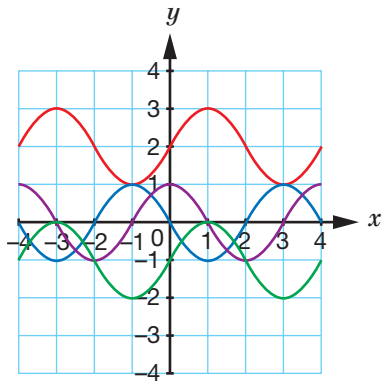
b $y = f(x - 2)$

7 ▶ $y = x^2 + 5x + 3$

8 ▶ $y = -x^2 - 5x - 3$

EXERCISE 3*

1 ▶



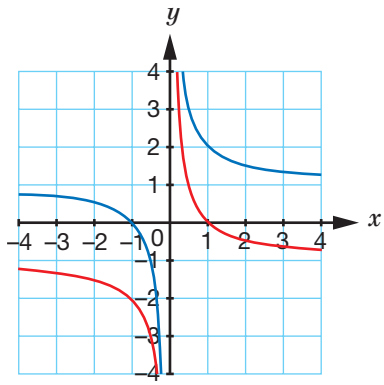
a Red curve **b** Blue curve

c Purple curve **d** Green curve

2 ▶ **a** $\begin{pmatrix}0\\5\end{pmatrix}$ **b** $\begin{pmatrix}13\\0\end{pmatrix}$

c $\begin{pmatrix}-11\\0\end{pmatrix}$ **d** $\begin{pmatrix}0\\-9\end{pmatrix}$

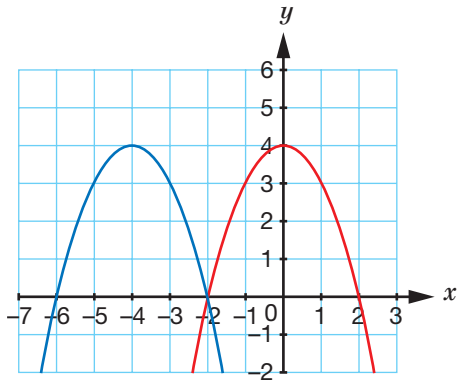
3 ▶



a Red curve **b** Blue curve

c $y = \frac{1}{x} + 1$

4 ▶



a Red curve **b** Blue curve

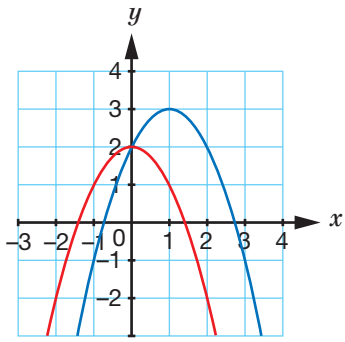
c $y = -x^2 - 8x - 12$

5 ▶ **a** $y = f(x + 3)$ **b** $y = f(x) + 3$

6 ▶ $y = 2x^3$

7 ▶ $y = 1 + 3x - x^2$

8 ▶ a Red curve **b** Blue curve

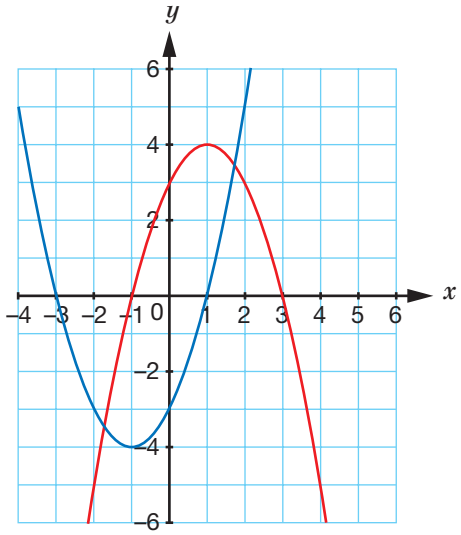


c $y = 2 + 2x - x^2$

ACTIVITY 1 $\sin(x + 90) = \cos x$, $a = 90$
$\cos(x - 90) = \sin x$, $a = -90$

EXERCISE 4

1 ▶

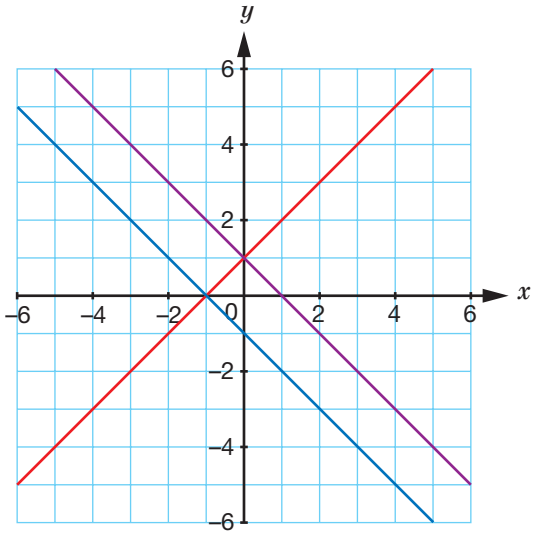


a Red curve

b Blue curve

2 ▶ a i Reflection in the x-axis
ii Reflection in the y-axis

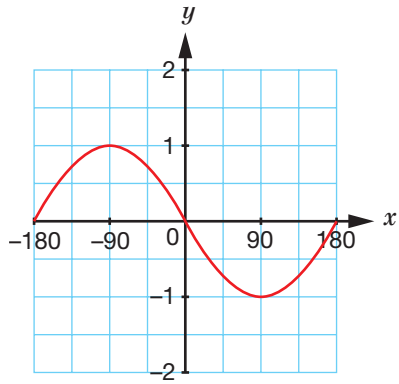
b Red line is $y = x + 1$, blue line is **a i**, purple line is **a ii**



