PEARSON EDEXCEL INTERNATIONAL A LEVEL

PURE MATHEMATICS 1

Student Book

Series Editors: Joe Skrakowski and Harry Smith

Authors: Greg Attwood, Jack Barraclough, Ian Bettison, Gordon Davies, Keith Gallick, Daniel Goldberg, Alistair Macpherson, Anne McAteer, Bronwen Moran, Su Nicholson, Diane Oliver, Joe Petran, Keith Pledger, Cong San, Joe Skrakowski, Harry Smith, Geoff Staley, Robert Ward-Penny, Dave Wilkins

Published by Pearson Education Limited, 80 Strand, London, WC2R 0RL.

www.pearsonglobalschools.com

Copies of official specifications for all Pearson qualifications may be found on the website: https://qualifications.pearson.com

Text © Pearson Education Limited 2018
Edited by Eric Pradel
Designed by © Pearson Education Limited 2018
Typeset by © Tech-Set Ltd, Gateshead, UK
Original illustrations © Pearson Education Limited 2018
Illustrated by © Tech-Set Ltd, Gateshead, UK
Cover design by © Pearson Education Limited 2018

The rights of Greg Attwood, Jack Barraclough, Ian Bettison, Gordon Davies, Keith Gallick, Daniel Goldberg, Alistair Macpherson, Anne McAteer, Bronwen Moran, Su Nicholson, Diane Oliver, Joe Petran, Keith Pledger, Cong San, Joe Skrakowski, Harry Smith, Geoff Staley, Robert Ward-Penny and Dave Wilkins to be identified as the authors of this work have been asserted by them in accordance with the Copyright, Designs and Patents Act 1988.

First published 2018

21 20 19 18
10 9 8 7 6 5 4 3 2 1

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

ISBN 978 1 292244 79 2

Printed by Neografia in Slovakia

Picture Credits
The publisher would like to thank the following for their kind permission to reproduce their photographs:

Alamy Stock Photo: Xinhua 36; **Getty Images:** Graiki 150, Henglein and Steets 18, mviamon 1; **NASA:** 170; **Science Photo Library Ltd:** CMS EXPERIMENT, CERN 57; **Shutterstock.com:** Dariush M. 85, DeReGe 133, fufu liu 104

Cover images: Front: **Getty Images:** Werner Van Steen
Inside front cover: **Shutterstock.com:** Dmitry Lobanov

All other images © Pearson Education
All artwork © Pearson Education

Endorsement Statement
In order to ensure that this resource offers high-quality support for the associated Pearson qualification, it has been through a review process by the awarding body. This process confirms that this resource fully covers the teaching and learning content of the specification or part of a specification at which it is aimed. It also confirms that it demonstrates an appropriate balance between the development of subject skills, knowledge and understanding, in addition to preparation for assessment.

Endorsement does not cover any guidance on assessment activities or processes (e.g. practice questions or advice on how to answer assessment questions) included in the resource, nor does it prescribe any particular approach to the teaching or delivery of a related course.

While the publishers have made every attempt to ensure that advice on the qualification and its assessment is accurate, the official specification and associated assessment guidance materials are the only authoritative source of information and should always be referred to for definitive guidance.

Pearson examiners have not contributed to any sections in this resource relevant to examination papers for which they have responsibility.

Examiners will not use endorsed resources as a source of material for any assessment set by Pearson. Endorsement of a resource does not mean that the resource is required to achieve this Pearson qualification, nor does it mean that it is the only suitable material available to support the qualification, and any resource lists produced by the awarding body shall include this and other appropriate resources.

ABOUT THIS BOOK

The following three themes have been fully integrated throughout the Pearson Edexcel International Advanced Level in Mathematics series, so they can be applied alongside your learning.

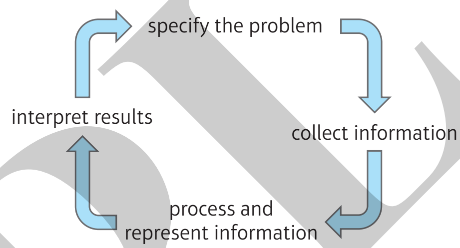
1. Mathematical argument, language and proof

- Rigorous and consistent approach throughout
- Notation boxes explain key mathematical language and symbols

2. Mathematical problem-solving

- Hundreds of problem-solving questions, fully integrated into the main exercises
- Problem-solving boxes provide tips and strategies
- Challenge questions provide extra stretch

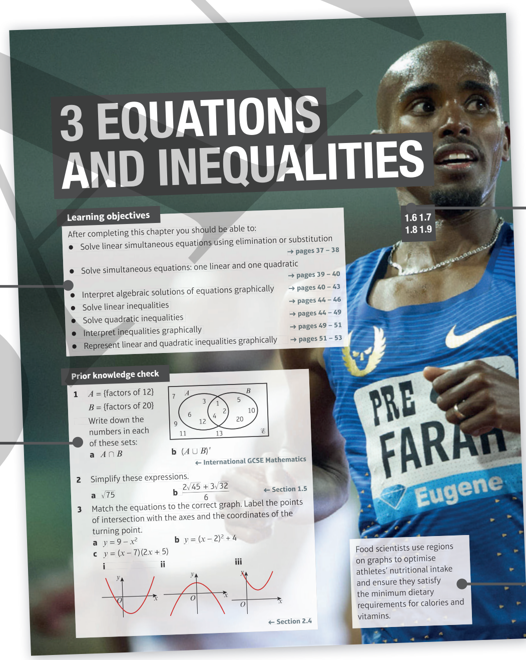
The Mathematical Problem-Solving Cycle



3. Transferable skills

- Transferable skills are embedded throughout this book, in the exercises and in some examples
- These skills are signposted to show students which skills they are using and developing

Finding your way around the book



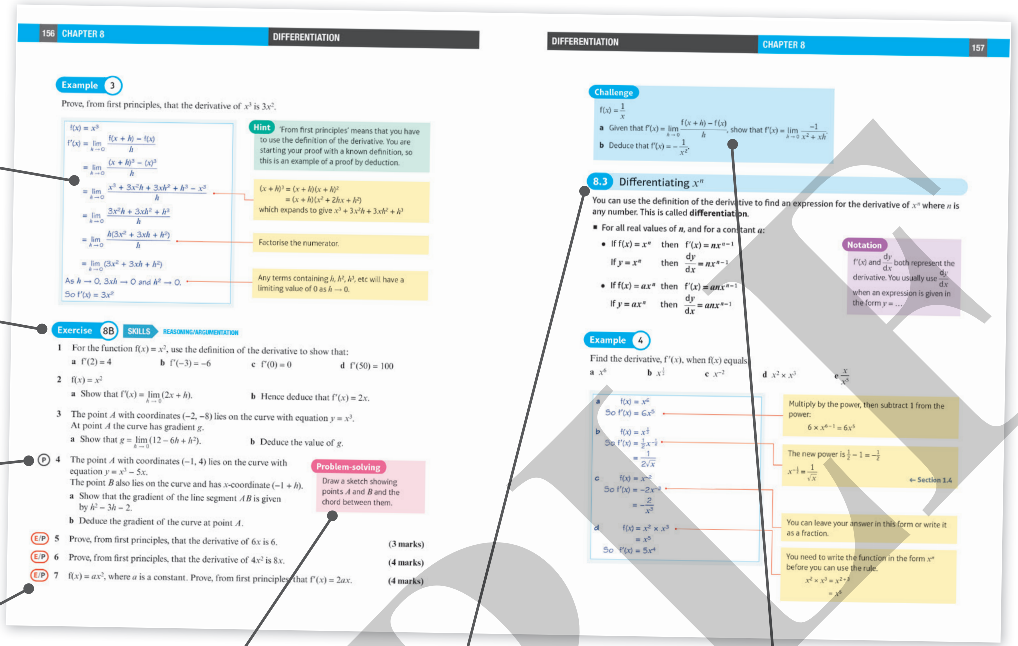
Each chapter starts with a list of *Learning objectives*

