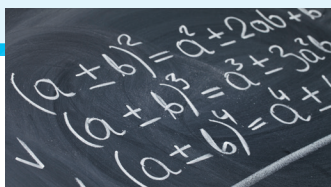## 3.1 Simplifying algebraic expressions

You will learn to:
- Simplify expressions by collecting like terms.

Confidence

Why learn this?
Algebra is a language that people in every country in the world can understand. It doesn't need to be translated into Japanese, Spanish or any other language.

Fluency
Write these additions as multiplications:
- 5 + 5 + 5
- 9 + 9 + 9 + 9 + 9
- 10 + 10
- 18 + 18 + 18 + 18 + 18

Explore
Why do we 'simplify' in algebra?

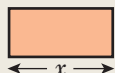## Exercise 3.1

Warm up

1 Write using index notation.
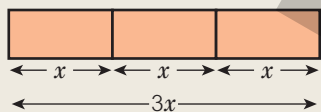   a $3 \times 3 \times 3 \times 3$
   b $2 \times 2 \times 2$
   c $5 \times 5 \times 5 \times 5 \times 5 \times 5$

> **Key point**
> An **algebraic expression** e.g. $3x + 2y$, contains numbers and letters.
> Each part of an algebraic expression is called a **term**.

Worked example

Simplify $x + x + x$

Think of a rod that is x cm long.

When you put three rods together the total length is $3x$ cm.

$\leftarrow x \rightarrow \leftarrow x \rightarrow \leftarrow x \rightarrow$
$\leftarrow 3x \rightarrow$

$3x$ — So $x + x + x = 3x$

2 Simplify
   a $n + n$
   b $y + y + y + y + y$
   c $2a + 3a$
   d $5b + 6b$
   e $5a - 3a$
   f $8b - 3b$
   g $7y + 2y - 3y$
   Discussion Why is $x + x + x + x$ the same as $4x$?

> **Key point**
> **Like terms** contain the same letter (or do not contain a letter).
> You simplify an expression by collecting like terms.

3 Simplify by collecting **like terms**.
   a $2x + 4x + 2 = 6x + \square$
   b $2b + 6c - 3c$
   c $6y - 2y + 8 - 3b$
   d $4y - 2 + 3y$
   e $9x + 3 - 3y - 7x$
   f $9a - 7b + 2a + 5$
   Discussion Are the two expressions $3x + 2y$ and $2y + 3x$ equivalent?

> **Q3e hint**
> $9x - 7x = \square$

4 Copy and complete these addition pyramids. Each brick is the sum of the two bricks below.

a

| 2a | 3b | a |

b

| 5x | 7 | −2x |

c

| 2x + 3 | 10 | 4y + 7 |
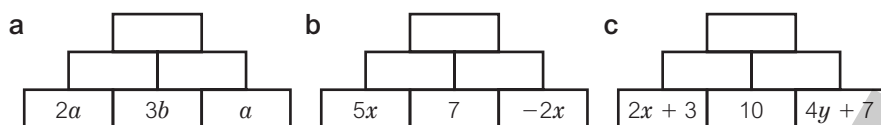
5 Simplify

  a $b × b$

  b $t × t × t$

  c $m × m × m × m$

6 Simplify

  a $2x^2 + 3x^2 = \square x^2$ **b** $4a + 2b^2 + 3b^2$

  c $2b^2 + 3b + b^2$ **d** $5x + 2x^2 + 7x$

  e $8x^4 − x^4$ **f** $12x^2 + 3x^3 − 2x^3$

7 Simplify

  a $a × b$ **b** $t × t × b$

  c $p × p × p × y × y$ **d** $m × 2$

  e $x × 5$ **f** $q × 7 × p$

Worked example

Simplify

a $3b × 2b$

$3b × 2b = 3 × b × 2 × b$
$\qquad\quad = 3 × 2 × b × b$ ——— The order of multiplication does not matter.
$\qquad\quad = 6b^2$

b $\dfrac{8b}{4}$

$\dfrac{8b}{4} = 2b$ ——— $\dfrac{8b}{4}$ means $8b ÷ 4$. Work out $8 ÷ 4$.

8 Simplify

  a $2b × 5b$ **b** $9a × 3a$ **c** $3a × 2a × 3a$

  d $\dfrac{12b}{4}$ **e** $\dfrac{9a}{2}$ **f** $\dfrac{36b}{12}$

9 Match the equivalent expressions.

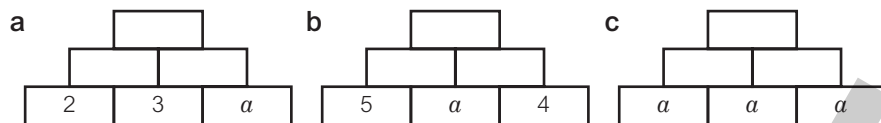| $2x$ | $4x − 3x$ | $x × x$ | x | $3x + 4x$ |

| $x + x$ | $4x^2$ | $3x$ | $2x × 2x$ | x^2 |

| $x × 2x$ | $7x$ | $\dfrac{9x}{3}$ | $2x^2$ |

10 Copy and complete these multiplication pyramids.
Each brick is the product of the two bricks below.

a

| 2 | 3 | a |

b

| 5 | a | 4 |

c

| a | a | a |

11 In between which pairs of expressions can you write $\equiv$?
  a $a + b \;\square\; b + a$
  b $a - b \;\square\; b - a$
  c $ab \;\square\; ba$
  d $a \div b \;\square\; b \div a$

Investigation   **Problem-solving**

1 This is an addition pyramid. Work out the missing values.

	$5a + 4b$	
$3a + 2b$		
		a
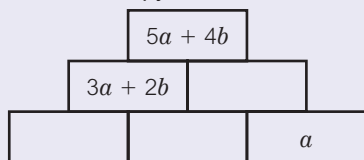
2 How many different possibilities can you find for this addition pyramid?

	$5a + 4b$	
$3a + 2b$		
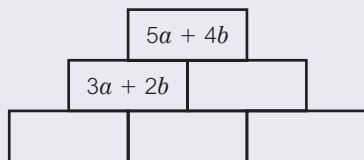
3 This is a multiplication pyramid. How many different possibilities can you find?

