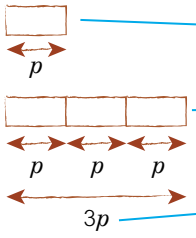
Simplifying algebraic expressions

Guided

1 Simplify

a $p + p + p$



Think of a rod that is p cm long.

When you put three rods together the total length is $3p$ cm.

So $p + p + p = 3p$

An algebraic expression, for example $3x + 2y$, contains numbers and letters. Each part of an algebraic expression is called a term.
Like terms contain the same letter (or do not contain a letter).

Worked example

b $w + w + w + w + w$ ..... **c** $3p + 4p$ ..... **d** $5w - 2w$ .....

You simplify an expression by collecting like terms.

Guided

2 Simplify by collecting like terms.

a $3t + 2t + 5 = 5t +$ .....

b $5w - 3w + 2y$ .....

c $5h + 2h + 5j + 2 = 7h +$ ..... $+$ .....

d $7h + 1 - 4h + 2$ .....

3 Simplify

a $t \times t$ .....

b $n \times n \times n$ .....

c $i \times i \times i \times i$ .....

$2 \times 2 \times 2 = 2^3$
In the same way we can write $t \times t \times t = t^3$

Guided

4 Simplify

a $3d^2 + 5d^2 = 8$ .....

b $5b^2 + 4b^2 + 3b =$ ..... $+ 3b$

c $7h^3 + 3h + 2h^3$ .....

d $4c + 4c^2 + 4c$ .....

e $6k^5 - k^5$.....

f $6m^2 + 4m^3 - 3m^3$ .....

Like terms must have *exactly* the same letters and powers.
For example, $2x^2$ and $3x^3$ are *not* like terms as the powers of x are different.

5 Simplify

a $b \times c$ .....

b $a \times a \times c \times c$ .....

c $s \times 3$ .....

d $d \times 3 \times c$ .....

Write letters in alphabetical order.
$n \times m = mn$
Write numbers before letters.
$a \times 2 = 2 \times a = 2a$

Guided

6 Simplify

a $5c \times 3c = 5 \times c \times 3 \times c$
$= 5 \times 3 \times c \times c$
$=$ ..... c^2

The order of multiplication does not matter.

b $2d \times 7d$ .....

c $\frac{10b}{5} = 2b$

$\frac{10b}{5}$ means $10b \div 5$.
Work out $10 \div 5$

d $\frac{18t}{6}$ .....

7 Write $\equiv$ or $\neq$ between equivalent expressions.

a $x + y$ ..... $y + x$

b $x - y$ ..... $y - x$

c xy ..... yx

d $x \div y$ ..... $y \div x$

Test with some numerical values for x and y.

The identity symbol ($\equiv$) shows that two expressions are always equivalent.
For example, $a + 2b \equiv 2b + a$

Check

Tick each box as your **confidence** in this topic improves.

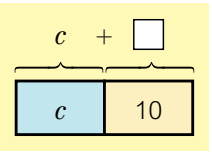
☹ 😐 ☺

Need extra help? Go to page 27 and tick the boxes next to Q1, 2, 4, 5 and 7. Then try them once you've finished 3.1–3.5.

Writing algebraic expressions

1 Hannah collects football cards. She has c cards.
Write an expression for how many she has when there are

a 10 more ……..
b 7 fewer ……..
c 12 fewer ……..
d twice as many ……..
e 4 times as many ……..
f half as many. ……..



Finding half is the same as dividing by 2.

2 a Barney has f football cards and r rugby cards.
Write an expression for the total number of football and rugby cards he has. ……..

b Barney gives away 5 football cards and is given 3 rugby cards.
Write an expression for the total number of football and rugby cards he has now. ……..

Try it with numbers. How would you write 5 more than 3?

3 Write an algebraic expression for

a d more than c ……..
b c multiplied by d ……..
c d less than c ……..
d d more than 7 times c ……..
e 5 times d add 3 times c ……..
f c multiplied by itself ……..
g 9 times d multiplied by itself ……..
h 1 less than c multiplied by itself ……..
i c divided by d ……..
j 7 more than 9 divided by c. ……..

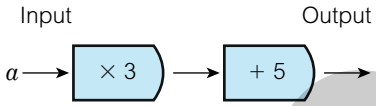
4 Write an expression for the output of each function machine.

a is multiplied by 3 then 5 is added.

