1 Significant figures, powers and standard form

1.1 STEM: Powers of 10

You will learn to:

- Use and understand powers of 10.
- Use the prefixes associated with powers of 10.
- Understand the effect of multiplying and dividing by any integer power of 10.

Confidence

Why learn this?

A byte is a unit of digital information stored on a computer. A megabyte is 10^6 bytes and a gigabyte is 10^9 bytes.

$10^2 \times 10^3 = 10^{\square}$
$10^4 \times 10 = 10^{\square}$
$10^7 \div 10^5 = 10^{\square}$
$10^8 \div 10^2 = 10^{\square}$

Explore

How many photographs can you store on a 1 terabyte server?

Exercise 1.1

Warm up

1 Match each value in the top row to the equivalent value from the bottom row.

10^2	10^4	10^3	10^5
1000	100 000	100	10 000

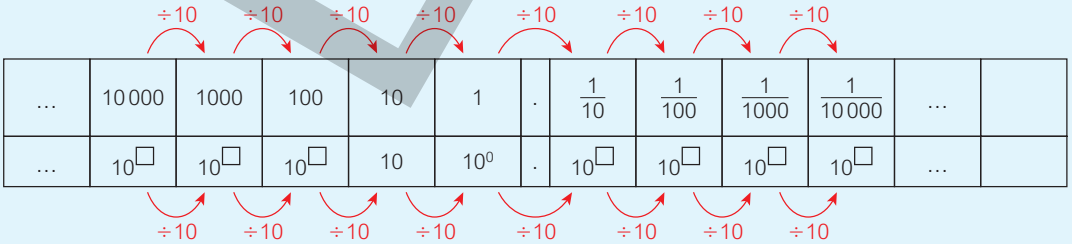
2 Work out

a 4.5×10
b 2.36×1000
c 0.843×100
d $1.45 \times 10\,000$
e $270 \div 10$
f $4685 \div 1000$
g $35 \div 100$
h $450 \div 10\,000$

Key point

Each of the headings in the place-value table is a power of 10. This is because we have a *dec*imal system (*dec* = 10).

3 Copy and complete this place-value table.



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4 STEM

This table shows the **prefixes** for powers of 10.

Prefix	Letter	Power	Number
tera	T	10^{12}	1 000 000 000 000
giga	G	10^{9}	1 000 000 000
mega	M	10^{6}	1 000 000
kilo	k	10^{3}	1000
deci	d	10^{-1}	0.1
centi	c	10^{-2}	0.01
milli	m	10^{-3}	0.001
micro	μ	10^{-6}	0.000 001
nano	n	10^{-9}	0.000 000 001
pico	p	10^{-12}	0.000 000 000 001

a Add the prefixes to your place-value table in Q3.
b A gram (g) is a unit of mass. How many grams are in a kilogram (kg)?
c A joule (J) is a unit of energy. How many joules are in a megajoule (MJ)?
d A watt (W) is a unit of power. How many watts are in a gigawatt (GW)?

5 STEM Convert
a 4 kg to g
b 2.4 MJ to J
c 12.5 GW to W.

6 STEM How many times bigger is
a a millimetre than a nanometre
b a gigawatt than a megawatt
c a kilojoule than a joule
d a megagram than a kilogram?

Discussion What other name do we use for a megagram?

Key point

Some powers of 10 have a name called a **prefix**. Each prefix is represented by a letter. The prefix for 10^6 is mega (M) as in megabyte (MB).

Literacy hint

The prefix for micro is the Greek letter μ, pronounced mu.

Worked example

The average distance of Venus from the Sun is 1.08×10^8 km.
Write this distance as an ordinary number.

First write 10^8 as an ordinary number.

$1.08 \times 10^8 = 1.08 \times 100\,000\,000 = 108\,000\,000$ km

7 STEM The table shows information about some planets.

Name of planet	Diameter of planet (km)	Average distance from Sun (km)
Mercury	4.9×10^3	5.79×10^7
Earth	1.28×10^4	1.5×10^8
Saturn	1.2×10^5	1.427×10^9

a Copy the table. Write all the distances as ordinary numbers.
b Which planet has the greatest diameter?
c Which planet is closest to the Sun?

Discussion How can you answer parts **b** and **c** without writing the distances as ordinary numbers?

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8 **STEM / Problem-solving** The Space Shuttle had a lift-off mass of 1.1×10^5 kg. How many tonnes is this?

9 **STEM** The table shows the dimensions of some small organisms.

Name of organism	Length	Width
dust mite	0.42 millimetres	0.25 millimetres
bacteria	2 micrometres	0.5 micrometres
virus	0.3 micrometres	15 nanometres

a Write all the **dimensions** in metres.
b Which organism has the greatest length?
c Which organism has the smallest width?
Discussion How can you answer parts **b** and **c** without writing the dimensions as ordinary numbers?

10 **STEM / Reasoning** An atom is the smallest object that you can see with an electron microscope. The width of an atom is about 0.1 nanometres.
What is this distance in millimetres?

11 **Explore** How many photographs can you store on a 1 terabyte server?
What have you learned in this lesson to help you to answer this question?
What other information do you need?

12 **Reflect** After this lesson Jaina says, 'I understood this lesson well because it's all about place value.' Look back at the work you have done in this lesson.
How has place value helped you?
What other maths skills have you used in this lesson?

Q8 hint

1 tonne = 1000 kg

Q9 Literacy hint

The **dimensions** of an object are its measurements.

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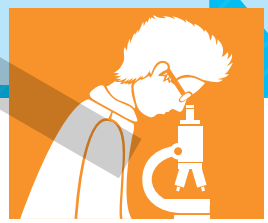
1.2 Calculating and estimating

You will learn to:
- Calculate with powers.
- Round to a number of significant figures.

Why learn this?
The organisers of sporting events often round the number of spectators to estimate the income from ticket sales.

Fluency
What is
2^3
2^4
2^5
2^6?



Explore
When is it a good idea to round numbers? When is it not a good idea?

Confidence

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Exercise 1.2

1 Simplify
a $5^2 \times 5^4$ **b** $\frac{8^5}{8^3}$ **c** $\frac{7^4 \times 7^6}{7^7}$

2 Work out
a -4×-4 **b** -7×-7 **c** $(-3)^2$ **d** $(-10)^2$

3 Use rounding to estimate the answers.
a $97 \div 4$ **b** 12.3×10.2 **c** $18.6 \div 5$

4 Evaluate $\frac{2 \times 3^9}{3^7}$

$\frac{2 \times 3^9}{3^7} = 2 \times 3^{\square}$
$= 2 \times \square$
$= \square$

5 Work out
a $\frac{5 \times 2^{12}}{2^9}$ **b** $\frac{3^2 \times 4^4}{4^3}$ **c** $\frac{2 \times 5^3 \times 5^5}{5^4}$ **d** $\frac{6^{15} \times 10}{6^7 \times 6^6}$

6 **Problem-solving** Work out $\frac{2^8 \times 16 \times 32 \times 7}{8 \times 2^{10}}$.