The *Prior knowledge check* helps make sure you are ready to start the chapter

Glossary terms will be identified by bold blue text, on their first appearance

Each chapter is mapped to the specification content for easy reference

The real world applications of the maths you are about to learn are highlighted at the start of the chapter



Step-by-step worked examples focus on the key types of questions you'll need to tackle

Transferable skills are signposted where they naturally occur in the exercises and examples

Exercise questions are carefully graded so they increase in difficulty and gradually bring you up to exam standard

Exercises are packed with exam-style questions to ensure you are ready for the exams

Problem-solving boxes provide hints, tips and strategies, and *Watch out* boxes highlight areas where students often lose marks in their exams

Each section begins with an explanation and key learning points

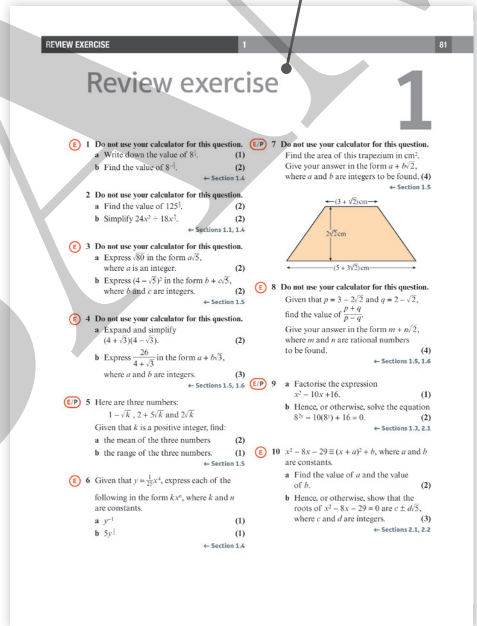
Challenge boxes give you a chance to tackle some more difficult questions

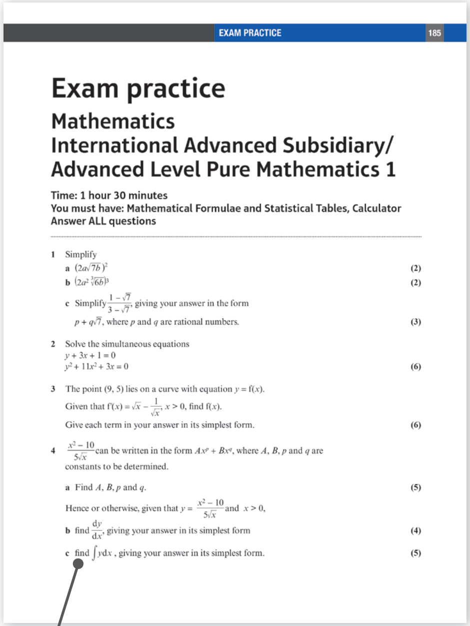
Exam-style questions are flagged with Ⓔ

Problem-solving questions are flagged with Ⓟ

Each chapter ends with a *Chapter review* and a *Summary of key points*

After every few chapters, a *Review exercise* helps you consolidate your learning with lots of exam-style questions





A full practice paper at the back of the book helps you prepare for the real thing

QUALIFICATION AND ASSESSMENT OVERVIEW

Qualification and content overview

Pure Mathematics 1 (P1) is a **compulsory** unit in the following qualifications:

International Advanced Subsidiary in Mathematics

International Advanced Subsidiary in Pure Mathematics

International Advanced Level in Mathematics

International Advanced Level in Pure Mathematics

Assessment overview

The following table gives an overview of the assessment for this unit.

We recommend that you study this information closely to help ensure that you are fully prepared for this course and know exactly what to expect in the assessment.

Unit	Percentage	Mark	Time	Availability
P1: Pure Mathematics 1 Paper code WMA11/01	$33\frac{1}{3}$% of IAS $16\frac{2}{3}$% of IAL	75	1 hour 30 mins	January, June and October First assessment January 2019

IAS – International Advanced Subsidiary, IAL – International Advanced A Level

Assessment objectives and weightings

		Minimum weighting in IAS and IAL
AO1	Recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of contexts.	30%
AO2	Construct rigorous mathematical arguments and proofs through use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions, including the construction of extended arguments for handling substantial problems presented in unstructured form.	30%
AO3	Recall, select and use their knowledge of standard mathematical models to represent situations in the real world; recognise and understand given representations involving standard models; present and interpret results from such models in terms of the original situation, including discussion of the assumptions made and refinement of such models.	10%
AO4	Comprehend translations of common realistic contexts into mathematics; use the results of calculations to make predictions, or comment on the context; and, where appropriate, read critically and comprehend longer mathematical arguments or examples of applications.	5%
AO5	Use contemporary calculator technology and other permitted resources (such as formulae booklets or statistical tables) accurately and efficiently; understand when not to use such technology, and its limitations. Give answers to appropriate accuracy.	5%

Relationship of assessment objectives to units

P1	Assessment objective				
	AO1	AO2	AO3	AO4	AO5
Marks out of 75	30–35	25–30	5–15	5–10	1–5
%	$40\text{-}46\frac{2}{3}$	$33\frac{1}{3}$–40	$6\frac{2}{3}$–20	$6\frac{2}{3}$–$13\frac{1}{3}$	$1\frac{1}{3}$–$6\frac{2}{3}$

Calculators

Students may use a calculator in assessments for these qualifications. Centres are responsible for making sure that calculators used by their students meet the requirements outlined below.

Students are expected to have available a calculator with at least the following keys: +, –, ×, ÷, π, x^2, $\sqrt{x}$, $\frac{1}{x}$, x^y, ln x, e^x, $x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory.

Prohibitions

Calculators with any of the following facilities are prohibited in all examinations:

- databanks
- retrieval of text or formulae
- built-in symbolic algebra manipulations
- symbolic differentiation and/or integration
- language translators
- communication with other machines or the internet

Extra online content

Whenever you see an *Online* box, it means that there is extra online content available to support you.

SolutionBank

SolutionBank provides worked solutions for questions in the book. Download all the solutions as a PDF or quickly find the solution you need online.

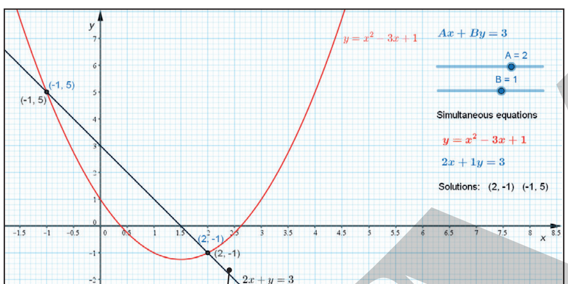
Use of technology

Explore topics in more detail, visualise problems and consolidate your understanding. Use pre-made GeoGebra activities or Casio resources for a graphic calculator.

Online Find the point of intersection graphically using technology.

GeoGebra

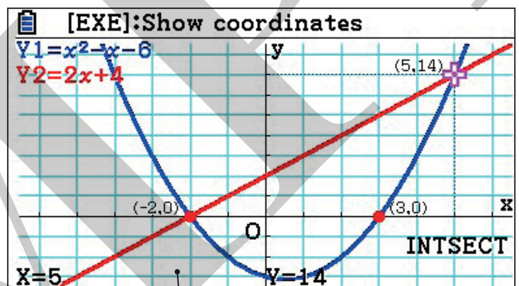
GeoGebra-powered interactives



Interact with the maths you are learning using GeoGebra's easy-to-use tools

CASIO®

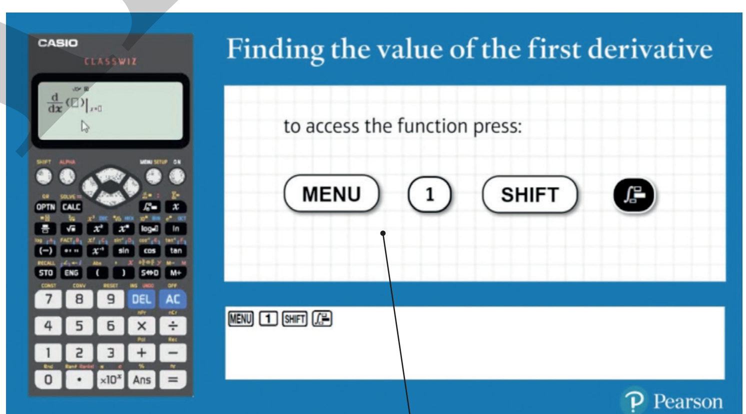
Graphic calculator interactives



Explore the maths you are learning and gain confidence in using a graphic calculator

Calculator tutorials

Our helpful video tutorials will guide you through how to use your calculator in the exams. They cover both Casio's scientific and colour graphic calculators.