To show that the whole expression is divided by 5 draw a long division line.

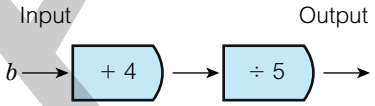
Guided

a



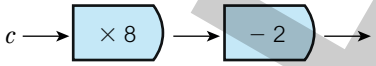
$a \times 3 + 5 = 3a + 5$

b

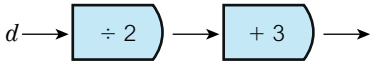


$(b + 4) \div 5 = \frac{b + 4}{5}$

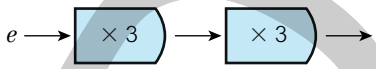
c



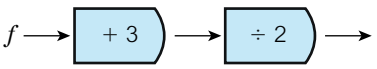
d



e



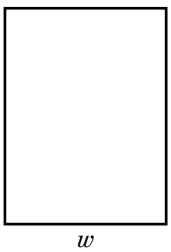
f



5 A rectangle has width w. The height is 2 more than the width.

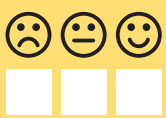
a Write an expression for the height. ……..
b Write and simplify an expression for the perimeter.

c Calculate the perimeter of the rectangle when w = 8 cm.



Check

Tick each box as your **confidence** in this topic improves.



Need extra help? Go to page 28 and tick the boxes next to Q12–14. Then try them once you've finished 3.1–3.5.

STEM: Using formulae

Guided

1 Work out the value of each expression when $a = 5$.

a $3a = 3 \times a = 3 \times 5 = 15$ **b** $7a$ ..... **c** $a + 7$ .....

d $a - 7$ ..... **e** a^2 ..... **f** $9 - a$ .....

Worked example

2 Given $a = 3$, $b = 4$ and $c = 6$ work out the value of

a ab ..... **b** $ac + 2$ ..... **c** $3(b + 1)$ .....

d $2(a + c)$ ..... **e** $\frac{c}{2}$ ..... **f** $\frac{b + c}{5}$ .....

3 STEM Use the formula density $= \frac{\text{mass}}{\text{volume}}$ to work out the density of

a a piece of plastic with a mass of 12 g and a volume of 6 cm^3

Guided

density $= \frac{12}{6}$

$= 2$ g/cm^3

Substitute the values into the formula.
Write the units.
g/cm^3 means grams per cubic cm.

A **formula** is a general rule for a relationship between quantities. You use a formula to work out an unknown quantity by substituting.

Literacy hint

Density is the mass (in grams) of 1 cm^3 of a substance.

b a lump of metal with a mass of 28 g and a volume of 8 cm^3

c a sample of liquid with a mass of 250 g and a volume of 200 cm^3.

4 STEM Use the formula distance = speed × time to work out the distance travelled when

Guided

a speed = 20 m/s, time = 4 seconds

distance $= 20 \times 4 =$ ..... metres

m/s means metres per second.
km/h means kilometres per hour.
mph means miles per hour.

b speed = 5 m/s, time = 30 seconds .....

c speed = 50 km/h, time = 2 hours .....

d speed = 4 mph, time = $\frac{1}{2}$ hour. .....

5 STEM The formula to calculate pressure (P) in N/m^2, is $P = \frac{F}{A}$, where F is the force in N and A is the area in m^2

Work out the pressure when $F = 40$ and $A = 5$

6 STEM The formula for converting from temperature in Celsius (C) to Fahrenheit (F) is

$F = 1.8C + 32$.

Convert these temperatures into °F.

a 100 °C ..... **b** 0 °C .....

c –100 °C ..... **d** –40 °C .....

Check

Tick each box as your **confidence** in this topic improves.

☹ 😐 ☺

Need extra help? Go to pages 27 and 28 and tick the boxes next to Q8–11. Then try them once you've finished 3.1–3.5.

Writing formulae

1 A mobile phone company charges £0.25 per minute for talk time and £2.50 per gigabyte (GB) for downloads. It uses the formula $C = 0.25t + 2.5d$.

a What do you think t stands for? .....

b What do you think d stands for? .....

c How much would 100 minutes of talk time and 2 GB of downloads cost? .....