7 **Reasoning** Sarka and Rasheed both work out the same calculation. Here is what they write.

Sarka

$32 - (-5)^2 = 32 - -25$
$= 32 + 25$
$= 57$

Rasheed

$32 - (-5)^2 = 32 - +25$
$= 32 - 25$
$= 7$

Who is correct? Explain the mistake that the other one has made.

Key point
You can simplify expressions containing powers to make calculations easier.

Q4 Literacy hint
Evaluate means 'work out the value'.

Q4 hint
Simplify the powers of 3, $\frac{3^9}{3^7} = 3^{\square}$, then multiply by 2.

Q6 Strategy hint
Write as many numbers as possible as powers of 2.

Warm up

8 Sort these cards into matching pairs.

$14 + 4^2$	$14 - 4^2$	$14 + (-4)^2$	$14 - (-4)^2$
$25 - 2^2 - 6^2$	$25 - (-2)^2 + 6^2$	$25 - 2^2 + (-6)^2$	$25 - 2^2 - (-6)^2$

Discussion What method did you use?

Investigation

Reasoning

1 a Work out **i** $(2 \times 5)^2$ **ii** $2^2 \times 5^2$
b Work out **i** $(2 \times 5)^3$ **ii** $2^3 \times 5^3$
2 What do you notice about your answers to Q1?
3 a Write a rule for calculating the power of the product of two numbers.
Check that this rule works using two numbers of your own.
b Will this same rule work for three or more numbers?
4 a Work out **i** $(10 \div 2)^2$ **ii** $10^2 \div 2^2$
b What do you notice about your answers to part **a**?
5 Write a rule for calculating the power of the quotient of two numbers.
Check this rule works using two numbers of your own.
6 a Work out **i** $(3 + 4)^2$ **ii** $3^2 + 4^2$
b What do you notice about your answers to part **a**?
Discussion Is there a rule for calculating the power of the sum or difference of two numbers?

Investigation Q5 hint

Make sure that the second number divides exactly into the first, and that the power is greater than 2.

9 Work out

a $\dfrac{(3 \times 4)^2}{2^2 \times 3}$

b $\dfrac{(3 \times 4)^3}{2^2 \times 9}$

c $\dfrac{32 \times 5^3}{(5 \times 4)^2}$

d $\dfrac{(6 \times 2 \times 8)^2}{4^3 \times 3}$

Q9 hint

$\dfrac{(3 \times 4)^2}{2^2 \times 3} = \dfrac{3^2 \times 4^2}{2^2 \times 3} = \dfrac{3 \times {}^{1}\cancel{3} \times {}^{2}\cancel{4} \times {}^{2}\cancel{4}}{{}^{1}\cancel{2} \times {}^{1}\cancel{2} \times {}^{1}\cancel{3}} = \ldots$

Worked example

Round these numbers to the given number of **significant figures**.

a 42.038 (4 s.f.)
b 0.05713 (3 s.f.)
c 21 561 (2 s.f.)

a 42.04

When the next digit is 5 or above, round the previous digit up. Here the fifth significant figure is an 8, so round the 3 up to a 4.

b 0.0571

The fourth significant figure is 3, so leave the third digit as 1.

c 22 000

2 and 1 are the first 2 significant figures. The third is 5, so round the 1 up to 2.

Key point

You can round numbers to a given number of **significant figures (s.f.)**. The first significant figure is the one with the highest place value. It is the first non-zero digit in the number, counting from the left.

10 Round these numbers to the given number of significant figures.
a 47.368 (4 s.f.)
b 0.00662 (1 s.f.)
c 579 452 (2 s.f.)

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Topic links: Negative numbers, Volume, Range, Order of operations

Subject links: Science (Q15)

11 Estimate the answer to each calculation by rounding each number to 1 significant figure.

a 37×492

b 6230×26

c $897 \div 28$

d $45\,239 \div 183$

12 Estimate the answer to each calculation by rounding each number to 1 significant figure.

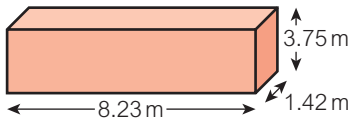
a $\dfrac{(1.2 + 3.5)^2}{1.8^3}$

b $\dfrac{(27 - 14)^3}{7.3^2}$

c $\dfrac{(3.3^2 \times 2)}{(2.3 + 4.2)^2}$

d $\dfrac{(786 - 529)^2}{7.4^2}$

13 The diagram shows a cuboid.



Work out the volume of the cuboid.
Give your answer in m^3 correct to 3 significant figures.

14 **Problem-solving** Sarita starts with a whole number.
She rounds it to 2 significant figures.
Her answer is 670.

a Write down two different numbers she could have started with.

b What is the largest number she could have started with?

c What is the smallest number she could have started with?

15 **STEM** The table shows the diameters of five planets.

Planet	Mercury	Venus	Earth	Mars	Uranus
Diameter (km)	4878	12 104	12 756	6794	51 118

a Round each diameter correct to 1 significant figure.

b Work out an estimate of the range in diameters.

16 **Real** A football stadium seats 42 785.
The average price of a ticket is £32.
Estimate the total money taken from ticket sales for one match.

17 **Explore** When is it a good idea to round numbers?
When is it not a good idea?
Look back at the maths you have learned in this lesson.
How can you use it to answer this question?

18 **Reflect** Look back at Q12.
Use a calculator to work out the exact answer to each part.
How can your estimate help you to check your calculator answer?

Q11a hint

$37 \times 492 \approx 40 \times 500 = \square$

Q11 Literacy hint

≈ means approximately equal to.

Q12a hint

$\dfrac{(1.2 + 3.5)^2}{1.8^3} \approx \dfrac{5^2}{8}$

What number is a multiple of 8 and close to 5^2? Use this to estimate the final answer.

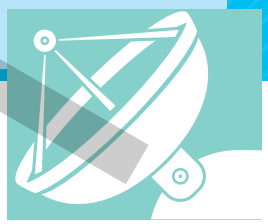
DRAFT

1.3 Indices

You will learn to:
- Use negative indices.
- Work out powers of fractions.

Why learn this?
Carbon dating uses negative indices to describe the decay of carbon-14.

Fluency
- Work out $3^3 - 4^2$
- Which is larger: $\frac{1}{2} \times \frac{1}{2}$ or $\frac{1}{3} \times \frac{1}{3}$?
- Work out -15×-3



Explore
Does raising a number to a power always make the number bigger?

