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ABOUT THIS BOOK

The following three themes have been fully integrated throughout the Pearson Edexcel International Advanced Level in Mathematics series, so they can be applied alongside your learning.

1. **Mathematical argument, language and proof**
   - Rigorous and consistent approach throughout
   - Notation boxes explain key mathematical language and symbols

2. **Mathematical problem-solving**
   - Hundreds of problem-solving questions, fully integrated into the main exercises
   - Problem-solving boxes provide tips and strategies
   - Challenge questions provide extra stretch

3. **Transferable skills**
   - Transferable skills are embedded throughout this book, in the exercises and in some examples
   - These skills are signposted to show students which skills they are using and developing

Finding your way around the book

Each chapter starts with a list of Learning objectives

The Prior knowledge check helps make sure you are ready to start the chapter

Glossary terms will be identified by bold blue text on their first appearance

Each chapter is mapped to the specification content for easy reference

The real world applications of the maths you are about to learn are highlighted at the start of the chapter
ABOUT THIS BOOK

Each chapter ends with a Chapter review and a Summary of key points

Exercise questions are carefully graded so they increase in difficulty and gradually bring you up to exam standard

Transferable skills are signposted where they naturally occur in the exercises and examples

Step-by-step worked examples focus on the key types of questions you’ll need to tackle

Problem-solving boxes provide hints, tips and strategies, and Watch out boxes highlight areas where students often lose marks in their exams

Each section begins with an explanation and key learning points

Challenge boxes give you a chance to tackle some more difficult questions

Exercises are packed with exam-style questions to ensure you are ready for the exams

Exam-style questions are flagged with

Problem-solving questions are flagged with

Exam practice
Mathematics
International Advanced Subsidiary/ Advanced Level Pure Mathematics 1

Time: 1 hour 30 minutes
You may have: Mathematical Formulae and Statistical Tables, Calculator

Answer ALL questions.

1. Simplify:
   a) \( \frac{5x^2}{2xy} \)
   b) \( \frac{3x^3}{4y^2} \)

2. Simplify \( \frac{2x^2y}{3xy^2} \), where \( x \) and \( y \) are non-zero real numbers.

3. Given \( x = 2 \) and \( y = 3 \), find \( xy \).

4. Given \( x = \frac{1}{2} \) and \( y = -1 \), find \( xy \).

5. Given \( x = \frac{1}{2} \) and \( y = -1 \), find \( x^2 + y^2 \).

A full practice paper at the back of the book helps you prepare for the real thing
QUALIFICATION AND ASSESSMENT OVERVIEW

Qualification and content overview

Pure Mathematics 1 (P1) is a compulsory unit in the following qualifications:
- International Advanced Subsidiary in Mathematics
- International Advanced Subsidiary in Pure Mathematics
- International Advanced Level in Mathematics
- International Advanced Level in Pure Mathematics

Assessment overview

The following table gives an overview of the assessment for this unit.
We recommend that you study this information closely to help ensure that you are fully prepared for this course and know exactly what to expect in the assessment.

<table>
<thead>
<tr>
<th>Unit</th>
<th>Percentage</th>
<th>Mark</th>
<th>Time</th>
<th>Availability</th>
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<tr>
<td>P1: Pure Mathematics 1</td>
<td>33 1/3 %</td>
<td>75</td>
<td>1 hour 30 mins</td>
<td>January, June and October</td>
</tr>
<tr>
<td>Paper code WMA11/01</td>
<td>16 2/3 %</td>
<td></td>
<td></td>
<td>First assessment January 2019</td>
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IAS – International Advanced Subsidiary, IAL – International Advanced A Level

Assessment objectives and weightings

<table>
<thead>
<tr>
<th>AO1</th>
<th>Recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of contexts.</th>
<th>Minimum weighting in IAS and IAL</th>
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<tr>
<td></td>
<td>30%</td>
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<td>AO2</td>
<td>Construct rigorous mathematical arguments and proofs through use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions, including the construction of extended arguments for handling substantial problems presented in unstructured form.</td>
<td>30%</td>
</tr>
<tr>
<td>AO3</td>
<td>Recall, select and use their knowledge of standard mathematical models to represent situations in the real world; recognise and understand given representations involving standard models; present and interpret results from such models in terms of the original situation, including discussion of the assumptions made and refinement of such models.</td>
<td>10%</td>
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<td>AO4</td>
<td>Comprehend translations of common realistic contexts into mathematics; use the results of calculations to make predictions, or comment on the context; and, where appropriate, read critically and comprehend longer mathematical arguments or examples of applications.</td>
<td>5%</td>
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<tr>
<td>AO5</td>
<td>Use contemporary calculator technology and other permitted resources (such as formulae booklets or statistical tables) accurately and efficiently; understand when not to use such technology, and its limitations. Give answers to appropriate accuracy.</td>
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Relationship of assessment objectives to units

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<td>6(\frac{1}{3})–20</td>
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Calculators

Students may use a calculator in assessments for these qualifications. Centres are responsible for making sure that calculators used by their students meet the requirements outlined below.

Students are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π, \(x^2\), √, \(\frac{1}{x}\), \(x^x\), ln, e^x, x!, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory.

Prohibitions

Calculators with any of the following facilities are prohibited in all examinations:

• databanks
• retrieval of text or formulae
• built-in symbolic algebra manipulations
• symbolic differentiation and/or integration
• language translators
• communication with other machines or the internet
**Extra online content**

Whenever you see an *Online* box, it means that there is extra online content available to support you.

**SolutionBank**

SolutionBank provides worked solutions for questions in the book. Download all the solutions as a PDF or quickly find the solution you need online.

**Use of technology**

Explore topics in more detail, visualise problems and consolidate your understanding. Use pre-made GeoGebra activities or Casio resources for a graphic calculator.

**GeoGebra**

GeoGebra-powered interactives

- Interact with the maths you are learning using GeoGebra’s easy-to-use tools

**CASIO**

Graphic calculator interactives

- Explore the maths you are learning and gain confidence in using a graphic calculator

**Calculator tutorials**

Our helpful video tutorials will guide you through how to use your calculator in the exams. They cover both Casio’s scientific and colour graphic calculators.