3 ▶ a $y = f(-x)$

b $-f(x) y = -x^3 - x^2 - x - 1$

4 ▶ **a** and **b** are the same.

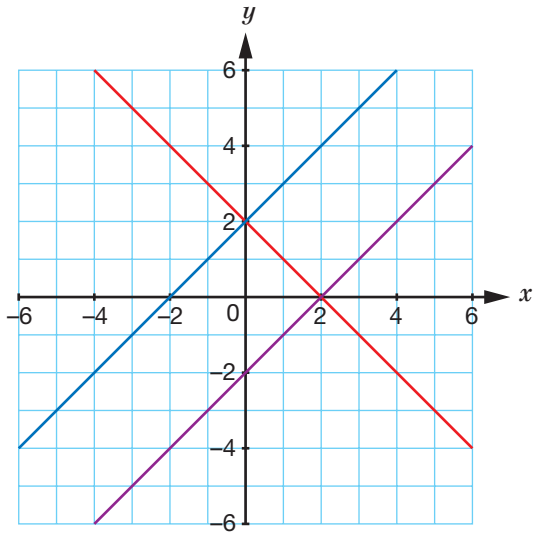


5 ▶ (0, 4), (–2, 0), (–4, –2)

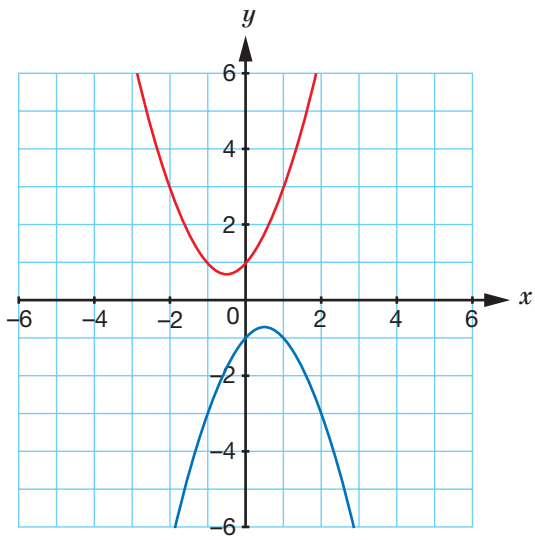
EXERCISE 4*

1 ▶ a i Reflection in y-axis
ii Reflection in the x-axis.