Confidence

Warm up

Exercise 1.3

1 Write as a single power.

a $3^4 \times 3^5$ **b** $7^9 \div 7^5$

c $4^{13} \div 4^{10}$ **d** $(2^4)^3$

e $(11^7)^3$ **f** $(5^5)^5$

g 8×2^6 **h** $3^5 \times 81$

i $5^{10} \div 125$

Q1 hint

To multiply powers, add the indices.
To divide powers, subtract the indices.
To work out the power of a power, multiply the indices.

Investigation

Reasoning

1 Copy and complete the sequence of powers. Write your numbers as integers or fractions of 10.

2 Repeat part **1** for powers of 2.

3 Copy and complete.

a $10^{-2} = \frac{1}{10^{\square}}$ **b** $2^{-3} = \frac{1}{2^{\square}}$

c $2^{-5} = \frac{1}{2^{\square}}$ **d** The reciprocal of 10^4 is $\square$

4 Copy and complete the rules.

$2^{-n} = \frac{1}{2^{\square}}$

$10^{-n} = \frac{1}{10^{\square}}$

5 Write down the value of 5^{-2} as a decimal. Check your answer with a calculator.

Discussion What is the value of any number raised to the power 0?

$10^5 = 100\,000$ ÷10
$10^4 = 10\,000$ ÷10
$10^3 =$
$10^2 =$
$10^1 =$
$10^0 =$
$10^{-1} =$
$10^{-2} =$
$10^{-3} =$
$10^{-4} =$
$10^{-5} =$

2 Copy and complete.

a $3^{-2} = \square$ **b** $\frac{1}{3} = 3^{\square}$

c $6^{\square} = \frac{1}{36}$ **d** $\square^{-2} = \frac{1}{169}$

e $4^{-3} = \square$ **f** $\frac{1}{81} = 3^{\square} = 9^{\square}$

g $\frac{1}{64} = 8^{\square} = 4^{\square} = 2^{\square}$ **h** $5^{-1} = \square$

Key point

A number raised to a negative power is the same as the reciprocal of that number to the power.

Topic links: Calculations with fractions, Laws of indices

DRAFT

3 Write each calculation as a single power.

a $10^5 \times 10^{-2}$ **b** $4^3 \times 4^{-1}$

c $11^{-2} \times 11^{-5}$ **d** $7^2 \div 7^{-5}$

e $6^{-2} \div 6^4$ **f** $8^{-7} \div 8^{-3}$

g $(9^{-2})^5$ **h** $(12^{-4})^{-2}$

4 Write each calculation as

i a single power

ii an integer or a fraction.

a $3^2 \times 3^{-1} \times 3^{-4}$ **b** $4^2 \times 4^{-1} \div 4^{-2}$

c $5^{-3} \div 5 \div 5^{-2}$ **d** $\dfrac{2^{-3} \times 2^{-5}}{2^{-4}}$

5 Write each calculation as a fraction.

a $\frac{1}{3} \times \frac{1}{3}$ **b** $\left(\frac{3}{4}\right)^2$

c $\left(\frac{3}{5}\right)^3$ **d** $\left(\frac{2}{3}\right)^4$

6 Write each number as a fraction raised to a power.

a $\frac{16}{100}$ **b** $\frac{9}{49}$

c $\frac{25}{64}$ **d** $\frac{8}{64}$

e $\frac{16}{625}$ **f** $\frac{1}{27}$

7 **Problem-solving** Jamal eats half a cheesecake, his brother eats half of what is left and his sister eats half of what is then left. How much cheesecake remains?

Write your answer

a as a fraction

b as a fraction raised to a power.

8 **Explore** Does raising a number to a power always make the number bigger?

Choose some sensible numbers to help you explore this situation. Then use what you've learned in this lesson to help you to answer the question.

9 **Reflect** Rhiannon says, 'Mathematics is often about spotting patterns'. Do you agree? Explain.

When else have you used pattern spotting in maths?

Q3 hint

The laws of indices still apply with negative numbers.

Q5 hint

The brackets show that the whole fraction (the numerator and denominator) is squared.

Q9 hint

Look back at this lesson and the previous lesson. Can you find any questions where you were spotting a pattern?

Explore

Reflect

DRAFT

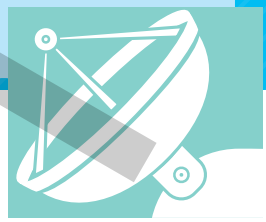
1.4 Standard form

You will learn to:
- Write numbers using standard form.
- Order numbers written in standard form.

Why learn this?
Scientists describing the universe need to write down very large numbers in a way that is easy to read.

Fluency
Multiply 3.05 by
- 10
- 1000
- 0.1

Work out $10^3 \times 0.15$



Explore
What units are used to measure distances in the Universe?

Confidence

Warm up

Exercise 1.4

1 Work out

a 2.5×100 **b** 7.3×0.01

c 4.06×10^{-1} **d** 9.55×10^{-3}

2 Copy and complete.

a $23.4 \times 10^3 = \square$ **b** $2.35 \times 10^{\square} = 235$

c $34 \times 10^{\square} = 34\,000$ **d** $0.067 \times 10^2 = \square$

3 Which of these numbers are written in **standard form**?

a 2.8×10^3 **b** 7×10^5

c 0.2×10^2 **d** 27×10^{-5}

e 3.3×10 **f** 5.022×10^{-6}

4 These numbers are written in standard form. Write them as ordinary numbers.

a 7×10^2 **b** 2.5×10^{-5}

c 5.4×10^6 **d** 3.04×10^{-3}

Key point

A number written in **standard form** is a number between 1 and 10 multiplied by a power of 10.
4.2×10^6 is written in standard form.
Using algebra, standard form is $A \times 10^n$, where $1 \leqslant A < 10$ and n is an integer.

Q4 hint

$7 \times 10^2 = 7 \times 100 = \square$

Worked example

Write each number using standard form.

a 41 000

b 0.003 94

$41\,000 = 4.1 \times 10^4$

4.1 lies between 1 and 10. Multiply by the power of 10 needed to give the original number: 4.1000

$0.003\,94 = 3.94 \times 10^{-3}$

3.94 lies between 1 and 10.
Multiply by the power of 10 needed to give the original number.
The number is less than 1 so the power of 10 is negative.
This is the same as dividing by a power of 10: 0.00394

Topic links: Powers, Laws of indices, Priority of operations

Subject links: Science (Explore, Q6, Q8–10)

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5 Write each number in standard form.

a 23 500 **b** 315 **c** 12 000 000
d 0.04 **e** 0.000 35 **f** 0.000 000 090 1

6 **STEM** The distance light travels in a year is called a light-year.

a Write each of the distances in the table in standard form.

