

PEARSON EDEXCEL INTERNATIONAL A LEVEL

MECHANICS 1

Student Book

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ABOUT THIS BOOK

The following three overarching themes have been fully integrated throughout the Pearson Edexcel International Advanced Level in Mathematics series, so they can be applied alongside your learning and practice.

1. Mathematical argument, language and proof

- Rigorous and consistent approach throughout
- Notation boxes explain key mathematical language and symbols
- Opportunities to critique arguments and justify methods

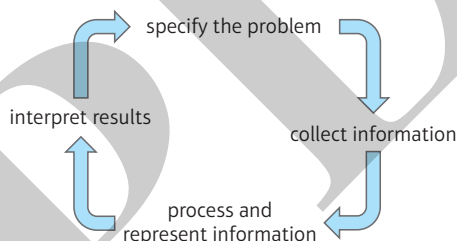
2. Mathematical problem-solving

- Hundreds of problem-solving questions, fully integrated into the main exercises
- Problem-solving boxes provide tips and strategies
- Structured and unstructured questions to build confidence
- Challenge questions provide extra stretch

3. Mathematical modelling

- Dedicated modelling sections in relevant topics provide plenty of practice where you need it
- Examples and exercises include qualitative questions that allow you to interpret answers in the context of the model

The Mathematical Problem-Solving Cycle



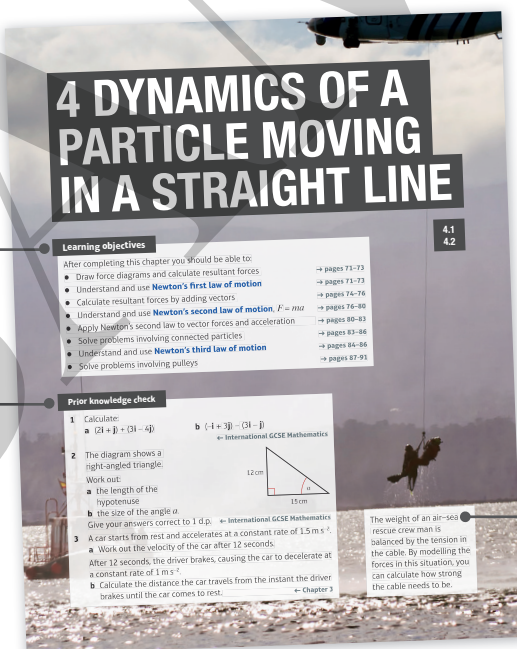
Finding your way around the book

Access an online digital edition using the code at the front of the book.



Each chapter starts with a list of *Learning objectives*

The *Prior knowledge check* helps make sure you are ready to start the chapter



The real world applications of the maths you are about to learn are highlighted at the start of the chapter

Problem-solving boxes provide hints, tips and strategies, and Watch out boxes highlight areas where students often lose marks in their exams

Exercises are packed with exam-style questions to ensure you are ready for the exams

Exercise questions are carefully graded so they increase in difficulty and gradually bring you up to exam standard

Exam-style questions are flagged with **E**

Problem-solving questions are flagged with **P**

Challenge boxes give you a chance to tackle some more difficult questions

Transferable skills are signposted where they naturally occur in the exercises and examples.

Each section begins with an explanation and key learning points

Step-by-step worked examples focus on the key types of questions you'll need to tackle

Each chapter ends with a Chapter review and a Summary of key points

After every few chapters, a Review exercise helps you consolidate your learning with lots of exam-style questions

REVIEW EXERCISE

Review exercise

1 The figure shows the velocity-time graph of a cyclist moving on a straight road over a 7 s period. The sections of the graph from $t = 0$ to $t = 3$ s and from $t = 3$ to $t = 7$ s are straight lines. The section from $t = 3$ to $t = 7$ s is parallel to the t -axis.