	$8a^2$	
$2a$		
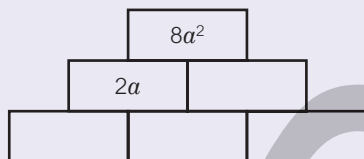
12 **Explore** Why do we 'simplify' in algebra?
Is it easier to explore this question now that you have completed the lesson?
What further information do you need to be able to answer this?

13 **Reflect** In algebra, letters are used to represent values we do not know. This lesson may be the first time you have done algebra.
Choose **A**, **B** or **C** to complete each statement.
In this lesson, I did...   **A** well   **B** ok   **C** not very well
So far, I think algebra is...   **A** easy   **B** ok   **C** difficult
When I think about the next lesson,
    I feel...   **A** confident   **B** ok   **C** unsure
If you answered mostly **A**s and **B**s, did your experience surprise you? Why?
If you answered mostly **C**s, then look back at the questions you found most difficult. Ask a friend or your teacher to explain them to you. Then answer the statements above again.

Explore

Reflect

3.2 Writing algebraic expressions

You will learn to:
- Write expressions using four operations.

Why learn this?
Computers are programmed using a computer algebra system (CAS).

Fluency
Work out:
- 3^2
- 5^3
- 1^4

Explore
Think of a number. Double it. Add 10. Divide by 2. Subtract your original number. Try this with different numbers. What answer do you get? Why?

Confidence

Exercise 3.2

1 Simplify

  a $2x + 3x - 5x$ **b** $3x^2 - 4x + 2x^2$ **c** $3x + 5 - 2x + 4$

2 Simplify

  a $y \times y$ **b** $b \times b \times b$ **c** $4 \times 2n$

  d $4b \times 2b$ **e** $\dfrac{16c}{4}$

3 John collects coins. He has b coins. Write an expression for how many he has when there are

  a 2 more   **b** 4 fewer   **c** 17 more

  d 5 times as many   **e** half as many.

Q3a hint

| b + ☐ |
| b | 2 |

Q3d hint

☐b ; b b b b b

Q3e hint
Finding half is the same as dividing by 2.

4 Haruto is m years old. Write expressions for the ages of each of these people.

  a Laila is 4 times as old as Haruto.

  b Maggie is 5 years older than Haruto.

  c Ami is 6 years younger than Haruto.

  d Iman is half the age of Haruto.

  e Rashid is 5 years older than twice Haruto's age.

  f Ruth is 3 years younger than 5 times Haruto's age.

5 Write an algebraic expression for

  a y more than x **b** x multiplied by y

  c y less than x **d** x more than 2 times y

  e 3 times y add 4 times x **f** y multiplied by itself

  g 4 times x multiplied by itself   **h** 7 less than y multiplied by itself

  i x divided by y **j** 2 more than 20 divided by x.

Q5a hint
Try it with numbers. How would you write 5 more than 3?

Topic links: Order of operations, Graphs

6 t represents a number. Write and simplify an expression for

  a 2 more than triple the number   **b** 5 less than double the number

  c 4 more than double the number   **d** the number added to itself

  e the number subtract 5   **f** the number multiplied by itself

  g the number divided by 3   **h** 3 divided by the number.

Worked example

Write an expression for each function machine.

a Input   Output

$$m \rightarrow \boxed{\times 2} \rightarrow \boxed{+3} \rightarrow$$

$m \times 2 + 3 = 2m + 3$ m is multiplied by 2 then 3 is added.

b Input   Output

$$x \rightarrow \boxed{-2} \rightarrow \boxed{\div 3} \rightarrow$$
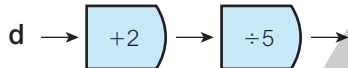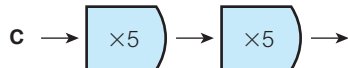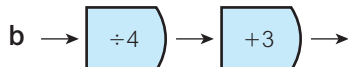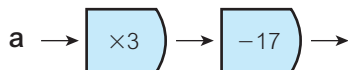
$\dfrac{x-2}{3}$ To show that the whole expression is divided by 3 draw a long division line.

---

7 Write an expression for the output of each function machine.

  a $\rightarrow \boxed{\times 3} \rightarrow \boxed{-17} \rightarrow$

  b $\rightarrow \boxed{\div 4} \rightarrow \boxed{+3} \rightarrow$

  c $\rightarrow \boxed{\times 5} \rightarrow \boxed{\times 5} \rightarrow$

  d $\rightarrow \boxed{+2} \rightarrow \boxed{\div 5} \rightarrow$

8 A rectangle has width b. The length is 5 more than the height.

             b

  a Write an expression for the length.

  b Write and simplify an expression for the perimeter.

  c Calculate the perimeter of the rectangle when $b = 10$ cm.

9 **Explore** Think of a number. Double it. Add 10. Halve it. Take away your original number. Try this with different numbers. What answer do you get? Why?

  Is it easier to explore this question now that you have completed the lesson?

  What further information do you need to be able to answer this?

10 **Reflect** This lesson suggested bar modelling and function machines to help you with writing expressions. Did they help you? How?

  Did you use any other methods? Explain the method(s) you used.

Explore

Reflect

3.3 STEM: Using formulae

Confidence

You will learn to:
- Substitute into formulae.

Why learn this?
You can substitute into formulae to work out all sorts of things – from the volume of the Earth to the cooking time of a meal.

Fluency
Work out:
- $2 + 3 \times 4$
- $\dfrac{4 + 5}{2}$
- $\dfrac{4 \times (5 + 2)}{2}$

Explore
How can you predict your adult height?

Exercise 3.3

Warm up

1 A recipe gives this formula to work out how long it takes to cook a chicken:

    $(50 \times \text{mass (in kg)} + 40)$ minutes

How long does it take to cook a 2 kg chicken?

2 **STEM** To calculate the maximum heart rate when exercising, use

    maximum heart rate = 220 – age in years.

