A.1 Kinematics

When can you think of a steam train as a particle?

Guiding Questions

How can the motion of a body be described quantitatively and qualitatively?

How can the position of a body in space and time be predicted?

How can the analysis of motion in one and two dimensions be used to solve real-life problems?

The photograph at the start of this chapter shows a train, but we will not be dealing with complicated systems like trains in their full complexity. In physics, we try to understand everything on the most basic level. Understanding a physical system means being able to predict its final conditions given its initial conditions. To do this for a train, we would have to calculate the position and motion of every part – and there are a lot of parts. In fact, if we considered all the particles that make up all the parts, then we would have a huge number of particles to deal with.

In this course, we will be dealing with one particle of matter at a time. This is because the ability to solve problems with one particle makes us able to solve problems with many particles. We may even pretend a train is one particle.

The initial conditions of a particle describe where it is and what it is doing. These can be defined by a set of numbers, which are the results of measurements. As time passes, some of these quantities might change. What physicists try to do is predict their values at any given time in the future. To do this, they use mathematical models.

Nature of Science

From the definitions of velocity and acceleration, we can use mathematics to derive a set of equations that predict the position and velocity of a particle at any given time. We can show by experiment that these equations give the correct result for some examples, then make the generalization that the equations apply in all cases.

Students should understand:

that the motion of bodies through space and time can be described and analyzed in terms of position, velocity, and acceleration
velocity is the rate of change of position, and acceleration is the rate of change of velocity
the change in position is the displacement
the difference between distance and displacement
the difference between instantaneous and average values of velocity, speed and acceleration, and how to determine them

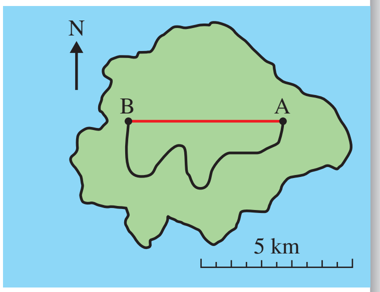
the equations of motion for solving problems with uniformly accelerated motion as given by: $s = \frac{u+v}{2}t$ $v = u + at$ $s = ut + \frac{1}{2}at^2$ $v^2 = u^2 + 2as$
motion with uniform and non-uniform acceleration
the behavior of projectiles in the absence of fluid resistance, and the application of the equations of motion resolved into vertical and horizontal components
the qualitative effect of fluid resistance on projectiles, including time of flight, trajectory, velocity, acceleration, range and terminal speed.

Further information about the fluid resistance force can be found in A.2.

Nature of Science

In the Tools chapter, we observed that things move and now we are going to mathematically model that movement. Before we do that, we must define some quantities.

Displacement and distance



A.1 Figure 1

Note: since displacement is a vector, you should always say what the direction is.

It is important to understand the difference between distance traveled and displacement. To explain this, consider the route marked out on the map shown in Figure 1.

Displacement is the shortest path moved in a particular direction.

The unit of displacement is the meter (m). Displacement is a vector quantity.

On the map, the displacement is the length of the straight line from A to B, which is a distance of 5 km west.

Distance is how far you have traveled from A to B.

The unit of distance is also the meter (m). Distance is a scalar quantity.

In this example, the distance traveled is the length of the path taken, which is about 10 km.

Sometimes, this difference leads to a surprising result. For example, if you run all the way round a running track, you will have traveled a distance of 400 m but your displacement will be 0 m.

In everyday life, it is often more important to know the distance traveled. For example, if you are going to travel from Paris to Lyon by road, you will want to know that the distance by road is 450 km, not that your final displacement will be 336 km SE. However, in physics, we break everything down into its simplest parts, so we start by considering motion in a straight line only. In this case, it is more useful to know the displacement, since that also has information about which direction you have traveled in.

Velocity and speed

Both speed and velocity are a measure of how fast a body is moving.

Velocity is defined as the rate of change of position. Since 'change of position' is displacement and 'rate of change' requires division by time taken:

$$\text{velocity} = \frac{\text{displacement}}{\text{time}}$$

$$\text{velocity} = \frac{\text{displacement}}{\text{time}}$$

The unit of velocity is m s^{-1}.

Velocity is a vector quantity.

Speed is defined as the distance traveled per unit time:

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

The unit of speed is also m s^{-1}.

Speed is a scalar quantity.

Exercise

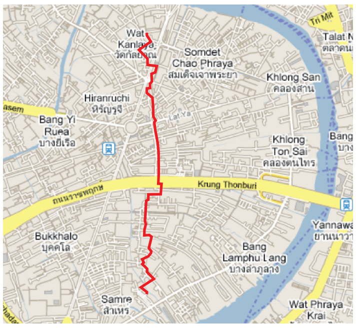
Q1. Convert the following speeds into m s^{-1}:

(**a**) a car traveling at 100 km h^{-1}

(**b**) a runner running at 20 km h^{-1}.

Average velocity and instantaneous velocity

Consider traveling by car from the north of Bangkok to the south – a distance of about 16 km. If the journey takes 4 hours, you can calculate your velocity to be $\frac{16}{4}$ = 4 km h^{-1} in a southward direction. This does not tell you anything about the journey, just the difference between the beginning and the end (unless you managed to travel at a constant speed in a straight line). The value calculated is the **average velocity** and in this example it is quite useless. If we broke the trip down into lots of small pieces, each lasting only one second, then for each second the car could be considered to be traveling in a straight line at a constant speed. For these short stages, we could quote the car's **instantaneous velocity** – which is how fast it is going at that moment in time and in which direction.



A.1 Figure 2 It is not possible to take this route across Bangkok with a constant velocity.

The bus in the photo has a constant velocity for a very short time.

Exercise

Q2. A runner runs once around a circular track of length 400 m with a constant speed in 96 s. Calculate:

(**a**) the average speed of the runner

(**b**) the average velocity of the runner

(**c**) the instantaneous velocity of the runner after 48 s

(**d**) the displacement after 24 s.

Constant velocity

If the velocity is constant, then the instantaneous velocity is the same all the time so:

$$\text{instantaneous velocity} = \text{average velocity}$$

Since velocity is a vector, this also implies that the direction of motion is constant.

What does Newton's first law tell us about the forces on a body traveling at constant velocity? (A.2)

Measuring a constant velocity

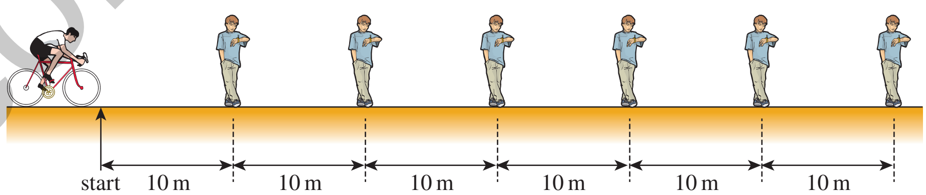
From the definition of velocity, we see that:

$$\text{velocity} = \frac{\text{displacement}}{\text{time}}$$

Rearranging this gives:

$$\text{displacement} = \text{velocity} \times \text{time}$$

So, if velocity is constant, displacement is proportional to time. To test this relationship and find the velocity, we can measure the displacement of a body at different times. To do this, you either need a lot of clocks or a stop clock that records many times. This is called a **lap timer**. In this example, a bicycle was ridden at constant speed along a straight road past six students standing 10 m apart, each operating a stop clock as in Figure 3. The clocks were all started when the bike, already moving, passed the start marker and stopped as the bike passed each student.



A.1 Figure 3 Measuring the time for a bike to pass.

The results achieved are shown in Table 1.

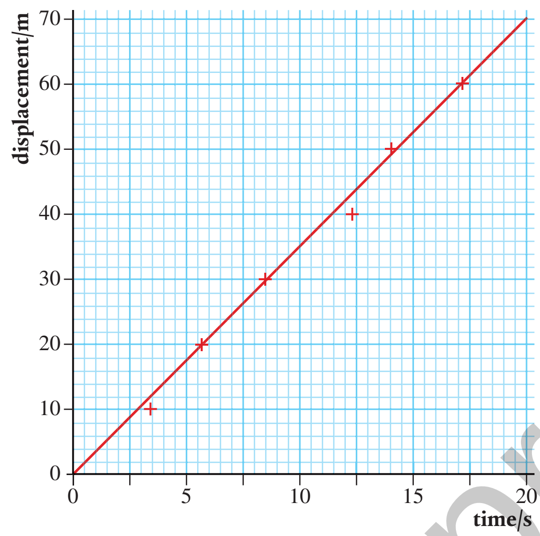
Displacement/m ±0.1 m	Time/s ±0.02 s
10.0	3.40
20.0	5.62
30.0	8.55
40.0	12.31
50.0	14.17
60.0	17.21

A.1 Table 1

The uncertainty in displacement is given as 0.1 m since it is difficult to decide exactly when the bike passed the marker.

