PEARSON EDEXCEL INTERNATIONAL A LEVEL

MECHANICS 3

Student Book

Series Editors: Joe Skrakowski and Harry Smith

Authors: Dave Berry, Keith Gallick, Susan Hooker, Michael Jennings, Jean Littlewood, Lee McKelvey, Bronwen Moran, Su Nicholson, Laurence Pateman, Keith Pledger, Joe Skrakowski, Harry Smith, Jack Williams

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ABOUT THIS BOOK

The following three themes have been fully integrated throughout the Pearson Edexcel International Advanced Level in Mathematics series, so they can be applied alongside your learning.

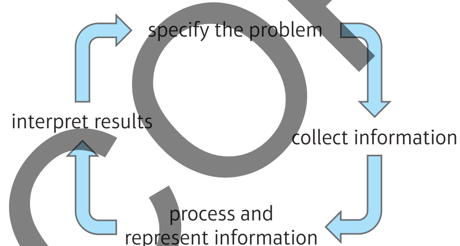
1. Mathematical argument, language and proof

- Rigorous and consistent approach throughout
- Notation boxes explain key mathematical language and symbols

2. Mathematical problem-solving

- Hundreds of problem-solving questions, fully integrated into the main exercises
- Problem-solving boxes provide tips and strategies
- Challenge questions provide extra stretch

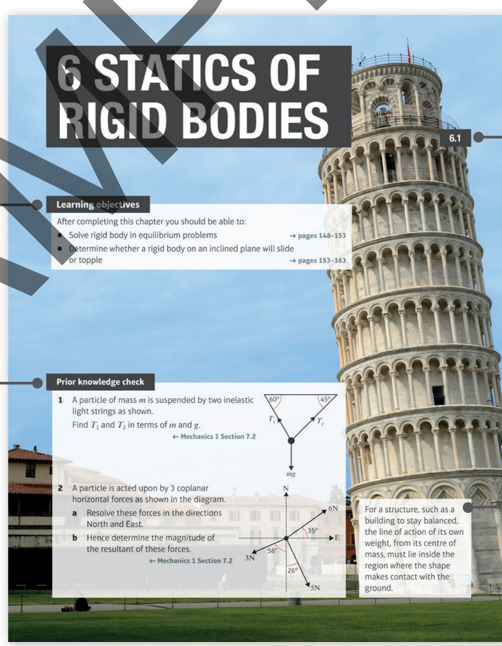
The Mathematical Problem-Solving Cycle



3. Transferable skills

- Transferable skills are embedded throughout this book: in the exercises and in some examples
- These skills are signposted to show students which skills they are using and developing

Finding your way around the book



Each chapter starts with a list of *Learning objectives*

The *Prior knowledge check* helps make sure you are ready to start the chapter

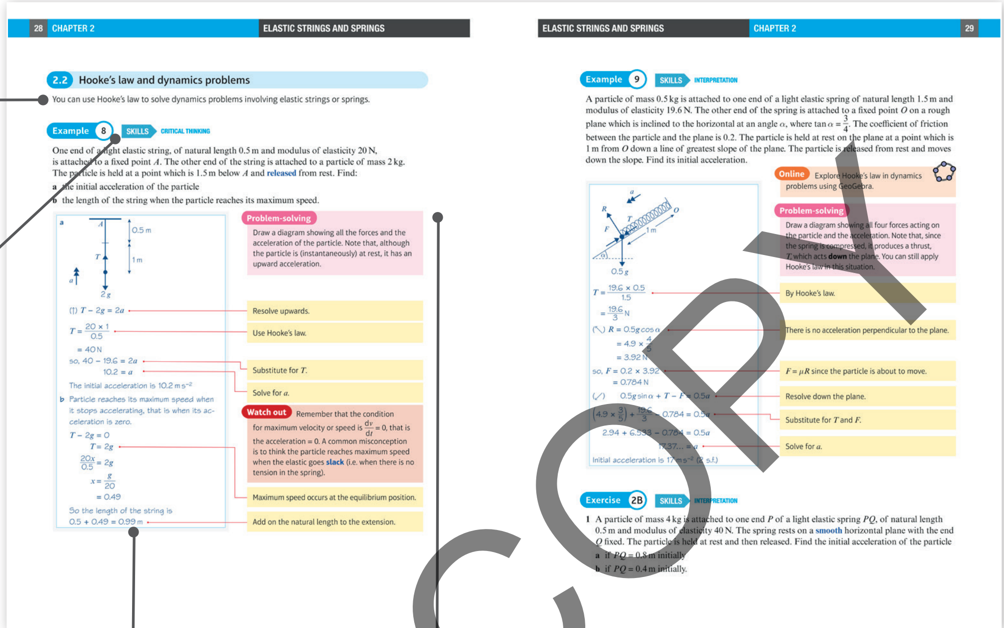
Glossary terms will be identified by bold blue text on their first appearance

Each chapter is mapped to the specification content for easy reference

The real world applications of the maths you are about to learn are highlighted at the start of the chapter

Each section begins with explanation and key learning points

Transferable skills are signposted where they naturally occur in the exercises and examples



Step-by-step worked examples focus on the key types of questions you'll need to tackle

Problem-solving boxes provide hints, tips and strategies, and *Watch out* boxes highlight areas where students often lose marks in their exams

Exercises are packed with exam-style questions to ensure you are ready for the exams

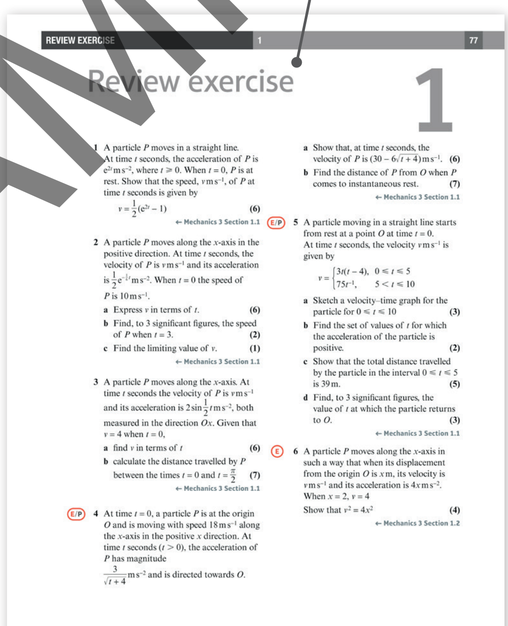
Exam-style questions are flagged with E

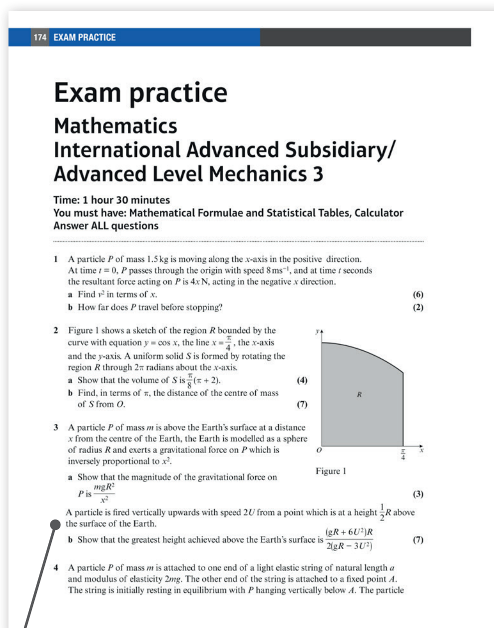
Problem-solving questions are flagged with P

Each chapter ends with a *Chapter review* and a *Summary of key points*

Exercise questions are carefully graded so they increase in difficulty and gradually bring you up to exam standard

After every few chapters, a *Review exercise* helps you consolidate your learning with lots of exam-style questions





A full practice paper at the back of the book helps you prepare for the real thing

QUALIFICATION AND ASSESSMENT OVERVIEW

Qualification and content overview

Mechanics 3 (M3) is an **optional** unit in the following qualifications:

International Advanced Subsidiary in Further Mathematics

International Advanced Level in Further Mathematics

Assessment overview

The following table gives an overview of the assessment for this unit.