2 Renting a car costs £15 per day.

Guided

a How much does it cost to rent a car for 4 days?

£15 × 4 = £60

Write down the cost per day. Multiply the cost by the number of days.

Literacy hint

'per day' means each day.

b How much does it cost to rent a car for a week? .....

c Write an expression for how much a car costs to rent for d days. $15d$

d Write a formula for the cost, C, of renting a car for d days. $C =$ .....

3 Davina organises graduation parties. She always orders 5 more party bags than the number of guests.
Write a formula that connects the number of guests, g, to the number of party bags, b.

4 **Modelling**

a Write an algebraic expression for finding the mean of 4 numbers a, b, c and d. .....

b Write a formula for the mean of 4 numbers.

mean of 4 numbers = $\frac{\text{sum of 4 numbers}}{4}$

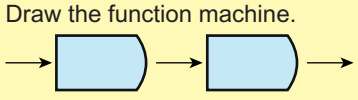
c Use your formula to work out the mean when $a = 2$, $b = 5$, $c = 6$ and $d = 7$.

Write $m = \square$

5 **Real / STEM** The mean total lung volume is worked out from 3 different readings.
The readings are p, q and r.
Write a formula to work out the mean total lung volume, T.

6 A function machine multiplies each input by 2 and then adds 7.

Draw the function machine.



a What is the output if the input is

i 5 ..... **ii** –4 ..... **iii** a? .....

b Write a formula which connects the output, b, with the input, a. .....

7 A taxi charges £1.25 a mile plus an extra £4.

a How much does a 3-mile taxi ride cost? .....

b Write a formula which connects the charge, C, to the distance travelled, d. .....

Brackets and formulae

Guided

1 Expand

a $3(y + 5) = 3 \times y + 3 \times 5 = 3y +$ .....

b $7(h - 2) = 7 \times h - 7 \times 2 =$ .....

c $2(d + 10)$ .....

d $5(p - 5)$ .....

e $2(2 + k)$ .....

f $5(3 - a)$ .....

g $4(2 - c)$ .....

h $3(3 - n)$ .....

i $4(6 + t)$ .....

j $5(x + 6)$. .....

Expand a bracket means multiply every number inside the bracket by the number or letter outside the bracket.

2 A company making hockey sticks works out their profit, P, by subtracting £17 from the cost of a hockey stick, H, and multiplying the answer by the number of hockey sticks sold, a.
Write a formula for calculating P. .....

Guided

3 Expand

a $t(t + 2) = t \times t + t \times 2 = t^2 +$ .....

b $d(d + 5)$ .....

c $s(s - 3)$ .....

d $y(5y + 3) = y \times 5y + y \times 3 = 5y^2$ .....

e $j(5 - 7j)$ .....

f $3q(2q + 4) = 3q \times 2q + 3q \times 4 = 6q^2$ .....

g $5t(5t - 1)$ .....

h $3x(10 - 3x)$ .....

4 Naima has £x. Kate has the square of Naima's amount. Meinir has £10 more than Kate.

a Write an expression for Kate's money. .....

b Write an expression for Meinir's money. .....

c Write and simplify an expression for the sum of all their money. .....

d Naima has £5. What is the sum of all their money?.....

Check

Tick each box as your **confidence** in this topic improves.

☹ 😐 ☺

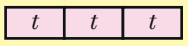
Need extra help? Go to page 27 and tick the boxes next to Q3 and 6. Then try them once you've finished 3.1–3.5.

Simplifying expressions

1 Complete

a $t + t + t =$ ..... t **b** $p + p + p + p =$ ..... p

Draw bars to help.



2 Simplify

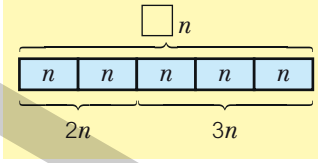
Guided

a $2n + 3n = 5$ ..... **b** $6a + 3a$ .....

c $9q - q = 9q - 1q =$ ..... **d** $3b + b$ .....

e $3g + 2 + 4g = 3g + 4g + 2 = 7g$ + ...

f $6s + 5t - 2s$ .....