The digital stop clock has a scale with 2 decimal places, so the uncertainty is 0.01 s. However, the uncertainty given is 0.02 s since the clocks all had to be started at the same time.

Since displacement (*s*) is proportional to time (*t*), then a graph of *s* vs *t* should give a straight line with gradient = velocity as shown in Figure 4.



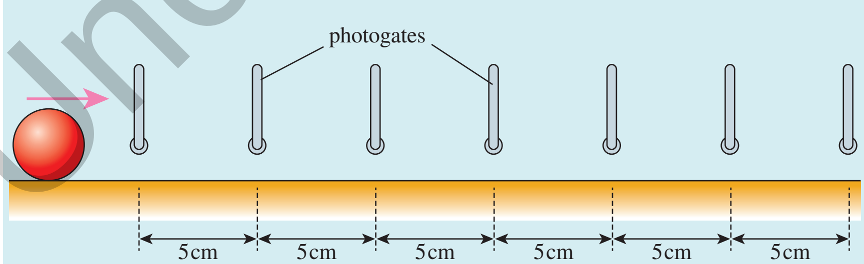
A.1 Figure 4 Graph of displacement vs time for a bike.

Notice that in this graph the line does not pass through all the points. This is because the uncertainty in the measurement in time is almost certainly bigger than the uncertainty in the clock (±0.02 s) due to the reaction time of the students stopping the clock. To get a better estimate of the uncertainty, we would need to have several students standing at each 10 m position. Repeating the experiment is not possible in this example since it is very difficult to ride at the same velocity several times.

The gradient indicates that: velocity = 3.5 m s^{-1}

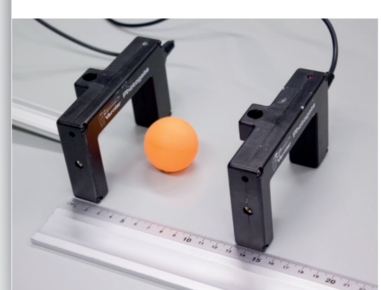
SKILLS

Most school laboratories are not large enough to ride bikes in so when working indoors, we need to use shorter distances. This means that the times are going to be shorter so hand-operated stop clocks will have too great a percentage uncertainty. One way of timing in the lab is by using photogates. These are connected to a computer via an interface and record the time when a body passes in or out of the gate. So, to replicate the bike experiment in the lab using a ball, we would need seven photogates as in Figure 5, with one extra gate to represent the start.



A.1 Figure 5 How to measure the time for a rolling ball if you have seven photogates.

This would be quite expensive so we compromise by using just two photogates and a motion that can be repeated. An example could be a ball moving along a horizontal section of track after it has rolled down an inclined plane. Provided the ball starts from the same point, it should have the same velocity. So, instead of using seven photogates, we can use two – one is at the start of the motion and the other is moved to different positions along the track as in Figure 6.



A.1 Figure 6 The ball interrupts the infrared light transmitted across each gate as it passes through them. The times of these interruptions are measured and recorded by a data logger.

A.1 Figure 7 Measuring the velocity of a ball with two photogates.

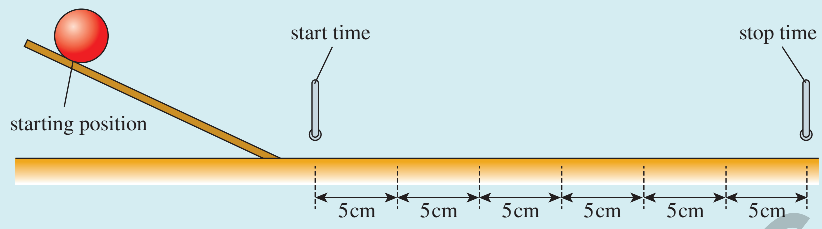
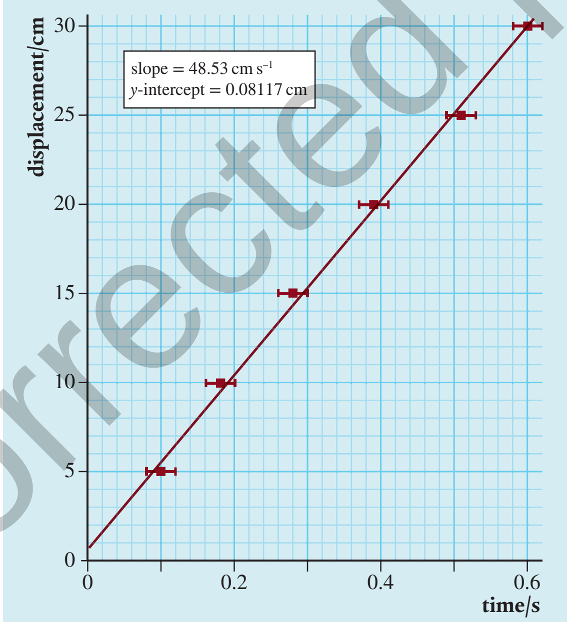


Table 2 shows the results obtained using this arrangement.

A.1 Table 2

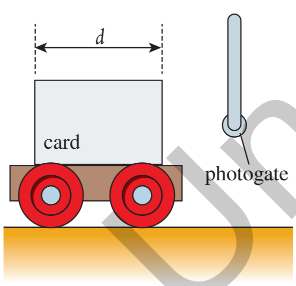
Displacement/ cm ± 0.1 cm	**Time(t)/s ± 0.0001 s**					**Mean t/s**	**Δt/s**
5.0	0.0997	0.0983	0.0985	0.1035	0.1040	0.101	0.003
10.0	0.1829	0.1969	0.1770	0.1824	0.1825	0.18	0.01
15.0	0.2844	0.2800	0.2810	0.2714	0.2779	0.28	0.01
20.0	0.3681	0.3890	0.3933	0.3952	0.3854	0.39	0.01
25.0	0.4879	0.5108	0.5165	0.4994	0.5403	0.51	0.03
30.0	0.6117	0.6034	0.5978	0.6040	0.5932	0.60	0.01

A.1 Figure 8 Graph of displacement vs time for a rolling ball.



Notice that the uncertainty calculated from $\frac{(\text{max} - \text{min})}{2}$ is much more than the instrument uncertainty. A graph of displacement vs time gives Figure 8.

From this graph, we can see that within the limits of the experiment's uncertainties the displacement could be proportional to time, so we can conclude that the velocity may have been constant. However, if we look closely at the data, we see that there seems to be a slight curve, indicating that perhaps the ball was slowing down. To verify this, we would have to collect more data.



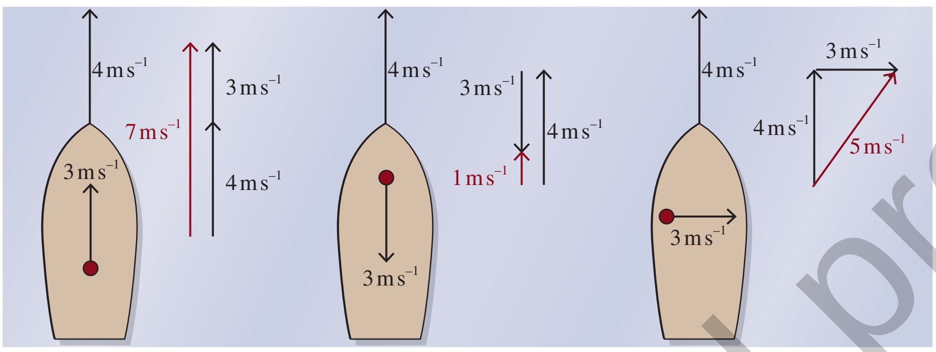
A.1 Figure 9 A card and photogate used to measure instantaneous velocity.