State what can be deduced about the motion of the cyclist from the fact that:

- the graph from $t = 0$ to $t = 3$ s is a straight line. (1)
- the graph from $t = 3$ to $t = 7$ s is parallel to the t -axis. (1)
- the distance travelled by the cyclist during this 7 s period. (4)

2 A train stops at two stations 7.5 km apart. Between the stations it takes 75 s to accelerate uniformly to a speed of 24 m s^{-1} , then travels at this speed for a time T seconds before decelerating uniformly for the final 0.6 km.

- Draw a velocity-time graph to illustrate this journey. (3)
- Hence, or otherwise, find:
 - the deceleration of the train during the final 0.6 km (3)
 - the value of T . (9)
 - the total time for the journey. (4)

3 An electric train starts from rest at a station A and moves along a straight level track. The train accelerates uniformly at 0.4 m s^{-2} for a speed of 16 m s^{-1} . This speed is then maintained for a distance of 2000 m . Finally, the train retards uniformly at 0.2 m s^{-2} before coming to rest at a station B . For this journey from A to B :

- find the total time taken. (5)
- find the distance from A to B . (5)
- sketch the displacement-time graph, showing clearly the shape of the graph for each stage of the journey. (3)

4 A small ball is projected vertically upward from a point A . The greatest height reached by the ball is 40 m above A . Calculate:

- the speed of projection. (3)
- the time between the instant that the ball is projected and the instant it returns to A . (3)

5 A ball is projected vertically upward and takes 3 seconds to reach its highest point. At time t seconds, the ball is 39.2 m above its point of projection. Find the possible values of t . (5)

6 A light object is acted upon by a horizontal force of $p \text{ N}$ and a vertical force of $q \text{ N}$ as shown in the diagram.

EXAM-STYLE PRACTICE

Exam-style practice

Mathematics

International Advanced Subsidiary/ Advanced Level Mechanics 1

Time: 1 hour 30 minutes
You must have: Mathematical Formulae and Statistical Tables, Calculator

1 Two particles P and Q , of mass 4 kg and 2 kg respectively, move in the same direction on a smooth horizontal surface with constant speeds of 0.2 m s^{-1} and 0.1 m s^{-1} respectively. At $t = 0$, P and Q are 0.8 m apart.

- Work out the time taken for P and Q to collide. (6)
- After the collision, the speed of P is halved, and both P and Q continue to move in the same direction. (4)

2 The velocity-time graph shows the motion of a train during a particular journey.

Describe what is happening between:

- t_1 and t_2 (1)
- t_2 and t_3 (1)
- t_3 and t_4 (1)

The train travels 120 km between t_1 and t_2 . Work out:

- the total length of the journey. (2)
- the total distance travelled by the train. (4)

A full practice paper at the back of the book helps you prepare for the real thing

QUALIFICATION AND ASSESSMENT OVERVIEW

Qualification and content overview

Mechanics 1 (M1) is an **optional** unit in the following qualifications:

International Advanced Subsidiary in Mathematics

International Advanced Subsidiary in Further Mathematics

International Advanced Level in Mathematics

International Advanced Level in Further Mathematics

Assessment overview

The following table gives an overview of the assessment for this unit.

We recommend that you study this information closely to help ensure that you are fully prepared for this course and know exactly what to expect in the assessment.

Unit	Percentage	Mark	Time	Availability
M1: Mechanics 1	$33\frac{1}{3}\%$ of IAS	75	1 hour 30 mins	January, June and October
Paper code WME01/01	$16\frac{2}{3}\%$ of IAL			First assessment June 2019

IAS – International Advanced Subsidiary, IAL – International Advanced A Level

Assessment objectives and weightings

		Minimum weighting in IAS and IAL
A01	Recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of contexts.	30%
A02	Construct rigorous mathematical arguments and proofs through use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions, including the construction of extended arguments for handling substantial problems presented in unstructured form.	30%
A03	Recall, select and use their knowledge of standard mathematical models to represent situations in the real world; recognise and understand given representations involving standard models; present and interpret results from such models in terms of the original situation, including discussion of the assumptions made and refinement of such models.	10%
A04	Comprehend translations of common realistic contexts into mathematics; use the results of calculations to make predictions, or comment on the context; and, where appropriate, read critically and comprehend longer mathematical arguments or examples of applications.	5%
A05	Use contemporary calculator technology and other permitted resources (such as formulae booklets or statistical tables) accurately and efficiently; understand when not to use such technology, and its limitations. Give answers to appropriate accuracy.	5%

