Topic	Pure mathematics
Element	Teaching guide: Lesson 1
Timing	1 hr

Students have already met the quadratic formula in IG and should be able to use it fluently. A good starting point has been given to students in the Passage A.

- Solve $x^2 - 2x + 5 = 0$

This example demonstrates the use of the discriminant in order to test whether an equation has two distinct real roots, one unique real root or non-real roots. 'The discriminant' in the lesson explains the basics and has supporting graphs, which students should become familiar with.

The two examples in Passage C demonstrate the method clearly, and students should work through them noting carefully the explanatory notes in the yellow and pink boxes.

As always in any work in mathematics, but especially algebraic work, it is crucial for students to show their working at all times. This not only helps teachers to understand the thought process, but also gets students used to showing working, so when the time comes to sit examinations, good practice is well embedded.

Passage D has seven questions, which are ramped in difficulty so that by the end of the exercise fluency will have been achieved in applying this mathematical method.

The second section covers one application of the use of the discriminant. Once again, students should read the introductory work carefully and absorb the graphical explanation.

Example 3 (Passage H) is a detailed question that is in many ways a typical IAL Pure 1 question. It may be a good idea for students to attempt the question themselves first, and then look at the model answer with the accompanying explanatory notes to check and elaborate on how the solution was achieved.

It is a typical question because two aspects of the specification are tested in this one question, and it requires careful and relevant algebraic manipulation as well as the basic knowledge of the behaviour of roots of a quadratic equation.

Skills that are tested are as follows:

- knowledge of roots of a quadratic equations
- finding solutions of quadratic equations – NOT the original equation, which can confuse students!

The second exercise (Passage I) has three questions that will really test for thorough understanding of this method. All of them are very typical Pure 1 questions.

Topic	Pure mathematics
Element	Teaching guide: Lesson 2
Timing:	1 hr

This lesson is in three sections: The basic straight line, finding an equation of a straight line in a specified form and consideration of perpendicular and parallel lines.

Students are first asked to read through the material in '$y = mx + c$' (Passage A) and complete the first two exercises.

It is vital that students are very conversant with this topic because it will come up time and time again all throughout pure mathematics A level, in coordinate geometry and also in questions involving differentiation.

In 'Equations of straight lines' (Passage B), students are asked to find the equations of straight lines.

Students tend to use either $y = mx + c$ or the formula $y - y_1 = m(x - x_1)$in equal measure.

It is probably worth mentioning to students using $y = mx + c$, that given the gradient and a pair of coordinates they have to find the value of c in order to gain the method mark in a question. This is of course a perfectly valid method and students should always be encouraged to think creatively in mathematics.

Students who use the formula will automatically gain the method mark by substituting the value of m and the coordinates (into the correct place) in the formula.

For example;

The line l passes through the point A with coordinates (2, 5) and is parallel to the line with equation $3y = x - 7$.

Find an equation of l in the form $ax + by + c = 0$ **(4 marks)**

Using $y = mx + c$

The gradient of l is $\frac{1}{3}$ **B1**

$x = 2 \;\; y = 5 \quad m = \frac{1}{3} \Rightarrow 5 = \frac{1}{3} \times 2 + c$

$c = 5 - \frac{2}{3} = \frac{13}{3}$ **M1** is earned at this point

So the equation of the line is $y = \frac{1}{3}x - \frac{13}{3}$ **A1**

Writing l in the required form $x - 3y - 13 = 0$ **A1**

Using $y - y_1 = m(x - x_1)$

The gradient of l is $\frac{1}{3}$ **B1**

$x = 2 \;\; y = 5 \quad m = \frac{1}{3} \Rightarrow y - 5 = \frac{1}{3}(x - 2)$ **M1A1** is earned at this point

Writing l in the required form $x - 3y - 13 = 0$ **A1**

There is less processing in the latter method, but that, of course, depends on the circumstances of the question.

Finally, students are asked to complete the third exercise (Passage G) and then read through the final section, 'Parallel and perpendicular lines' (Passage H). To complete this lesson, students should then attempt the final exercise in Passage J.