Measuring instantaneous velocity

To measure instantaneous velocity, a very small displacement must be used. This could be achieved by placing two photogates close together or attaching a piece of card to the moving body as shown in Figure 9. The time taken for the card to pass through the photogate is recorded and the instantaneous velocity calculated from: $\frac{\text{length of card}}{\text{time taken}} \left(\frac{d}{t}\right)$

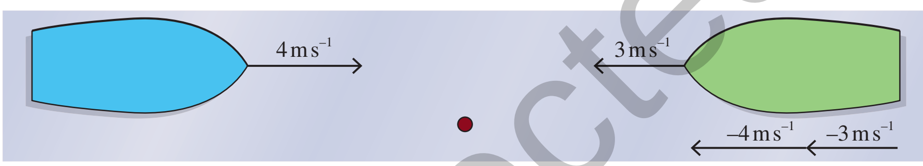
Relative velocity

Velocity is a vector so velocities must be added as vectors. Imagine you are running north at $3\,\mathrm{m\,s^{-1}}$ on a ship that is also traveling north at $4\,\mathrm{m\,s^{-1}}$ as shown in Figure 10. Your velocity relative to the ship is $3\,\mathrm{m\,s^{-1}}$ but your velocity relative to the water is $7\,\mathrm{m\,s^{-1}}$. If you turn around and run due south, your velocity will still be $3\,\mathrm{m\,s^{-1}}$ relative to the ship but $1\,\mathrm{m\,s^{-1}}$ relative to the water. Finally, if you run toward the east, the vectors add at right angles to give a resultant velocity of magnitude $5\,\mathrm{m\,s^{-1}}$ relative to the water. You can see that the velocity vectors have been added.



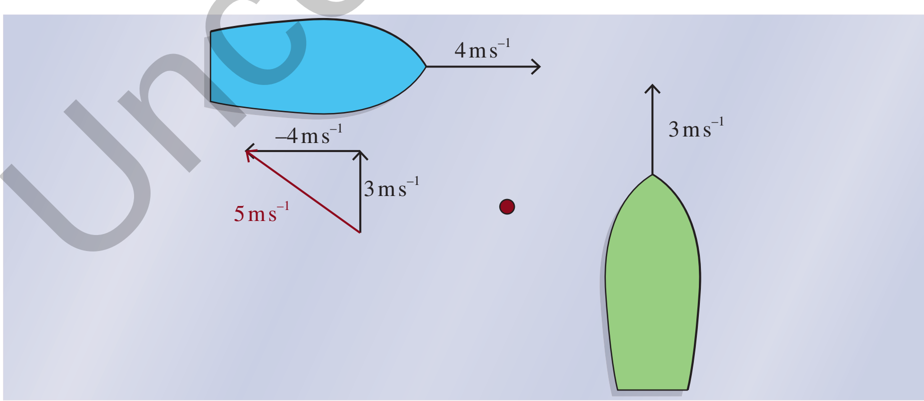
A.1 Figure 10 Running on board a ship.

Imagine that you are floating in the water watching two boats traveling toward each other as in Figure 11.



A.1 Figure 11 Two boats approach each other. The vector addition for the velocity of the green boat from the perspective of the blue boat is shown.

The blue boat is traveling east at $4\,\mathrm{m\,s^{-1}}$ and the green boat is traveling west at $-3\,\mathrm{m\,s^{-1}}$. Remember that the sign of a vector in one dimension gives the direction. So, if east is positive, then west is negative. If you were standing on the blue boat, you would see the water going past at $-4\,\mathrm{m\,s^{-1}}$ so the green boat would approach with the velocity of the water plus its velocity in the water: $-4 + -3 = -7\,\mathrm{m\,s^{-1}}$. This can also be done in two dimensions as in Figure 12.



A.1 Figure 12 Two boats traveling perpendicular to each other. The vector addition for the velocity of the green boat from the perspective of the blue boat is shown.

According to the swimmer floating in the water, the green boat travels north and the blue boat travels east, but an observer on the blue boat will see the water traveling toward the west and the green boat traveling due north. Adding these two velocities gives a velocity of $5\,\mathrm{m\,s^{-1}}$ in an approximately northwest direction.

What is the relative speed of the light from a star measured by a rocket traveling at 0.5 times the speed of light toward the star? (A.5)

How effectively do the equations of motion model Newton's laws of dynamics? (A.2)

Exercise

Q3. An observer standing on a road watches a bird flying east at a velocity of $10\,\mathrm{m\,s^{-1}}$. A second observer, driving a car along the road northward at $20\,\mathrm{m\,s^{-1}}$, sees the bird. What is the velocity of the bird relative to the driver?

Q4. A boat travels along a river heading north with a velocity $4\,\mathrm{m\,s^{-1}}$ as a woman walks across a bridge from east to west with velocity of $1\,\mathrm{m\,s^{-1}}$. Calculate the velocity of the woman relative to the boat.

Acceleration

In everyday usage, the word **accelerate** means to go faster. However, in physics, acceleration is defined as the rate of change of velocity:

$$\text{acceleration} = \frac{\text{change of velocity}}{\text{time}}$$

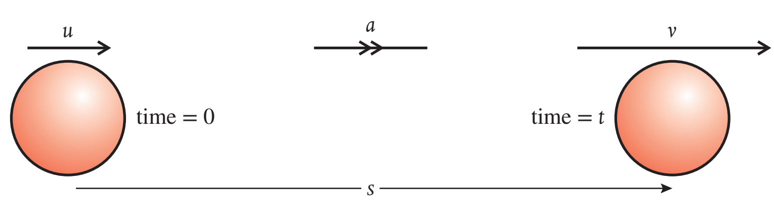
The unit of acceleration is $\mathrm{m\,s^{-2}}$.

Acceleration is a vector quantity.

This means that whenever a body changes its velocity, it accelerates. This could be because it is getting faster, slower, or just changing direction. In the example of the journey across Bangkok, the car would have been slowing down, speeding up and going round corners almost the whole time so it would have had many different accelerations. However, this example is far too complicated for us to consider in this course (and probably any physics course). For most of this chapter, we will only consider the simplest example of accelerated motion, which is constant acceleration.

Constant acceleration in one dimension

In one-dimensional motion, acceleration, velocity and displacement are all in the same direction. This means they can be added without having to draw triangles. Figure 13 shows a body that is starting from an initial velocity u and accelerating at a constant rate a to velocity v in t seconds. The distance traveled in this time is s. Since the motion is in a straight line, this is also the displacement.



A.1 Figure 13 A red ball is accelerated at a constant rate.

Using the definitions already stated, we can write equations related to this example.

Average velocity

From the definition, average velocity $= \dfrac{\text{displacement}}{\text{time}}$

$$\text{average velocity} = \frac{s}{t} \qquad (1)$$

Since the velocity changes at a constant rate from the beginning to the end, we can also calculate the average velocity by adding the initial and final velocities and dividing by two:

$$\text{average velocity} = \frac{(u+v)}{2} \qquad (2)$$

Acceleration

Acceleration is defined as the rate of change of velocity:

$$a = \frac{(v - u)}{t} \qquad (3)$$

We can use these equations to solve any problem involving constant acceleration. However, to make problem solving easier, we can derive two more equations by substituting from one into the other.

Equating equations (1) and (2):

$$\frac{s}{t} = \frac{(u + v)}{2}$$

$$s = \frac{(u + v)\,t}{2} \qquad (4)$$

Rearranging (3) gives: $v = u + at$

If we substitute for v in equation (4), we get: $s = ut + \frac{1}{2}at^2$ (5)

Rearranging (3) again gives: $t = \frac{(v - u)}{a}$

If t is now substituted in equation (4), we get: $v^2 = u^2 + 2as$ (6)

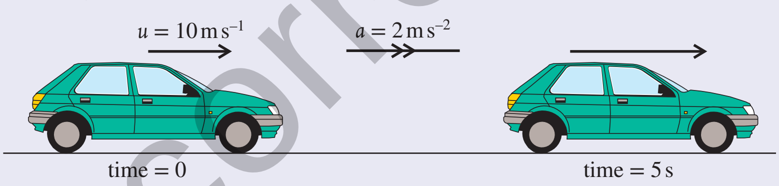
These equations are sometimes known as the *suvat* equations. If you know any three of s, u, v, a, and t, you can find either of the other two in one step.