Object	Distance from Earth (light-years)
Centre of our galaxy	26 000
Andromeda (a neighbouring galaxy)	2 500 000
Betelgeuse (a star of Orion)	600

b The Triangulum Galaxy is the furthest galaxy you can see without a telescope. It is 3×10^6 light-years away. Is it closer to Earth than Betelgeuse?

7 Put these sets of numbers in order, from smallest to largest.

a 9.87×10^2 8.65×10^4 1.9×10^3 3.59×10^2 1.95×10^4
b 5.3×10^{-3} 4.8×10^{-2} 3.99×10^{-5} 8.05×10^{-6} 8.76×10^{-3}
c 3.22×10^{-2} 3.02×10^2 3.2×10^{-3} 3.22×10^2 3.22×10^{-3}

8 **STEM / Problem-solving** The table shows the masses of the planets in our Solar System.

a Rewrite each mass in standard form (some already are).

Planet	Mass (kg)
Earth	5.97×10^{24} kg
Jupiter	1899×10^{24} kg
Mars	0.642×10^{24} kg
Mercury	0.33×10^{24} kg
Neptune	102×10^{24} kg
Saturn	568.5×10^{24} kg
Uranus	86.8×10^{24} kg
Venus	4.87×10^{24} kg

b Approximately how many times heavier is Earth than Mars?

c Which planet is approximately 1000 times heavier than Mars?

9 **STEM** Write these atoms in order of the size of their nucleus, largest first.

Atom	Size of nucleus (m)
gold	1.4×10^{-14}
helium	3.8×10^{-15}
aluminum	7.2×10^{-15}

10 **STEM** Write the following measurements as ordinary numbers
i in metres **ii** in millimetres.

a The diameter of the core in an optical fibre: 6.25×10^{-5} m

b The line width on a microprocessor: 1.8×10^{-7} m

11 **Explore** What units are used to measure distances in the Universe? Is it easier to explore this question now that you have completed the lesson? What further information do you need to be able to answer this?

12 **Reflect** Look back at your answer to Q7. You could answer this using ordinary numbers or standard form. Which method did you choose? Explain your choice.

Q5 Literacy hint

Standard form is sometimes called **standard index form**.

Q7 hint

Write each number in full to help you to order them.

Q10 hint

There are 1000 mm in 1 metre, so multiply by $10^{\square}$.

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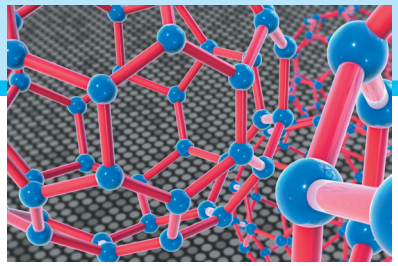
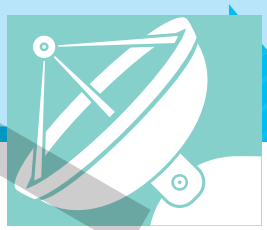
1.5 STEM: Calculating with standard form

You will learn to:
- Calculate with numbers written in standard form.



Why learn this?
Scientists exploring nanotechnology need to describe very small numbers in a way that is easy to read.

Fluency
Write as a single power of 10
- $10^4 \times 10^3$
- 10×10^{-5}
- $(10^2)^{-3}$
- $10^2 \times 10^3 \times 10^3$



Explore
What is the smallest organism you can see?

Confidence

Warm up

Exercise 1.5: Orders of magnitude

1 Write each number in standard form.

a 59 000
b 0.0601
c 0.000 000 072
d 5323

2 Write as a single power of 10

a $10^{-3} \times 10^2$
b $10^{-3} \times 10^{-2}$
c $10^3 \div 10^{-1}$
d $10^{-1} \div 10^2$

Worked example

Write $(2.7 \times 10^3) \times (4 \times 10^2)$ in standard form.

$(2.7 \times 10^3) \times (4 \times 10^2) = 2.7 \times 4 \times 10^3 \times 10^2$

Rearrange so that the numbers are together and the powers of 10 are together.

$= 10.8 \times 10^5$

Calculate the product of the numbers and use laws of indices to simplify the powers of 10.

$= 1.08 \times 10 \times 10^5$

$= 1.08 \times 10^6$

Rewrite the answer in standard form, if necessary: $10.8 = 1.08 \times 10^1$

3 Work out each calculation. Give your answers in standard form.

a $(1.2 \times 10^2) \times (3 \times 10^3)$
b $(1.5 \times 10^5) \times (5 \times 10^3)$
c $(4 \times 10^4) \times (6.25 \times 10)$
d $(1.2 \times 10^3)^2$

4 Work out each calculation. Give your answers in standard form.

a $\dfrac{6 \times 10^8}{3 \times 10^2}$
b $\dfrac{8 \times 10^5}{2 \times 10^3}$
c $\dfrac{1.2 \times 10^6}{3 \times 10}$
d $\dfrac{2 \times 10^5}{1.25 \times 10^4}$

Q4 hint
Divide the number parts. Use the laws of indices to divide the powers of 10.

Topic links: Ratio, Enlargement

Subject links: Science (Q6–15, Q18)

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5 Use a calculator to work out

a $(9.6 \times 10^7) \times (6.41 \times 10^3)$ **b** $\dfrac{1.342 \times 10^{11}}{6.1 \times 10^5}$

Discussion Which buttons do you use on your calculator? Is the answer in standard form?

6 **STEM** Light travels at 299 792 458 metres per second (m/s).

a Write down the speed of light in km/s, correct to 1 significant figure

i as an ordinary number **ii** using standard form.

The distance from the Sun to the Earth is 1.496×10^8 km.

b Use your answer to part **a** to work out how long it takes light to travel from the Sun to the Earth. Give your answer to the nearest minute.

7 **STEM / Problem-solving** Sound travels at 3.4×10^2 m/s.
What is the ratio of the speed of light to the speed of sound?
Give your answer in the form $n:1$, to 2 decimal places.

Q7 hint

Use the speed of light given in Q6.

8 **STEM / Reasoning** A human hair has a diameter of approximately 1×10^{-1} mm. The human eye cannot easily see anything smaller than a human hair without a microscope.
An optical microscope can enlarge an image to 1000 times the size of the object. Is it possible to see these organisms with a microscope?

a polio virus 2×10^{-5} mm
b human red blood cell 1×10^{-2} mm
c staphylococcus 5×10^{-4} mm

Q8c Literacy hint

Staphylococcus is a bacteria that causes food poisoning.