**Online**

- Work out each coefficient quickly using the $^nC_r$ and power functions on your calculator.
- Step-by-step guide with audio instructions on exactly which buttons to press and what should appear on your calculator’s screen.
1 ALGEBRAIC EXPRESSIONS

Learning objectives

After completing this chapter you should be able to:

- Multiply and divide integer powers → pages 2–4
- Expand a single term over brackets and collect like terms → pages 2–4
- Expand the product of two or three expressions → pages 4–6
- Factorise linear, quadratic and simple cubic expressions → pages 6–9
- Know and use the laws of indices → pages 9–11
- Simplify and use the rules of surds → pages 12–13
- Rationalise denominators → pages 13–15

Prior knowledge check

1. Simplify:
   a. \(4m^3n + 5mn^2 - 2m^2n + mn^2 - 3mn^2\)
   b. \(3x^2 - 5x + 2 + 3x^2 - 7x - 12\)

2. Write as a single power of 2:
   a. \(2^5 \times 2^3\)
   b. \(2^6 \div 2^2\)
   c. \((2^3)^2\)

3. Expand:
   a. \(3(x + 4)\)
   b. \(5(2 - 3x)\)
   c. \(6(2x - 5y)\)

4. Write down the highest common factor of:
   a. 24 and 16
   b. 6x and 8x^2
   c. 4xy^2 and 3xy

5. Simplify:
   a. \(\frac{10x}{5}\)
   b. \(\frac{20x}{2}\)
   c. \(\frac{40x}{24}\)

Computer scientists use indices to describe very large numbers. A quantum computer with 1000 qubits (quantum bits) can consider \(2^{1000}\) values simultaneously. This is greater than the number of particles in the observable universe.
1.1 Index laws

You can use the laws of indices to simplify powers of the same base.

- \( a^m \times a^n = a^{m+n} \)
- \( a^m \div a^n = a^{m-n} \)
- \((a^m)^n = a^{mn}\)

\((ab)^n = a^n b^n\)

Example 1

Simplify these expressions:

- \(a^2 \times x^5\)
- \(2r^2 \times 3r^3\)
- \(\frac{b^7}{b^3}\)
- \(6x^3 \div 3x^3\)
- \((a^3)^2 \times 2a^2\)
- \((3x^3)^3 \div x^4\)

\[a^2 \times x^5 = x^{2+5} = x^7\]
\[2r^2 \times 3r^3 = 2 \times 3 \times r^2 \times r^3 = 6r^{2+3} = 6r^5\]
\[\frac{b^7}{b^3} = b^{7-3} = b^4\]
\[6x^3 \div 3x^3 = \frac{6}{3} \times \frac{x^3}{x^3} = 2x^2\]
\[(a^3)^2 \times 2a^2 = 2 \times a^6 \times a^2 = 2a^8\]
\[\frac{(3x^3)^3}{x^4} = 3^3 \times \frac{(x^3)^3}{x^4} = 27 \times x^9 \times x^{-4} = 27x^5\]

Example 2

Expand these expressions and simplify if possible:

- \(-3x(7x - 4)\)
- \(y^2(3 - 2y^3)\)
- \(4x(3x - 2x^2 + 5x^3)\)
- \(2x(5x + 3) - 5(2x + 3)\)

- \(-3x(7x - 4) = -21x^2 + 12x\)
- \(y^2(3 - 2y^3) = 3y^2 - 2y^5\)
- \(4x(3x - 2x^2 + 5x^3) = 12x^2 - 8x^4 + 20x^4\)
- \(2x(5x + 3) - 5(2x + 3) = 10x^2 + 6x - 10x - 15 = 10x^2 - 4x - 15\)
ALGEBRAIC EXPRESSIONS

CHAPTER 1

Example 3

Simplify these expressions:

a \( \frac{x^7 + x^4}{x^3} \)

b \( \frac{3x^2 - 6x^5}{2x} \)

c \( \frac{20x^7 + 15x^3}{5x^2} \)

\( a = \frac{x^7}{x^3} + \frac{x^4}{x^3} \)

\( = x^7 \div x^3 + x^4 \div x^3 \)

\( = x^4 + x^1 \)

\( x^1 \) is the same as \( x \).

\( b = \frac{3x^2}{2x} - \frac{6x^5}{2x} \)

\( = \frac{3}{2} x^2 - \frac{3}{2} x^5 \)

\( = \frac{3}{2} x^{2-1} - \frac{3}{2} x^{5-1} \)

\( = \frac{3}{2} x - \frac{3}{2} x^4 \)

\( c = \frac{20x^7}{5x^2} + \frac{15x^3}{5x^2} \)

\( = \frac{4x^7}{5x^2} + \frac{3x^3}{5x^2} \)

\( = 4x^{7-2} + 3x^{3-2} \)

\( = 4x^5 + 3x^1 \)

Exercise 1A

1 Simplify these expressions:

a \( x^3 \times x^4 \)

b \( 2x^3 \times 3x^2 \)

c \( \frac{k^3}{k^2} \)

d \( \frac{4p^3}{2p} \)

\( e = \frac{3x^3}{3x^2} \)

\( f = (y^2)^5 \)

\( g = 10x^5 \div 2x^3 \)

\( h = (p^3)^2 \div p^4 \)

\( i = (2a^2)^2 \div 2a^3 \)

\( j = 8p^4 \div 4p^3 \)

\( k = 2a^4 \times 3a^5 \)

\( l = \frac{21a^3b^7}{7ab^4} \)

\( m = 9x^2 \times (3x^2)^3 \)

\( n = 3x^3 \times 2x^2 \times 4x^6 \)

\( o = 7a^4 \times (3a^3)^2 \)

\( p = (4y^3)^3 \div 2y^3 \)

\( q = 2a^3 \div 3a^2 \times 6a^5 \)