Work out the maximum heart rate for these ages.

  a 18   **b** 45   **c** 79

3 Work out the value of each expression when $a = 3$.

  a $2a$ **b** $a + 3$ **c** $a - 5$ **d** a^2 **e** $10 - a$

4 Given $x = 3$, $y = 5$, $z = 8$ work out the value of

  a xy **b** $xz + 5$ **c** $2(x + 1)$ **d** $\dfrac{z}{2}$ **e** $\dfrac{x + y}{2}$

> **Q3a hint**
>
> $2a = 2 \times a = 2 \times \square$

> **Q4a hint**
>
> $\square \times \square$

> **Key point**
>
> A **formula** is a general rule for a relationship between quantities. You use a formula to work out an unknown quantity by substituting.

Worked example

The **formula** used to calculate speed is: $\text{speed} = \dfrac{\text{distance}}{\text{time}}$

Work out the speed of a cyclist who travels 1000 metres in 20 seconds.

$Speed = \dfrac{1000}{20}$

$= 50 \, m/s$

> Substitute the values into the formula.
> Write the units.
> m/s means metres per second.

5 **STEM** Use the formula $\text{speed} = \dfrac{\text{distance}}{\text{time}}$ to work out the speed of each of these cyclists in metres per second.

  a distance = 3000 m   time = 360 seconds

  b distance = 600 m   time = 50 seconds

  c distance = 10 000 m   time = 640 seconds

Topic links: Order of operations, Negative numbers   **Subject links:** Science (Q5–12)

6 **STEM** In physics the formula $F = ma$ is used to calculate force, F, where m = mass and a = acceleration.
Work out the value of the force (F) when
   a $m = 2$, $a = 27$ b $m = 5$, $a = 32$ c $m = 25$, $a = 7$
   Discussion When $F = 20$ and $a = 5$ could you work out the value of m?

7 **STEM** Weight (W) in newtons (N) is calculated using the formula
$W = mg$, where m = mass in kg and g = acceleration due to gravity in m/s^2.
   a On Earth $g = 10$ m/s^2. Work out the weight, in newtons, of
      i a 5 kg dog
      ii a 70 kg man
      iii a 30 kg monkey.
   b On the Moon $g = 1.6$ m/s^2.
      Work out the weight of the dog, the man and the monkey on the Moon.

8 **STEM** The formula to calculate pressure (P) in N/m^2 is $P = \dfrac{F}{A}$, where
F = force in N and A = area in m^2. Work out the pressure when
   a $F = 20$, $A = 2$
   b $F = 100$, $A = 25$

9 **STEM** An engineer uses the formula $V = IR$ to work out the voltage in a
circuit, where I is the current (in amps) and R is the resistance (in ohms).
Work out the voltage, V, of a circuit with
   a current 4 amps and resistance 10 ohms
   b current 3.1 amps and resistance 15 ohms
   c current 7.2 amps and resistance 20 ohms.

10 The formula for the perimeter of a rectangle is $P = 2l + 2w$.
Work out the perimeter when
   a $l = 12$ cm and $w = 2$ cm
   b $l = 4$ m and $w = 5$ m
   c $w = 10.5$ cm and $l = 6$ cm.

11 **STEM** To convert from °C (C) to Kelvin (K) scientists use the
formula $K = C + 273$.
Convert these temperatures to Kelvin.
   a 100°C   b −20°C   c 0°C   d −100°C

12 **STEM** The formula for converting a temperature from Fahrenheit (F) to
Celsius (C) is $C = \dfrac{5(F - 32)}{9}$.
Convert these temperatures into °C.
   a 41°F   b 59°F   c 77°F   d 23°F

13 **Explore** How can you predict your adult height?
Is it easier to explore this question now you have completed the lesson?
What further information do you need to be able to answer this?

14 **Reflect** Look back at the formula in Q6.
   a Would it matter if this formula used the letters x and y instead of
      m and a?
   b Do the letters help you to understand a formula?

Q14 hint
If you used different letters would
your answers be different?

3.4 Writing formulae

Confidence

You will learn to:
• Write formulae from a description.

	A	B	C	D	E
1	The Cupcake Shop - First Quarter Sales				
2		January	February	March	Total
3	Red velvet	£1,292	£1,156	£1,208	£3,656
4	Lemon drizzle	£2,047	£1,987	£1,999	£6,033
5	Vanilla	£1,795	£1,896	£1,689	£5,380
6	Fudge	£1,250	£1,346	£1,287	£3,883
7	Total revenues	£6,384	£6,385	£6,183	£18,952

E3 | fx | =B3+C3+D3

Fluency
Work out the mean of 6, 12, 12.

Explore
What is the formula to convert weeks into minutes?

Why learn this?
You can write formulae into a spreadsheet so that it automatically does all the calculations for you.

Exercise 3.4

Warm up

1 Write algebraic expressions for
  a 2 more than x
  b 5 less than y
  c the cost of x apples at 20p each.

2 When $x = 2$ and $y = 7$ work out the value of
  a xy **b** $2x + 4$ **c** $9x - 2y$ **d** $3y + 2x$

3 An online company charges \$5 to rent a film and \$10 to download a film. It uses the formula $C = 5r + 10d$.
  a What do you think r stands for?
  b What do you think d stands for?
  c How much would 3 rentals and 2 downloads cost?

4 Alika earns £9 per hour.
  a How much does she earn in 5 hours?
  b How much does she earn in 12 hours?
  c Write an expression for how much she earns in x hours.
  d Write a formula for her earnings, E, in x hours.
  Discussion What is the difference between an expression and a formula?

Q4 Literacy hint
To 'earn' money means to make money.

Q4d hint
E = your expression from part **c**.

Literacy hint
'per week' means each week.

Worked example
Storing furniture in a warehouse costs \$12 per week.
Write a formula for the cost, C, of storing furniture for y weeks.

$12y$ ——— Write down the cost each week. Multiply the cost by the number of weeks.

$C = 12y$ ——— Write C = your expression.

Topic links: Order of operations, Negative numbers

Subject links: PE (Q8), ICT(Q12), Food technology (Q14)

5 Renting a go-cart costs £15 per hour.
   a Write an expression for the cost for x hours.
   b Write a formula, $C = \square$.