As always, students need to read through the examples, noting both the exemplar methods and the explanations in the yellow boxes before attempting the exercises.

Topic	Pure mathematics
Element	Teaching guide: Lesson 3
Timing:	1 hr

Students will be well versed in differentiating simple functions as this forms part of the IG Specifications A and B.

Questions on differentiation also present an opportunity for examiners to test simplification of indices and surds, so fluency in this fundamental work is essential (which students will find in the first exercise).

A common error seen in examinations is as follows:

Given $\mathrm{f}(x) = 3x^2 - \frac{1}{4\sqrt{x}} + 1$

(a) Show that $\mathrm{f}(x) = 3x^2 - ax^{-b} + 1$ where a and b are rational numbers.

(b) Find $\mathrm{f}'(x)$.

A typical error in part (a) would be: $\mathrm{f}(x) = 3x^2 - (4x)^{-\frac{1}{2}} + 1$ or $\mathrm{f}(x) = 3x^2 - 4x^{-\frac{1}{2}} + 1$.

This lesson not only refreshes the basic technique for differentiating functions but also introduces the function notation $\mathrm{f}(x)$ and $\mathrm{f}'(x)$, and students should be encouraged to use this notation where it is appropriate to do so.

They should complete 'Differentiating x^n ' (to revise what was learned in IG Spec A or B) and then complete the first exercise, which will also refresh knowledge of simplifying indices and surds.

'Differentiating quadratics' in Passage D introduces the concept of a gradient. The examples are all quadratics because the gradient function will be linear and so the concept of a gradient of a straight line should be well understood. (Lesson 2 dealt with that in detail). Students should read through the section and then complete the second exercise in passage F.

This lesson also introduces the application of differentiation to find the equations of tangents and normals to curves at any given point in 'Gradients, tangents and normals (Passage G)'. As always, encourage students to read through the explanations and exemplar questions in detail including reading all the notes in the yellow boxes. This section ties in with what was learned in the previous lesson on straight lines, and the exercise in this section will ask students to differentiate a function and then find the equation of either a tangent or normal (or both) at a given point. Students should read through 'Gradients, tangents and normals' and then complete the third exercise in Passage I.

Topic	Pure mathematics
Element	Teaching guide: Lesson 4
Timing:	1 hr

Integration is the only new topic introduced in this short course of study for preparation for A level.

Because of this, only the basic principle of integration is covered here, with the emphasis on plenty of fundamental practice. No attempt has been made in this lesson to introduce students to finding a function given the gradient function and a pair of coordinates, which is part of P1. Students who have gained fluency in the basic process of varying difficulties of functions will find the next step very straightforward.

The following errors are often seen in Pure 1 examinations.

- Forgetting to include $+c$ following indefinite integration. This will always lose candidates the final mark in a simple integration, but in a question where candidates are required to find a function given coordinates, they will lose at least half of the available marks.
- Attempting to simplify and integrate in one operation.

Students need to begin by working through the basic method in 'Integrating xn' (Passage A) and then complete the first exercise. The notation used in the text is formal, but students need to be familiar with it as it is the same notation used in the formula booklet issued in A level examinations.

It is imperative that students complete all examples, including $+c$ every time.

Here is an example of how marks are awarded in the Pure 1 examination.

Find $\int \left(3x - \frac{2}{\sqrt[3]{x}}\right) \mathrm{d}x$

Give each term in simplest form. **(4 marks)**

Solution

$\int \left(3x - 2x^{-\frac{1}{3}}\right) \mathrm{d}x = \frac{3x^2}{2} - \frac{2x^{-\frac{1}{3}+1}}{-\frac{1}{3}+1} + c$. This will score **M1A1** unsimplified.

If, however, a candidate makes an error simplifying while integrating, at least one and possibly two marks will be lost.

Please emphasise to your students the need to show **full** working.

Students then need to read through 'Indefinite integrals' in Passage C, which now uses the formal integral notation that students must also be familiar with. They should then complete the second exercise.

Once again emphasise the importance of writing $+c$ every time.