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PEARSON EDEXCEL INTERNATIONAL AS / A LEVEL

PHYSICS

STUDENT BOOK 1

MILES HUDSON



PEARSON EDEXCEL INTERNATIONAL AS/A LEVEL

PHYSICS

Student Book 1

Miles Hudson

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Text

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ABOUT THIS BOOK

This book is written for students following the Pearson Edexcel International Advanced Subsidiary (IAS) Physics specification. This book covers the full IAS course and the first year of the International A Level (IAL) course.

The book contains full coverage of IAS units (or exam papers) 1 and 2. Each unit in the specification has two topic areas. The topics in this book, and their contents, fully match the specification. You can refer to the Assessment Overview on page X for further information. Students can prepare for the written Practical Skills Paper (unit 3) by using the IAL Physics Lab Book (see page viii of this book).

Each Topic is divided into chapters and sections to break the content down into manageable chunks. Each section features a mix of learning and activities.

Learning objectives
Each chapter starts with a list of key assessment objectives.

Specification reference
The exact specification references covered in the section are provided.

Subject vocabulary
Key terms are highlighted in blue in the text. Clear definitions are provided at the end of each section for easy reference, and are also collated in a glossary at the back of the book.

Exam hints
Tips on how to answer exam-style questions and guidance for exam preparation. **Worked examples** also show you how to work through questions and set out calculations.

1A 1 VELOCITY AND ACCELERATION

SPECIFICATION REFERENCE
1.3.1 1.3.4

LEARNING OBJECTIVES

- Explain the distinction between scalar and vector quantities.
- Distinguish between speed and velocity and define acceleration.
- Calculate values using equations for velocity and acceleration.



fig A These runners are accelerating to a high speed.

Movement is fundamental to the functioning of our universe. Whether you are running to catch a bus or want to calculate the speed required for a rocket to travel to Mars or the kinetic energy of an electron in an X-ray machine, you need to be able to work out how fast things are moving.

RATE OF MOVEMENT

One of the simplest things we can measure is how fast an object is moving. You can calculate an object's **speed** if you know the amount of time taken to move a certain distance:

$$\text{speed (m s}^{-1}\text{)} = \frac{\text{distance (m)}}{\text{time (s)}}$$

$$v = \frac{d}{t}$$

However, the calculation for speed will only tell you how fast an object is moving. Often it is also vitally important to know in what direction this movement is taking the object. When you include the direction in the information, about the rate of movement of an object, this is then known as the **velocity**. So, the velocity is the rate of change of **displacement**, where the distance in a particular direction is called the 'displacement'.

$$\text{velocity (m s}^{-1}\text{)} = \frac{\text{displacement (m)}}{\text{time (s)}}$$

OR

$$v = \frac{\Delta s}{\Delta t}$$

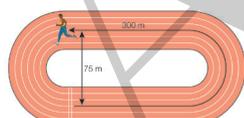


fig B The displacement due north is only 75 m, whilst the actual distance this athlete has run is 300 m. So the velocity due north is much less than the actual speed.

LEARNING TIP
The upper case Greek letter delta, Δ , is used in mathematics to indicate a change in a quantity. For example, Δs means the change in the displacement of an object, to be used here to calculate the velocity of the object.

DID YOU KNOW?
The froghopper, a 6 mm long insect, can accelerate at $4\,000\text{ m s}^{-2}$.

LEARNING TIP
Vector notation means that vectors are written in bold type to distinguish them from scalar variables.

A quantity for which the direction must be stated is known as a **vector**. If direction is not important, the measurement is referred to as a **scalar** quantity. Therefore, velocity is a vector and speed is a scalar; distance is a scalar, and displacement is a vector.

Scalar and vector quantities are not limited to measurements related to movement. Every measured quantity can be classified to include the direction (vector, e.g. force) or as being sufficiently stated by its magnitude only (scalar, e.g. mass).

MOTION

1A.1 VELOCITY AND ACCELERATION

11

AVERAGE AND INSTANTANEOUS SPEED

In most journeys, it is unlikely that speed will remain constant throughout. As part of his training programme, an athlete in **fig A** wants to keep a record of his speed for all races. From rest, before the starting gun starts the race, he accelerates to a top speed. However, the race timing will be made from start to finish, and so it is most useful to calculate an average speed over the whole race. **Average speed** is calculated by dividing the total distance for a journey by the total time for the journey. This it averages out the slower and faster parts of the journey, and even includes stops.

Instantaneous speed can be an important quantity, and we will look at how to measure it in the next topic.



fig C Most speed checks look at instantaneous speed, but CCTV allows police to monitor average speed over a long distance.

EXAM HINT

While accelerations can (very briefly) be extraordinarily high. Note that for the electron in question 3(b), no speed or velocity can ever be greater than the speed of light, which is $3 \times 10^8\text{ m s}^{-1}$. If you calculate a speed that is higher than this, check your calculation again as it must be wrong.

CHECKPOINT

SKILLS PROBLEM SOLVING

- The athlete in **fig B** has taken 36 seconds from the start to reach the 300 m mark as shown. Calculate:
 - his average speed during this 36 seconds
 - his average velocity due north during this 36 seconds
 - his average velocity due east during this 36 seconds.
- A driver in a car travelling at about 40.2 km h^{-1} sees a cat run onto the road ahead.
 - Convert 40.2 km h^{-1} into a speed in m s^{-1} .
 - The car travels 16.5 m whilst the driver is reacting to the danger. What is his reaction time?
 - The car comes to a stop in 2.5 s. What is its deceleration?
- An electron in an X-ray machine is accelerated from rest to half the speed of light in $1.7 \times 10^{-15}\text{ s}$. Calculate:
 - the speed the electron reaches in m s^{-1}
 - the acceleration the electron experiences.

SUBJECT VOCABULARY

speed the rate of change of distance:

$$\text{speed (m s}^{-1}\text{)} = \frac{\text{distance (m)}}{\text{time (s)}}$$

$$v = \frac{d}{t}$$

velocity the rate of change of displacement:

$$\text{velocity (m s}^{-1}\text{)} = \frac{\text{displacement (m)}}{\text{time (s)}}$$

$$v = \frac{\Delta s}{\Delta t} \text{ OR } v = \frac{\Delta x}{\Delta t}$$

displacement the vector measurement of distance in a certain direction

vector a quantity that must have both magnitude and direction

scalar a quantity that has magnitude only

average speed speed for a whole journey, calculated by dividing the total distance for a journey by the total time for the journey:

$$\text{average speed (m s}^{-1}\text{)} = \frac{\text{total distance (m)}}{\text{total time (s)}}$$

instantaneous speed the speed at any particular instant in time on a journey, which can be found from the gradient of the tangent to a distance-time graph at that time

acceleration the vector defined as the rate of change of velocity:

$$\text{acceleration (m s}^{-2}\text{)} = \frac{\text{change in velocity (m s}^{-1}\text{)}}{\text{time taken to change the velocity (s)}}$$

$$a = \frac{v - u}{t} \text{ OR } a = \frac{\Delta v}{\Delta t}$$

ACCELERATION

Acceleration is defined as the rate of change of velocity. Therefore, it must include the direction in which the speed is changing, and so acceleration is a vector quantity. The equation defining acceleration is:

$$\text{acceleration (m s}^{-2}\text{)} = \frac{\text{change in velocity (m s}^{-1}\text{)}}{\text{time taken to change the velocity (s)}}$$

$$a = \frac{v - u}{t}$$

OR

$$a = \frac{\Delta v}{\Delta t}$$

where u is the initial velocity and v is the final velocity.

The vector nature of acceleration is very important. One of the consequences is that if an object changes only the direction of its velocity, it is accelerating, while remaining at a constant speed.

Similarly, deceleration represents a negative change in velocity, and so could be stated as a negative acceleration.

Learning tips
These help you focus your learning and avoid common errors.

Did you know?
Interesting facts help you to remember the key concepts.

Checkpoint
Questions at the end of each section check understanding of the key learning points in each chapter.

Your learning, chapter by chapter, is always put in context:

- Links to other areas of Physics include previous knowledge that is built on in the topic, and future learning that you will cover later in your course.
- A checklist details maths knowledge required. If you need to practise these skills, you can use the **Maths Skills** reference at the back of the book as a starting point.

TOPIC 1 MECHANICS

CHAPTER 1A MOTION

How can we calculate how fast a plane is flying, in what direction it is going and how long it will take to reach a certain destination? If you were a pilot, how would you know what force to make the engines produce and where to direct that force so your plane moves to your destination? An incredible number of intricate calculations need to be done to enable a successful flight, and the basis for all of them is simple mechanics.

This chapter explains the multiple movements of objects. It looks at how movement can be described and recorded, and then moves on to explaining why movement happens. It covers velocity and acceleration, including how to calculate these in different situations.

We only consider objects moving at speeds that could be encountered in everyday life. At these speeds (much less than the speed of light) Sir Isaac Newton's laws of motion accurately describe three laws of motion. With knowledge of basic geometry, we can identify aspects of movement in each dimension.

Newton's laws of motion have been constantly under test by scientists ever since he published them in 1687. Within the constraints established by Einstein in the early twentieth century, Newton's laws have always correctly described the relationships between data collected. You may have a chance to confirm Newton's laws in experiments of your own. With modern ICT recording of data, the reliability of such experiments is now much improved over traditional methods.

MATHS SKILLS FOR THIS CHAPTER

- Units of measurement (e.g. the newton, N)
- Using Pythagoras' theorem, and the angle sum of a triangle (e.g. finding a resultant vector)
- Using sin, cos and tan in physical problems (e.g. resolving vectors)
- Using angles in regular 2D structures (e.g. interpreting force diagrams to solve problems)
- Changing the subject of an equation (e.g. rearranging the kinematics equations)
- Substituting numerical values into algebraic equations (e.g. calculating the acceleration)
- Plotting two variables from experimental or other data, understanding that $y = mx + c$ represents a linear relationship and determining the slope of a linear graph (e.g. verifying Newton's second law experimentally)
- Estimating, by graphical methods as appropriate, the area between a curve and the x-axis and realising the physical significance of the area that has been determined (e.g. using a speed-time graph)

What prior knowledge do I need?

- Using a stopwatch to measure time
- Measuring and calculating the speed of objects
- Gravity making things fall down
- Measuring forces, calculating resultant forces
- The motion of objects as a result of forces acting on them

What will I study in this chapter?

- The definitions of and equations for speed, distance, displacement, time, velocity, acceleration
- Graphs of distance over time
- The classification of scalars and vectors
- Adding and resolving vectors
- Newton's laws of motion
- Kinematics equations
- Moments (turning forces)

What will I study later?

- Topic 1B
 - Kinetic energy and gravitational potential energy
 - Transferring between gravitational potential energy and kinetic energy
- Work and power
- Topic 1C
 - Momentum and the principle of conservation of momentum
- Topic 2A
 - Fluid movements and terminal velocity
- Topic 3
 - Wave movements
- Topic 5 (Book 2: IAL)
 - The meaning and calculation of impulse (A level)

1A THINKING BIGGER

THE BATTLE OF AGRA

Agra Fort was built in the 15th Century, although the present structure was built in 1573. In this activity, you need to imagine attacking the fort using a cannon that fires a cannonball as a projectile.

STUDENT ESSAY



Fig. A Agra Fort is now an UNESCO World Heritage Site.

In this section, it will use some basic mechanics to answer a question: could the Mughal Empire artillery really have attacked Agra Fort in the way described previously? The nineteenth-century source material suggests that the fort was under siege by the Mughals for three months and 'battered by artillery'. However, the correct walk here links in the way of obvious battle scars.

Looking at Fig. B, the question that needs to be answered here is: 'How high up the front wall of the fort will the cannonball hit?' This height is marked on Fig. B as 'H'.

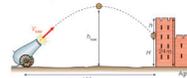


Fig. B The trajectory of a cannonball fired towards Agra Fort. We assume the cannonball leaves the cannon at ground level.

In addition to the layout shown in Fig. B, we need information about the initial velocity of the cannonball. The cannon explosion could act for 0.05 s to accelerate the cannonball

(mass = 12 kg) with a force of 9300 N. It causes the cannonball to leave the cannon at an angle of 45° to the horizontal.

Steps to the answer

We can work out what calculations are required to solve this problem, by working back from the answer we want to find. The fundamental idea is that the parabolic trajectory would be symmetrical if the flight was not interrupted by crashing into the fortress wall.

- 1 To find the height up the wall from the ground, we will need to work out how far down from the cannonball's maximum height it falls: $H_{\text{max}} - H$
- 2 To find H_{max} , we need to know the time of flight, t_{max} , so we can divide the time in two to reach H_{max} , and time left to fall height H . We will use vertical gravitational acceleration to calculate the vertical drop in that remaining time: $H_{\text{max}} - H = \frac{1}{2}gt^2$
- 3 From Fig. B, we can see that $t = 1.50$ s.

H_{max} can be found by resolving the velocity to give the horizontal component:

$$v_{\text{horizontal}} = v \cos 45^\circ$$

The overall velocity will come from the cannon's acceleration of the cannonball:

$$v = at$$

where $a = 0.05 \text{ s}^{-1}$, and the question tells us that the explosion acts for 0.05 seconds.

The overall velocity will come from the cannon's acceleration of the cannonball:

$$v = at$$

$$a = \frac{F}{m}$$

MOTION
THINKING BIGGER
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SCIENCE COMMUNICATION

1 The extract opposite is a draft for a university essay about the Mughal siege of 1857. Consider the extract and comment on the type of writing being used. For example, think about whether this is a scientific reporting the results of their experiments, a scientific review of data, a newspaper or a magazine-style article for a specific audience. Try and answer the following questions:

- (a) How can you tell that the author is doubtful about the historical source material?
- (b) What is the purpose of this mathematical analysis, for its inclusion in this essay?

PHYSICS IN DETAIL

How we will look at the physics in detail. You may need to combine concepts from different areas of physics to work out the answers.

- 2 Complete the calculation steps, in reverse as suggested, in order to find out the answer: H
 - (a) the acceleration caused by the explosion
 - (b) overall velocity that the cannonball is projected from the cannon
 - (c) horizontal and vertical components of the velocity
 - (d) time of flight based from the horizontal layer
 - (e) time to reach maximum height using vertical motion
 - (f) remaining flight time from maximum height
 - (g) height fallen from the maximum in the remaining flight time
 - (h) final answer: H
- 3 State two assumptions that have been made in these calculations.
- 4 Calculate what difference there would be in the answer if the cannon was loaded with different cannonballs of masses 10 kg and 14 kg. Note from Fig. B that the fortress walls are 24 m high. Comment on this difference.
- 5 If the available supply of cannonballs offered very variable masses, how might the Mughals be able to overcome the problems shown in question 4?

ACTIVITY

Imagine the writer of this essay is a friend of yours, and he has come to you for help with the calculations as he is not an experienced scientist. His section 'Steps to the answer' was taken from a research source about a different fortress under siege. Write an email to Chua to explain the calculations required in each step.

INTERPRETATION NOTE

Once you have gathered the calculation questions below, decide whether you think the Mughal siege happened as the author suggests.

THINKING BIGGER TIP

You can assume that an explosion exerts a force on the cannonball to fire it out of the cannon.

INTERPRETATION NOTE

You can assume that the writer understands mathematics, and is generally intelligent – a student who could have done A level physics but preferred arts subjects.

Thinking Bigger

At the end of each topic, there is an opportunity to read and work with real-life research and writing about science. The activities help you to read real-life material that's relevant to your course, analyse how scientists write, think critically and consider how different aspects of your learning piece together.

Skills

These sections will help you develop transferable skills, which are highly valued in further study and the workplace.

Exam Practice

Exam-style questions at the end of each chapter are tailored to the Pearson Edexcel specification to allow for practice and development of exam writing technique. They also allow for practice responding to the command words used in the exams (see the **command words glossary** at the back of this book).

1A EXAM PRACTICE

- 1 Questions can be scalar or vector. Select the row of the table that correctly states a scalar quantity and a vector quantity.

Scalar quantity	Vector quantity
A acceleration	mass
B mass	weight
C speed	distance
D displacement	speed

(Total for Question 1 = 1 mark)
- 2 How is the kinetic energy E_k of a car related to its speed, v ?
 - A $E_k \propto v$
 - B $E_k \propto v^2$
 - C $E_k \propto v^3$
 - D $E_k \propto v^4$

(Total for Question 2 = 1 mark)
- 3 The unit of force is the newton. One newton is equivalent to:
 - A 0.1 kg
 - B 1 kg m s^{-1}
 - C 1 kg m s^{-2}
 - D 1 m s^{-2}

(Total for Question 3 = 1 mark)
- 4 A ball is thrown vertically upwards at a speed of 11.0 m s^{-1} . What is the maximum height it reaches?
 - A 0.56 m
 - B 1.12 m
 - C 6.17 m
 - D 12.3 m

(Total for Question 4 = 1 mark)
- 5 Calculate the moment exerted on the nut by the spanner shown in the diagram.
 - A 2.4 N m
 - B 4.2 N m
 - C 4.8 N m
 - D 420 N m

(Total for Question 5 = 1 mark)
- 6 (a) What is meant by a vector quantity? [1]

(b) A car is driven around a bend at a constant speed. Explain what happens to its velocity. [2]

(Total for Question 6 = 3 marks)
- 7 You are asked to determine the acceleration of free fall at the surface of the Earth, g , using a free fall method in the laboratory.
 - (a) Describe the apparatus you would use, the measurements you would take and explain how you would use them to determine g . [6]
 - (b) Give one precaution you would take to ensure the accuracy of your measurements. [1]

(Total for Question 7 = 7 marks)
- 8 The graph shows how displacement varies with time for an object that starts from rest with constant acceleration.
 - (a) Use the distance-time graph to determine the speed of the object at a time of 4.0 s. [3]
 - (b) Calculate the acceleration. [2]

(Total for Question 8 = 5 marks)
- 9 The photograph shows a sequence of images of a bouncing tennis ball.

A student plots the following graph and claims that it shows the vertical motion of the ball in the photograph.

MOTION
EXAM PRACTICE
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(a) Without carrying out any calculations, describe how the following can be found from the graph.

- (i) the vertical distance travelled by the ball between 0.5 s and 1.0 s. [2]
- (ii) the acceleration at 1 s. [2]

(b) The graph contains several errors in its representation of the motion of the ball. Explain two of these errors. [4]

(Total for Question 9 = 6 marks)

- 10 There has been a proposal to build a train tunnel underneath the Atlantic Ocean from England to America. The suggestion is that in the future the trip of 5000 km could take as little as one hour. Assume that half the time is spent accelerating uniformly and the other half is spent decelerating uniformly with the same magnitude as the acceleration.
- (a) Show that the acceleration would be about 2 m s^{-2} . [2]
- (b) Calculate the maximum speed. [2]
- (c) Calculate the maximum force required to decelerate the train. mass of train = $4.5 \times 10^7 \text{ kg}$. [2]

(Total for Question 10 = 6 marks)
- 11 During a lesson on Newton's laws of motion, a student says, 'We don't really need to bother with Newton's first law because it is included in his second law'. State Newton's first two laws of motion and explain how Newton's second law includes the first law. [5]

(Total for Question 11 = 5 marks)
- 12 The diagram shows an arrangement used to launch a light beam rocket at a school science competition.
- (a) The rocket travels 1.88 m in a time of 0.88 s. [1]
- (b) Show that the horizontal component of the initial velocity of the rocket is about 2 m s^{-1} . [2]
- (c) Show that the vertical component of the initial velocity of the rocket is about 4 m s^{-1} . [2]
- (d) Calculate the initial velocity of the rocket. [4]
- (e) The students obtained their data by filming the flight. When they checked the maximum height reached by the rocket they found it was less than the height predicted using this velocity. [1]
- (f) Suggest why the maximum height reached was less than predicted. [1]
- (g) Give two advantages of filming the flight to obtain the data. [2]

(Total for Question 12 = 13 marks)

PRACTICAL SKILLS

Practical work is central to the study of physics. The Pearson Edexcel International Advanced Subsidiary (IAS) Physics specification includes eight Core Practicals that link theoretical knowledge and understanding to practical scenarios.

Your knowledge and understanding of practical skills and activities will be assessed in all examination papers for the IAS Level Physics qualification.

- Papers 1 and 2 will include questions based on practical activities, including novel scenarios.
- Paper 3 will test your ability to plan practical work, including risk management and selection of apparatus.

In order to develop practical skills, you should carry out a range of practical experiments related to the topics covered in your course. Further suggestions in addition to the Core Practicals are included below.

STUDENT BOOK TOPIC	IAS CORE PRACTICALS	UNIT 1 (TOPICS 1 AND 2) MECHANICS AND MATERIALS
TOPIC 1 MECHANICS	CP1 Determine the acceleration of a freely-falling object	Possible further practicals include: <ul style="list-style-type: none"> • Strobe photography or the use of a video camera to analyse projectile motion • Determine the centre of gravity of an irregular rod • Investigate the conservation of momentum using light gates and air track • Hooke's law and the Young modulus experiments for a variety of materials
TOPIC 2 MATERIALS	CP2 Use a falling-ball method to determine the viscosity of a liquid	
	CP3 Determine the Young modulus of a material	
TOPIC 3 WAVES AND THE PARTICLE NATURE OF LIGHT	CP4 Determine the speed of sound in air using a two-beam oscilloscope, signal generator, speaker and microphone	
	CP5 Investigate the effects of length, tension and mass per unit length on the frequency of a vibrating string or wire	Possible further practicals include: <ul style="list-style-type: none"> • Estimating power output of an electric motor • Using a digital voltmeter to investigate the output of a potential divider and investigating current/voltage graphs for a filament bulb, thermistor and diode • Determining the refractive index of solids and liquids, demonstrating progressive and stationary waves on a slinky
	CP6 Determine the wavelength of light from a laser or other light source using a diffraction grating	
TOPIC 4 ELECTRIC CIRCUITS	CP7 Determine the electrical resistivity of a material	
	CP8 Determine the e.m.f. and internal resistance of an electrical cell	

4A 4 RESISTIVITY

2.4.71 2.4.72 CP7

LEARNING OBJECTIVES

- Define resistivity.
- Explain how to measure resistivity experimentally.
- Make calculations of resistance using resistivity.

Resistance is the result of collisions between charge carriers and atoms in the current's path. This effect will vary depending on the density of charge carriers and the density of fixed atoms, as well as the strength of the forces between them. So, pieces of different materials with identical dimensions will have differing resistances. The general property of a material to resist the flow of electric current is called **resistivity**, which has the symbol ρ , and SI units of ohm metres, $\Omega \text{ m}$.

LEARNING TIP

Resistivity is a property of a material. All samples of the same material, regardless of their shape and size, will have the same resistivity, whilst their resistances may vary different.

The resistance of an object is dependent on its dimensions and the material from which it is made. This gives rise to an equation for calculating the resistance of a uniform sample of material if we know the resistivity of the material:

$$\text{resistance (}\Omega\text{)} = \frac{\text{resistivity (}\Omega \text{ m)} \times \text{sample length (m)}}{\text{cross-sectional area (m}^2\text{)}}$$

$$R = \frac{\rho l}{A}$$



Fig A With knowledge of the resistivity, ρ , of a material, we can calculate the resistance of objects made from it.

RESISTIVITY EQUATION EXAMPLE

See Fig A and Table A. What is the resistance of a piece of copper fuse wire if it is 0.40 mm in diameter and 2 cm long?

wire radius = 0.20 mm = $2.0 \times 10^{-4} \text{ m}$
 cross-sectional area, $A = \pi r^2 = \pi \times (2.0 \times 10^{-4})^2 = 1.3 \times 10^{-7} \text{ m}^2$

$$R = \frac{\rho l}{A}$$

$$R = \frac{1.7 \times 10^{-8} \times 0.02}{1.3 \times 10^{-7}}$$

$$R = 2.6 \times 10^{-1} \Omega$$

MATERIAL	RESISTIVITY ρ ($\Omega \text{ m}$) AT 20 °C	$\frac{\Delta \rho}{\Delta T}$ / $\% \text{ } ^\circ\text{C}^{-1}$
silver	1.6×10^{-8}	+0.38
copper	1.7×10^{-8}	+0.40
aluminium	2.8×10^{-8}	+0.38
constantan	4.9×10^{-7}	+0.003
germanium	4.2×10^{-1}	-5.0
silicon	2.6×10^3	-7.0
polyethylene	2×10^{11}	
glass	$\sim 10^{12}$	
epoxy resin	$\sim 10^{15}$	

Table A Resistivity varies greatly between materials, and is also dependent on temperature. Note the small change in resistivity with temperature for constantan, an alloy of copper and nickel; this information is used where accurately known resistance is important.

PRACTICAL SKILLS CP7

Investigating resistivity
 You can investigate the resistivity for a metal in the school laboratory using a simple circuit.

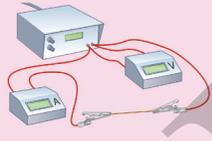


Fig B Measuring the resistance for various lengths of a wire will allow us to plot a graph to find its resistivity.

We will need to use a micrometer screw gauge to measure the wire's diameter. For improved accuracy, this is done in right-angled pairs at several places along the length of the wire, and then we take the mean diameter measurement.

For several different lengths of the wire, the wire's resistance should be measured using the voltmeter-ammeter method ($R = \frac{V}{I}$). The resistance will be small, so care must be taken to ensure currents are safely low.

The equation involving resistivity means that we could calculate a value for it by re-arranging the equation and taking one of the results and making the calculation. However, it is always more reliable to

In the **Student Book**, the Core Practical specification and Lab Book references are supplied in the relevant sections.

Practical Skills
 Practical skills boxes explain techniques used in the Core Practicals, and also detail useful skills and knowledge gained in other related investigations.

CORE PRACTICAL 1: DETERMINE THE ACCELERATION OF A FREELY-FALLING OBJECT

SPECIFICATION REFERENCE 1.3.11

Procedure

- Drop the object from rest and record the time taken, t , for:
 - the sphere to fall through the trap door
 - the dowl to pass through the light gate.
- Repeat step 1 twice more and calculate the mean value of t for each method.
- Measure and record the height, h , fallen by the object.
- Vary the height and repeat steps 1-3. You should take readings at at least six different heights.
- Use half the range in your readings for t as the uncertainty in t . Calculate the percentage uncertainty in t .
- For method (b), you should measure the length of the dowl.

Learning tips

- Make sure that points plotted on a graph take up more than half of the available space on each scale. You do not always need to include the origin.
- Keep scales simple: one large square as 5, 10 or 20 is ideal. A scale where one large square represents 3 or 7 units (or similar) is very difficult to plot and can often lead to errors.
- Always consider whether the graph line should go through the origin.
- Straight lines should be drawn with the aid of a rule long enough to cover the full length of the line.
- Since the object is falling at constant acceleration, use the appropriate kinematics equation:
 - $s = ut + \frac{1}{2}at^2$ where $u = 0$, $a = g$, and $s = h$
 This can be rearranged to: $t^2 = \frac{2h}{g}$
 Comparison with $y = mx + c$ shows that plotting t^2 against h should give a straight line passing through the origin with gradient $\frac{2}{g}$.
 - $v^2 = u^2 + 2as$ where $u = 0$, $a = g$, and $s = h$
 Therefore: $v^2 = 2gh$
 Comparison with $y = mx + c$ shows that plotting v^2 against h should give a straight line passing through the origin with gradient $2g$.

Objectives

- To measure the acceleration due to gravity, g , of an object falling freely and to consider the following alternative methods:
 - object falling through a trap door
 - object falling through a light gate

Equipment

- metre rule or tape measure with millimetre resolution
- For (a):
 - steel sphere
 - electronic timer
 - electromagnet to retain steel sphere
 - trap door switch
 - clamp and stand
 - low voltage power supply
- For (b):
 - falling object, such as a dowl with 2 cm diameter, 30 cm long means to guide dowl through light gate
 - light gate and datalogger

Safety

- Make sure the stand cannot topple over by clamping it securely.
- Keep hands and face away from the falling objects.
- Turn off the electromagnet 'drops' so that it doesn't overheat and cause burns.

CORE PRACTICAL 1: DETERMINE THE ACCELERATION OF A FREELY-FALLING OBJECT

SPECIFICATION REFERENCE 1.3.11

- The percentage uncertainty ($\%U$) in t^2 is twice that in t . Use this to draw error bars onto your graph for method (a) – in the t direction only. You can use a typical mid-range value to calculate uncertainties – you do not need to work out a separate error bar for each value. Draw a new line of best fit and use this to calculate the $\%U$ in your value for g .

- Calculate the percentage difference ($\%D$) between your value and the accepted value of g (9.81 ms^{-2}) and comment on the accuracy of method (a).

Questions

- Describe an advantage of using light gates in this experiment.
- Discuss the effect of air resistance on your value for g .
- Explain why the graph should be a straight line.

This Student Book is accompanied by a **Lab Book**, which includes instructions and writing frames for the Core Practicals for students to record their results and reflect on their work. Practical skills checklists, practice questions and answers are also provided. The Lab Book records can be used as preparation and revision for the Practical Skills Paper.

ASSESSMENT OVERVIEW

The following tables give an overview of the assessment for Pearson Edexcel International Advanced Subsidiary course in Physics. You should study this information closely to help ensure that you are fully prepared for this course and know exactly what to expect in each part of the examination. More information about this qualification, and about the question types in the different papers, can be found on page 200 of this book.

PAPER / UNIT 1	PERCENTAGE OF IAS	PERCENTAGE OF IAL	MARK	TIME	AVAILABILITY
MECHANICS AND MATERIALS Written exam paper Paper code WPH11/01 Externally set and marked by Pearson Edexcel Single tier of entry	40%	20%	80	1 hour 30 minutes	January, June and October First assessment : January 2019
PAPER / UNIT 2	PERCENTAGE	PERCENTAGE OF IAL	MARK	TIME	AVAILABILITY
WAVES AND ELECTRICITY Written exam paper Paper code WPH12/01 Externally set and marked by Pearson Edexcel Single tier of entry	40%	20%	80	1 hour 30 minutes	January, June and October First assessment June 2019
PAPER / UNIT 3	PERCENTAGE	PERCENTAGE OF IAL	MARK	TIME	AVAILABILITY
PRACTICAL SKILLS IN PHYSICS 1 Written examination Paper code WPH13/01 Externally set and marked by Pearson Edexcel Single tier of entry	20%	10%	50	1 hour 20 minutes	January, June and October First assessment : June 2019

ASSESSMENT OBJECTIVES AND WEIGHTINGS

ASSESSMENT OBJECTIVE	DESCRIPTION	% IN IAS	% IN IA2	% IN IAL
A01	Demonstrate knowledge and understanding of science	34–36	29–31	32–34
A02	(a) Application of knowledge and understanding of science in familiar and unfamiliar contexts.	34–36	33–36	34–36
	(b) Analysis and evaluation of scientific information to make judgments and reach conclusions.	9–11	14–16	11–14
A03	Experimental skills in science, including analysis and evaluation of data and methods	20	20	20

RELATIONSHIP OF ASSESSMENT OBJECTIVES TO UNITS

UNIT NUMBER	ASSESSMENT OBJECTIVE			
	A01	A02 (a)	A02 (b)	A03
UNIT 1	17–18	17–18	4.5–5.5	0.0
UNIT 2	17–18	17–18	4.5–5.5	0.0
UNIT 3	0.0	0.0	0.0	20
TOTAL FOR INTERNATIONAL ADVANCED SUBSIDIARY	34–36	34–36	9–11	20

WORKING AS A PHYSICIST

Throughout your study of physics, you will develop knowledge and understanding of what it means to work scientifically. You will develop confidence in key scientific skills, such as handling and controlling quantities and units and making estimates. You will also learn about the ways in which the scientific community functions and how society as a whole uses scientific ideas.

At the end of each chapter in this book, there is a section called Thinking Bigger. These sections are based broadly on the content of the chapter just completed, but they will also draw on your previous learning from earlier in the course or from your previous studies and point towards future learning and less familiar contexts. The Thinking Bigger sections will also help you to develop transferable skills. By working through these sections, you will:

- read real-life scientific writing in a variety of contexts and aimed at different audiences
- develop an understanding of how the professional scientific community functions
- learn to think critically about the nature of what you have read
- understand the issues, problems and challenges that may be raised
- gain practice in communicating information and ideas in an appropriate scientific way
- apply your knowledge and understanding to unfamiliar contexts.

You will also gain scientific skills through the hands-on practical work that forms an essential part of your course. As well as understanding the experimental methods of the practicals, it is important that you develop the skills necessary to plan experiments and analyse and evaluate data. Not only are these very important scientific skills, but they will be assessed in your examinations.

MATHS SKILLS FOR PHYSICISTS

- **Recognise and make use of appropriate units in calculations** (*e.g. knowing the difference between base and derived units*)
- **Estimate results** (*e.g. estimating the speed of waves on the sea*)
- **Make order of magnitude calculations** (*e.g. estimating approximately what an answer should be before you start calculating, including using standard form*)
- **Use algebra to rearrange and solve equations** (*e.g. finding the landing point of a projectile*)
- **Recognise the importance of the straight line graph as an analysis tool for the verification and development of physical laws by experimentation** (*e.g. choosing appropriate variables to plot to generate a straight line graph with experimental data*)
- **Determine the slope and intercept of a linear graph** (*e.g. finding acceleration from a velocity–time graph*)
- **Calculate the area under the line on a graph** (*e.g. finding the energy stored in a stretched wire*)
- **Use geometry and trigonometry** (*e.g. finding components of vectors*)

A full-page background image of an astronaut in a white space suit floating in space. The astronaut's helmet and visor are prominent in the upper right, and their gloved hands are visible in the lower left. The background is a deep blue with some light rays or star trails.

What prior knowledge do I need?

- Understanding and knowledge of physical facts, terminology, concepts, principles and practical techniques
- Applying the concepts and principles of physics, including the applications of physics, to different contexts
- Appreciating the practical nature of physics and developing experimental and investigative skills based on the use of correct and safe laboratory techniques
- Communicating scientific methods, conclusions and arguments using technical and mathematical language
- Consideration of the implications, including benefits and risks, of scientific and technological developments
- Understanding how society uses scientific knowledge to make decisions about the implementation of technological developments
- Understanding of how scientific ideas change over time, and the systems in place to validate these changes

What will I study in this section?

- The difference between base and derived quantities and their SI units
- How to estimate values for physical quantities and use these estimates to solve problems

What will I study later?

- Knowledge and understanding of further physical facts and terminology, deeper concepts, principles and more complex practical techniques
- Practical skills and techniques for some key physics experiments
- How to communicate information and ideas in appropriate ways using appropriate terminology
- The implications of science and their associated benefits and risks
- The role of the scientific community in validating new knowledge and ensuring integrity
- The ways in which society uses science to inform decision making

1 STANDARD UNITS IN PHYSICS

LEARNING OBJECTIVES

- Understand the distinction between base and derived quantities.
- Understand the idea of a fixed system of units, and explain the SI system.

BASE AND DERIVED QUANTITIES



▲ **fig A** The international standard kilogram, officially known as the International Prototype Kilogram, is made from a mixture of platinum and iridium and is held at the Bureau International des Poids et Mesures in Paris. All other masses are defined by comparing with this metal cylinder.