b Red line is $y = 2 - x$, blue line is **a i**, purple line is **a ii**



2 ▶ a Red curve **b** Blue curve

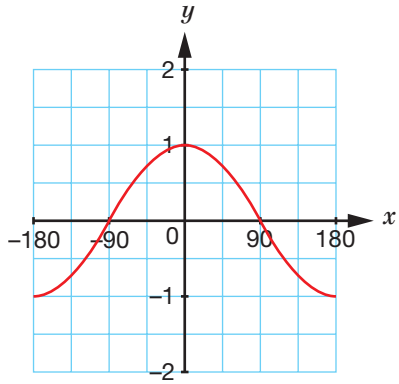


3 ▶ **a** $(a, 0)$ **b** $(0, -b)$ **c** $(c, -d)$

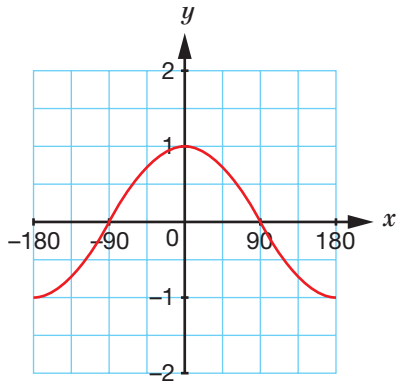
4 ▶ **a** $y = -x^3 - x^2 - x - 1$

b $y = -x^3 + x^2 - x + 1$

5 ▶ **a**

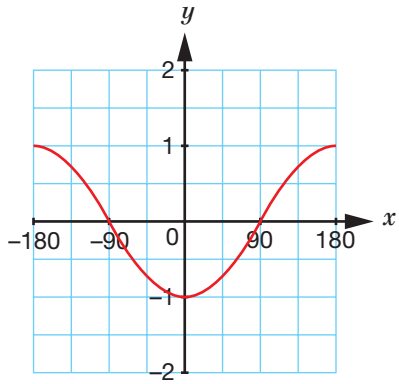


b

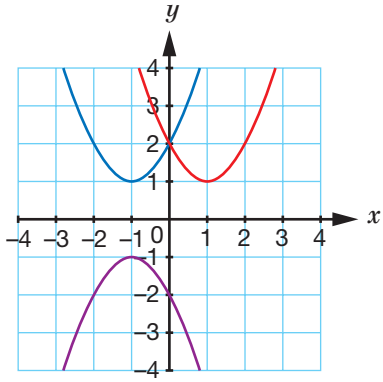


c The curves are the same, so $\cos(x) = \cos(-x)$

d



6 ▶



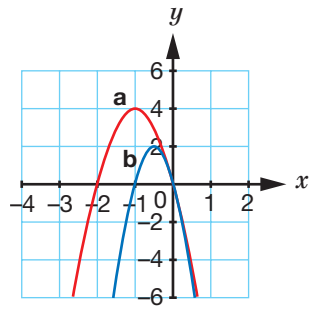
a Red curve is $y = f(x)$, blue curve is $y = f(-x)$, purple curve is $y = -f(-x)$

b Rotation of 180° about the origin

c No

EXERCISE 5

1 ▶

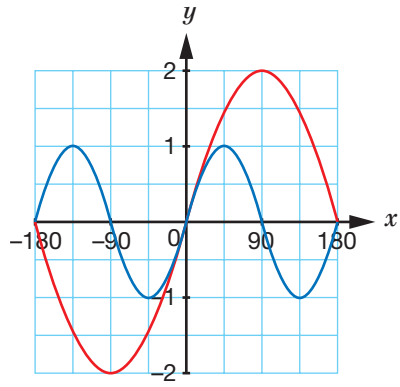


a (−1, 4) **b** (−0.5, 2)

2 ▶ **a** Stretch in the x direction, scale factor $\frac{1}{3}$

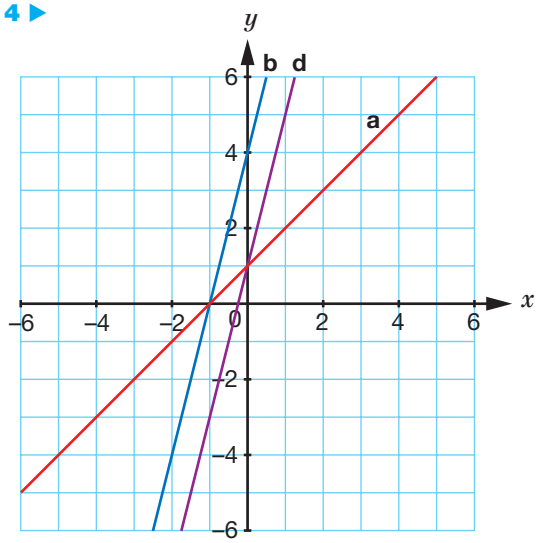
b Stretch in the y direction, scale factor 3

3 ▶



a Red curve **b** Blue curve

4 ▶



c $y = 4 + 4x$ **e** $y = 1 + 4x$

5 ▶ **a** Stretch in the y direction, scale factor 2

b $y = 2f(x)$

c Stretch in the x direction, scale factor $\frac{1}{2}$

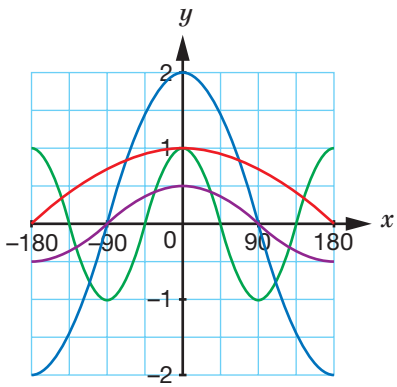
d $y = f(2x)$

6 ▶ **a** $y = 5x^2 + 5x + 5$

b $y = x^2 - 2x + 2$

EXERCISE 5*

1 ▶



a Red curve **b** Blue curve

c Purple curve **d** Green curve

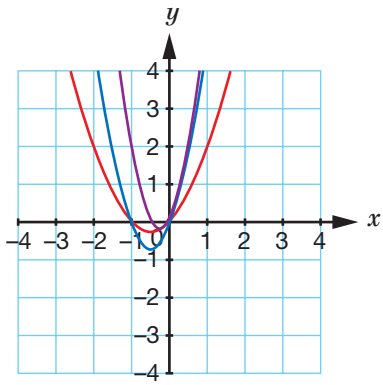
2 ▶ **a** Stretch in the y direction, scale factor $\frac{1}{4}$

b Stretch in the x direction, scale factor $\frac{1}{5}$

c Stretch in the y direction, scale factor 7

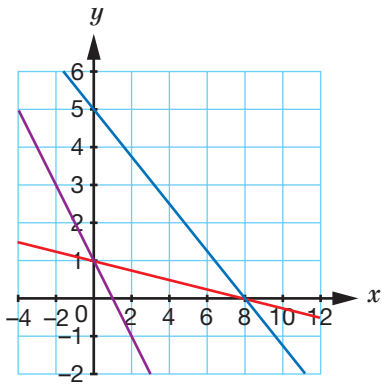
d Stretch in the x direction, scale factor 2