We recommend that you study this information closely to help ensure that you are fully prepared for this course and know exactly what to expect in the assessment.

Unit	Percentage	Mark	Time	Availability
M3: Mechanics 3 Paper code WME03/01	$33\frac{1}{3}$ % of IAS $16\frac{2}{3}$ % of IAL	75	1 hour 30 mins	January and June First assessment June 2020

IAS: International Advanced Subsidiary, IAL: International Advanced A Level.

Assessment objectives and weightings

		Minimum weighting in IAS and IAL
AO1	Recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of contexts.	30%
AO2	Construct rigorous mathematical arguments and proofs through use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions, including the construction of extended arguments for handling substantial problems presented in unstructured form.	30%
AO3	Recall, select and use their knowledge of standard mathematical models to represent situations in the real world; recognise and understand given representations involving standard models; present and interpret results from such models in terms of the original situation, including discussion of the assumptions made and refinement of such models.	10%
AO4	Comprehend translations of common realistic contexts into mathematics; use the results of calculations to make predictions, or comment on the context; and, where appropriate, read critically and comprehend longer mathematical arguments or examples of applications.	5%
AO5	Use contemporary calculator technology and other permitted resources (such as formulae booklets or statistical tables) accurately and efficiently; understand when not to use such technology, and its limitations. Give answers to appropriate accuracy.	5%

Relationship of assessment objectives to units

M3	Assessment objective				
	AO1	AO2	AO3	AO4	AO5
Marks out of 75	20–25	25–30	10–15	5–10	5–10
%	$26\frac{2}{3}$–$33\frac{1}{3}$	$33\frac{1}{3}$–40	$13\frac{1}{3}$–20	$6\frac{2}{3}$–$13\frac{1}{3}$	$6\frac{2}{3}$–$13\frac{1}{3}$

Calculators

Students may use a calculator in assessments for these qualifications. Centres are responsible for making sure that calculators used by their students meet the requirements given in the table below.

Students are expected to have available a calculator with at least the following keys: +, –, ×, ÷, π, x^2, $\sqrt{x}$, $\frac{1}{x}$, x^y, $\ln x$, e^x, $x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory.

Prohibitions

Calculators with any of the following facilities are prohibited in all examinations:

- databanks
- retrieval of text or formulae
- built-in symbolic algebra manipulations
- symbolic differentiation and/or integration
- language translators
- communication with other machines or the internet

Extra online content

Whenever you see an *Online* box, it means that there is extra online content available to support you.

SolutionBank

SolutionBank provides worked solutions for questions in the book. Download the solutions as a PDF or quickly find the solution you need online.

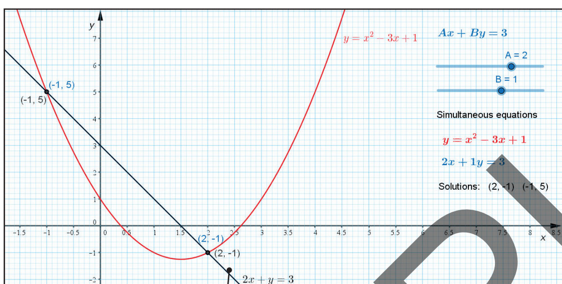
Use of technology

Explore topics in more detail, visualise problems and consolidate your understanding. Use pre-made GeoGebra activities or Casio resources for a graphic calculator.

Online Find the point of intersection graphically using technology.

GeoGebra

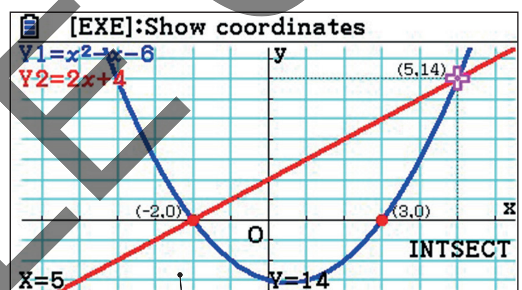
GeoGebra-powered interactives



Interact with the maths you are learning using GeoGebra's easy-to-use tools

CASIO

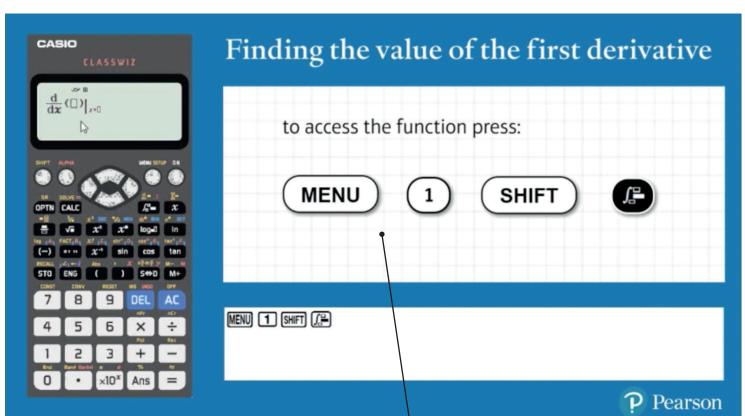
Graphic calculator interactives



Explore the maths you are learning and gain confidence in using a graphic calculator

Calculator tutorials

Our helpful video tutorials will guide you through how to use your calculator in the exams. They cover both Casio's scientific and colour graphic calculators.



Step-by-step guide with audio instructions on exactly which buttons to press and what should appear on your calculator's screen

Online Work out each coefficient quickly, using the ${}^nC_r$ and power functions on your calculator.

2 ELASTIC STRINGS AND SPRINGS

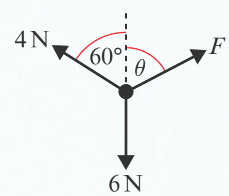
Learning objectives

After completing this chapter you should be able to:

- Use Hooke's law to solve equilibrium problems involving elastic strings or springs → pages 20–28
- Use Hooke's law to solve dynamics problems involving elastic strings or springs → pages 28–30
- Find the energy stored in an elastic string or spring → pages 31–33
- Solve problems involving elastic energy using the principle of conservation of mechanical energy and the work–energy principle → pages 33–40

Prior knowledge check

1 Three forces act on a particle. Given that the particle is in equilibrium, calculate the exact values of F and $\tan\theta$.



← Mechanics 1 Section 7.1

2 A particle of mass 4 kg is pulled along a rough horizontal table by a horizontal force of magnitude 12 N. Given that the mass moves with constant velocity, work out the coefficient of friction between the particle and the table.

← Mechanics 1 Section 5.3

3 A smooth plane is inclined at 30° to the horizontal. A particle of mass 0.4 kg slides down a line of greatest slope of the plane. The particle starts from rest at point P and passes point Q with a speed 5 m s^{-1}. Use the principle of conservation of mechanical energy to find the distance PQ.

← Mechanics 2 Section 4.2

Bungee jumping is an activity that involves jumping from a high point whilst attached to a long elastic cord. When the person jumps, their gravitational potential energy is converted (i.e. changed) into kinetic energy. As the bungee cord extends, this kinetic energy (K.E.) is converted into elastic potential energy.

2.1 Hooke's law and equilibrium problems

You can use Hooke's law to solve **equilibrium** problems involving **elastic** strings or springs.

The **tension** (T) produced when an elastic string or spring is **stretched** is **proportional** to the **extension** (x).