9 **STEM / Modelling** The table gives the sizes of eggs from different animals.

Object	Approximate diameter (m)
Extinct elephant bird	2.5×10^{-1}
Ostrich	1.5×10^{-1}
Hummingbird	1×10^{-2}
Sea star	9×10^{-4}
Human	1.2×10^{-4}

A model of an ostrich egg is built for an exhibition. Its diameter is 10 m.
The same scale is used to build models of the other objects.

a Work out the length of each model. Choose an appropriate unit of length for each answer.

b Is this a good scale to use? Would you be able to hold each model in your hand?

10 **STEM** A 3D electron microscope magnifies objects 1 000 000 times.
A water molecule has a diameter of 2×10^{-10} m.
How large will it appear in the microscope?
Give your answer in millimetres.

11 **STEM** Graphite is made up of layers of graphene sheets. Each sheet of graphene is one atom thick. There are 3×10^6 layers of graphene in 1 mm thickness of graphite. If you ignore the thickness of the layers, what is the gap between the layers? Give your answer in standard form.

Q11 hint

Layers of graphene are so thin, about 1.4×10^{-10} m thick, that they can be ignored in this calculation.

DRAFT

12 **STEM** Here are some wavelengths in the electromagnetic spectrum.

Gamma ray	1×10^{-12} m
Red light	6.8×10^{-7} m
Microwave	1.22×10^{-1} m
VHF radio wave	3 m
Low frequency radio wave	10 km

a How many gamma ray wavelengths fit into the length of one red light wave?

b How many times larger are low frequency radio waves than microwaves?

c Which is longer: 10^2 VHF radio waves or 3×10^{10} red light rays?

13 **STEM / Problem-solving** The mass of a proton is about 2000 times larger than the mass of an electron.
Copy and complete this sentence, using standard form.
The mass of an electron = the mass of a proton $\times \square \times 10^{\square}$

14 **Real / STEM** Sunglasses are coated with very thin layers to cut out ultraviolet radiation. These layers are about 4×10^2 nm thick. Give the thickness in metres.

Literacy hint

1 nm = 1 nanometer = 10^{-9} m

15 **Real** Your fingernail grows about 1 nm per second. How much could your fingernail grow in 4 weeks? Give your answer in millimetres.

16 Work out each calculation. Give your answers in standard form.

a $5.1 \times 10^8 + 1.45 \times 10^8$

b $9.05 \times 10^5 + 7.8 \times 10^5$

c $6.75 \times 10^{-4} + 4.25 \times 10^{-4}$

d $3.9 \times 10^7 + 4.2 \times 10^6$

e $5.6 \times 10^{-4} + 2.07 \times 10^{-3}$

Q16d hint

Both numbers need to have the same power of 10 before you add them.
$4.2 \times 10^6 = \square \times 10^7$

17 Work out

a $9.6 \times 10^{-7} - 6.3 \times 10^{-7}$

b $8.88 \times 10^4 - 8.37 \times 10^4$

c $5.33 \times 10^6 - 2.8 \times 10^5$

d $7.02 \times 10^{-3} - 6.1 \times 10^{-4}$

18 **STEM** The wavelengths in the visible light spectrum extend from 3.8×10^{-7} m to 7.5×10^{-7} m.
What is the range of wavelengths in the visible light spectrum?

19 **Explore** What is the smallest organism you can see?
Is it easier to explore this question now that you have completed the lesson?
What further information do you need to be able to answer this?

20 **Reflect** The title of this lesson is 'Orders of magnitude'. Why do you think scientists find it useful to know about orders of magnitude?

Explore

Reflect

DRAFT

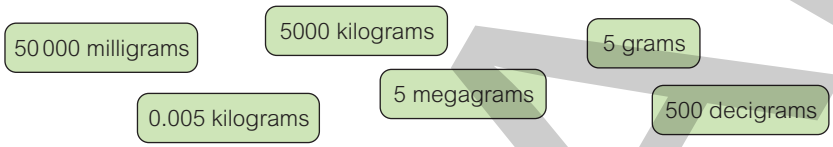
1 Check up

Powers of 10

1 a Complete this table of prefixes.

Prefix	Power of 10	Number
giga	$10^{\square}$	
mega	$10^{\square}$	
kilo	$10^{\square}$	1000
deci	$10^{\square}$	
centi	$10^{\square}$	
milli	$10^{\square}$	0.001
micro	$10^{\square}$	

b Match the cards that show the same value.



2 Write these numbers in order of size, starting with the smallest.

4.6×10^4 $8.9 \div 10^5$ 2.1×10^5 $2.4 \div 10^7$

Calculating and estimating

3 Use rounding to estimate the answers to

a 13.3×12.8

b $24.8 \div 5.2$

4 Evaluate

a $\dfrac{3^3 \times 5}{2}$

b $\dfrac{4^2 \times 3}{2^3}$

Indices

5 Write each of these numbers as a fraction.

a 5^{-3}

b 6^{-1}

c 2^{-4}

6 Write as a single power.

a $10^3 \times 10^{-4}$

b $3^{-2} \div 3$

c $(3^{-2})^3$

d $7^{-5} \div 7^{-2}$

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Standard form

7 Write each number in standard form.

a 345

c 34.5×10^3

d 0.005×10^6

8 Write 0.007 231 in standard form.

9 Write these numbers in order, from smallest to largest.

3.1×10^{-2} 3.2×10^{-3} 3.22×10^3 3.022×10^4 3.2×10^{-5}

Calculating with standard form

10 Work out each calculation. Give your answers in standard form.

a $(4.1 \times 10^{-6}) \times (2 \times 10^3)$

b $\dfrac{6 \times 10^3}{1.5 \times 10^2}$

11 Work out each calculation. Give your answers

i in standard form

ii as ordinary numbers.

a $\dfrac{23.31 \times 10^5}{3.7 \times 10^7}$

b $(7.09 \times 10^2) \times (6.3 \times 10^3)$

12 **Real** The mass of iron in planet Earth is 2.090×10^{24} kg. Given that the Earth has a mass of 5.972×10^{24} kg, find the percentage of the Earth's mass that is iron.

13 **How sure are you of your answers? Were you mostly**

☹ **Just guessing** 😐 **Feeling doubtful** 🙂 **Confident**

What next? Use your results to decide whether to strengthen or extend your learning.

Challenge

14 $a = 2.3 \times 10^6$ and $b = 2.3 \times 10^{-3}$

Calculate

a a^2 **b** b^3

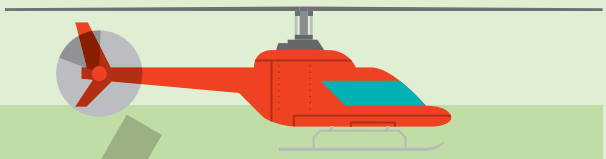
c a^3b^4 **d** $\dfrac{a}{b^6}$

Give your answers in standard form.