3 ▶



a Blue curve **b** Purple curve

4 ▶



a Red line **b** Blue line

c $y = \dfrac{5(8 - x)}{8}$ **d** Purple line

e $y = 1 - x$

5 ▶ **a** Stretch in the x direction, scale factor 3

b $y = f\left(\frac{x}{3}\right)$

c Stretch in the y direction, scale factor $\frac{1}{4}$

d $y = \frac{1}{4}f(x)$

6 ▶ **a** $y = 64x^3 + 32x^2 - 12x$

b $y = x^2 - 4x + 8$

ACTIVITY 2

$y = (x - 1)^2, y = -(x - 1)^2, y = (x + 1)^2, y = -(x + 1)^2$

$y = x^3, y = -x^3, y = \frac{1}{x}, y = -\frac{1}{x}$

$y = \cos(2x) + 1, y = -\cos(2x) - 1$

EXERCISE 6 REVISION

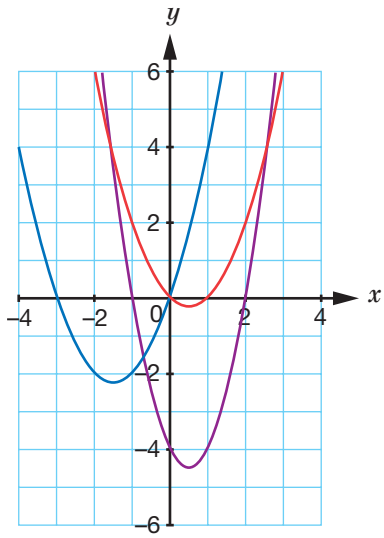
1 ▶ **b** −3, 0, 5 **c** $y = -3x - 1$

2 ▶ **b** 2.6 mm/s, 6.3 mm/s

3 ▶ **a** Graph 1, **b** Graph 2

4 ▶ **a** D **b** C **c** B **d** A

5 ▶



a Red curve **b** Blue curve

c Purple curve

6 ▶ **a** (2, −4) **b** (−2, 4)

7 ▶ **a** $y = 4 - x^2$ **b** (−2, 0) and (2, 0)

c $y = 4 - 4x^2$

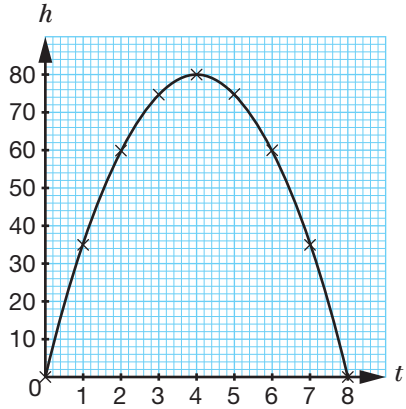
EXERCISE 6* REVISION

1 ▶ **a**

x	0	1	2	3	4
y	1	3	9	27	81

b 3.3, 9.9 **c** $y = 3.3x - 0.3$

2 ▶ **a** (1, 35), (2, 64), (3, 75), (4, 80), (5, 75), (6, 60), (7, 35), (8, 0)



b

t (s)	0	1	2	3	4
v (m/s)	40	30	20	10	0

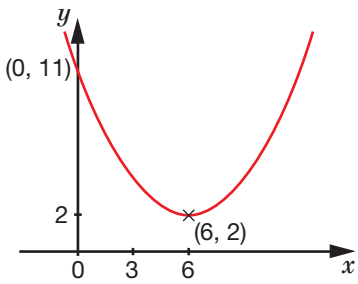
t (s)	5	6	7	8
v (m/s)	–10	–20	–30	–40

c Straight-line graph passing through (0, 40) and (8, –40)

d Acceleration is constant (–10 m/s), i.e. constant deceleration (10 m/s)

3 ▶ a D **b** C **c** B **d** A

4 ▶ a



b (6, 2) **c** (1.5, 2)

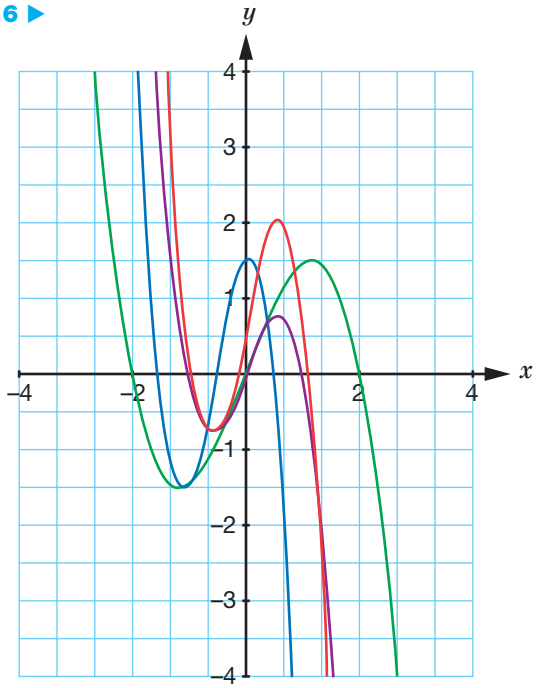
d $x = 2x = \frac{1}{2}x = 0$ hence stretching horizontally does not affect the point where the graphs intersect the y-axis.