Step-by-step guide with audio instructions on exactly which buttons to press and what should appear on your calculator's screen

Online Work out each coefficient quickly using the ${}^nC_r$ and power functions on your calculator.

1 ALGEBRAIC EXPRESSIONS

1.1
1.2
1.10

Learning objectives

After completing this chapter you should be able to:

- Multiply and divide integer powers → pages 2–4
- Expand a single term over brackets and collect like terms → pages 2–4
- Expand the product of two or three expressions → pages 4–6
- Factorise linear, quadratic and simple cubic expressions → pages 6–9
- Know and use the laws of indices → pages 9–11
- Simplify and use the rules of surds → pages 12–13
- Rationalise denominators → pages 13–15

Prior knowledge check

1 Simplify:

a $4m^2n + 5mn^2 - 2m^2n + mn^2 - 3mn^2$

b $3x^2 - 5x + 2 + 3x^2 - 7x - 12$

← International GCSE Mathematics

2 Write as a single power of 2:

a $2^5 \times 2^3$ **b** $2^6 \div 2^2$

c $(2^3)^2$ ← International GCSE Mathematics

3 Expand:

a $3(x + 4)$ **b** $5(2 - 3x)$

c $6(2x - 5y)$ ← International GCSE Mathematics

4 Write down the highest common factor of:

a 24 and 16 **b** $6x$ and $8x^2$

c $4xy^2$ and $3xy$ ← International GCSE Mathematics

5 Simplify:

a $\frac{10x}{5}$ **b** $\frac{20x}{2}$ **c** $\frac{40x}{24}$

← International GCSE Mathematics

Computer scientists use indices to describe very large numbers. A quantum computer with 1000 qubits (quantum bits) can consider 2^{1000} values simultaneously. This is greater than the number of particles in the observable universe.

1.1 Index laws

- You can use the laws of **indices** to **simplify** powers of the same **base**.
  - $a^m \times a^n = a^{m+n}$
  - $a^m \div a^n = a^{m-n}$
  - $(a^m)^n = a^{mn}$

$(ab)^n = a^n b^n$

Notation

x^5 — This is the **index**, **power** or **exponent**. This is the base.

Example 1

Simplify these **expressions**:

a $x^2 \times x^5$ **b** $2r^2 \times 3r^3$ **c** $\frac{b^7}{b^4}$ **d** $6x^5 \div 3x^3$ **e** $(a^3)^2 \times 2a^2$ **f** $(3x^2)^3 \div x^4$

a $x^2 \times x^5 = x^{2+5} = x^7$

Use the rule $a^m \times a^n = a^{m+n}$ to simplify the index.

b $2r^2 \times 3r^3 = 2 \times 3 \times r^2 \times r^3$
$= 6 \times r^{2+3} = 6r^5$

Rewrite the expression with the numbers together and the r **terms** together.
$2 \times 3 = 6$
$r^2 \times r^3 = r^{2+3}$

c $\frac{b^7}{b^4} = b^{7-4} = b^3$

Use the rule $a^m \div a^n = a^{m-n}$ to simplify the index.

d $6x^5 \div 3x^3 = \frac{6}{3} \times \frac{x^5}{x^3}$
$= 2 \times x^2 = 2x^2$

$x^5 \div x^3 = x^{5-3} = x^2$

e $(a^3)^2 \times 2a^2 = a^6 \times 2a^2$
$= 2 \times a^6 \times a^2 = 2a^8$

Use the rule $(a^m)^n = a^{mn}$ to simplify the index.

$a^6 \times a^2 = a^{6+2} = a^8$

f $\frac{(3x^2)^3}{x^4} = 3^3 \times \frac{(x^2)^3}{x^4}$
$= 27 \times \frac{x^6}{x^4} = 27x^2$

Use the rule $(ab)^n = a^n b^n$ to simplify the **numerator**.
$(x^2)^3 = x^{2 \times 3} = x^6$

$\frac{x^6}{x^4} = x^{6-4} = x^2$

Example 2

Expand these expressions and simplify if possible:

a $-3x(7x - 4)$ **b** $y^2(3 - 2y^3)$
c $4x(3x - 2x^2 + 5x^3)$ **d** $2x(5x + 3) - 5(2x + 3)$

Watch out A minus sign outside brackets changes the sign of every term inside the brackets.

a $-3x(7x - 4) = -21x^2 + 12x$

$-3x \times 7x = -21x^{1+1} = -21x^2$
$-3x \times (-4) = +12x$

b $y^2(3 - 2y^3) = 3y^2 - 2y^5$

$y^2 \times (-2y^3) = -2y^{2+3} = -2y^5$

c $4x(3x - 2x^2 + 5x^3)$
$= 12x^2 - 8x^3 + 20x^4$

d $2x(5x + 3) - 5(2x + 3)$
$= 10x^2 + 6x - 10x - 15$
$= 10x^2 - 4x - 15$

Remember: a minus sign outside the brackets changes the signs within the brackets.

Simplify $6x - 10x$ to give $-4x$.

Example 3

Simplify these expressions:

a $\dfrac{x^7 + x^4}{x^3}$ b $\dfrac{3x^2 - 6x^5}{2x}$ c $\dfrac{20x^7 + 15x^3}{5x^2}$

a $\dfrac{x^7 + x^4}{x^3} = \dfrac{x^7}{x^3} + \dfrac{x^4}{x^3}$

Divide each term of the numerator by x^3.

$= x^{7-3} + x^{4-3} = x^4 + x$

x^1 is the same as x.

b $\dfrac{3x^2 - 6x^5}{2x} = \dfrac{3x^2}{2x} - \dfrac{6x^5}{2x}$

Divide each term of the numerator by $2x$.

$= \dfrac{3}{2}x^{2-1} - 3x^{5-1} = \dfrac{3x}{2} - 3x^4$

Simplify each fraction:
$\dfrac{3x^2}{2x} = \dfrac{3}{2} \times \dfrac{x^2}{x} = \dfrac{3}{2} \times x^{2-1}$
$-\dfrac{6x^5}{2x} = -\dfrac{6}{2} \times \dfrac{x^5}{x} = -3 \times x^{5-1}$

c $\dfrac{20x^7 + 15x^3}{5x^2} = \dfrac{20x^7}{5x^2} + \dfrac{15x^3}{5x^2}$

Divide each term of the numerator by $5x^2$.