Relationship of assessment objectives to units

M1	Assessment objective				
	A01	A02	A03	A04	A05
Marks out of 75	20–25	20–25	15–20	6–11	4–9
%	$26\frac{2}{3}$ – $33\frac{1}{3}$	$26\frac{2}{3}$ – $33\frac{1}{3}$	20– $26\frac{2}{3}$	8 – $14\frac{2}{3}$	$5\frac{1}{3}$ –12

Calculators

Students may use a calculator in assessments for these qualifications. Centres are responsible for making sure that calculators used by their students meet the requirements given in the table below.

Students are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π, x^2 , \sqrt{x} , $\frac{1}{x}$, x^y , ln x , e^x , $x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory.

Prohibitions

Calculators with any of the following facilities are prohibited in all examinations:

- databanks
- retrieval of text or formulae
- built-in symbolic algebra manipulations
- symbolic differentiation and/or integration
- language translators
- communication with other machines or the internet

Extra online content

Whenever you see an *Online* box, it means that there is extra online content available to support you.



SolutionBank

SolutionBank provides a full worked solution for every question in the book.

Download all the solutions as a PDF or quickly find the solution you need online.

Use of technology

Explore topics in more detail, visualise problems and consolidate your understanding. Use pre-made GeoGebra activities or Casio resources for a graphic calculator.

Online

Find the point of intersection graphically using technology.



GeoGebra

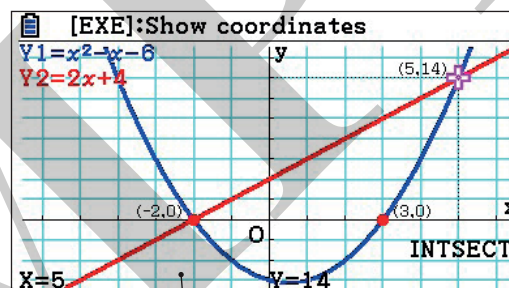
GeoGebra-powered interactives



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CASIO®

Graphic calculator interactives



Explore the maths you are learning and gain confidence in using a graphic calculator

Calculator tutorials

Our helpful video tutorials will guide you through how to use your calculator in the exams. They cover both Casio's scientific and colour graphic calculators.

Finding the value of the first derivative

to access the function press:

MENU 1 SHIFT

MENU 1 SHIFT

Pearson

Online

Work out each coefficient quickly using the nC_r and power functions on your calculator.



Step-by-step guide with audio instructions on exactly which buttons to press and what should appear on your calculator's screen

1 MATHEMATICAL MODELS IN MECHANICS

1.1

Learning objectives

After completing this chapter you should be able to:

- Understand how the concept of a mathematical model applies to mechanics → pages 2–3
- Understand and be able to apply some of the common assumptions used in mechanical models → pages 4–5
- Know SI units for quantities and derived quantities used in mechanics → pages 6–8

Prior knowledge check

Give your answers correct to 3 significant figures (s.f.) where appropriate.

1 Solve these equations:

a $5x^2 - 21x + 4 = 0$

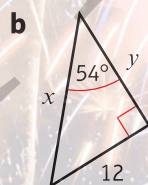
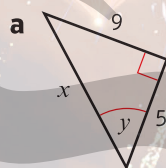
b $6x^2 + 5x = 21$