5 ▶ a (0, 4), (3, 0), (2, –8)

b (0, 2), (–3, 0), (–2, –4)

c (–1, 2), (2, 0), (1, –4)

6 ▶



a Red curve **b** Blue curve

c Purple curve **d** Green curve

7 ▶ a Stretch scale factor 2 in the x direction

b $y = \sin\left(\frac{1}{2}x\right)$

c Stretch scale factor 2 in the y direction followed by a translation of $\begin{pmatrix}0\\3\end{pmatrix}$

d $y = 2\sin\left(\frac{1}{2}x\right) + 3$

e 4.73 m (3 s.f.)

EXAM PRACTICE: GRAPHS 8

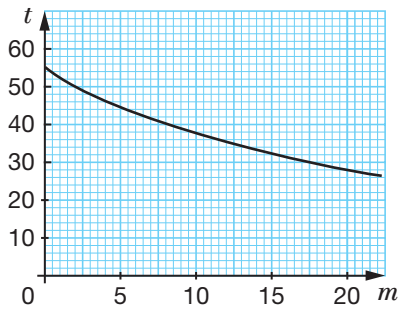
1 ▶ a i 5 **ii** –3

b $x = -0.39$ or $x = 1.72$

2 ▶ a

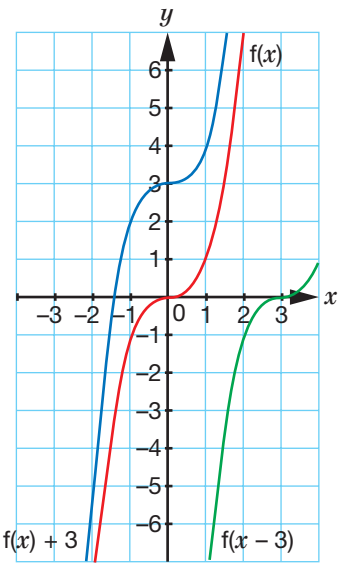
m	1	2	4	6	10	15	20
t	52	50	46	42	37	31	28

b



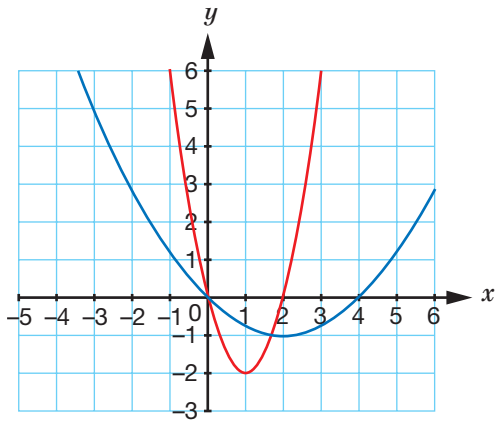
c –1.5 °C/min

3 ▶ a



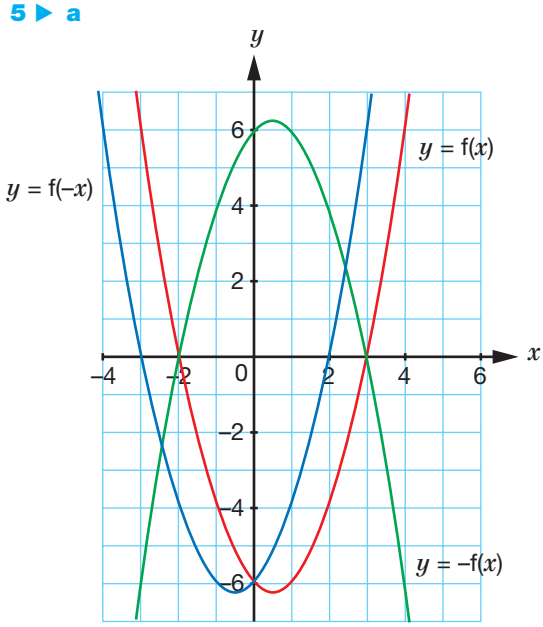
b i (0, 3) **ii** (3, 0)