- $T \propto x$
- $T = kx$, where k is a constant

The constant k depends on the unstretched length of the string or spring, l, and the **modulus of elasticity** of the string or spring, λ.

- $T = \dfrac{\lambda x}{l}$

This relationship is called **Hooke's law**.

T is a force measured in newtons, and x and l are both lengths, so the units of λ are also newtons. The value of λ depends on the material from which the elastic string or spring is made, and is a measure of the 'stretchiness' (i.e. how far something can **stretch**) of the string or spring. In this chapter you may assume that Hooke's law applies for the values given in a question. In reality, Hooke's law only applies for values of x up to a maximum value, known as the elastic limit of the spring or string.

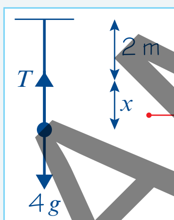
Watch out An elastic spring can also be **compressed**. Instead of a tension this will produce a **thrust** (or **compression**) **force**. Hooke's law still works for compressed elastic springs.

Notation In this chapter, all elastic strings and springs are modelled as being **light**. This means they have **negligible mass** and do not stretch under their own weight

Example 1

SKILLS PROBLEM-SOLVING

An elastic string of **natural length** 2 m and modulus of elasticity 29.4 N has one end fixed. A particle of mass 4 kg is attached to the other end and hangs at rest. Find the extension of the string.



Draw a diagram showing all the forces acting on the particle.

Note that the elastic string is in tension.

$(\uparrow)\ T - 4g = 0$

$T = 4g$

The particle is in equilibrium. **Resolve** vertically upwards to find T. ← Mechanics 1 Section 5.1

$T = \dfrac{29.4x}{2}$

Using Hooke's law, with $\lambda = 29.4$ and $l = 2$

so $4g = \dfrac{29.4x}{2}$

Equating the two expressions for T.

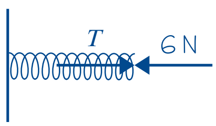
$x = \dfrac{8}{3}$ m

The string stretches by $\dfrac{8}{3}$ m.

Watch out This is the extension in the spring. The total length will be $4\frac{2}{3}$ m.

Example 2 SKILLS PROBLEM-SOLVING

An elastic spring of natural length 1.5 m has one end attached to a fixed point. A horizontal force of magnitude 6 N is applied to the other end and compresses the spring to a length of 1 m. Find the modulus of elasticity of the spring.



Draw a diagram showing the applied force 6 N and the thrust force T produced in the spring.

$(\rightarrow)\ T - 6 = 0$
$T = 6\text{ N}$

The forces are in equilibrium.

$$T = \frac{\lambda \times 0.5}{1.5} = \frac{\lambda}{3}$$

Use Hooke's law. The compression of the spring is $1.5 - 1 = 0.5$ m.

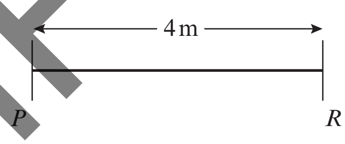
so $\frac{\lambda}{3} = 6$
$\lambda = 18\text{ N}$

Equating the two expressions for T.

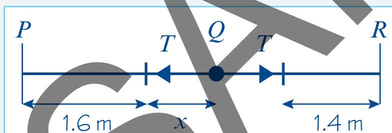
The modulus of elasticity is 18 N.

Example 3 SKILLS INTERPRETATION

The elastic springs PQ and QR are joined together at Q to form one long spring. The spring PQ has natural length 1.6 m and modulus of elasticity 20 N. The spring QR has natural length 1.4 m and modulus of elasticity 28 N. The ends, P and R, of the long spring are attached to two fixed points which are 4 m apart, as shown in the diagram.



Find the tension in the combined spring.



Problem-solving

Since Q is at rest the tension in each spring must be the same.

Let the extension in spring PQ be x.
The extension in $QR = 1 - x$

Since $PR = 4$ m, total extension of the springs is $4 - 1.6 - 1.4 = 1$ m

For PQ: $T = \frac{20x}{1.6}$

For QR: $T = \frac{28(1 - x)}{1.4}$

Use Hooke's law.

so $\frac{20x}{1.6} = \frac{28(1 - x)}{1.4}$

Equate the tensions.

$$\frac{20x}{1.6} = 20(1 - x)$$
$$12.5x = 20 - 20x$$
$$32.5x = 20$$
$$x = \frac{8}{13}$$
Solve for x.

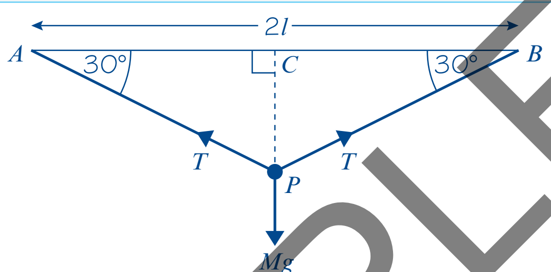
so $$T = \frac{20}{1.6} \times \frac{8}{13}$$
Substitute for x into tension equation for PQ.

$$= \frac{100}{13}\text{ N}$$

The tension in the combined spring is 7.69 N (3 s.f.)

Example 4 SKILLS ANALYSIS

An elastic string of natural length $2l$ and modulus of elasticity $4mg$ is stretched between two points A and B. The points A and B are on the same horizontal level and $AB = 2l$. A particle P is attached to the midpoint of the string and hangs in equilibrium with both parts of the string making an angle of 30° with the line AB. Find, in terms of m, the mass of the particle.



Online Explore Hooke's law in equilibrium problems involving two elastic springs using GeoGebra.

Problem-solving

Draw a large clear diagram showing the forces acting on the particle. It is useful to label the midpoint of A and B as well.

Let the mass of the particle be M.

$(\uparrow)\ 2T\cos 60° = Mg$
The particle is in equilibrium.

$T = Mg$

$$AP = \frac{l}{\cos 30°} = \frac{2l}{\sqrt{3}}$$
Use $\triangle APC$.

so the stretched length of the string is

$$\frac{4l}{\sqrt{3}}$$
Since $AP = PB$

$\therefore$ Extension of string is $\left(\frac{4l}{\sqrt{3}} - 2l\right)$

$$\therefore T = \frac{4mg}{2l}\left(\frac{4l}{\sqrt{3}} - 2l\right)$$
Use Hooke's law.

$$= 2mg\left(\frac{4}{\sqrt{3}} - 2\right)$$
Cancel the ls.

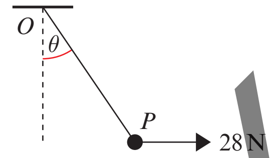
$= 0.62mg$ (2 s.f.)

Hence, $0.62mg = Mg$
Use $T = Mg$.

The mass of the particle is $0.62m$ (2 s.f.).

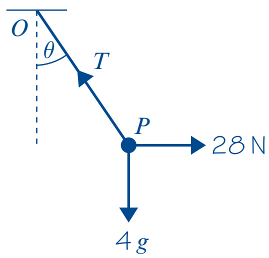
Example 5 SKILLS PROBLEM-SOLVING

An elastic string has natural length 2 m and modulus of elasticity 98 N. One end of the string is attached to a fixed point O and the other end is attached to a particle P of mass 4 kg. The particle is held in equilibrium by a horizontal force of magnitude 28 N, with OP making an angle θ with the **vertical**, as shown. Find



a the value of θ

b the length OP.



a $(\leftarrow)$ $T\sin\theta = 28$

$(\uparrow)$ $T\cos\theta = 4g$

$\tan\theta = \frac{28}{4g} = \frac{5}{7}$

so, $\theta = 35.5°$

$= 36°$ (2 s.f.)

b $T = \frac{28}{\sin\theta}$

so $\frac{28}{\sin\theta} = \frac{98x}{2}$

so $x = \frac{4}{7\sin\theta}$

$= 0.983...$

$OP = 2 + 0.983... = 2.983...$

$\therefore$ Length of OP is 2.98 m (3 s.f.)