\( r = 3a^4 \times 2a^5 \times a^3 \)
2 Expand and simplify if possible:

a \( 9(x - 2) \)  
b \( x(x + 9) \)  
c \( -3y(4 - 3y) \)  
d \( xy + 5 \)  
e \( -x(3x + 5) \)  
f \( -5x(4x + 1) \)  
g \( (4x + 5)x \)  
h \( -3y(5 - 2y)^2 \)  
j \( (3x - 5)x^2 \)  
k \( 3(x + 2) + (x - 7) \)  
m \( 4(c + 3d^2) - 3(2c + d^2) \)  
n \( (r^2 + 3r^2 + 9) - (2r^2 + 3r^2 - 4) \)  
o \( x(3x^2 - 2x + 5) \)  
p \( 7y^2(2 - 5y + 3y^2) \)  
q \( -2y^2(5 - 7y + 3y^2) \)  
r \( 7(x - 2) + 3(x + 4) - 6(x - 2) \)  
t \( 3x^2 - x(3 - 4x) + 7 \)  
u \( 4x(x + 3) - 2x(3x - 7) \)  
v \( 3x^2(2x + 1) - 5x^2(3x - 4) \)

3 Simplify these fractions:

a \( \frac{6x^4 + 10x^6}{2x} \)  
b \( \frac{3x^5 - x^7}{x} \)  
c \( \frac{2x^4 - 4x^2}{4x} \)  
d \( \frac{8x^3 + 5x}{2x} \)  
e \( \frac{7x^7 + 5x^2}{5x} \)  
f \( \frac{9x^5 - 5x^3}{3x} \)

\[ \text{Example 4 Skills Interpretation} \]

1.2 Expanding brackets

To find the product of two expressions you multiply each term in one expression by each term in the other expression.

Multiplying each of the 2 terms in the first expression by each of the 3 terms in the second expression gives \( 2 \times 3 = 6 \) terms.

\[(x + 5)(4x - 2y + 3) = x(4x - 2y + 3) + 5(4x - 2y + 3)\]
\[= 4x^2 - 2xy + 3x + 20x - 10y + 15\]
\[= 4x^2 - 2xy + 23x - 10y + 15\]
Simplify your answer by collecting like terms.

Example 4

Expand these expressions and simplify if possible:

a \( (x + 5)(x + 2) \)  
b \( (x - 2y)(x^2 + 1) \)  
c \( (x - y)^2 \)  
d \( (x + y)(3x - 2y - 4) \)

- **a** \( (x + 5)(x + 2) \)
  
  \[= x^2 + 2x + 5x + 10\]
  
  \[= x^2 + 7x + 10\]

- **b** \( (x - 2y)(x^2 + 1) \)
  
  \[= x^3 + x - 2x^2y - 2y\]

- **c** \( (x - y)^2 \)
  
  \[\text{Multiply } x \text{ by } (x + 2) \text{ and then multiply 5 by } (x + 2).\]

- **d** \( (x + y)(3x - 2y - 4) \)
  
  \[\text{Simplify your answer by collecting like terms.}\]

\[\text{There are no like terms to collect.}\]
**ALGEBRAIC EXPRESSIONS**

**CHAPTER 1**

**Example 5**

Expand these expressions and simplify if possible:

**a** \( (2x + 3)(x - 7) \)

\[
= (2x^2 + 3x)(x - 7)
\]

\[
= 2x^3 - 14x^2 + 3x^2 - 21x
\]

\[
= 2x^3 - 11x^2 - 21x
\]

**b** \( (5x - 3y)(2x - y + 4) \)

\[
= (5x^2 - 3xy)(2x - y + 4)
\]

\[
= 10x^3 - 5x^2y + 20x^2 - 6xy + 3xy^2 - 12xy
\]

\[
= 10x^3 - 11x^2y + 20x^2 + 3xy^2 - 12xy
\]

**c** \( (x - 4)(x + 3)(x + 1) \)

\[
= (x^2 - x - 12)(x + 1)
\]

\[
= x^3 + x - 12x - x - 12 + 12
\]

\[
= x^3 + 13x - 12
\]

**Exercise 1B**

1 Expand and simplify if possible:

- **a** \( (x + 4)(x + 7) \)
- **b** \( (x - 3)(x + 2) \)
- **c** \( (x - 2)^2 \)
- **d** \( (x - y)(2x + 3) \)
- **e** \( (x + 3y)(4x - y) \)
- **f** \( (x - 4y)(3x + y) \)
- **g** \( (2x + 3)(x - 4) \)
- **h** \( (3x + 2y)^2 \)
- **i** \( (2x + 8y)(2x + 3) \)
- **j** \( (x + 5)(2x + 3y - 5) \)
- **k** \( (x - 1)(3x - 4y - 5) \)
- **l** \( (x - 4y)(2x + y + 5) \)
- **m** \( (x + 2y - 1)(x + 3) \)
- **n** \( (2x + 2y + 3)(x + 6) \)
- **o** \( (4 - y)(4y - x + 3) \)
- **p** \( (4y + 5)(3x - y + 2) \)
- **q** \( (5y - 2x + 3)(x - 4) \)
- **r** \( (4y - x - 2)(5 - y) \)
2 Expand and simplify if possible:
   a \( 5(x + 1)(x - 4) \)  
   b \( 7(x - 2)(2x + 5) \)  
   c \( 3(x - 3)(x - 3) \)  
   d \( x(x - y)(x + y) \)  
   e \( x(2x + y)(3x + 4) \)  
   f \( y(x - 5)(x + 1) \)  
   g \( y(3x - 2y)(4x + 2) \)  
   h \( y(7 - x)(2x - 5) \)  
   i \( x(2x + y)(5x - 2) \)  
   j \( x(x + 2)(x + 3y - 4) \)  
   k \( y(2x + y - 1)(x + 5) \)  
   l \( y(3x + 2y - 3)(2x + 1) \)  
   m \( x(2x + 3)(x + y - 5) \)  
   n \( 2x(3x - 1)(4x - y - 3) \)  
   o \( 3x(2x + y)(2x + 3y + 5) \)  
   p \( (x + 3)(x + 2)(x - 1) \)  
   q \( (x + 2)(x - 4)(x + 3) \)  
   r \( (x + 3)(x - 1)(x - 5) \)  
   s \( (x - 5)(x - 4)(x - 3) \)  
   t \( (2x + 1)(x - 2)(x + 1) \)  
   u \( (2x + 3)(3x - 1)(x + 2) \)  
   v \( (x + y)(x - y)(x - 1) \)  
   w \( (3x - 2)(2x + 1)(3x - 2) \)  
   x \( (2x - 3y)^3 \)

3 The diagram shows a rectangle with a square cut out. The rectangle has length \( 3x - y + 4 \) and width \( x + 7 \). The square has side length \( x - 2 \). Find an expanded and simplified expression for the area shaded green.