c $3x^2 - 5x - 4 = 0$

d $8x^2 - 18 = 0$

← International GCSE Mathematics

2 Find the value of x and y in these right-angled triangles.



← International GCSE Mathematics

3 Convert:

a 30 km h^{-1} to cm s^{-1}

b 5 g cm^{-3} to kg m^{-3}

← International GCSE Mathematics

4 Write in standard form:

a 7 650 000

b 0.003 806

← International GCSE Mathematics

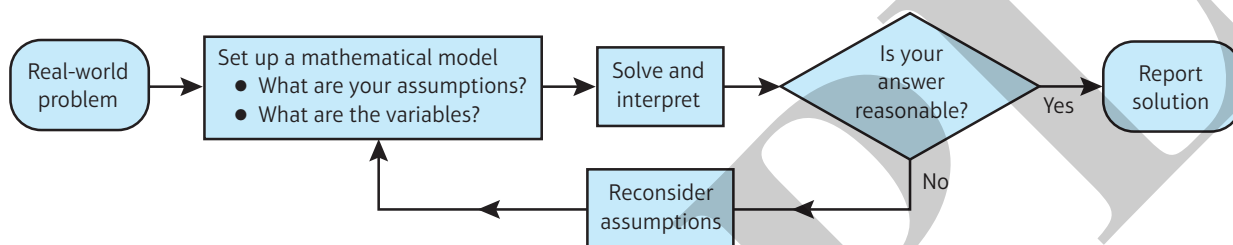
Mathematical models can be used to find solutions to **real-world problems** in many everyday situations. If you model a firework as a particle you can ignore the effects of wind and air resistance.

1.1 Constructing a model

Mechanics deals with **motion** and the action of **forces** on objects. Mathematical **models** can be constructed to simulate real-life situations (i.e. using models to create conditions that exist in real life, in order to study those conditions). However, in many cases it is necessary to simplify a problem by making one or more **assumptions**. This allows you to describe the problem using equations or graphs in order to solve it.

The solution to a mathematical model needs to be interpreted in the context of the original problem. It is possible that your model may need to be refined (improved with small changes) and your assumptions reconsidered.

This flow chart summarises the mathematical modelling process:



Example 1 SKILLS PROBLEM-SOLVING

The motion of a basketball as it leaves a player's hand and passes through the net can be modelled using the equation $h = 2 + 1.1x - 0.1x^2$, where h m is the height of the basketball above the ground and x m is the horizontal distance travelled.

- Find the height of the basketball:
 - when it is released
 - at a horizontal distance of 0.5 m.
- Use the model to predict the height of the basketball when it is at a horizontal distance of 15 m from the player.
- Comment on the **validity** of this prediction.

- a** i $x = 0$: $h = 2 + 0 - 0$
Height = 2 m
- ii $x = 0.5$: $h = 2 + 1.1 \times 0.5 - 0.1 \times (0.5)^2$
Height = 2.525 m
- b** $x = 15$: $h = 2 + 1.1 \times 15 - 0.1 \times (15)^2$
Height = -4 m
- c** Height cannot be **negative** so the model is not valid when $x = 15$ m.

When the basketball is released at the start of the motion $x = 0$. Substitute $x = 0$ into the equation for h .

Substitute $x = 0.5$ into the equation for h .

Substitute $x = 15$ into the equation for h .

h represents the height of the basketball above the ground, so it is only valid if $h \geq 0$.