$= 4x^{7-2} + 3x^{3-2} = 4x^5 + 3x$

Exercise 1A

SKILLS INTERPRETATION

1 Simplify these expressions:

a $x^3 \times x^4$ b $2x^3 \times 3x^2$ c $\dfrac{k^3}{k^2}$

d $\dfrac{4p^3}{2p}$ e $\dfrac{3x^3}{3x^2}$ f $(y^2)^5$

g $10x^5 \div 2x^3$ h $(p^3)^2 \div p^4$ i $(2a^3)^2 \div 2a^3$

j $8p^4 \div 4p^3$ k $2a^4 \times 3a^5$ l $\dfrac{21a^3b^7}{7ab^4}$

m $9x^2 \times 3(x^2)^3$ n $3x^3 \times 2x^2 \times 4x^6$ o $7a^4 \times (3a^4)^2$

p $(4y^3)^3 \div 2y^3$ q $2a^3 \div 3a^2 \times 6a^5$ r $3a^4 \times 2a^5 \times a^3$

2 Expand and simplify if possible:

a $9(x-2)$ **b** $x(x+9)$ **c** $-3y(4-3y)$

d $x(y+5)$ **e** $-x(3x+5)$ **f** $-5x(4x+1)$

g $(4x+5)x$ **h** $-3y(5-2y^2)$ **i** $-2x(5x-4)$

j $(3x-5)x^2$ **k** $3(x+2)+(x-7)$ **l** $5x-6-(3x-2)$

m $4(c+3d^2)-3(2c+d^2)$ **n** $(r^2+3t^2+9)-(2r^2+3t^2-4)$

o $x(3x^2-2x+5)$ **p** $7y^2(2-5y+3y^2)$ **q** $-2y^2(5-7y+3y^2)$

r $7(x-2)+3(x+4)-6(x-2)$ **s** $5x-3(4-2x)+6$

t $3x^2-x(3-4x)+7$ **u** $4x(x+3)-2x(3x-7)$ **v** $3x^2(2x+1)-5x^2(3x-4)$

3 Simplify these fractions:

a $\dfrac{6x^4+10x^6}{2x}$ **b** $\dfrac{3x^5-x^7}{x}$ **c** $\dfrac{2x^4-4x^2}{4x}$

d $\dfrac{8x^3+5x}{2x}$ **e** $\dfrac{7x^7+5x^2}{5x}$ **f** $\dfrac{9x^5-5x^3}{3x}$

1.2 Expanding brackets

To find the product of two expressions you multiply each term in one expression by each term in the other expression.

Multiplying each of the 2 terms in the first expression by each of the 3 terms in the second expression gives 2 × 3 = 6 terms.

(x ×, 5 ×)

$$(x+5)(4x-2y+3) = x(4x-2y+3)+5(4x-2y+3)$$
$$= 4x^2-2xy+3x+20x-10y+15$$
$$= 4x^2-2xy+23x-10y+15$$

Simplify your answer by **collecting like terms**.

Example 4 SKILLS INTERPRETATION

Expand these expressions and simplify if possible:

a $(x+5)(x+2)$ **b** $(x-2y)(x^2+1)$ **c** $(x-y)^2$ **d** $(x+y)(3x-2y-4)$

a $(x+5)(x+2)$
$= x^2+2x+5x+10$
$= x^2+7x+10$

Multiply x by $(x+2)$ and then multiply 5 by $(x+2)$.

Simplify your answer by collecting like terms.

b $(x-2y)(x^2+1)$
$= x^3+x-2x^2y-2y$

$-2y \times x^2 = -2x^2y$

There are no like terms to collect.

c $(x-y)^2$
$= (x-y)(x-y)$
$= x^2 - xy - xy + y^2$
$= x^2 - 2xy + y^2$

$(x-y)^2$ means $(x-y)$ multiplied by itself.

$-xy - xy = -2xy$

d $(x+y)(3x-2y-4)$
$= x(3x-2y-4) + y(3x-2y-4)$
$= 3x^2 - 2xy - 4x + 3xy - 2y^2 - 4y$
$= 3x^2 + xy - 4x - 2y^2 - 4y$

Multiply x by $(3x-2y-4)$ and then multiply y by $(3x-2y-4)$.

Example 5

Expand these expressions and simplify if possible:

a $x(2x+3)(x-7)$ **b** $x(5x-3y)(2x-y+4)$ **c** $(x-4)(x+3)(x+1)$

a $x(2x+3)(x-7)$
$= (2x^2+3x)(x-7)$
$= 2x^3 - 14x^2 + 3x^2 - 21x$
$= 2x^3 - 11x^2 - 21x$

Start by expanding one pair of brackets: $x(2x+3) = 2x^2 + 3x$

You could also have expanded the second pair of brackets first: $(2x+3)(x-7) = 2x^2 - 11x - 21$ Then multiply by x.

b $x(5x-3y)(2x-y+4)$
$= (5x^2-3xy)(2x-y+4)$
$= 5x^2(2x-y+4) - 3xy(2x-y+4)$
$= 10x^3 - 5x^2y + 20x^2 - 6x^2y + 3xy^2 - 12xy$
$= 10x^3 - 11x^2y + 20x^2 + 3xy^2 - 12xy$

Be careful with minus signs. You need to change every sign in the second pair of brackets when you multiply it out.

c $(x-4)(x+3)(x+1)$
$= (x^2-x-12)(x+1)$
$= x^2(x+1) - x(x+1) - 12(x+1)$
$= x^3 + x^2 - x^2 - x - 12x - 12$
$= x^3 - 13x - 12$

Choose one pair of brackets to expand first, for example:
$(x-4)(x+3) = x^2 + 3x - 4x - 12$
$= x^2 - x - 12$

You multiplied together three **linear** terms, so the final answer contains an x^3 term.

Exercise 1B SKILLS INTERPRETATION

1 Expand and simplify if possible:

a $(x+4)(x+7)$ **b** $(x-3)(x+2)$ **c** $(x-2)^2$
d $(x-y)(2x+3)$ **e** $(x+3y)(4x-y)$ **f** $(2x-4y)(3x+y)$
g $(2x-3)(x-4)$ **h** $(3x+2y)^2$ **i** $(2x+8y)(2x+3)$
j $(x+5)(2x+3y-5)$ **k** $(x-1)(3x-4y-5)$ **l** $(x-4y)(2x+y+5)$
m $(x+2y-1)(x+3)$ **n** $(2x+2y+3)(x+6)$ **o** $(4-y)(4y-x+3)$
p $(4y+5)(3x-y+2)$ **q** $(5y-2x+3)(x-4)$ **r** $(4y-x-2)(5-y)$

2 Expand and simplify if possible:

a $5(x+1)(x-4)$ **b** $7(x-2)(2x+5)$ **c** $3(x-3)(x-3)$

d $x(x-y)(x+y)$ **e** $x(2x+y)(3x+4)$ **f** $y(x-5)(x+1)$

g $y(3x-2y)(4x+2)$ **h** $y(7-x)(2x-5)$ **i** $x(2x+y)(5x-2)$

j $x(x+2)(x+3y-4)$ **k** $y(2x+y-1)(x+5)$ **l** $y(3x+2y-3)(2x+1)$

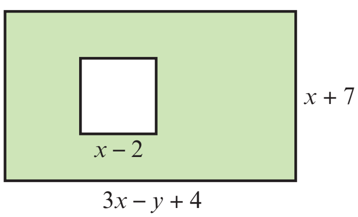
m $x(2x+3)(x+y-5)$ **n** $2x(3x-1)(4x-y-3)$ **o** $3x(x-2y)(2x+3y+5)$

p $(x+3)(x+2)(x+1)$ **q** $(x+2)(x-4)(x+3)$ **r** $(x+3)(x-1)(x-5)$

s $(x-5)(x-4)(x-3)$ **t** $(2x+1)(x-2)(x+1)$ **u** $(2x+3)(3x-1)(x+2)$

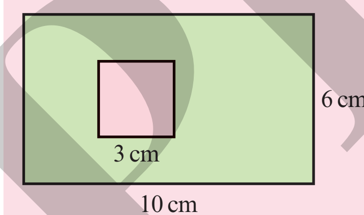
v $(3x-2)(2x+1)(3x-2)$ **w** $(x+y)(x-y)(x-1)$ **x** $(2x-3y)^3$

P **3** The diagram shows a rectangle with a square cut out. The rectangle has length $3x-y+4$ and width $x+7$. The square has side length $x-2$. Find an expanded and simplified expression for the area **shaded** green.



Problem-solving

Use the same strategy as you would use if the lengths were given as numbers:



P **4** A **cuboid** has **dimensions** $(x+2)$ cm, $(2x-1)$ cm and $(2x+3)$ cm. Show that the volume of the cuboid is $(4x^3+12x^2+5x-6)$ cm^3.

E/P **5** **Given that** $(2x+5y)(3x-y)(2x+y) = ax^3 + bx^2y + cxy^2 + dy^3$, where a, b, c and d are **constants**, find the values of a, b, c and d. **(2 marks)**

Challenge

Expand and simplify $(x+y)^4$.

1.3 Factorising

You can write expressions as a **product** of their **factors**.

- Factorising is the opposite of expanding brackets.