6 The amount of bread a restaurant orders depends on the number of bookings each day. They order 10 more loaves of bread than they have bookings.
   Write a formula that connects the number of bookings, b, to the number of loaves of bread, L.

7 A library buys bookcases. Each bookcase has 6 shelves.
   Write a formula that connects the total number of shelves in the library, L, to the number of bookcases, B.

8 Modelling
   a Write an algebraic expression for finding the mean of three numbers x, y and z.
   b Write a formula for the mean of three numbers.
   c Use your formula to work out the mean when $x = 5$, $y = 22$ and $z = 12$.

9 Real A mobile phone company charges £12.50 per month and £4 per gigabyte (GB) of data.
   a Work out the cost when
      i 10 GB are used
      ii no data is used.
   b Write a formula for the monthly cost, C, when n gigabytes of data are used.

10 A function machine multiplies each input by 5 and then adds 3.
   Work out the output when the input is
   a **i** 5
      ii −2
      ii −7
      iv x
   b Write a formula that connects the output, y, with the input, x.

11 The cooking time for a piece of meat is 45 minutes per kg plus an extra 20 minutes.
   a Work out the cooking time for a 3 kg piece.
   b Write a formula that connects the cooking time (in minutes), C, to the mass in kg, k.

12 Explore What is the formula to convert weeks into minutes?
   Is it easier to explore this question now that you have completed the lesson?
   What further information do you need to be able to answer this?

13 Reflect In this lesson you wrote your own formulae. In lesson 3.3 you were given formulae to work with. Which did you find more difficult? What made it more difficult?
   Are there particular kinds of questions you need more practice on? If so, what kinds?

Explore

Reflect

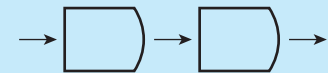# 3.5 Brackets and formulae

You will learn to:
• Expand expressions involving brackets

Confidence

Why learn this?
Science formulae often contain brackets.
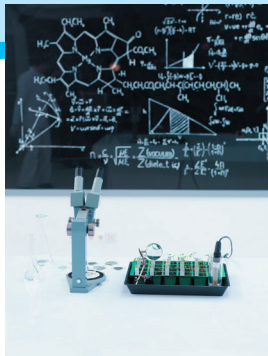
Fluency
Work out:
• $5(2 + 3)$
• $2(10 - 4)$
• $4(3 - 6)$

Explore
How do you convert temperatures in degrees Celsius to degrees Fahrenheit?

Exercise 3.5

Warm up

1 Simplify
  a $3 \times y$ **b** $a \times 6$ **c** $p \times -3$ **d** $7 \times 2s$ **e** $9z \times 5$

2 Simplify
  a $b \times b$ **b** $m \times 2m$ **c** $a \times a \times a$ **d** $3b \times 2b$ **e** $6n \times 3n$

3 a Work out
  i $3(2 + 5)$ **ii** $3 \times 2 + 3 \times 5$
  What do you notice?
 b Work out
  i $4(6 - 1)$ **ii** $4 \times 6 + 4 \times -1$
  What do you notice?

Key point

Expand a bracket means multiply every number inside the bracket by the number or letter outside the bracket.

Worked example

Expand
a $2(x + 3)$

$2(x + 3) = 2 \times x + 2 \times 3$
$\qquad\quad = 2x + 6$

b $5(y - 2)$

$5(y - 2) = 5 \times y + 5 \times -2$
$\qquad\quad = 5y - 10$

4 Expand
  a $3(x + 4)$ **b** $2(n + 12)$
  c $3(p - 7)$ **d** $4(y - 5)$
  e $2(2 + r)$ **f** $5(8 - b)$
  g $8(2 - q)$ **h** $10(10 - a)$
  Discussion How could you check that your answers are correct?
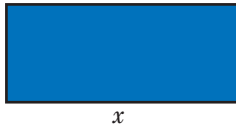
Topic links: Order of operations

5 Anne earns x pounds per hour. Jamil earns £3 more per hour than Anne. On Sunday he gets paid double.
Write an algebraic expression in terms of x for the amount Jamil earns per hour on Sunday.

6 The length of a rectangle is x cm.

x

Its width is 5 cm less than its length.
a Write an expression for the width of the rectangle.
b Write and simplify an expression for the area of the rectangle.
c Copy and complete the formula for calculating the area, $A = \square$.
d Use the formula to work out the area of the rectangle when
 i $x = 10$
 ii $x = 12$

7 Ravi is x years old. Ana's age is the square of Ravi's.
Bryony is 5 years older than Ana.
a Write an expression for Ana's age.
b Write an expression for Bryony's age.
c Write and simplify an expression for the sum of all their ages.
d Ravi is 5 years old. What is the sum of their ages?
Discussion How did you answer part **d**? Is there more than one way? Which is the quickest way?

8 Expand
a $b(b + 4)$ **b** $y(y - 2)$
c $t(10 + t)$ **d** $r(2 - r)$
e $w(3w + 2)$ **f** $p(10 + 4p)$
g $q(15 - 2q)$ **h** $2r(3r + 1)$
i $8m(2m - 3)$ **j** $2b(20 - 4b)$

9 **Explore** How do you convert temperatures in degrees Celsius to degrees Fahrenheit?
Is it easier to explore this question now that you have completed the lesson?
What further information do you need to be able to answer this?

10 **Reflect** Write a definition, in your own words, for
 • expand
 • simplify.
Compare your definitions with those written by others in your class.
Can you improve your definitions?

Explore

Reflect

3 Check up

Simplifying expressions

1 Expand
  a $3(x + 4)$ **b** $2(a - w)$ **c** $5(11 - x)$

2 Simplify
  a $x + x$ **b** $4x + 7x$ **c** $10c - 5c$
  d $4t - t$ **e** $7x + 2b - 5x$

3 Simplify
  a $y \times y \times y$ **b** $x \times x$ **c** $3 \times t \times t$
  d $2 \times r \times r \times r \times 5$ **e** $5r \times r$ **f** $7t \times 2t$
  g $y \div 7$ **h** $\dfrac{12y}{6}$

4 Simplify
  a $x^2 + 3x^2$ **b** $x + x^2 + x$ **c** $2 + x^2 + 2x^2 - 5$

5 Expand
  a $x(x + 3)$ **b** $b(b - 2)$ **c** $a(10 - a)$
  d $2x(3x + 1)$ **e** $4t(10 - 2t)$

Substitution

6 Area of rectangle = length × width
Work out the area of a rectangle with width = 12 cm and length = 7 cm.

7 $T = 5B$ What is the value of T when $B = 12$?