Online Explore Hooke's law in equilibrium problems involving one elastic spring using GeoGebra.

Problem-solving

Since the particle is in equilibrium, you can resolve horizontally and vertically to find θ. You could also use these two equations to find an exact value for T, but it is easier to use your calculator and an unrounded value for θ to find x.

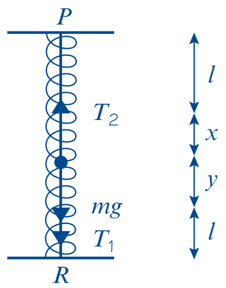
Divide the equations to eliminate T.

Give answer to 2 s.f. as value for g is correct to 2 s.f.

Use Hooke's law, with x as the extension of the string.

Example 6 SKILLS INTERPRETATION

Two identical elastic springs PQ and QR each have natural length l and modulus of elasticity $2mg$. The springs are joined together at Q. Their other ends, P and R, are attached to fixed points, with P being $4l$ vertically above R. A particle of mass m is attached at Q and hangs at rest in equilibrium. Find the distance of the particle below P.



Problem-solving

Draw a diagram showing the forces acting on the particle. Note that we have assumed that the lower spring is **stretched** and is therefore in **tension**. If the extension of the lower spring turns out to be negative, then it means the lower spring is in compression.

$l + x + y + l = 4l$

Since P is $4l$ above R.

$y = 2l - x$

$(\uparrow)\ T_2 - mg - T_1 = 0$

Since the mass is in equilibrium.

$$\Rightarrow \frac{2mgx}{l} = mg + \frac{2mg(2l - x)}{l}$$

Use Hooke's law.

$2x = l + 2\ (2l - x)$

Divide both sides by mg and multiply through by l.

$2x = l + 4l - 2x$

$4x = 5l$

$x = \frac{5l}{4}$

Solve for x. The extension x is positive and so the top spring is in tension. The value of y is also positive. This demonstrates that the bottom spring is also in tension.

The distance of the particle below P is $\frac{9l}{4}$

Add on the natural length, l, of the spring.

Example 7

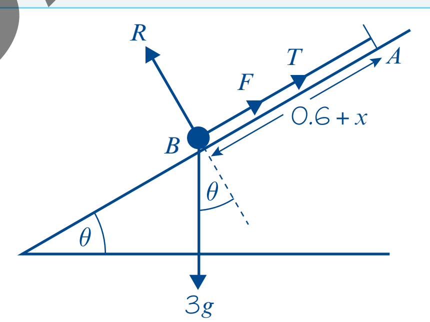
SKILLS CRITICAL THINKING

One end, A, of a light elastic string AB, of natural length 0.6 m and modulus of elasticity 10 N, is attached to a point on a fixed rough **plane**. The plane is **inclined** at an angle θ to the horizontal, where $\sin\theta = \frac{4}{5}$. A ball of mass 3 kg is attached to the end, B, of the string. The **coefficient of friction**, μ, between the ball and the plane is $\frac{1}{3}$. The ball rests in **limiting equilibrium**, on the point of sliding down the plane, with AB along the line of greatest **slope**.

a Find
 i the tension in the string
 ii the length of the string.

b If $\mu > \frac{1}{3}$, without doing any further calculation, state how your answer to part **a ii** would change.

a i



Problem-solving

Draw a clear diagram showing all the forces. The ball is on the point of sliding **down** the plane, so the frictional force acts up the plane.

Let extension of string be x m.

$\sin\theta = \frac{4}{5}$ so $\cos\theta = \frac{3}{5}$

($\nwarrow$) $R = 3g\cos\theta = \frac{9g}{5}$

Resolving forces **perpendicular** to the plane.

$F = \mu R = \frac{3g}{5}$

Ball is in limiting equilibrium so $F = \mu R$

($\nearrow$) $T + F = 3g\sin\theta$

Resolving forces up the plane and substituting for T and F.

$T = 3g\sin\theta - F$

$T = \left(3g \times \frac{4}{5}\right) - \frac{3g}{5} = \frac{9g}{5} = 17.6$ N (3 s.f.)

ii $T = \frac{\lambda x}{l}$

$17.6 = \frac{10x}{0.6}$

so $x = 1.06$ m (3 s.f.)

Length of string $= 0.6 + 1.06$

$= 1.66$ m (3.s.f.)

Watch out x is the extension in the string, so add the natural length to find the total length.

b If $\mu > \frac{1}{3}$ then

F would be greater as $F = \mu R$

T would be less as $T = 3g\sin\theta - F$

x would be less as $T = \frac{\lambda x}{l}$

so answer to part **a ii** would be less than 1.66 m

Problem-solving

The **coefficient of friction** is greater, which means that there is a greater force due to **friction** acting up the plane. The string has to produce less force to keep the ball in equilibrium, so less extension is required.

Exercise 2A

SKILLS PROBLEM-SOLVING

1 One end of a light elastic string is attached to a fixed point. A force of 4 N is applied to the other end of the string so as to stretch it. The natural length of the string is 3 m and the modulus of elasticity is λ N. Find the total length of the string when

a $\lambda = 30$ **b** $\lambda = 12$ **c** $\lambda = 16$

2 The length of an elastic spring is reduced to 0.8 m when a force of 20 N compresses it. Given that the modulus of elasticity of the spring is 25 N, find its natural length.

(P) **3** An elastic spring of modulus of elasticity 20 N has one end fixed. When a particle of mass 1 kg is attached to the other end and hangs at rest, the total length of the spring is 1.4 m. The particle of mass 1 kg is removed and replaced by a particle of mass 0.8 kg. Find the new length of the spring.

(P) **4** A light elastic spring, of natural length a and modulus of elasticity λ, has one end fixed. A scale pan of mass M is attached to its other end and hangs in equilibrium. A mass m is gently placed in the scale pan. Find the distance of the new equilibrium position below the old one.

P **5** An elastic string has length a_1 when supporting a mass m_1 and length a_2 when supporting a mass m_2. Find the natural length and modulus of elasticity of the string.

P **6** When a weight, W N, is attached to a light elastic string of natural length l m the extension of the string is 10 cm. When W is increased by 50 N, the extension of the string is increased by 15 cm. Find W.

E/P **7** An elastic spring has natural length $2a$ and modulus of elasticity $2mg$. A particle of mass m is attached to the midpoint of the spring. One end of the spring, A, is attached to the floor of a room of height $5a$ and the other end is attached to the ceiling of the room at a point B vertically above A. The spring is modelled as light.

a Find the distance of the particle below the ceiling when it is in equilibrium. **(8 marks)**

b In reality the spring may not be light. What effect will the model have had on the calculation of the distance of the particle below the ceiling? **(1 mark)**

E/P **8** A **uniform** rod PQ, of mass 5 kg and length 3 m, has one end, P, smoothly hinged to a fixed point. The other end, Q, is attached to one end of a light elastic string of modulus of elasticity 30 N. The other end of the string is attached to a fixed point R which is on the same horizontal level as P with $RP = 5$ m. The system is in equilibrium and $\angle PQR = 90°$. Find

a the tension in the string **(5 marks)**

b the natural length of the string. **(3 marks)**

Problem-solving

First take **moments** about P.

