Topic	Pure mathematics
Element	Learning guide: Lesson 1
Timing:	1 hr

You have already used the quadratic formula in IG. The quadratic formula is of course:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Now let us try to solve the following equation:

$$x^2-2x+5=0$$

Solution

The first thing to notice is that this quadratic equation will not factorise. Therefore, you can solve the equation by using the formula or by completing the square.

- Using the quadratic formula

If you use the formula you get: $x=\frac{-(-2)\pm\sqrt{(-2)^2-4\times1\times5}}{2\times1}=\frac{2\pm\sqrt{-16}}{2}=\frac{2\pm\sqrt{4}\sqrt{-4}}{2}$

and when you divide through by 2 you obtain $x=1\pm\sqrt{-4}$ with a negative square root.

Trying to find the square root of a negative number is problematic. No real number (p) exists such that $p^2=-4$ and so the equation has no real solution.

- Completing the square

Always look carefully at the quadratic equation you need to solve. If it will not factorise, the coefficient of x^2 is 1 and the coefficient of x is an even number, completing the square is the far simpler method.

$x^2-2x+5=(x-1)^2-1+5=0\Rightarrow x=1\pm\sqrt{-4}$

However, just by testing using the discriminant we could arrive at this conclusion quickly and easily: $b^2-4ac=(-2)^2-4\times1\times6$ $=-16$, $-16<0$, so you can conclude that there are no real roots.

To start the lesson you should read 'The discriminant' (Passage 1) at the beginning of the lesson carefully as it explains the discriminant very clearly, with graphs to clarify the three distinct conditions.

Work through the two subsequent examples, and make sure that you also read the notes in the yellow and pink boxes.

Then complete the first exercise in Passage D. The answers to the questions in the exercise are included for you to check your own work.

'Simultaneous equations on graphs' (Passage E) deals with an application of the discriminant to test whether two lines or curves intersect. You will need to apply this theory all the way through your A level studies.

Once again, read the section, noting the graphical explanations. The third example (Passage H) has a typical question that really tests understanding of this application.

- There are three questions for you to complete on your own in the final exercise. Again, check the answers after you have completed them.

Topic	Pure mathematics
Element	Learning guide: Lesson 2
Timing:	1 hr

This lesson is about the equations of straight lines. Complete as many of the examples in the four exercises as you need until you feel completely confident.

Knowledge of straight lines and their equations will form a part of your A level studies from P1 through to P4, and if you choose to study Further Mathematics, you will also need to use this knowledge here as well.

Start by reading the theoretical work and have a go at the examples for yourself before you look at the solutions in the first section and the example below, and then attempt the questions in the first exercise. Be sure to read the explanations in the yellow boxes in the notes.

Straight line graphs are usually specified in one of two forms:

$y = mx + c$, which you will be familiar with from your studies in IG.

A new form, $ax + by + c = 0$, which is very useful as it avoids the use of awkward fractions; it's also useful if you want to draw the line quickly as you can easily find the intersections with the coordinate axes.

For example:

When $x = 0 \Rightarrow -6y + 12 = 0 \Rightarrow y = 2$

When $y = 0 \Rightarrow 3x + 12 = 0 \Rightarrow x = -4$

You will need to be very familiar with the formula for the equation of a straight line when you have been given the gradient m and a pair of coordinates:

$y - y_1 = m(x - x_1)$– **this formula should be committed to memory.**

You should also know the formula for finding the gradient of a line from two sets of coordinates:

$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{y_1 - y_2}{x_1 - x_2}$– **this formula should also be committed to memory.**

Now attempt the second exercise on finding gradients in Passage D and then read 'Equations of straight lines' on finding equations of straight lines in passage E. You should then attempt the third exercise. As before, start by reading the theoretical work and have a go at the examples for yourself before you look at the solutions in the second section. Again, be sure to read the explanations in the yellow boxes in the notes.

<u>'Parallel and perpendicular lines'</u>

Parallel lines all have the same gradient.

Perpendicular lines cross at an angle of 90°.

You can convert between parallel and perpendicular lines by using the relationship

$m_1 \times m_2 = -1$ or $m_1 = -\frac{1}{m_2}$.

The line perpendicular to the tangent to a curve is called the normal to the curve.

Now read through 'Parallel and perpendicular lines' (Passage H) in your notes and attempt the final exercise in Passage J. As before, start by reading the theoretical work and have a go at the examples for yourself before you look at the solutions in the third section and be sure to read the explanations in the yellow boxes in the notes.

Topic	Pure mathematics
Element	Learning guide: Lesson 3
Timing:	1 hr

You have already studied differentiation in your IG syllabus, and learned how to differentiate simple functions.

In A level mathematics you will learn how to differentiate much more complex functions, and you will also learn about applications of differentiation. In International A level Pure, this is limited to finding gradients and equations of tangents and normals to a curve.