Exercise 1A**SKILLS** **PROBLEM-SOLVING**

- 1** The motion of a golf ball after it is struck by a golfer can be modelled using the equation $h = 0.36x - 0.003x^2$, where h m is the height of the golf ball above the ground and x m is the horizontal distance travelled.
- Find the height of the golf ball when it is:
 - struck
 - at a horizontal distance of 100 m.
 - Use the model to predict the height of the golf ball when it is at a horizontal distance of 200 m from the golfer.
 - Comment on the validity of this prediction.
- 2** A stone is thrown into the sea from the top of a cliff. The height of the stone above sea level, h m at time t s after it is thrown can be modelled by the equation $h = -5t^2 + 15t + 90$.
- Write down the height of the cliff above sea level.
 - Find the height of the stone:
 - when $t = 3$
 - when $t = 5$.
 - Use the model to predict the height of the stone after 20 seconds.
 - Comment on the validity of this prediction.
- (P) 3** The motion of a basketball as it leaves a player's hand and passes through the net is modelled using the equation $h = 2 + 1.1x - 0.1x^2$, where h m is the height of the basketball above the ground and x m is the horizontal distance travelled.
- Find the two values of x for which the basketball is exactly 4 m above the ground.
This model is valid for $0 \leq x \leq k$, where k m is the horizontal distance of the net from the player.
 - Given that the height of the net is 3 m, find the value of k .
 - Explain why the model is not valid for $x > k$.
- (P) 4** A car accelerates from rest to 60 km h^{-1} in 10 seconds. A quadratic equation of the form $d = kt^2$ can be used to model the distance travelled, d metres in time t seconds.
- Given that when $t = 1$ second the distance travelled by the car is 13.2 metres, use the model to find the distance travelled when the car reaches 60 km h^{-1} .
 - Write down the range of values of t for which the model is valid.
- (P) 5** The model for the motion of a golf ball given in question 1 is valid only when h is **positive**. Find the range of values of x for which the model is valid.
- (P) 6** The model for the height of the stone above sea level given in question 2 is valid only from the time the stone is thrown until the time it enters the sea. Find the range of values of t for which the model is valid.

Problem-solving

Use the information given to work out the value of k .

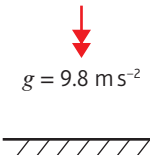
1.2 Modelling assumptions

Modelling assumptions can simplify a problem and allow you to analyse a real-life situation using known mathematical techniques. You need to understand the significance of different modelling assumptions and how they affect the calculations in a particular problem.

Watch out

Modelling assumptions can affect the validity of a model. For example, when modelling the landing of an aeroplane flight, it would not be appropriate to ignore the effects of wind and air resistance.

This table shows some common models and modelling assumptions that you need to know.

Model	Modelling assumptions
Particle – Dimensions of the object are negligible.	<ul style="list-style-type: none"> • mass of the object is concentrated at a single point • rotational forces (i.e. moving around a central fixed point) and air resistance can be ignored
Rod – All dimensions but one are negligible, like a pole or a beam.	<ul style="list-style-type: none"> • mass is concentrated along a line • no thickness • rigid (does not bend or buckle)
Lamina – Object with area but negligible thickness, like a sheet of paper.	<ul style="list-style-type: none"> • mass is distributed across a flat surface
Uniform body – Mass is distributed evenly.	<ul style="list-style-type: none"> • mass of the object is concentrated at a single point at the geometric centre of the body – the centre of mass
Light object – Mass of the object is small compared to other masses, like a string or a pulley.	<ul style="list-style-type: none"> • treat object as having zero mass • tension the same at both ends of a light string
Inextensible string – A string that does not stretch under load.	<ul style="list-style-type: none"> • acceleration is the same in objects connected by a taut inextensible string
Smooth surface – a surface on which it can be assumed there is no friction.	<ul style="list-style-type: none"> • assume that there is no friction between the surface and any object on it
Rough surface – a surface on which there is friction.	<ul style="list-style-type: none"> • objects in contact with the surface experience a frictional force if they are moving, or are acted on by a force
Wire – Rigid thin length of metal.	<ul style="list-style-type: none"> • treated as one-dimensional
Smooth and light pulley – All pulleys you consider will be smooth and light.	<ul style="list-style-type: none"> • pulley has no mass • tension is the same on either side of the pulley
Bead – Particle with a hole in it for threading on a wire or string (i.e. passing the wire or string through the hole).	<ul style="list-style-type: none"> • a smooth bead moves freely along a wire or string • for a smooth bead, tension is the same on either side of the bead
Peg – A support from which a body can be suspended or rested.	<ul style="list-style-type: none"> • dimensionless and fixed • can be rough or smooth as specified in the question
Air resistance – Resistance experienced as an object moves through the air.	<ul style="list-style-type: none"> • usually modelled as being negligible
Gravity – Force of attraction between all objects. Acceleration due to gravity is denoted by g . 	<ul style="list-style-type: none"> • assume all objects with mass are attracted toward the Earth • acceleration due to Earth's gravity is uniform (i.e. the same in all parts, at all times) and acts vertically downward • g is constant and is taken as 9.8 m s^{-2}, unless otherwise stated in the question

Example 2**SKILLS** ANALYSIS

A mass is attached to a length of string which is fixed to the ceiling.