4 ▶



a Red curve **b** Blue curve

5 ▶ a



b Reflection in the x-axis

c Reflection in the y-axis

UNIT 9: SHAPE AND SPACE 9

EXERCISE 1

1 ▶ a 11.7 cm **b** 14.2 cm **c** 34.4°

2 ▶ a 18.6 cm **b** 28.1 cm **c** 48.5°

3 ▶ a 14.1 cm **b** 17.3 cm **c** 35.3°

4 ▶ a 28.3 cm **b** 34.6 cm

c 35.3° **d** 19.5°

5 ▶ a 4.47 m **b** 4.58 m

c 29.2° **d** 12.6°

6 ▶ a 407 m **b** 402 m

c 8.48° **d** 13.3°

7 ▶ a 43.3 cm **b** 68.7 cm **c** 81.2 cm

8 ▶ a 28.9 cm **b** 75.7 cm **c** 22.4°

EXERCISE 1*

1 ▶ a 16.2 cm **b** 67.9° **c** 55.3 cm^2

2 ▶ a 26.5 cm **b** 61.9° **c** 1530 cm^2

3 ▶ a 30.3° **b** 31.6° **c** 68.9°

4 ▶ a 36.9° **b** 828 cm^2

5 ▶ a 15 m **b** 47.7° **c** £91 300

6 ▶ a 66.4° **b** 32.9°

7 ▶ 46.5 m

8 ▶ a OW = 4290 m, OS = 2760 m

b 36.0° **c** 197 km/h

ACTIVITY 1

$d^2 = a^2 + b^2 + c^2$

EXERCISE 2 REVISION

1 ▶ a i 15 cm **ii** 20.5 cm **iii** 16.6 cm **iv** 20.5 cm

b 43.0° **c** 43.0° **d** 35.8°

2 ▶ a 19.3 cm **b** 21.2°

3 ▶ a AC = 42.4 cm

b 33.9 cm **c** 68.0° **d** 58.0°

4 ▶ a 18.4° **b** 500 m **c** 11.3°

5 ▶ a 18.7 m **b** 31.9° **c** 20.5°

EXERCISE 2* REVISION

1 ▶ a 21.9 cm

b Angle ADB = 33.2°

c Angle DAE = 24.2°

2 ▶ a AC = 70.7 cm

b 98.7 cm

c 27.9°

d 216 000 cm^2

3 ▶ a 40 cm

b 18.1 cm

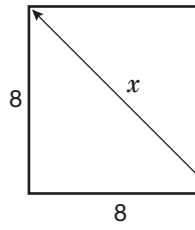
c 42.2°

d 24.2 cm

e 48.6°

4 ▶ 25.5°

5 ▶ a x is the length of the diagonal of the square that is the top face of the cube.



Using Pythagoras' theorem:

$x^2 = 8^2 + 8^2 = 128$

$x = \sqrt{128} = \sqrt{64 \times 2} = \sqrt{64} \times \sqrt{2} = 8\sqrt{2}$

b 50.5°

EXAM PRACTICE: SHAPE AND SPACE 9

1 ▶ a 43.3 cm

b 35.3°

2 ▶ a ii 21.2 cm **ii** 10.6 cm **iii** 14.5 cm

b 54°

c 27°

3 ▶ 61.1°

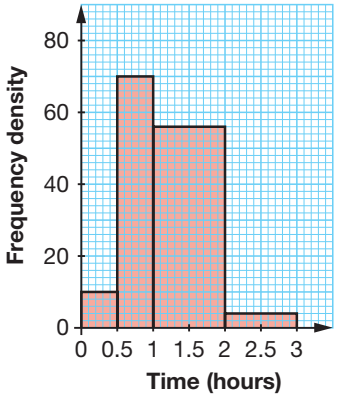
UNIT 9: HANDLING DATA 6

EXERCISE 1

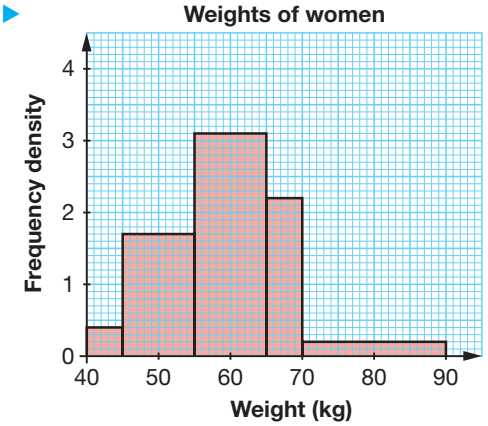
1 ▶ a, b

Age, a (years)	Frequency	Class width	Frequency density
$0 < a \leqslant 10$	3	10	0.3
$10 < a \leqslant 20$	14	10	1.4
$20 < a \leqslant 40$	17	20	0.85
$40 < a \leqslant 60$	19	20	0.95
$60 < a \leqslant 80$	7	20	0.35