Some measurements we make are of fundamental qualities of things in the universe. For example, the length of a pencil is a fundamental property of the object. Compare this with the pencil's speed if you drop it. To give a value to the speed, we have to consider a distance moved, and the rate of motion over that distance – we also need to measure time and then do a calculation. You can see that there is a fundamental difference between the types of quantity that are length and speed. We call the length a base unit, whilst the speed is a derived unit. At present, the international scientific community uses seven base units, and from these all other units are derived. Some derived units have their own names. For example, the derived unit of force should be kg m s^{-2} , but this has been named the newton (N). Other derived units do not get their own name, and we just list the base units that went together in deriving the quantity. For example, speed is measured in m s^{-1} .

BASIC QUANTITY	UNIT NAME	UNIT SYMBOL
mass	kilogram	kg
time	second	s
length	metre	m
electric current	ampere	A
temperature	kelvin	K
amount of substance	mole	mol
light intensity	candela	cd

table A The base units.

The choice of which quantities are the base ones is somewhat a matter of choice. The scientists who meet to decide on the standard unit system have chosen these seven. You might think that electric current is not a fundamental property, as it is the rate of movement of charge. So it could be derived from measuring charge and time. However, scientists had to pick what was fundamental and they chose current. This means that electric charge is a derived quantity found by multiplying current passing for a given time.

SI UNITS

For each of the base units, a meeting is held every four or six years of the General Conference on Weights and Measures, under the authority of the Bureau International des Poids et Mesures in Paris. At this meeting, they either alter the definition, or agree to continue with the current definition. As we learn more and more about the universe, these definitions are gradually moving towards the fundamental constants of nature.



▲ **fig B** A standard metre, made to be exactly the length that light could travel in $1/299\,792\,458$ of a second.

The current definition of each of the seven base units is listed below:

- The kilogram is the unit of mass; it is equal to the mass of the International Prototype Kilogram, as in **fig A**.
- The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium-133 atom.
- The metre is the length of the path travelled by light in vacuum during a time interval of $\frac{1}{299\,792\,458}$ of a second (see **fig B**).
- The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 m apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length.
- The kelvin, unit of thermodynamic temperature, is the fraction $\frac{1}{273.16}$ of the thermodynamic temperature of the triple point of water.
- The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kg of carbon-12. (When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles or specified groups of such particles.)

EXAM HINT

Table A has the complete list of SI base units. You will not be asked questions about the candela in the exam.

LEARNING TIP

'Metrology' is the study of the science of measurement, and 'metrics' refers to ways of standardising measuring techniques.

DERIVED UNITS

In **table B** you will see many of the derived units that we will study in this book, but this is only a list of those that have their own name.

DERIVED QUANTITY	UNIT NAME	UNIT SYMBOL	BASE UNITS EQUIVALENT
force	newton	N	kg m s^{-2}
energy (work)	joule	J	$\text{kg m}^2 \text{s}^{-2}$
power	watt	W	$\text{kg m}^2 \text{s}^{-3}$
frequency	hertz	Hz	s^{-1}
charge	coulomb	C	A s
voltage	volt	V	$\text{kg m}^2 \text{s}^{-3} \text{A}^{-1}$
resistance	ohm	Ω	$\text{kg m}^2 \text{s}^{-3} \text{A}^{-2}$

table B Some well known derived units.

POWER PREFIXES

Sometimes the values we have to work with for some quantities mean that the numbers involved are extremely large or small. For example, the average distance from the Earth to the sun, measured in metres, is 150 000 000 000 m. Scientists have made an easier system for writing such large values by adding a prefix to the unit which tells us that it has been multiplied by a very large or very small amount. In the Earth orbit example, the distance is equivalent to 150 billion metres, and the prefix giga- means multiply by a billion. So the Earth-sun distance becomes 150 gigametres, or 150 Gm.

FACTOR	NAME	SYMBOL	FACTOR	NAME	SYMBOL
10^1	deca-	da	10^{-1}	deci-	d
10^2	hecto-	h	10^{-2}	centi-	c
10^3	kilo-	k	10^{-3}	milli-	m
10^6	mega-	M	10^{-6}	micro-	μ
10^9	giga-	G	10^{-9}	nano-	n
10^{12}	tera-	T	10^{-12}	pico-	p
10^{15}	peta-	P	10^{-15}	femto-	f
10^{18}	exa-	E	10^{-18}	atto-	a
10^{21}	zetta-	Z	10^{-21}	zepto-	z
10^{24}	yotta-	Y	10^{-24}	yocto-	y

table C Prefixes used with SI units.

CHECKPOINT

SKILLS PROBLEM SOLVING

1. Refer to **table B** and answer the following questions:
 - (a) Pick any quantity that you have studied before and explain how its base unit equivalent is shown.
 - (b) All of the derived quantity units are named after scientists. Compare their names and abbreviations. What do you notice?
2. Write the following in standard form:
 - (a) 9.2 GW
 - (b) 43 mm
 - (c) 6400 km
 - (d) 44 ns.
3. Write the following using an appropriate prefix and unit symbol:
 - (a) 3 600 000 joules
 - (b) 31 536 000 seconds
 - (c) 10 millionths of an ampere
 - (d) 105 000 hertz.

2 ESTIMATION

LEARNING OBJECTIVES

- Estimate values for physical quantities.
- Use your estimates to solve problems.

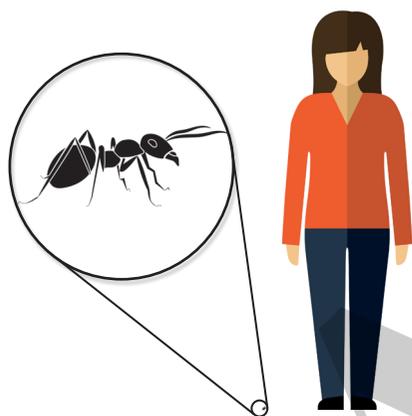
ORDER OF MAGNITUDE

In physics, it can be very helpful to be able to make approximate estimates of values to within an order of magnitude. This means that the power of ten of your estimate is the same as the true value. For example, you are the same height as the ceiling in your classroom, if we consider the order of magnitude. The ceiling may be twice your height, but it would need to be ten times bigger to reach the next order of magnitude.

This is made clearer if we express all values in standard form and then compare the power of ten. You are likely to be a thousand times taller than an ant, so we would say you are three orders of magnitude larger.

typical ant height: $1.7 \text{ mm} = 1.7 \times 10^{-3} \text{ m}$

typical human height: $1.7 \text{ m} = 1.7 \times 10^0 \text{ m}$



▲ **fig A** We are three orders of magnitude taller than an ant.

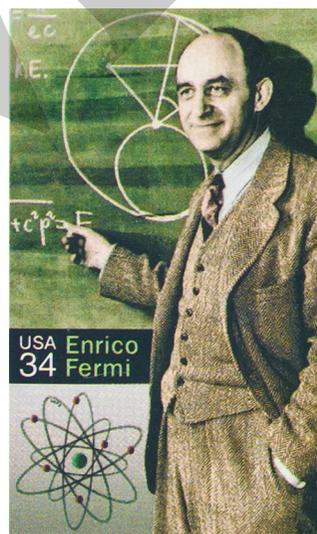
In many situations, physicists are not interested in specific answers, as circumstances can vary slightly and then the specific answer is incorrect. An order of magnitude answer will always be correct, unless you change the initial conditions by more than an order of magnitude. So a physicist could easily answer the question 'What is the fastest speed of a car?' because we don't really want to know the exact true value. To give an exact answer would depend on knowing the model of car, and the weather and road conditions, and this answer would only be correct for that car on that day. By estimating important quantities, like a typical mass for cars, we can get an approximate – order of magnitude – answer. The reason for doing so would be that it allows us to develop ideas as possible or impossible. Also it helps us focus on developing the ideas along lines that will eventually be feasible when we get to developing a specific solution. This reduces time and money wasted by following ideas that are impossible. It also

helps us quickly notice any miscalculations in an answer to a question. If we used an equation to calculate the answer to the fastest speed of a particular car in particular conditions, and the answer came out as 300 000 000 metres per second (the speed of light), we would immediately know that the answer is wrong, and re-check the calculation.

ORDER OF MAGNITUDE SCALE	TYPICAL OBJECT
$1 \times 10^{13} \text{ m}$	size of the solar system
$1 \times 10^{11} \text{ m}$	size of Earth's orbit around the sun
$1 \times 10^8 \text{ m}$	size of Moon's orbit around Earth
$1 \times 10^4 \text{ m}$	diameter of Manchester
$1 \times 10^0 \text{ m}$	human height
$1 \times 10^{-3} \text{ m}$	ant height
$1 \times 10^{-5} \text{ m}$	biological cell diameter
$1 \times 10^{-8} \text{ m}$	wavelength of ultraviolet light
$1 \times 10^{-10} \text{ m}$	diameter of an atom
$1 \times 10^{-14} \text{ m}$	diameter of an atomic nucleus

table A Examples of object scales changing with powers of ten.

FERMI QUESTIONS



▲ **fig B** Enrico Fermi was one of the developers of both nuclear reactors and nuclear bombs, along with other work on particle physics, quantum physics and statistical mechanics. He was awarded the 1938 Nobel Prize for Physics for the discovery of new radioactive elements and induced radioactivity.

Enrico Fermi was an Italian physicist who lived from 1901 to 1954. He was a pioneer of estimation. What have become known as Fermi questions are seemingly specific questions, to which

only an order of magnitude answer is expected. It is common for the question to appear very difficult, as we do not have enough information to work out the answer. One of Fermi's most interesting thought experiments was a consideration of whether or not alien life exists. Over a lunch with other scientists in 1950, Fermi surprised the group by asking 'Where is everybody?' referring to extraterrestrials. There seems to be no evidence of the existence of alien life. That is still as true today as it was in 1950. However, when Fermi made an estimation of what would be necessary for an extraterrestrial civilisation to travel to visit us, his estimate came out at a much shorter amount of time than the age of our galaxy.

Conditions and likelihood for a visit by extraterrestrials

- A planet that will support life – the galaxy holds about 100 billion stars, so there is a high probability that some other solar systems will have an Earth-like planet.
- Time to develop life – many of the stars in the galaxy are much older than the sun, so alien life developing at the same rate as our own should have been established as long as a billion years before ours.
- Time to develop interstellar travel – even if humans have to live our entire history again in order to develop spaceships that can travel to other stars, that means we will reach other stars in less than a million years of human existence.
- Time to explore the whole galaxy – from the nearest star to exploration of the whole galaxy is, by extrapolation, only a matter of a few million years.

▲ **fig C** The Fermi Paradox: even the most conservative estimates of the requirements of exploring the galaxy mean that aliens should reach Earth within ten million years of their life beginning. If they existed, they would be here.

You need to work out what steps are needed to make an estimation. First, think about what steps you would take to reach an answer, if you could have any information you wanted. Then, when the necessary data is not all available, make an estimate for the missing numbers. Making sensible assumptions is the key to solving Fermi questions.

WORKED EXAMPLE

Probably the most famous example of a Fermi question was this challenge to a class:

'How many piano tuners are there in Chicago?'

The only piece of information he provided was that the population of Chicago was 3 million.

Step 1: How many pianos in Chicago?

If each household is 4 people, then there are:

$$\frac{3\,000\,000}{4} = 750\,000 \text{ households}$$

If one household in ten owns a piano, then there are:

$$\frac{750\,000}{10} = 75\,000 \text{ pianos}$$

Step 2: How many pianos per piano tuner?

Assume each piano needs tuning once a year. Further assume a piano tuner works 200 days a year, and can service 4 pianos a day. Each tuner can service: $200 \times 4 = 800$ pianos.

Step 3: How many tuners?

Each piano tuner works with 800 pianos, and there are 75 000 pianos in total. So there are: $\frac{75\,000}{800} = 94$ piano tuners.

Your answer to Fermi would be 'There are 100 piano tuners in Chicago'. This is not expected to be the exactly correct answer, but it will be correct to order of magnitude. We would not expect to find that Chicago has only 10 piano tuners, and it would be very surprising if there were 1000.

CHECKPOINT

SKILLS ADAPTIVE LEARNING

1. Give an order of magnitude estimate for the following quantities:
 - (a) the height of a giraffe
 - (b) the mass of an apple
 - (c) the reaction time of a human
 - (d) the diameter of a planet
 - (e) the temperature in this room.
2. Answer the following Fermi questions, showing all the steps and the assumptions and estimates you make.
 - (a) How many tennis balls would fit into a soccer stadium?
 - (b) How many atoms are there in your body?
 - (c) How many drops of water are there in a swimming pool?
 - (d) In your lifetime, how much money will you make in total?
 - (e) How many Fermi questions could Enrico Fermi have answered whilst flying from Rome to New York?

TOPIC 1 MECHANICS

CHAPTER

1A

MOTION

How can we calculate how fast a plane is flying, in what direction it is going and how long it will take to reach a certain destination? If you were a pilot, how would you know what force to make the engines produce and where to direct that force so your plane moves to your destination?

An incredible number of intricate calculations need to be done to enable a successful flight, and the basis for all of them is simple mechanics.

This chapter explains the multiple movements of objects. It looks at how movement can be described and recorded, and then moves on to explaining why movement happens. It covers velocity and acceleration, including how to calculate these in different situations.

We only consider objects moving at speeds that could be encountered in everyday life. At these speeds (much less than the speed of light) Sir Isaac Newton succinctly described three laws of motion. With knowledge of basic geometry, we can identify aspects of movement in each dimension.

Newton's laws of motion have been constantly under test by scientists ever since he published them in 1687. Within the constraints established by Einstein in the early twentieth century, Newton's laws have always correctly described the relationships between data collected. You may have a chance to confirm Newton's laws in experiments of your own. With modern ICT recording of data, the reliability of such experiments is now much improved over traditional methods.

MATHS SKILLS FOR THIS CHAPTER

- Units of measurement (*e.g. the newton, N*)
- Using Pythagoras' theorem, and the angle sum of a triangle (*e.g. finding a resultant vector*)
- Using sin, cos and tan in physical problems (*e.g. resolving vectors*)
- Using angles in regular 2D structures (*e.g. interpreting force diagrams to solve problems*)
- Changing the subject of an equation (*e.g. re-arranging the kinematics equations*)
- Substituting numerical values into algebraic equations (*e.g. calculating the acceleration*)
- Plotting two variables from experimental or other data, understanding that $y = mx + c$ represents a linear relationship and determining the slope of a linear graph (*e.g. verifying Newton's second law experimentally*)
- Estimating, by graphical methods as appropriate, the area between a curve and the x -axis and realising the physical significance of the area that has been determined (*e.g. using a speed-time graph*)

What prior knowledge do I need?

- Using a stopwatch to measure times
- Measuring and calculating the speed of objects
- Gravity making things fall down
- Measuring forces, calculating resultant forces
- The motion of objects as a result of forces acting on them

What will I study in this chapter?

- The definitions of and equations for: speed, distance, displacement, time, velocity, acceleration
- Graphs of motion over time
- The classification of scalars and vectors
- Adding and resolving vectors
- Newton's laws of motion
- Kinematics equations
- Moments (turning forces)

What will I study later?

Topic 1B

- Kinetic energy and gravitational potential energy
- Transferring between gravitational potential energy and kinetic energy
- Work and power

Topic 1C

- Momentum and the principle of conservation of momentum

Topic 2A

- Fluid movements and terminal velocity

Topic 3A

- Wave movements

Topic 5 (Book 2: IAL)

- The meaning and calculation of impulse (A level)

LEARNING OBJECTIVES

- Explain the distinction between scalar and vector quantities.
- Distinguish between speed and velocity and define acceleration.
- Calculate values using equations for velocity and acceleration.



▲ **fig A** These runners are accelerating to a high speed.

Movement is fundamental to the functioning of our universe. Whether you are running to catch a bus or want to calculate the speed required for a rocket to travel to Mars or the kinetic energy of an electron in an X-ray machine, you need to be able to work out how fast things are moving.

RATE OF MOVEMENT

One of the simplest things we can measure is how fast an object is moving. You can calculate an object's **speed** if you know the amount of time taken to move a certain distance:

$$\text{speed (m s}^{-1}\text{)} = \frac{\text{distance (m)}}{\text{time (s)}}$$

$$v = \frac{d}{t}$$

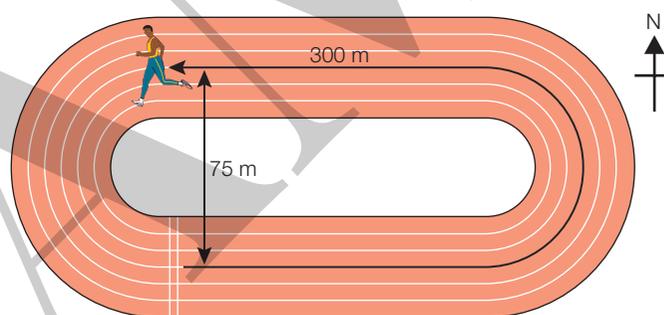
However, the calculation for speed will only tell you how fast an object is moving. Often it is also vitally important to know in what direction this movement is taking the object. When you include the direction in the information about the rate of movement of an object, this is then known as the **velocity**. So, the velocity is the rate of change of **displacement**, where the distance in a particular direction is called the 'displacement'.

$$\text{velocity (m s}^{-1}\text{)} = \frac{\text{displacement (m)}}{\text{time (s)}}$$

$$v = \frac{s}{t}$$

OR

$$v = \frac{\Delta s}{\Delta t}$$



▲ **fig B** The displacement due north is only 75 m, whilst the actual distance this athlete has run is 300 m. So the velocity due north is much less than the actual speed.

LEARNING TIP

The upper case Greek letter delta, Δ , is used in mathematics to indicate a change in a quantity. For example, Δs means the change in the displacement of an object, to be used here to calculate the velocity of the object.

DID YOU KNOW?

The froghopper, a 6 mm long insect, can accelerate at $4\,000\text{ m s}^{-1}$.

LEARNING TIP

Vector notation means that vectors are written in **bold** type to distinguish them from scalar variables.

A quantity for which the direction must be stated is known as a **vector**. If direction is not important, the measurement is referred to as a **scalar** quantity. Therefore, velocity is a vector and speed is a scalar; distance is a scalar and displacement is a vector.

Scalar and vector quantities are not limited to measurements related to movement. Every measured quantity can be classified to include the direction (vector, e.g. force) or as being sufficiently stated by its magnitude only (scalar, e.g. mass).

AVERAGE AND INSTANTANEOUS SPEED

In most journeys, it is unlikely that speed will remain constant throughout. As part of his training programme, an athlete in **fig A** wants to keep a record of his speed for all races. From rest, before the starting gun starts the race, he accelerates to a top speed. However, the race timing will be made from start to finish, and so it is most useful to calculate an average speed over the whole race. **Average speed** is calculated by dividing the total distance for a journey by the total time for the journey. Thus it averages out the slower and faster parts of the journey, and even includes stops.

Instantaneous speed can be an important quantity, and we will look at how to measure it in the next topic.



▲ **fig C** Most speed checks look at instantaneous speed, but CCTV allows police to monitor average speed over a long distance.

ACCELERATION

Acceleration is defined as the rate of change of velocity. Therefore, it must include the direction in which the speed is changing, and so acceleration is a vector quantity. The equation defining acceleration is:

$$\text{acceleration (m s}^{-2}\text{)} = \frac{\text{change in velocity (m s}^{-1}\text{)}}{\text{time taken to change the velocity (s)}}$$

$$a = \frac{v - u}{t}$$

OR

$$a = \frac{\Delta v}{\Delta t}$$

where **u** is the initial velocity and **v** is the final velocity.

The vector nature of acceleration is very important. One of the consequences is that if an object changes only the direction of its velocity, it is accelerating, *while remaining at a constant speed*. Similarly, deceleration represents a negative change in velocity, and so could be stated as a negative acceleration.

EXAM HINT

Whilst accelerations can (very briefly) be extraordinarily high, like that for the electron in question 3(b), no speed or velocity can ever be greater than the speed of light, which is $3 \times 10^8 \text{ m s}^{-1}$. If you calculate a speed that is higher than this, check your calculation again as it must be wrong.

CHECKPOINT

SKILLS PROBLEM SOLVING

- The athlete in **fig B** has taken 36 seconds from the start to reach the 300 m mark as shown. Calculate:
 - his average speed during this 36 seconds
 - his average velocity due north during this 36 seconds
 - his average velocity due east during this 36 seconds.
- A driver in a car travelling at about 40.2 km h^{-1} sees a cat run onto the road ahead.
 - Convert 40.2 km h^{-1} into a speed in m s^{-1} .
 - The car travels 16.5 m whilst the driver is reacting to the danger. What is his reaction time?
 - The car comes to a stop in 2.5 s. What is its deceleration?
- An electron in an X-ray machine is accelerated from rest to half the speed of light in $1.7 \times 10^{-15} \text{ s}$. Calculate:
 - the speed the electron reaches in m s^{-1}
 - the acceleration the electron experiences.

SUBJECT VOCABULARY

speed the rate of change of distance:

$$\text{speed (m s}^{-1}\text{)} = \frac{\text{distance (m)}}{\text{time (s)}}$$

$$v = \frac{d}{t}$$

velocity the rate of change of displacement:

$$\text{velocity (m s}^{-1}\text{)} = \frac{\text{displacement (m)}}{\text{time (s)}}$$

$$v = \frac{s}{t} \text{ OR } v = \frac{\Delta s}{\Delta t}$$

displacement the vector measurement of distance in a certain direction

vector a quantity that must have both magnitude and direction

scalar a quantity that has magnitude only

average speed speed for a whole journey, calculated by dividing the total distance for a journey by the total time for the journey:

$$\text{average speed (m s}^{-1}\text{)} = \frac{\text{total distance (m)}}{\text{total time (s)}}$$

instantaneous speed the speed at any particular instant in time on a journey, which can be found from the gradient of the tangent to a distance–time graph at that time

acceleration the vector defined as the rate of change of velocity:

$$\text{acceleration (m s}^{-2}\text{)} = \frac{\text{change in velocity (m s}^{-1}\text{)}}{\text{time taken to change the velocity (s)}}$$

$$a = \frac{v - u}{t} \text{ OR } a = \frac{\Delta v}{\Delta t}$$

LEARNING OBJECTIVES

- Interpret displacement–time graphs, velocity–time graphs and acceleration–time graphs.
- Make calculations from these graphs.
- Understand the graphical representations of accelerated motion.

One of the best ways to understand the movements of an object whilst on a journey is to plot a graph of the position of the object over time. Such a graph is known as a displacement–time graph. A **velocity–time graph** will also provide detail about the movements involved. A velocity–time graph can be produced from direct measurements of the velocity or generated from calculations made using the displacement–time graph.

DISPLACEMENT–TIME GRAPHS

If we imagine a boat trip on a river, we could monitor the location of the boat over the hour that it travels for and plot the displacement–time graph for these movements. Depending on what information we want the graph to provide, it is often simpler to draw a distance–time graph in which the direction of movement is ignored.

The graphs shown in **fig A** are examples of plotting position against time, and show how a distance–time graph cannot decrease with time. A displacement–time graph could have parts of it in the negative portions of the y -axis, if the movement went in the opposite direction at some points in time.

The simplest thing we could find from these graphs is how far an object has moved in a certain time. For example, in **fig A**, both the graphs show that in the first 15 minutes the boat moved 150 m. Looking at the time from 40 to 48 minutes, both show that the boat travelled 120 m, but the displacement–time graph is in the negative region of the y -axis, showing the boat was moving down river from the starting point – the opposite direction to the places it had been in the first 40 minutes.

During the period from 20 to 25 minutes, both graphs have a flat line at a constant value, that shows no change in the distance or displacement. This means the boat was not moving – a flat line on a distance–time ($d-t$) graph means the object is stationary. From 20 to 25 minutes on the velocity–time ($v-t$) graph of this journey (see **fig B**) the line would be at a velocity of 0 m s^{-1} .

SPEED AND VELOCITY FROM $d-t$ GRAPHS

The **gradient** of the $d-t$ graphs in **fig A** will tell us how fast the boat was moving. Gradient is found from the ratio of changes in the y -axis divided by the corresponding change on the x -axis, so:

for a distance–time graph:

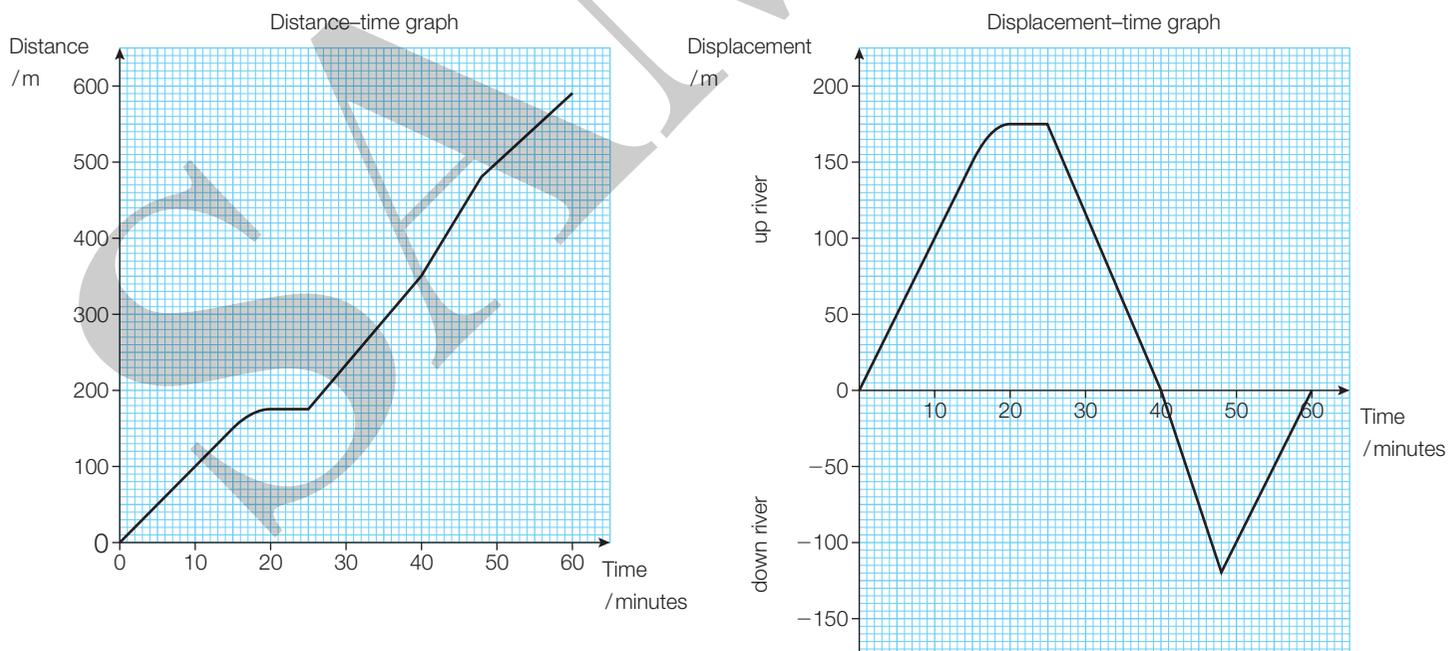
$$\text{gradient} = \frac{\text{distance (m)}}{\text{time (s)}} = \text{speed (m s}^{-1}\text{)}$$

$$v = \frac{d}{t}$$

for a displacement–time graph:

$$\text{gradient} = \frac{\text{displacement (m)}}{\text{time (s)}} = \text{velocity (m s}^{-1}\text{)}$$

$$v = \frac{\Delta s}{\Delta t}$$



▲ **fig A** A comparison of the displacement–time graph of the boat trip up and down a river with its corresponding distance–time graph.

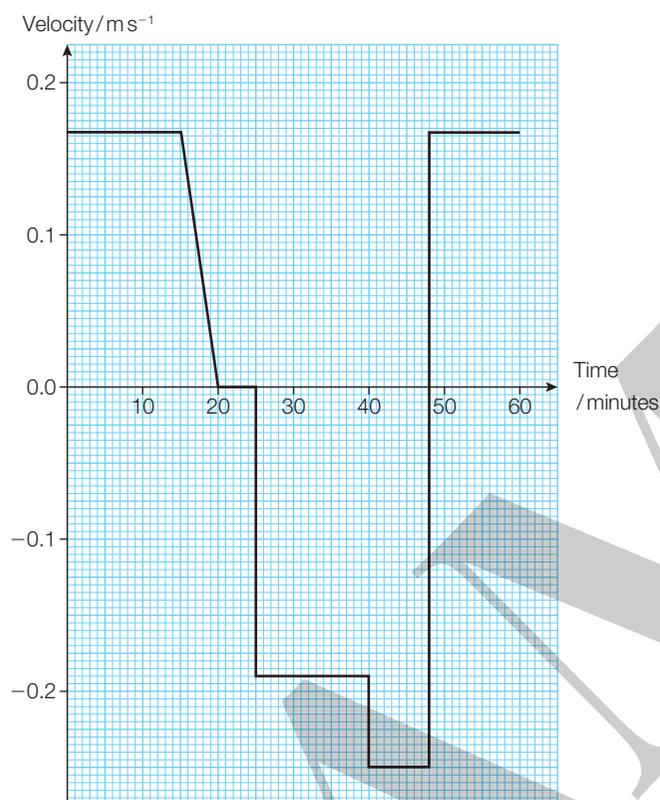
For example, the first 15 minutes of the boat trip in **fig A** represents a time of 900 seconds. In this time, the boat travelled 150 m. Its velocity is:

$$v = \frac{\Delta s}{\Delta t} = \frac{150}{900} = 0.167 \text{ m s}^{-1} \text{ up river}$$

VELOCITY–TIME GRAPHS

A velocity–time graph will show the velocity of an object over time. We calculated that the velocity of the boat on the river was 0.167 m s^{-1} up river for the first 15 minutes of the journey. Looking at the graph in **fig B**, you can see that the line is constant at $+0.167 \text{ m s}^{-1}$ for the first 15 minutes.

Also notice that the velocity axis includes negative values, so that the difference between travelling up river (positive y -axis values) and down river (negative y -axis values) can be represented.



▲ **fig B** Velocity–time graph of the boat trip.

ACCELERATION FROM v – t GRAPHS

Acceleration is defined as the rate of change in velocity.

In order to calculate the gradient of the line on a v – t graph, we must divide a change in velocity by the corresponding time difference. This exactly matches with the equation for acceleration:

$$\text{gradient} = \frac{\Delta v}{\Delta t} = \frac{v - u}{t} = \text{acceleration}$$

For example, between 15 and 20 minutes on the graphs, the boat slows evenly to a stop. The acceleration here can be calculated as the gradient:

$$\text{gradient} = \frac{\Delta v}{\Delta t} = \frac{v - u}{t} = \frac{0 - 0.167}{5 \times 60} = \frac{-0.167}{300} = -0.0006 \text{ m s}^{-2}$$

So the acceleration is: $a = -0.6 \times 10^{-3} \text{ m s}^{-2}$.

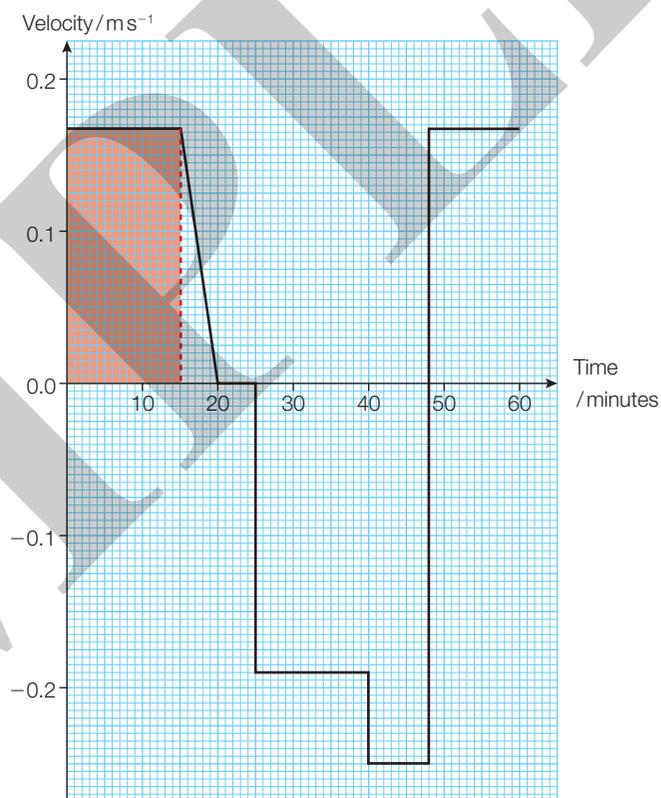
DISTANCE TRAVELLED FROM v – t GRAPHS

Speed is defined as the rate of change in distance:

$$v = \frac{d}{t}$$

$$\therefore d = v \times t$$

As the axes on the v – t graph represent velocity and time, an area on the graph represents the multiplication of velocity \times time, which gives distance. So to find the distance travelled from a v – t graph, find the area between the line and the x -axis.



▲ **fig C** In the first 15 minutes (900 seconds) the distance travelled by the boat moving at 0.167 m s^{-1} is given by the area between the line and the x -axis: $d = v \times t = 0.167 \times 900 = 150 \text{ m}$.

If we are only interested in finding the distance moved, this also works for a negative velocity. You find the area from the line up to the time axis. This idea will still work for a changing velocity. Find the area under the line and you have found the distance travelled. For example, from 0 to 20 minutes, the area under the line, all the way down to the x -axis, is a trapezium, so we need to find that area. To calculate the whole distance travelled in the journey for the first 40 minutes, we would have to find the areas under the four separate stages (0–15 minutes; 15–20 minutes; 20–25 minutes; and 25–40 minutes) and then add these four answers together.

PRACTICAL SKILLS

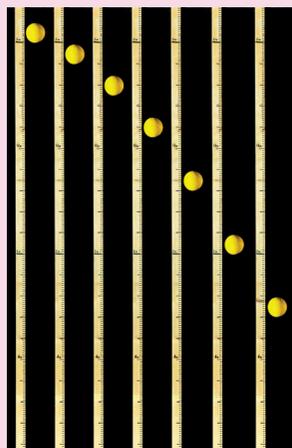
CP1

Finding the acceleration due to gravity by multiflash photography

Using a multiflash photography technique, or a video recording that can be played back frame by frame, we can observe the falling motion of a small object such as a marble (see **fig D**). We need to know the time between frames.

From each image of the falling object, measure the distance it has fallen from the scale in the picture. A carefully drawn distance–time graph will show a curve as the object accelerates. From this curve, take regular measurements of the gradient by drawing tangents to the curve. These gradients show the instantaneous speed at each point on the curve.

Plotting these speeds on a velocity–time graph should show a straight line, as the acceleration due to gravity is a constant value. The gradient of the line on this v – t graph will be the acceleration due to gravity, g .