← Mechanics 1 Section 8.1

E/P **9** A light elastic string AB has natural length l and modulus of elasticity $2mg$. Another light elastic string CD has natural length l and modulus of elasticity $4mg$. The strings are joined at their ends B and C and the end A is attached to a fixed point. A particle of mass m is hung from the end D and is at rest in equilibrium. Find the length AD. **(7 marks)**

E/P **10** An elastic string PA has natural length 0.5 m and modulus of elasticity 9.8 N. The string PB is **inextensible**. The end A of the elastic string and the end B of the inextensible string are attached to two fixed points which are on the same horizontal level. The end P of each string is attached to a 2 kg particle. The particle hangs in equilibrium below AB, with PA making an angle of 30° with AB and PA perpendicular to PB. Find

a the length of PA **(7 marks)**

b the length of PB **(2 marks)**

c the tension in PB. **(2 marks)**

E/P **11** A particle of mass 2 kg is attached to one end P of a light elastic string PQ of modulus of elasticity 20 N and natural length 0.8 m. The end Q of the string is attached to a point on a rough plane which is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$. The coefficient of friction between the particle and the plane is $\frac{1}{2}$. The particle rests in limiting equilibrium, on the point of sliding down the plane, with PQ along a line of greatest slope. Find

a the tension in the string **(5 marks)**

b the length of the string. **(2 marks)**

2.2 Hooke's law and dynamics problems

You can use Hooke's law to solve dynamics problems involving elastic strings or springs.

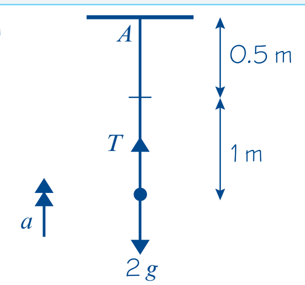
Example 8 SKILLS CRITICAL THINKING

One end of a light elastic string, of natural length 0.5 m and modulus of elasticity 20 N, is attached to a fixed point A. The other end of the string is attached to a particle of mass 2 kg. The particle is held at a point which is 1.5 m below A and **released** from rest. Find:

a the initial acceleration of the particle

b the length of the string when the particle reaches its maximum speed.

a



$(\uparrow)\ T - 2g = 2a$

$T = \dfrac{20 \times 1}{0.5}$

$= 40\,\text{N}$

so, $40 - 19.6 = 2a$

$10.2 = a$

The initial acceleration is $10.2\,\text{m}\,\text{s}^{-2}$

b Particle reaches its maximum speed when it stops accelerating, that is when its acceleration is zero.

$T - 2g = 0$

$T = 2g$

$\dfrac{20x}{0.5} = 2g$

$x = \dfrac{g}{20}$

$= 0.49$

So the length of the string is

$0.5 + 0.49 = 0.99\,\text{m}$

Problem-solving

Draw a diagram showing all the forces and the acceleration of the particle. Note that, although the particle is (instantaneously) at rest, it has an upward acceleration.

Resolve upwards.

Use Hooke's law.

Substitute for T.

Solve for a.

Watch out Remember that the condition for maximum velocity or speed is $\dfrac{dv}{dt} = 0$, that is the acceleration = 0. A common misconception is to think the particle reaches maximum speed when the elastic goes **slack** (i.e. when there is no tension in the spring).

Maximum speed occurs at the equilibrium position.

Add on the natural length to the extension.

Example 9

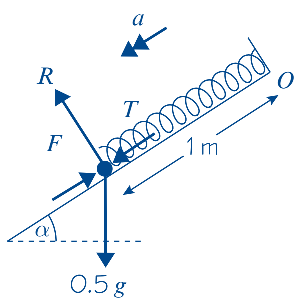
SKILLS INTERPRETATION

A particle of mass 0.5 kg is attached to one end of a light elastic spring of natural length 1.5 m and modulus of elasticity 19.6 N. The other end of the spring is attached to a fixed point O on a rough plane which is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$. The coefficient of friction between the particle and the plane is 0.2. The particle is held at rest on the plane at a point which is 1 m from O down a line of greatest slope of the plane. The particle is released from rest and moves down the slope. Find its initial acceleration.

Online Explore Hooke's law in dynamics problems using GeoGebra.

Problem-solving

Draw a diagram showing all four forces acting on the particle and the acceleration. Note that, since the spring is compressed, it produces a thrust, T, which acts **down** the plane. You can still apply Hooke's law in this situation.



$T = \frac{19.6 \times 0.5}{1.5}$ — By Hooke's law.

$= \frac{19.6}{3}$ N

($\nwarrow$) $R = 0.5g\cos\alpha$ — There is no acceleration perpendicular to the plane.

$= 4.9 \times \frac{4}{5}$

$= 3.92$ N

so, $F = 0.2 \times 3.92$ — $F = \mu R$ since the particle is about to move.

$= 0.784$ N

($\swarrow$) $0.5g\sin\alpha + T - F = 0.5a$ — Resolve down the plane.

$\left(4.9 \times \frac{3}{5}\right) + \frac{19.6}{3} - 0.784 = 0.5a$ — Substitute for T and F.

$2.94 + 6.533 - 0.784 = 0.5a$

$17.37\ldots = a$ — Solve for a.

Initial acceleration is 17 m s^{-2} (2 s.f.)

Exercise 2B

SKILLS INTERPRETATION

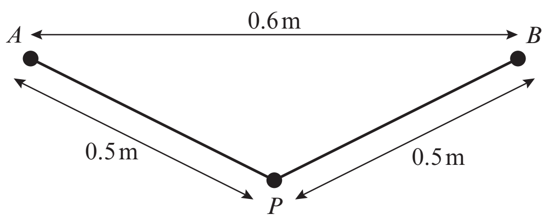
1 A particle of mass 4 kg is attached to one end P of a light elastic spring PQ, of natural length 0.5 m and modulus of elasticity 40 N. The spring rests on a **smooth** horizontal plane with the end Q fixed. The particle is held at rest and then released. Find the initial acceleration of the particle

a if $PQ = 0.8$ m initially

b if $PQ = 0.4$ m initially.

E **2** A particle of mass 0.4 kg is fixed to one end A of a light elastic spring AB, of natural length 0.8 m and modulus of elasticity 20 N. The other end B of the spring is attached to a fixed point. The particle hangs in equilibrium. It is then pulled vertically downwards through a distance 0.2 m and released from rest. Find the initial acceleration of the particle. **(4 marks)**

E/P **3**



A particle P of mass 2 kg is attached to the midpoint of a light elastic string, of natural length 0.4 m and modulus of elasticity 20 N. The ends of the elastic string are attached to two fixed points A and B which are on the same horizontal level, with $AB = 0.6$ m. The particle is held in the position shown, with $AP = BP = 0.5$ m, and released from rest. Find the initial acceleration of the particle and state its direction. **(5 marks)**

E/P **4** A particle of mass 2 kg is attached to one end P of a light elastic spring. The other end Q of the spring is attached to a fixed point O. The spring has natural length 1.5 m and modulus of elasticity 40 N. The particle is held at a point which is 1 m vertically above O and released from rest. Find the initial acceleration of the particle, stating its magnitude and direction. **(5 marks)**

E/P **5** A particle of mass 1 kg is attached to one end of a light elastic spring of natural length 1.6 m and modulus of elasticity 21.5 N. The other end of the spring is attached to a fixed point O on a rough plane which is inclined to the horizontal at an angle α where $\tan \alpha = \frac{5}{12}$. The coefficient of friction between the particle and the plane is $\frac{1}{2}$. The particle is held at rest on the plane at a point which is 1.2 m from O down a line of greatest slope of the plane. The particle is released from rest and moves down the slope.

a Find its initial acceleration. **(6 marks)**

b Without doing any further calculation, state how your answer to part **a** would change if the coefficient of friction between the particle and the plane was greater than $\frac{1}{2}$. **(1 mark)**

Challenge

Two light elastic strings each have natural length l and modulus of elasticity λ.
A particle P of mass 3 kg is attached to one end of each string. The other ends of the strings are attached to fixed points A and B, where AB is horizontal and $AB = 2x$ m.
The particle is held at a point x m below the midpoint of AB and released from rest.
The initial acceleration of the particle is $\frac{g}{2}$ m s^{-2}.