The mass is drawn to one side with the string stretched tightly and allowed to swing.

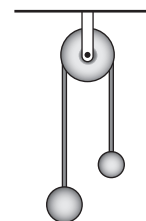
State the effect of the following assumptions on any calculations made using this model.

- a** The string is **light and inextensible** (unable to be stretched further).
- b** The mass is modelled as a particle.

- a** Ignore the mass of the string and any stretching effect caused by the mass.
- b** Ignore the rotational effect of any external forces that are acting on it, and the effects of air resistance.

Exercise 1B**SKILLS** ANALYSIS

- 1** A football is kicked by the goalkeeper from one end of the football pitch.
State the effect of the following assumptions on any calculations made using this model.
 - a** The football is modelled as a particle.
 - b** Air resistance is negligible.
- 2** An ice hockey puck is hit and slides across the ice.
State the effect of the following assumptions on any calculations made using this model.
 - a** The ice hockey puck is modelled as a particle.
 - b** The ice is smooth.
- 3** A parachutist wants to model her descent from an aeroplane to the ground. She models herself and her parachute as particles connected by a light inextensible string. Explain why this may not be a suitable modelling assumption for this situation.
- 4** A fishing rod manufacturer constructs a mathematical model to predict the behaviour of a particular fishing rod. The fishing rod is modelled as a light rod.
 - a** Describe the effects of this modelling assumption.
 - b** Comment on its validity in this situation.
- 5** Make a list of the assumptions you might make to create simple models of the following:
 - a** the motion of a golf ball after it is hit
 - b** the motion of a child on a sledge going down a snow-covered hill
 - c** the motion of two objects of different masses connected by a string that passes over a pulley
 - d** the motion of a suitcase on wheels being pulled along a path by its handle.



1.3 Quantities and units

The International System of Units, (abbreviated SI from the French, *Système international d'unités*) is the modern form of the metric system. These **base SI units** are most commonly used in mechanics:

Quantity	Unit	Symbol
Mass	kilogram	kg
Length/ displacement	metre	m
Time	second	s

Watch out

A common misunderstanding is that kilograms measure **weight**, not mass. However, **weight** is a **force** which is measured in **newtons** (N).

These **derived** units are compound units built from the base units:

Quantity	Unit	Symbol
Speed/velocity	metres per second	m s^{-1}
Acceleration	metres per second per second	m s^{-2}
Weight/force	newton	$\text{N} (= \text{kg m s}^{-2})$

Example 3

SKILLS REASONING/ARGUMENTATION

Write the following quantities in SI units.

a 4 km b 0.32 grams c $5.1 \times 10^6 \text{ km h}^{-1}$

a $4 \text{ km} = 4 \times 1000 = 4000 \text{ m}$

b $0.32 \text{ g} = 0.32 \div 1000 = 3.2 \times 10^{-4} \text{ kg}$

c $5.1 \times 10^6 \text{ km h}^{-1} = 5.1 \times 10^6 \times 1000$
 $= 5.1 \times 10^9 \text{ m h}^{-1}$

$5.1 \times 10^9 \div (60 \times 60) = 1.42 \times 10^6 \text{ m s}^{-1}$

The SI unit of length is the metre; $1 \text{ km} = 1000 \text{ m}$.