Expanding brackets →

$4x(2x+y) = 8x^2 + 4xy$

$(x+5)^3 = x^3 + 15x^2 + 75x + 125$

$(x+2y)(x-5y) = x^2 - 3xy - 10y^2$

← Factorising

Example 6 SKILLS ANALYSIS

Factorise these expressions completely:

a $3x + 9$ **b** $x^2 - 5x$ **c** $8x^2 + 20x$ **d** $9x^2y + 15xy^2$ **e** $3x^2 - 9xy$

a $3x + 9 = 3(x + 3)$ — 3 is a **common factor** of $3x$ and 9.

b $x^2 - 5x = x(x - 5)$ — x is a common factor of x^2 and $-5x$.

c $8x^2 + 20x = 4x(2x + 5)$ — 4 and x are common factors of $8x^2$ and $20x$, so take $4x$ outside the brackets.

d $9x^2y + 15xy^2 = 3xy(3x + 5y)$ — 3, x and y are common factors of $9x^2y$ and $15xy^2$, so take $3xy$ outside the brackets.

e $3x^2 - 9xy = 3x(x - 3y)$ — x and $-3y$ have no common factors so this expression is completely factorised.

- A **quadratic** expression has the form $ax^2 + bx + c$ where a, b and c are real numbers and $a \neq 0$.

Notation **Real** numbers are all the positive and negative numbers, or zero, including fractions and **surds**.

To factorise a quadratic expression:

- Find two factors of ac that add up to b — For the expression $2x^2 + 5x - 3$, $ac = -6 = -1 \times 6$ and $-1 + 6 = 5 = b$.
- Rewrite the b term as a sum of these two factors — $2x^2 - x + 6x - 3$
- Factorise each pair of terms — $= x(2x - 1) + 3(2x - 1)$
- Take out the common factor — $= (2x - 1)(x + 3)$

- $x^2 - y^2 = (x + y)(x - y)$

Notation An expression in the form $x^2 - y^2$ is called the **difference** of two squares.

Example 7

Factorise:

a $x^2 - 5x - 6$ **b** $x^2 + 6x + 8$ **c** $6x^2 - 11x - 10$ **d** $x^2 - 25$ **e** $4x^2 - 9y^2$

a $x^2 - 5x - 6$ — Here $a = 1$, $b = -5$ and $c = -6$.

$ac = -6$ and $b = -5$

So $x^2 - 5x - 6 = x^2 + x - 6x - 6$ — (1) Find the two factors of $ac = -6$ which add to give $b = -5$. $-6 + 1 = -5$ (2) Rewrite the b term using these two factors.

$= x(x + 1) - 6(x + 1)$ — (3) Factorise the first two terms and the last two terms.

$= (x + 1)(x - 6)$ — (4) $x + 1$ is a factor of both terms, so take that outside the brackets. This is now completely factorised.

b $x^2 + 6x + 8$
$= x^2 + 2x + 4x + 8$ — $ac = 8$ and $2 + 4 = 6 = b$.
$= x(x + 2) + 4(x + 2)$ — Factorise.
$= (x + 2)(x + 4)$

c $6x^2 - 11x - 10$
$= 6x^2 - 15x + 4x - 10$ — $ac = -60$ and $4 - 15 = -11 = b$.
$= 3x(2x - 5) + 2(2x - 5)$ — Factorise.
$= (2x - 5)(3x + 2)$

d $x^2 - 25$ — This is the difference of two squares as the two terms are x^2 and 5^2.
$= x^2 - 5^2$
$= (x + 5)(x - 5)$ — The two x terms, $5x$ and $-5x$, **cancel** each other out.

e $4x^2 - 9y^2$ — This is the same as $(2x)^2 - (3y)^2$.
$= 2^2x^2 - 3^2y^2$
$= (2x + 3y)(2x - 3y)$

Example 8

Factorise completely:

a $x^3 - 2x^2$ **b** $x^3 - 25x$ **c** $x^3 + 3x^2 - 10x$

a $x^3 - 2x^2 = x^2(x - 2)$ — You can't factorise this any further.

b $x^3 - 25x = x(x^2 - 25)$ — x is a common factor of x^3 and $-25x$, so take x outside the brackets. $x^2 - 25$ is the difference of two squares.
$= x(x^2 - 5^2)$
$= x(x + 5)(x - 5)$

c $x^3 + 3x^2 - 10x = x(x^2 + 3x - 10)$ — Write the expression as a product of x and a quadratic factor.
$= x(x + 5)(x - 2)$ — Factorise the quadratic to get three linear factors.

Exercise 1C SKILLS PROBLEM-SOLVING

1 Factorise these expressions completely:

a $4x + 8$	**b** $6x - 24$	**c** $20x + 15$
d $2x^2 + 4$	**e** $4x^2 + 20$	**f** $6x^2 - 18x$
g $x^2 - 7x$	**h** $2x^2 + 4x$	**i** $3x^2 - x$
j $6x^2 - 2x$	**k** $10y^2 - 5y$	**l** $35x^2 - 28x$
m $x^2 + 2x$	**n** $3y^2 + 2y$	**o** $4x^2 + 12x$
p $5y^2 - 20y$	**q** $9xy^2 + 12x^2y$	**r** $6ab - 2ab^2$
s $5x^2 - 25xy$	**t** $12x^2y + 8xy^2$	**u** $15y - 20yz^2$
v $12x^2 - 30$	**w** $xy^2 - x^2y$	**x** $12y^2 - 4yx$

2 Factorise:

a $x^2 + 4x$ **b** $2x^2 + 6x$ **c** $x^2 + 11x + 24$
d $x^2 + 8x + 12$ **e** $x^2 + 3x - 40$ **f** $x^2 - 8x + 12$
g $x^2 + 5x + 6$ **h** $x^2 - 2x - 24$ **i** $x^2 - 3x - 10$
j $x^2 + x - 20$ **k** $2x^2 + 5x + 2$ **l** $3x^2 + 10x - 8$
m $5x^2 - 16x + 3$ **n** $6x^2 - 8x - 8$
o $2x^2 + 7x - 15$ **p** $2x^4 + 14x^2 + 24$
q $x^2 - 4$ **r** $x^2 - 49$
s $4x^2 - 25$ **t** $9x^2 - 25y^2$ **u** $36x^2 - 4$
v $2x^2 - 50$ **w** $6x^2 - 10x + 4$ **x** $15x^2 + 42x - 9$

Hint For part **n**, take 2 out as a common factor first. For part **p**, let $y = x^2$.

3 Factorise completely:

a $x^3 + 2x$ **b** $x^3 - x^2 + x$ **c** $x^3 - 5x$
d $x^3 - 9x$ **e** $x^3 - x^2 - 12x$ **f** $x^3 + 11x^2 + 30x$
g $x^3 - 7x^2 + 6x$ **h** $x^3 - 64x$ **i** $2x^3 - 5x^2 - 3x$
j $2x^3 + 13x^2 + 15x$ **k** $x^3 - 4x$ **l** $3x^3 + 27x^2 + 60x$

E/P **4** Factorise completely $x^4 - y^4$. **(2 marks)**

Problem-solving
Watch out for terms that can be written as a function of a function, for example: $x^4 = (x^2)^2$.

E **5** Factorise completely $6x^3 + 7x^2 - 5x$. **(2 marks)**

Challenge
Write $4x^4 - 13x^2 + 9$ as the product of four linear factors.

1.4 Negative and fractional indices

Indices can be negative numbers or fractions.

$$x^{\frac{1}{2}} \times x^{\frac{1}{2}} = x^{\frac{1}{2}+\frac{1}{2}} = x^1 = x,$$

similarly $\underbrace{x^{\frac{1}{n}} \times x^{\frac{1}{n}} \times \ldots \times x^{\frac{1}{n}}}_{n \text{ terms}} = x^{\frac{1}{n}+\frac{1}{n}+\ldots+\frac{1}{n}} = x^1 = x$

■ You can use the laws of indices with any rational power.