Worked example

A car traveling at $10\,\text{m s}^{-1}$ accelerates at $2\,\text{m s}^{-2}$ for 5 s. What is its displacement?

Solution

The first thing to do is draw a simple diagram:



This enables you to see what is happening at a glance rather than reading the text. The next stage is to make a list of *suvat*.

$s = ?$

$u = 10\,\text{m s}^{-1}$

$v = ?$

$a = 2\,\text{m s}^{-2}$

$t = 5\,\text{s}$

To find s, you need an equation that contains *suat*. The only equation with all four of these quantities is: $s = ut + \frac{1}{2}at^2$

Using this equation gives: $s = 10 \times 5 + \frac{1}{2} \times 2 \times 5^2$

$s = 75\,\text{m}$

These equations are known as the *suvat* equations:

$$a = \frac{(v - u)}{t}$$

$$s = \frac{(v + u)t}{2}$$

$$s = ut + \tfrac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

How are the equations for rotational motion related to those for linear motion? (A.4)

When the units are consistent, you do not need to include units in all stages of a calculation, just in the answer.

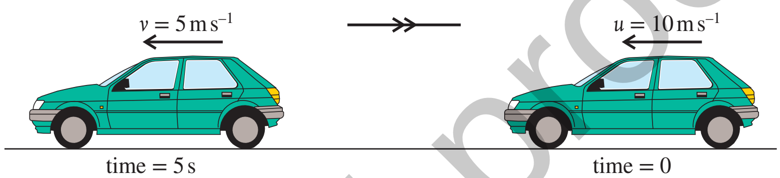
IB Physics Standard.indb 13 22/12/2022 16:30

The signs of displacement, velocity, and acceleration

We must not forget that displacement, velocity and acceleration are vectors. This means that they have direction. However, since this is a one-dimensional example, there are only two possible directions, forward and backward. We know which direction the vector is in from its sign.

If we take right to be positive:

- A positive displacement means that the body has moved to the right.
- A positive velocity means the body is moving to the right.
- A positive acceleration means that the body is either moving to the right and getting faster or moving to the left and getting slower. This can be confusing so consider the following example.



A.1 Figure 14 A car moves to the left with decreasing speed

The car is traveling in a negative direction so the velocities are negative.

$$u = -10\,\text{m s}^{-1}$$

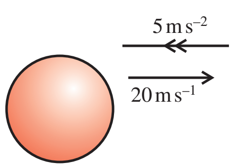
$$v = -5\,\text{m s}^{-1}$$

$$t = 5\,\text{s}$$

The acceleration is therefore given by:

$$a = \frac{(v - u)}{t} = \frac{-5 - (-10)}{5} = 1\,\text{m s}^{-2}$$

The positive sign tells us that the acceleration is in a positive direction (right) even though the car is traveling in a negative direction (left).



A.1 Figure 15 The acceleration is negative so points to the left.

The acceleration due to gravity is not constant all over the Earth. $9.81\,\text{m s}^{-2}$ is the average value. The acceleration also gets smaller the higher you go. However, we ignore this change when conducting experiments in the lab since labs are not that high.

To make the examples easier to follow, $g = 10\,\text{m s}^{-2}$ is used throughout. However, you should only use this approximate value in exam questions if told to do so.

Worked example

A body with a constant acceleration of $-5\,\text{m s}^{-2}$ is traveling to the right with a velocity of $20\,\text{m s}^{-1}$. What will its displacement be after 20 s?

Solution

$s = ?$

$u = 20\,\text{m s}^{-1}$

$v = ?$

$a = -5\,\text{m s}^{-2}$

$t = 20\,\text{s}$

To calculate s, we can use the equation: $s = ut + \frac{1}{2}at^2$

$$s = 20 \times 20 + \tfrac{1}{2}(-5) \times 20^2 = 400 - 1000 = -600\,\text{m}$$

This means that the final displacement of the body is to the left of the starting point. It has gone forward, stopped, and then gone backward.

Exercise

Q5. Calculate the final velocity of a body that starts from rest and accelerates at $5\,\mathrm{m\,s^{-2}}$ for a distance of 100 m.

Q6. A body starts with a velocity of $20\,\mathrm{m\,s^{-1}}$ and accelerates for 200 m with an acceleration of $5\,\mathrm{m\,s^{-2}}$. What is the final velocity of the body?

Q7. A body accelerates at $10\,\mathrm{m\,s^{-2}}$ and reaches a final velocity of $20\,\mathrm{m\,s^{-1}}$ in 5 s. What is the initial velocity of the body?

Free fall motion

Although a car has been used in the previous examples, the acceleration of a car is not usually constant so we should not use the *suvat* equations. The only example of constant acceleration that we see in everyday life is when a body is dropped. Even then, the acceleration is only constant for a short distance.

How does the motion of an object change within a gravitational field? (D.1)

Acceleration of free fall

When a body is allowed to fall freely, we say it is in free fall. Bodies falling freely on the Earth fall with an acceleration of about $9.81\,\mathrm{m\,s^{-2}}$ (depending where you are). The body falls because of gravity. For that reason, we use the letter g to denote this acceleration. Since the acceleration is constant, we can use the *suvat* equations to solve problems.

If you jump out of a plane (with a parachute on), you will feel the push of the air as it rushes past you. As you fall faster and faster, the air will push upward more and more until you cannot go any faster. At this point, you have reached terminal velocity. We will come back to this after introducing forces.

Exercise

In these calculations, use $g = 10\,\mathrm{m\,s^{-2}}$.

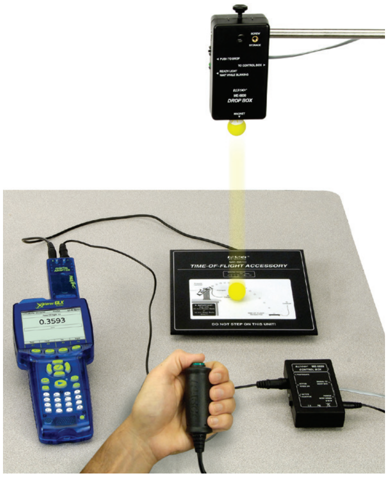
Q8. A ball is thrown upward with a velocity of $30\,\mathrm{m\,s^{-1}}$. What is the displacement of the ball after 2 s?

Q9. A ball is dropped. What will its velocity be after falling 65 cm?

Q10. A ball is thrown upward with a velocity of $20\,\mathrm{m\,s^{-1}}$. After how many seconds will the ball return to its starting point?

Measuring the acceleration due to gravity

When a body falls freely under the influence of gravity, it accelerates at a constant rate. This means that time to fall t and distance s are related by the equation: $s = ut + \frac{1}{2}at^2$. If the body starts from rest, then $u = 0$ so the equation becomes: $s = \frac{1}{2}at^2$. Since s is directly proportional to t^2, a graph of s vs t^2 would therefore be a straight line with gradient $\frac{1}{2}g$. It is difficult to measure the time for a ball to pass different markers, but if we assume the ball falls with the same acceleration when repeatedly dropped, we can measure the time taken for the ball to fall from different heights. There are many ways of doing this. All involve some way of starting a clock when the ball is released and stopping it when it hits the ground. Table 3 shows a set of results from a 'ball drop' experiment.



Apparatus for measuring g.

A.1 Table 3

Height(h)/m ± 0.001 m	Time(t)/s ± 0.001 s					Mean t/s	t^2/s^2	$\Delta(t^2)$/s^2
0.118	0.155	0.153	0.156	0.156	0.152	0.154	0.024	0.001
0.168	0.183	0.182	0.183	0.182	0.184	0.183	0.0334	0.0004
0.218	0.208	0.205	0.210	0.211	0.210	0.209	0.044	0.001
0.268	0.236	0.235	0.237	0.239	0.231	0.236	0.056	0.002
0.318	0.250	0.254	0.255	0.250	0.256	0.253	0.064	0.002
0.368	0.276	0.277	0.276	0.278	0.276	0.277	0.077	0.001
0.418	0.292	0.293	0.294	0.291	0.292	0.292	0.085	0.001
0.468	0.310	0.310	0.303	0.300	0.311	0.307	0.094	0.003
0.518	0.322	0.328	0.330	0.328	0.324	0.326	0.107	0.003
0.568	0.342	0.341	0.343	0.343	0.352	0.344	0.118	0.004

If a parachutist kept accelerating at a constant rate, they would break the sound barrier after about 30 s of flight. By understanding the forces involved, scientists have been able to design wing suits so that base jumpers can achieve forward velocities greater than their rate of falling.