In this module you will learn how to differentiate more complex functions and learn a new notation, $f(x)$, which means 'function of x', and $f'(x)$, which means $\frac{dy}{dx}$.

For example, given $f(x) = 3x^2 - \frac{1}{4\sqrt{x}} + 1$, find $f'(x)$.

To start with make sure that all terms in x are in the numerator so rewrite the expression as follows: $f(x) = 3x^2 - \frac{1}{4\sqrt{x}} + 1 = 3x^2 - \frac{x^{-\frac{1}{2}}}{4} + 1$

Be very careful here because an often seen error is $\frac{1}{4\sqrt{x}} \Rightarrow 4x^{-\frac{1}{2}}$.

$f(x) = 3x^2 - \frac{x^{-\frac{1}{2}}}{4} + 1 \Rightarrow f'(x) = 6x - \left(-\frac{1}{2}\right)\frac{x^{-\frac{3}{2}}}{4}$.

Do not attempt to simplify at the same time as you differentiate. Show **every** step of your working and simplify at the end.

This lesson starts with basic differentiation (including simplifying expressions using the rules of indices). Make absolutely sure that you can differentiate fluently – this will form a very large part of your A level studies.

'Differentiating quadratics' is really an extension of 'Differentiating x^n', but introduces the concept of a gradient, including a zero gradient that is also a **turning point**. Read Passage A carefully and then have a go at the four questions in the second exercise.

Make sure that you are really confident in this basic work before you move on to 'Gradients, tangents and normals' in Passage G, which is an application of differentiation in Pure Mathematics.

Gradients, tangents and normals' will call upon the work you learned in Lesson 3 because this section ties differentiation and straight lines together. As always, read the worked examples very carefully and be sure to also read the notes in the yellow boxes, even if you are clear on the method. Then complete the questions in the third exercise (Passage I), which has questions on equations of tangents and normals to curves.

Topic	Pure mathematics
Element	Learning guide: Lesson 4
Timing:	1 hr

This section is completely new to you because integration is not in the IG syllabus for Specification A or B.

Integration is the reverse (or inverse) of differentiation. When you differentiate you subtract 1 from the power of the variable (x); when you integrate you increase the power of the variable (x) by 1, **and divide** by the **new** power.

This is very straightforward when you integrate positive integer powers such as $\int x^3\ dx$, which becomes $\int x^3\ dx = \frac{x^{3+1}}{3+1} + c$, but is much trickier when you have negative powers or fractional powers.

Begin by reading through the basic work in 'Integrating x^n' in Passage A. Work through Examples 1, 2 and 3. Although the notation looks a little complicated, it is in fact quite simple.

Remember to **add** 1 to the power and then **divide** by the new power.

When you have completed these examples, go through the examples in the first exercise. As always, the answers are provided for you to check.

For example: Find $\int \left(3x - \frac{2}{\sqrt[3]{x}}\right) dx$.

<u>Solution</u>

As your first step, rewrite the integral with all terms in x in the numerator.

$$\int \left(3x - \frac{2}{\sqrt[3]{x}}\right) dx = \int \left(3x - 2x^{-\frac{1}{3}}\right) dx$$

Now integrate, **without** attempting to simplify in one operation. Always show every step.

$$\int \left(3x - 2x^{-\frac{1}{3}}\right) dx = \frac{3x^2}{2} - \frac{2x^{-\frac{1}{3}+1}}{-\frac{1}{3}+1} + c = \frac{3x^2}{2} - \frac{2x^{\frac{2}{3}}}{\frac{2}{3}} + c = \frac{3x^2}{2} + 3x^{\frac{2}{3}} + c.$$ **Don't forgetting + c.**

The '+ c' is important and is sometimes forgotten. So that you understand why it is important, consider the following function.

$f(x) = 2x^5 + 9x^2 - 2$, therefore $f'(x) = 10x^4 + 18x$. You can see that the -2 disappears in differentiation.

Because integration is the reverse of differentiation, let us now integrate $10x^4 + 18x$ to return to the original function.

$$\int (10x^4 + 18x)\ dx = \frac{10x^5}{5} + \frac{18x^2}{2} = 2x^5 + 9x^2 = 2x^5 + 9x^2.$$

You can see that the -2 is missing from the integrated expression. Therefore, it is most important that the final integrated expression is given as $\int (10x^4 + 18x)\ dx = 2x^5 + 9x^2 + c$.

Now read through the notes in 'Indefinite integrals' (Passage C). Don't be put off by the formal notation used, but you should be familiar with it as you will see it in the formula booklet used in A level examinations. Remember that g(x) is just another function of x. We could have used h(x) just as well.

Work your way through Example 4 in Passage E. Have a go at all of the examples yourself first and then check your solutions with the model answers in the text.

Then complete the second exercise and check your answers.