Reflect

DRAFT

1 Strengthen



You will:
- Strengthen your understanding with practice.

Powers of 10

1 Copy and complete
- **a** kilo (k) = 10^3 = 1000
- **b** mega (M) = 10^6 = ☐
- **c** giga (G) = 10^9 = ☐

Key point

To convert bigger units to smaller units, multiply ←

×1000 ×1000 ×1000 ×1000 ×1000 ×1000 ×1000 ×1000

pm nm µm mm m km Mm Gm Tm

÷1000 ÷1000 ÷1000 ÷1000 ÷1000 ÷1000 ÷1000 ÷1000

To convert smaller units to bigger units, divide →

Literacy hint

To write an abbreviation, put the letter for the prefix before the letter for the measure. So pm means picometre, nm means nanometre and so on.

2 Convert
- **a** 6.5 Tm to km
- **b** 0.014 m to nm
- **c** 50 000 nm to mm
- **d** 2200 km to Mm
- **e** 0.000 0006 Gm to mm

Q2a hint

6.5 × 1000 × 1000 × 1000 = ☐

3 Convert
- **a** 5 kilojoules (kJ) to joules (J)
- **b** 0.021 megawatts (MW) to watts (W)
- **c** 270 000 l to ml
- **d** 720 µg to mg

4 STEM
- **a** Safia's computer processor has a speed of 6.1 megahertz (MHz). What is its speed in kilohertz (kHz)?
- **b** The wavelength of a red light is 690 nm. Convert this length to µm.

Calculating and estimating

1 Copy these numbers. Circle the first significant figure. Write down its value.

a 32.45 **b** 0.64 **c** 25 800 **d** 0.0782

Q2a hint

Circle the first significant figure. It's in the 10s column, so round to the nearest 10.

2 Write the numbers in Q1 to 1 significant figure.

3 Round these numbers to the given number of significant figures.
- **a** 53 876 (2 s.f.) **b** 0.735 (2 s.f.)
- **c** 56.554 (3 s.f.) **d** 0.002 410 6 (3 s.f.)

Q3a hint

Circle the second significant figure. What place-value column is it in?

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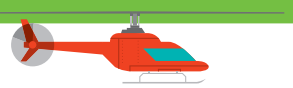


4 Round each number in these calculations to 1 significant figure. Then estimate the answer to each calculation.

a 44×273 **b** 67×534 **c** $421 \div 18$ **d** $(585 \div 33)^2$

Q4a hint

④4 → 40, ②73 → 300, $40 \times 300 = \square$.

Indices

1 a i $3^2 \div 3^5 = 3^{\square}$

ii $3^2 \div 3^5 = \frac{1}{2} = \frac{\cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3} \times 3 \times 3 \times 3} = \frac{\square}{\square}$

iii Use your answers to copy and complete: $3^{-3} = \frac{1}{3^{\square}}$

Q1a hint

Which index rule can you use?

b Copy and complete.

i $7^{-2} = \frac{1}{7^{\square}}$ **ii** $4^{-5} = \frac{1}{4^{\square}}$ **iii** $\frac{1}{9^3} = 9^{\square}$ **iv** $\frac{1}{5^7} = 5^{\square}$

2 Write each calculation as a single power.

a $7^2 \times 7^{-4} = 7^{2 + -4} = 7^{\square}$

b $3^{-1} \times 3^5 = 3^{\square + \square} = 3^{\square}$

c $5^{-4} \times 5$

d $8^2 \div 8^6 = 8^{\square - \square} = 8^{\square}$

e $4^{-3} \div 4^{-5}$

f $\frac{10^3}{10^7}$

g $(5^{-2})^3 = 5^{-2 \times \square} = 5^{\square}$

h $(6^5)^{-4}$

Q2c hint

$5 = 5^1$

3 Which calculations in Q2 have answers that are less than 1?

Q3 hint

Write each answer as a fraction. Which fractions have a numerator smaller than the denominator?

Standard form

1 Work out

a 3.7×10^3
b 2.5×10^4
c 8.1×10^2
d 5.4×10^7

Q1a hint

3.7

3 7 0 0

3.7×10^3 means multiply 3.7 by 10 three times.

2 Work out

a 9.3×10^{-3}
b 7.3×10^{-2}
c 1.5×10^{-4}
d 4.9×10^{-6}

Q2a hint

9.3

0.0 0 9 3

9.3×10^{-3} means divide 9.3 by 10 three times.

3 A number written using standard form looks like this:

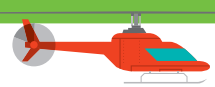
$A \times 10^n$

number between 1 and 10 — times sign — power of 10

Write each number using standard form.

a $3100 = 3.1 \times 10^{\square}$ **b** 29 000
c 7 150 000 **d** 69 000 000 000

DRAFT



4 Write each number using standard form.

a $0.0064 = 6.4 \times 10^{\square}$

b 0.072

c 0.000 004

d 0.000 000 021

5 Write the numbers in each list in order, from smallest to largest.

a 1.8×10^{5} 3.7×10^{-2} 9.4×10^{2} 6.9×10^{-7}

b 4×10^{-1} 4.2×10^{-2} 4.22×10^{2} 2.4×10^{2} 2.44×10^{-1}

Calculating with standard form

1 Work out each calculation. Give your answers in standard form.

a $(3 \times 10^{4}) \times (2.6 \times 10^{5}) = 3 \times 2.6 \times 10^{4} \times 10^{5} = \square \times 10^{\square}$

b $(1.7 \times 10^{5}) \times (2 \times 10^{3})$

c $(5 \times 10^{2}) \times (2.5 \times 10^{8})$

d $\dfrac{6.6 \times 10^{6}}{2.2 \times 10^{2}} = \dfrac{6.6}{2.2} \times \dfrac{10^{6}}{10^{3}} = \square \times 10^{\square}$

e $\dfrac{7.8 \times 10^{3}}{3 \times 10^{7}}$

f $\dfrac{2 \times 10^{9}}{8 \times 10^{5}}$

2 Work out each calculation. Give your answers in standard form.

a $(6.41 \times 10^{5}) \times (1.8 \times 10^{7})$

b $(3.7 \times 10^{-3}) \times (9.3 \times 10^{10})$

c $\dfrac{55.8 \times 10^{9}}{6.2 \times 10^{2}}$

d $\dfrac{2.136 \times 10^{3}}{3.56 \times 10^{8}}$

3 The average distance from the Sun to the Earth is approximately 1.5×10^{8} km.
Light travels at $3 \times 10^{8}\,\text{m s}^{-1}$.
Find the time taken for light to travel from the Sun to the Earth.