SKILLS

A worksheet with full details of how to carry out this experiment is available in your eBook.

Notice that the uncertainty in t^2 is calculated from: $\frac{(t_{max}^2 - t_{min}^2)}{2}$

Notice how the line in Figure 16 is very close to the points and that the uncertainties reflect the actual random variation in the data. The gradient of the line is equal to $\frac{1}{2}g$ so $g = 2 \times$ gradient.

$$g = 2 \times 4.814 = 9.624\ \mathrm{m\,s^{-2}}$$

The uncertainty in this value can be estimated from the steepest and least steep lines:

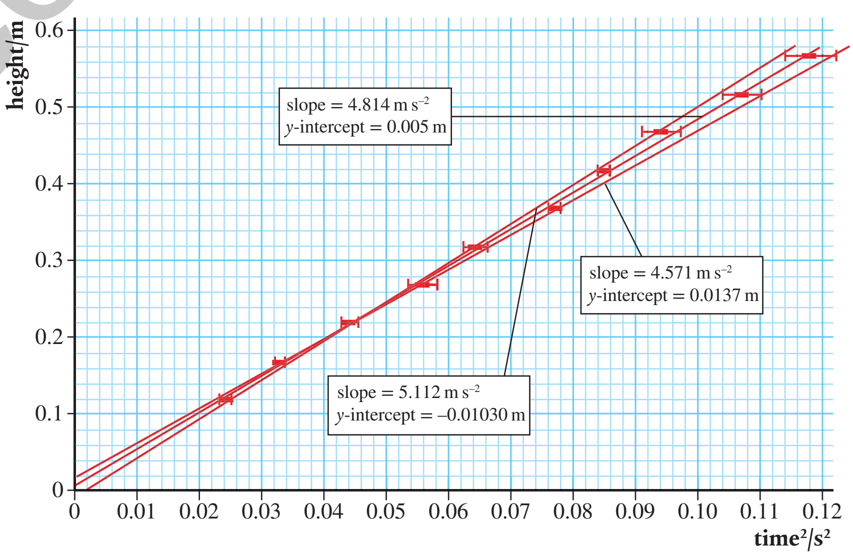
$$g_{max} = 2 \times 5.112 = 10.224\ \mathrm{m\,s^{-2}}$$

$$g_{min} = 2 \times 4.571 = 9.142\ \mathrm{m\,s^{-2}}$$

$$\Delta g = \frac{(g_{max} - g_{min})}{2} = \frac{(10224 - 9.142)}{2} = 0.541\ \mathrm{m\,s^{-2}}$$

So, the final value including uncertainty is $9.6 \pm 0.5\ \mathrm{m\,s^{-2}}$.

This is in agreement with the accepted average value which is $9.81\ \mathrm{m\,s^{-2}}$.



A.1 Figure 16 Height vs time^2 for a falling object.

Why would it not be appropriate to apply the *suvat* equations to the motion of a body falling freely from a distance of 2 times the Earth's radius to the surface of the Earth? (D.1)

Graphical representation of motion

Graphs are used in physics to give a visual representation of relationships. In kinematics, they can be used to show how displacement, velocity and acceleration change with time. Figure 17 shows the graphs for four different examples of motion.

The best way to sketch graphs is to split the motion into sections then plot where the body is at different times. Joining these points will give the displacement–time graph. Once you have done that, you can work out the *v–t* and *a–t* graphs by looking at the *s–t* graph rather than the motion.

> You need to be able to:
> - work out what kind of motion a body has by looking at the graphs
> - sketch graphs for a given motion.

Gradient of displacement–time graph

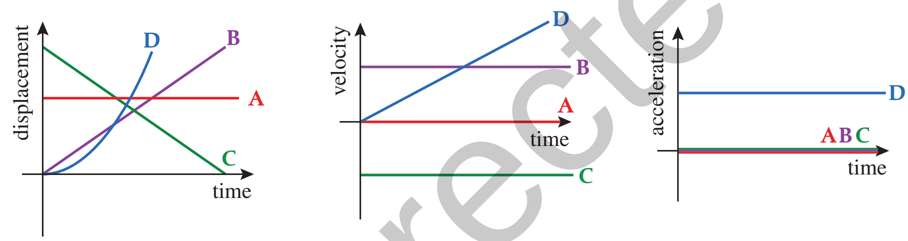
The gradient of a graph is: $\dfrac{\text{change in } y}{\text{change in } x} = \dfrac{\Delta y}{\Delta x}$

In the case of the displacement–time graph, this will give:

$$\text{gradient} = \frac{\Delta s}{\Delta t}$$

This is the same as velocity.

We can represent the motion of a body on displacement–time graphs, velocity–time graphs and acceleration–time graphs. The three graphs of these types shown in Figure 17 display the motion of four bodies, which are labeled A, B, C and D.



A.1 Figure 17 Graphical representation of motion.

Body A

A body that is not moving.
Displacement is always the same.
Velocity is zero.
Acceleration is zero.

Body B

A body that is traveling with a constant positive velocity.
Displacement increases linearly with time.
Velocity is a constant positive value.
Acceleration is zero.

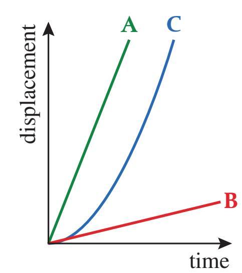
Body C

A body that has a constant negative velocity.
Displacement is decreasing linearly with time.
Velocity is a constant negative value.
Acceleration is zero.

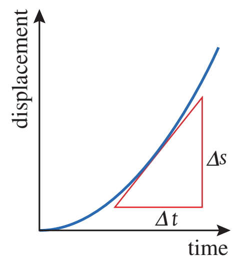
Body D

A body that is accelerating with constant acceleration.
Displacement is increasing at a non-linear rate. The shape of this line is a parabola since displacement is proportional to t^2 ($s = ut + \frac{1}{2}at^2$).
Velocity is increasing linearly with time.
Acceleration is a constant positive value.

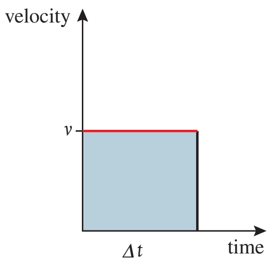
So, the gradient of the displacement–time graph equals the velocity. Using this information, we can see that line A in Figure 18 represents a body with a greater velocity than line B, and that since the gradient of line C is increasing, this must be the graph for an accelerating body.



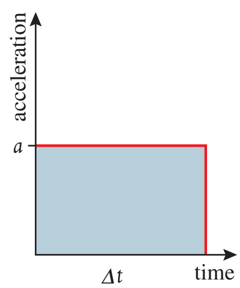
A.1 Figure 18 Three new bodies to compare.



▲ **A.1 Figure 19** Finding the gradient of the tangent



▲ **A.1 Figure 20** The area is displacement



▲ **A.1 Figure 21** The area is change in velocity

How does analyzing graphs allow us to determine other physical quantities? (NOS)

Instantaneous velocity

When a body accelerates, its velocity is constantly changing. The displacement–time graph for this motion is therefore a curve. To find the instantaneous velocity from the graph, we can draw a tangent to the curve and find the gradient of the tangent as shown in Figure 19.

Area under velocity–time graph

The area under the velocity–time graph for the body traveling at constant velocity v shown in Figure 20 is given by:

$$\text{area} = v\Delta t$$

But we know from the definition of velocity that: $v = \frac{\Delta s}{\Delta t}$

Rearranging gives $\Delta s = v\Delta t$ so the area under a velocity–time graph gives the displacement.

This is true, not only for simple cases such as this, but for all examples.

Gradient of velocity–time graph

The gradient of the velocity–time graph is given by $\frac{\Delta v}{\Delta t}$. This is the same as acceleration.