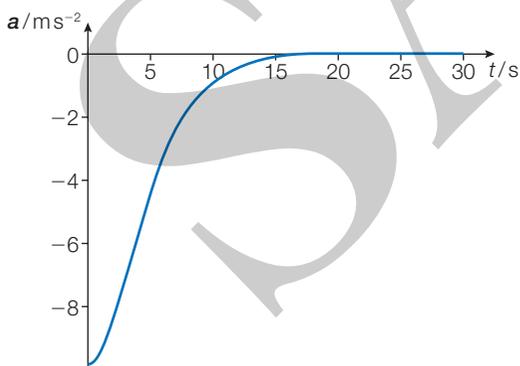
▲ **fig D** Multiflash photography allows us to capture the accelerating movement of an object falling under gravity.

Safety Note: Persons with medical conditions such as epilepsy or migraine may be adversely affected by multiflash photography.

ACCELERATION–TIME GRAPHS

Acceleration–time graphs show how the acceleration of an object changes over time. In many instances the acceleration is zero or a constant value, in which case an acceleration–time (a – t) graph is likely to be of relatively little interest. For example, the object falling in our investigation above will be accelerated by gravity throughout. Assuming it is relatively small, air resistance will be minimal, and the a – t graph of its motion would be a horizontal line at $a = -9.81 \text{ m s}^{-2}$. Compare this with your results to see how realistic it is to ignore air resistance.

For a larger object falling for a long period, such as a skydiver, then the acceleration will change over time as the air resistance increases with speed.



▲ **fig E** Acceleration–time graph for a skydiver.

The weight of a skydiver is constant, so the resultant force will be decreasing throughout, which means that the acceleration will also reduce (see **Section 1A.5**). The curve would look like that in **fig E**. See **Section 2A.4** for more details on falling objects and terminal velocity.

EXAM HINT

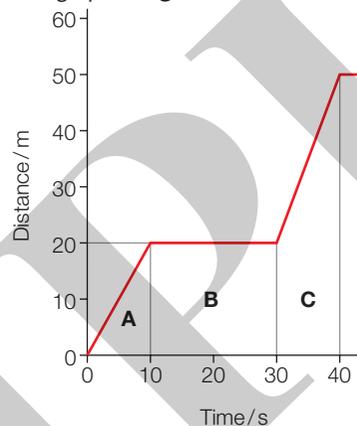
Remember that the gradient of a distance–time graph represents speed or velocity. So if the line is curved, the changing gradient indicates a changing speed, which you can describe as the same as the changes in gradient.

CHECKPOINT

SKILLS

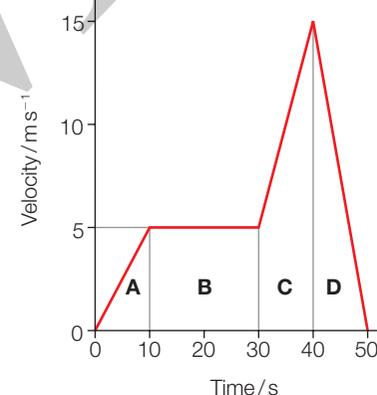
INTERPRETATION
(OF GRAPHICAL DATA)

- Describe in as much detail as you can, including calculated values, what happens in the bicycle journey shown on the d – t graph in **fig F**.



▲ **fig F** Distance–time graph of a bike journey.

- Describe in as much detail as you can, including calculated values, what happens in the car journey shown on the v – t graph in **fig G**.



▲ **fig G** Velocity–time graph of a car journey.

- From **fig B**, calculate the distance travelled by the boat from 40 to 60 minutes.

SUBJECT VOCABULARY

displacement–time graph a graph showing the positions visited on a journey, with displacement on the y -axis and time on the x -axis.

velocity–time graph a graph showing the velocities on a journey, with velocity on the y -axis and time on the x -axis.

gradient the slope of a line or surface

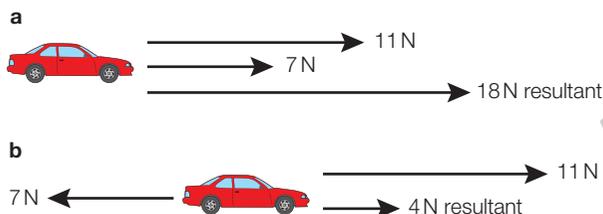
LEARNING OBJECTIVES

- Add two or more vectors by drawing.
- Add two perpendicular vectors by calculation.

Forces are vectors. This means that measuring their magnitude is important, but equally important is knowing the direction in which they act. In order to calculate the overall effect of multiple forces acting on the same object, we can use vector addition to work out the **resultant force**. This resultant force can be considered as a single force that has the same effect as all the individual forces combined.

ADDING FORCES IN THE SAME LINE

If two or more forces are acting along the same line, then combining them is simply a case of adding or subtracting their magnitudes depending on their directions.



▲ **fig A** Adding forces in the same line requires a consideration of their comparative directions.

ADDING PERPENDICULAR FORCES

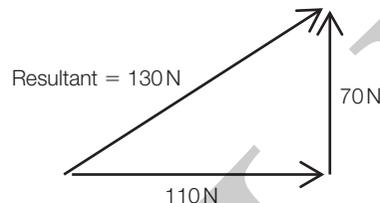
The effect on an object of two forces that are acting at right angles (perpendicular) to each other will be the vector sum of their individual effects. We need to add the sizes with consideration for the directions in order to find the resultant.



▲ **fig B** These two rugby players are each putting a force on their opponent. The forces are at right angles, so the overall effect would be to move him in a third direction, which we could calculate.

MAGNITUDE OF THE RESULTANT FORCE

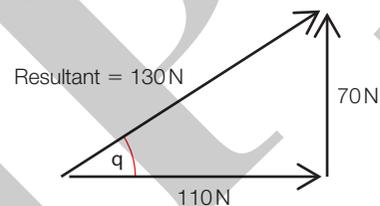
To calculate the resultant magnitude of two perpendicular forces, we can draw them, one after the other, as the two sides of a right-angled triangle and use Pythagoras' theorem to calculate the size of the hypotenuse.



▲ **fig C** The resultant force here is calculated using Pythagoras' theorem:
 $F = \sqrt{(70^2 + 110^2)} = 130 \text{ N}$

DIRECTION OF THE RESULTANT FORCE

As forces are vectors, when we find a resultant force it must have both magnitude and direction. For perpendicular forces (vectors), trigonometry will determine the direction.



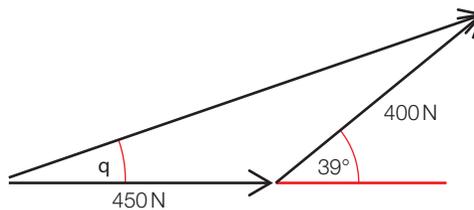
▲ **fig D** The resultant force here is at an angle up from the horizontal of:
 $\theta = \tan^{-1}\left(\frac{70}{110}\right) = 32^\circ$

EXAM HINT

Always take care to state where the angle for a vector's direction is measured. For example, in **fig D**, the angle should be stated as 32° up from the horizontal. This is most easily expressed on a diagram of the situation, where you draw in the angle.

ADDING TWO NON-PERPENDICULAR FORCES

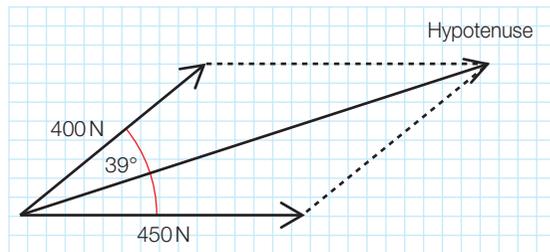
The geometry of perpendicular vectors makes the calculation of the resultant simple. We can find the resultant of any two vectors by drawing one after the other, and then the resultant will be the third side of the triangle from the start of the first one to the end of the second one. A scale drawing of the vector triangle will allow measurement of the size and direction of the resultant.



▲ **fig E** The resultant force here can be found by scale drawing the two forces, and then measurement of the resultant on the drawing using a ruler and a protractor.

THE PARALLELOGRAM RULE

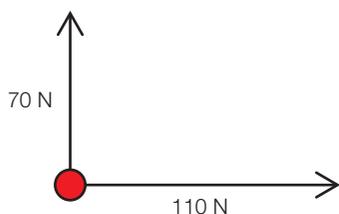
There is another method for finding the resultant of two non-perpendicular forces (or vectors) by scale drawing, which can be easier to use. This is called the parallelogram rule. Draw the two vectors to scale – at the correct angle and scaled so their length represents the magnitude – starting from the same point. Then draw the same two vectors again parallel to the original ones, so that they form a parallelogram, as shown in **fig F**. The resultant force (or vector) will be the diagonal across the parallelogram from the starting point.



▲ **fig F** Finding the resultant vector using the parallelogram rule.

EXAM HINT

The vector addition rules shown on these pages work for all vectors, not just forces. They are useful only for co-planar vectors, which means vectors that are in the same plane. If we have more than two vectors that are in more than one plane, add two vectors together first, in their plane, and then add the resultant to the next vector using these rules again. Keep doing this until all the vectors have been added in.



▲ **fig G** Free-body force diagram of a rugby player (red circle). The forces from the tacklers are marked on as force arrows.

FREE-BODY FORCE DIAGRAMS

If we clarify what forces are acting on an object, it can be simpler to calculate how it will move. To do this, we usually draw a **free-body force diagram**, which has the object isolated, and all the forces that act on it drawn in at the points where they act. Forces acting on other objects, and those other objects, are not drawn. For example, **fig G** could be said to be a free-body force diagram of the rugby player being tackled in **fig B**, and this would lead us to draw **fig C** and **fig D** to make our resultant calculations.

SKILLS ANALYSIS

CHECKPOINT

- Work out the resultant force on a toy car if it has the following forces acting on it:
 - rubber band motor driving forwards 8.4 N
 - air resistance 0.5 N
 - friction 5.8 N
 - child's hand pushing forward 10 N.
- As a small plane accelerates to take off, the lift force on it is 6000 N vertically upwards, whilst the thrust is 2800 N horizontally forwards. What is the resultant of these forces on the plane?
- Draw a free-body force diagram of yourself sitting on your chair.
- Draw the scale diagram of **fig E**, and work out what the resultant force would be.
 - Use the parallelogram rule, as in **fig F**, to check your answer to part (a).
- In order to try and recover a car stuck in a muddy field, two tractors pull on it. The first acts at an angle of 20° left of the forwards direction with a force of 2250 N. The second acts 15° to the right of the forwards direction with a force of 2000 N. Draw a scale diagram of the situation and find the resultant force on the stuck car.

SUBJECT VOCABULARY

resultant force the total force (vector sum) acting on a body when all the forces are added together accounting for their directions

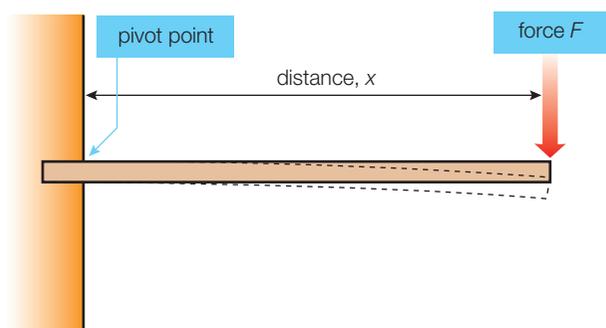
free-body force diagram diagram showing an object isolated, and all the forces that act on it are drawn in at the points where they act, using arrows to represent the forces

LEARNING OBJECTIVES

- Calculate the moment of a force.
- Apply the principle of moments.
- Find the centre of gravity of an object.

Forces on an object could act so that the object does not start to move along, but instead rotates about a fixed pivot. If the object is fixed so that it cannot rotate, it will bend.

THE MOMENT OF A FORCE

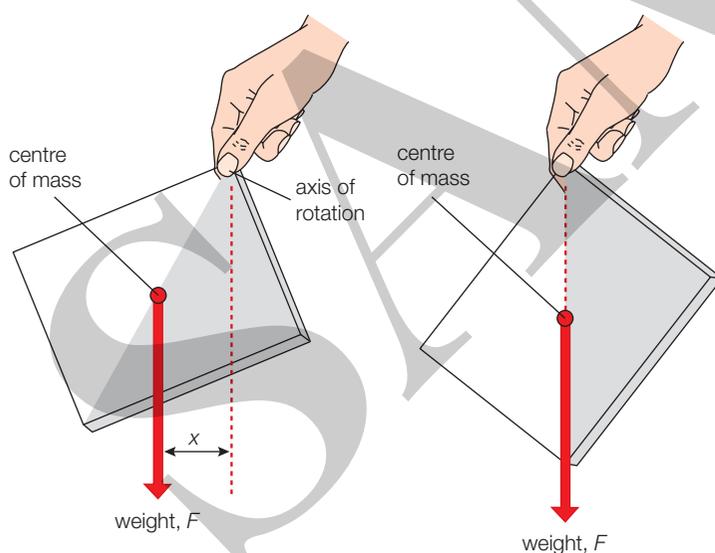


▲ **fig A** A force acts on a beam fixed at a point. The moment of a force causes rotation or, in this case, bending.

The tendency to cause rotation is called the moment of a force. It is calculated from:

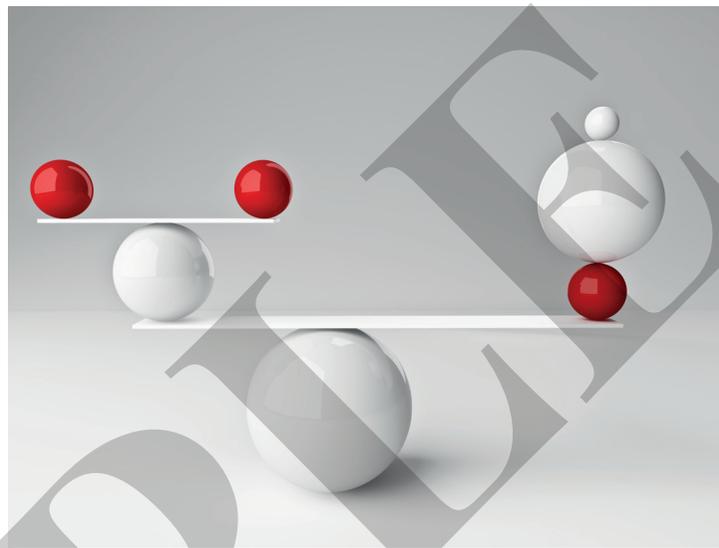
$$\text{moment (Nm)} = \text{force (N)} \times \text{perpendicular distance from the pivot to the line of action of the force (m)}$$

$$\text{moment} = Fx$$



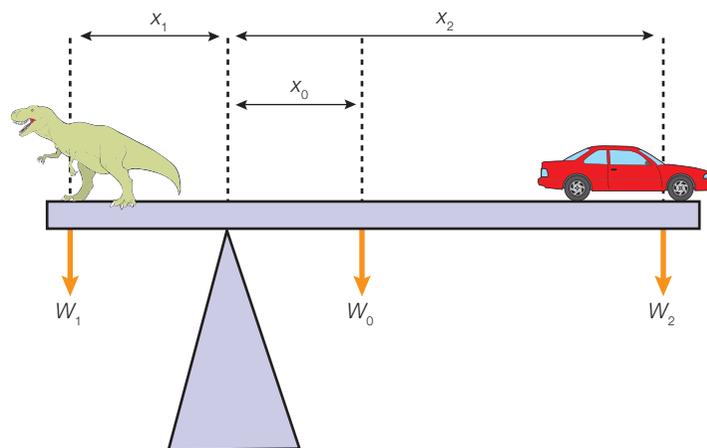
▲ **fig B** The calculation of moment only considers the perpendicular distance between the line of action of the force and the axis of rotation, through the pivot point. When free to rotate, a body will turn in the direction of any net moment.

PRINCIPLE OF MOMENTS



▲ **fig C** Balanced moments create an equilibrium situation.

If we add up all the forces acting on an object and the resultant force, accounting for their directions, is zero, then the object will be in **equilibrium**. Therefore it will remain stationary or, if it is already moving, it will carry on moving at the same velocity. The object could keep a constant velocity, but if the moments on it are not also balanced, it could be made to start rotating. The **principle of moments** tells us that if the total of all the moments trying to turn an object clockwise is equal to the total of all moments trying to turn an object anticlockwise, then it will be in rotational equilibrium. This means it will either remain stationary, or if it is already rotating it will continue at the same speed in the same direction.



▲ **fig D** As the metre-long beam is balanced, the sum of all the clockwise moments must equal the sum of all the anticlockwise moments.

LEARNING TIP

The clockwise moments and the anticlockwise moments must all be taken about the same pivot point.

EXAM HINT

Note that the steps and layout of the solution in this worked example are suitable for moments questions in the exam.

WORKED EXAMPLE

In **fig D**, we can work out the weight of the beam if we know all the other weights and distances. The beam is uniform, so its weight will act from its centre. The length of the beam is 100 cm. So if $x_1 = 20$ cm, then x_0 must be 30 cm, and $x_2 = 80$ cm. The dinosaur (W_1) weighs 5.8 N and the toy car's weight (W_2) is 0.95 N.

In equilibrium, principle of moments: sum of anticlockwise moments = sum of clockwise moments

$$\begin{aligned} W_1 x_1 &= W_0 x_0 + W_2 x_2 \\ 5.8 \times 0.20 &= W_0 \times 0.30 + 0.95 \times 0.80 \\ \therefore W_0 &= \frac{1.16 - (0.76)}{0.30} \\ W_0 &= 1.3 \text{ N} \end{aligned}$$

LEARNING TIP

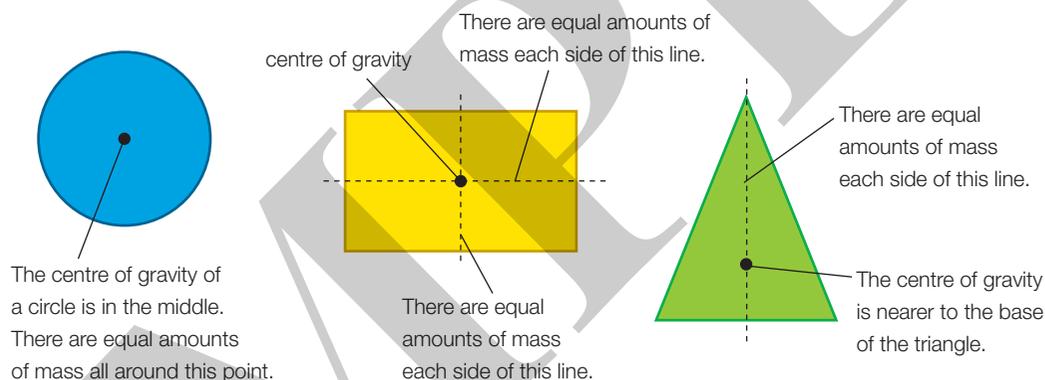
In order to calculate the sum of the moments in either direction, each individual moment must be calculated first and these individual moments then added together. The weights and/or distances *cannot* be added together and this answer used to calculate some sort of combined moment.

LEARNING TIP

You can consider the terms 'centre of gravity' and 'centre of mass' to mean the same thing. They are exactly the same for objects that are small compared to the size of the Earth.

CENTRE OF GRAVITY

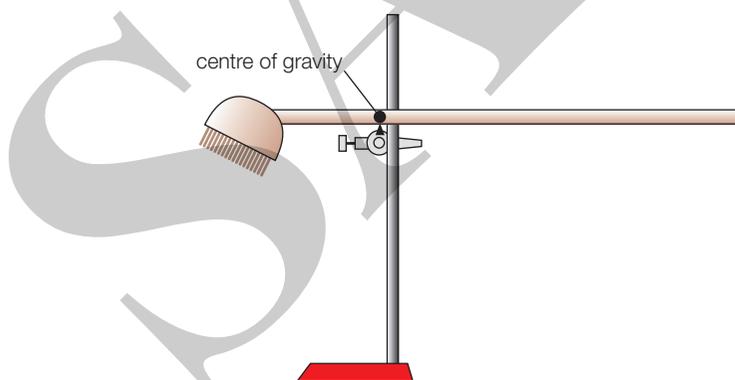
The weight of an object is caused by the gravitational attraction between the Earth and each particle contained within the object. The sum of all these tiny weight forces appears to act from a single point for any object, and this point is called the **centre of gravity**. For a symmetrical object, we can calculate the position of its centre of gravity, as it must lie on every line of symmetry. The point of intersection of all lines of symmetry will be the centre of gravity. **Fig E** illustrates this with two-dimensional shapes, but the idea can be extended into three dimensions. For example, the centre of gravity of a sphere is at the sphere's centre.



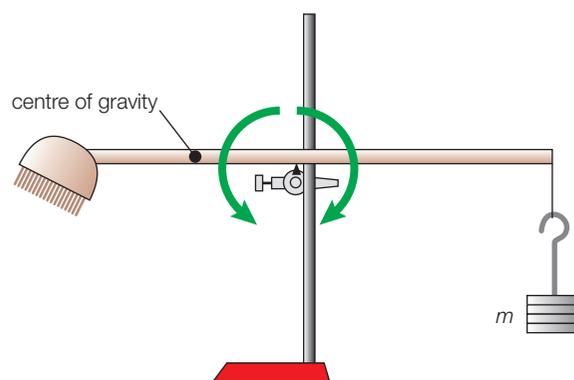
▲ **fig E** The centre of gravity of a symmetrical object lies at the intersection of all lines of symmetry.

IRREGULAR OBJECTS

The centre of gravity of an irregularly shaped object will still follow the rule that it is the point at which its weight appears to act on the object. A Bunsen burner, for example, has a heavy base. As such, the centre of gravity is low down near that concentration of mass, as there will be a greater attraction by the Earth's gravity to this large mass.



▲ **fig F** Balancing a broom on its centre of gravity.



▲ **fig G** Finding the centre of gravity of an irregular rod (broom).

PRACTICAL SKILLS

Finding the centre of mass of an irregular rod

In this investigation, we use the principle of moments to find the centre of gravity of a broom. It is not easy to estimate/determine the (location of the) centre of gravity by looking at a broom because it is not a symmetrical object. With the extra mass at the brush head end, the centre of gravity will be nearer that end.

If you can balance the broom on the edge of a thick metal ruler, then the centre of gravity must lie above the ruler edge. As the perpendicular distance from the line of action to the weight is zero, the moment is zero so the broom sits in equilibrium.

You will probably find it difficult to balance the broom exactly, so you can use an alternative method. First you measure the mass of the broom (M) using a digital balance. Then you use a set of hanging masses (of mass m) to balance the broom more in the middle of the handle, as in **fig G**. When the broom is balanced, you measure the distance (d) from the hanging masses to the pivot. You calculate the distance (x) from the pivot to the centre of gravity of the broom using the principle of moments:

clockwise moment = anticlockwise moment

$$mg \times d = Mg \times x$$

$$\therefore x = \frac{md}{M}$$

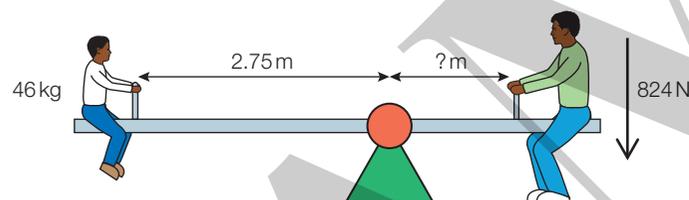
Note: Do not get into the habit of using only the mass in moments calculations, as the definition is *force* times distance. It just happens that in this case g cancels on each side.



Safety Note: Securely clamp the base of the stand to the bench. Do not use a very heavy broom as you would need a heavier counterweight which could fall and cause injury.

CHECKPOINT

1. What is the moment of a 252 N force acting on a solid object at a perpendicular distance of 1.74 m from an axis of rotation of the object?
2. A child and his father are playing on a seesaw, see **fig H**. They are exactly balanced when the boy (mass 46 kg) sits at the end of the seesaw, 2.75 m from the pivot. If his father weighs 824 N, how far is he from the pivot?



▲ **fig H**

3. The weight of the exercise book in the left-hand picture in **fig B** causes a rotation so it moves towards the second position. Explain why it does not continue rotating but comes to rest in the position of the second picture.
4. If the same set-up as shown in **fig D** was used again, but the toy car was replaced with a banana weighing 1.4 N, find out where the banana would have to be positioned for the beam to balance – calculate the new x_3 .

SUBJECT VOCABULARY

equilibrium the situation for a body where there is zero resultant force and zero resultant moment. It will have zero acceleration

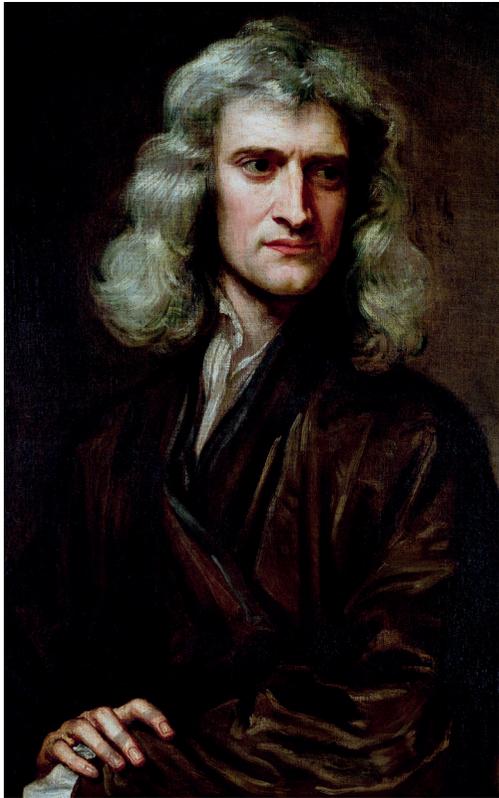
principle of moments a body will be in equilibrium if the sum of clockwise moments acting on it is equal to the sum of the anticlockwise moments

centre of gravity the point through which the weight of an object appears to act

LEARNING OBJECTIVES

- Recall Newton's laws of motion and use them to explain the acceleration of objects.
- Make calculations using Newton's second law of motion.
- Identify pairs of forces involved in Newton's third law of motion.

Sir Isaac Newton was an exceptional thinker and scientist. His influence over science in the West is still enormous, despite the fact that he lived from 1642 to 1727. He was a professor at Cambridge University, a member of the British Parliament, and a president of the respected scientific organisation, the Royal Society, in London. Probably his most famous contribution to science was the development of three simple laws governing the movement of objects subject to forces.



▲ **fig A** A portrait painting of Sir Isaac Newton

NEWTON'S FIRST LAW OF MOTION

If an object is stationary there needs to be a resultant force on it to make it move. We saw how to calculate resultant forces in **Section 1A.3**. If the object is already moving then it will continue at the same speed in the same direction unless a resultant force acts on it. If there is no resultant force on an object – either because there is zero force acting or all the forces balance out – then the object's motion is not changed.

NEWTON'S SECOND LAW OF MOTION

This law tells us how much an object's motion will be changed by a resultant force. For an object with constant mass, it is usually written mathematically:

$$\Sigma F = ma$$

$$\text{resultant force (N)} = \text{mass (kg)} \times \text{acceleration (m s}^{-2}\text{)}$$

For example, this relationship allows us to calculate the acceleration due to gravity (g) if we measure the force (F) accelerating a mass (m) downwards.

$$F = ma = mg$$

$$\therefore g = \frac{F}{m}$$



▲ **fig B** A stationary object will not move unless it is acted upon by a resultant force.

PRACTICAL SKILLS

Newton's second law investigation

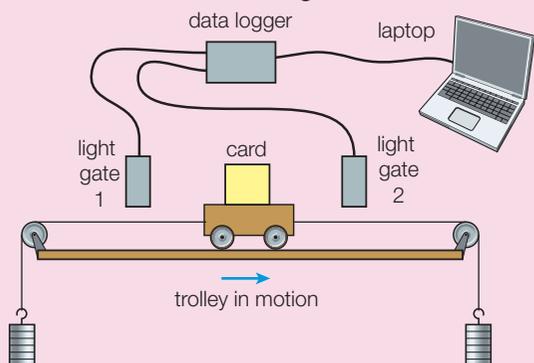


fig C Experimental set-up for investigating the relationship between F , m and a .

You can use the set-up shown in **fig C**, or a similar experiment on an air track. This set-up measures the acceleration for various values of the resultant force that acts on the trolley whilst you keep its mass constant (**table A**). By plotting a graph of acceleration against resultant force, a straight line will show that acceleration is proportional to the resultant force. You could also plot a graph for varying masses of trolley whilst you keep the resultant force constant (**table B**).



Safety Note: Place heavy trolleys and runways where they cannot slide or fall off benches and cause injuries. Place air track 'blowers' on the floor. Secure the hose so that it cannot come loose and blow dirt and dust into eyes.

FORCE / N	ACCELERATION / m s^{-2}
0.1	0.20
0.2	0.40
0.3	0.60
0.4	0.80
0.5	1.00
0.6	1.20

table A Values of acceleration for different forces acting on a trolley.

MASS / kg	ACCELERATION / m s^{-2}
0.5	1.00
0.6	0.83
0.7	0.71
0.8	0.63
0.9	0.55
1.0	0.50

table B Values of acceleration resulting from an applied force of 0.5 N when the mass of the trolley is varied.

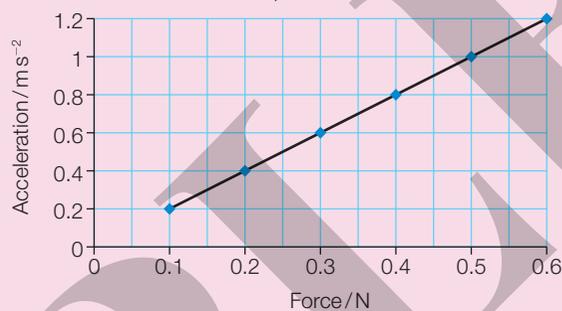


fig D Graph of results from the first investigation into Newton's second law. Acceleration is directly proportional to force.

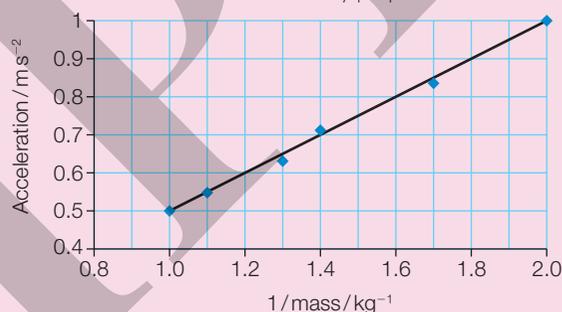


fig E Graph of results from the second investigation into Newton's second law. The x-axis represents the derived data of '1/mass', so the straight best-fit line shows that acceleration is inversely proportional to mass. Note that there is no need for a graph to start at the origin: choose axes scales that will best show the pattern in the data by making the points fill the graph paper.

Experimental verification of Newton's second law is well established. The investigation shown in **fig C** demonstrates that: $a = \frac{F}{m}$

LEARNING TIP

Straight-line graphs

Physicists always try to arrange their experimental data into graphs that produce a straight best-fit line. This proves a linear relationship between the experimental variables, and can also give us numerical information about the quantities involved.

The equation for a straight line is:

$$y = mx + c$$

' m ' in the equation is the gradient of the straight line, and ' c ' is the value on the y-axis where the line crosses it, known as the y-intercept.

If we plot experimental data on a graph and get a straight best-fit line, then this proves the quantities we plotted on x and y have a linear relationship. It is referred to as 'directly proportional' (or simply 'proportional') only if the line also passes through the origin, meaning that $c = 0$.

The graphs in **fig D** and **fig E** demonstrate experimental verification of Newton's second law. In each case, the third variable in the equation was kept constant as a control variable. For example, in **fig D** the straight best-fit line, showing that $y \propto x$, proves that $a \propto F$, and the gradient of the best-fit line would represent the reciprocal of the mass that was accelerated and was kept constant throughout, as a control variable. In this example $c = 0$, which means that the proportional relationships are simple:

$$y = mx$$

$$a = \frac{F}{m}$$

As both the above equations represent the graph in **fig D**, it follows that the gradient, m , equals the reciprocal of the mass, $1/m$. It is just coincidence that the symbol ' m ' for gradient, and ' m ' for mass are the same letter in this example.

Note that graphs in physics are causal relationships. In **fig D**, the acceleration is caused by the force. It is very rare in physics that a graph would represent a statistical correlation, and so phrases such as 'positive correlation' do not correctly describe graphs of physics experiments.

Similarly, it is very rare that a graph of a physics experiment would be correctly drawn if the points are joined 'dot-to-dot'. In most cases a best-fit line should be drawn, as has been done in **fig E**.

LEARNING TIP

Identifying Newton's third law of force pairs can confuse people. To find the two forces, remember they must always act on different objects, and must always have the same cause. In **fig F**, it is the mutual repulsion of electrons in the atoms of the football and the boot that cause the action and reaction.

NEWTON'S THIRD LAW OF MOTION

'When an object A causes a force on another object B, then object B causes an equal force in the opposite direction to act upon object A'. For example, when a skateboarder pushes off from a wall, they exert a force on the wall with their hand. At the same time, the wall exerts a force on the skateboarder's hand. This equal and opposite reaction force is what they can feel with the sense of touch, and as the skateboard has very low friction, the wall's push on them causes acceleration away from the wall. As the wall is connected to the Earth, the Earth and wall combination will accelerate in the opposite direction. The Earth has such a large mass that its acceleration can't be noticed. That's Newton's second law again: acceleration is inversely proportional to mass; huge mass means tiny acceleration.



▲ **fig F** When the boot puts a force on the football, the football causes an equal and opposite force on the boot. The footballer can feel the kick because they feel the reaction force from the ball on their toe.

SKILLS ANALYSIS

CHECKPOINT

- In terms of Newton's laws of motion:
 - Explain why this book will sit stationary on a table.
 - Describe and explain what will happen if your hands then put an upwards force on the book that is greater than its weight.
 - Explain why you feel the book when your hands put that upwards force on it.
- the test for a sulfate
 - the test for a chloride.
- Calculate the acceleration in each of the following cases:
 - A mass of 12.0 kg experiences a resultant force of 785 N.
 - A force of 22.2 N acts on a 3.1 kg mass.
 - A 2.0 kg bunch of bananas is dropped. The bunch weighs 19.6 N.
 - During a tackle, two footballers kick a stationary ball at the same time, with forces acting in opposite directions. One kick has a force of 210 N, the other has a force of 287 N. The mass of the football is 430 g.