4 A cuboid has dimensions \((x + 2)\) cm, \((2x - 1)\) cm and \((2x + 3)\) cm. Show that the volume of the cuboid is \((4x^3 + 12x^2 + 5x - 6)\) cm\(^3\).

5 Given that \((2x + 5y)(3x - y)(2x + y) = ax^3 + bx^2y + cxy^2 + dy^3\), where \(a\), \(b\), \(c\) and \(d\) are constants, find the values of \(a\), \(b\), \(c\) and \(d\). (2 marks)

Challenge

Expand and simplify \((x + y)^6\).

1.3 Factorising

You can write expressions as a **product** of their **factors**.

- Factorising is the opposite of expanding brackets.

   - Expanding brackets
     
     \[
     4x(2x + y) = 8x^2 + 4xy \\
     (x + 5)^3 = x^3 + 15x^2 + 75x + 125 \\
     (x + 2y)(x - 5y) = x^2 - 3xy - 10y^2
     \]

   - Factorising

   - Expanding brackets
**Example 6**

**SKILLS**

**ANALYSIS**

Factorise these expressions completely:

a. \(3x + 9\)

b. \(x^2 - 5x\)

c. \(8x^2 + 20x\)

d. \(9x^2y + 15xy^2\)

e. \(3x^2 - 9xy\)

- **a.** \(3x + 9 = 3(x + 3)\)
  - 3 is a **common factor** of \(3x\) and 9.

- **b.** \(x^2 - 5x = x(x - 5)\)
  - \(x\) is a common factor of \(x^2\) and \(-5x\).

- **c.** \(8x^2 + 20x = 4x(2x + 5)\)
  - 4 and \(x\) are common factors of \(8x^2\) and \(20x\), so take \(4x\) outside the brackets.

- **d.** \(9x^2y + 15xy^2 = 3xy(3x + 5y)\)
  - 3, \(x\) and \(y\) are common factors of \(9x^2y\) and \(15xy^2\), so take \(3xy\) outside the brackets.

- **e.** \(3x^2 - 9xy = 3(x - 3y)\)
  - \(x\) and \(-3y\) have no common factors so this expression is completely factorised.

- **Notation**

Real numbers are all the positive and negative numbers, or zero, including fractions and surds.

For the expression \(2x^2 + 5x - 3\), \(ac = -6 = -1 \times 6\)

\(2x^2 - x + 6x - 3\)

\(= x(2x - 1) + 3(2x - 1)\)

\(= (2x - 1)(x + 3)\)

**Notation**

An expression in the form \(x^2 - y^2\) is called the **difference** of two squares.

**Example 7**

Factorise:

a. \(x^2 - 5x - 6\)

b. \(x^2 + 6x + 8\)

c. \(6x^2 - 11x - 10\)

d. \(x^2 - 25\)

e. \(4x^2 - 9y^2\)

- **a.** \(x^2 - 5x - 6\)
  - \(ac = -6\) and \(b = -5\)
  - So \(x^2 - 5x - 6 = x^2 + x - 6x - 6\)
  - \(= x(x + 1) - 6(x + 1)\)
  - \(= (x + 1)(x - 6)\)

Here \(a = 1\), \(b = -5\) and \(c = -6\).

1. Find the two factors of \(ac = -6\) which add to give \(b = -5\).
   \(-6 + 1 = -5\)

2. Rewrite the \(b\) term using these two factors.

3. Factorise the first two terms and the last two terms.

4. \(x + 1\) is a factor of both terms, so take that outside the brackets. This is now completely factorised.
### Example 8

Factorise completely:

**a** \( x^3 - 2x^2 \)

\[
\begin{align*}
  x^3 - 2x^2 &= x^2(x - 2) \\
  \text{You can't factorise this any further.}
\end{align*}
\]

**b** \( x^3 - 25x \)

\[
\begin{align*}
  x^3 - 25x &= x(x^2 - 25) \\
  &= x(x - 5)(x + 5) \\
  x^2 - 25 &= (x - 5)(x + 5) \\
  \text{x is a common factor of } x^3 \text{ and } -25x, \text{ so take } x \text{ outside the brackets.}
\end{align*}
\]

**c** \( x^3 + 3x^2 - 10x \)

\[
\begin{align*}
  x^3 + 3x^2 - 10x &= x(x^2 + 3x - 10) \\
  &= x(x + 5)(x - 2) \\
  x^2 = 25 \text{ is the difference of two squares.}
\end{align*}
\]

Write the expression as a product of \( x \) and a quadratic factor.

Factorise the quadratic to get three linear factors.

### Exercise 1C

Factorise these expressions completely:

**a** \( 4x + 8 \)

**b** \( 6x - 24 \)

**c** \( 20x + 15 \)

**d** \( 2x^2 + 4 \)

**e** \( 4x^2 + 20 \)

**f** \( 6x^2 - 18x \)

**g** \( x^2 - 7x \)

**h** \( 2x^2 + 4x \)

**i** \( 3x^2 - x \)

**j** \( 6x^2 - 2x \)

**k** \( 10y^2 - 5y \)

**l** \( 35x^2 - 28x \)

**m** \( x^2 + 2x \)

**n** \( 3y^2 + 2y \)

**o** \( 4x^2 + 12x \)

**p** \( 5y^2 - 20y \)

**q** \( 9xy^2 + 12x^2y \)

**r** \( 6ab - 2ab^2 \)

**s** \( 5x^2 - 25xy \)

**t** \( 12x^2y + 8xy^2 \)

**u** \( 15y - 20y^2 \)

**v** \( 12x^2 - 30 \)

**w** \( xy^2 - x^2y \)

**x** \( 12y^2 - 4yx \)

---

**Notes:**

- **Example 8:**
  - **a:** Factorise \( x^3 - 2x^2 \) to \( x^2(x - 2) \).
  - **b:** Factorise \( x^3 - 25x \) to \( x(x - 5)(x + 5) \).
  - **c:** Factorise \( x^3 + 3x^2 - 10x \) to \( x(x + 5)(x - 2) \).