- $a^{\frac{1}{m}} = \sqrt[m]{a}$
- $a^{\frac{n}{m}} = \sqrt[m]{a^n}$
- $a^{-m} = \dfrac{1}{a^m}$
- $a^0 = 1$

Notation **Rational** numbers are those that can be written as $\frac{a}{b}$ where a and b are **integers**, and $b \neq 0$.

Notation $a^{\frac{1}{2}} = \sqrt{a}$ is the positive **square root** of a. For example: $9^{\frac{1}{2}} = \sqrt{9} = 3$, but $9^{\frac{1}{2}} \neq -3$.

Example 9

Simplify:

a $\frac{x^3}{x^{-3}}$ **b** $x^{\frac{1}{2}} \times x^{\frac{3}{2}}$ **c** $(x^3)^{\frac{2}{3}}$ **d** $2x^{1.5} \div 4x^{-0.25}$ **e** $\sqrt[3]{125x^6}$ **f** $\frac{2x^2 - x}{x^5}$

a $\frac{x^3}{x^{-3}} = x^{3-(-3)} = x^6$

Use the rule $a^m \div a^n = a^{m-n}$.

b $x^{\frac{1}{2}} \times x^{\frac{3}{2}} = x^{\frac{1}{2}+\frac{3}{2}} = x^2$

This could also be written as $\sqrt{x}$.
Use the rule $a^m \times a^n = a^{m+n}$.

c $(x^3)^{\frac{2}{3}} = x^{3 \times \frac{2}{3}} = x^2$

Use the rule $(a^m)^n = a^{mn}$.

d $2x^{1.5} \div 4x^{-0.25} = \frac{1}{2}x^{1.5-(-0.25)} = \frac{1}{2}x^{1.75}$

Use the rule $a^m \div a^n = a^{m-n}$.
$1.5 - (-0.25) = 1.75$

e $\sqrt[3]{125x^6} = (125x^6)^{\frac{1}{3}}$

$= (125)^{\frac{1}{3}}(x^6)^{\frac{1}{3}} = \sqrt[3]{125}(x^{6 \times \frac{1}{3}}) = 5x^2$

Using $a^{\frac{1}{m}} = \sqrt[m]{a}$.

f $\frac{2x^2 - x}{x^5} = \frac{2x^2}{x^5} - \frac{x}{x^5}$

$= 2 \times x^{2-5} - x^{1-5} = 2x^{-3} - x^{-4}$

Divide each term of the numerator by x^5.

$= \frac{2}{x^3} - \frac{1}{x^4}$

Using $a^{-m} = \frac{1}{a^m}$

Example 10 SKILLS INTERPRETATION

Evaluate:

a $9^{\frac{1}{2}}$ **b** $64^{\frac{1}{3}}$ **c** $49^{\frac{3}{2}}$ **d** $25^{-\frac{3}{2}}$

a $9^{\frac{1}{2}} = \sqrt{9} = 3$

Using $a^{\frac{1}{m}} = \sqrt[m]{a}$. Thus, $9^{\frac{1}{2}} = \sqrt{9}$.

b $64^{\frac{1}{3}} = \sqrt[3]{64} = 4$

This means the cube root of 64.

c $49^{\frac{3}{2}} = (\sqrt{49})^3$

$= 7^3 = 343$

Using $a^{\frac{n}{m}} = \sqrt[m]{a^n}$.
This means the square root of 49, cubed.

d $25^{-\frac{3}{2}} = \frac{1}{25^{\frac{3}{2}}} = \frac{1}{(\sqrt{25})^3}$

$= \frac{1}{5^3} = \frac{1}{125}$

Using $a^{-m} = \frac{1}{a^m}$

Online Use your calculator to enter negative and **fractional** powers.

Example 11

Given that $y = \frac{1}{16}x^2$, **express** each of the following in the form kx^n, where k and n are constants.

a $y^{\frac{1}{2}}$ **b** $4y^{-1}$

a $y^{\frac{1}{2}} = \left(\frac{1}{16}x^2\right)^{\frac{1}{2}}$

$= \frac{1}{\sqrt{16}}x^{2 \times \frac{1}{2}} = \frac{x}{4}$

b $4y^{-1} = 4\left(\frac{1}{16}x^2\right)^{-1}$

$= 4\left(\frac{1}{16}\right)^{-1}x^{2 \times (-1)} = 4 \times 16x^{-2}$

$= 64x^{-2}$

Substitute $y = \frac{1}{16}x^2$ into $y^{\frac{1}{2}}$.

$\left(\frac{1}{16}\right)^{\frac{1}{2}} = \frac{1}{\sqrt{16}}$ and $(x^2)^{\frac{1}{2}} = x^{2 \times \frac{1}{2}}$

$\left(\frac{1}{16}\right)^{-1} = 16$ and $x^{2 \times (-1)} = x^{-2}$

Problem-solving

Check that your answers are in the correct form. If k and n are constants they could be positive or negative, and they could be integers, fractions or surds.

Exercise 1D SKILLS PROBLEM-SOLVING

1 Simplify:

a $x^3 \div x^{-2}$ **b** $x^5 \div x^7$ **c** $x^{\frac{3}{2}} \times x^{\frac{5}{2}}$

d $(x^2)^{\frac{3}{2}}$ **e** $(x^3)^{\frac{5}{3}}$ **f** $3x^{0.5} \times 4x^{-0.5}$

g $9x^{\frac{2}{3}} \div 3x^{\frac{1}{6}}$ **h** $5x^{\frac{7}{5}} \div x^{\frac{2}{5}}$ **i** $3x^4 \times 2x^{-5}$

j $\sqrt{x} \times \sqrt[3]{x}$ **k** $(\sqrt{x})^3 \times (\sqrt[3]{x})^4$ **l** $\frac{(\sqrt[3]{x})^2}{\sqrt{x}}$

2 Evaluate, without using your calculator:

a $25^{\frac{1}{2}}$ **b** $81^{\frac{3}{2}}$ **c** $27^{\frac{1}{3}}$

d 4^{-2} **e** $9^{-\frac{1}{2}}$ **f** $(-5)^{-3}$

g $\left(\frac{3}{4}\right)^0$ **h** $1296^{\frac{3}{4}}$ **i** $\left(\frac{25}{16}\right)^{\frac{3}{2}}$

j $\left(\frac{27}{8}\right)^{\frac{2}{3}}$ **k** $\left(\frac{6}{5}\right)^{-1}$ **l** $\left(\frac{343}{512}\right)^{-\frac{2}{3}}$

3 Simplify:

a $(64x^{10})^{\frac{1}{2}}$ **b** $\frac{5x^3 - 2x^2}{x^5}$ **c** $(125x^{12})^{\frac{1}{3}}$ **d** $\frac{x + 4x^3}{x^3}$

e $\frac{2x + x^2}{x^4}$ **f** $\left(\frac{4}{9}x^4\right)^{\frac{3}{2}}$ **g** $\frac{9x^2 - 15x^5}{3x^3}$ **h** $\frac{5x + 3x^2}{15x^3}$

E **4** **a** Find the value of $81^{\frac{1}{4}}$. **(1 mark)**

b Simplify $x(2x^{-\frac{1}{3}})^4$. **(2 marks)**

E **5** Given that $y = \frac{1}{8}x^3$, express each of the following in the form kx^n, where k and n are constants.

a $y^{\frac{1}{3}}$ **(2 marks)**

b $\frac{1}{2}y^{-2}$ **(2 marks)**

1.5 Surds

If n is an integer that is *not* a **square number**, then any multiple of $\sqrt{n}$ is called a surd.
Examples of surds are $\sqrt{2}$, $\sqrt{19}$ and $5\sqrt{2}$.

Surds are examples of **irrational** numbers.
The decimal expansion of a surd is never-ending and never repeats, for example $\sqrt{2} = 1.414213562\ldots$

Notation Irrational numbers cannot be written in the form $\frac{a}{b}$ where a and b are integers.

You can use surds to write exact answers to calculations.