SUBJECT VOCABULARY

Newton's first law of motion an object will remain at rest, or in a state of uniform motion, until acted upon by a resultant force

Newton's second law of motion if an object's mass is constant, the resultant force needed to cause an acceleration is given by the equation:

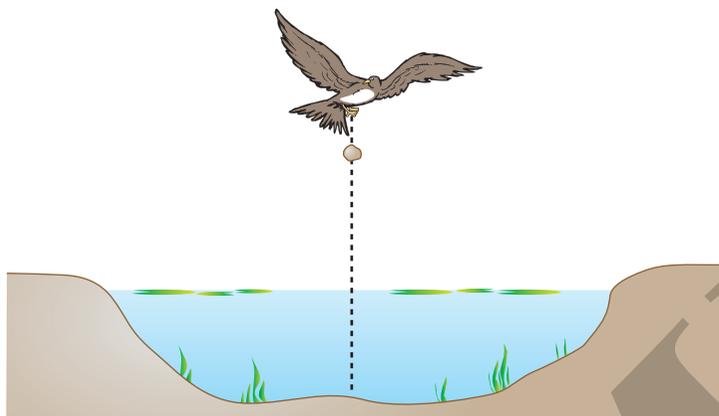
$$\Sigma F = ma$$

Newton's third law of motion for every action, there is an equal and opposite reaction

LEARNING OBJECTIVES

- Recall the simple kinematics equations.
- Calculate unknown variables using the kinematics equations.

Kinematics is the study of the movement of objects. We can use equations to find out details about the motion of objects accelerating in one dimension.



▲ **fig A** Kinematics is the study of the description of the motion of objects. The equations could be used to make calculations about the stone falling through the air, and separately about its motion through the water. The acceleration in air and in water will be different as the resultant force acting in each will be different.

ZERO ACCELERATION

If an object has no resultant force acting on it then it does not accelerate. This is **uniform motion**. In this situation, calculations on its motion are very easy, as they simply involve the basic velocity equation:

$$v = \frac{s}{t}$$

The velocity is the same at the beginning and end of the motion, and if we need to find the displacement travelled, it is a simple case of multiplying velocity by time:

$$s = v \times t$$

CONSTANT ACCELERATION

There are equations that allow us to work out the motion of an object that is moving with a constant acceleration. For these kinematics equations, the first step is to define the five variables used:

- s – displacement (m)
- u – initial velocity (m s^{-1})
- v – final velocity (m s^{-1})
- a – acceleration (m s^{-2})
- t – time (s)

Each equation uses four of the variables, which means that if we know the values of any three variables, we can find out the other two.

ACCELERATION REDEFINED

By re-arranging the equation that defined acceleration, we come to the usual expression of the first kinematics equation:

$$v = u + at$$

For example, if a stone is dropped off a cliff (see **fig B**) and takes three seconds to hit the ground, what is its speed when it does hit the ground?

Identify the three things we know:

- falling under gravity, so $a = g = 9.81 \text{ m s}^{-2}$ (constant acceleration, so the kinematics equations can be used)
- starts at rest, so $u = 0 \text{ m s}^{-1}$
- time to fall $t = 3 \text{ s}$

$$v = u + at = 0 + 9.81 \times 3 = 29.43$$

$$v = 29.4 \text{ m s}^{-1}$$

EXAM HINT

Often the acceleration will not be clearly stated, but the object is falling under gravity, so:

$$a = g = 9.81 \text{ m s}^{-2}$$

EXAM HINT

Often the initial velocity will not be explicitly stated, but the object starts 'at rest'. This means it is stationary at the beginning, so:

$$u = 0 \text{ m s}^{-1}$$

DISTANCE FROM AVERAGE SPEED

As the kinematics equations only work with *uniform* acceleration, the average speed during any acceleration will be halfway from the initial velocity to the final velocity. Therefore the distance travelled is the average speed multiplied by the time:

$$s = \frac{(u + v)}{2} \times t$$

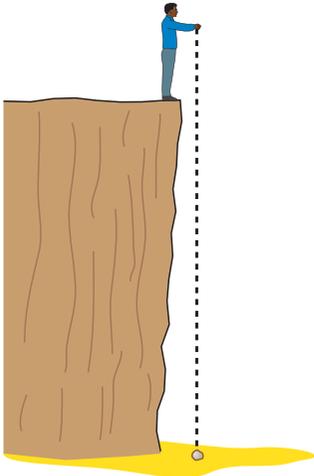
For example, for the same stone dropping off the cliff as in the previous example, we could work out how high the cliff is.

Identify the three things we know:

- final velocity came to be $v = 29.4 \text{ m s}^{-1}$
- starts at rest, so $u = 0 \text{ m s}^{-1}$
- time to fall $t = 3 \text{ s}$

$$s = \frac{(u + v)}{2} \times t = \frac{(0 + 29.4)}{2} \times 3 = 44.1$$

$$s = 44.1 \text{ m}$$



▲ **fig B** Calculations about falling objects are very common.

COMBINING EQUATIONS A

We can combine the equations

$$v = u + at$$

and

$$s = \frac{(u + v)}{2} \times t$$

By substituting the first of these equations into the second, we get the combination equation:

$$s = \frac{(u + (u + at))}{2} \times t = \frac{(2ut + at^2)}{2}$$

$$s = ut + \frac{1}{2}at^2$$

We can use this equation to check again how high the stone was dropped from:

- falling under gravity, so $a = g = 9.81 \text{ m s}^{-2}$
 - starts at rest, so $u = 0 \text{ m s}^{-1}$
 - time to fall $t = 3 \text{ s}$
- $$s = ut + \frac{1}{2}at^2 = (0 \times 3) + \left(\frac{1}{2} \times 9.81 \times 3^2\right) = 44.1$$
- $$s = 44.1 \text{ m}$$

Notice that the answer must come out the same, as we are calculating for the same cliff. This highlights the fact that we can use whichever equation is most appropriate for the information given.

COMBINING EQUATIONS B

$$s = \frac{(u + v)}{2} \times t$$

$$\therefore t = \frac{2s}{(u + v)}$$

and $v = u + at$

By substituting the first of these equations into the second, we get the combination equation:

$$v = u + a \times \frac{2s}{(u + v)}$$

$$\therefore v(u + v) = u(u + v) + 2as$$

$$\therefore vu + v^2 = u^2 + uv + 2as \quad vu = uv \text{ so subtract from each side}$$

$$v^2 = u^2 + 2as$$

Check again what the stone's final velocity would be:

- falling under gravity, so $a = g = 9.81 \text{ m s}^{-2}$
 - starts at rest, so $u = 0 \text{ m s}^{-1}$
 - height to fall $s = 44.1 \text{ m}$
- $$v^2 = u^2 + 2as = 0^2 + (2 \times 9.81 \times 44.1) = 865$$
- $$\therefore v = \sqrt{865} = 29.4$$
- $$v = 29.4 \text{ m s}^{-1}$$

Notice that the answer must come out the same as previously calculated, and this again highlights that there are many ways to reach the answer.

KINEMATICS EQUATION	QUANTITY NOT USED
$v = u + at$	distance
$s = \frac{(u + v)}{2} \times t$	acceleration
$s = ut + \frac{1}{2}at^2$	final velocity
$v^2 = u^2 + 2as$	time

table A Each of the kinematics equations is useful, depending on the information we are given. If you know three quantities, you can always find a fourth by identifying which equation links those four quantities and re-arranging that equation to find the unknown.

EXAM HINT

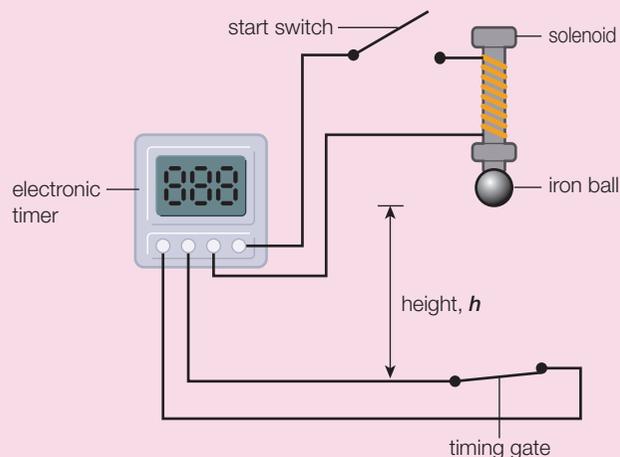
The kinematics equations are only valid if there is a constant acceleration. If the acceleration is changing they cannot be used.

PRACTICAL SKILLS

CP1

Finding the acceleration due to gravity by freefall

A system for timing the fall of an object under gravity can allow us to measure the acceleration due to gravity. In this experiment, we measure the time taken by a falling object to drop under gravity from a certain height, and then alter the height and measure again.



▲ **fig C** The freefall time of an object from different heights allows us to find the acceleration due to gravity, g .

If we vary the height from which the object falls, the time taken to land will vary. The kinematics equations tell us that:

$$s = ut + \frac{1}{2}at^2$$

As it always starts from rest, $u = 0$ throughout; the acceleration is that caused by gravity, g ; and the distance involved is the height from which it is released, h . Thus:

$$h = \frac{1}{2}gt^2$$

$$\therefore t^2 = \frac{2h}{g}$$

Compare this equation with the equation for a straight-line graph:

$$y = mx + c$$

We plot a graph of h on the x -axis and t^2 on the y -axis to give a straight best-fit line. The gradient of the line on this graph will be $2/g$, from which we can find g .

We could find a value for g by taking a single measurement from this experiment and using the equation to calculate it:

$$g = \frac{2h}{t^2}$$

However, a single measurement in any experiment is subject to uncertainty from both random and systematic errors. We can reduce such uncertainties significantly by taking many readings and plotting a graph, which leads to much more reliable conclusions.



Safety Note: Secure the tall stand holding the solenoid so that it cannot topple over.

CHECKPOINT

1. What is the final velocity of a bike that starts at 4 m s^{-1} and has zero acceleration act on it for 10 seconds?
2. How far will the bike in question 1 travel in the 10-second time period?
3. Calculate the acceleration in each of the following cases:
 - (a) $v = 22 \text{ m s}^{-1}$; $u = 8 \text{ m s}^{-1}$; $t = 2.6 \text{ s}$
 - (b) a ball starts at rest and after 30 m its velocity has reached 4.8 m s^{-1}
 - (c) in 15 s, a train moves 100 m from rest.
4. The bird in **fig A** drops the stone from a height of 88 m above the water surface. Initially, the stone has zero vertical velocity. How long will it take the stone to reach the surface of the pond? Assume air resistance is negligible.
5. If the stone in **fig A** enters the water at 41.6 m s^{-1} , and takes 0.6 s to travel the 3 metres to the bottom of the pond, what is its average acceleration in the pond water?

SUBJECT VOCABULARY

kinematics the study of the description of the motion of objects

uniform motion motion when there is no acceleration:

$$\text{velocity (m s}^{-1}\text{)} = \frac{\text{displacement (m)}}{\text{time (s)}}$$

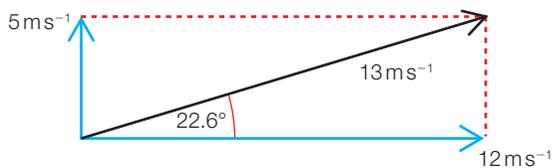
$$v = \frac{s}{t}$$

1A 7 RESOLVING VECTORS

LEARNING OBJECTIVES

- Explain that any vector can be split into two components at right angles to each other.
- Calculate the values of the component vectors in any such right-angled pair (resolution).

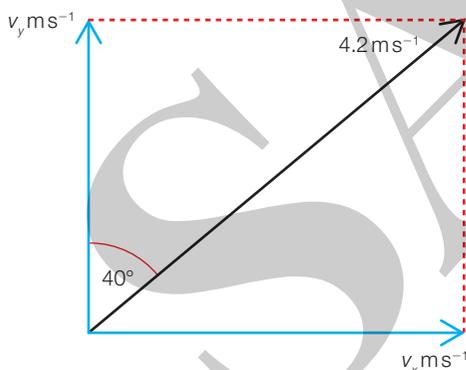
We have seen that vectors that act perpendicular to each other can be combined to produce a single vector that has the same effect as the two separate vectors. If we start with a single vector, we can find a pair of vectors at right angles to each other that would combine to give our single original vector. This reverse process is called **resolution** or **resolving vectors**. The resolved pair of vectors will both start at the same point as the original single vector.



▲ **fig A** The original velocity vector of 13 m s^{-1} has been resolved into a horizontal and a vertical velocity vector, which would separately be the same overall effect. An object moving right at 12 m s^{-1} and up at 5 m s^{-1} will move 13 metres each second at an angle of 22.6° up and right from the horizontal.

RESOLVING VECTORS CALCULATIONS

In order to resolve a vector into a pair at right angles, we must know all the details of the original vector. This means we must know its size and direction. The direction is most commonly given as an angle to either the vertical or the horizontal. This is useful, as we most commonly want to split the vector up into a horizontal and vertical pair.



▲ **fig B** Resolving a velocity vector into vertical and horizontal components.

In the example of **fig B**, a basketball is thrown into the air at an angle of 40° to the vertical. In order to find out if it will go high enough to reach the hoop, kinematics calculations could be done on its vertical motion. However, this can only be done if we

can isolate the vertical component of the basketball's motion. Similarly, we could find out if it will travel far enough horizontally to reach the hoop if we know how fast it is moving horizontally.

The basketball's velocity must be resolved into horizontal and vertical components. This will require some trigonometrical calculations.

THE VERTICAL COMPONENT OF VELOCITY

Redrawing the components in **fig B** to show how they add up to produce the 4.2 m s^{-1} velocity vector, shows again that they form a right-angled triangle, as in **fig C**. This means we can use the relationship:

$$\begin{aligned} \cos 40^\circ &= \frac{v_y}{4.2} \\ \therefore v_y &= 4.2 \times \cos 40^\circ \\ &= 4.2 \times 0.766 \\ \therefore v_y &= 3.2 \text{ m s}^{-1} \end{aligned}$$

▲ **fig C** Finding the components of velocity.

THE HORIZONTAL COMPONENT OF VELOCITY

Similarly, for the horizontal component, we can use the relationship:

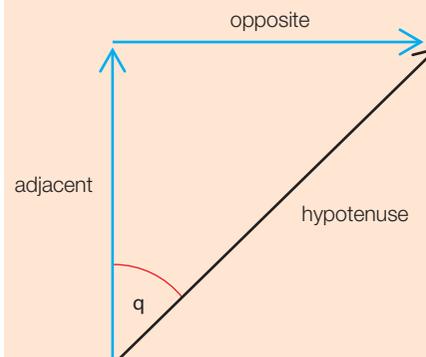
$$\begin{aligned} \sin 40^\circ &= \frac{v_x}{4.2} \\ \therefore v_x &= 4.2 \times \sin 40^\circ = 4.2 \times 0.643 \\ \therefore v_x &= 2.7 \text{ m s}^{-1} \end{aligned}$$

LEARNING TIP

$\sin \theta^\circ = \text{opposite/hypotenuse}$

$\cos \theta^\circ = \text{adjacent/hypotenuse}$

$\tan \theta^\circ = \text{opposite/adjacent}$



▲ **fig D** Trigonometry reminder.

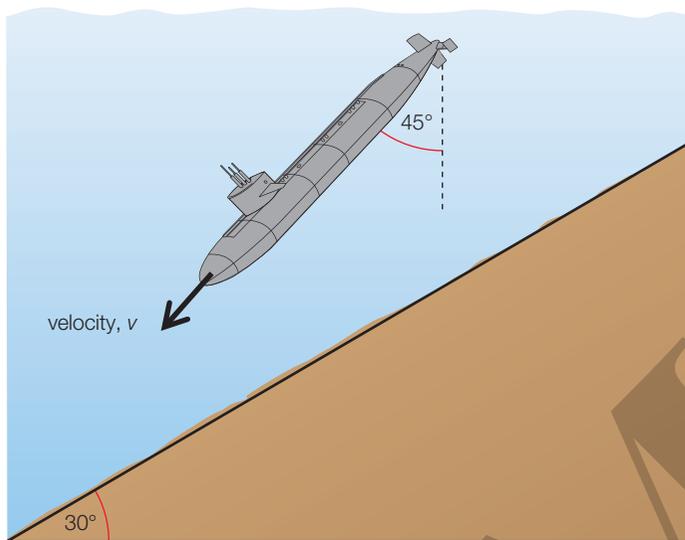
LEARNING TIP

Resolving vectors could also be achieved using a scale diagram. Using this method in rough work may help you to get an idea as to what the answer should be in order to check that your calculations are about right.

ALTERNATIVE RESOLUTION ANGLES

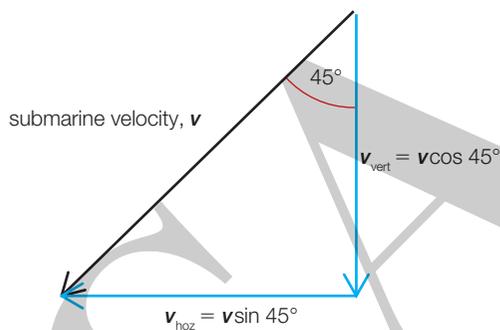
If we know the velocity of an object, we have seen that we can resolve this into a pair of velocity vectors at right angles to each other. The choice of direction of the right-angle pair, though, is arbitrary, and can be chosen to suit a given situation.

Imagine a submarine descending underwater close to an angled seabed.



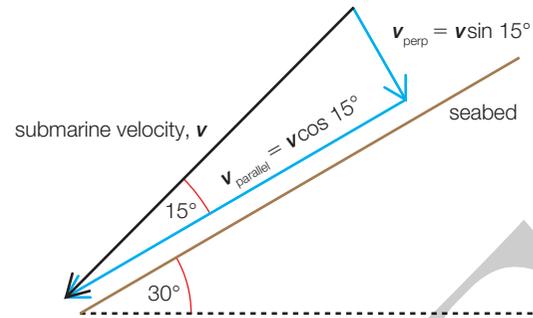
▲ **fig E** Resolving submarine velocity vectors helps to avoid collision.

The velocity could be resolved into a pair of vectors, horizontal and vertical.



▲ **fig F** Submarine velocity resolved into horizontal and vertical components.

However, the submarine commander is likely to be most interested in knowing how quickly the submarine is approaching the seabed. This could be found by resolving the velocity of the submarine into a component parallel with the seabed, and the right-angled pair with that will be a component perpendicular to the seabed. It is this v_{perp} that will tell the submarine commander how quickly he is approaching the seabed.



▲ **fig G** Submarine velocity resolved into components parallel and perpendicular to the seabed.

LEARNING TIP

The vectors produced by resolution can be at any angle with respect to the original vector, as long as they are perpendicular to each other and their resultant equals the original vector. In order to allow for every pair to add up to the original vector, their magnitudes will vary as the angle changes.

CHECKPOINT

SKILLS INTERPRETATION

- (a) On graph paper, draw a velocity vector for a stone fired from a catapult at 45° to the horizontal. Your arrow should be 10 cm long, representing a velocity of 10 m s^{-1} . Draw onto your diagram the horizontal and vertical components that would make up the overall velocity. Use a ruler to measure the size of the horizontal and vertical components, and convert these lengths into metres per second using the same scaling.

(b) Find the horizontal and vertical velocity components for this catapult stone by calculation, and compare with your answers from part (a).
- A javelin is thrown at 16 m s^{-1} at an angle of 35° up from the horizontal. Calculate the horizontal and vertical components of the javelin's motion.
- A ladder is leant against a wall, at an angle of 28° to the wall. The 440 N force from the floor acts along the length of the ladder. Calculate the horizontal and vertical components of the force from the floor that act on the bottom of the ladder.
- A plane is flying at 240 m s^{-1} , on a bearing of 125° from due north. Calculate its velocity component due south, and its velocity component due east.

SUBJECT VOCABULARY

resolution or **resolving vectors** the determination of a pair of vectors, at right angles to each other, that sums to give the single vector they were resolved from

catapult a device that can throw objects at high speed

LEARNING OBJECTIVES

- Apply kinematics equations to moving objects.
- Apply the independence of horizontal and vertical motion to objects moving freely under gravity.
- Combine horizontal and vertical motion to calculate the movements of projectiles.

Objects thrown or fired through the air generally follow **projectile** motion. Here we are going to combine ideas from the various earlier sections in order to solve questions about projectiles. Resolving vectors showed that the actions in each of two perpendicular directions are wholly independent. This means we can use Newton's laws of motion and the kinematics equations separately for the horizontal and vertical motions of *the same object*. This will allow us to calculate its overall motion in two dimensions.

We only consider the motion after the force projecting an object has finished – for example, after a cannonball has left the cannon. Air resistance is ignored in these calculations, so the only force acting will be the object's weight. Thus, all vertical motion will follow kinematics equations, with gravity as the acceleration. There will be no horizontal force at all, which means no acceleration and therefore $v = s/t$.

LEARNING TIP

Vectors acting at right angles to each other act independently. Their combined effect can be calculated using vector addition, but they can also be considered to act separately and at the same time. This would cause independent effects, which themselves could then be combined to see an overall effect.

LEARNING TIP

Physics is holistic: you can apply ideas from one area of physics to other areas of physics. Remember that physics is a set of rules that attempts to explain everything in the universe. Every rule should therefore be universally applicable.

HORIZONTAL THROWS

If an object is thrown horizontally, it will start off with zero vertical velocity. However, gravity will act on it so that its motion will curve downwards in a parabola shape, like the stone in **fig A**.

In the example of **fig A**, a stone is kicked horizontally off a cliff with a velocity of 8.2 m s^{-1} . How much time is the stone in flight? How far does it get away from the cliff by the time it lands?

Horizontal and vertical motions are totally independent. Here the vertical component of velocity is initially zero, but the stone accelerates under gravity. Uniform acceleration means the kinematics equations can be used.

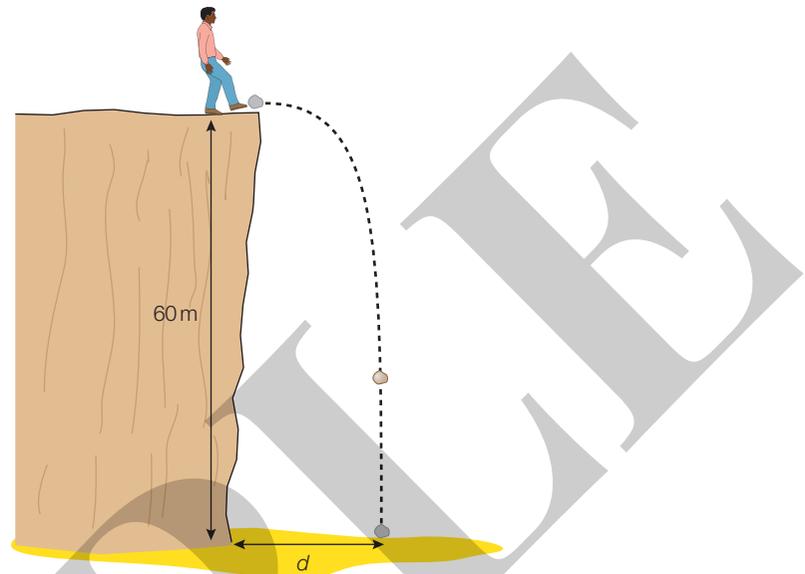


fig A Vertical acceleration on a horizontally moving stone.

The time to hit the beach, t , will be the same as if the stone was simply dropped. We know $u = 0 \text{ m s}^{-1}$; $a = -9.81 \text{ m s}^{-2}$; and the height fallen, $s = -60 \text{ m}$.

$$s = ut + \frac{1}{2}at^2$$

$$u = 0 \therefore ut = 0$$

$$\therefore s = \frac{1}{2}at^2$$

$$\therefore t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{(2 \times -60)}{-9.81}}$$

$$t = 3.5 \text{ s}$$

Horizontally, there is no accelerating force once the stone is in flight, so it has a constant speed. Thus, to find the distance travelled horizontally, d :

$$v = \frac{d}{t}$$

$$\therefore d = v \times t$$

$$d = 8.2 \times 3.5$$

$$d = 28.7 \text{ m}$$

RECOMBINING VELOCITY COMPONENTS

In the example of the stone kicked from the cliff to the beach, we might also want to calculate the final velocity of the stone on landing. This means adding vertical and horizontal components into their resultant.

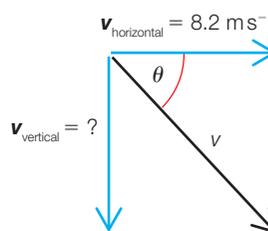


fig B The stone's final velocity is the resultant of its vertical and horizontal components.

In the example of **fig A**, what is the velocity of the stone when it hits the beach?

The horizontal velocity was given as 8.2 m s^{-1} . To calculate the vertical velocity: $u = 0 \text{ m s}^{-1}$; $a = -9.81 \text{ m s}^{-2}$; and $s = -60 \text{ m}$.

$$v_{\text{vertical}}^2 = u^2 + 2as = 0^2 + (2 \times -9.81 \times -60) = 1177.2$$

$$\therefore v_{\text{vertical}} = \sqrt{1177.2} = -34.3$$

$$v_{\text{vertical}} = -34.3 \text{ m s}^{-1}$$

Pythagoras' theorem gives the magnitude of the final velocity:

$$v = \sqrt{(8.2^2 + 34.3^2)}$$

$$\therefore v = 35.3 \text{ m s}^{-1}$$

Trigonometry will give the angle at which the stone is flying on impact with the beach:

$$\tan \theta = \frac{34.3}{8.2}$$

$$\therefore \theta = 77^\circ$$

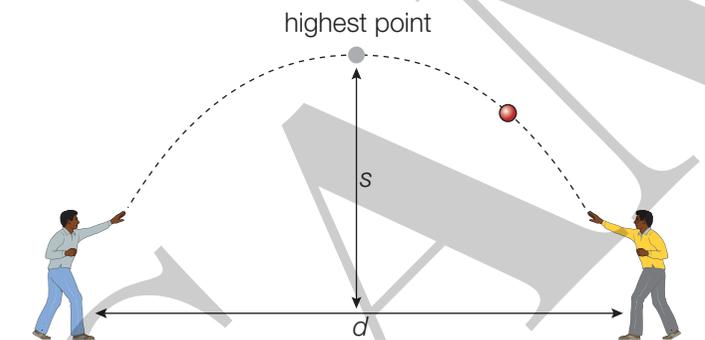
The stone's velocity when it lands on the beach is 35.3 metres per second at an angle of 77° down from the horizontal.

EXAM HINT

We use the usual approach that up is positive throughout this section. You can choose any direction to be negative, but you must refer to that direction as negative at every step of the calculation. If you are not consistent, your calculation will be incorrect.

VERTICAL THROWS

Imagine throwing a ball to a friend. The ball goes up as well as forwards. One common idea in these calculations is that an object thrown with a vertical upwards component of motion will have a symmetrical trajectory. At the highest point, the vertical velocity is momentarily zero. Getting to this point will take half of the time of the whole flight.



▲ **fig C** Considering only the vertical component of velocity.

In the example of **fig C**, a ball is thrown with a vertical velocity component of 5.5 m s^{-1} . How much time is it in flight? How high does it get?

These questions would have the same answer if a person threw the ball vertically up and caught it again themselves. This again highlights the *independence* of horizontal and vertical motions. It may be that an initial velocity at an angle is quoted, so that we need to resolve the velocity vector into its horizontal and vertical components in order to know that here $v_{\text{vertical}} = 5.5 \text{ m s}^{-1}$.

Consider the second question first: uniform acceleration under gravity means $a = -9.81 \text{ m s}^{-2}$ and the kinematics equations can be used. We know $u = 5.5 \text{ m s}^{-1}$; at the top of the path, $v = 0 \text{ m s}^{-1}$; and we want to find the height, s .

$$v^2 = u^2 + 2as$$

$$\therefore s = \frac{v^2 - u^2}{2a} = \frac{0^2 - (5.5)^2}{2 \times -9.81}$$

$$s = 1.54 \text{ m}$$

Note that 1.54 metres is actually the height the ball reaches above the point of release at which it left the hand – the point where its initial speed was 5.5 m s^{-1} – but this is often ignored in projectiles calculations.

The time of flight for the ball will be just the time taken to rise and fall vertically. We find the time to reach the highest point, and then double that value. We know $u = 5.5 \text{ m s}^{-1}$; at the top of the path, $v = 0 \text{ m s}^{-1}$; and we want to find the time, t .

$$v = u + at$$

$$\therefore t = \frac{v - u}{a} = \frac{0 - (5.5)}{-9.81}$$

$$t = 0.56 \text{ s}$$

So the overall time of flight will be 0.56 seconds doubled: total time = 1.12 s.

EXAM HINT

For virtually all projectile motion calculations, we assume that there is no air resistance, so the only force acting is gravity, vertically downwards.

CHECKPOINT

SKILLS PROBLEM SOLVING

- A boy throws a ball vertically at a velocity of 4.8 m s^{-1} .
 - How long is it before he catches it again?
 - What will be the ball's greatest height above the point of release?
- The boy in question 1 now throws his ball horizontally out of a high window with a velocity of 3.1 m s^{-1} .
 - How long will it take to reach the ground 18 m below?
 - How far away, horizontally, should his friend stand in order to catch the ball?
- A basketball is thrown with a velocity of 6.0 m s^{-1} at an angle of 40° to the vertical, towards the hoop.
 - If the hoop is 0.90 m above the point of release, will the ball rise high enough to go in the hoop?
 - If the centre of the hoop is 3.00 m away, horizontally, from the point of release, explain whether or not you believe this throw will score in the hoop. Support your explanation with calculations.

SUBJECT VOCABULARY

projectile a moving object on which the only force of significance acting is gravity. The trajectory is thus pre-determined by its initial velocity

1A THINKING BIGGER

THE BATTLE OF AGRA

SKILLS

CRITICAL THINKING, PROBLEM SOLVING, ANALYSIS, INTERPRETATION, ADAPTIVE LEARNING, PRODUCTIVITY, ETHICS, COMMUNICATION, ASSERTIVE COMMUNICATION

Agra Fort was built in the 11th Century, although the present structure was built in 1573. In this activity, you need to imagine attacking the fort using a cannon that fires a cannonball as a projectile.

STUDENT ESSAY

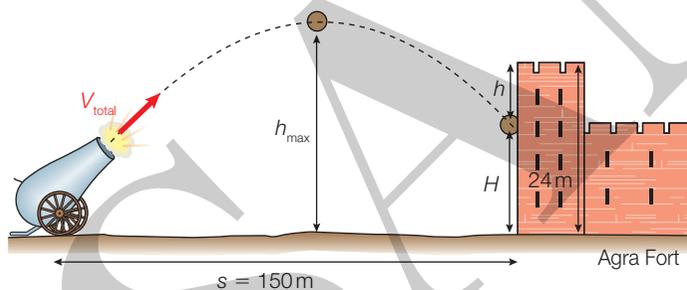


▲ **fig A** Agra Fort is now an UNESCO World Heritage Site.

In this section, I will use some basic mechanics to answer a question: could the Mughal Empire artillery really have attacked Agra Fort in the way described previously? The nineteenth-century source material suggests that the fortress was under siege by the Mughals for three months and ‘battered by artillery’. However, the current walls bear little in the way of obvious battle scars.

Looking at **fig B**, the question that needs to be answered here is:

‘How high up the front wall of the fortress will the cannonball hit?’ This height is marked on **fig B** as ‘ H ’.



▲ **fig B** The trajectory of a cannonball fired towards Agra Fort. We assume the cannonball leaves the cannon at ground level.

In addition to the layout shown in **fig B**, we need information about the initial velocity of the cannonball. The cannon explosion could act for 0.05 s to accelerate the cannonball

(mass = 12 kg) with a force of 9300 N. It causes the cannonball to leave the cannon at an angle of 45° to the horizontal.

Steps to the answer

We can work out what calculations are required to solve this problem, by working back from the answer we want to find. The fundamental idea is that the parabola trajectory would be symmetrical if the flight was not interrupted by crashing into the fortress wall.

- 1 To find the height up the wall from the ground, we will need to work out how far down from the cannonball’s maximum height it falls:

$$H = h_{\max} - h$$

- 2 To find h , we need to know the time of flight, t_{total} so we can divide this into a time to reach h_{\max} , and time left to fall height h . We will use vertical gravitational acceleration to calculate the vertical drop in that remaining time:

$$t_{\text{total}} = \frac{s}{v_{\text{horizontal}}}$$

From **fig B**, we can see that $s = 150$ m.

- 3 $v_{\text{horizontal}}$ can be found by resolving the velocity to give the horizontal component:

$$v_{\text{horizontal}} = v_{\text{total}} \times \cos 45^\circ$$

- 4 The overall velocity will come from the cannon’s acceleration of the cannonball:

$$v = u + at$$

where $u = 0 \text{ m s}^{-1}$, and the question tells us that the explosion acts for 0.05 seconds.

- 5 Newton’s second law of motion gives us the acceleration caused by the sling:

$$a = \frac{F}{m}$$

Calculate the answer by reversing these steps:

The acceleration caused by the explosion:

$$a = \frac{F}{m}$$

$$a =$$

SCIENCE COMMUNICATION

- 1 The extract opposite is a draft for a university essay about the Mughal siege of 1857. Consider the extract and comment on the type of writing being used. For example, think about whether this is a scientist reporting the results of their experiments, a scientific review of data, a newspaper or a magazine-style article for a specific audience. Try and answer the following questions:
- How can you tell that the author is doubtful about the historical source material?
 - What is the purpose of this mathematical analysis, for its inclusion in this essay?

PHYSICS IN DETAIL

Now we will look at the physics in detail. You may need to combine concepts from different areas of physics to work out the answers.

- 2 Complete the calculation steps, in reverse as suggested, in order to find out the answer, H :
- the acceleration caused by the explosion
 - overall velocity that the cannonball is projected from the cannon
 - horizontal and vertical components of the velocity
 - time of flight found from the horizontal travel
 - time to reach maximum height using vertical motion
 - remaining flight time from maximum height
 - height fallen from the maximum in the remaining flight time
 - final answer, H .
- 3 State **two** assumptions that have been made in these calculations.
- 4 Calculate what difference there would be in the answer if the cannon was loaded with different cannonballs of masses 10 kg and 14 kg. Note from **fig B** that the fortress walls are 24 m high. Comment on these answers.
- 5 If the available supply of cannonballs offered very variable masses, how might the Mughals be able to overcome the problems shown in question 4.

ACTIVITY

Imagine the writer of this essay is a friend of yours, and he has come to you for help with the calculations as he is not an experienced scientist. His section 'Steps to the answer' was taken from a research source about a different fortress under siege. Write an email to Claus to explain the calculations required in each step.

INTERPRETATION NOTE

Once you have answered the calculation questions below, decide whether you think the Mughal siege happened as the author suggests.

THINKING BIGGER TIP

Inside a cannon, an explosion exerts a force on the cannonball to fire it out of the cannon.

INTERPRETATION NOTE

You can assume that the writer understands mathematics, and is generally intelligent – a student who could have done A Level physics but preferred arts subjects.

1A EXAM PRACTICE

- 1 Quantities can be scalar or vector. Select the row of the table that correctly states a scalar quantity and a vector quantity.

	Scalar quantity	Vector quantity
A	acceleration	mass
B	mass	weight
C	speed	distance
D	weight	speed

[1]

(Total for Question 1 = 1 mark)

- 2 How is the kinetic energy, E_k , of a car related to its speed, v ?