- **Exercise 1C:**
  - Factorise each expression completely.

- **SKILLS PROBLEM-SOLVING:**
  - Uncorrected proof, all content subject to change at publisher discretion. Not for resale, circulation or distribution in whole or in part. © Pearson 2019
2 Factorise:
  a  \( x^2 + 4x \)
  b  \( 2x^2 + 6x \)
  c  \( x^2 + 11x + 24 \)
  d  \( x^2 + 8x + 12 \)
  e  \( x^2 + 3x - 40 \)
  f  \( x^2 - 8x + 12 \)
  g  \( x^2 + 5x + 6 \)
  h  \( x^2 - 2x - 24 \)
  i  \( x^2 - 3x - 10 \)
  j  \( x^2 + x - 20 \)
  k  \( 2x^2 + 5x + 2 \)
  l  \( 3x^2 + 10x - 8 \)
  m  \( 5x^2 - 16x + 3 \)
  n  \( 6x^2 - 8x - 8 \)
  o  \( 2x^2 + 7x - 15 \)
  p  \( 2x^4 + 14x^2 + 24 \)
  q  \( x^2 - 4 \)
  r  \( x^2 - 49 \)
  s  \( 4x^2 - 25 \)
  t  \( 9x^2 - 25y^2 \)
  v  \( 2x^2 - 50 \)
  w  \( 6x^2 - 10x + 4 \)

3 Factorise completely:
  a  \( x^3 + 2x \)
  b  \( x^3 - x^2 + x \)
  c  \( x^3 - 5x \)
  d  \( x^3 - 9x \)
  e  \( x^3 - x^2 - 12x \)
  f  \( x^3 + 11x^2 + 30x \)
  g  \( x^3 - 7x^2 + 6x \)
  h  \( x^3 - 64x \)
  i  \( 2x^3 - 5x^2 - 3x \)
  j  \( 2x^3 + 13x^2 + 15x \)
  k  \( x^3 - 4x \)
  l  \( 3x^3 + 27x^2 + 60x \)

4 Factorise completely \( x^4 - y^4 \).  \( \text{(2 marks)} \)

5 Factorise completely \( 6x^3 + 7x^2 - 5x \).  \( \text{(2 marks)} \)

Challenge
Write \( 4x^4 - 13x^2 + 9 \) as the product of four linear factors.

1.4 Negative and fractional indices

Indices can be negative numbers or fractions.

\[ x^{\frac{1}{2}} \times x^{\frac{1}{3}} = x^{\frac{1}{2} + \frac{1}{3}} = x^{\frac{5}{6}} = x \]

similarly \[ x^{\frac{1}{n}} \times x^{\frac{1}{n}} \times \ldots \times x^{\frac{1}{n}} = x^{\frac{1}{n} + \frac{1}{n} + \ldots + \frac{1}{n}} = x^{\frac{n}{n}} = x \]

\( n \) terms

- You can use the laws of indices with any rational power.
  
  \[ a^{\frac{1}{m}} = \frac{1}{\sqrt[m]{a}} \]
  
  \[ a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m \]
  
  \[ a^{-m} = \frac{1}{a^m} \]
  
  \[ a^0 = 1 \]

Notation
Rational numbers are those that can be written as \( \frac{a}{b} \) where
\( a \) and \( b \) are integers, and \( b \neq 0 \).

Notation
\( a^{\frac{1}{2}} = \sqrt{a} \) is the positive square root of \( a \).

For example: \( 9^{\frac{1}{2}} = \sqrt{9} = 3 \), but \( 9^\frac{1}{2} \neq -3 \).
Example 9

Simplify:

a) \( \frac{x^3}{x^{-3}} \)  
   - Use the rule \( \frac{a^m}{a^n} = a^{m-n} \).
   - This could also be written as \( \sqrt[3]{x} \).

b) \( x^\frac{1}{2} \times x^\frac{1}{3} \)  
   - Use the rule \( a^m \times a^n = a^{m+n} \).

Use the rule \( (a^m)^n = a^{mn} \).

This means the cube root of \( 64 \).

Using \( a^\frac{m}{n} = \sqrt[n]{a} \).

This means the square root of 49, cubed.

Using \( a^{-m} = \frac{1}{a^m} \).

Online  
Use your calculator to enter negative and fractional powers.
**Example 11**

Given that $y = \frac{1}{16}x^2$, express each of the following in the form $kx^n$, where $k$ and $n$ are constants.

a) $y^{\frac{1}{2}}$

b) $4y^{-1}$

---

**Substitute** $y = \frac{1}{16}x^2$ into $y^{\frac{1}{2}}$.

$\left(\frac{1}{16}\right)^{\frac{1}{2}} = \frac{1}{4}$ and $(x^2)^{\frac{1}{2}} = x$

$\left(\frac{1}{16}\right)^{-1} = 16$ and $x^2 \times (-1) = x^{-2}$

---

**Problem-solving**

Check that your answers are in the correct form. If $k$ and $n$ are constants they could be positive or negative, and they could be integers, fractions or surds.

---

**Exercise 1D**

**Skills**

1. Simplify:
   - a) $x^3 \div x^2$
   - b) $x^5 \div x^7$
   - c) $x^3 \times x^\frac{1}{2}$
   - d) $(x^2)^\frac{1}{2}$
   - e) $(x^3)^\frac{1}{2}$
   - f) $3x^{0.5} \times 4x^{-0.5}$
   - g) $9x^{\frac{1}{2}} \div 3x^{\frac{1}{3}}$
   - h) $5x^{\frac{3}{2}} \div x^{\frac{1}{2}}$
   - i) $3x^4 \times 2x^{-5}$
   - j) $\sqrt{x} \times \sqrt{x}$
   - k) $(\sqrt{x})^4 \times (\sqrt{x})^4$
   - l) $\frac{\sqrt{x}}{\sqrt{x}}$