8 Density = $\dfrac{\text{mass}}{\text{volume}}$, where mass is in kg, volume is in m^3 and density is in kg/m^3.

Work out the density of a block with mass 20 kg and volume 4 m^3.

9 The approximate perimeter, P, of a semicircle can be calculated using the formula $P = a + \dfrac{3a}{2}$
Work out the approximate perimeter when $a = 4$ cm.
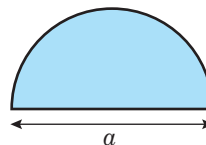
10 Use the formula $b = 10t - c$ to work out the value of b when
  a $t = 3$, $c = 5$
  b $t = 1$, $c = 7$
 **c** $t = 4$, $c = -2$
  d $t = 3$, $c = -4$

11 Work out the value of the expression $ab + 2c$ when $a = 2$, $b = 5$, $c = 9$.

**12** What is the value of x^2 when $x = 7$?

Writing expressions and formulae

13 Mia has x stamps. Write expressions for the number of stamps each person has.
   a Carl has 7 fewer than Mia.
   b Onick has 12 times as many as Mia.
   c Mehmet has half as many as Mia.

14 Jack is paid $5 for each hour he babysits.
   Write a formula that connects the total amount he is paid, T, and the number of hours he babysits, x.

15 Write an algebraic expression for
   a a more than b
   b 3 more than a, multiplied by b
   c a multiplied by itself
   d b divided by 5.

16 A regular pentagon has 5 sides of equal length.
   Write a formula that connects the perimeter, P, to the length of one of the sides, a.

17 A class has 30 students. A teacher buys sweets to share between them.
   Write a formula that connects the number of sweets each student receives, S, and the number of sweets the teacher buys, p.

18 A square has sides a cm long.
   Write a formula for finding the area of the square, A, using the length of the side, a.

19 **How sure are you of your answers? Were you mostly**
   🙁 **Just guessing**   😐 **Feeling doubtful**   🙂 **Confident**
   What next? Use your results to decide whether to strengthen or extend your learning.
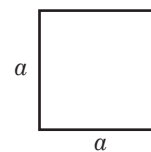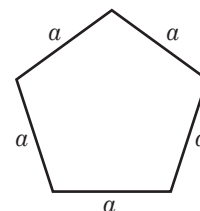
Challenge

**20** A pattern is made of squares and rectangles.

   Pattern 1   Pattern 2   Pattern 3
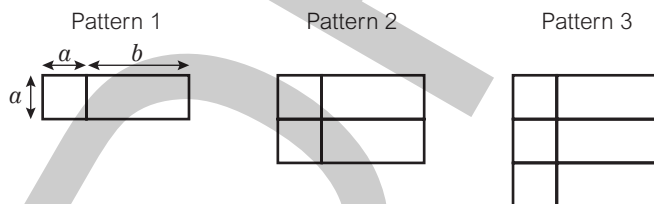
   Write and simplify an algebraic expression for the area of
   a Pattern 1   **b** Pattern 2   **c** Pattern 3
   d Pattern 10   **e** Pattern n.

21 Find a value of x so that
   a x^2 is equal to $2x$ **b** x^2 is equal to x^3.

22 $a + b = -2$ and $a - b = -6$.
   a and b are whole numbers. What are the values of a and b?

3 Strengthen

You will:
- Strengthen your understanding with practice.
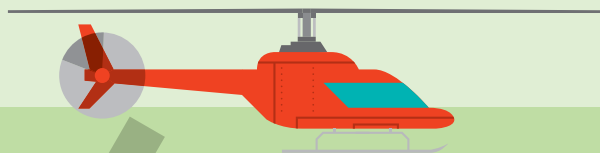
Simplifying expressions

1 Copy and complete.
  a $p + p + p = \square p$
  b $m + m + m + m = \square m$
  c $d + d$
  d $t + t + t + t + t$

2 Simplify
  a $2t + 3t$
  b $5g + 7g$
  c $10y - 3y$
  d $5p - p$
  e $10y + 2b + 3y$
  f $6m + n + 5m$
  g $4a + 3b - a$
  h $3q + 2b - 3b$
  i $4t + 7 - 2t$
  j $4y + 8 - 2 + 3y$

3 Expand $3(2 + 4)$

> **Q3 hint**
>
> $3 \times (2 + 4)$
>
> | 2 | 4 | | 2 | 4 | | 2 | 4 |
>
> $= 3 \times 2$ + 3×4
>
> | 2 | 2 | 2 | | 4 | 4 | 4 |

4 Copy and complete.
  a $2(x + 3) = \square x + \square$
  b $3(x + 4) = (x + 4) + (x + 4) + (x + 4) = \square + \square$
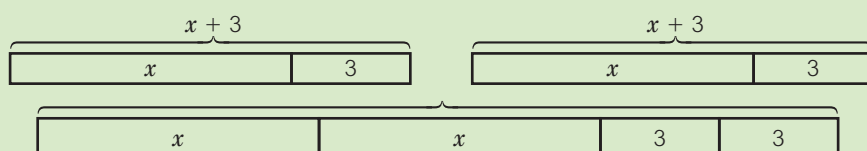 **c** $4(b + 2)$
  d $5(t + 3) = \square \times t + \square \times 3 = \square t + \square$
  e $3(6 + a)$
  f $2(r - 3) = \square \times r + \square \times -3$
  g $6(10 - b)$

> **Q4a hint**
>
> | $x + 3$ | | $x + 3$ | |
> | x | 3 | x | 3 |
>
> | x | x | 3 | 3 |

Hints (sidebar)
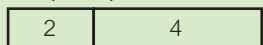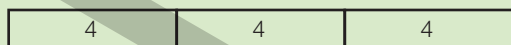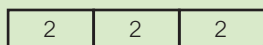
> **Q1 hint**
>
> Draw bars to help.
>
> | p | p | p |
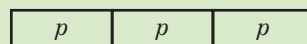
> **Q2a hint**
>
> $\square t$
>
> | t | t | t | t | t |
>
> $\underbrace{2t}$ $\underbrace{3t}$
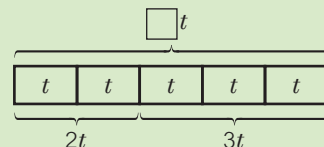
> **Q2e hint**
>
> Add the y terms first.
>
> | $10y$ | $3y$ |
>
> $\square y$
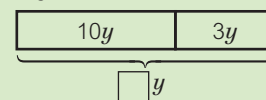
> **Q2i hint**
>
> Numbers, e.g. 7, can only be added to other numbers.