4 **Problem-solving** How thick is a single page in this book?
Follow these steps to find out.

a Find out how many sheets of paper are in the book.

b Use a ruler to measure the total thickness of the pages in the book (in mm).

c Use your calculator to find the thickness of one sheet in mm. Write the answer in standard form.

d Convert your answer in part **c** to nanometres (nm). Give your answer to 1 decimal place.

Q4a hint

6.4
0.0 0 6 4

Divide by how many 10s to get 0.0064?

6.4 lies between 1 and 10.

Q5 hint

Look at the powers of 10 first.
If numbers have the same power of 10, sort them by the decimal number.

Q2a hint

Use the 10^{x} key on your calculator.

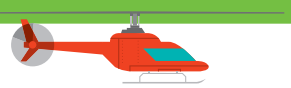
Q3 hint

Ensure that you use the same units.

Q4d hint

1 nm = 10^{-6} mm

DRAFT



5 **Real** Write these countries in order of population size, from smallest to largest.

Country	Population (July 2014)
UK	6.411×10^7
New Zealand	4.540×10^6
Iceland	3.263×10^5
Japan	1.271×10^8
St Lucia	1.8×10^5
Brazil	2.028×10^8

6 **Real** Use the information in the table in Q6 to answer these questions.

a Write the population of Iceland as an ordinary number.

b Calculate the total population of all the countries in the table.

c How many times bigger is the population of the UK than the population of St Lucia?

7 **STEM** An electron has a mass of 9.109×10^{-31} kg.

a How many electrons are there in 1 kg of electrons?
A proton has a mass of 1.673×10^{-27} kg.

b How many electrons are equivalent to one proton?

Enrichment

8 On Earth, the oceans cover an area of 3.62×10^8 km^2 with a mean depth of 3.68×10^4 m.
Use this information to estimate the volume of water in the oceans of the Earth. Give your answer in standard form in m^3.

9 **Reflect** Nandini says, 'Working with indices, powers and roots is all about adding, subtracting, multiplying and dividing'.
Look back at the questions you answered in these Strengthen lessons. Describe when you had to:

- add
- subtract
- multiply
- divide.

Do you agree with Nandini statement? Give some examples to explain why.

Reflect

DRAFT

Subject link: Geography (Q5 and 6), Science (Q3, 7 and 8)

1 Extend

You will:
- Extend your understanding with problem-solving.

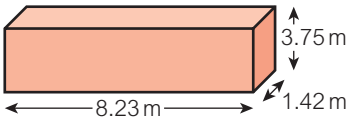
1 a Match each prefix to its correct power of 10.

centi	micro	giga	pico	kilo		
10^{3}	10^{-3}	10^{9}	10^{-2}	10^{-6}	10^{-1}	10^{-12}

b Write the prefixes for the remaining powers of 10.

2 Work out these conversions.
- **a** 1 kilogram (kg) = ☐g
- **b** 1 megajoule (MJ) = ☐J
- **c** 1 gigatonne (Gt) = ☐t
- **d** 1 terawatt (TW) = ☐W
- **e** 1 decilitre (dl) = ☐l

3 The diagram shows a cuboid.



What is the surface area of the cuboid?
Give your answer to 3 significant figures.

4 Write as a single power.

- **a** $11^{7} \times 11^{-3} \div 11^{-2}$
- **b** $3^{-13} \times 3^{4} \div 3^{-5} \div 3^{2}$
- **c** $\dfrac{7^{-7} \times 7^{-2}}{7^{-1} \div 7^{8}}$
- **d** $\dfrac{5^{-3} \div 5^{3}}{5^{10} \times 5^{-3}}$

5 Work out

- **a** $3^{-3} \times 2^{-2} \times 2^{-1} \times 3^{4}$
- **b** $\left(\frac{1}{2}\right)^{-3} \times \left(\frac{1}{3}\right)^{-1} \times \left(\frac{1}{2}\right)^{2} \times \left(\frac{1}{3}\right)^{-1}$
- **c** $\left(\frac{1}{2}\right)^{-2} \times \left(\frac{1}{3}\right) \times \left(\frac{1}{2}\right)^{3} \times \left(\frac{1}{3}\right)^{-2}$
- **d** $\left(\frac{1}{2}\right)^{-3} \times \left(\frac{1}{2}\right)^{3}$

6 **Real / STEM** Scientists often use units written with negative indices.
For example, 30 m/s = 30 ms^{-1}.
Write these units using negative indices.
- **a** km/h
- **b** m/s^2
- **c** kg/m^3
- **d** mi/h

7 **STEM** There are 100 trillion microorganisms in the human intestines.
This is 10 times the number of cells in a human body.
Write the number of cells in a human body, in standard form.

8 **Problem-solving** Write these numbers in order, from smallest to largest.

1.26×10^{-3} 0.12×10^{-2} 0.00124 1205×10^{-6} $\dfrac{1}{8 \times 10^{2}}$

Q2e hint
1 decimetre = 0.1 m

Q7 Literacy hint
1 trillion = 1 000 000 000 000

Q8 Strategy hint
Write each number in standard form first.

DRAFT

9 **Real** The table shows how people accessed information about a football tournament.

	Number of people (to 3 s.f.)
Connected with the official site	1.12×10^8
Apps downloaded	2.20×10^7
Facebook users	4.55×10^8
Tweets during the match	3.66×10^7

Use the data to work out the missing number in each sentence.

a There were roughly _____ times more Facebook users than people connected to the official site.

b There were roughly _____ times more Facebook users than tweets in the match.

c There were roughly _____ times more tweets in the match than people who downloaded apps.

10 Put the answers to these calculations in order, from smallest to largest.

A $(2.3 \times 10^{-3}) \times (7.4 \times 10^{-2})$

B $(1.3 \times 10^{-2})^2$

C $(5.3 \times 10^{-2}) \div (3.2 \times 10^2)$

D $(1.091 \times 10^{-4}) + (6 \times 10^{-5})$

E $(1.8 \times 10^{-4}) - (1.8 \times 10^{-5})$

11 Work out the reciprocals of these numbers. Give your answers in standard form.

a 2×10^9

b 8×10^7

c 4×10^{-5}

d 1.6×10^{-4}

12 **Real** The number of app downloads in July 2008 was 1.0×10^7. There were 10 times as many in September 2008 and 10 times as many again in April 2009.
In June 2014 there were 7.5×10^{10} app downloads.

a How many downloads were there in April 2009?

b What was the increase from July 2008 to June 2014?