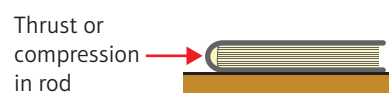
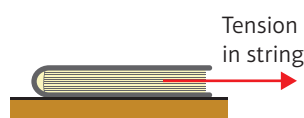
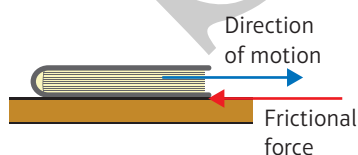
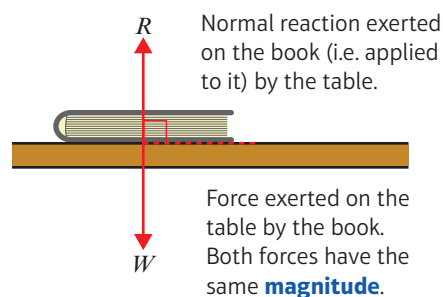
The SI unit of mass is the kg; $1 \text{ kg} = 1000 \text{ g}$.
The answer is given in standard form.

The SI unit of speed is m s^{-1} . Convert from km h^{-1} to m h^{-1} by multiplying by 1000.

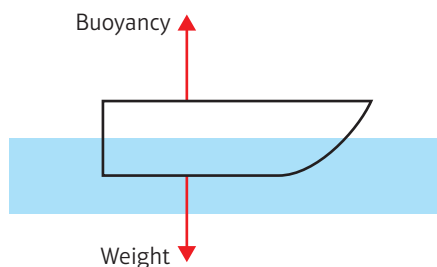
Convert from m h^{-1} to m s^{-1} by dividing by 60×60 .
The answer is given in standard form to 3 s.f.

You will encounter a variety of forces in mechanics. These **force diagrams** show some of the most common forces.

- The **weight** (or gravitational force) of an object acts vertically downward.
- The **normal reaction** is the force which acts perpendicular (i.e. at a 90° angle to it) to a surface when an object is in contact with the surface. In this example the normal reaction is due to the weight of the book resting on the surface of the table.
- Friction** is a force which opposes the motion between two rough surfaces.
- If an object is being pulled along by a string, the force acting on the object is called the **tension** in the string.
- If an object is being pushed along using a light rod, the force acting on the object is called the **thrust** or **compression** in the rod.



- **Buoyancy** is the upward force on a body that allows it to float or rise when submerged (i.e. underneath the surface) in a liquid. In this example buoyancy acts to keep the boat afloat in the water.

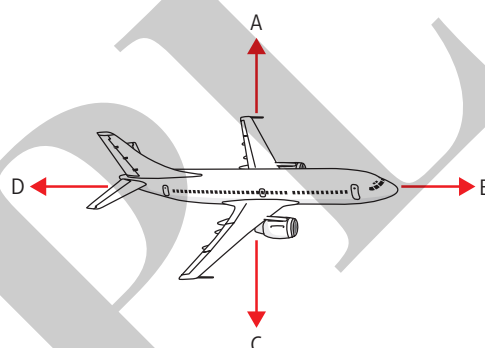


- **Air resistance** opposes motion. In this example the weight of the parachutist acts vertically downward and the air resistance acts vertically upward.



Example 4

The force diagram shows an aircraft in flight. Write down the names of the four forces shown on the diagram.



Also known as 'lift', this is the upward force that keeps the aircraft up in the air.

A upward thrust

B forward thrust

C weight

D air resistance

Also known as 'thrust', this is the force that propels the aircraft forward.

This is the gravitational force acting downward on the aircraft.

Also known as 'drag', this is the force that acts in the **opposite** direction to the forward thrust.

Exercise 1C

1 Convert to SI units:

a 65 km h^{-1}

b 15 g cm^{-2}

c 30 cm per minute

d 24 g m^{-3}

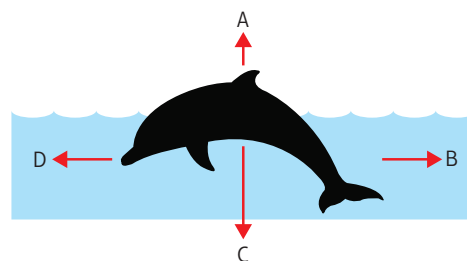
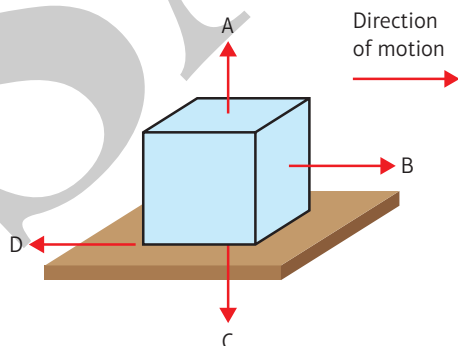
e $4.5 \times 10^{-2} \text{ g cm}^{-3}$