- A $E_k \propto v$
 B $E_k \propto v^2$
 C $E_k \propto \sqrt{v}$
 D $E_k \propto \frac{1}{v}$

[1]

(Total for Question 2 = 1 mark)

- 3 The unit of force is the newton. One newton is equivalent to:

- A 0.1 kg
 B 1 kg m s^{-1}
 C 1 kg m s^{-2}
 D 1 m s^{-2}

[1]

(Total for Question 3 = 1 mark)

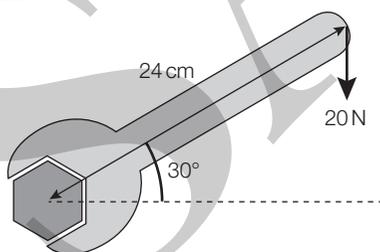
- 4 A ball is thrown vertically upwards at a speed of 11.0 m s^{-1} . What is the maximum height it reaches?

- A 0.561 m
 B 1.12 m
 C 6.17 m
 D 12.3 m

[1]

(Total for Question 4 = 1 mark)

- 5 Calculate the moment exerted on the nut by the spanner shown in the diagram.



- A 2.4 Nm
 B 4.2 Nm
 C 4.8 Nm
 D 420 Nm

[1]

(Total for Question 5 = 1 mark)

- 6 (a) What is meant by a vector quantity? [1]
 (b) A car is driven around a bend at a constant speed. Explain what happens to its velocity. [2]

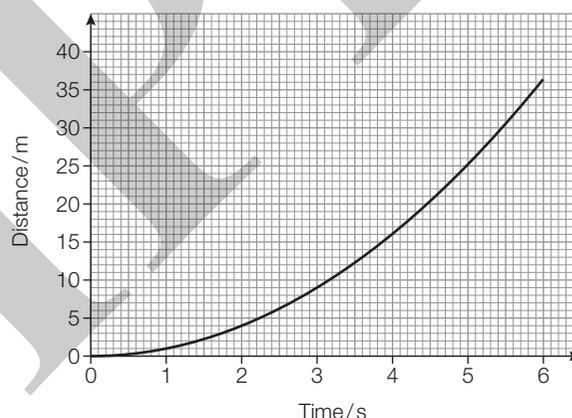
(Total for Question 6 = 3 marks)

- 7 You are asked to determine the acceleration of free fall at the surface of the Earth, g , using a free fall method in the laboratory.

- (a) Describe the apparatus you would use, the measurements you would take and explain how you would use them to determine g . [6]
 (b) Give **one** precaution you would take to ensure the accuracy of your measurements. [1]

(Total for Question 7 = 7 marks)

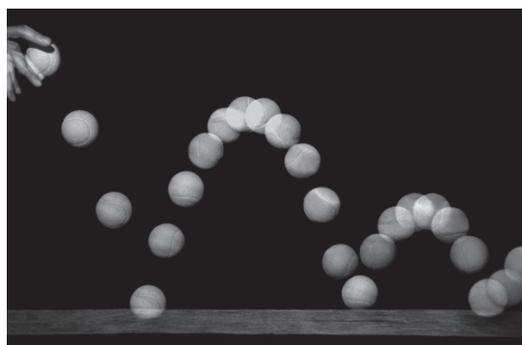
- 8 The graph shows how displacement varies with time for an object that starts from rest with constant acceleration.



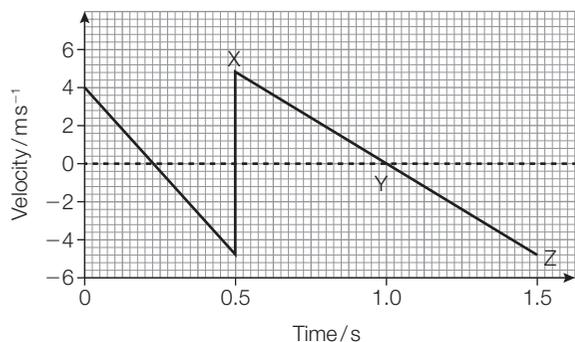
- (a) Use the distance–time graph to determine the speed of the object at a time of 4.0 s. [3]
 (b) Calculate the acceleration. [2]

(Total for Question 8 = 5 marks)

- 9 The photograph shows a sequence of images of a bouncing tennis ball.



A student plots the following graph and claims that it shows the vertical motion of the ball in the photograph.



- (a) Without carrying out any calculations, describe how the following can be found from the graph
- the vertical distance travelled by the ball between 0.5 s and 1.0 s
 - the acceleration at Y. [2]
- (b) The graph contains several errors in its representation of the motion of the ball
Explain two of these errors. [4]

(Total for Question 9 = 6 marks)

- 10** There has been a proposal to build a train tunnel underneath the Atlantic Ocean from England to America. The suggestion is that in the future the trip of 5000 km could take as little as one hour.

Assume that half the time is spent accelerating uniformly and the other half is spent decelerating uniformly with the same magnitude as the acceleration.

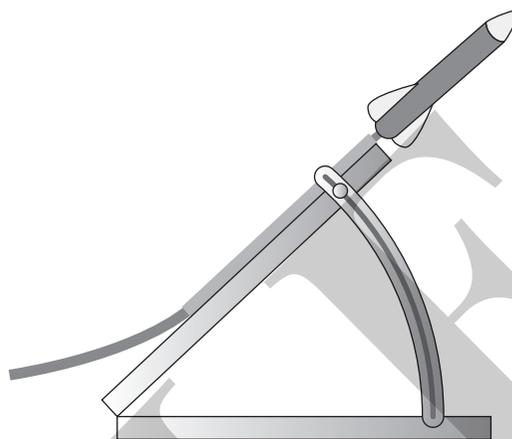
- Show that the acceleration would be about 2 m s^{-2} . [2]
- Calculate the maximum speed. [2]
- Calculate the resultant force required to decelerate the train.
mass of train = $4.5 \times 10^5 \text{ kg}$ [2]

(Total for Question 10 = 6 marks)

- 11** During a lesson on Newton's laws of motion, a student says, 'We don't really need to bother with Newton's first law because it is included in his second law'. State Newton's first two laws of motion and explain how Newton's second law includes the first law. [5]

(Total for Question 11 = 5 marks)

- 12** The diagram shows an arrangement used to launch a light foam rocket at a school science competition.



The rocket is launched at the level of one end of a long table and lands at the other end at the same level. The students measure the horizontal distance travelled by the rocket and the time of flight.

- The rocket travels 1.88 m in a time of 0.88 s.
 - Show that the horizontal component of the initial velocity of the rocket is about 2 m s^{-1} . [2]
 - Show that the vertical component of the initial velocity of the rocket is about 4 m s^{-1} . [2]
 - Calculate the initial velocity of the rocket. [4]
- The students obtained their data by filming the flight. When they checked the maximum height reached by the rocket they found it was less than the height predicted using this velocity.
 - Suggest why the maximum height reached was less than predicted. [1]
 - Give **two** advantages of filming the flight to obtain the data. [2]

(Total for Question 12 = 11 marks)

TOPIC 1 MECHANICS

CHAPTER 1B ENERGY

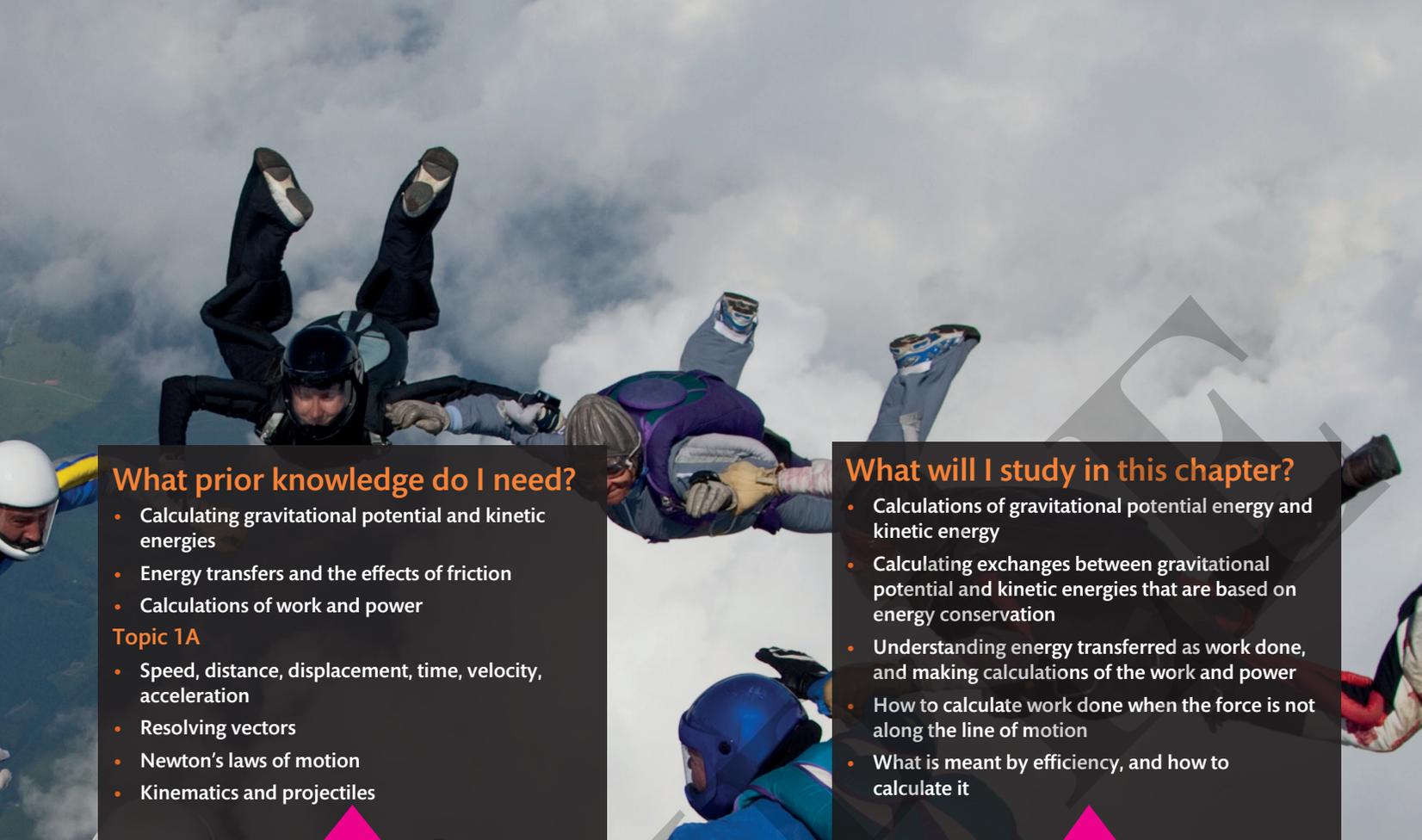
Chapter 1A finished with a discussion of the motion of a projectile cannonball's flight. An alternative way of considering changes in movements of objects affected by gravity is to follow what happens to their energy. An object held up has an amount of gravitational potential energy as a result of its position. This energy can be transferred to kinetic, or movement, energy if the object falls. Humans have evolved over millions of years to avoid situations in which they might fall a great height, as such falls are generally dangerous. The kinetic energy that humans have gained from falling could be transferred to other stores of energy in their bodies, through large forces, and cause injuries.

The effect of gravity on the movement of an object should be considered in relation to the energy a body may possess or transfer. There are equations for calculating kinetic energy and gravitational potential energy, and the transfer of energy when a force is used to cause the transfer. These formulae and Newton's laws can be used together to work out everything we might want to know about the movement of any everyday object in any everyday situation.

Whilst it is difficult for scientists to describe or identify the exact nature of energy, the equations that describe energy relationships have always worked correctly.

MATHS SKILLS FOR THIS CHAPTER

- Units of measurement (*e.g. the joule, J*)
- Changing the subject of an equation (*e.g. finding the velocity of a falling object*)
- Substituting numerical values into algebraic equations (*e.g. calculating the power used*)
- Plotting two variables from experimental or other data, understanding that $y = mx + c$ represents a linear relationship and determining the slope of a linear graph (*e.g. finding the acceleration due to gravity experimentally*)
- Using angles in regular 2D and 3D structures (*e.g. finding the angle with which to calculate work done*)
- Using sin, cos and tan in physical problems (*e.g. calculating the work done by a force acting at an angle*)



What prior knowledge do I need?

- Calculating gravitational potential and kinetic energies
- Energy transfers and the effects of friction
- Calculations of work and power

Topic 1A

- Speed, distance, displacement, time, velocity, acceleration
- Resolving vectors
- Newton's laws of motion
- Kinematics and projectiles

What will I study in this chapter?

- Calculations of gravitational potential energy and kinetic energy
- Calculating exchanges between gravitational potential and kinetic energies that are based on energy conservation
- Understanding energy transferred as work done, and making calculations of the work and power
- How to calculate work done when the force is not along the line of motion
- What is meant by efficiency, and how to calculate it



What will I study later?

Topic 1C

- Momentum and the principle of conservation of momentum
- Elastic and inelastic collisions

Topic 2A

- Fluid movements and terminal velocity

Topic 2B

- Elastic potential energy

Topic 4B

- Energy conservation in electrical circuits
- Power in electric circuits

Topic 6A (Book 2: IAL)

- Potential difference modelled on gravitational potential energy

LEARNING OBJECTIVES

- Calculate transfers of gravitational potential energy near the Earth's surface.
- Calculate the kinetic energy of a body.
- Calculate exchanges between gravitational potential and kinetic energies, based on energy conservation.



▲ **fig A** The gravitational potential energy transferred to kinetic energy for a falling coconut can be a significant hazard in tropical countries.

DID YOU KNOW?

The strength of the Earth's gravitational field at the top of the Eiffel Tower is less than a hundredth of a per cent smaller than at the foot of the Eiffel Tower.



▲ **fig B** A jumbo jet plane has a lot of kinetic energy.

Gravitational potential energy (E_{grav}) is the energy an object has by virtue of its position in a gravitational field. **Kinetic energy** (E_k) is the energy an object has by virtue of its movement. As objects rise or fall, gravitational potential energy can be transferred to kinetic energy and kinetic energy can be transferred to gravitational potential energy.

GRAVITATIONAL POTENTIAL ENERGY

Gravitational potential energy (gpe) can be calculated using the equation:

$$\text{gpe (J)} = \text{mass (kg)} \times \text{gravitational field strength (N kg}^{-1}\text{)} \times \text{height (m)}$$

$$E_{grav} = mgh$$

Usually, the gpe is considered as a change caused by a change in height, for example, the change in gpe when you lift an object onto a shelf. This alters the equation slightly to consider transfers to or from gpe:

$$\Delta E_{grav} = mg\Delta h$$

A brick of mass 2.2 kg is lifted vertically through a height of 1.24 m. The gpe gained is calculated as follows:

$$\Delta E_{grav} = mg\Delta h$$

$$\Delta E_{grav} = 2.2 \times 9.81 \times 1.24$$

$$\Delta E_{grav} = 26.8 \text{ J}$$

Writing the formula this way suggests that the gravitational field strength is a fixed value. The gravitational field strength is a measure of the pull of gravity by a planet at a distance from its centre. This is not actually constant, as the strength of the gravitational field experienced by a mass is inversely proportional to the square of the distance from the planet's centre. However, close to the Earth's surface, over small scales, such as the heights that humans deal with in everyday life, it is an acceptably close approximation to say g is fixed at 9.81 N kg^{-1} .

KINETIC ENERGY

Kinetic energy can be calculated using the equation:

$$\text{kinetic energy (J)} = \frac{1}{2} \times \text{mass (kg)} \times (\text{speed})^2 (\text{m}^2 \text{s}^{-2})$$

$$E_k = \frac{1}{2} \times m \times v^2$$

For example, a large jumbo jet plane might have a cruising speed of 900 km h^{-1} and a flight mass of 400 tonnes. What would its kinetic energy be?

First convert into SI units:

$$v = 900 \text{ km h}^{-1} = 900\,000 \text{ m h}^{-1} = \frac{9 \times 10^5}{60 \times 60} = 250 \text{ m s}^{-1}$$

$$m = 400 \times 1000 = 4 \times 10^5 \text{ kg}$$

$$E_k = \frac{1}{2} \times m \times v^2$$

$$\therefore E_k = \frac{1}{2} \times 4 \times 10^5 \times 250^2$$

$$E_k = 1.25 \times 10^{10} \text{ J} = 12.5 \text{ GJ}$$

TRANSFER BETWEEN E_{grav} AND E_K

The principle of conservation of energy tells us that we can never lose any energy or gain energy out of nowhere. In any energy transfer, we must have the same total energy before and after the transfer. Gravitational potential energy can be transferred to kinetic energy if an object falls to a lower height. Alternatively, an object thrown upwards will slow down as its kinetic energy is transferred to gpe. In either case:

$$\Delta E_{grav} = mg\Delta h = \frac{1}{2}mv^2 = E_k$$

Depending on the situation, it can often be useful to divide out the mass that appears on both sides of this equation. This allows a convenient calculation of how fast an object will be travelling after falling a certain distance from rest:

$$v = \sqrt{2g\Delta h}$$

or how high an object could rise if projected upwards at a certain speed:

$$\Delta h = \frac{v^2}{2g}$$

LEARNING TIP

The fact that mass divides out to give the relationship $v = \sqrt{2gh}$ confirms Galileo's idea that objects will all fall to the ground at the same rate regardless of their mass.

However, remember that all of the relationships shown in this section assume that no energy is lost through friction or air resistance, and that this can be an important factor when some objects fall.



▲ **fig C** The Burj Khalifa tower in Dubai.

For example, how fast would a coin hit the ground if it were dropped from the top of Burj Khalifa tower in Dubai, which is 830 m tall?

$$v = \sqrt{2g\Delta h}$$

$$v = \sqrt{2(9.81 \times 830)} = \sqrt{16285}$$

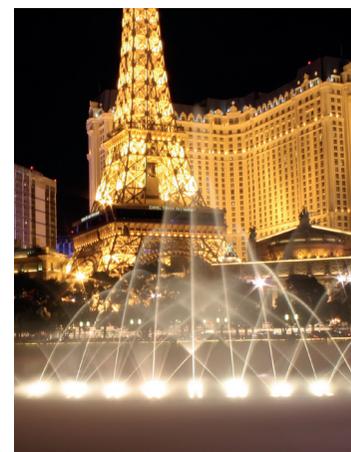
$$v = 128 \text{ m s}^{-1}$$

Another example: how high would water from a fountain rise if it were ejected vertically upwards from a spout at 13.5 m s^{-1} ?

$$\Delta h = \frac{v^2}{2g}$$

$$\Delta h = \frac{13.5^2}{2(9.81)} = \frac{182.25}{19.62}$$

$$h = 9.29 \text{ m}$$



▲ **fig D** Fountain designers need to be able to calculate gpe and kinetic energy.

PRACTICAL SKILLS

CP1

Finding g from energy conservation

There are a number of different experimental methods for finding the gravitational field strength. One example of these relies on the transfer of gravitational potential energy to kinetic energy. In this experiment, we measure the velocity that has been reached by a falling object after it has fallen under gravity from a certain height, and then alter the height and measure the velocity again.

If we vary the height from which the object falls, the gravitational potential energy is different at each height. This gpe will all be transferred to kinetic energy.

$$mg\Delta h = \frac{1}{2}mv^2$$

As we saw previously, this equation can be simplified to give:

$$v^2 = 2g\Delta h$$

Compare this equation with the equation for a straight-line graph: $y = mx + c$

If we plot a graph of Δh on the x -axis and v^2 on the y -axis, we will get a straight best-fit line. The gradient of the line on this graph will be twice the gravitational field strength, $2g$, from which we can find g .

We could find a value for g by taking a single measurement from this experiment and using the equation to calculate it:

$$g = \frac{v^2}{2\Delta h}$$

However, as mentioned in **Section 1A.6**, a single measurement in any experiment is prone to uncertainty from both random and systematic errors. The reliability of the conclusions is significantly improved with multiple readings and graphical analysis.



fig E The freefall velocity of an object from different heights allows us to find the gravitational field strength, g .

Safety Note: Secure the tall stand so that it cannot topple over. The object must be positioned so that it cannot cause injury as it falls.

SKILLS CREATIVITY

CHECKPOINT

1. Estimate the speed at which a coconut from the tree in **fig A** would hit the sand.
2. How fast would a fountain need to squirt its water upwards to reach a height of 15 m?
3. How fast would a snowboarder be moving if he slid down a slope dropping a vertical height of 45 m?
4. How high will a 48 kg trampolinist rise if he leaves the trampoline at a speed of 6.1 m s^{-1} ?
5. What assumption must you make in order to answer all of the above questions?

SUBJECT VOCABULARY

gravitational potential energy (E_{grav}) the energy an object stores by virtue of its position in a gravitational field:

$$\text{gpe (J)} = \text{mass (kg)} \times \text{gravitational field strength (N kg}^{-1}\text{)} \times \text{height (m)}$$

$$E_{grav} = mgh \text{ OR } \Delta E_{grav} = mg\Delta h$$

kinetic energy (E_k) the energy an object stores by virtue of its movement:

$$\text{kinetic energy (J)} = \frac{1}{2} \times \text{mass (kg)} \times (\text{speed})^2 \text{ (m}^2 \text{ s}^{-2}\text{)}$$

$$E_k = \frac{1}{2} \times m \times v^2$$

LEARNING OBJECTIVES

- Calculate energy transferred as work done, including when the force is not along the line of motion.
- Calculate the power of an energy transfer.
- Explain efficiency and be able to calculate.

We often make assumptions that allow simplification of calculations in physics. In general, these assumptions make little difference as they are chosen to ignore effects which have a very small impact on the actual real world answers. An example of this is with the transfer of gravitational potential energy to kinetic energy, where the effects of air resistance are ignored.

It is important to remember that the principle of **conservation of energy** insists that no energy can be lost in any scenario. Even if we were to consider the loss of kinetic energy to heating of the air through air resistance, the total amount of energy would be constant; it would simply have transferred to different stores.

WORK

In physics, the phrase 'doing work' has a specific meaning to do with energy use. The amount of **work done** means the amount of **energy** transferred, so work is measured in joules.

In general terms, we can express any energy transfer as work done. For example, a 15 W light bulb working for 10 seconds transfers 150 J of electrical energy as heat and light – it does 150 J of work. In any situation where we know how to calculate the energy before and after, we can calculate the energy transferred and thus the work done.

FORCING WORK

If energy is transferred mechanically by means of a force, then the amount of work done can be calculated simply:

$$\text{work done (J)} = \text{force (N)} \times \text{distance moved in the direction of the force (m)}$$

$$\Delta W = F\Delta s$$



▲ **fig B** 'Work' on a building site.

In the example of **fig B**, a brick of mass 2.2 kg is lifted vertically against its weight through a height of 1.24 m. The work done is:

$$\Delta W = F\Delta s$$

$$\Delta W = mg \times \Delta s$$

$$\therefore W = 2.2 \times 9.81 \times 1.24$$

$$W = 26.8 \text{ J}$$

Note that this is the same amount of energy as we calculated for the gravitational potential energy of this same brick undergoing the same lift in **Section 1B.1**.



▲ **fig A** Work is done by transferring gravitational potential energy to a heavy object.

EXAM HINT

Beware of confusing ΔW for work with W for weight, and W as the abbreviation for watts.

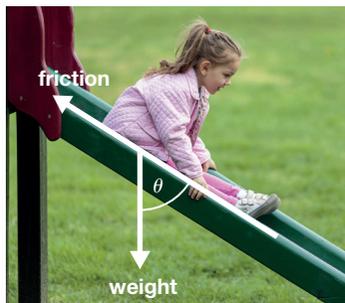
LEARNING TIP

An object that gains gravitational potential energy is having work done on it against its weight force.

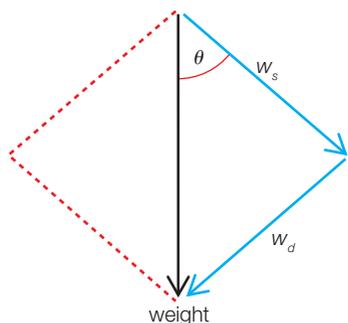
weight = mass \times gravitational field strength

$$W = mg$$

work = force \times distance = weight \times height = $mg\Delta h$



▲ **fig C** Gravity working against friction.



▲ **fig D** The weight force is the resultant of its components along the slide and perpendicular to it.

WORK DONE BY FORCES AT AN ANGLE

In the bricklaying example on the previous page, the direction of movement was exactly in line with the force lifting the brick. This is an unusual situation, and usually the force doing work will be at an angle to the direction of movement. In **fig C**, gravity pulls the child down the slide but it must work against friction. The weight acts vertically downwards, but the friction acts up the slide. Friction always acts in the exact opposite direction to movement.

Assuming that the child slides at constant velocity, Newton's first law of motion tells us that this means the friction is exactly balanced by the component of gravity pulling the child down the slope. So to find the component of the weight force that is acting down the slope, we will need to resolve it in the directions of down the slope and perpendicular to it.

The weight component working down the slope equals the friction, F .

$$F = mg\cos\theta$$

The work done is force multiplied by the distance travelled along the line of the force, so here:

$$\text{work} = \Delta s \times mg\cos\theta$$

This example shows us the general formula for calculating the work done when there is an angle between the force and the distance along which we are measuring:

$$\Delta W = F\Delta s\cos\theta$$

POWER

Power is defined as the rate of energy transfer. This may be done with reference to work done.

$$\text{power (W)} = \frac{\text{energy transferred (J)}}{\text{time for the energy transfer (s)}} \quad P = \frac{E}{t}$$

$$\text{power (W)} = \frac{\text{work done (J)}}{\text{time for the work to be done (s)}} \quad P = \frac{\Delta W}{t}$$

Remember:

$$\text{work (J)} = \text{force (N)} \times \text{distance moved in the direction of the force (m)}$$

$$\Delta W = F\Delta s$$

So:

$$\text{power (W)} = \frac{\text{force (N)} \times \text{distance moved (m)}}{\text{time for the force to move (s)}}$$

$$P = \frac{F\Delta s}{t}$$

For example, the power of a forklift truck lifting a 120 kg crate vertically up 5.00 m in 4.0 seconds would be calculated as:

$$P = \frac{F\Delta s}{t} = \frac{mg\Delta s}{t}$$

$$P = \frac{120 \times 9.81 \times 5}{4.0}$$

$$P = 1470 \text{ W} = 1.47 \text{ kW}$$

EXAM HINT

1 kilowatt (kW) = 1000 watts (W) = 1×10^3 W

1 megawatt (MW) = 1000 kW = 1×10^6 W

1 gigawatt (GW) = 1000 MW = 1×10^6 kW = 1×10^9 W

1 terawatt (TW) = 1000 GW = 1×10^6 MW = 1×10^9 kW = 1×10^{12} W

EFFICIENCY

From the equations above, we can calculate the work and power generated in different situations. If most of the energy is not actually transferred to a store that is useful to us, then the activity may be a waste of energy. The ability of a machine to transfer energy usefully is called **efficiency**.

Efficiency is defined mathematically as:

$$\text{efficiency} = \frac{\text{useful work done}}{\text{total energy input}}$$

$$\text{efficiency} = \frac{\text{useful energy output}}{\text{total energy input}}$$

If we remember that power is energy divided by time, then measuring the energy flows in a machine for a fixed amount of time means that we can write a power version of the efficiency equation:

$$\text{efficiency} = \frac{\text{useful energy output/time}}{\text{total energy input/time}}$$

$$\text{efficiency} = \frac{\text{useful power output}}{\text{total power input}}$$

The answer will be a decimal between zero and one. It is common to convert this to a percentage value (multiply the decimal by 100).

WORKED EXAMPLE

If the forklift truck referred to above lifted the crate when supplied with electrical energy from its battery at a rate of 3000 joules per second, what is its efficiency?

$$\text{efficiency} = \frac{\text{useful power output}}{\text{total power input}}$$

$$\text{efficiency} = \frac{1470}{3000}$$

$$\text{efficiency} = 0.49 = 49\%$$

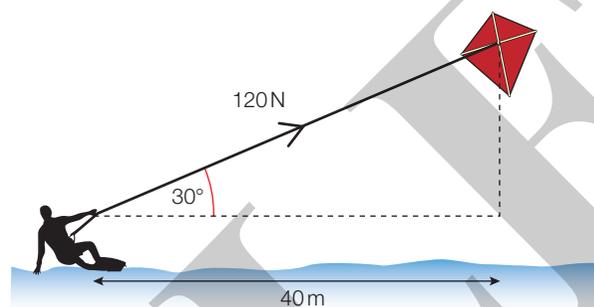
EXAM HINT

Note that the steps and layout of the solution in this worked example are suitable for efficiency questions in the exam.

CHECKPOINT

SKILLS PROBLEM SOLVING

- In these two situations, who does more work, and by how much?
 - A lioness carries a 2.8 kg cub 4.60 m up a tree.
 - An eagle lifts a 1.4 kg rabbit 8.25 m up to her nest.
- Calculate the work done by the tension in the kite string in **fig E** over the distance shown.



▲ fig E Moving at an angle to the force doing work.

- What is the power of an electric motor, in watts, if it lifts 500 g, through 80 cm in 20 seconds?
 - What is the motor's efficiency if the electricity supplied it with a total of 12 J to make the lift?
- In loading a delivery van, the driver pushed a 15 kg crate for 5 metres up a ramp. He had to push with a force of 132 N and the vertical height gained was 1.3 m. What was the efficiency of pushing this crate up the ramp onto the back of the van?

SUBJECT VOCABULARY

conservation of energy the rule that requires that energy can never be created or destroyed

work done in a mechanical system. This is the product of a force and the distance moved in the direction of the force:

$$\text{work done (J)} = \text{force (N)} \times \text{distance moved in the direction of the force (m)}$$

$$\Delta W = F\Delta s$$

energy the property of an object that gives it the capacity to do work. A change in the amount of energy of an object could be equated to work being done, even if this is not mechanical – a change in the heat energy of a sample of gas, for example

power the rate of energy transfer:

$$P = \frac{E}{t} = \frac{\Delta W}{t}$$

efficiency the ability of a machine to transfer energy usefully:

$$\text{efficiency} = \frac{\text{useful energy output}}{\text{total energy input}}$$

$$\text{efficiency} = \frac{\text{useful power output}}{\text{total power input}}$$

1B THINKING BIGGER

THE MECHANICS OF SOCCER

SKILLS

CRITICAL THINKING, PROBLEM SOLVING,
ANALYSIS, INTERPRETATION, ADAPTIVE
LEARNING, PRODUCTIVITY, COMMUNICATION

Soccer is the most popular sport in the world in terms of numbers of people playing. It is a fast moving, skilful sport in which the ball may fly at over 200 kilometres per hour.

In this activity, we will consider how mechanics can apply to events in soccer.

COACHING BOOKLET

GOALSCORING

In this section, we will look at some of the science behind shots on goal.



▲ **fig A** Antoine Griezmann kicks a shot at goal.

If we want to calculate how fast a soccer ball is moving after it has been kicked from stationary (for example, after a penalty shot), we need to think about its acceleration by using Newton's second law. A standard soccer ball has a mass of 0.40 kg. If the foot applies a force of 350 N for a twentieth of a second, we can work out the answer:

$$F = ma \text{ so } a = \frac{F}{m}$$
$$a = \frac{350}{0.40} \therefore a = 875 \text{ m s}^{-2}$$
$$v = u + at$$

The ball is stationary before it is kicked

$$\text{so } u = 0 \text{ m s}^{-1}, a = 875 \text{ m s}^{-2}, t = 0.05 \text{ s}$$

$$v = 0 + 875 (0.05)$$

$$v = 44 \text{ m s}^{-1}$$

From *Soccer is Mad Easy*, a booklet aimed at new coaches, particularly teachers who may also need to teach International A Level Physical Education



▲ **fig B** Tiago Volpi tries to save a penalty kick.

A penalty kick is taken from a spot 11 metres from the goal. If we assume zero drag forces, we can calculate the longest time the goalkeeper has to react to this shot after it leaves the foot.

We know the start velocity and the distance, so this is a straightforward question.

$$v = \frac{s}{t} \quad \text{so } vt = s$$
$$\therefore t = \frac{11}{44} \quad t = 0.25 \text{ s}$$

If the goalkeeper reacts quickly enough to catch the ball, how far will his hands be pushed backwards in order to stop the ball? The maximum decelerating force his arms can provide to slow the ball is 2000 newtons. Here we should consider the removal of all the ball's kinetic energy as work being done.

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}(0.40) \times (44)^2 = 387 \text{ J}$$

$$\Delta W = F\Delta s$$

$$\Delta s = \frac{\Delta W}{F} = \frac{387}{2000} = 0.194 \text{ m}$$

So the goalkeeper's hands are pushed back just under 20 cm in order to stop the ball. According to Newton's third law, when the hands produce a force of 2000 N on the ball, the ball exerts a force of 2000 N on the hands.

SCIENCE COMMUNICATION

1 The extract opposite is from a booklet to teach soccer coaches the theory behind the sport, especially for reference in International A Level Physical Education lessons, which have a significant amount of theory in them. Consider the extract and comment on the type of writing that is used. Try and answer the following questions:

- How has the author maintained the relevance of the calculations for the reader?
- Discuss the level of difficulty of the calculations, with reference to the target audience and the purpose of the text.

INTERPRETATION NOTE

Think about the complexity and level of difficulty of these calculations compared with those that you have to do in this physics course. Also consider that the numbers used in the examples are for top players – what level of player are these coaches probably dealing with?

PHYSICS IN DETAIL

Now we will look at the physics in detail. Some of these questions link to topics earlier in this book, so you may need to combine concepts from different areas of physics to work out the answers.

- Identify, and comment on the validity of, the assumptions that have been made in these calculations.
- A free kick is awarded 32 m from the goal line. The striker kicks it so that the ball leaves the ground at a 5° angle and a speed of 40 m s^{-1} . How long does the goalkeeper have to react before the ball reaches the goal line?
- The study of mechanics in sport is a popular and often profitable new area of scientific study.
 - Describe how a sports scientist could use electronic equipment to collect data to study the movement of players and equipment over time.
 - Explain why technological developments have made the data collected more valid and reliable than other traditional methods of studying mechanics.

ACTIVITY

Junior soccer rules for young children use a reduced pitch, 27.5 m by 18 m, and a ball with a mass of 320 g. Write a similar section for a new version of this coaching manual which is aimed at junior soccer coaches.

THINKING BIGGER TIP

You will need to consider the strength of small children in order to estimate the forces they can give with a kick.

1B EXAM PRACTICE

1 The definition of the watt comes from which equation?

- A $P = \frac{E}{t}$
- B $P = E \times t$
- C $\Delta W = F\Delta s$
- D $\Delta W = F\Delta \cos\theta$

[1]

(Total for Question 1 = 1 mark)

2 A 305 g ball is thrown vertically upwards to reach a height of 5.55 m. How much potential energy has it gained at that height?