5 Write the missing numbers.
  a $6 \times 6 \times 6 = 6^{\square}$
  b $5 \times 5 \times 5 \times 5 = 5^{\square}$
  c $11 \times 11 = 11^{\square}$

6 Match each expression on a blue card to one on a yellow card.

a $a \times a \times a$	**i** a^4
b $a \times a$	**ii** a^6
c $a \times a \times a \times a \times a$	**iii** a^2
d $a \times a \times a \times a$	**iv** a^3
e $a \times a \times a \times a \times a \times a$	**v** a^5

7 Copy and complete.
  a $2w \times 3w =$
  b $4a \times 2a =$
  c $3b \times 5b =$
  d $8m \times 3m =$
  e $9n \times 11n =$

8 Simplify these. Which is the 'odd one out'?
  a $n \times n$
  b $n + n$
  c $2 \times n$
  d $n \times 2$

9 Copy and complete.
  a $m(m + 1) =$
  b $b(b + 2) =$
  c $d(3 + d) =$
  d $r(r - 1) =$
  e $m(m - 3) =$
  f $t(10 - t) =$

10 a Complete the pattern.
  $t + t = 2t$
  $t^2 + t^2 = 2t^2$
  $t^3 + t^3 = 2t^3$
  $t^4 + t^4 = \square$
  b Simplify by collecting like terms.
  i $p^2 + p^2$
  ii $x^3 + x^3$
  iii $m^2 + m^2 + m^2$
  iv $2x^2 + 3x^2$

11 Simplify by collecting like terms.
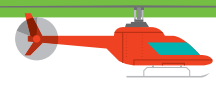 **a** $t^2 + t^2 + 3t = \square t^2 + \square$
  b $p^3 + p + p$
  c $3x + x^2 + 2x$

Substitution

1 Molly earns £9 per hour. She uses this formula to work out her pay.
Pay = 9 × number of hours
Work out how much she is paid for 8 hours.

2 The formula to work out the distance a car travels is
distance = speed × time
A car travels at a speed of 50 km per hour for 2 hours.
How far does it travel?

3 Work out the value of each expression when $m = 2$ and $n = 6$.

a $m + 3$ 　　　　　 **b** $n - 5$ 　　　　　 **c** $m + n$

d $4m$ 　　　　　 **e** $3n$ 　　　　　 **f** $\dfrac{n}{2}$

g mn 　　　　　 **h** $mn + 2$ 　　　　　 **i** $\dfrac{n}{m}$

4 The formula for the area of a rectangle is $l \times w$ where l = length and w = width.
Work out the area of a rectangle when

a $l = 3$, $w = 5$
b $l = 9$, $w = 7$
c $l = 4$, $w = 4$
d $l = 20$, $w = 1$

5 $P = 10(a + b)$
Work out the value of P when

a $a = 9$, $b = 3$
b $a = 2$, $b = 5$
c $a = 3$, $b = 12$

6 Use the formula $P = 10 + m$ to work out the value of P when

a $m = -2$ 　　　　 **b** $m = -5$ 　　　　 **c** $m = -10$

7 Work out the value of n^2 when

a $n = 5$ 　　　　 **b** $n = 1$ 　　　　 **c** $n = 7$

8 Copy and complete the calculations when $m = 2$.

a $m^4 = \square \times \square \times \square \times \square = \square$ 　　　 **b** $m^2 + 1 = \square \times \square + 1 = \square$
c $m^2 - 2 = \square \times \square - 2 = \square$ 　　　 **d** $3m^2 = 3 \times \square \times \square = \square$

Writing expressions and formulae

1 Match each algebraic expression to its description.

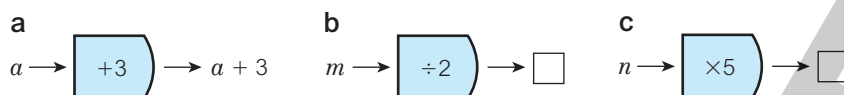|	
a $x + 3$	**i** 3 less than x
b $x - 3$	**ii** x less than 3
c $3x$	**iii** 3 more than x
d $\dfrac{x}{3}$	**iv** one third of x
e $3 - x$	**v** 3 times x

2 To estimate the number of bulbs that will grow, S, Eddie divides the number planted, p, by 3.
Copy and complete the formula $S = \dfrac{\square}{\square}$

3 Write an expression for each function machine. The first one has been done for you.

a

$a \longrightarrow \boxed{+3} \longrightarrow a + 3$

b

$m \longrightarrow \boxed{\div 2} \longrightarrow \square$

c

$n \longrightarrow \boxed{\times 5} \longrightarrow \square$

4 To convert a decimal, m, to a percentage, p, multiply by 100.
Choose the correct formula connecting m and p.

$p = m + 100 \qquad p = 100m \qquad p = \dfrac{m}{100} \qquad p = 100 - m$

5 Write a formula to convert mm, x, into cm, y.

6 Write a description for each expression.

a $x + y$ **b** xy **c** $x - y$ **d** $\dfrac{x}{y}$

e $y + x$ **f** $y - x$ **g** $\dfrac{y}{x}$ **h** yx

7 To convert from km, K, to miles, M, divide by 8 then multiply by 5. Write the formula.

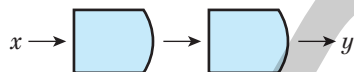**8** To find the mean, M, of two values, a and b, add them together and divide by 2.
a Write an expression for a add b divided by 2.
b Write a formula for finding the mean of a and b. $M = $ _____

9 I think of a number, add 3, and then divide by 8.
a What would the result be if the original number was 21?
b Copy and complete the function machine.