f $6.3 \times 10^{-3} \text{ kg cm}^{-2}$

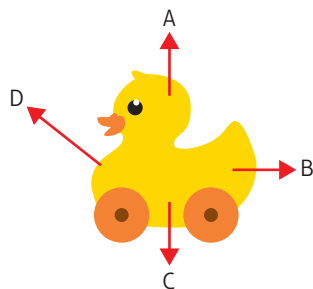
2 Write down the names of the forces shown in each of these diagrams.

a A box being pushed along rough ground

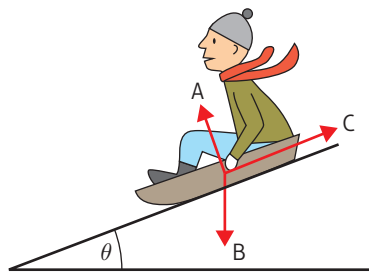
b A dolphin swimming through the water



c A toy duck being pulled along by a string



d A man sliding down a hill on a sledge



Chapter review 1

- P 1** The motion of a cricket ball after it is hit until it lands on the cricket pitch can be modelled using the equation $h = \frac{1}{10}(24x - 3x^2)$, where h m is the vertical height of the ball above the cricket pitch and x m is the horizontal distance from where it was hit. Find:

- a the vertical height of the ball when it is at a horizontal distance of 2 m from where it was hit
- b the two horizontal distances for which the height of the ball was 2.1 m.

Given that the model is valid from when the ball is hit to when it lands on the cricket pitch:

- c find the values of x for which the model is valid
- d work out the **maximum** height of the cricket ball.

Hint The path of the cricket ball is modelled as a quadratic curve. Draw a sketch for the model and use the symmetry of the curve.

- P 2** A diver dives from a diving board into a swimming pool with a depth of 4.5 m. The height of the diver above the water, h m, can be modelled using $h = 10 - 0.58x^2$ for $0 \leq x \leq 5$, where x m is the horizontal distance from the end of the diving board.

- a Find the height of the diver when $x = 2$ m.
- b Find the horizontal distance from the end of the diving board to the point where the diver enters the water.

In this model the diver is modelled as a particle.

- c Describe the effects of this modelling assumption.
- d Comment on the validity of this modelling assumption for the motion of the diver after she enters the water.

- 3** Make a list of the assumptions you might make to create simple models of the following:

- a the motion of a man skiing down a snow-covered slope
- b the motion of a yo-yo on a string.

In each case, describe the effects of the modelling assumptions.

- 4 Convert to SI units:
- a 2.5 km per minute b 0.6 kg cm^{-2} c $1.2 \times 10^3 \text{ g cm}^{-3}$
- 5 A man throws a bowling ball in a bowling alley.
- a Make a list of the assumptions you might make to create a simple model of the motion of the bowling ball.
- b Taking the direction in which the ball travels as the positive direction, state with a reason whether each of the following are likely to be positive or negative:
- i the velocity ii the acceleration.

Summary of key points

- 1 Mathematical models can be constructed to simulate real-life situations.
- 2 Modelling assumptions can be used to simplify your calculations.
- 3 The base SI units most commonly used in mechanics are:

Quantity	Unit	Symbol
Mass	kilogram	kg
Length/displacement	metre	m
Time	second	s

- 4 The derived SI units most commonly used in mechanics are:

Quantity	Unit	Symbol
Speed/velocity	metres per second	m s^{-1}
Acceleration	metres per second per second	m s^{-2}
Weight/force	newton	N (= kg m s^{-2})