- A 16.6 J
- B 16.6 kJ
- C 1690 J
- D 16 600 J

[1]

(Total for Question 2 = 1 mark)

3 Efficiency can be calculated using values of energy, or values of power.

Select the row of the table that correctly gives the expressions for calculating efficiency in terms of either energy or power:

	Energy expression	Power expression
A	efficiency = $\frac{\text{total energy input}}{\text{useful energy output}}$	$\frac{\text{useful power input}}{\text{total power output}}$
B	efficiency = $\frac{\text{useful energy output}}{\text{total energy input}}$	$\frac{\text{total power input}}{\text{useful power output}}$
C	efficiency = $\frac{\text{total energy input}}{\text{useful energy output}}$	$\frac{\text{useful power input}}{\text{total power output}}$
D	efficiency = $\frac{\text{useful energy output}}{\text{total energy input}}$	$\frac{\text{useful power output}}{\text{total power input}}$

[1]

(Total for Question 3 = 1 mark)

4 A horse pulls a carriage of weight 5600 N with a force of 80 N for a distance of 1.2 km around New York's Central Park. How much work is done by the horse?

- A 6.7 kJ
- B 96 kJ
- C 538 kJ
- D 6720 kJ

[1]

(Total for Question 4 = 1 mark)

5 The photograph shows a wind turbine. Kinetic energy of the wind is transferred to electrical energy as the turbine blades rotate.



- (a) Explain why we can say that the wind is doing work on the blades. [2]
- (b) The area swept out by one blade, as it turns through 360°, is 6000 m². Wind at a speed of 9 m s⁻¹ passes the turbine.
 - (i) Show that the volume of air passing through this area in 5 seconds is about 300 000 m³. [2]
 - (ii) Calculate the mass of this air. Density of air = 1.2 kg m⁻³ [2]
 - (iii) Calculate the kinetic energy of this mass of air. [2]
 - (iv) Betz's law states that a turbine cannot usefully transfer more than 59% of the kinetic energy of the wind. Use this law to find the maximum output of the wind turbine. [2]
- (c) Suggest a reason why it is not possible to usefully transfer 100% of the kinetic energy of the wind. [1]
- (d) Suggest the limitations of using wind turbines to provide power. [2]

(Total for Question 5 = 13 marks)

6 One account of the origin of the term *horsepower* is as follows. In the eighteenth century, James Watt manufactured steam engines. He needed a way to demonstrate the benefits of these compared to the horses they replaced. He did some calculations based on horses walking in circles to turn a mill wheel.

Watt observed that a horse could turn the wheel 144 times in one hour. The horse travelled in a circle of radius 3.7 m and exerted a force of 800 N.

- (a) Show that the work done by the horse in turning the wheel through one revolution was about 20 000 J. [3]
 (b) Calculate the average power of the horse in SI units. [3]

(Total for Question 6 = 6 marks)

7 In a demonstration of energy transfer, a large pendulum is made by suspending a 7.0 kg bowling ball on a long piece of wire.

A student is invited to pull back the ball until it just touches her nose and then to release it and stand perfectly still while waiting for the ball to return.



The following instructions are given:

Do not push the ball – just release it.
 Do not move your face before the ball returns.

Explain this demonstration and the need for these instructions. [6]

(Total for Question 7 = 6 marks)

8 The photograph shows a lawnmower being used to cut grass.



- (a) (i) In order to push the lawnmower, a minimum force of 650 N must be applied to the handle of the lawnmower at an angle of 42° to the horizontal. Show that the horizontal component of the force is about 500 N. [2]
 (ii) The lawnmower is used to cut 15 strips of grass, each 7 m long. Calculate the work done by the person pushing the lawnmower. [2]
 (b) This photograph shows a lawnmower with the top section of the handle horizontal.



Explain how this changes the minimum force required to push the lawnmower. [2]

(Total for Question 8 = 6 marks)

9 Metrology is the science of measurement and World Metrology Day is May 20th. In 2010, the day was used to celebrate the 50th anniversary of the SI system.

A metrologist from the National Physical Laboratory said on a radio programme that the SI system uses units that everyone can understand. He stated the following example.

'If you hold an apple in your hand it's about a *newton*, if you raise it through one metre that's about a *joule* and if you do it in one second that's about a *watt*.'

Assuming that the apple has a mass of 100 g, explain and justify the statements made about the three words in italics. [6]

(Total for Question 9 = 6 marks)

TOPIC 1 MECHANICS

CHAPTER

1C

MOMENTUM

Collisions can be devastating. Vehicle safety is a very important area of technological research, and much of the science is based on the concept of momentum and momentum transfer. Momentum is a property of a moving object. It is larger if the mass is greater and if the object moves faster. To change the momentum of an object requires a force, and this is how car crashes are so damaging. Larger forces cause more damage, whether to the vehicle or its passengers.

To reduce the forces involved in transferring momentum we need to know how momentum transfers and how to calculate the forces depending on the momentum transfer needed.

Not all momentum changes are dangerous. Rocket science is generally based on maximising the forces caused by a transfer of momentum, by maximising that momentum transfer. This will create maximum acceleration in order to move the rocket fast enough to gain enough gravitational potential energy to leave the Earth. This chapter will show you how the properties of a moving object can tell us its momentum, and how to calculate the transfer of momentum, and the forces involved in changing the movement of an object.

MATHS SKILLS FOR THIS CHAPTER

- Units of measurement (*e.g. the unit for momentum, kg m s^{-1}*)
- Changing the subject of an equation (*e.g. finding the velocity of an object after collision*)
- Substituting numerical values into algebraic equations (*e.g. calculating the momentum*)
- Plotting two variables from experimental data (*e.g. observing changes in momentum over time*)
- Using sin, cos and tan in physical problems, and making calculations using them (*e.g. resolving a momentum vector*)

What prior knowledge do I need?

Topic 1A

- Ideas about stopping distances of cars, and the safety features in vehicles
- Speed, distance, displacement, time, velocity, acceleration
- Resolving vectors
- Newton's laws of motion and the kinematics equations

What will I study in this chapter?

- Calculations of momentum
- What we mean by collisions and explosions, and how momentum is related to them
- The principle of conservation of linear momentum
- How to apply the principle of conservation of linear momentum in one dimension
- The relationship between Newton's laws and changing momentum

What will I study later?

Topic 2A

- Fluid movements and terminal velocity

Topic 2B

- Stress and strain and the deformation of solids

Topic 5A (Book 2: IAL)

- The conservation of momentum in two dimensions
- Elastic and inelastic collisions

Topic 7B (Book 2: IAL)

- The relationship between momentum and the kinetic energy of a particle
- The importance of particle momentum in the design of accelerators

LEARNING OBJECTIVES

- Calculate the momentum of an object.
- Explain how momentum is gained or lost.

MOMENTUM

Momentum is a measure of an object's motion. It is quite difficult to define momentum in words, but it gives an idea of what will be required to stop the object moving. The best definition is mathematical:

$$\text{momentum (kg m s}^{-1}\text{)} = \text{mass (kg)} \times \text{velocity (m s}^{-1}\text{)}$$

$$p = m \times v$$

As momentum is the product of mass (a scalar) and velocity (a vector), momentum is a vector. This means its direction is very important and must be remembered. The direction will be the same as that of its velocity.



▲ **fig A** Which object moves with the greatest momentum?

EXAM HINT

Note that the steps and layout of the solution in this worked example are suitable for momentum questions in the exam.

WORKED EXAMPLE

An athletics hammer (see **fig A**) has a mass of 7.26 kg (men's competition standard) and can be released at speeds in excess of 25 m s⁻¹. Its momentum at 25.0 m s⁻¹ would be:

$$p = m \times v$$

$$p = 7.26 \times 25.0$$

$$p = 182 \text{ kg m s}^{-1}$$

A baseball has a mass of 145 grams. A fast pitcher can throw it at 40 m s⁻¹. If this baseball is released at 40 m s⁻¹, its momentum is:

$$p = m \times v$$

$$p = 0.145 \times 40$$

$$p = 5.8 \text{ kg m s}^{-1}$$

These examples show that it is much more difficult to stop a well-thrown athletics hammer than to stop a baseball. Think about what 'more difficult' means in this case.

NEWTON'S SECOND LAW OF MOTION

If we want to bring an object to rest or to accelerate it up to a certain velocity, the requirements will be different for different situations. A golf ball is accelerated in a very different way to a ferry. Think of the forces needed and the time for which they act. This brings us to another way of thinking about momentum. It is a measure of the accelerating force, and the time it is applied for, that is required to bring an object up to the speed it is moving at. Alternatively, it is the force required, and for how long, to bring a moving object to rest.

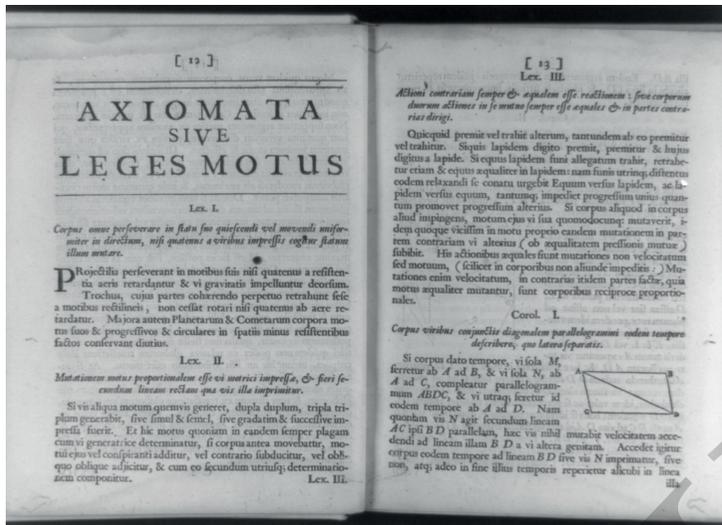


fig B Newton's laws of motion as he originally wrote them.

Newton's second law can be written mathematically as $F = ma$. In fact, that formula is only true if the mass remains constant. When Newton originally wrote his second law in the 1687 book, *Philosophiae naturalis principia mathematica*, he actually wrote it as:

The rate of change of momentum of a body is directly proportional to the resultant force applied to the body, and is in the same direction as the force.

This can be written mathematically as:

$$F = \frac{dp}{dt} = \frac{d(mv)}{dt}$$

Here, F is the applied force, and $\frac{dp}{dt}$ is the rate of change of momentum in the direction of the force.

The $\frac{d(x)}{dt}$ term is a mathematical expression meaning the rate of change of x , or how quickly x changes. However, if the quantities are not being measured over a very short timescale, we can express this using average changes:

$$F = \frac{\Delta p}{\Delta t}$$

PRACTICAL SKILLS

Investigating momentum change

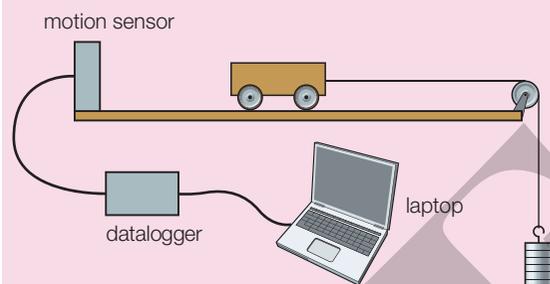


fig C Measuring how a force changes the momentum of a trolley.

You can investigate the rate of change of momentum in the school laboratory. A trolley starts from rest and as a force acts upon it its velocity increases. If you record the trolley's movement over time, you can find the velocity each second. If you then calculate the momentum each second, you will be able to plot a graph of momentum against time. It should be a straight line. As $p = Ft$, the gradient of this line will be equal to the accelerating force.

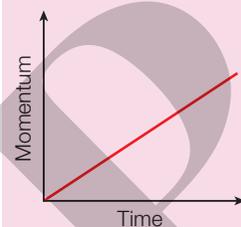


fig D Accelerating from rest, momentum will be proportional to time.

! Safety Note: Place trolleys and runways so they cannot fall and cause injuries. For large masses, place a 'catch box' filled with crumpled paper or bubbled plastic in the 'drop zone' to avoid injury to feet.

CHECKPOINT

SKILLS **PROBLEM SOLVING**

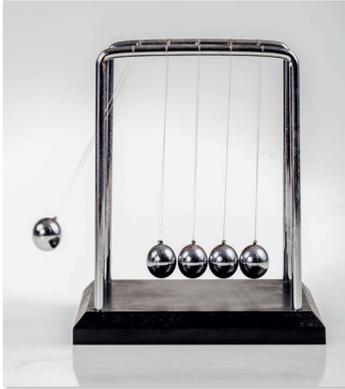
- Calculate the momentum in each of these examples:
 - an ice skater with a mass of 64 kg glides at 3.75 m s⁻¹
 - a rugby player of mass 120 kg runs at a speed of 4.9 m s⁻¹
 - an ant of mass 5 milligrams moves at a speed of 5 centimetres per second.
- Estimate the momentum of the motorcyclist and the skateboarder shown in **fig A**.
- Using the ideas of Newton's second law, explain why hitting an airbag will cause less injury than if a passenger hits the dashboard.
- Estimate the force applied by a person throwing a Frisbee.

SUBJECT VOCABULARY

momentum (kg m s⁻¹) = mass (kg) × velocity (m s⁻¹)
 $p = m \times v$

LEARNING OBJECTIVES

- Explain the principle of conservation of linear momentum.
- Make calculations based on the conservation of linear momentum.



▲ **fig A** Newton's cradle: each time the balls collide, momentum is transferred from one to another, but the total momentum remains constant.

LEARNING TIP

Total momentum is only conserved when no external forces (such as friction) act on the system.

COLLISIONS

When objects collide, we can use the laws of physics to calculate where the objects will go after the collision. We can use the principle of **conservation of linear momentum** to predict the motion of objects after a collision. This principle tells us that if we calculate the momentum of each object before they collide, the sum total of these momenta (accounting for their direction) will be the same as the sum total afterwards.

LEARNING TIP

The word 'linear' appears here to remind us that this is all about objects moving in straight lines. There are similar physics principles about rotating objects, but they use different equations for the calculations. In this book we will only consider linear momentum.

This principle depends on the condition that no external force acts on the objects in question. An external force would provide an additional acceleration, and the motion of the objects would not be dependent on the collision alone. As we saw in the previous section, a resultant force will cause a change in momentum, so it makes sense that momentum is only conserved if no external force acts. Imagine if a juggler's ball moving upward collided with one coming down. Momentum conservation would suggest that the one falling down would bounce back with an upward velocity after the collision. Common sense tells us that all balls will still end up back on the ground. The external force of gravity means that the principle of conservation of momentum alone cannot be used to predict the motions after the collision.



▲ **fig B** David Porter of the TCU Horned Frogs feels the full force of the conservation of momentum.

WORKED EXAMPLE

In an American football match, the stationary quarterback is tackled by a defender who dives through the air at 4 m s^{-1} and, in mid-air, grabs the quarterback and the two move quickly backwards together. Ignoring any friction effects, calculate how fast the two will move back if the tackler has a mass of 140 kg and the stationary player has a mass of 95 kg . Consider the entire situation to be happening horizontally.

Before:

Quarterback stationary so zero momentum

$$p_{\text{tackler}} = mv = 140 \times 4 = 560$$

$$\text{momentum before} = 560 \text{ kg m s}^{-1}$$

After:

$$\text{momentum after} = \text{momentum before} = 560 \text{ kg m s}^{-1}$$

$$p_{\text{both}} = m_{\text{both}} \times v_{\text{both}}$$

$$v_{\text{both}} = \frac{p_{\text{both}}}{m_{\text{both}}} = \frac{560}{(140 + 95)} = \frac{560}{235}$$

$$v_{\text{both}} = 2.4 \text{ m s}^{-1}$$

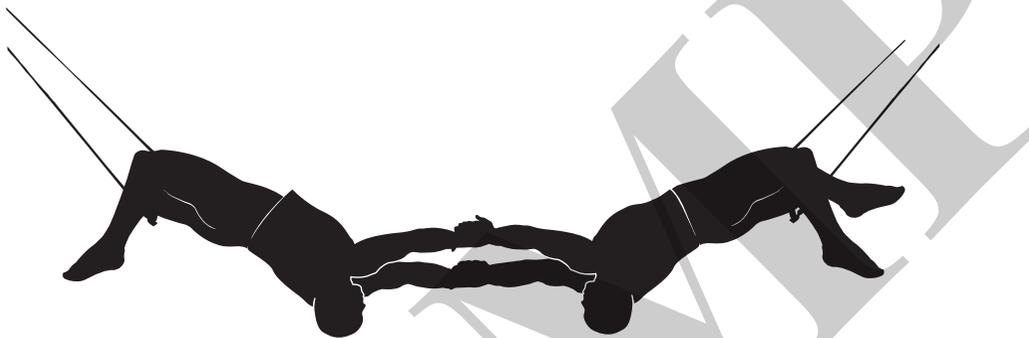
EXAM HINT

For any exam question about momentum in collisions and explosions, make sure you state: 'total momentum before = total momentum after'.

LEARNING TIP

In a collision in which two objects join together to become one and move off together, they are often said to 'coalesce'.

EXPLOSIONS



▲ **fig C** In this illustration, these trapeze artists are stationary. If they let go of each other, they will 'explode' – they will fly apart with equal and opposite momenta.

If a stationary object explodes, then the total momentum of all the shrapnel parts added up (taking account of the direction of their movements) must be zero. The object had zero momentum at the start, so the law of conservation of linear momentum tells us this must be the same total after the **explosion**. In physics, any such event is termed an explosion, although it may not be very dramatic. For example, if the two trapeze artists in **fig C** simply let go their hands and swing apart, they have zero total momentum before and will have equal and opposite momenta afterwards, which when added together will total zero again.

EXAM HINT

Note that the steps and layout of the solution in this worked example are suitable for conservation of linear momentum questions in the exam.

WORKED EXAMPLE

If the boy has a mass of 55 kg and steps forward at a speed of 1.5 m s^{-1} , what will happen to the boat which has a mass of 36 kg? (Ignore friction effects.)

This situation is an explosion, so:

total momentum before = total momentum after = zero

$$\therefore p_{\text{boat}} + p_{\text{boy}} = 0$$

$$\therefore p_{\text{boat}} = -p_{\text{boy}}$$

So when the two are added up, the total momentum is still zero.

$$\therefore p_{\text{boat}} = -(55 \times 1.5) = -82.5 \text{ kg m s}^{-1}$$

$$m_{\text{boat}} \times v_{\text{boat}} = -82.5 \text{ kg m s}^{-1}$$

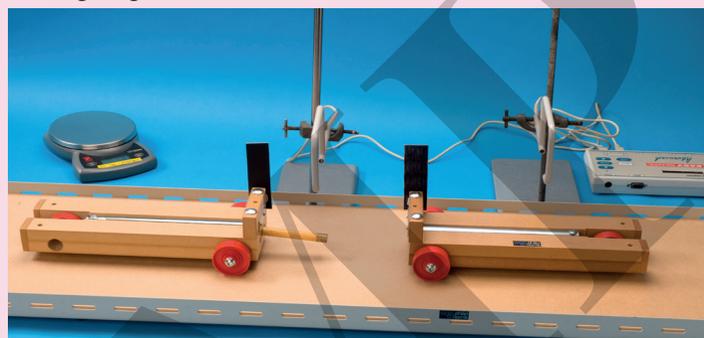
$$v_{\text{boat}} = \frac{-82.5}{m_{\text{boat}}} = \frac{-82.5}{36}$$

$$v_{\text{boat}} = -2.3 \text{ m s}^{-1}$$

So the boat moves at 2.3 m s^{-1} in the opposite direction to the boy.



▲ **fig D** Caution: explosions may make you wet!

PRACTICAL SKILLS**Investigating transfer of momentum**

▲ **fig E** Verifying the principle of conservation of linear momentum.

You can investigate the transfer of momentum in collisions in the school laboratory using trolleys, or sliders on an airtrack. By recording the movement of one trolley crashing into another, you can find the momentum of each one before and after the collision. The calculation of adding up the total momenta before and after collision will allow you to prove the principle of conservation of linear momentum. Try different types of collision and trolleys with different masses. You could also try an explosion in which the trolleys come apart from a stationary position.

In experiments using trolleys, we often find that momentum is actually not conserved in the measurements we make. With airtrack collisions, the measurements match very closely or exactly with the conservation of momentum theory. What might be the reasons for this difference between the two types of experiment?



▲ **fig F** A trolley 'explosion'.



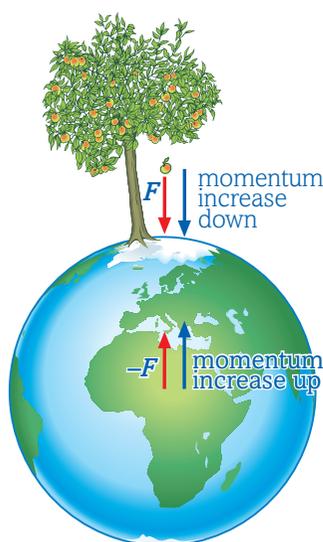
! Safety Note: Place trolleys and runways so they cannot fall and cause injuries.

NEWTON'S THIRD LAW

Conservation of momentum is directly responsible for Newton's third law. Remember, this told us that for every force, there is an equal and opposite force. If we think of a force as a way to change

momentum ($F = \frac{dp}{dt}$) then a force changing momentum in one direction must be countered

by an equal and opposite one to ensure that overall momentum is conserved. For example, if the gravitational pull of the Earth causes an apple to fall from a tree, the apple gains momentum towards the Earth. For conservation of momentum, the Earth must gain an equal and opposite momentum. This is then caused by an equal and opposite gravitational force on the Earth from the apple. The huge mass of the Earth means that its acceleration cannot be noticed by us.



▲ **fig G** Conservation of momentum causes equal and opposite forces, as Newton explained in his third law of motion.

EXAM HINT

Make sure you answer the question that has been asked. In Q3, no marks will be awarded for answers that do not refer to Newton's third law.

CHECKPOINT

SKILLS ANALYSIS

1. A movie stuntman with a mass of 90 kg stands on a stationary 1 kg skateboard. An actor throws a 3.4 kg brick at the stuntman who catches it. The brick is travelling at 4.1 m s^{-1} when caught.
2. A boy in a stationary boat on a still pond has lost his oars in the water. In order to get the boat moving again, he throws his rucksack horizontally out of the boat with a speed of 4 m s^{-1} .
Mass of boat = 60 kg; mass of boy = 40 kg; mass of rucksack = 5 kg
 - (a) How fast will this action make the boat move?
 - (b) If he throws the rucksack by exerting a force on it for 0.2 s, how much force does he exert?
3. How can Newton's third law explain the problem suffered by the boy stepping out of the boat in **fig D**?
4. In a stunt for an action movie, the 100 kg actor jumps from a train that is crossing a river bridge. On the river below, the heroine tied to a small boat is drifting towards a waterfall at 3 m s^{-1} . The small boat and heroine have a total mass of 200 kg.
 - (a) If the hero times his jumps perfectly so as to land on the small boat, and his velocity is 12 m s^{-1} at an angle of 80° to the river current, what will be the velocity of the small boat immediately after his landing? Draw a vector diagram to show the momentum addition. Ignore any vertical motion.
 - (b) If the waterfall is 100 m downstream, and the hero landed when the small boat was 16 m from the bank, would they drop over the fall? Assume the velocity remains constant after the hero has landed. The small boat and the waterfall are on the same side of the bridge as he jumps.

SUBJECT VOCABULARY

conservation of linear momentum the vector sum of the momenta of all objects in a system is the same before and after any interaction (collision) between the objects

explosion a situation in which a stationary object (or system of joined objects) separates into component parts, which move off at different velocities. Momentum must be conserved in explosions

1C THINKING BIGGER

SAVING HOCKEY GOALKEEPERS

SKILLS

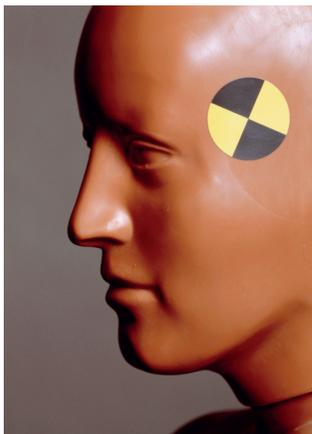
CRITICAL THINKING, PROBLEM SOLVING, ANALYSIS, REASONING/ARGUMENTATION, INTERPRETATION, ADAPTIVE LEARNING, PRODUCTIVITY, SELF-EVALUATION, COMMUNICATION, TEAMWORK

OBO is a New Zealand based company that manufactures hockey goalkeeping equipment. The following extracts from their website explain some of their testing laboratory's abilities, and report on a potential new material used in leg guards.

COMPANY WEBSITE

A LOOK INSIDE THE O LAB

In order to design and build the world's most protective and best performing goalkeeper equipment we need the facts. The O Lab is packed full of the world's most advanced impact test equipment... and a few very clever people to test and help evaluate the results.



▲ **fig A** Anatomically correct crash test dummy head.



▲ **fig B** Data capture software simultaneously showing video and concussion data.

Every detail sorted by a small group of smart committed people. Video capture at speeds up to 2000 frames per second, skin contact analysis, and accurate concussion measurements.



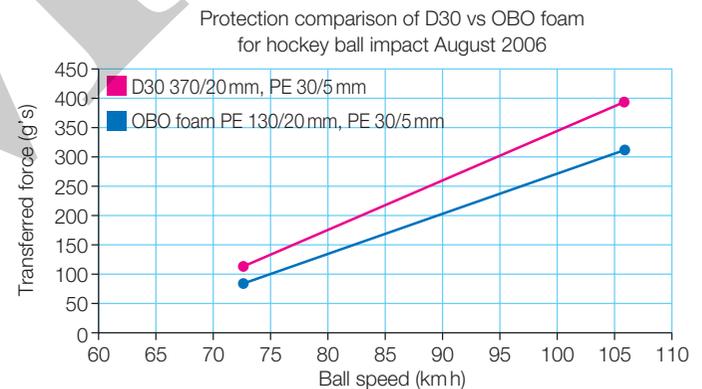
▲ **fig C** Ball cannon capable of speeds in excess of 200 km h.

WHY OBO DON'T USE D30

A while ago, an important new protection polymer called D30 was offered to OBO. Because we are always trying to improve our products we were excited by the potential of D30 so our designer made a special trip from New Zealand to England to meet with the D30 creators. He returned home with lots of information and some samples which we tested in the purpose-built OBO impact lab... The O lab.

Our impact lab testing showed that while D30 weighed more than two and a half times the OBO polyethylene and EVA foams, it provided significantly less protection when dealing with the high speed and highly localised impact encountered with a hockey ball.

Have a look at the results on the impact graph below (the horizontal axis is speed, the vertical axis is transferred force – the higher the transferred force, the less the protection).



▲ **fig D** Protection comparison of D30 vs OBO foam for hockey ball impact.

SCIENCE COMMUNICATION

- 1 The website opposite was written to support the business activities of a sports supply company. Consider the article and comment on the type of writing being used. Think about, for example, how the physics involved is explained, and the degree of detail included. Try and answer the following questions:
- What range of people do you expect might read the OBO website? What would be the likely scientific background of their customers?
 - On the actual OBO webpage, there are many more pictures, including brief videos, that we could not fit in this book. How has the ratio of text to images been chosen for the intended audience, and the website as a medium?
 - Where units have been included, comment on the actual units chosen to measure the quantities involved.

INTERPRETATION NOTE

As you read the articles, consider where they have come from, who wrote them and whom they were written for. Established scientific publications are good sources of reliable information, whereas other resources might be less reliable for a number of reasons. Think about what makes a source reliable and why.

PHYSICS IN DETAIL

Now we will look at the physics in, or connected to, these website extracts. Some of these questions link to topics in much earlier sections of this book, so you may need to combine concepts from different areas of physics to work out the answers.

- For the testing of the polymer D3O, calculate the range of momenta for the test balls fired from the cannon. (A standard hockey ball has a mass of 160 grams.)
- Explain, with reference to Newton's laws:
 - why the 'transferred force', i.e. the force felt through the foam by a goalkeeper wearing it, would be the same as the force needed to decelerate the ball
 - why the lines on the graph show a linear relationship with a positive gradient.
- In light of the principle of conservation of linear momentum, how can a goalkeeper remain stationary, whilst the ball's momentum is completely removed in collision with the leg pads?

ACTIVITY

Imagine you work for OBO as an international sales representative and you have to prepare a presentation to delegates at a trade show. Your presentation will need to explain in much greater scientific detail the testing that the equipment has been through in The O Lab. Prepare a questionnaire, for OBO head office in New Zealand, of questions that will give you the details you need to prepare your presentation.

THINKING BIGGER TIP

You do not need to prepare the presentation for the trade show, just your questionnaire designed to get the information you need from OBO head office to be able to prepare such a presentation.

DID YOU KNOW?

OBO use one unusual testing procedure they call the DTH test. This involves a real goalkeeper wearing the item under test. A ball is fired at the test subject and the lab researchers ask the question, 'Did That Hurt?'

1C EXAM PRACTICE

1 Which is the correct expression for calculating momentum?

- A Mass \times speed
- B Mass \div speed
- C Mass \times velocity
- D Mass \div speed

[1]

(Total for Question 1 = 1 mark)

2 Which of the following is the correct unit for momentum?

- A kg s^{-1}
- B kg m s^{-1}
- C kg m s^{-2}
- D $\text{kg m}^{-1} \text{s}^{-1}$

[1]

(Total for Question 2 = 1 mark)

3 A hockey ball of mass 158 g is hit with a force of 2000 N so that it travels at 28.1 ms^{-1} . What is the ball's momentum?

- A 4.4 kg m s^{-1}
- B 62.4 kg m s^{-2}
- C $4440 \text{ kg m}^{-1} \text{s}^{-1}$
- D 8880 kg m s^{-1}

[1]

(Total for Question 3 = 1 mark)

4 An ice skater and his coach begin stationary and push apart from each other with a force of 75 N. The skater has a mass of 64 kg, whilst the coach weighs 804 N.

If the skater moves off to the left with a speed of 3.6 m s^{-1} , what is the velocity of the coach?

- A 0.022 m s^{-1} to the right
- B 2.8 m s^{-1} to the right
- C 3.1 m s^{-1} to the left
- D 3.1 m s^{-1} to the right

[1]

(Total for Question 4 = 1 mark)

5 In an explosion, a stationary object of mass M splits into two objects which move in opposite directions, left and right, at the same speed.

Which row in the table correctly gives the mass of each of the two objects after the explosion?

	Mass of left moving object	Mass of right moving object
A	$\frac{M}{2}$	$\frac{M}{2}$
B	$\frac{M}{2}$	$2M$
C	$2M$	$2M$
D	M	M

[1]

(Total for Question 5 = 1 mark)

6 How tiny bacteria move is of interest in nanotechnology. Mycobacteria move by ejecting slime from nozzles in their bodies.

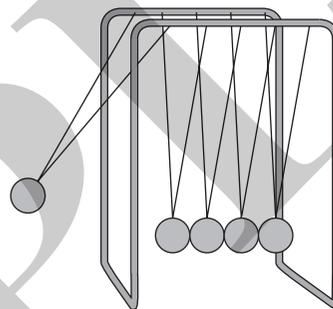
Explain the physics principles behind this form of propulsion.

[4]

(Total for Question 6 = 4 marks)

7 A student is using a 'Newton's Cradle'. This consists of a set of identical solid metal balls hanging by threads from a frame so that they are in contact with each other.

He initially pulls one ball to the side as shown.



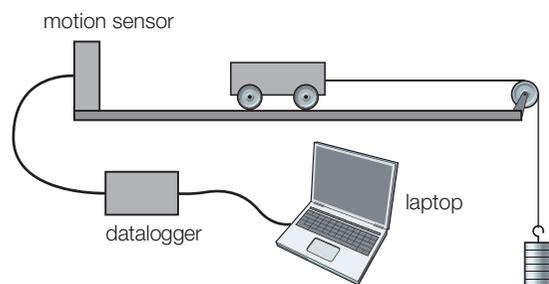
He releases the ball, it collides with the nearest stationary ball and stops. The ball furthest to the right immediately moves away. The middle three balls remain stationary.

- (a) Explain what measurements the student would take and describe how he would use them to investigate whether momentum had been conserved in this event. [4]
- (b) The student makes the following observations:
 - the ball on the right returns and collides with a similar result; this repeats itself a number of times
 - after a while, the middle balls are also moving
 - shortly afterwards, the balls all come to rest.

Discuss these observations in terms of energy. [3]

(Total for Question 7 = 7 marks)

- 8 A student uses a motion sensor and a datalogging computer to investigate the momentum changes when a trolley is accelerated by a falling weight connected to it. Assume the trolley suffers no friction on the desk, and there is also no friction in the pulley wheel.



- (a) Explain how Newton's second law of motion predicts that the momentum of this trolley will change when the weight is allowed to fall freely. [2]
- (b) When the weight is released, the trolley experiences a resultant accelerating force of 2.85 N and has a mass of 350 g. Calculate the rate of change of velocity of the trolley. [3]
- (c) The trolley reaches a velocity of 11.1 m s^{-1} , calculate its momentum at this velocity, including the correct unit. [2]
- (d) Explain what is meant by the principle of conservation of momentum. [2]

The student changes the experiment so that he can collide a moving trolley with an identical stationary one.

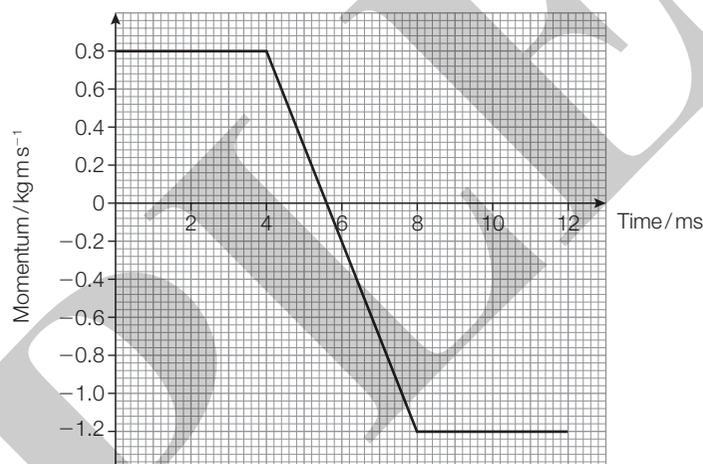


- (e) Explain the momentum and speed changes for each trolley if the moving trolley stops on collision, and the stationary one moves away. [4]
- (f) Explain the momentum and speed changes involved if the trolleys join together on collision, and both move away together. [4]
- (g) Explain how this experiment could be changed to investigate the momentum and speed changes in an explosion. [3]

(Total for Question 8 = 20 marks)

- 9 Explain how the principle of conservation of momentum in collisions is a consequence of Newton's third law of motion. [6]
- (Total for Question 9 = 6 marks)

- 10 A hockey ball is travelling horizontally with a momentum of 0.8 kg m s^{-1} just before it hits a goalkeeper's leg pad. It rebounds horizontally from the leg pad with a momentum of -1.2 kg m s^{-1} . The graph shows the variation in the momentum of the ball during this process.