2. Evaluate, without using your calculator:
   - a) $25^{\frac{1}{2}}$
   - b) $81^{\frac{1}{2}}$
   - c) $27^{\frac{1}{3}}$
   - d) $4^{-2}$
   - e) $9^{-\frac{1}{2}}$
   - f) $(-5)^{-3}$
   - g) $\left(\frac{3}{4}\right)^0$
   - h) $1296^{\frac{1}{4}}$
   - i) $\left(\frac{25}{16}\right)^{\frac{1}{2}}$
   - j) $\left(\frac{27}{8}\right)^{\frac{1}{3}}$
   - k) $\left(\frac{6}{5}\right)^{-1}$

3. Simplify:
   - a) $(64x^{10})^{\frac{1}{2}}$
   - b) $\frac{5x^3 - 2x^2}{x^5}$
   - c) $(125x^{13})^{\frac{1}{3}}$
   - d) $\frac{x + 4x^3}{x^3}$
   - e) $\frac{9x^2 - 15x^5}{3x^3}$
   - f) $\frac{\left(\frac{4}{9}\right)^4}{x^4}$

---

**E 4 a)** Find the value of $81^{\frac{1}{2}}$.

b) Simplify $x(2x^{-1})^4$.

---

**E 5** Given that $y = \frac{1}{8}x^3$, express each of the following in the form $kx^n$, where $k$ and $n$ are constants.

a) $y^{\frac{1}{2}}$

b) $\frac{1}{2}y^{-2}$

(1 mark)

(2 marks)
1.5 Surds

If \( n \) is an integer that is not a square number, then any multiple of \( \sqrt{n} \) is called a surd. Examples of surds are \( \sqrt{2}, \sqrt{19} \) and \( 5\sqrt{2} \).

Surds are examples of irrational numbers. The decimal expansion of a surd is never-ending and never repeats, for example \( \sqrt{2} = 1.414213562... \)

You can use surds to write exact answers to calculations.

- You can manipulate surds using these rules:
  - \( \sqrt{ab} = \sqrt{a} \times \sqrt{b} \)
  - \( \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}} \)

Example 12

Simplify:

\[ a \sqrt{12} \quad b \frac{\sqrt{20}}{2} \quad c 5\sqrt{6} - 2\sqrt{24} + \sqrt{294} \]

\[ a \sqrt{12} = \sqrt{4 \times 3} \]
\[ = \sqrt{4} \times \sqrt{3} = 2\sqrt{3} \]

\[ b \frac{\sqrt{20}}{2} = \frac{\sqrt{4 \times 5}}{2} \]
\[ = \frac{2 \times \sqrt{5}}{2} = \sqrt{5} \]

\[ c 5\sqrt{6} - 2\sqrt{24} + \sqrt{294} \]
\[ = 5\sqrt{6} - 2\sqrt{6 \times 4} + \sqrt{6 \times 49} \]
\[ = \sqrt{5 \times 2 \times 2 + 49} \]
\[ = \sqrt{6(5 - 2 \times 2 + 7)} \]
\[ = \sqrt{6(8)} \]
\[ = 8\sqrt{6} \]

Look for a factor of 12 that is a square number. Use the rule \( \sqrt{ab} = \sqrt{a} \times \sqrt{b} \). \( \sqrt{4} = 2 \)

\( \sqrt{20} = \sqrt{4 \times 5} \)

\( \sqrt{4} = 2 \)

Cancel by 2.

\( \sqrt{6} \) is a common factor.

Work out the square roots \( \sqrt{4} \) and \( \sqrt{49} \).

\( 5 - 4 + 7 = 8 \)
### Example 13 SKILLS PROBLEM-SOLVING

Expand and simplify if possible:

a. \( \sqrt{2}(5 - \sqrt{3}) \)

\[
= 5\sqrt{2} - \sqrt{2}\sqrt{3} \\
= 5\sqrt{2} - \sqrt{6}
\]

b. \( (2 - \sqrt{3})(5 + \sqrt{3}) \)

\[
= 2(5 + \sqrt{3}) - \sqrt{3}(5 + \sqrt{3}) \\
= 10 + 2\sqrt{3} - 5\sqrt{3} - 3 \\
= 7 - 3\sqrt{3}
\]

### Exercise 1E SKILLS PROBLEM-SOLVING

Do not use your calculator for this exercise.

1. Simplify:
   a. \( \sqrt{28} \)
   b. \( \sqrt{72} \)
   c. \( \sqrt{50} \)
   d. \( \sqrt{32} \)
   e. \( \sqrt{90} \)
   f. \( \frac{\sqrt{12}}{2} \)
   g. \( \frac{\sqrt{27}}{3} \)
   h. \( \sqrt{20} + \sqrt{80} \)
   i. \( \sqrt{200} + \sqrt{18} - \sqrt{72} \)
   j. \( \sqrt{175} + \sqrt{63} + 2\sqrt{28} \)
   k. \( \sqrt{28} - 2\sqrt{63} + \sqrt{7} \)
   l. \( \sqrt{80} - 2\sqrt{20} + 3\sqrt{45} \)
   m. \( \frac{\sqrt{80} - 2\sqrt{20} + 5\sqrt{45}}{\sqrt{11}} \)
   n. \( \frac{\sqrt{44}}{\sqrt{11}} \)
   o. \( \sqrt{12} + 3\sqrt{48} + \sqrt{75} \)

2. Expand and simplify if possible:
   a. \( \sqrt{3}(2 + \sqrt{3}) \)
   b. \( \sqrt{5}(3 - \sqrt{3}) \)
   c. \( \sqrt{2}(4 - \sqrt{5}) \)
   d. \( (2 - \sqrt{2})(3 + \sqrt{5}) \)
   e. \( (2 - \sqrt{3})(3 - \sqrt{7}) \)
   f. \( (4 + \sqrt{5})(2 + \sqrt{5}) \)
   g. \( (5 - \sqrt{3})(1 - \sqrt{3}) \)
   h. \( (4 + \sqrt{3})(2 - \sqrt{3}) \)
   i. \( (7 - \sqrt{11})(2 + \sqrt{11}) \)

3. Simplify \( \sqrt{75} - \sqrt{12} \) giving your answer in the form \( a\sqrt{3} \), where \( a \) is an integer. (2 marks)

### 1.6 Rationalising denominators

If a fraction has a surd in the denominator, it is sometimes useful to rearrange it so that the denominator is a rational number. This is called rationalising the denominator.