$x \longrightarrow \boxed{} \longrightarrow \boxed{} \longrightarrow y$

c Which of the formulae below connects x with y?

$y = \dfrac{x + 3}{8} \qquad y = 8(x + 3) \qquad y = \dfrac{x}{3} + 8 \qquad y = 3x + 8$

Enrichment

1 a When $x = 1$ work out the value of
 i x^2 **ii** x^3 **iii** x^4 **iv** x^5
 b Predict the value of x^{119}.

2 Victoria says, '$x + 2 < x + 3$ for any value of x'.
Is she correct? Explain your answer.

3 Reflect Look back at the questions you answered in this section.
Which hints were most useful to you? What made them more useful?
Which hints were least useful to you? What made them less useful?
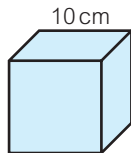What do your answers tell you about how you learn maths best?

3 Extend

You will:
• Extend your understanding with problem-solving.

1 A square has sides of length x. Write and simplify an expression for its
  a perimeter   **b** area

**2** A cube has edges of length 10 cm.

10 cm

  a Work out the area of one of the square faces.
  b The cube is painted. Work out the total area that is painted.
  Another cube has edges of length x.
  c Write an algebraic expression for the area of one of the faces.
  d Write an algebraic expression for the total area of all the faces.

3 **Finance** Company 1 uses the formula $C = 0.05M + 0.02T$ for calculating the cost of a mobile phone bill, where M = number of minutes of calls, T is the number of texts and C is in dollars.
  a Work out the cost of bills for each of these customers.
    Customer A: 10 minutes of calls, 1000 texts
    Customer B: 300 minutes of calls, 20 texts
    Customer C: 1000 minutes of calls
  b Company 2 uses the formula $C = 0.1M + 0.01T$.
    Work out the bill for each of the customers if they used this company.
  c Which company should each customer use?

4 **Problem-solving** Jasmin is working out coordinates using a rule.
  She takes the x-coordinate and puts it into the function machine to get a y-coordinate:

x-coordinate y-coordinate
→ $+3$ →

x-coordinate y-coordinate
0 → $+3$ → ☐

Q4 hint
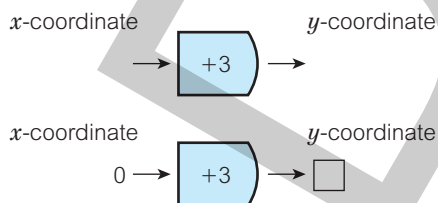
Start with $x = 0$.

  Work out several pairs of coordinates and plot them on a coordinate grid. Join them with a line. What do you notice? Design your own function machine and generate coordinates. Plot them and join them with a line. What do you notice?
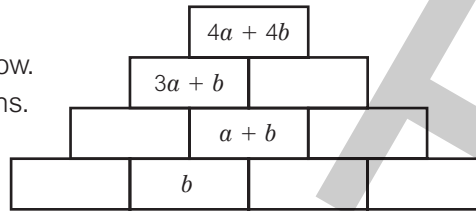
5 A triangle has side length n cm.
The second side is 5 less than double this length.
The third side is twice the length of the second side.
Write an expression for the perimeter of the triangle. Simplify your expression as much as possible.

6 In the pyramid, each brick is the sum of the two bricks below.
Work out the missing expressions.

7 A magician uses this number trick: *Think of a number. Add 3. Multiply it by 2. Subtract double the number you first thought of. The number you have is 6.*
Explain the trick.

8 In a magic square the diagonals, rows and columns all sum to the same total.

a Write the numbers 1–9 in the magic square (using each number only once) so that all the diagonals, rows and columns sum to 15.
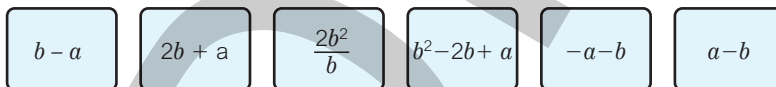Three numbers have been written for you.

2 3 6 7 8 9

b Write the algebraic expressions in the magic square so that all the rows, columns and diagonals sum to $3c$.

$c - a - b$ $c + b$ $c - a + b$ $c + a - b$ $c + a$ $c + a + b$
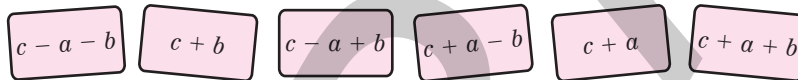
9 When $a = -2$ and $b = 4$ all but one of these expressions have the same value.
Which is the 'odd one out'?

$b - a$ $2b + a$ $\dfrac{2b^2}{b}$ $b^2 - 2b + a$ $-a - b$ $a - b$

10 This is part of a spreadsheet a shop uses to calculate wages.

a What value will be calculated in cell D2?
b What expressions should be written in cells D3 and D4 to calculate the wages of Mr Gupta and Mrs Alam?
c The value in cell B4 is changed to £19. What value will show in cell D4?
d The expression in C5 calculates the mean number of hours worked.
What is this value?
e What does the expression in cell B5 calculate?

	A	B	C	D
1		Pay per hour	Number of hours	Pay
2	Mrs Badri	8	25	= B2 * C2
3	Mr Gupta	7	17	
4	Mrs Alam	15	15	
5		= (B2+B3+B4)/3	= (C2+C3+C4)/3	
6				

11 The length of a rectangle is three times the width. Write and simplify an expression for
   a the area
   b the perimeter.
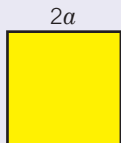
Worked example

Draw a pair of number lines from 1 to 10.
Show the mapping: $x \rightarrow 2x + 3$

0 1 2 3 4 5 6 7 8 9 10

0 1 2 3 4 5 6 7 8 9 10

> Substitute each number on the top number line into the function $2x + 3$.