- (a) Describe how the ball's momentum changes over time from 0 to 10 ms. [3]
- (b) Explain in terms of Newton's laws why the momentum changes from positive to negative during the ball's collision with the leg pad. [2]
- (c) What is the resultant force on the ball during the following time periods? [4]
- 0–4 ms
 - 4–8 ms
 - 8–12 ms
- (d) Draw a new version of the graph for a collision in which the ball is initially travelling at half the speed, and for which the impact time is also halved, but the force provided by the leg pads is the same. [3]

(Total for Question 10 = 12 marks)

TOPIC 2 MATERIALS

CHAPTER 2A FLUIDS

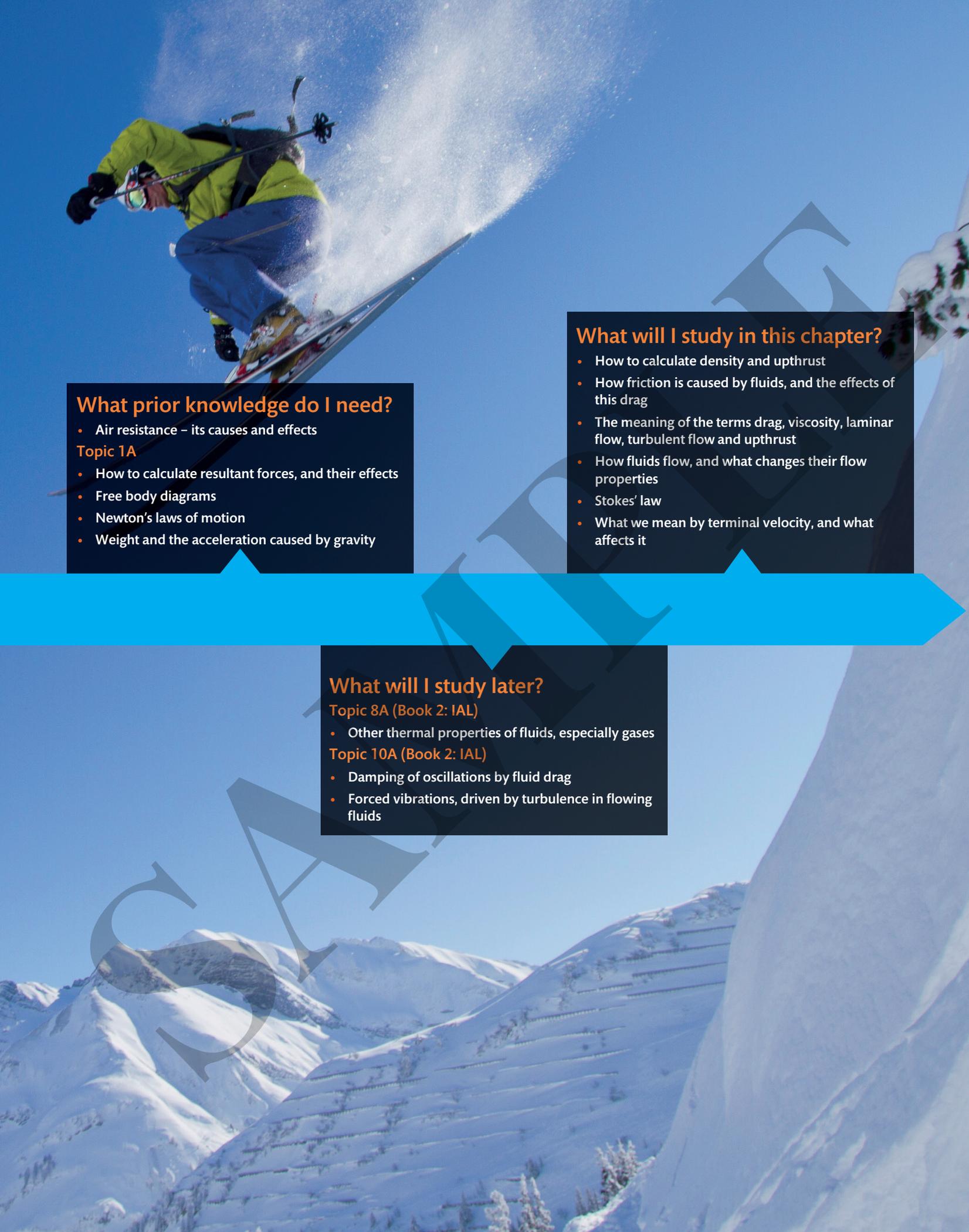
In Topic 1A, we calculated the motion of objects. We followed Newton's laws, and assumed zero air resistance. How safe was that assumption? Urban myths describe coins dropped from high buildings that cut deep into the concrete below – are these myths actually true? A very approximate calculation, using estimates of average coin size and mass, and the strength of the concrete, suggests that from a height of 100 metres, a falling coin could penetrate tens of centimetres ... if it fell in a vacuum.

The falling coin might tumble in flight, depending upon the exact effects of air resistance. This resistance will slow the coin significantly, and it will hit the ground at a much lower speed than the theoretical estimate in the calculation mentioned above. Most likely, it will land with an impact similar to that of a hailstone.

In this chapter, you will see how gases and liquids behave when flowing, or when causing friction with moving solid objects. You will also learn about some of the factors that can affect these movements.

MATHS SKILLS FOR THIS CHAPTER

- Units of measurement (*e.g. the unit for density, kg m^{-3}*)
- Visualising and representing 3D forms and finding volumes of rectangular blocks, cylinders and spheres (*e.g. finding the volume of an object as a step towards finding its density*)
- Changing the subject of an equation (*e.g. re-arranging the Stokes' law equation*)
- Solving algebraic equations (*e.g. finding the depth at which a barge floats in a river*)
- Substituting numerical values into algebraic equations (*e.g. an upthrust calculation*)
- Determining the slope of a linear graph (*e.g. finding the viscosity from terminal velocity data*)



What prior knowledge do I need?

- Air resistance – its causes and effects

Topic 1A

- How to calculate resultant forces, and their effects
- Free body diagrams
- Newton's laws of motion
- Weight and the acceleration caused by gravity

What will I study in this chapter?

- How to calculate density and upthrust
- How friction is caused by fluids, and the effects of this drag
- The meaning of the terms drag, viscosity, laminar flow, turbulent flow and upthrust
- How fluids flow, and what changes their flow properties
- Stokes' law
- What we mean by terminal velocity, and what affects it

What will I study later?

Topic 8A (Book 2: IAL)

- Other thermal properties of fluids, especially gases

Topic 10A (Book 2: IAL)

- Damping of oscillations by fluid drag
- Forced vibrations, driven by turbulence in flowing fluids

LEARNING OBJECTIVES

- Use the equation defining density.
- Understand how the upthrust force in a fluid can be calculated.

FLUIDS

Have you ever wondered why it is sometimes so difficult to get thick sauce out of a bottle? The answer is that the manufacturers make it thick on purpose. Market research shows that consumers enjoy a certain consistency of ketchup or mayonnaise on their fries, and producing it that thick makes the sauce flow very slowly.

This chapter will explain various aspects of the movements of fluids, including some of the ways in which fluid properties are measured. A **fluid** is defined as any substance that can flow. Normally this means any gas or liquid, but solids made up of tiny particles can sometimes behave as fluids; an example is the flow of sand through an hourglass.

DENSITY

One of the key properties of a fluid is its **density**. Density is a measure of the mass per unit volume of a substance – this is technically called ‘volumic mass’. Its value depends on the mass of the particles from which the substance is made, and how closely those particles are packed:

$$\text{density (kg m}^{-3}\text{)} = \frac{\text{mass (kg)}}{\text{volume (m}^3\text{)}}$$

$$\rho = \frac{m}{V}$$



▲ **fig A** Density is very important in determining the weight of an object.

The equation for calculating density works for mixtures and pure substances, and for all states of matter. Thus, fluid density is also mass per unit volume.

WORKED EXAMPLE

A house brick is 23 cm long, 10 cm wide and 7 cm high. Its mass is 3.38 kg.

What is the brick's density?

$$\rho = \frac{m}{V}$$

$$\begin{aligned} \text{volume } V &= 0.23 \times 0.10 \times 0.07 \\ &= 1.61 \times 10^{-3} \text{ m}^3 \end{aligned}$$

$$\text{mass } m = 3.38 \text{ kg}$$

$$\begin{aligned} \text{density } \rho &= \frac{3.38}{1.61 \times 10^{-3}} \\ \rho &= 2100 \text{ kg m}^{-3} \end{aligned}$$

EXAM HINT

Note that the steps and layout of the solution in this worked example are suitable for density questions in the exam.

At 20 °C, a child's balloon filled with helium is a sphere with a radius of 20 cm. The mass of helium in the balloon is 6 grams.

What is the density of helium at this temperature?

$$\rho = \frac{m}{V}$$

$$r = 0.20 \text{ m}$$

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi \times (0.20)^3 \\ &= 0.0335 \text{ m}^3 \end{aligned}$$

$$m = 0.006 \text{ kg}$$

$$\rho = \frac{0.006}{0.0335}$$

$$\rho = 0.179 \approx 0.18 \text{ kg m}^{-3}$$

LEARNING TIP

As objects expand when they get hotter, the volume depends on the temperature, and so the density must also be affected by changes in temperature.

Here is a table showing densities for different materials.

MATERIAL	STATE	DENSITY / kg m ⁻³
air	gas (sea level, 20 °C)	1.2
pure water	liquid (4 °C)	1000
sulfuric acid (95% conc)	liquid (20 °C)	1839
cork	solid	240
ice	solid	919
window glass	solid	2579
iron	solid	7850
gold	solid	19 320

table A Examples of density values for solids, liquids and gases.

UP THRUST

When an object is submerged in a fluid, it feels an upwards force caused by the fluid pressure – the **upthrust**. It turns out that the size of this force is equal to the weight of the fluid that has been displaced by the object. This is known as **Archimedes' principle**. If the object is completely submerged, the mass of fluid displaced is equal to the volume of the object multiplied by the density of the fluid:

$$m = V\rho$$

The weight of fluid displaced (i.e. upthrust) is then found using the relationship:

$$W = mg$$

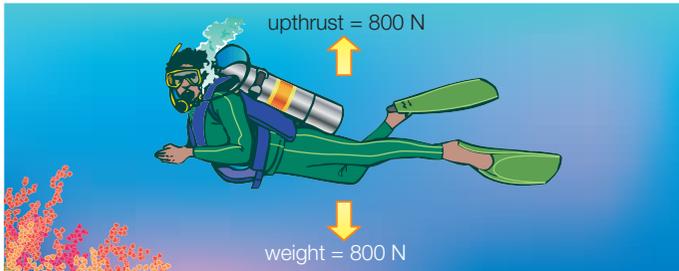


fig B Scuba diving equipment includes a buoyancy control device that can change volume to displace more or less water. This varies the upthrust and so helps the diver move up or down.

WHY DOES A BRICK SINK?

If the house brick from the example calculation of density above were dropped in a pond, it would experience an upthrust equal to the weight of water it displaced. This is simply the weight of an equal volume of water. As the density of water is 1000 kg m^{-3} , the mass of water displaced by the brick would be:

$$m = 1000 \text{ kg m}^{-3} \times 1.61 \times 10^{-3} \text{ m}^3 = 1.61 \text{ kg}$$

The water has a weight of:

$$W = 1.61 \times 9.81 = 15.8 \text{ N}$$

so there is an upward force on the brick of 15.8 N.

If we compare the weight of the brick with the upthrust when it is submerged, the resultant force will be downwards:

$$\text{weight} = 3.38 \times 9.81 = 33.2 \text{ N downwards}$$

$$\text{upthrust} = 15.8 \text{ N upwards}$$

$$\text{resultant force} = 33.2 - 15.8 = 17.6 \text{ N downwards}$$

So, the brick will accelerate downwards within the water until it reaches the bottom of the pond, which then exerts an extra upwards force to balance the weight so the brick rests stationary on the bottom with zero resultant force.

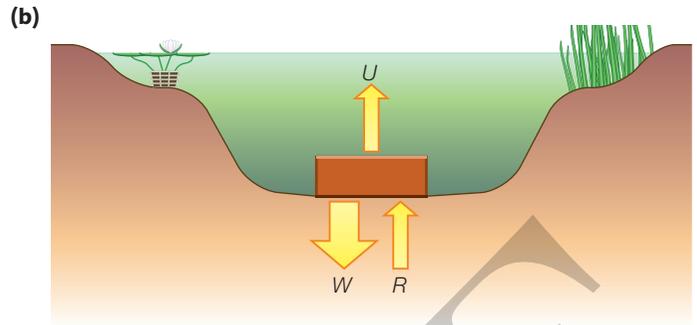
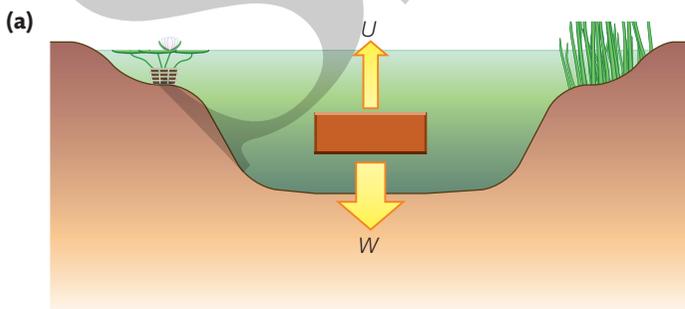


fig C (a) If the upthrust on an object is less than its weight, then the object will sink through a fluid; (b) an object will remain at rest when balanced forces act on it.

FLOATING

Imagine an object lowering into a fluid. The instant the object touches the surface of the fluid there is no upthrust, because no fluid has been displaced. As the object sinks deeper into the fluid, it displaces an increasing volume of the fluid, so increasing the upthrust acting upon it. If a point is reached when the upthrust and weight are balanced exactly, the object will stop sinking further – it will float there. So, for an object to float, it will have to sink until it has displaced its own weight of fluid.

WORKED EXAMPLE

A giant garbage barge on New York's Hudson River is 60 m long and 10 m wide. What depth of the hull will be under water if it and its cargo have a combined mass of 1500 tonnes? (Assume that the density of water in the Hudson River = 1000 kg m^{-3} .)

To float:

$$\text{upthrust} = \text{weight}$$

$$\text{weight} = mg = 1500 \times 1000 \times 9.81 = 1.47 \times 10^7 \text{ N}$$

$$\therefore \text{upthrust} = 1.47 \times 10^7 \text{ N}$$

The upthrust is equal to the weight of the volume of water displaced by the hull:

$$\text{upthrust} = \rho \times V \times g$$

where:

volume $V = \text{length of hull, } l \times \text{width of hull, } w \times \text{depth of hull under water, } d$

So:

$$\text{upthrust} = 1000 \times 60 \times 10 \times d \times 9.81$$

$$= 5.89 \times 10^6 \times d$$

$$d = \frac{1.47 \times 10^7}{5.89 \times 10^6}$$

$$d = 2.5 \text{ m}$$

The hull will be 2.5 m underwater.

EXAM HINT

Note that the steps and layout of the solution in this worked example are suitable for upthrust questions in the exam.



▲ **fig D** A hydrometer can check the density of battery fluids, which tells us if the fluids need changing.

THE HYDROMETER

The idea of floating at different depths is the principle behind the **hydrometer**, an instrument used to determine the density of a fluid. The device has a constant weight, so it will sink lower in fluids of lesser density. This is because a greater volume of a less-dense fluid must be displaced to balance the weight of the hydrometer. Scale markings on the narrow stem of the hydrometer indicate the density of liquid.

Some car batteries use a sulfuric acid solution. The density of this solution tells us the charge level of the battery, and should be checked by a mechanic when the car is serviced.

HYDROMETER READING (DENSITY COMPARED TO WATER)	STATE OF CHARGE
1.255–1.275	100%
1.215–1.235	75%
1.180–1.200	50%
1.155–1.165	25%
1.110–1.130	0%

table B For a particular car battery, the hydrometer readings can be compared to a table to tell us how charged the battery is.

SKILLS PROBLEM SOLVING

CHECKPOINT

- A car battery contains 1 litre of sulfuric acid solution. The mass of the liquid in the battery is 1.265 kg. What is the density of this battery fluid? (1000 litres = 1 m³)
- The radius of a bowling ball is 0.11 m and its mass is 7.26 kg. What is its density
 - in kg m⁻³?
 - in g cm⁻³?
- ▶ Estimate the mass of air in this room.
- A golf ball has a diameter of 4.27 cm.
 - If a golf player hits the ball into a stream, what upthrust does it experience when it is completely submerged? (Assume density of water = 1000 kg m⁻³.)
 - If the mass of the ball is 45 g, what is the resultant force on it when underwater?
 - Referring to Newton's laws of motion, explain what will happen to the submerged golf ball.
- A ball bearing of mass 180 g is hung on a thread in oil of density 800 kg m⁻³. Calculate the tension in the string if the density of the ball bearing is 8000 kg m⁻³.
- Estimate your own density.

SUBJECT VOCABULARY

fluid any substance that can flow

density a measure of the mass per unit volume of a substance

upthrust an upwards force on an object caused by the object displacing fluid

Archimedes' principle the upthrust on an object is equal to the weight of fluid displaced

hydrometer an instrument used to determine the density of a fluid

LEARNING OBJECTIVES

- Understand the terms laminar flow and turbulent flow.

If you ski down a hill, you can go faster by tucking your body into a crouching position. By presenting a smaller area to air resistance, you reduce the force slowing you down. However, speed skiers chasing world record speeds go further in their efforts to increase their speeds.



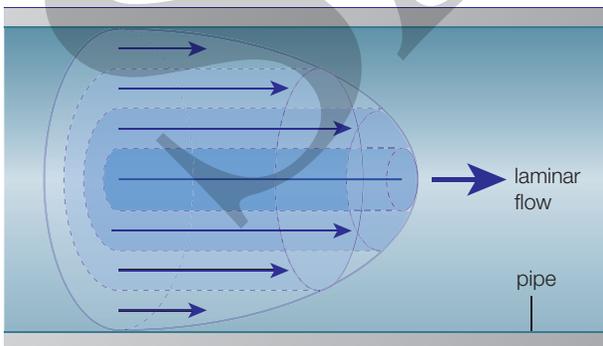
▲ **fig A** Why does this skier have such an oddly shaped helmet and adaptations to the suit's lower legs?

LAMINAR FLOW

When a fluid moves, there are two ways this can happen: **laminar flow** (also called **streamline flow**) and **turbulent flow**. In general, laminar flow occurs at lower speeds, and will change to turbulent flow as the fluid velocity increases past a certain value. The velocity at which this changeover occurs will vary depending upon the fluid in question and the shape of the area through which it is flowing.

If we take a simple example such as water flowing slowly through a pipe, it will be laminar flow. Think of the water in the pipe as several concentric cylinders from the central axis outwards to the layer of water in contact with the pipe itself. Friction between the outermost layer and the pipe wall means this layer will only be able to move slowly. The next layer in will experience friction with the slow-moving outermost layer, but this will be less than the friction between the outermost layer and the pipe. Thus, this inner layer will move faster than the outermost layer. The next layer in moves faster again, with the velocity of each layer increasing nearer the centre, where a central cylinder of water is moving the fastest.

As with most areas of scientific investigation, Isaac Newton produced much work on the subject of fluid flow. In particular, he is credited with the development of equations to describe the frictional force between the layers in streamline flow. If a liquid follows his formulae, as most common liquids do, it is known as a *Newtonian* fluid.



▲ **fig B** Laminar flow in a pipe shows streamlines of different but unchanging velocities.

STREAMLINING

The lines of laminar fluid flow are called **streamlines**. At any point on any one of these streamlines, the velocity of the flow will be constant over time. Remember that velocity is a vector, so this means that the water at any point in the pipe will always move in the same direction and at the same speed. The direction and/or speed may be different in different places, but at any given place direction and speed must stay constant.

In the wind tunnel in **fig C**, the smoke would flow over the car in exactly the same pattern forever if all the wind tunnel factors were kept constant. Changing the speed of the airflow in the tunnel allows designers to test how the prototype would behave at faster speeds, and at what point laminar flow changes to turbulent flow.



▲ **fig C** Smoke streamlines show laminar flow of air over a well-designed car.

In turbulent flow the fluid velocity in any given place changes over time. The flow becomes chaotic and swirling eddies form (you see eddies when water runs away through a plug hole). A poorly designed car would cause turbulent flow of air over it. In the wind tunnel the smoke trails over the car would be seen to swirl in ever-changing patterns. Turbulent flow increases the drag on a vehicle and so increases fuel consumption.



▲ **fig D** Increased speeds change streamline flow to turbulent flow.

PRACTICAL SKILLS

Investigating types of flow

Turbulent flow was first demonstrated by Osborne Reynolds in 1883 in an experiment showing coloured water flowing in a glass tube. You can set up a similar experiment to show turbulence caused by faster fluid flow, or by different shapes of obstacles. At most speeds, a smooth, curved obstacle will produce less turbulence than a squarer one.



▲ **fig E** A few crystals of potassium manganate(VII) will produce purple trails in the water flow which can then be made to pass around objects made from clay. You can alter the flow rate and the obstacle shapes in order to see how the flow changes.



Safety Note: Avoid skin contact with the potassium manganate (vii), and the clay after it has been used.

CHECKPOINT

SKILLS ADAPTIVE LEARNING

1. Give three examples of objects that are designed to reduce the amount of turbulent flow of air or water over them.
2. Sketch diagrams to illustrate the basic definitions of streamline flow and turbulent flow. Explain how your diagrams show each type of flow.
3. The text on streamlining above says the water flowing at a particular point in a pipe 'will always move in the same direction and at the same speed'. Explain why the smoke in the wind tunnel can change direction and move up and over the car.
4. Describe and explain the differences in the water surfaces in the two pictures in **fig D**.

SUBJECT VOCABULARY

laminar flow/streamline flow a fluid moves with uniform lines in which the velocity is constant over time

turbulent flow fluid velocity in a particular place changes over time, often in an unpredictable manner

streamlines lines of laminar flow in which the velocity is constant over time

LEARNING OBJECTIVES

- Understand the concept of viscosity.
- Know how viscosity is related to temperature.

VISCOSITY



fig A The 'sharkskin' suit helped break many swimming world records at the Beijing Olympics – it was later banned by swimming's governing body as 'technology doping'. Despite the manufacturer's claim that this was due to the material's extremely low viscous drag, scientists discovered this to be false and put the success down to its physiological benefits for athletes.

When you walk in a swimming pool, you find it much harder than walking through air. The friction acting against you is greater in water than it is in air. This frictional force in fluids is due to **viscosity**. If the frictional force caused by movement through the fluid is small, we say the viscosity is low.

Newton developed a formula for the friction in liquids that includes several factors. One of these factors relates to the particular liquid in question. It would be even harder to wade through a swimming pool of oil than one full of water. This fluid-dependent factor is called the **coefficient of viscosity** and has the symbol η , the Greek letter eta.

As viscosity determines the friction force acting within a fluid, it has a direct effect on the rate of flow of the fluid. Consider the differing rates of flow of a river of lava (see **table A**) compared with the viscosity of lava.

LAVA TYPE	SILICA CONTENT	VISCOSITY	APPROXIMATE FLOW RATE / km h ⁻¹
basaltic	least	least	30–60
andesitic	in between	in between	10
rhyolitic	most	most	1

table A How is the rate of flow related to the viscosity of the fluid?

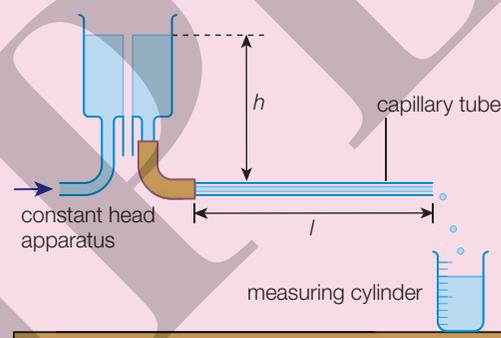
The rate of flow of a fluid through a pipe is inversely proportional to the viscosity of the fluid. In 1838, Jean Poiseuille, a French doctor and physiologist, investigated the flow of fluids in pipes and proved the connection between flow rate and viscosity. Poiseuille was interested in blood flow through the body, but his law is very important in industrial design. For example, the rate of flow of liquid chocolate through pipes in the manufacture of

sweets will vary with the chocolate's viscosity, which will vary depending on the exact recipe used to produce it. More sugar may mean greater viscosity and thus slower flow through the pipes, and so less chocolate per sweet.

PRACTICAL SKILLS

Investigating flow rates

You can investigate how fluid flow rate depends on the fluid's viscosity by doing an experiment very similar to those carried out by Poiseuille in the mid-nineteenth century. Using a constant pressure, water forced through a narrow pipe will flow at a certain rate, inversely proportional to its viscosity. By varying the height of the water tank, you can record measurements of this 'head of pressure', h , against the flow rate. The gradient of the best-fit line will allow you to calculate the viscosity of the water.



Height, h/m	Volume water collected in 1 minute/cm ³	Flow rate /cm ³ s ⁻¹
0.1	120	2.0
0.2	245	4.1
0.3	350	5.8
0.4	475	7.9
0.5	590	9.8

Tube internal diameter $2r = 2\text{ mm}$; tube length $l = 20\text{ cm}$

fig B Experimental set up and sample results for an investigation into Poiseuille flow.

You can plot a graph of the flow rate (F) against height (h) and hence calculate the viscosity of water (η).

Poiseuille's equations tells us that the gradient of the graph = $\frac{\pi\rho g r^4}{8\eta l}$

where r is the internal radius of the capillary tube, ρ is the density of water, and $g = 9.81\text{ N kg}^{-1}$.



Safety Note: Secure the constant head apparatus to a stand so that it cannot fall over even when filled with water.

An even greater variation in viscosity of liquid chocolate is caused by changes in its temperature. The sweet manufacturer can account for variation in a recipe (which might come from something as minor as a change in supplier of cocoa beans) by adjusting the flow rate by altering the temperature. Viscosity is directly related to fluid temperature. In general, liquids have a lower coefficient of viscosity at higher temperatures. For gases, viscosity increases with temperature.

FLUID	TEMPERATURE / °C	VISCOSITY / PaS
air	0	0.000017
air	20	0.000018
air	100	0.000022
water	0	0.0018
water	20	0.0010
water	100	0.0003
glycerine	-40	6700
glycerine	20	1.5
glycerine	30	0.63
chocolate	30	100
chocolate	50	60

table B The viscosities of different fluids at different temperatures.

PRACTICAL SKILLS

Investigating how viscosity changes with temperature

You can investigate how the viscosity of a liquid changes with temperature using a re-sealable tin or bottle half-full of a test fluid (such as syrup). The temperature of the liquid is varied using a water bath. The viscosity of the liquid will affect the rate at which the tin or bottle rolls down a fixed ramp.

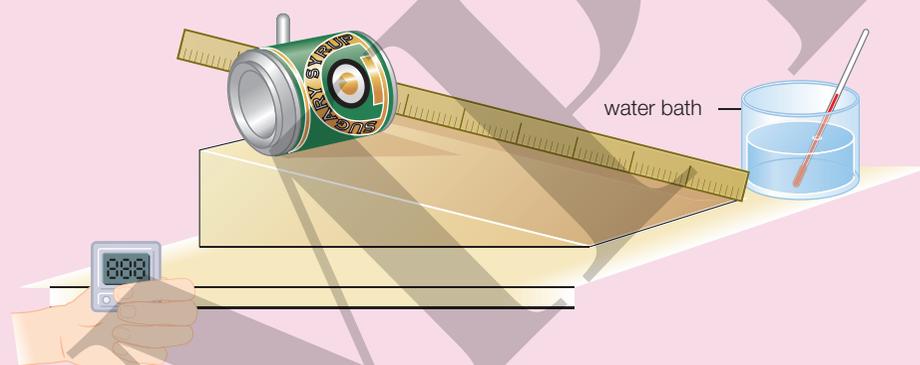


fig C How does the viscosity of syrup change with its temperature?

Safety Note: If edible syrup or oils are used, do not taste. Avoid skin contact with mineral oil or motor oil. If the liquid is very hot it will stick to skin and cause severe burns.

SKILLS ANALYSIS

CHECKPOINT

1. Why is the world record for 100 m swimming a longer time than that for 100 m sprinting?
2. Describe how temperature affects viscosity for liquids and gases.
3. How and why would holding a swimming competition in a warmer pool affect the times achieved by swimmers?
4. Why might a chocolate manufacturer alter their machinery so it functioned at a higher temperature?
5. Draw a graph of the experimental results shown in **fig B** in order to find the viscosity of water. How does your value compare with the figures in **table B**?

SUBJECT VOCABULARY

viscosity how resistant a fluid is to flowing

coefficient of viscosity a numerical value given to a fluid to indicate how much it resists flow

LEARNING OBJECTIVES

- Use the equation for viscous drag.
- Use a falling ball method to determine the viscosity of a liquid.

You have previously learned that acceleration due to gravity near the surface of the Earth is about 9.81 m s^{-2} . An object falling in a vacuum does accelerate at this rate. However, it is unusual for objects to be dropped near the surface of the Earth in a vacuum (in nearly all such cases a physics teacher is likely to be demonstrating to a class). In reality, in order to calculate an object's actual acceleration when falling, we need to take account of all the forces acting on it, combine these to find a resultant force, and then use Newton's second law ($\mathbf{a} = \frac{\Sigma \mathbf{F}}{m}$) to calculate the resulting acceleration.



▲ **fig A** A skydiver will fall at a constant speed if the forces acting on him are balanced.

For a falling object such as a skydiver, this means we need to include the weight, the upthrust caused by the object being in the fluid air, and the viscous drag force caused by the movement. The difficult part is that the viscous drag varies with speed through the fluid, and speed is constantly changing as a result of the acceleration. Usually, we consider the equilibrium situation, in which the weight exactly balances the sum of upthrust and drag, which means that the falling velocity remains constant. This constant velocity is the **terminal velocity**.

EXAM HINT

The phrase 'terminal velocity' is only defined to be for objects falling under gravity with a constant weight. For a similar situation horizontally, for example a car using a constant thrust force, an alternative phrase such as 'maximum velocity' should be used.

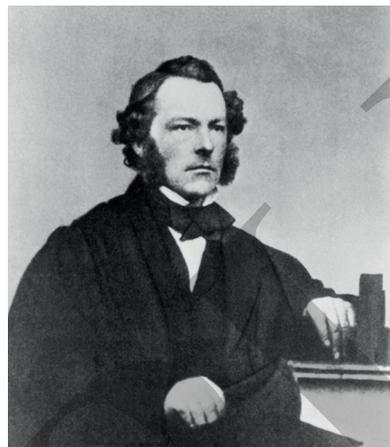
STOKES' LAW

In the mid-nineteenth century, Sir George Gabriel Stokes, an Irish mathematician and physicist at Cambridge University, investigated fluid dynamics and came up with an equation for the viscous drag (\mathbf{F}) on a small sphere at low speeds. This formula is now called Stokes' law:

$$\mathbf{F} = 6\pi r\eta\mathbf{v}$$

where r is the radius of sphere (m), \mathbf{v} is the velocity of sphere (m s^{-1}), and η is the coefficient of viscosity of the fluid (Pa s).

Thus, in such a simple situation, the drag force is directly proportional to the radius of the sphere, and directly proportional to the velocity.



▲ **fig B** Along with Lord Kelvin and James Clerk Maxwell, Sir George Gabriel Stokes helped to build the reputation of Cambridge University in many areas of mathematical physics.

For simplicity, we will only consider simple situations, such as a solid sphere moving slowly in a fluid. Imagine a ball bearing dropping through a column of oil, for example. If you consider the terminal velocity of the sphere in terms of the forces in detail, then:

$$\text{weight} = \text{upthrust} + \text{Stokes' force}$$

$$m_s g = \text{weight of fluid displaced} + 6\pi r\eta v_{\text{term}}$$

where m_s is the mass of the sphere and v_{term} is its terminal velocity.

For the sphere, the mass m_s is given by:

$$m_s = \text{volume} \times \text{density of sphere} = \frac{4}{3}\pi r^3 \times \rho_s$$

so the weight of the sphere W_s is given by:

$$W_s = m_s g = \frac{4}{3}\pi r^3 \rho_s g$$

For the sphere, the upthrust is equal to the weight of fluid displaced. The mass m_f of fluid displaced is given by:

$$m_f = \text{volume} \times \text{density of fluid} = \frac{4}{3}\pi r^3 \times \rho_f$$

so the weight of fluid displaced W_f is given by:

$$W_f = m_f g = \frac{4}{3}\pi r^3 \rho_f g$$

Overall then:

$$\frac{4}{3}\pi r^3 \rho_s g = \frac{4}{3}\pi r^3 \rho_f g + 6\pi r\eta v_{\text{term}}$$

We can rearrange the equation to find the terminal velocity:

$$v_{\text{term}} = \frac{\frac{4}{3}\pi r^3 g(\rho_s - \rho_f)}{6\pi r\eta}$$

Cancelling the π and the radius term:

$$v_{\text{term}} = \frac{2r^2 g(\rho_s - \rho_f)}{9\eta}$$

So terminal velocity is proportional to the square of the radius. This means that a larger sphere falls faster. Furthermore, because the radius is squared, it falls much faster.

EXAM HINT

At a fixed temperature, viscosity is constant.

'Viscous drag' increases with velocity, but 'viscosity' does not change.

EXAM HINT

Note that the steps and layout of the solution in this worked example are suitable for terminal velocity questions in the exam.

WORKED EXAMPLE

Find the terminal velocity of (a) a steel ball bearing of radius 1 mm and (b) a steel ball bearing of radius 2 mm falling through glycerine in a measuring cylinder.

The viscosity of glycerine is highly temperature dependent: at 20 °C we can take $\eta = 1.5 \text{ Pa s}$

$$\text{density of steel} = 7800 \text{ kg m}^{-3}$$

$$\text{density of glycerine} = 1200 \text{ kg m}^{-3}$$

$$g = 9.81 \text{ m s}^{-2}$$

(a) For a 1 mm radius ball bearing:

$$v_{\text{term}} = \frac{2r^2g(\rho_s - \rho_f)}{9\eta}$$

$$v_{\text{term}} = \frac{2(1 \times 10^{-3})^2 \times 9.81 \times (7800 - 1200)}{9 \times 1.5}$$

$$v_{\text{term}} = 9.6 \times 10^{-3} \text{ m s}^{-1}$$

(b) For a 2 mm radius ball bearing:

$$v_{\text{term}} = \frac{2r^2g(\rho_s - \rho_f)}{9\eta}$$

$$v_{\text{term}} = \frac{2(2 \times 10^{-3})^2 \times 9.81 \times (7800 - 1200)}{9 \times 1.5}$$

$$v_{\text{term}} = 3.8 \times 10^{-2} \text{ m s}^{-1}$$

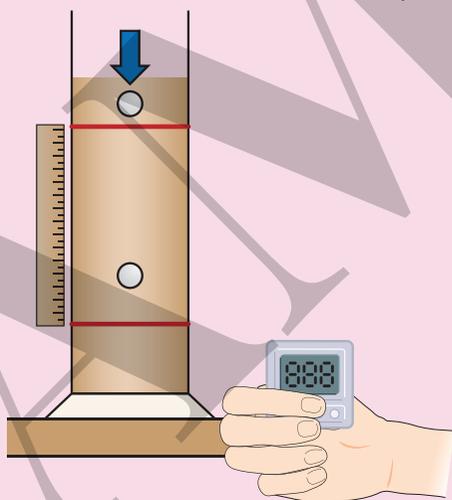
Comparing the values, you can see that doubling the radius of the ball makes its terminal velocity four times as great.