3 Expand

Guided

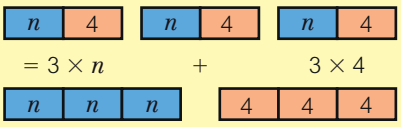
a $3(n + 4) = (n + 4) + (n + 4) + (n + 4) = 3n +$ .....

b $2(p + 5) = (p + 5) + (p + 5) =$ .....

c $4(a + 3)$ .....

d $5(4 - b)$ .....

$3 \times (n + 4)$



4 Fill in the missing numbers.

a $5 \times 5 \times 5 = 5^{\cdots}$ **b** $t \times t \times t \times t \times t = t^{\cdots}$

5 Simplify $6p \times 3p$ .....

6 Expand

Guided

a $n(n + 2) = n \times n + n \times 2 = n^2 +$ ..... **b** $t(t + 3)$ .....

c $p(5 + p)$ ..... **d** $g(1 - g)$ .....

$n \times n$

$n(n \times 2)$

$n \times 2$

7 Simplify by collecting like terms.

You can only add terms with the same letters and powers.

Guided

a $a^2 + a^2 + 5a =$ ..... $a^2 +$ .....

b $5b + b^2 + 4b$ .....

Substitution

8 The formula to work out the distance a train travels is distance = speed × time.
A train travels at a speed of 100 km per hour for 3 hours.
How far does it travel?

Guided

distance = speed × time

= 100 × 3 = ... km

9 Work out the value of each expression when $x = 4$ and $y = 8$.

a $x + 6$ ..... $\underbrace{x + 6}_{4 + 6 =}$ **b** $y - 5$ ..... **c** $x + y$ .....

d $10x$ ..... **e** $xy + 3$ ..... **f** $\frac{y}{2}$ .....

10 $T = 4(p + q)$. Work out the value of T when $p = 5$ and $q = 3$.

11 Use the formula $T = 4 + p$ to work out the value of T when

a $p = -2$ ..... **b** $p = -4$ .....

$T = 4 + -2$
$= 4 - 2$

Writing expressions and formulae

12 Match each algebraic expression to its description.

$x + 5$	$x - 5$	$5 - x$	$5x$	$\frac{x}{5}$

5 times x	5 less than x	5 more than x	One fifth of x	x less than 5

13 Write an expression for each function machine.

a $a \rightarrow$ [+ 5] $\rightarrow$ .....

b $r \rightarrow$ [× 2] $\rightarrow$ .....

c $x \rightarrow$ [− 4] $\rightarrow$ .....

14 Write a description for each expression.

a $a + b$ .....

b ab .....

c $b - a$ .....

d $\frac{a}{b}$ .....

Use these phrases:
more than, less than, multiplied by, divided by.
For example, b more than ....

15 To convert from cm, C, to inches, I, multiply by 4 then divide by 10. Write the formula. .....

16 I think of a number, add 12, and then divide by 9.

a What would the result be if the original number was 15? .....

b Complete the function machine.

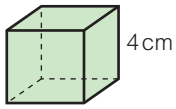
$x \rightarrow$ [□] $\rightarrow$ [□] $\rightarrow y$

c Which of these formulae correctly connects x with y?

$y = 12x + 9$ $y = \frac{x + 12}{9}$ $y = 9(x + 12)$ $y = \frac{x}{12} + 9$

1 A hexagon has sides of length x.
Write and simplify an expression for its perimeter.

2 A cube has edges of length 4 cm.



a Work out the area of one of the square faces. .....

b The cube is painted.
Work out the total area that is painted.

Another cube has edges of length e.

c Write an algebraic expression for the area of one of the faces. .....

d Write an algebraic expression for the total area of all the faces. .....

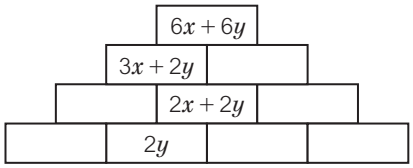
3 **Finance** A party organising company uses the formula $P = 4.5c + 6a$ for calculating profit (financial gain), where c is the number of children and a is the number of adults.
Work out the profit when there are

a 20 children and 10 adults ..... **b** 30 adults. .....

4 A quadrilateral has one side of length y cm.
The second side is 2 cm more than double this length.
The other two sides are each 3 times the length of the second side.
Write an expression for the perimeter of the quadrilateral.
Simplify your expression as much as possible.

Sketch and label the quadrilateral.