■ You can **manipulate** surds using these rules:

- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Example 12

Simplify:

a $\sqrt{12}$ **b** $\frac{\sqrt{20}}{2}$ **c** $5\sqrt{6} - 2\sqrt{24} + \sqrt{294}$

a $\sqrt{12} = \sqrt{(4 \times 3)}$

$= \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

Look for a factor of 12 that is a square number. Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$. $\sqrt{4} = 2$

b $\frac{\sqrt{20}}{2} = \frac{\sqrt{4} \times \sqrt{5}}{2}$

$\sqrt{20} = \sqrt{4} \times \sqrt{5}$

$= \frac{2 \times \sqrt{5}}{2} = \sqrt{5}$

$\sqrt{4} = 2$

Cancel by 2.

c $5\sqrt{6} - 2\sqrt{24} + \sqrt{294}$

$= 5\sqrt{6} - 2\sqrt{6} \times \sqrt{4} + \sqrt{6} \times \sqrt{49}$

$\sqrt{6}$ is a common factor.

$= \sqrt{6}(5 - 2\sqrt{4} + \sqrt{49})$

Work out the square roots $\sqrt{4}$ and $\sqrt{49}$.

$= \sqrt{6}(5 - 2 \times 2 + 7)$

$5 - 4 + 7 = 8$

$= \sqrt{6}(8)$

$= 8\sqrt{6}$

Example 13 SKILLS PROBLEM-SOLVING

Expand and simplify if possible:

a $\sqrt{2}(5 - \sqrt{3})$ **b** $(2 - \sqrt{3})(5 + \sqrt{3})$

a $\sqrt{2}(5 - \sqrt{3})$ — $\sqrt{2} \times 5 - \sqrt{2} \times \sqrt{3}$

$= 5\sqrt{2} - \sqrt{2}\sqrt{3}$

$= 5\sqrt{2} - \sqrt{6}$ — Using $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

b $(2 - \sqrt{3})(5 + \sqrt{3})$

$= 2(5 + \sqrt{3}) - \sqrt{3}(5 + \sqrt{3})$ — Expand the brackets completely before you simplify.

$= 10 + 2\sqrt{3} - 5\sqrt{3} - \sqrt{9}$ — Collect like terms: $2\sqrt{3} - 5\sqrt{3} = -3\sqrt{3}$

$= 7 - 3\sqrt{3}$ — Simplify any roots if possible: $\sqrt{9} = 3$

Exercise 1E SKILLS PROBLEM-SOLVING

Do not use your calculator for this exercise.

1 Simplify:

a $\sqrt{28}$ **b** $\sqrt{72}$ **c** $\sqrt{50}$

d $\sqrt{32}$ **e** $\sqrt{90}$ **f** $\frac{\sqrt{12}}{2}$

g $\frac{\sqrt{27}}{3}$ **h** $\sqrt{20} + \sqrt{80}$ **i** $\sqrt{200} + \sqrt{18} - \sqrt{72}$

j $\sqrt{175} + \sqrt{63} + 2\sqrt{28}$ **k** $\sqrt{28} - 2\sqrt{63} + \sqrt{7}$ **l** $\sqrt{80} - 2\sqrt{20} + 3\sqrt{45}$

m $3\sqrt{80} - 2\sqrt{20} + 5\sqrt{45}$ **n** $\frac{\sqrt{44}}{\sqrt{11}}$ **o** $\sqrt{12} + 3\sqrt{48} + \sqrt{75}$

2 Expand and simplify if possible:

a $\sqrt{3}(2 + \sqrt{3})$ **b** $\sqrt{5}(3 - \sqrt{3})$ **c** $\sqrt{2}(4 - \sqrt{5})$

d $(2 - \sqrt{2})(3 + \sqrt{5})$ **e** $(2 - \sqrt{3})(3 - \sqrt{7})$ **f** $(4 + \sqrt{5})(2 + \sqrt{5})$

g $(5 - \sqrt{3})(1 - \sqrt{3})$ **h** $(4 + \sqrt{3})(2 - \sqrt{3})$ **i** $(7 - \sqrt{11})(2 + \sqrt{11})$

E **3** Simplify $\sqrt{75} - \sqrt{12}$ giving your answer in the form $a\sqrt{3}$, where a is an integer. **(2 marks)**

1.6 Rationalising denominators

If a fraction has a surd in the **denominator**, it is sometimes useful to **rearrange** it so that the denominator is a rational number. This is called rationalising the denominator.

- The rules to rationalise denominators are:
  - For fractions in the form $\frac{1}{\sqrt{a}}$, multiply the numerator and denominator by $\sqrt{a}$.
  - For fractions in the form $\frac{1}{a + \sqrt{b}}$, multiply the numerator and denominator by $(a - \sqrt{b})$.
  - For fractions in the form $\frac{1}{a - \sqrt{b}}$, multiply the numerator and denominator by $(a + \sqrt{b})$.

Example 14

Rationalise the denominator:

a $\frac{1}{\sqrt{3}}$ **b** $\frac{1}{3+\sqrt{2}}$ **c** $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ **d** $\frac{1}{(1-\sqrt{3})^2}$

a $\frac{1}{\sqrt{3}} = \frac{1 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$ — Multiply the numerator and denominator by $\sqrt{3}$.

$= \frac{\sqrt{3}}{3}$ — $\sqrt{3} \times \sqrt{3} = (\sqrt{3})^2 = 3$

b $\frac{1}{3+\sqrt{2}} = \frac{1 \times (3-\sqrt{2})}{(3+\sqrt{2})(3-\sqrt{2})}$ — Multiply numerator and denominator by $(3-\sqrt{2})$.

$= \frac{3-\sqrt{2}}{9-3\sqrt{2}+3\sqrt{2}-2}$ — $\sqrt{2} \times \sqrt{2} = 2$

$= \frac{3-\sqrt{2}}{7}$ — $9 - 2 = 7$, $-3\sqrt{2} + 3\sqrt{2} = 0$

c $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{(\sqrt{5}+\sqrt{2})(\sqrt{5}+\sqrt{2})}{(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})}$ — Multiply numerator and denominator by $(\sqrt{5}+\sqrt{2})$.

$= \frac{5+\sqrt{5}\sqrt{2}+\sqrt{2}\sqrt{5}+2}{5-2}$ — $-\sqrt{2}\sqrt{5}$ and $\sqrt{5}\sqrt{2}$ cancel each other out.

$= \frac{7+2\sqrt{10}}{3}$ — $\sqrt{5}\sqrt{2} = \sqrt{10}$

d $\frac{1}{(1-\sqrt{3})^2} = \frac{1}{(1-\sqrt{3})(1-\sqrt{3})}$ — Expand the brackets.

$= \frac{1}{1-\sqrt{3}-\sqrt{3}+\sqrt{9}}$ — Simplify and collect like terms. $\sqrt{9} = 3$

$= \frac{1}{4-2\sqrt{3}}$

$= \frac{1 \times (4+2\sqrt{3})}{(4-2\sqrt{3})(4+2\sqrt{3})}$ — Multiply the numerator and denominator by $(4+2\sqrt{3})$.

$= \frac{4+2\sqrt{3}}{16+8\sqrt{3}-8\sqrt{3}-12}$ — $\sqrt{3} \times \sqrt{3} = 3$

$= \frac{4+2\sqrt{3}}{4} = \frac{2+\sqrt{3}}{2}$ — $16 - 12 = 4$, $8\sqrt{3} - 8\sqrt{3} = 0$

Exercise 1F

SKILLS ANALYSIS

Do not use your calculator for this exercise.