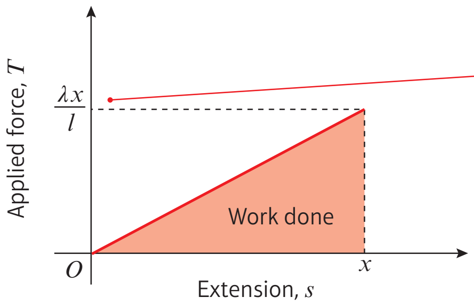
a Show that when the particle is released the tension, T, in each string is $\frac{3\sqrt{2}g}{4}$ N.

b Given that at the point the particle is released, each string has extended by $\frac{1}{4}$ of its natural length, find the modulus of elasticity for each string.

2.3 Elastic energy

You can find the energy stored in an elastic string or spring.

You can draw a **force–distance** diagram to show the extension x in an elastic string as a gradually increasing force is applied. The area under the force–distance graph is the **work done** in stretching the elastic string.



The applied force is always equal and opposite to the tension in the elastic string, T. This value increases as the string stretches.

Using the formula for the area of a triangle:

$$\text{Area} = \frac{1}{2}x\left(\frac{\lambda x}{l}\right) = \frac{\lambda x^2}{2l}$$

Using integration:

$$\text{Area} = \int_0^x T\,\mathrm{d}s = \int_0^x \frac{\lambda s}{l}\,\mathrm{d}s = \left[\frac{\lambda s^2}{2l}\right]_0^x = \frac{\lambda x^2}{2l}$$

- **The work done in stretching an elastic string or spring of modulus of elasticity λ from its natural length l to a length $(l + x)$ is $\frac{\lambda x^2}{2l}$.**

Watch out This rule applies as long as the extension is within the **elastic limit** of the string or spring.

When λ is measured in newtons and x and l are measured in metres, the work done is in **joules (J)**.

When a stretched string is released it will 'ping' back (i.e. return) to its natural length. In its stretched position it has the potential to do work, or elastic **potential energy** (this is also called **elastic energy**).

- **The elastic potential energy (E.P.E.) stored in a stretched string or spring is exactly equal to the amount of work done to stretch the string or spring.**

Links This is an application of the work–energy principle.
← Mechanics 2 Section 4.3

- **The E.P.E. stored in an elastic string or spring of modulus of elasticity λ which has been stretched from its natural length l to a length $(l + x)$ is $\frac{\lambda x^2}{2l}$.**

You can apply the same formulae for work done and elastic potential energy when an elastic string or spring is compressed.

Example 10

SKILLS PROBLEM-SOLVING

An elastic string has natural length 1.4 m and modulus of elasticity 6 N. Find the energy stored in the string when its length is 1.6 m.

$$\text{Energy stored} = \frac{6 \times 0.2^2}{2 \times 1.4}$$
$$= 0.0857 \text{ J (3 s.f.)}$$

Use $\frac{\lambda x^2}{2l}$ with $x = 1.6 - 1.4 = 0.2$

Example 11

SKILLS PROBLEM-SOLVING

A light elastic spring has natural length 0.6 m and modulus of elasticity 10 N. Find the work done in compressing the spring from a length of 0.5 m to a length of 0.3 m.

$$\text{Work done in compression} = \text{Energy stored when length is 0.3 m} - \text{Energy stored when length is 0.5 m}$$
$$= \frac{10 \times 0.3^2}{2 \times 0.6} - \frac{10 \times 0.1^2}{2 \times 0.6}$$
$$= \frac{10}{1.2}(0.3^2 - 0.1^2)$$
$$= \frac{25}{3}(0.3 + 0.1)(0.3 - 0.1)$$
$$= \frac{25}{3} \times 0.4 \times 0.2$$
$$= \frac{2}{3} \text{ J}$$

The spring is being compressed, so it will have greater stored energy at the shorter length.

Use the **compression** in the formula, not the length.

Watch out A common error to use $\frac{10 \times (0.3 - 0.1)^2}{2 \times 0.6}$ which is not the same.

Exercise 2C

SKILLS PROBLEM-SOLVING

1 An elastic spring has natural length 0.6 m and modulus of elasticity 8 N. Find the work done when the spring is stretched from its natural length to a length of 1 m.

2 An elastic spring, of natural length 0.8 m and modulus of elasticity 4 N, is compressed to a length of 0.6 m. Find the elastic potential energy stored in the spring.

3 An elastic string has natural length 1.2 m and modulus of elasticity 10 N. Find the work done when the string is stretched from a length of 1.5 m to a length 1.8 m.

4 An elastic spring has natural length 0.7 m and modulus of elasticity 20 N. Find the work done when the spring is stretched from a length

a 0.7 m to 0.9 m

b 0.8 m to 1.0 m

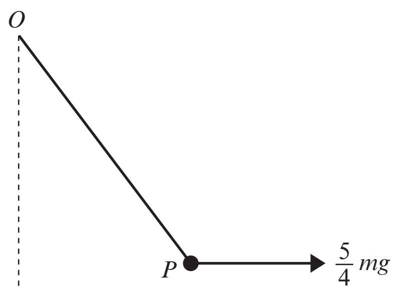
c 1.2 m to 1.4 m

Hint Note that your answers to **a**, **b** and **c** are all different.

E **5** A light elastic spring has natural length 1.2 m and modulus of elasticity 10 N. One end of the spring is attached to a fixed point. A particle of mass 2 kg is attached to the other end and hangs in equilibrium. Find the energy stored in the spring. **(3 marks)**

E/P **6** An elastic string has natural length a. One end is fixed. A particle of mass $2m$ is attached to the free end and hangs in equilibrium, with the length of the string $3a$. Find the elastic potential energy stored in the string. **(3 marks)**

E/P **7**



A particle P of mass m is attached to one end of a light elastic string, of natural length a and modulus of elasticity $2mg$. The other end of the string is attached to a fixed point O.

The particle P is held in equilibrium by a horizontal force of magnitude $\frac{5}{4}$mg applied to P.

This force acts in the vertical plane containing the string, as shown in the diagram.

Find

a the tension in the string **(5 marks)**

b the elastic energy stored in the string. **(4 marks)**

2.4 Problems involving elastic energy

You can solve problems involving elastic energy using the principle of conservation of mechanical energy and the work–energy principle.

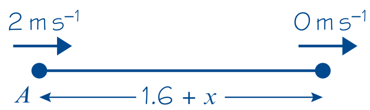
- When no external forces (other than gravity) act on a particle, then the sum of its **kinetic energy**, **gravitational** potential energy and elastic potential energy remains constant.

Links This is an application of the principle of conservation of mechanical energy.
← Mechanics 2 Section 4.3

If a particle which is attached to an elastic spring or string is subject to a **resistance** as it moves, you will need to apply the work–energy principle.

Example 12 SKILLS INTERPRETATION

A light elastic string, of natural length 1.6 m and modulus of elasticity 10 N, has one end fixed at a point A on a smooth horizontal table. A particle of mass 2 kg is attached to the other end of the string. The particle is held at the point A and **projected** horizontally along the table with speed $2\,\text{m s}^{-1}$. Find how far it travels before first coming to instantaneous rest.



Draw a simple diagram showing the initial and final positions of the particle.

Suppose that the extension of the string when the particle comes to rest is x.