- The rules to rationalise denominators are:
  - For fractions in the form \( \frac{1}{\sqrt{a}} \), multiply the numerator and denominator by \( \sqrt{a} \).
  - For fractions in the form \( \frac{1}{a + \sqrt{b}} \), multiply the numerator and denominator by \( (a - \sqrt{b}) \).
  - For fractions in the form \( \frac{1}{a - \sqrt{b}} \), multiply the numerator and denominator by \( (a + \sqrt{b}) \).
### Example 14

Rationalise the denominator:

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<tbody>
<tr>
<td>a</td>
<td>$\frac{1}{\sqrt{3}}$</td>
<td>b</td>
<td>$\frac{1}{3 + \sqrt{2}}$</td>
<td>c</td>
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<tr>
<td>a</td>
<td>$\frac{1}{\sqrt{3}} = \frac{1 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$</td>
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<td>b</td>
<td>$\frac{1}{3 + \sqrt{2}} = \frac{1 \times (3 - \sqrt{2})}{(3 + \sqrt{2})(3 - \sqrt{2})}$</td>
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<td>c</td>
<td>$\frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} - \sqrt{2}} = \frac{(\sqrt{5} + \sqrt{2})(\sqrt{5} + \sqrt{2})}{(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})}$</td>
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<td>d</td>
<td>$\frac{1}{(1 - \sqrt{3})^2} = \frac{1}{(1 - \sqrt{3})(1 - \sqrt{3})}$</td>
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- Multiply the numerator and denominator by $\sqrt{3}$.
- $\sqrt{3} \times \sqrt{3} = (\sqrt{3})^2 = 3$

- Multiply numerator and denominator by $(3 - \sqrt{2})$.
- $\sqrt{2} \times \sqrt{2} = 2$

- Multiply numerator and denominator by $(\sqrt{5} + \sqrt{2})$.
- $-\sqrt{2} \sqrt{5}$ and $\sqrt{5} \sqrt{2}$ cancel each other out.
- $\sqrt{5} \sqrt{2} = \sqrt{10}$

- Expand the brackets.
- Simplify and collect like terms. $\sqrt{9} = 3$

- Multiply the numerator and denominator by $(4 + 2\sqrt{3})$.
- $\sqrt{3} \times \sqrt{3} = 3$

- $16 - 12 = 4, 8\sqrt{3} - 8\sqrt{3} = 0$
Exercise 1F
SKILLS   ANALYSIS

Do not use your calculator for this exercise.

1 Simplify:
   a \( \frac{1}{\sqrt{5}} \)  \( \quad \) b \( \frac{1}{\sqrt{11}} \)  \( \quad \) c \( \frac{1}{\sqrt{2}} \)  \( \quad \) d \( \frac{\sqrt{3}}{15} \)
   e \( \frac{\sqrt{12}}{48} \)  \( \quad \) f \( \frac{\sqrt{5}}{80} \)  \( \quad \) g \( \frac{\sqrt{12}}{156} \)  \( \quad \) h \( \frac{\sqrt{7}}{63} \)

2 Rationalise the denominators and simplify:
   a \( \frac{1}{1 + \sqrt{3}} \)  \( \quad \) b \( \frac{1}{2 + \sqrt{5}} \)  \( \quad \) c \( \frac{1}{3 - \sqrt{7}} \)  \( \quad \) d \( \frac{4}{3 - \sqrt{5}} \)  \( \quad \) e \( \frac{1}{\sqrt{5} + \sqrt{3}} \)
   f \( \frac{3 - \sqrt{2}}{4 - \sqrt{5}} \)  \( \quad \) g \( \frac{5}{2 + \sqrt{5}} \)  \( \quad \) h \( \frac{5\sqrt{2}}{8 - \sqrt{7}} \)  \( \quad \) i \( \frac{11}{3 + \sqrt{11}} \)
   k \( \frac{\sqrt{17} - \sqrt{11}}{\sqrt{17} + \sqrt{11}} \)  \( \quad \) l \( \frac{\sqrt{41} + 29}{\sqrt{41} - 29} \)  \( \quad \) m \( \frac{\sqrt{2} - \sqrt{3}}{\sqrt{3} - \sqrt{2}} \)

3 Rationalise the denominators and simplify:
   a \( \frac{1}{(3 - \sqrt{2})^2} \)  \( \quad \) b \( \frac{1}{(2 + \sqrt{5})^2} \)  \( \quad \) c \( \frac{1}{(5 + \sqrt{2})(3 - \sqrt{2})} \)  \( \quad \) d \( \frac{3}{(5 + \sqrt{2})^2} \)

E/P 4 Simplify \( \frac{3 - 2\sqrt{5}}{\sqrt{5} - 1} \) giving your answer in the form \( p + q\sqrt{5} \), where \( p \) and \( q \) are rational numbers.
(4 marks)

Problem-solving
You can check that your answer is in the correct form by writing down the values of \( p \) and \( q \) and checking that they are rational numbers.

Chapter review 1
SKILLS   EXECUTIVE FUNCTION

1 Simplify:
   a \( y^3 \times y^5 \)  \( \quad \) b \( 3x^2 \times 2x^5 \)  \( \quad \) c \( (4x^2)^3 \div 2x^5 \)  \( \quad \) d \( 4b^2 \times 3b^3 \times b^4 \)

2 Expand and simplify if possible:
   a \( (x + 3)(x - 5) \)  \( \quad \) b \( (2x - 7)(3x + 1) \)  \( \quad \) c \( (2x + 5)(3x - y + 2) \)

3 Expand and simplify if possible:
   a \( x(x + 4)(x - 1) \)  \( \quad \) b \( (x + 2)(x - 3)(x + 7) \)  \( \quad \) c \( (2x + 3)(x - 2)(3x - 1) \)