When $x = 0$, $2x + 3 = 0 + 3 = 3$

0 1 2 3 4 5 6 7 8 9 10

0 1 2 3 4 5 6 7 8 9 10
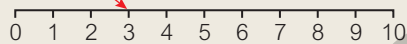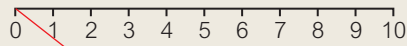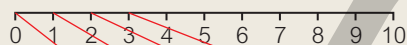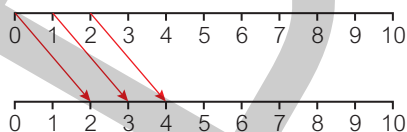
When $x = 1$, $2x + 3 = 2 + 3 = 5$
When $x = 2$, $2x + 3 = 4 + 3 = 7$
When $x = 3$, $2x + 3 = 6 + 3 = 9$

12 The function $x \rightarrow x + 2$ is shown on a **mapping diagram**.

0 1 2 3 4 5 6 7 8 9 10

0 1 2 3 4 5 6 7 8 9 10

Copy and complete the mapping diagram.

Key point

A mapping diagram is a visual representation of a function.

Topic links: Calculate the mean, Calculations involving negative numbers.

Subject links: ICT (Q11).

13 Show each mapping on a pair of number lines from 0 to 10.

a $x \rightarrow x - 5$ **b** $x \rightarrow 3x - 3$ **c** $x \rightarrow \dfrac{2x + 4}{2}$

14 Show each mapping on a pair of number lines from –5 to 5.

a $x \rightarrow x + 2$ **b** $x \rightarrow 4 + x$ **c** $x \rightarrow 2 - x$

15 a Copy and complete the mapping diagram for $x \rightarrow \frac{1}{2}x$

0 1 2 3 4 5 6 7 8 9 10

0 1 2 3 4 5 6 7 8 9 10

b What value maps to 10?

16 The first of three **consecutive** whole numbers is x.
a Write expressions for the next two numbers: x, $x + \square$, $x + \square$.
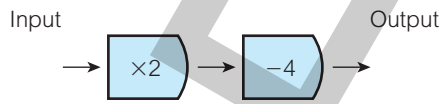**b** Write and simplify an expression for the sum of the three numbers.

17 Here is a function machine.
a Write an expression for the function machine.

Input → ×2 → −4 → Output

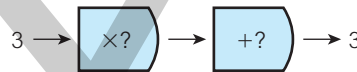**b** What is the value of the output when the input is
i 5 **ii** 1 **iii** 0
c What number, when you input it, will give the same output as the input?

18 Work out the missing values in the function machine.

Is there more than one solution to this question?

3 → ×? → +? → 3

19 Look at these advertisements.

2U Taxis	A 2 B Taxis
Fare: $2 per mile	Fare: $4 per passenger + $1 per mile

a Work out the cost of a 10-mile journey with 3 passengers with each company.
b For each company write a formula for calculating the cost of a journey, T, with distance d and number of passengers p.
c Which company is cheaper for 2 passengers to travel 15 miles?
d A journey with 2U costs $40. What distance was the journey?
e A journey for 2 passengers with A2B costs $40.
What distance was the journey?

20 Reflect What kinds of jobs involve using formulae?
What careers are you interested in? Do you think you will need to use formulae in your job? How?
What professionals are you likely to meet who might use formulae in their work?

Reflect

3 Unit test

1 To convert between hours and minutes use the formula
     minutes = number of hours × 60
   Work out the number of minutes in 7 hours.

2 The formula for calculating the perimeter of a shape, P, is $P = 2a + 3b$.
   Work out the value of P when $a = 5$ and $b = 7$.

3 Use the formula $m = \dfrac{c}{100}$ for converting centimetres, c, to metres, m.
   Work out the value of m when $c = 325$.

4 Use the formula $D = \dfrac{n(n - 3)}{2}$ to work out the value of D when $n = 4$.

5 Expand
   a $3(x + 4)$
   b $5(x - 7)$
   c $7(10 - x)$

6 Write an expression for
   a 2 less than y
   b 5 times m
   c y divided by 10
   d x more than y.

7 Angela is paid £10 more than Imogen.
   Write a formula connecting the amount Angela is paid, A, and the amount
   Imogen is paid, I.

8 Write an expression for
   a b multiplied by itself
   b double b
   c a divided by b.

9 Work out the value of these expressions when $p = 3$, $q = 6$.
   a $2(p + 3)$
   b $5(2p + q)$

10 Simplify by collecting like terms.
    a $x + 2x$
    b $6x + 2y - 3x$
    c $10 + 12y + 7 - 14y$

11 When $a = 5$, $b = 11$ and $c = 9$ work out the value of
    a $4a + 2c$
    b $20 - 3a$
    c $10c - 2b + a$

12 Use the formula $z = 2m - a$ to work out the value of z when
    a $m = 3$, $a = 5$
    b $m = 1$, $a = -7$

13 Simplify
    a $r \times r \times r \times r \times r$
    b $2 \times y \times 7 \times y \times y$
    c $3y \times y$
    d $3m \times 5m$
    e $18x \div 3$

14 Simplify by collecting like terms.
    a $3r^3 + 10r^3$
    b $12x + 3x^2 - 5x$

15 Expand
    a $x(x + 7)$
    b $r(r - 5)$
    c $2b(b + 5)$
    d $3b(2b - 4)$

16 Find the value of each expression when $b = 2$ and $m = 9$.
    a b^3
    b $b^2 - m$
    c $\dfrac{b + 2m}{2}$
    d $m^2 - b^2$
    e $3(m - b)$

Challenge

17 Are there any values of x that make these pairs of expressions equal?
    a $2x^2$ and $2x$
    b $6x - 3$ and $3x + 6$
    c $\dfrac{3x}{2}$ and $\dfrac{2x}{3}$
    d $2(3x + 5)$ and $2(3x - 5)$

18 **Reflect** Look back at the work you have done in this unit. Find a question
that you could not answer immediately, but that you worked hard at, and then
answered correctly.
How do you feel when you find it difficult to answer a maths question?
Write down the strategies you use to help you when you have difficulty.
How do you feel when you eventually understand and get the correct answer?

Reflect