5 In the pyramid each brick is the sum of the two bricks below.
Work out the missing expressions.



6 A maths teacher uses this number puzzle: *Think of a number. Double it. Add 2. Multiply by 5. Subtract 10 times the number you first thought of. Your answer is 10.*
Explain the teacher's trick.

Call the unknown number 'x' and construct an algebraic expression.

7 When $x = 4$ all but one of these expressions have the same value.
Which is the odd one out?

$x^2 - 20$ $x - 8$ $\frac{x}{4} - 5$ $8 - 2x$ $-7 + \frac{12}{x}$

8 The product of two terms is $12x^2$.

a What could the two terms be? .....

b Give two other possible terms. .....

Literacy hint
Product means multiply.

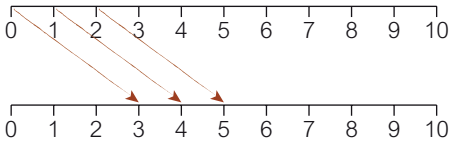
9 The sum of two terms is $10x$. Their product is $24x^2$.
What are the terms?

10 The length of a rectangle is 5 times the width. Write and simplify an expression for

a the area ..... **b** the perimeter. .....

Guided

11 Show the mapping $x \rightarrow x + 3$ on a pair of number lines from 0 to 10.



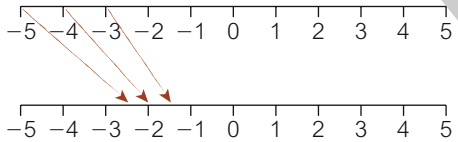
When $x = 0$, $x + 3 = 0 + 3 = 3$
When $x = 1$, $x + 3 = 1 + 3 = 4$
When $x = 2$, $x + 3 = 2 + 3 = 5$

Substitute each number on the top number line into the function $x + 3$.

12 Show the mapping $x \rightarrow 2x - 2$ on a pair of number lines from −5 to 5.

Guided

13 **a** Complete the mapping diagram for $x \rightarrow \frac{1}{2}x$



b What value maps to 5? .....

Literacy hint

Consecutive numbers follow each other.
4, 5, 6 are consecutive.

Guided

14 The first of five consecutive whole numbers is x.

a Write expressions for the next three numbers.
x, $x + 1$, $x +$ ....., $x +$ ....., $x +$ .....

b Write and simplify an expression for the sum of the five numbers.

c Factorise the expression. .....

d Write an expression for the mean of the five numbers. .....
What do you notice?

15 The first of three consecutive whole even numbers is x.
Write an expression for the mean of the three numbers.

Look at Q14.

16 What value(s) of x would make each of these statements true?

a $x^2 = 3x$ ..... **b** $x^2 > 3x$ ..... **c** $x^2 < 3x$ .....

1 To convert between days and hours use the formula
Hours = number of days × 24
Work out the number of hours in 5 days. .....

2 The formula for calculating the perimeter of a shape, P, is $P = 2x + 5y$.
Work out the value of P when $x = 12$ and $y = 3$.

3 Use the formula $k = \frac{m}{1000}$ for changing meters, m, to kilometers, k.
Work out the value of m when $k = 4325$.

4 Expand $5(p - 3)$. .....

5 Write an expression for

a 5 less than t .....

b 4 times w .....

c p divided by 3. .....

6 Sophia jogs one day and swims the next. When she jogs she covers 1 mile. When she swims she covers 3 miles. Write a formula connecting the total distance she travels, T, with the number of days she jogs, j, and the number of days she swims, s, over two days.

7 Work out the value of each expression when $x = 2$ and $y = 5$.

a $3(x + 4)$ .....

b $2(5x + y)$ .....

8 By collecting like terms, simplify $4 + 3e - 1 + 2e$. .....

9 Simplify

a $t \times t \times t \times t$ .....

b $2p \times p$ .....

c $2y \times 3y$ .....

10 By collecting like terms, simplify $2v^3 + 3v^2 + 4v^3$. .....

11 Expand $5d(3d + 3)$. .....

12 Work out the value of x^2 when $x = 8$. .....

13 Find the value of each expression when $p = 3$ and $q = 7$.

a p^2 .....

b $2p + q$ .....

c $3q - p$ .....

d pq .....

e $\frac{p + q}{2}$ .....