2 ▶

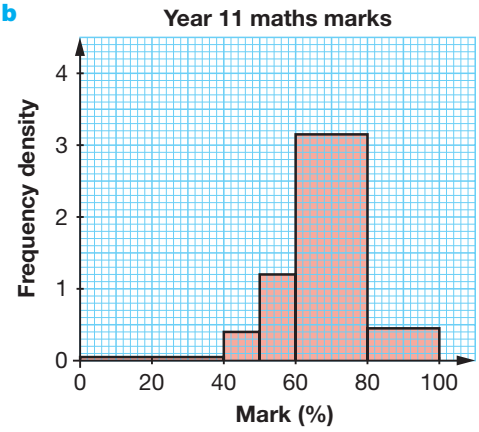


3 ▶



4 ▶ a 67.8%

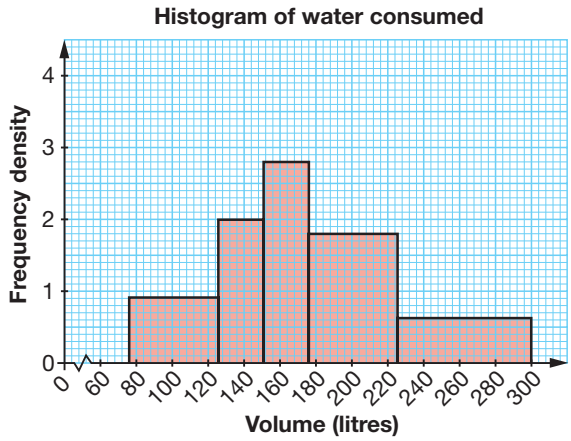
b



EXERCISE 1*

1 ▶ a 175 litres/day

b



2 ▶ 102

3 ▶ a The number waiting for less than 20 minutes is 150. The total number of patients is 310, so Rachel is wrong.

b 38 patients

4 ▶ a Bar for $30 < t \leqslant 60$ class drawn with a frequency density of 1.8

Bar for $100 < t \leqslant 200$ class drawn with a frequency density of 2.5

Frequency for the $60 < t \leqslant 100$ class = 64

b 112 minutes

EXERCISE 2

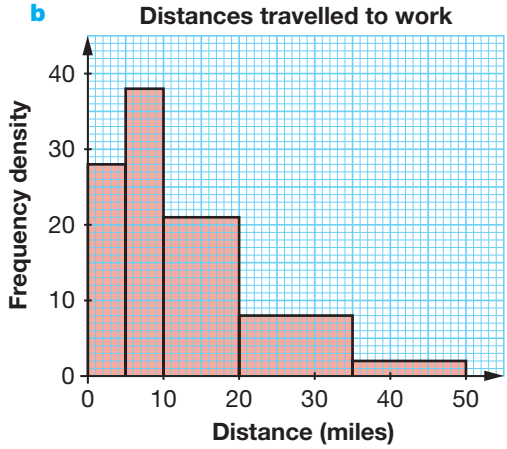
1 ▶ a 2 **b** 17 **c** 40

2 ▶ a 27 **b** 59 **c** 87

3 ▶ a

Distance, d (miles)	Frequency
$0 < d \leqslant 5$	140
$5 < d \leqslant 10$	190
$10 < d \leqslant 20$	210
$20 < d \leqslant 35$	120
$35 < d \leqslant 50$	30

b



4 ▶ **a** 25 **b** 49 **c** 3.6 kg

d

Weight, w (kg)	Frequency
$2.8 < w \leqslant 3.0$	5
$3.0 < w \leqslant 3.5$	15
$3.5 < w \leqslant 4.0$	25
$4.0 < w \leqslant 4.2$	4

e 3.54 kg **f** 27

EXERCISE 2*

1 ▶ **a** 52 **b** 172 cm **c** 38

2 ▶ **a** 90 plants **b** 22 cm **c** 16 plants

3 ▶ **a** 23.8 g

b

Mass, m (grams)	Frequency
$16 < m \leqslant 20$	8
$20 < m \leqslant 22$	14
$22 < m \leqslant 24$	20
$24 < m \leqslant 25$	19
$25 < m \leqslant 26$	13
$26 < m \leqslant 29$	6

c 23.25 g

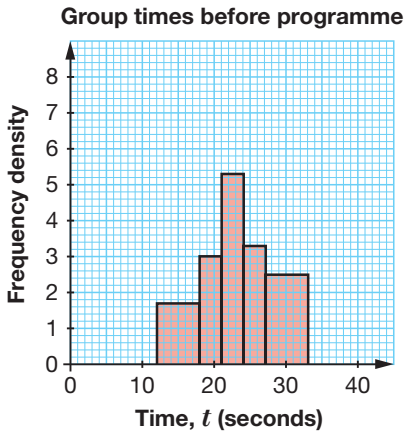
4 ▶ **a** 250 runners **b** 40 runners

c **i**

Time, t (minutes)	Frequency
$35 < t \leqslant 45$	18
$45 < t \leqslant 50$	54
$50 < t \leqslant 60$	110
$60 < t \leqslant 70$	40
$70 < t \leqslant 90$	28