4 Expand the brackets:
   a \( 3(5y + 4) \)  \( \quad \) b \( 5x^2(3 - 5x + 2x^2) \)  \( \quad \) c \( 5x(2x + 3) - 2x(1 - 3x) \)  \( \quad \) d \( 3x^2(1 + 3x) - 2x(3x - 2) \)
5 Factorise these expressions completely:
   a. $3x^2 + 4x$
   b. $4y^2 + 10y$
   c. $x^2 + xy + xy^2$
   d. $8xy^2 + 10x^2y$

6 Factorise:
   a. $x^2 + 3x + 2$
   b. $3x^2 + 6x$
   c. $x^2 - 2x - 35$
   d. $2x^2 - x - 3$
   e. $5x^2 - 13x - 6$
   f. $6 - 5x - x^2$

7 Factorise:
   a. $2x^3 + 6x$
   b. $x^3 - 36x$
   c. $2x^3 + 7x^2 - 15x$

8 Simplify:
   a. $9x^3 + 3x^{-3}$
   b. $(4\frac{1}{3})$
   c. $3x^{-2} \times 2x^4$
   d. $3x^{\frac{4}{3}} + 6x^{\frac{1}{3}}$

9 Evaluate, without using your calculator:
   a. $\left(\frac{8}{27}\right)^{\frac{3}{2}}$
   b. $\left(\frac{225}{289}\right)^{\frac{3}{2}}$

10 Simplify, without using your calculator:
   a. $\frac{3}{\sqrt{63}}$
   b. $\sqrt{20} + 2\sqrt{45} - \sqrt{80}$

11 a. Find the value of $35x^2 + 2x - 48$ when $x = 25$.  
   b. By factorising the expression, show that your answer to part a can be written as the product of two prime factors.

12 Expand and simplify if possible, without using your calculator:
   a. $\sqrt{2}(3 + \sqrt{5})$
   b. $(2 - \sqrt{5})(5 + \sqrt{3})$
   c. $(6 - \sqrt{2})(4 - \sqrt{7})$

13 Rationalise the denominator and simplify:
   a. $\frac{1}{\sqrt{3}}$
   b. $\frac{1}{\sqrt{2} - 1}$
   c. $\frac{3}{\sqrt{3} - 2}$
   d. $\frac{\sqrt{23} - \sqrt{37}}{\sqrt{23} + \sqrt{37}}$
   e. $\frac{1}{(2 + \sqrt{3})^2}$
   f. $\frac{1}{(4 - \sqrt{7})^2}$

14 Do not use your calculator for this question.
   a. Given that $x^3 - x^2 - 17x - 15 = (x + 3)(x^2 + bx + c)$, where $b$ and $c$ are constants, work out the values of $b$ and $c$.
   b. Hence, fully factorise $x^3 - x^2 - 17x - 15$.

15 Given that $y = \frac{1}{64}x^3$, express each of the following in the form $kx^n$, where $k$ and $n$ are constants.
   a. $y^\frac{1}{3}$
   b. $4y^{-1}$

16 Show that $\frac{5}{\sqrt{75} - \sqrt{50}}$ can be written in the form $\sqrt{a} + \sqrt{b}$, where $a$ and $b$ are integers. (5 marks)

17 Expand and simplify $(\sqrt{11} - 5)(5 - \sqrt{11})$, without using your calculator. (2 marks)

18 Factorise completely $x - 64x^3$. (3 marks)

19 Express $27^{x + 1}$ in the form $3^y$, stating $y$ in terms of $x$. (2 marks)
20 Solve the equation \(8 + x\sqrt{12} = \frac{8x}{\sqrt{3}}\).

Give your answer in the form \(a\sqrt{b}\), where \(a\) and \(b\) are integers. (4 marks)

21 Do not use your calculator for this question.

A rectangle has a length of \((1 + \sqrt{3})\) cm and area of \(\sqrt{12}\) cm\(^2\).

Calculate the width of the rectangle in cm.

Express your answer in the form \(a + b\sqrt{3}\), where \(a\) and \(b\) are integers to be found. (2 marks)

22 Show that \(\frac{(2 - \sqrt{x})^2}{\sqrt{x}}\) can be written as \(4x^{\frac{1}{2}} - 4 + x^{\frac{1}{2}}\). (2 marks)

23 Given that \(243 = 3^a\), find the value of \(a\). (3 marks)

24 Given that \(\frac{4x^3 + x^{\frac{1}{2}}}{\sqrt{x}}\) can be written in the form \(4x^a + x^b\), write down the value of \(a\) and the value of \(b\). (2 marks)

Challenge

a Simplify \((\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})\).

b Hence show that \(\frac{1}{\sqrt{1 + \sqrt{2}}} + \frac{1}{\sqrt{2 + \sqrt{3}}} + \frac{1}{\sqrt{3 + \sqrt{4}}} + \ldots + \frac{1}{\sqrt{24 + \sqrt{25}}} = 4\)

Summary of key points

1 You can use the laws of indices to simplify powers of the same base.
   - \(a^m \times a^n = a^{m+n}\)
   - \(a^m \div a^n = a^{m-n}\)
   - \((a^n)^m = a^{mn}\)
   - \((ab)^n = a^n b^n\)

2 Factorising is the opposite of expanding brackets.

3 A quadratic expression has the form \(ax^2 + bx + c\) where \(a\), \(b\) and \(c\) are real numbers and \(a \neq 0\).

4 \(x^2 - y^2 = (x + y)(x - y)\)

5 You can use the laws of indices with any rational power.
   - \(a^{\frac{1}{m}} = \sqrt[m]{a}\)
   - \(a^{\frac{n}{m}} = \sqrt[m]{a^n}\)
   - \(a^{-m} = \frac{1}{a^m}\)
   - \(a^0 = 1\)

6 You can manipulate surds using these rules:
   - \(\sqrt{ab} = \sqrt{a} \times \sqrt{b}\)
   - \(\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}\)

7 The rules to rationalise denominators are:
   - For fractions in the form \(\frac{1}{\sqrt{a}}\), multiply the numerator and denominator by \(\sqrt{a}\).
   - For fractions in the form \(\frac{1}{a + \sqrt{b}}\), multiply the numerator and denominator by \((a - \sqrt{b})\).
   - For fractions in the form \(\frac{1}{a - \sqrt{b}}\), multiply the numerator and denominator by \((a + \sqrt{b})\).