1 Simplify:

a $\frac{1}{\sqrt{5}}$ **b** $\frac{1}{\sqrt{11}}$ **c** $\frac{1}{\sqrt{2}}$ **d** $\frac{\sqrt{3}}{\sqrt{15}}$

e $\frac{\sqrt{12}}{\sqrt{48}}$ **f** $\frac{\sqrt{5}}{\sqrt{80}}$ **g** $\frac{\sqrt{12}}{\sqrt{156}}$ **h** $\frac{\sqrt{7}}{\sqrt{63}}$

2 Rationalise the denominators and simplify:

a $\frac{1}{1+\sqrt{3}}$ **b** $\frac{1}{2+\sqrt{5}}$ **c** $\frac{1}{3-\sqrt{7}}$ **d** $\frac{4}{3-\sqrt{5}}$ **e** $\frac{1}{\sqrt{5}-\sqrt{3}}$

f $\frac{3-\sqrt{2}}{4-\sqrt{5}}$ **g** $\frac{5}{2+\sqrt{5}}$ **h** $\frac{5\sqrt{2}}{\sqrt{8}-\sqrt{7}}$ **i** $\frac{11}{3+\sqrt{11}}$ **j** $\frac{\sqrt{3}-\sqrt{7}}{\sqrt{3}+\sqrt{7}}$

k $\frac{\sqrt{17}-\sqrt{11}}{\sqrt{17}+\sqrt{11}}$ **l** $\frac{\sqrt{41}+\sqrt{29}}{\sqrt{41}-\sqrt{29}}$ **m** $\frac{\sqrt{2}-\sqrt{3}}{\sqrt{3}-\sqrt{2}}$

3 Rationalise the denominators and simplify:

a $\frac{1}{(3-\sqrt{2})^2}$ **b** $\frac{1}{(2+\sqrt{5})^2}$ **c** $\frac{4}{(3-\sqrt{2})^2}$

d $\frac{3}{(5+\sqrt{2})^2}$ **e** $\frac{1}{(5+\sqrt{2})(3-\sqrt{2})}$ **f** $\frac{2}{(5-\sqrt{3})(2+\sqrt{3})}$

E/P 4 Simplify $\frac{3-2\sqrt{5}}{\sqrt{5}-1}$ giving your answer in the form $p + q\sqrt{5}$, where p and q are rational numbers. **(4 marks)**

Problem-solving

You can check that your answer is in the correct form by writing down the values of p and q and checking that they are rational numbers.

Chapter review 1

SKILLS EXECUTIVE FUNCTION

1 Simplify:

a $y^3 \times y^5$ **b** $3x^2 \times 2x^5$ **c** $(4x^2)^3 \div 2x^5$ **d** $4b^2 \times 3b^3 \times b^4$

2 Expand and simplify if possible:

a $(x + 3)(x - 5)$ **b** $(2x - 7)(3x + 1)$ **c** $(2x + 5)(3x - y + 2)$

3 Expand and simplify if possible:

a $x(x + 4)(x - 1)$ **b** $(x + 2)(x - 3)(x + 7)$ **c** $(2x + 3)(x - 2)(3x - 1)$

4 Expand the brackets:

a $3(5y + 4)$ **b** $5x^2(3 - 5x + 2x^2)$ **c** $5x(2x + 3) - 2x(1 - 3x)$ **d** $3x^2(1 + 3x) - 2x(3x - 2)$

5 Factorise these expressions completely:

a $3x^2 + 4x$ **b** $4y^2 + 10y$ **c** $x^2 + xy + xy^2$ **d** $8xy^2 + 10x^2y$

6 Factorise:

a $x^2 + 3x + 2$ **b** $3x^2 + 6x$ **c** $x^2 - 2x - 35$ **d** $2x^2 - x - 3$

e $5x^2 - 13x - 6$ **f** $6 - 5x - x^2$

7 Factorise:

a $2x^3 + 6x$ **b** $x^3 - 36x$ **c** $2x^3 + 7x^2 - 15x$

8 Simplify:

a $9x^3 \div 3x^{-3}$ **b** $(4^{\frac{3}{2}})^{\frac{1}{3}}$ **c** $3x^{-2} \times 2x^4$ **d** $3x^{\frac{1}{3}} \div 6x^{\frac{2}{3}}$

9 Evaluate, without using your calculator:

a $\left(\frac{8}{27}\right)^{\frac{2}{3}}$ **b** $\left(\frac{225}{289}\right)^{\frac{3}{2}}$

10 Simplify, without using your calculator:

a $\frac{3}{\sqrt{63}}$ **b** $\sqrt{20} + 2\sqrt{45} - \sqrt{80}$

11 **a** Find the value of $35x^2 + 2x - 48$ when $x = 25$.

b By factorising the expression, show that your answer to part **a** can be written as the product of two **prime** factors.

12 Expand and simplify if possible, without using your calculator:

a $\sqrt{2}(3 + \sqrt{5})$ **b** $(2 - \sqrt{5})(5 + \sqrt{3})$ **c** $(6 - \sqrt{2})(4 - \sqrt{7})$

13 Rationalise the denominator and simplify:

a $\frac{1}{\sqrt{3}}$ **b** $\frac{1}{\sqrt{2} - 1}$ **c** $\frac{3}{\sqrt{3} - 2}$ **d** $\frac{\sqrt{23} - \sqrt{37}}{\sqrt{23} + \sqrt{37}}$ **e** $\frac{1}{(2 + \sqrt{3})^2}$ **f** $\frac{1}{(4 - \sqrt{7})^2}$

14 **Do not use your calculator for this question.**

a Given that $x^3 - x^2 - 17x - 15 = (x + 3)(x^2 + bx + c)$, where b and c are constants, work out the values of b and c.

b **Hence**, fully factorise $x^3 - x^2 - 17x - 15$.

E 15 Given that $y = \frac{1}{64}x^3$, express each of the following in the form kx^n, where k and n are constants.

a $y^{\frac{1}{3}}$ **(1 mark)**

b $4y^{-1}$ **(1 mark)**

E/P 16 Show that $\frac{5}{\sqrt{75} - \sqrt{50}}$ can be written in the form $\sqrt{a} + \sqrt{b}$, where a and b are integers. **(5 marks)**

E 17 Expand and simplify $(\sqrt{11} - 5)(5 - \sqrt{11})$, without using your calculator. **(2 marks)**

E 18 Factorise completely $x - 64x^3$. **(3 marks)**

E/P 19 Express 27^{2x+1} in the form 3^y, stating y in terms of x. **(2 marks)**

E/P **20** Solve the equation $8 + x\sqrt{12} = \frac{8x}{\sqrt{3}}$.

Give your answer in the form $a\sqrt{b}$, where a and b are integers. **(4 marks)**

P **21** **Do not use your calculator for this question.**
A rectangle has a length of $(1 + \sqrt{3})$ cm and area of $\sqrt{12}$ cm^2.
Calculate the width of the rectangle in cm.
Express your answer in the form $a + b\sqrt{3}$, where a and b are integers to be found.

E **22** Show that $\frac{(2 - \sqrt{x})^2}{\sqrt{x}}$ can be written as $4x^{-\frac{1}{2}} - 4 + x^{\frac{1}{2}}$. **(2 marks)**

E/P **23** Given that $243\sqrt{3} = 3^a$, find the value of a. **(3 marks)**

E/P **24** Given that $\frac{4x^3 + x^{\frac{5}{2}}}{\sqrt{x}}$ can be written in the form $4x^a + x^b$,

write down the value of a and the value of b. **(2 marks)**

Challenge

a Simplify $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$.

b Hence show that $\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \ldots + \frac{1}{\sqrt{24} + \sqrt{25}} = 4$

Summary of key points

1 You can use the laws of indices to simplify powers of the same base.
- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $(ab)^n = a^n b^n$

2 Factorising is the opposite of expanding brackets.

3 A quadratic expression has the form $ax^2 + bx + c$ where a, b and c are real numbers and $a \neq 0$.

4 $x^2 - y^2 = (x + y)(x - y)$

5 You can use the laws of indices with any rational power.
- $a^{\frac{1}{m}} = \sqrt[m]{a}$
- $a^{\frac{n}{m}} = \sqrt[m]{a^n}$
- $a^{-m} = \frac{1}{a^m}$
- $a^0 = 1$

6 You can manipulate surds using these rules:
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

7 The rules to rationalise denominators are:
- For fractions in the form $\frac{1}{\sqrt{a}}$, multiply the numerator and denominator by $\sqrt{a}$.
- For fractions in the form $\frac{1}{a + \sqrt{b}}$, multiply the numerator and denominator by $(a - \sqrt{b})$.
- For fractions in the form $\frac{1}{a - \sqrt{b}}$, multiply the numerator and denominator by $(a + \sqrt{b})$.