ii 56.7 minutes

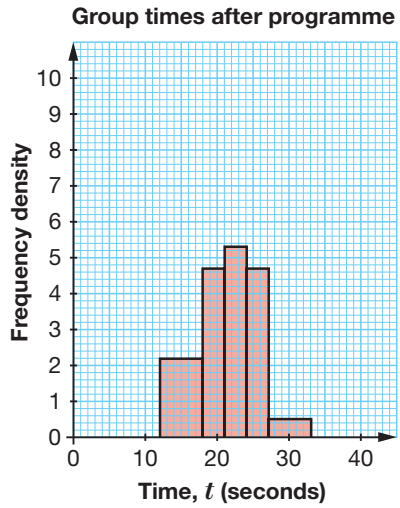
ACTIVITY 1



Median = 23.1 s

Mean = 23.2 s

IQR = 7.3 s



Median = 21.6 s

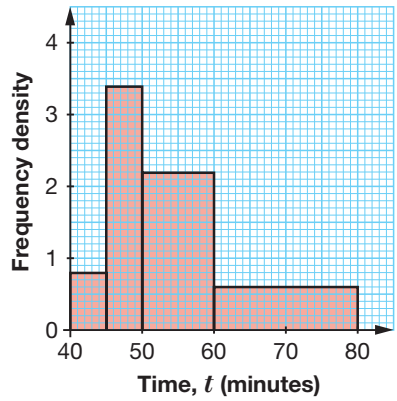
Mean = 21.3 s

IQR = 6 s

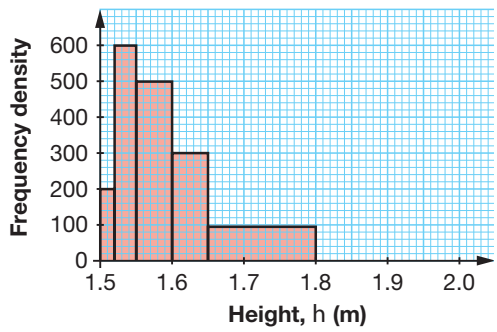
Evidence suggests that the Brain Training programme does have a positive impact. The mean and median times are both reduced for the group and there appears to be less dispersion of ability after the programme, as the IQR is also reduced. The second histogram is shifted slightly towards the shorter times.

EXERCISE 3 REVISION

1 ▶



2 ▶

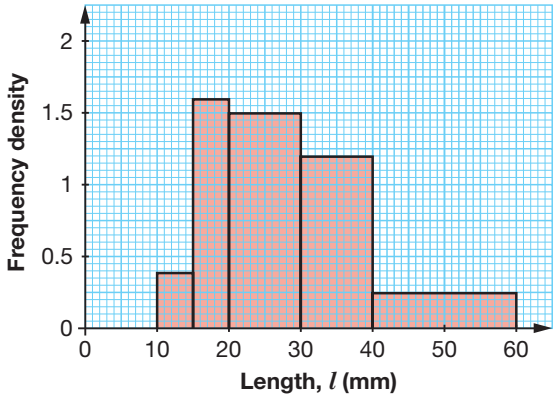


3 ▶ **a** 40 **b** 230 **c** 94

4 ▶ **a** 46 **b** 23.5th = 4.9375 **c** 15

EXERCISE 3* REVISION

1 ▶



2 ▶ a Frequency for the $30 < t \leqslant 60$ class = 33

Bar for $150 < t \leqslant 210$ class drawn with a frequency density of 1.5

b 105 vehicles

3 ▶ About 86 or 87 farms

4 ▶ a Frequency for the $160 < h \leqslant 165$ class = 18

Frequency for the $165 < h \leqslant 170$ class = 25

Bar for $140 < h \leqslant 160$ class drawn with a frequency density of 1.1

Bar for $170 < h \leqslant 185$ class drawn with a frequency density of 1.8

Bar for $185 < h \leqslant 200$ class drawn with a frequency density of 0.8

b There are 104 adults in total, so the median height is the mean of the 52nd and 53rd heights.

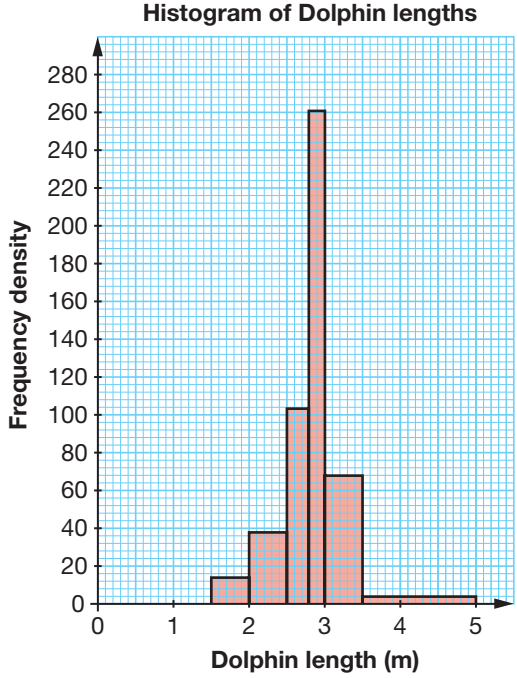
The number of people up to 167 cm is 22 + 18 + 10 = 50 and the number of people up to 168 cm is 22 + 18 + 15 = 55.

So the median lies between 167 cm and 168 cm, and Clare is correct.

c 168.4 cm

EXAM PRACTICE: HANDLING DATA 6

1 ▶ a



b i 2.85 m **ii** 2.87 m **iii** 24

c $2.8 < l \leqslant 3.0$ m

2 ▶ a 100 **b** 85 **c** 1.51 cm

d

Height, (m)	Frequency
$1.4 < m \leqslant 1.45$	5
$1.45 < m \leqslant 1.48$	15
$1.48 < m \leqslant 1.5$	20
$1.5 < m \leqslant 1.55$	20
$1.55 < m \leqslant 1.6$	15
$1.6 < m \leqslant 1.7$	10

e 1.524 cm **f** 36