Area under acceleration–time graph

The area under the acceleration–time graph in Figure 21 is given by $a\Delta t$. But we know from the definition of acceleration that: $a = \frac{(v - u)}{t}$

Rearranging this gives $v - u = a\Delta t$ so the area under the graph gives the change in velocity.

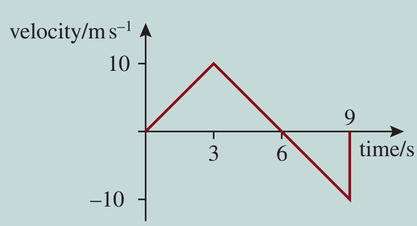
If you have covered calculus in your mathematics course, you may recognize these equations:

$$v = \frac{ds}{dt}, a = \frac{dv}{dt} = \frac{d^2s}{d^2t} \text{ and } s = \int v dt, v = \int a dt$$

Exercise

Q11. Sketch a velocity–time graph for a body starting from rest and accelerating at a constant rate to a final velocity of $25\,\mathrm{m\,s^{-1}}$ in 10 seconds. Use the graph to find the distance traveled and the acceleration of the body.

Q12. Describe the motion of the body whose velocity–time graph is shown. What is the final displacement of the body?

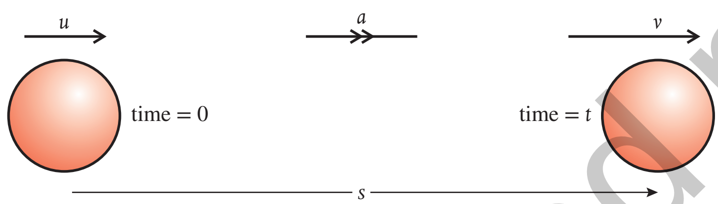


Q13. A ball is released from rest on the hill in the figure below. Sketch the *s–t, v–t,* and *a–t* graphs for its horizontal motion.

Q14. A ball rolls along a table then falls off the edge, landing on soft sand. Sketch the *s–t, v–t,* and *a–t* graphs for its vertical motion.

Example 1: The *suvat* example

As an example, let us consider the motion we looked at when deriving the *suvat* equations.

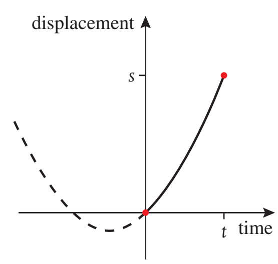


A.1 Figure 22 A body with constant acceleration.

Negative time does not mean going back in time – it means the time before you started the clock.

Displacement–time

The body starts with velocity u and travels to the right with constant acceleration a for a time t. If we take the starting point to be zero displacement, then the displacement–time graph starts from zero and rises to s in t seconds. We can therefore plot the two points shown in Figure 23. The body is accelerating so the line joining these points is a parabola. The whole parabola has been drawn to show what it would look like – the reason it is offset is because the body is not starting from rest. The part of the curve to the left of the origin tells us what the particle was doing before we started the clock.



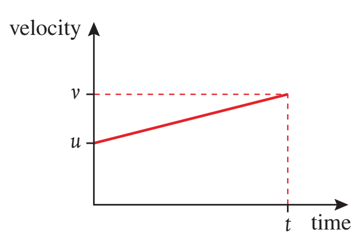
A.1 Figure 23 Constant acceleration.

Velocity–time

Figure 24 is a straight line with a positive gradient showing that the acceleration is constant. The line does not start from the origin since the initial velocity is u.

The gradient of this line is $\frac{(v-u)}{t}$, which we know from the *suvat* equations is acceleration.

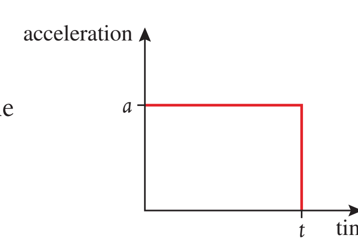
The area under the line makes the shape of a trapezium. The area of this trapezium is $\frac{1}{2}(v+u)t$. This is the *suvat* equation for s.



A.1 Figure 24 Constant acceleration.

Acceleration–time

The acceleration is constant so the acceleration–time graph is a horizontal line as shown in Figure 25. The area under this line is $a \times t$, which we know from the *suvat* equations equals $(v-u)$.

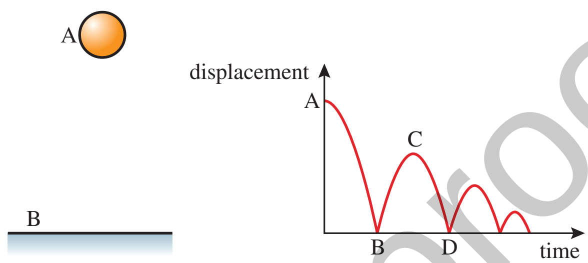


A.1 Figure 25 Constant acceleration.

Global query: some of the captions have full-stops, and others don't. Plse could you check these for us. Thks. - Design

Example 2: The bouncing ball

Consider a rubber ball dropped from position A above the ground onto hard surface B. The ball bounces up and down several times. Figure 26 shows the displacement–time graph for four bounces. From the graph, we see that the ball starts above the ground then falls with increasing velocity (as shown by the increasing negative gradient). When the ball bounces at B, the velocity suddenly changes from negative to positive as the ball begins to travel back up. As the ball goes up, its velocity decreases until it stops at C and begins to fall again.



A.1 Figure 26 Vertical displacement vs time.

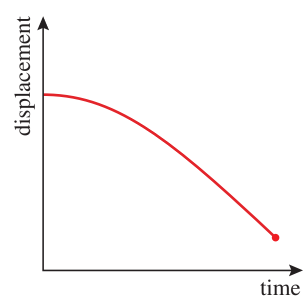
Why is the height reached by a bouncing ball less than the height of release? (A.3)

Exercise

Q15. By considering the gradient of the displacement–time graph in Figure 26, plot the velocity–time graph for the motion of the bouncing ball.

Example 3: A ball falling with air resistance

Figure 27 shows the motion of a ball that is dropped several hundred meters through the air. It starts from rest and accelerates for some time. As the ball accelerates, the air resistance increases, which stops the ball from getting any faster. At this point, the ball continues with constant velocity.



A.1 Figure 27 Vertical displacement vs time

Exercise

Q16. By considering the gradient of the displacement–time graph, plot the velocity–time graph for the motion of the falling ball in Figure 27.

Projectile motion

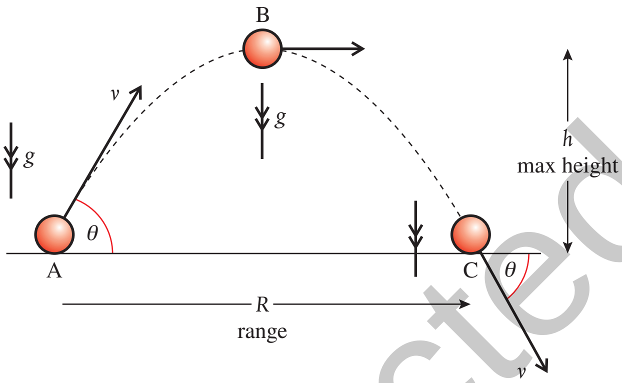
We all know what happens when a ball is thrown. It follows a curved path like the one in the photo. We can see from this photo that the path is parabolic and later we will show why that is the case.

A stroboscopic photograph of a projected ball.

Modeling projectile motion

All examples of motion up to this point have been in one dimension but projectile motion is two-dimensional. However, if we take components of all the vectors vertically and horizontally, we can simplify this into two simultaneous one-dimensional problems. The important thing to realize is that the vertical and horizontal components are independent of each other. You can test this by dropping an eraser off your desk and flicking one forward at the same time – they both hit the floor together. The downward motion is not changed by the fact that one stone is also moving forward.

When can problems on projectile motion be solved by applying conservation of energy instead of kinematic equations? (A.3)



A.1 Figure 28 A projectile launched at an angle θ.

Consider a ball that is projected at an angle θ to the horizontal, as shown in Figure 28. We can split the motion into three parts, beginning, middle and end, and analyze the vectors representing displacement, velocity and time at each stage. Notice that the path is symmetrical, so the motion on the way down is the same as on the way up.