K.E. lost by the particle = E.P.E. gained by the string

You can apply the **conservation of energy** principle since the table is smooth.

$$\frac{1}{2}mv^2 = \frac{\lambda x^2}{2l}$$

$$\frac{1}{2} \times 2 \times 2^2 = \frac{10x^2}{2 \times 1.6}$$

Note that you do not need to consider the energy of the particle in any intermediate positions.

$$1.28 = x^2$$

$$1.13\ldots = x$$

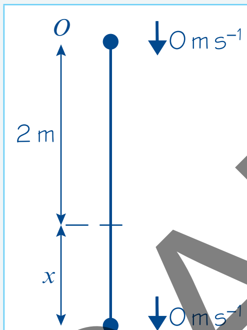
Total distance travelled is 2.73 m (3 s.f.)

Add on the natural length of the string.

It is important to realise that in the example above, the particle is not in equilibrium when it comes to instantaneous rest. Therefore you cannot use forces to solve this type of problem. The particle has an acceleration and will immediately 'spring' back towards A.

Example 13 **SKILLS** PROBLEM-SOLVING

A particle of mass 0.5 kg is attached to one end of an elastic string, of natural length 2 m and modulus of elasticity 19.6 N. The other end of the elastic string is attached to a point O. In fact, the particle is released from the point O. Find the greatest distance it will reach below O.



Draw a diagram showing the initial and final positions of the particle. Let the extension of the string be x when the particle comes to rest.

P.E. lost by particle = E.P.E. gained by string

There is no K.E. involved as the particle starts at rest and finishes at rest. Assuming no air resistance, energy will be conserved.

$$mgh = \frac{\lambda x^2}{2l}$$

$$0.5g(2 + x) = \frac{19.6x^2}{4}$$

$$2 + x = x^2$$

$$0 = x^2 - x - 2$$

$$0 = (x - 2)(x + 1)$$

$$x = 2 \text{ or } -1$$

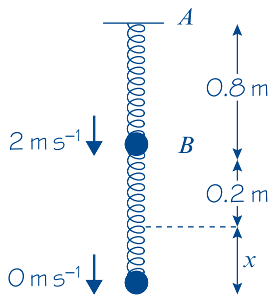
Hence greatest distance reached below O is 4 m.

Problem-solving

x is the extension in the string so it must be positive. Ignore the negative solution, and remember to add the natural length of the string.

Example 14 SKILLS INTERPRETATION

A light elastic spring, of natural length 1 m and modulus of elasticity 10 N, has one end attached to a fixed point A. A particle of mass 2 kg is attached to the other end of the spring and is held at a point B which is 0.8 m vertically below A. The particle is projected vertically downwards from B with speed 2 m s^{-1}. Find the distance it travels before first coming to rest.



Let the extension of the spring be x when the particle comes to rest.

K.E. loss + P.E. loss = E.P.E. gain

$$\frac{1}{2} \times 2 \times 2^2 + 2g(0.2 + x) = \frac{10x^2}{2} - \frac{10 \times 0.2^2}{2}$$

$$4 + 3.92 + 19.6x = 5x^2 - 0.2$$

$$0 = 5x^2 - 19.6x - 8.12$$

$$x = \frac{19.6 \pm \sqrt{19.6^2 + (4 \times 5 \times 8.12)}}{10}$$

$$= \frac{19.6 \pm 23.37\ldots}{10}$$

$$x = 4.298\ldots \text{ or } -0.378\ldots$$

Distance travelled $= 4.298\ldots + 0.2$

$= 4.498\ldots$

$= 4.5\text{ m}$ (2 s.f.)

Problem-solving

You will need to use the principle of conservation of mechanical energy with kinetic energy, gravitational potential energy **and** elastic potential energy.

Let the extension of the spring be x.
E.P.E. gain = final E.P.E. − initial E.P.E.

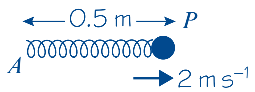
Write in the form $ax^2 + bx + c = 0$ to solve quadratic.

Ignore the negative solution.

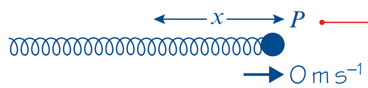
The spring is compressed by 0.2 m at the start.

Example 15 SKILLS CRITICAL THINKING

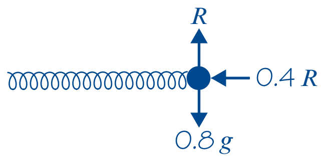
A light elastic spring, of natural length 0.5 m and modulus of elasticity 10 N, has one end attached to a point A on a rough horizontal plane. The other end is attached to a particle P of mass 0.8 kg. The coefficient of friction between the particle and the plane is 0.4. Initially the particle lies on the plane with $AP = 0.5\text{ m}$. It is then projected with speed 2 m s^{-1} away from A along the plane. Find the distance moved by P before it first comes to rest.



Draw a diagram showing the initial and final positions of the particle.



Let the extension of the spring be x.



Problem-solving

As it slides, P will be moving against friction, μR, from the plane.

$(\uparrow)$ $R = 0.8g$

First find the magnitude of the friction force.

Friction $= 0.4 \times 0.8g$

$= 0.32g$

work done against friction = overall loss in energy

Apply the work–energy principle.

$$\text{work done against friction} = \text{K.E. loss of } P - \text{E.P.E. gain of spring}$$

$$0.32gx = \frac{1}{2} \times 0.8 \times 2^2 - \frac{10x^2}{2 \times 0.5}$$

Use force × distance, $\frac{1}{2}mv^2$ and $\frac{\lambda x^2}{2l}$

$$10x^2 + 0.32gx - 1.6 = 0$$

Write in the form $ax^2 + bx + c = 0$ to solve quadratic.

$$x = \frac{-0.32g \pm \sqrt{(0.32g)^2 + 64}}{20}$$

Solve for x.

$x = 0.2728\ldots$ or $-0.586\ldots$

Ignore the negative solution.

P moves a distance 0.27 m (2 s.f.) before coming to rest.

Exercise 2D

SKILLS INTERPRETATION

(P) **1** An elastic string, of natural length l and modulus of elasticity mg, has one end fixed to a point A on a smooth horizontal table. The other end is attached to a particle P of mass m. The particle is held at a point on the table with $AP = \frac{3}{2}l$ and is released. Find the speed of the particle when the string reaches its natural length.

(P) **2** A particle of mass m is **suspended** from a fixed point O by a light elastic string, of natural length a and modulus of elasticity $4mg$. The particle is pulled vertically downwards a distance d from its equilibrium position and released from rest. If the particle just reaches O, find d.

(E/P) **3** A light elastic spring of natural length $2l$ has its ends attached to two points P and Q which are at the same horizontal level. The length PQ is $2l$. A particle of mass m is fastened to the midpoint of the spring and is held at the midpoint of PQ. The particle is released from rest and first comes to instantaneous rest when both parts of the spring make an angle of 60° with the line PQ.

a Find the modulus of elasticity of the spring. **(6 marks)**

b Suggest one way in which the model could be refined to make it more realistic. **(1 mark)**

E/P **4** A light elastic string, of natural length 1 m and modulus of elasticity 21.6 N, has one end attached to a fixed point O. A particle of mass 2 kg is attached to the other end. The particle is held at a point which is 3 m vertically below O and released from rest. Find

a the speed of the particle when the string first becomes slack **(5 marks)**

b the distance from O when the particle first comes to rest. **(3 marks)**

E/P **5** A particle P is attached to one end of a light elastic string of natural length a. The other end of the string is attached to a fixed point O. When P hangs at rest in equilibrium, the distance OP is $\frac{5a}{3}$. The particle is now projected vertically downwards from O with speed U and first comes to instantaneous rest at a distance $\frac{10a}{3}$ below O. Find U in terms of a and g. **(7 marks)**

E/P **6** A particle P of mass 1 kg is attached to the midpoint of a light elastic string, of natural length 3 m and modulus λ N. The ends of the string are attached to two points A and B on the same horizontal level with $AB = 3$ m. The particle is held at the midpoint of AB and released from rest. The particle falls vertically and comes to instantaneous rest at a point which is 1 m below the midpoint of AB. Find

a the value of λ **(5 marks)**

b the speed of P when it is 0.5 m below the initial position. **(5 marks)**

E/P **7** A light elastic string of natural length 2 m and modulus of elasticity 117.6 N has one end attached to a fixed point O. A particle P of mass 3 kg is attached to the other end. The particle is held at O and released from rest.

a Find the distance fallen by P before it first comes to rest. **(5 marks)**

b Find the greatest speed of P during the fall. **(4 marks)**

Problem-solving

P will be travelling at its greatest speed when the acceleration is zero.