13 **STEM / Reasoning** The smallest size the human eye can see is 10^{-4} m. The diameter of a virus particle is 170 nm.
Could you see a group of 1 million virus particles with the naked eye? Explain your answer.

14 **STEM** The formula for working out the frequency of a wave in the electromagnetic spectrum is
$f = \frac{c}{\lambda}$ where c is the speed of light and λ is the wavelength.
$c = 3 \times 10^8$ m/s
Work out the frequency of

a red light with wavelength 6.9×10^{-7} m

b blue light with wavelength 4.65×10^{-7} m.

Give your answers in standard form to 2 significant figures.

Q14 Literacy hint

λ is the Greek letter *lambda*. It is sometimes used instead of a letter from our alphabet.
The unit for frequency is hertz (Hz).

DRAFT

15 **Reasoning** **a** Which of these numbers have the same value?

$(0.5)^3$	8^{-2}	$\left(\frac{1}{4}\right)^2$	$\left(\frac{1}{2}\right)^3$	2^{-3}	$\left(\frac{1}{64}\right)^{\frac{1}{2}}$

b In how many different ways can you write $\frac{1}{9}$?

16 **Real** The populations of Bangladesh, China, India and Pakistan in 2014 are shown in the table.

Country	Population in 2014
Bangladesh	1.556×10^8
China	1.366×10^9
India	1.247×10^9
Pakistan	1.880×10^8

a Write these countries in order of population size, from smallest to largest.

b What is the difference between the population of India and China?

c How many times larger is the population of India than that of Pakistan?

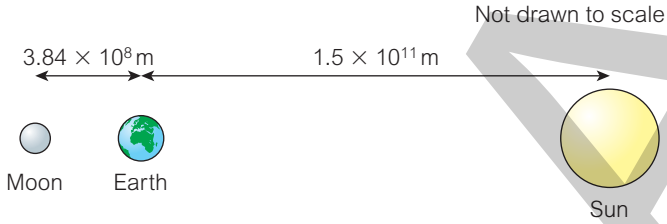
d What is the total population of these four countries?

The world population is 7.183×10^9.

e What proportion of the world's population lives in China or India?

Challenge

17 **Real / STEM / Modelling** A science museum wants to make a scale model of the Solar System.



The diagram shows the real distances between the Earth, Moon and Sun.
In the model the Earth and the Moon are 10 cm apart.
How far away from the Earth will the Sun need to be?

Discussion Is this a good scale for the model? Suggest some distances that might work better.

Investigation

Reasoning

The speed of light is 3×10^8 m/s.

a How many kilometres does light travel in one year? (Assume 1 year to be 365 days).

Neptune is approximately 4.5 billion km from the Sun.

b Work out how long it takes light from the Sun to reach Neptune.

18 **Reflect** In this unit you have learned a lot of new vocabulary.
Write a list of all the new vocabulary you have used.
Write, in your own words, a definition for each one.
Compare your definitions with those of your classmates.
Did you all learn the same thing?

Q18 hint

A light year is the distance travelled by light in one year.

Reflect

DRAFT

1 Unit test

1 **Real** The stadium that will host the 2020 Superbowl seats 98 025 people.
The average price of a ticket is $120.
Estimate the total money taken from ticket sales for the Superbowl.

2 Work out

a $\dfrac{4 + 3 \times 6 - 4}{3^2 - 4}$

b $25\left(3^3 + 2\right) \div 5 \times 3$

3 Use rounding to one significant figure to estimate

a 1875×5.36

b $\dfrac{285 \times 3.16}{11.2}$

4 Write each number in standard form.

a 820

b 0.000 091 5

5 Put these numbers in order, from smallest to largest.

1.24×10^{-2}
1.21×10^{-4}
1.2×10^{2}
1.23×10^{3}
1.24×10

6 Work out each calculation. Give your answers in standard form.

a $\dfrac{8.8 \times 10^8}{2.2 \times 10^3}$

b $(2.5 \times 10^4) \times (5 \times 10^{-7})$

7 Work out each calculation. Give your answers in standard form.

a $(1.505 \times 10^{-9}) \times (8.3 \times 10^4)$

b $\dfrac{48.96 \times 10^3}{5.1 \times 10^{-3}}$

8 Write as a single power.

a $6^3 \times 6^{-4}$

b $3^{-4} \div 3^{-2}$

c $(7^{-3})^2$

d $\dfrac{4^{-4} \div 4^{-2}}{4^{-1} \times 4^{-1}}$

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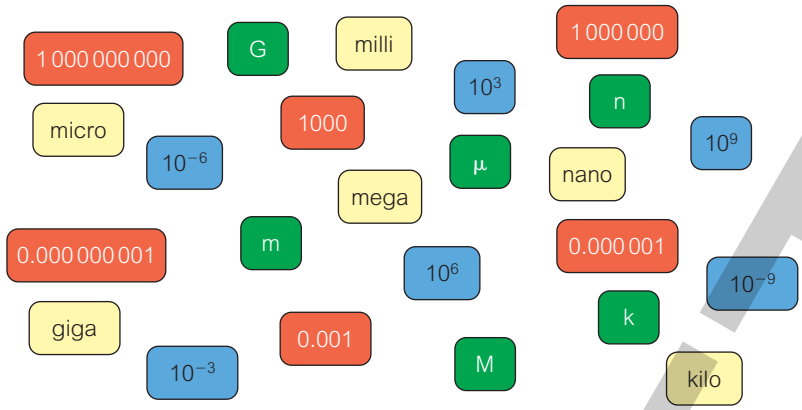
9 a Arrange these cards into their correct groups.
Each group must have one card of each colour.



b Convert
- **i** 9 GJ to joules
- **ii** 13 kW to watts
- **iii** 8.5 Ms to seconds.

10 An African elephant weighs about 6 tonnes.
The Earth weighs 5.97×10^{24} kg.
- **a** How many kg are in a tonne?
- **b** How many tonnes does the Earth weigh?
- **c** What is the mass of the Earth, measured in elephants? Give your answer in standard form.
- **d** Look back at the data in Exercise 1.4 Q8. How many elephants do you need to make the mass of each planet?

Challenge

11 Problem-solving A publisher prints 1.25×10^7 copies of a newspaper.
Each newspaper consists of 16 sheets of paper.
- **a** Calculate the number of sheets of paper needed to print all the newspapers. Give your answer in standard form.

To make the newspapers, the sheets of paper are folded in half.
- **b** The height of a pile of newspapers is 125 cm. The pile contains 420 newspapers.
Calculate the thickness of one sheet of paper. Give your answer in metres in standard form.

12 Reflect Which of the questions in this unit test:
- took the shortest time to answer? Why?
- took the longest time to answer? Why?
- were the most thought-provoking? Why?

Reflect

DRAFT