Horizontal components

At A (time = 0)	At B (time = $\frac{t}{2}$)	At C (time = t)
displacement = zero velocity = $v \cos \theta$ acceleration = 0	displacement = $\frac{R}{2}$ velocity = $v \cos \theta$ acceleration = 0	displacement = R velocity = $v \cos \theta$ acceleration = 0

Vertical components

At A	At B	At C
displacement = zero velocity = $v \sin \theta$ acceleration = $-g$	displacement = h velocity = zero acceleration = $-g$	displacement = zero velocity = $-v \sin \theta$ acceleration = $-g$

Note that, at C, we are using the magnitude of θ (which is unchanged from position A). Therefore the negative sign is in place to provide the correct velocity direction; the projectile is moving downward.

We can see that the vertical motion is constant acceleration and the horizontal motion is constant velocity. We can therefore use the *suvat* equations.

Since the horizontal displacement is proportional to t, the path has the same shape as a graph of vertical displacement plotted against time. This is parabolic since the vertical displacement is proportional to t^2.

suvat for horizontal motion

Since acceleration is zero, there is only one equation needed to define the motion.

suvat	A to C
$v = \frac{s}{t}$	$R = v\cos\theta t$

suvat for vertical motion

When considering the vertical motion, it is worth splitting the motion into two parts.

A.1 Table 4

suvat	At B	At C
$s = \frac{1}{2}(u+v)t$	$h = \frac{1}{2}(v\sin\theta)\frac{t}{2}$	$0 = \frac{1}{2}(v\sin\theta - v\sin\theta)t$
$v^2 = u^2 + 2as$	$0 = v^2\sin^2\theta - 2gh$	$(-v\sin\theta)^2 = (v\sin\theta)^2 - 0$
$s = ut + \frac{1}{2}at^2$	$h = v\sin\theta t - \frac{1}{2}g\left(\frac{t}{2}\right)^2$	$0 = v\sin\theta t - \frac{1}{2}gt^2$
$a = \frac{v-u}{t}$	$g = \frac{v\sin\theta - 0}{\frac{t}{2}}$	$g = \frac{v\sin\theta - (-v\sin\theta)}{t}$

For a given value of v, the maximum range is when $v\cos\theta t$ is a maximum value.

$$t = \frac{2v\sin\theta}{g}$$

If we substitute this for t we get:

$$R = \frac{2v^2\cos\theta\sin\theta}{g}$$

Now, $2\sin\theta\cos\theta = \sin^2\theta$ (a trigonometric identity)

So, $R = \frac{v^2\sin^2\theta}{g}$

This is maximum when $\sin^2\theta$ is a maximum ($\sin^2\theta = 1$), which is when $\theta = 45°$.

Some of these equations are not very useful since they simply state that 0 = 0. However, we do end up with three useful ones (highlighted):

$$R = v\cos\theta t \qquad (7)$$

$$0 = v^2\sin^2\theta - 2gh \quad \text{or} \quad h = \frac{v^2\sin^2\theta}{2g} \qquad (8)$$

$$0 = v\sin\theta t - \frac{1}{2}gt^2 \quad \text{or} \quad t = \frac{2v\sin\theta}{g} \qquad (9)$$

Solving problems

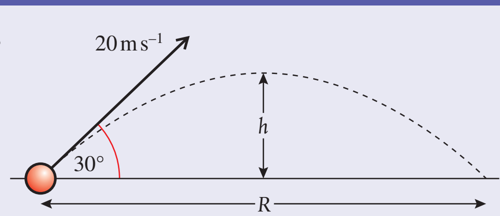
In a typical problem, you will be given the magnitude and direction of the initial velocity and asked to find either the maximum height or range. To calculate h, you can use equation (8), but to calculate R, you need to find the time of flight so must use (9) first. (You could also substitute for t into equation (6) to give another equation but we have enough equations already.)

How does the motion of a mass in a gravitational field compare to the motion of a charged particle in an electric field? (D.2)

You do not have to remember a lot of equations to solve a projectile problem. If you understand how to apply the *suvat* equations to the two components of the projectile motion, you only have to remember the *suvat* equations (and they are in the data booklet).

Worked example

A ball is thrown at an angle of 30° to the horizontal at a speed of 20 m s^{-1}. Calculate its range and the maximum height reached.



Solution

First, draw a diagram, including labels defining all the quantities known and unknown.

Now we need to find the time of flight. If we apply $s = ut + \frac{1}{2}at^2$ to the whole flight we get:

$$t = \frac{2v \sin \theta}{g} = \frac{(2 \times 20 \times \sin 30°)}{10} = 2\,\text{s}$$

We can now apply $s = vt$ to the whole flight to find the range:

$$R = v \cos \theta t = 20 \times \cos 30° \times 2 = 34.6\,\text{m}$$

Finally, to find the height, we apply $s = ut + \frac{1}{2}at^2$ to the vertical motion, but remember that this is only half the complete flight so the time is 1 *s*.

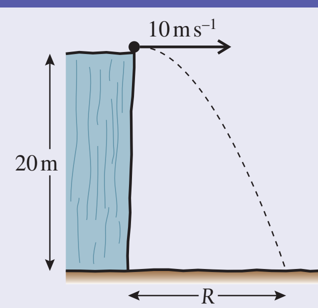
$$h = v \sin \theta t - \frac{1}{2}gt^2 = 20 \times \sin 30° \times 1 - \frac{1}{2} \times 10 \times 1^2 = 10 - 5 = 5\,\text{m}$$

When a bullet is fired at a distant target, it will travel in a curved path due to the action of gravity. Precision marksmen adjust their sights to compensate for this. The angle of this adjustment could be based on calculation or experiment (trial and error).

If you have ever played golf, you will know that it is not true that the maximum range is achieved with an angle of 45°. The angle is actually much less. This is because the ball is held up by the air like a plane is. In this photo, Alan Shepard is playing golf on the Moon. Here, the maximum range will be at 45°.

Worked example

A ball is thrown horizontally from a cliff top with a horizontal speed of $10\,\text{m}\,\text{s}^{-1}$.
If the cliff is 20 m high, what is the range of the ball?



Solution

This is an easy one since there are no angles to deal with. The initial vertical component of the velocity is zero and the horizontal component is $10\,\text{m}\,\text{s}^{-1}$.
To calculate the time of flight, we apply $s = ut + \frac{1}{2}at^2$ to the vertical component.
Knowing that the final displacement is −20 m, this gives:

$$-20\,\text{m} = 0 - \frac{1}{2}gt^2 \text{ so } t = \sqrt{\frac{(2 \times 20)}{10}} = 2\,\text{s}$$

We can now use this value to find the range by applying the equation $s = vt$ to the horizontal component: $R = 10 \times 2 = 20\,\text{m}$

Exercise

Q17. Calculate the range of a projectile thrown at an angle of 60° to the horizontal with a velocity of $30\,m\,s^{-1}$.

Q18. You throw a ball at a speed of $20\,m\,s^{-1}$.

(**a**) At what angle must you throw the ball so that it will just get over a wall that is 5 m high?

(**b**) How far away from the wall must you be standing?

Q19. A gun is aimed so that it points directly at the center of a target 200 m away. If the bullet travels at $200\,m\,s^{-1}$, how far below the center of the target will the bullet hit?

Q20. If you can throw a ball at $20\,m\,s^{-1}$, what is the maximum distance you can throw it?

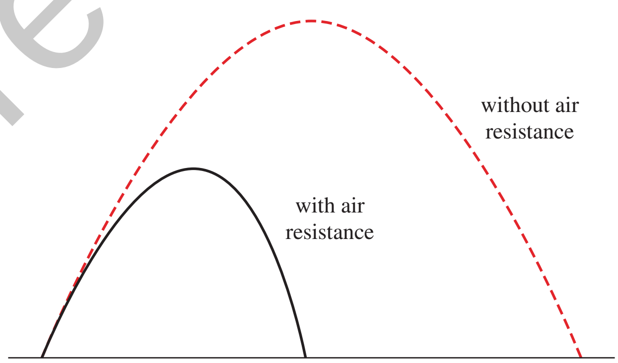
How does gravitational force allow for orbital motion? (A.2)

Challenge yourself

1. A projectile is launched perpendicular to a 30° slope at $20\,m\,s^{-1}$. Calculate the distance between the launching position and landing position.