E/P **8** A particle P of mass 2 kg is attached to one end of a light elastic string of natural length 1 m and modulus of elasticity 40 N. The other end of the string is fixed to a point O on a rough plane which is inclined at an angle α, where $\tan \alpha = \frac{3}{4}$. The particle is held at O and released from rest. Given that P comes to rest after moving 2 m down the plane, find the coefficient of friction between the particle and the plane. **(4 marks)**

Challenge

An elastic string of natural length l m is suspended from a fixed point O. When a mass of M kg is attached to the other end of the string, its extension is $\frac{l}{10}$ m. An additional M kg is then attached to the end of the string. Show that the work done in producing the additional extension is $\frac{3Mgl}{20}$ J.

Chapter review 2

SKILLS PROBLEM-SOLVING

P **1** A particle of mass m is supported by two light elastic strings, each of natural length a and modulus of elasticity $\frac{15mg}{16}$. The other ends of the strings are attached to two fixed points A and B where A and B are in the same horizontal line with $AB = 2a$. When the particle hangs at rest in equilibrium below AB, each string makes an angle θ with the vertical.

a Verify that $\cos\theta = \frac{4}{5}$.

b How much work must be done to raise the particle to the midpoint of AB?

2 A light elastic spring is such that a weight of magnitude W resting on the spring produces a compression a. The weight W is allowed to fall onto the spring from a height of $\frac{3a}{2}$ above it. Find the maximum compression of the spring in the subsequent motion.

3 A light elastic string of natural length 0.5 m is stretched between two points P and Q on a smooth horizontal table. The distance PQ is 0.75 m and the tension in the string is 15 N.

a Find the modulus of elasticity of the string.

A particle of mass 0.5 kg is attached to the midpoint of the string. The particle is pulled 0.1 m towards Q and released from rest.

b Find the speed of the particle as it passes through the midpoint of PQ.

P **4** A particle of mass m is attached to two strings AP and BP. The points A and B are on the same horizontal level and $AB = \frac{5a}{4}$.

The string AP is inextensible and $AP = \frac{3a}{4}$.

The string BP is elastic and $BP = a$.

The modulus of elasticity of BP is λ. Show that the natural length of BP is $\frac{5\lambda a}{3mg + 5\lambda}$

P **5** A light elastic string, of natural length a and modulus of elasticity $5mg$, has one end attached to the base of a vertical wall. The other end of the string is attached to a small ball of mass m. The ball is held at a distance $\frac{3a}{2}$ from the wall, on a rough horizontal plane, and released from rest. The coefficient of friction between the ball and the plane is $\frac{1}{5}$.

a Find, in terms of a and g, the speed V of the ball as it hits the wall.

The ball rebounds (i.e. bounces back) from the wall with speed $\frac{2V}{5}$. The string stays slack.

b Find the distance from the wall at which the ball comes to rest.

E/P **6** A light elastic string has natural length l and modulus $2mg$. One end of the string is attached to a particle P of mass m. The other end is attached to a fixed point C on a rough horizontal plane. Initially P is at rest at a point D on the plane where $CD = \frac{4l}{3}$

a Given that P is in limiting equilibrium, find the coefficient of friction between P and the plane. **(5 marks)**

The particle P is now moved away from C to a point E on the plane where $CE = 2l$

b Find the speed of P when the string returns to its natural length. **(5 marks)**

c Find the total distance moved by P before it comes to rest. **(4 marks)**

E/P **7** A light elastic string of natural length 0.2 m has its ends attached to two fixed points A and B which are on the same horizontal level with $AB = 0.2$ m. A particle of mass 5 kg is attached to the string at the point P where $AP = 0.15$ m. The system is released and P hangs in equilibrium below AB with $\angle APB = 90°$.

a If $\angle BAP = \theta$, show that the ratio of the extension of AP and BP is

$$\frac{4\cos\theta - 3}{4\sin\theta - 1}$$ **(4 marks)**

b Hence show that

$$\cos\theta\,(4\cos\theta - 3) = 3\sin\theta\,(4\sin\theta - 1)$$ **(4 marks)**

E/P **8** A particle of mass 3 kg is attached to one end of a light elastic string, of natural length 1 m and modulus of elasticity 14.7 N. The other end of the string is attached to a fixed point. The particle is held in equilibrium by a horizontal force of magnitude 9.8 N with the string inclined to the vertical at an angle θ.

a Find the value of θ. **(3 marks)**

b Find the extension of the string. **(3 marks)**

c If the horizontal force is removed, find the magnitude of the least force that will keep the string inclined at the same angle. **(2 marks)**

E/P **9** Two points A and B are on the same horizontal level with $AB = 3a$. A particle P of mass m is joined to A by a light inextensible string of length $4a$ and is joined to B by a light elastic string, of natural length a and modulus of elasticity $\frac{mg}{4}$. The particle P is held at a point C, such that $BC = a$ and both strings are **taut**. The particle P is released from rest.

a Show that when AP is vertical the speed of P is $2\sqrt{ga}$ **(6 marks)**

b Find the tension in the elastic string in this position. **(4 marks)**

Challenge

A bungee jumper attaches one end of an elastic rope to both ankles. The other end is attached to the platform on which he stands.

The bungee jumper is modelled as a particle of mass m kg attached to an elastic string of natural length l m with modulus of elasticity λ N.

a Show that the maximum distance the jumper **descends** after jumping off the platform is $l + k + \sqrt{2kl + k^2}$ where $k = \frac{mgl}{\lambda}$

b Suggest a **refinement** to this model that would result in
 i a greater maximum descent
 ii a smaller maximum descent.

Summary of key points

1 When an elastic string or spring is stretched, the tension, T, produced is proportional to the extension, x.

- $\boldsymbol{T \propto x}$
- $\boldsymbol{T = kx}$**, where k is a constant**

The constant k depends on the unstretched length of the spring or string, l, and the **modulus of elasticity** of the string or spring, λ.

- $\boldsymbol{T = \frac{\lambda x}{l}}$

This relationship is called **Hooke's law**.

2 The area under a **force–distance** graph is the **work done** in stretching an elastic string or spring. The work done in stretching or compressing an elastic string or spring with modulus of elasticity λ from its natural length, l to a length $(l + x)$ is $\frac{\lambda x^2}{2l}$

When λ is measured in newtons and x and l are measured in metres, the work done is in **joules (J)**.

3 The **elastic potential energy** (E.P.E.) stored in a stretched string or spring is exactly equal to the amount of work done to stretch the string or spring.

The E.P.E. stored in a string or spring of modulus of elasticity λ which has been stretched from its natural length l, to a length $(l + x)$ is $\frac{\lambda x^2}{2l}$

4 When no external forces (other than gravity) act on a particle, then the sum of its kinetic energy, gravitational potential energy and elastic potential energy remains constant.