Projectile motion with air resistance

In all the examples above, we have ignored the fact that the air will resist the motion of the ball. Air resistance opposes motion and increases with the speed of the moving object. The actual path of a ball including air resistance is likely to be as shown in Figure 29.



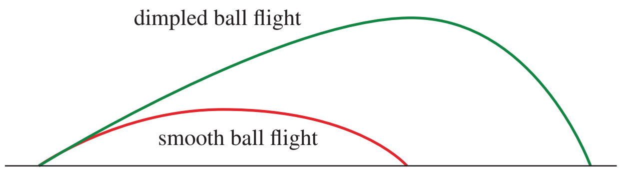
A.1 Figure 29 When air resistance is present, the projectile's motion is asymmetric

Notice that both the maximum height and the range are less. The path is also no longer a parabola – the way down is steeper than the way up.

The equation for this motion is complex. Horizontally, there is negative acceleration and so the horizontal component of velocity decreases. Vertically, there is increased magnitude of acceleration on the way up and a decreased magnitude of acceleration on the way down. None of these accelerations are constant so the *suvat* equations cannot be used. Luckily, all you need to know is the shape of the trajectory and the qualitative effects on range and time of flight.

Alternative air effects

The air does not always reduce the range of a projectile. A golf ball travels further than a ball projected in a vacuum. This is because the air holds the ball up, in the same way that it holds up a plane, due to the dimples in the ball and its spin.



A.1 Figure 30 The path of a smooth ball and a dimpled golf ball.

Guiding Questions revisited

How can the motion of a body be described quantitatively and qualitatively?

How can the position of a body in space and time be predicted?

How can the analysis of motion in one and two dimensions be used to solve real-life problems?

In this chapter, we have considered real-life examples to show that:

- Displacement is the straight-line distance between the start and end points of a body's motion and it has a direction.
- Velocity is the rate of change of displacement (and the vector equivalent of speed).
- Acceleration is the rate of change of velocity (and can therefore be treated as a vector).
- Motion graphs of displacement and velocity (or acceleration) against time enable qualitative changes in these quantities to be described and calculations of other quantities to be performed.
- The *suvat* equations of uniformly accelerated motion can be used to predict how position and velocity change with time (or one another) when a body experiences a constant acceleration.
- Vector quantities can be split into perpendicular components that can be treated independently, making it possible to solve problems in two dimensions using the *suvat* equations twice, for example, vertically and then horizontally for a projectile.
- Air resistance changes the acceleration in both perpendicular components, which means that the *suvat* equations cannot be used.

Practice questions

1. Police car P is stationary by the side of a road. Car S passes car P at a constant speed of 18 m s^{-1}. Car P sets off to catch car S just as car S passes car P. Car P accelerates at 4.5 m s^{-2} for 6.0 s and then continues at a constant speed. Car P takes *t* seconds to draw level with car S.

(**a**) State an expression, in terms of *t*, for the distance car S travels in *t* seconds. (1)

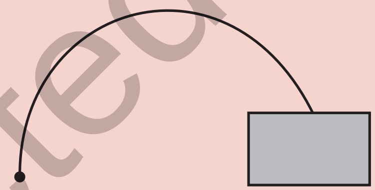
(**b**) Calculate the distance traveled by car P during the first 6.0 s of its motion. (1)

(**c**) Calculate the speed of car P after it has completed its acceleration. (1)

(**d**) State an expression, in terms of *t*, for the distance traveled by car P during the time that it is traveling at constant speed. (1)

(**e**) Using your answers to (**a**) to (**d**), determine the total time *t* taken by car P to draw level with car S. (2)

2. A ball is kicked with a speed of 14 m s^{-1} at 60° to the horizontal and lands on the roof of a 4 m high building.



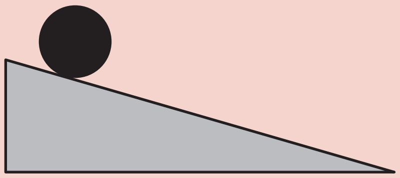
(**a**) (**i**) State the final vertical displacement of the ball. (1)

(**ii**) Calculate the time of flight. (3)

(**iii**) Calculate the horizontal displacement between the start point and the landing point on the roof. (2)

(**b**) The ball is kicked vertically upward. Explain the difference between the time to reach the highest point and the time from the highest point back to the ground. (3)

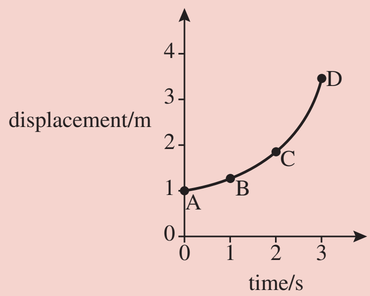
3. Two boys kick a football up and down a hill that is at an angle of 30° to the horizontal. One boy stands at the top of the hill and one boy stands at the bottom of the hill.



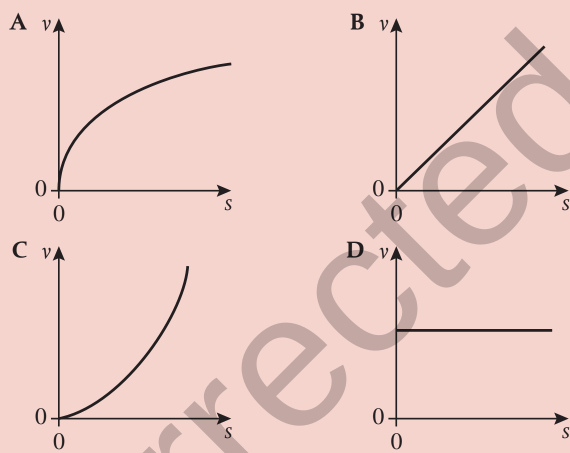
(**a**) Assuming that each boy kicks the ball perfectly to the other boy (without spin or bouncing), sketch a single path that the ball could take in either direction. (2)

(**b**) Compare the velocities with which each boy must strike the ball to achieve this path. (2)

4. The graph shows how the displacement of an object varies with time. At which point (A, B, C or D) does the instantaneous speed of the object equal its average speed over the interval from 0 to 3 s? (1)

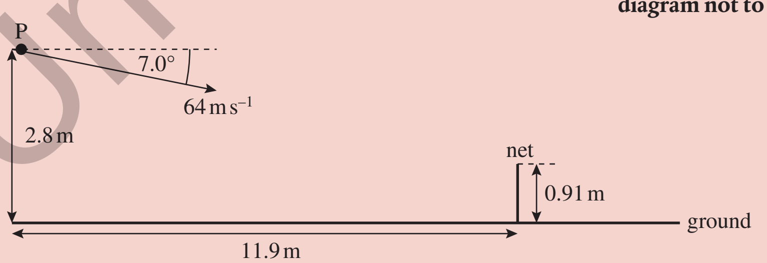


5. A runner starts from rest and accelerates at a constant rate. Which graph (A, B, C or D) shows the variation of the speed v of the runner with the distance traveled s? (1)



6. A student hits a tennis ball at point P, which is 2.8 m above the ground. The tennis ball travels at an initial speed of 64 m s^{-1} at an angle of 7.0° to the horizontal. The student is 11.9 m from the net and the net has a height of 0.91 m.

diagram not to scale



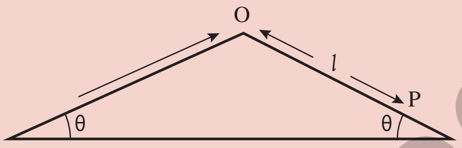
(**a**) Calculate the time it takes the tennis ball to reach the net. (2)

(**b**) Show that the tennis ball passes over the net. (3)

(**c**) Determine the speed of the tennis ball as it hits the ground. (2)

7. Estimate from what height, under free-fall conditions, a heavy stone would need to be dropped if it were to reach the surface of the Earth at the speed of sound ($330\,\mathrm{m\,s^{-1}}$). (2)

8. A motorbike is ridden up the left side of a symmetrical ramp. The bike reaches the top of the ramp at speed *u*, becomes airborne and falls to a point P on the other side of the ramp.



In terms of *u*, *l* and *g*, obtain expressions for:

(**a**) the time *t* for which the motorbike is in the air (2)

(**b**) the distance OP (= *l*) along the right side of the ramp. (3)