PRACTICAL SKILLS

CP2

Investigating terminal velocity

You can investigate the viscosity of a liquid by allowing various differently sized spheres to fall through it and then measure their terminal velocity and radii.



▲ **fig C** How does the terminal velocity of a falling sphere depend on its radius and the viscosity of the fluid?

You can plot a graph of the terminal velocity, v_{term} , against the square of the sphere radius, r^2 , and hence calculate the viscosity of water, η .

Stokes' law tells us that the gradient of the graph = $\frac{2g(\rho_s - \rho_f)}{9\eta}$

where ρ_s is the density of the material of the spheres and ρ_f is the density of the fluid they fall through, and $g = 9.81 \text{ N kg}^{-1}$. You may need to do an additional experiment to find the two densities.

Water is usually not viscous enough to give measurably different terminal velocities in this experiment. However, if you do use water, you can then compare the answer for its viscosity with that found from the Poiseuille flow experiment in **Section 2A.3**.

VISCOUS DRAG

You would find it difficult to wade through a swimming pool filled with oil because of the oil's viscous drag. This is the friction force between a solid and a fluid. Calculating this fluid friction force can be relatively simple. On the other hand, it can be very complicated for large objects, fast objects and irregularly shaped objects, as the turbulent flow creates an unpredictable situation.

It must be remembered that the simple slow-falling sphere of Stokes' law is not a common situation and in most real applications the terminal velocity value is a result of more complex calculations. However, the principle that larger objects generally fall faster holds true for most objects without a parachute.

FALLING OBJECT	TERMINAL VELOCITY / m s^{-1}
skydiver	60
golf ball	32
hail stone (0.5 cm radius)	14
raindrop (0.2 cm radius)	9

table A The terminal velocities of various objects falling in air. Note that the skydiver value varies greatly with the shape in which the body is held when falling.

CHECKPOINT

- Use Stokes' law to calculate the viscous drag on a ball bearing with a radius of 1 mm, falling at 1 mm s^{-1} through liquid chocolate at 30°C .
- Why is it difficult to calculate the terminal velocity for a cat falling from a high rooftop?
- A spherical meteorite, of radius 2 m and made of pure iron, falls towards Earth.
 - For its fall through the air, use Stokes' law to calculate the meteorite's terminal velocity.
 - The meteorite lands in a tropical freshwater lake that is at 20°C and continues sinking underwater. Use Stokes' law to calculate its new terminal velocity.
 - What assumptions have you made in order to make these calculations? (See tables of density data in **Section 2A.1** and viscosities in **Section 2A.3**.)
- Use **table A** to estimate the terminal velocity of the cat in question 2.
- The experiment shown in **fig C** was carried out using glycerine as the liquid through which the sphere was dropped, but the experiment was repeated at different temperatures from 10°C up to 50°C .
 - In what way would the density of the glycerine change as the temperature increased for each experiment.
 - In what way would the density of the ball bearing change as the temperature increased for each experiment.
 - In what way would the upthrust on the ball bearing change as the temperature of the glycerine was increased for each experiment.
 - Explain how the viscosity of glycerine changes as the temperature increases (see **table B** in **Section 2A.3**).
 - At each temperature, the student drew a graph of terminal velocity against the square of the radius of the ball bearing. Explain how the gradient of the graph would change with the temperature changes, and why.
 - Why is this experiment likely to be inconclusive if water were used instead of glycerine?

SKILLS CRITICAL THINKING

EXAM HINT

Students often confuse the terms 'drag' and 'upthrust'. Make sure you are clear on their definitions, and use the right word for the right upward force, depending on what is causing the force.

SUBJECT VOCABULARY

terminal velocity the velocity of a falling object when its weight is balanced by the sum of the drag and upthrust acting on it

2A THINKING BIGGER

THE PLIMSOLL LINE

SKILLS

CRITICAL THINKING, PROBLEM SOLVING, REASONING/ARGUMENTATION, INTERPRETATION, ADAPTIVE LEARNING, INITIATIVE, PRODUCTIVITY, ETHICS, COMMUNICATION, ASSERTIVE COMMUNICATION

Load lines are painted on the side of ships to show how low they may safely sit in the water. Although usually associated with the British Member of Parliament Samuel Plimsoll (1824–1898), such lines have been in use for hundreds of years.

In this activity, we will look at how the ‘Plimsoll line’ came to be required by British law.

OCEAN SERVICE WEBSITE

WHAT IS A PLIMSOLL LINE?

A commercial ship is properly loaded when the ship’s waterline is at the ship’s Plimsoll line.

The Plimsoll line is a reference mark located on a ship’s hull that indicates the maximum depth to which the vessel may be safely immersed when loaded with cargo. This depth varies with a ship’s dimensions, type of cargo, time of year, and the water densities encountered in port and at sea. Once these factors have been accounted for, a ship’s captain can determine the appropriate Plimsoll line needed for the voyage (see **fig B**).

Samuel Plimsoll (1824–1898) was a member of the British Parliament and he was concerned with the loss of ships and crews due to overloading. In 1876, he persuaded Parliament to pass the Unseaworthy Ships Bill, which required marking a ship’s sides with a line that would disappear below the waterline if the ship was overloaded. The line, also known as the Plimsoll mark, is found in the middle of the ship on both the port and starboard sides of cargo ships and is still used worldwide by the shipping industry.



LTF – Timber Tropical Fresh Water	TF – Tropical Fresh Water
LF – Timber Fresh Water	F – Fresh Water
LT – Timber Tropical Seawater	T – Tropical Seawater
LS – Timber Summer Seawater	S – Summer Temperate Seawater
LW – Timber Winter Seawater	W – Winter Temperate Seawater
LWNA – Timber Winter North Atlantic	WNA – Winter North Atlantic

▲ **fig B** Plimsoll mark on the hull of a floating ship, and a chart indicating variables such as water density.



▲ **fig A** Samuel Plimsoll.

From the website of the National Ocean Service, an office of the U.S. National Oceanic and Atmospheric Administration, <http://oceanservice.noaa.gov/facts/plimsoll-line.html>

SCIENCE COMMUNICATION

- 1 The text opposite is from the website of the National Ocean Service, an office of the U.S. National Oceanic and Atmospheric Administration. Consider the text and comment on the type of writing used. Try and answer the following questions:
 - (a) How has the author attempted to maintain the reader's interest?
 - (b) Discuss the level of the science presented, in relation to the intended audience.

INTERPRETATION NOTE

Think about the type of reader who is likely to visit the webpage. Also, think about why the National Ocean Service would include this webpage if it is 'a dull subject'.

PHYSICS IN DETAIL

Now we will look at the physics in detail. Some of these questions link to topics earlier in this book, so you may need to combine concepts from different areas of physics to work out the answers.

- 2 Explain the information in the first paragraph of the text, with relation to the heights of the marks in **fig B** and the table of the codes shown on a typical load line.
- 3 The highest load lines are for timber. These allow for timber to be loaded on to the deck of the ship. This can easily be thrown overboard if needed. Explain why the marks are higher, and how and when it could help if this extra deck cargo were thrown off the ship.
- 4 Explain why the ships in **fig B** have the shape that they do. Why are they designed differently from large square barges that do not travel on the ocean? Justify your answer.

THINKING BIGGER TIP

Think about how the density of the water leads to the depth that the ship sinks into the water. Would saltier water be more or less dense than freshwater? And how would the temperature of the water affect its density?

ACTIVITY

Write a comment piece for a magazine. Argue that although Plimsoll's campaign was 150 years ago, the capitalist approach to business has not changed very much. Include examples of at least one current business practice, anywhere in the world, where safety technology is not used enough because of its cost. Your article should include a paragraph explaining to a general audience the science behind the safety technology mentioned, and information on your sources of evidence.

DID YOU KNOW?

In the year 1873–74, 411 ships sank off the coast of the United Kingdom, with the loss of 506 lives. Overloading and poor repair made some ships so dangerous that they became known as 'coffin ships'.

2A EXAM PRACTICE

1 Which is the correct expression for calculating density?

- A Mass \times volume
- B Mass \div volume
- C Upthrust \times volume
- D Weight \div volume

[1]

(Total for Question 1 = 1 mark)

2 What is the mass of a spherical stone with a diameter 25 mm, if the density of the stone is 2900 kg m^{-3} ?

- A $2.82 \times 10^{-9} \text{ kg}$
- B 24 g
- C 190 g
- D $3.55 \times 10^8 \text{ kg}$

[1]

(Total for Question 2 = 1 mark)

3 Which is the correct definition of 'laminar flow'?

- A At a given point in a flowing fluid, the velocity of flow varies uniformly over time.
- B At a given point in a flowing fluid, the velocity of turbulent flow is proportional to the viscosity.
- C At a given point in a flowing fluid, the velocity of flow does not vary over time.
- D At a given point in a flowing fluid, the velocity of flow does not vary over the distance from the centre of flow.

[1]

(Total for Question 3 = 1 mark)

4 Which row in the table is correct?

	Gas viscosity	Liquid viscosity
A	increases at higher temperature	increases at higher temperature
B	increases at higher temperature	decreases at higher temperature
C	decreases at higher temperature	increases at higher temperature
D	decreases at higher temperature	decreases at higher temperature

[1]

(Total for Question 4 = 1 mark)

5 A stone is dropped into water. It has a mass of 288 g and there is an upthrust force of 0.58 N when it is released in the water. What is the resultant force on the stone at that moment of release, when it is initially not moving?

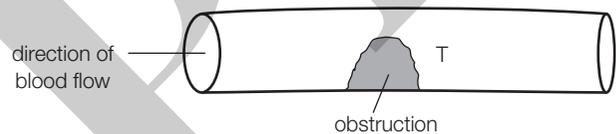
- A 287.4 N
- B 2.83 N
- C 2.25 N
- D zero

[1]

(Total for Question 5 = 1 mark)

6 Blood clots can lead to heart attacks. Blood flow through arteries is normally laminar, but an obstruction may cause the blood flow to become turbulent. This can lead to the formation of blood clots.

(a) The diagram shows an artery containing an obstruction.



After passing the obstruction the laminar flow becomes turbulent in the area marked T.

- (i) Add flow lines to the diagram to show laminar flow changing to turbulent flow after passing the obstruction. [2]
- (ii) Explain what is meant by laminar flow and turbulent flow. [2]

(b) In one experiment on blood flow, the viscosity of the blood and the velocity of blood flow were measured.

- (i) Describe how you would expect the velocity of blood flow to vary with the viscosity. [1]
- (ii) Suggest and explain how a rise in the temperature of the blood would affect the velocity of flow. [2]

(Total for Question 6 = 7 marks)

7 The photograph shows oil being poured into a cold frying pan and spreading out.

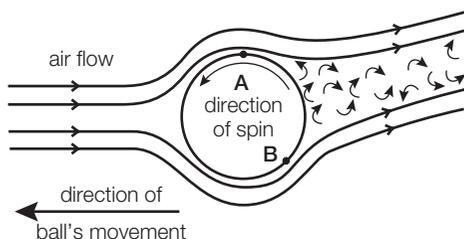
Explain the difference that using a hot pan would make to how the oil spreads. [2]



(Total for Question 7 = 2 marks)

8 In the game of table tennis, a ball is hit from one end of the table to the other over a small net.

(a) Making a table tennis ball spin when it is hit can affect its flight. The diagram shows the path of air around a spinning ball. It contains regions of laminar flow and turbulent flow. The flow changes from one to the other at points A and B.



(i) With reference to the diagram, explain what is meant by laminar flow and turbulent flow. [2]

(ii) The ball is spinning in the direction shown in the diagram. Suggest why there is a larger region of turbulent flow on the top of the ball than the bottom. [1]

(b) The diagram shows that the air is deflected upwards after passing the ball. Explain why this means there must be a downwards component of force on the ball in addition to its weight. [2]

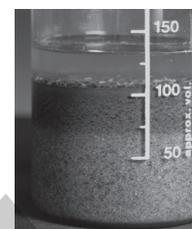
(c) Spinning a table tennis ball allows it to be hit harder and still hit the table on the other side of the net.

(i) A table tennis ball is hit, without any spin, from one end of a table so that it leaves the bat horizontally with a speed of 31 m s^{-1} . The length of the table is 2.7 m. Show that the ball falls a vertical distance of about 4 cm as it travels the length of the table. (3)

(ii) The net is 15 cm high. Explain how the spin helps the ball hit the table on the other side of the net. (3)

(Total for Question 8 = 11 marks)

9 Soil is usually made up of a variety of particles of different sizes. The photograph shows what happens when soil is mixed up with water and the particles are allowed to settle.



(a) The dot below represents a particle of the soil falling through water. (i) Add labelled arrows to show the three forces acting on the particle as it falls through the water. [2]



(ii) Explain why a particle held stationary in water and then released accelerates downwards at first but then reaches a steady downwards speed. [4]

(iii) Write an expression showing the relationship for these forces when the particle is falling at a steady speed. [1]

(b) A typical particle of sand in the sample has the following properties:

- diameter = $1.6 \times 10^{-3} \text{ m}$
- volume = $2.1 \times 10^{-9} \text{ m}^3$
- density = $2.7 \times 10^3 \text{ kg m}^{-3}$
- weight = $5.7 \times 10^{-5} \text{ N}$

(i) Show that the upthrust acting on the particle is about $2 \times 10^{-5} \text{ N}$. density of water = $1.0 \times 10^3 \text{ kg m}^{-3}$ [2]

(ii) Calculate the steady downwards speed this particle would achieve if allowed to fall through water. viscosity of water = $1.2 \times 10^{-3} \text{ Pa s}$ [3]

(c) The different types of particles in soil can be defined according to their diameters, as in the following table.

Soil particle	Particle diameter
clay	less than 0.002 mm
silt	0.002 mm – 0.05 mm
sand	0.05 mm – 2.00 mm
fine pebbles	2.00 mm – 5.00 mm
medium pebbles	5.00 mm – 20.00 mm
coarse pebbles	20.00 mm – 75.00 mm

The photograph shows that when soil is allowed to settle in water, the pebbles tend to be found towards the bottom, followed by sand, silt and clay in succession. Explain why this happens. Assume that all particles have the same density. [3]

(Total for Question 9 = 15 marks)

10 Explain how a stone dropped into a lake, from the surface, will reach a maximum velocity as it falls to the lake bottom, and how this will be different in summer and winter. [6]

(Total for Question 10 = 6 marks)

TOPIC 2 MATERIALS

CHAPTER 2B

SOLID MATERIAL PROPERTIES

You probably rarely consider the likelihood that a bridge will fail as you walk or drive over it. However, when designing the bridge, and selecting the materials to use in it, an engineer has to ensure that this likelihood is minimal.

Design processes for any new project need to include specifications indicating the extremes of conditions that may apply. A bridge needs to withstand an overloaded lorry that is not complying with weight restrictions; a snowboarding helmet might be used by a person on a snowmobile; or a motorist might try to use a climbing rope to tow their car. Some of these uses might be classed as unreasonable, and the designer would not expect to have to make their product strong enough to stand up to such extreme forces.

If you were designing a new harbour wall, what extreme weather conditions would you ensure that the wall could withstand? Would it be good enough to make sure it could withstand a storm with such strong winds and waves that only occurs once every 50 years? 100 years? 1000 years? And how do you achieve the strength required within budget?

In this chapter we will see how the strength of materials can be measured, and thus how to choose the right material for any given job.

MATHS SKILLS FOR THIS CHAPTER

- Units of measurement (*e.g. the pascal, Pa*)
- Calculating areas of circles (*e.g. finding the cross-sectional area of a wire in order to find stress*)
- Use of standard form and ordinary form (*e.g. calculating Young modulus*)
- Making order of magnitude calculations (*e.g. comparing Young modulus for different materials*)
- Changing the subject of an equation (*e.g. re-arranging the stress equation*)
- Substituting numerical values into algebraic equations (*e.g. calculating the strain*)
- Determining the slope of a linear graph (*e.g. finding the Young modulus from experimental results*)
- Estimating, by graphical methods as appropriate, the area between a curve and the x-axis, and realising the physical significance of the area that has been determined (*e.g. finding the energy stored in a stretched material from graphical data*)

What prior knowledge do I need?

- The deforming effects of forces
- Experimental measurements on wires
- The idea of general properties of materials, rather than properties of specific objects

Topic 1A

- How to calculate resultant forces

Topic 1C

- The testing of materials for use in hockey goalkeeping equipment

What will I study in this chapter?

- How an object deforms when a force is applied along its length
- How to calculate stress and strain
- The meaning of the Young modulus for a material, and how to calculate it
- How to do an experiment to measure the Young modulus for a material
- How to interpret stress–strain curves in order to analyse the strength of a solid material
- How these ideas can be applied to materials in use in real applications

What will I study later?

Topic 5A (Book 2: IAL)

- Non-conservation of momentum, and deformation in collisions

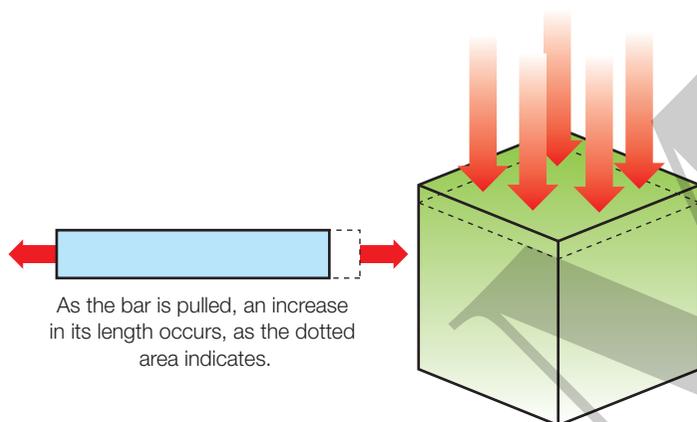
Topic 10A (Book 2: IAL)

- Oscillations of springs, with calculations using the spring constant
- How resonant vibration depends on the physical strength of an object

LEARNING OBJECTIVES

- Understand Hooke's law and be able to make calculations using it.
- Calculate the elastic strain energy stored in a deformed material sample.
- Estimate the elastic strain energy stored from a force–extension graph for a sample.

Whenever a force acts on a material sample, the sample will be deformed to a different size or shape. If it is made longer, the force is referred to as **tension**, and the extra length is known as the **extension**. For a material being squashed to a smaller size, both the force and the decrease in size are called **compression**, although the decrease in size could be referred to as a negative extension.



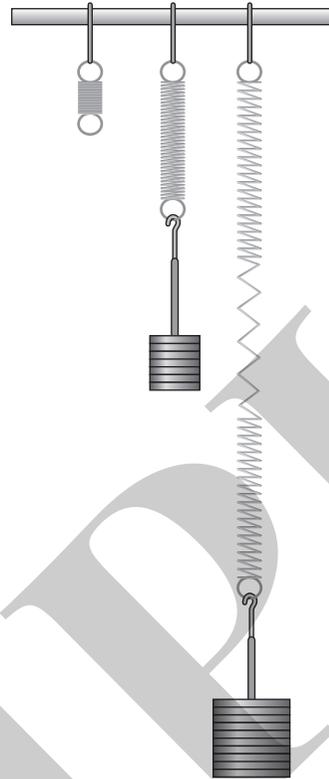
▲ **fig A** A tensile force causes extension. A compressive force causes a negative extension.

HOOKE'S LAW

Robert Hooke, a rival of Isaac Newton, was an exceptional experimental scientist. His interests were very diverse and he discovered many things that are more important than the rule called Hooke's law.

Hooke's law states that the force needed to extend a spring is proportional to the extension of the spring. A material only obeys Hooke's law if it has not passed what is called the **limit of proportionality**.

Up to a certain limit of proportionality, the force needed to extend a spring is proportional to the extension of the spring. If an object is subject to only a small force, it will deform elastically. When the force is removed, it returns to its original size and shape, as long as the **elastic limit** was not passed. We will look in much more detail at elastic and plastic **deformation** in **Section 2B.3**.



▲ **fig B** The middle spring is extended in proportion to the applied load, obeying Hooke's law. The right-hand spring has been stretched beyond its limit of proportionality, so no longer obeys Hooke's law.

Hooke's law is best described mathematically with the equation:

$$\text{force applied (N)} = \text{stiffness constant (N m}^{-1}\text{)} \times \text{extension (m)}$$

$$\Delta F = k\Delta x$$

EXAM HINT

The stiffness constant for a spring is usually referred to as the **spring constant**. Either phrase refers to k in the Hooke's law equation.

WORKED EXAMPLE

A spring has a stiffness constant of 50 N m^{-1} and is 3.0 cm long. How long would it be if a 200 g mass were hung from it?

Weight force:

$$W = mg = 0.200 \times 9.81$$

$$= 1.962 \text{ N}$$

$$\Delta F = k\Delta x$$

$$\therefore \Delta x = \frac{\Delta F}{k} = \frac{1.962}{50} = 0.03924 \text{ m}$$

$$= 3.924 \text{ cm}$$

$$\text{final length} = \text{original length} + \text{extension}$$

$$L = L_0 + \Delta x = 3.0 + 3.9$$

$$L = 6.9 \text{ cm}$$

EXAM HINT

Note that the steps and layout of the solution in this worked example are suitable for questions about Hooke's law in the exam.

PRACTICAL SKILLS

Investigating Hooke's law

You can perform a simple experiment to measure the stiffness constant for a spring. By hanging various masses on a spring and measuring the corresponding extensions, you can gather a set of results for ΔF and Δx . In each case, you could calculate the spring constant from a pair of readings, but you will get a more accurate final answer for k if you plot a graph of the results and find its gradient.

MASS / g	WEIGHT / N	ORIGINAL LENGTH / cm	LOADED LENGTH / cm	EXTENSION / cm
0	0.000	2.4	2.4	0.0
100	0.981	2.4	2.8	0.4
200	1.962	2.4	3.1	0.7
300	2.943	2.4	3.6	1.2
400	3.924	2.4	3.9	1.5
500	4.905	2.4	4.3	1.9
600	5.886	2.4	4.8	2.4
800	7.848	2.4	5.8	3.4
1000	9.810	2.4	6.5	4.1

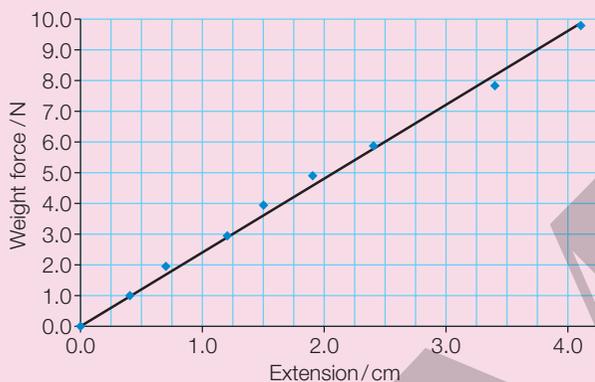


fig C The results of a Hooke's law investigation of a spring before it reaches its elastic limit. The fact that the best-fit line is straight shows that the spring obeys Hooke's law.

The graph in **fig C** has been plotted with the dependent variable on the y -axis, so that when the gradient of the line is calculated, this will give you the value for the spring constant:

$$\text{Gradient, } m = \frac{\Delta y}{\Delta x} = \frac{\Delta F}{\Delta x} = k$$



Safety Note: Use eye protection in case the spring becomes detached and flies back. Keep hands and feet clear of the 'drop zone' if large masses are used.

ELASTIC STRAIN ENERGY

The work done in deforming a material sample before it reaches its elastic limit will be stored within the material as elastic strain energy, E_{el} . We have previously seen that work done can be calculated by multiplying the force by the distance moved in the direction of the force. This is true in deforming materials too, but Hooke's law means that the force value varies for different extensions. If we plot the extension of a spring with increasingly large masses hanging on it, as in **fig C**, the graph will follow Hooke's law. To find the work done to extend the spring a certain amount, we must calculate using the average force over the distance of the extension.

$$\Delta E_{el} = \frac{1}{2}F\Delta x$$

WORKED EXAMPLE

In **fig C**, how much elastic strain energy is stored in the spring when it is extended by 2.5 cm?

At $\Delta x = 2.5$ cm, the corresponding force value is $F = 6.0$ N.

$$\Delta E_{el} = \frac{1}{2}F\Delta x$$

$$\Delta E_{el} = \frac{1}{2} \times 6.0 \times 0.025$$

$$\Delta E_{el} = 0.075 \text{ J}$$

WORK FROM FORCE–EXTENSION AND FORCE–COMPRESSION GRAPHS

The work done in deforming a material is calculated by multiplying the extension or compression by an appropriate average force value. If the force is varying in a non-linear way, which is common for some materials, it might not be a straightforward process to find the average force. However, the area between the line on a force–extension (F – Δx) graph and the extension axis will represent the work done. Finding the area under the line is easy with the linear type of relationship, shown in **fig C**. As the area under the line on **fig C** is a triangle, the formula for the area of a triangle gives us the same equation as was used above:

$$\Delta E_{el} = \frac{1}{2}F\Delta x$$

If the relationship is non-linear, as shown in the example of **fig D**, the work done can still be found from the area under the line.

WORKED EXAMPLE

What is the elastic strain energy stored in the material shown in **fig D** if it is extended by 12 cm?

The area under the line, up to the value $\Delta x = 0.12$ m, can be split into three sections:

- Triangle from origin to $x = 0.04$ m:
area = $\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 0.04 \times 4 = 0.08$ J
- Rectangle from $x = 0.04$ m to $x = 0.08$ m:
area = base \times height = $0.04 \times 4 = 0.16$ J
- Trapezium from $x = 0.08$ m to $x = 0.12$ m:
area = base \times average height = $0.04 \times (4 + 6)/2 = 0.20$ J
total work = $0.08 + 0.16 + 0.20$
elastic potential energy = 0.44 J

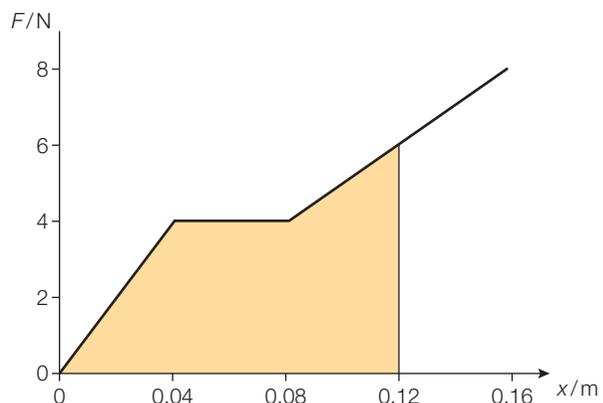


fig D The elastic potential energy stored in a material can be found from a non-linear force–extension graph by working out the area under the line up to the required extension.

If a non-linear force–extension or force–compression graph has a curved line, finding the area may involve estimating or counting the squares on the graph paper under the line. In this case, you will also need to multiply the number of squares by the elastic strain energy value ($F \times \Delta x$) for each individual square.

EXAM HINT

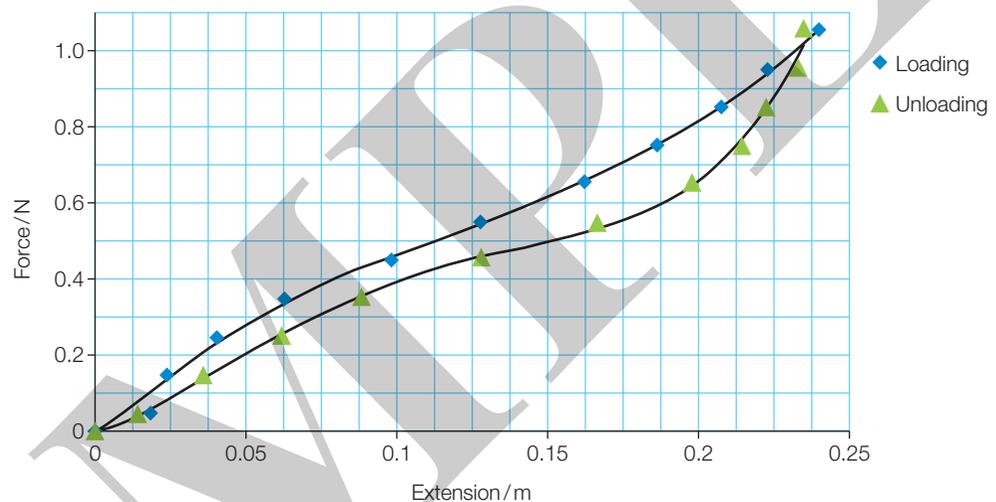
Be careful to distinguish between 'length' and 'extension'. You may be given a measurement for either length or extension. You should check carefully to see if you need to calculate the extension from information you have about lengths.

SKILLS REASONING / INTERPRETATION

SKILLS ANALYSIS

CHECKPOINT

1. What is the spring constant for a spring that starts at a length of 25 cm and extends to a length of 32 cm when a mass of 50 g is added?
2. In fig C, how would the line appear different if it were for a spring of constant $k = 280 \text{ N m}^{-1}$?
3. Explain how the elastic strain energy stored in a stretched spring could be calculated from the formula $\Delta E_{\text{el}} = \frac{1}{2} k(\Delta x)^2$.
4. Fig E shows the results of a Hooke's law experiment loading and unloading a rubber band to an extension of 24 cm.



▲ **fig E** Different extensions during loading and unloading is an example of **hysteresis**.

- (a) From the graph, estimate the work done in loading the rubber band.
- (b) When unloading, the rubber band releases the stored elastic strain energy. Estimate, from the graph, the area underneath the unloading curve, which will be the elastic strain energy released.
- (c) The difference and the answers to parts (a) and (b) represents energy lost in this process, which is mostly used in internal heating of the rubber band. How much **thermal** energy does the rubber gain from the complete cycle of loading and unloading?

SUBJECT VOCABULARY

tension a force acting within a material in a direction that would extend the material

extension an increase in size of a material sample caused by a tension force

compression a force acting within a material in a direction that would squash the material. Also the decrease in size of a material sample under a compressive force

limit of proportionality the maximum extension (or strain) that an object (or sample) can have, which is still proportional to the load (or stress) applied

deformation the process of alteration of form or shape

elastic limit the maximum extension or compression that a material can undergo and still return to its original dimensions when the force is removed

spring constant the Hooke's law constant of proportionality, k , for a spring under tension

hysteresis where the extension under a certain load will be different depending on its history of past loads and extensions

thermal connected with heat

LEARNING OBJECTIVES

- Calculate tensile/compressive stress.
- Calculate tensile/compressive strain.
- Calculate the Young modulus.

If you pull on the two ends of a metal bar, you are unlikely to deform it at all. However, pulling with the same force on the two ends of a very thin piece of wire made from the same metal may have very different results: it will probably extend elastically, and may pass its elastic limit and start to experience permanent deformation. Depending on the specific metal and the exact dimensions, it is possible that your force may be strong enough to break the wire. This example demonstrates that engineers need more information about the forces that may be encountered within the structures they are building than just the numbers of newtons. You used the same force and the same metal, but in the case of the metal bar you made no impression on it, whilst the thin wire broke. To make fair comparisons between samples, we need to consider the forces on them and their sizes as well as the materials from which they are made.

STRESS

Tensile (or compressive) **stress** is a measure of the force within a material sample, but it takes account of the cross-sectional area across the sample. This allows force comparisons to be made between samples of different sizes, so that they are measured under comparable conditions.

$$\text{stress (pascals, Pa, or N m}^{-2}\text{)} = \frac{\text{force (N)}}{\text{cross-sectional area (m}^2\text{)}}$$

$$\sigma = \frac{F}{A}$$

WORKED EXAMPLE

A cylindrical stone column has a diameter of 60 cm and supports a weight of 2500 N (fig A). What is the compressive stress in the column?

$$\text{Area} = \pi r^2 = 3.14 \times 0.30^2 = 0.2826 \text{ m}^2$$

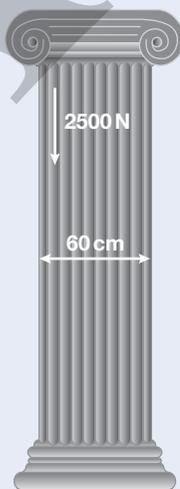
$$\sigma = \frac{F}{A} = \frac{2500}{0.2826}$$

$$\sigma = 8850 \text{ Pa}$$

EXAM HINT

Note that the steps and layout of the solution in this worked example are suitable for stress questions in the exam.

- ▶ **fig A** This support column is under a compressive stress of 8850 Pa.



STRAIN

Tensile (or compressive) **strain** is a measure of the extension (or compression) of a material sample, but it takes account of the original length of the sample. This allows extension comparisons to be made between samples of different sizes, so that they are measured under comparable conditions.

$$\text{strain (no units)} = \frac{\text{extension (m)}}{\text{original length (m)}}$$

$$\epsilon = \frac{\Delta x}{x}$$

As strain is a ratio, it has no units. However, it is often expressed as a percentage by multiplying the ratio by 100%.

WORKED EXAMPLE

A copper wire of length 1.76 m is stretched by a force to a length of 1.80 m. What is the tensile strain in the wire?

$$\epsilon = \frac{\Delta x}{x} = \frac{(1.80 - 1.76)}{1.76} = \frac{0.04}{1.76}$$

$$\epsilon = 0.023 = 2.3\%$$

EXAM HINT

Note that the steps and layout of the solution in this worked example are suitable for strain questions in the exam.



▶ **fig B** This copper wire has a tensile strain of 2.3%.

YOUNG MODULUS

If a material is deformed elastically, stress will be proportional to strain, with a constant of proportionality that is a measure of the stiffness of the material – how much it deforms under a certain stress. The stiffness constant is called the **Young modulus**. So, the Young modulus is a measure of the stiffness of a material, which takes account of the shape and size of the sample, so that different samples of the same material will all have the same value for the Young modulus. The idea of stiffness is a measure of how much the material deforms when forces are applied to it.

$$\text{Young modulus (Pa)} = \frac{\text{stress (Pa)}}{\text{strain (no units)}}$$

$$E = \frac{\sigma}{\epsilon}$$

LEARNING TIP

The stiffness constant, k , from Hooke's law relates to a particular object, such as a spring. For a material, the Young modulus, E , is the stiffness constant for the material in general, regardless of sample size.

The definition for the Young modulus also includes the fact that the material must be undergoing elastic deformation. Beyond the limit of proportionality, this equation will no longer work to calculate the stiffness of the material.

LEARNING TIP

From the original definitions of stress and strain, the Young modulus can also be calculated from:

$$E = \frac{Fx}{A\Delta x}$$

EXAM HINT

Note that the steps and layout of the solution in this worked example are suitable for Young modulus questions in the exam.

WORKED EXAMPLE

The copper wire from above has a diameter of 0.22 mm and was stretched using a force of 100 N. What is the Young modulus of copper?

$$A = \pi r^2 = 3.14 \times (1.1 \times 10^{-4})^2 = 3.80 \times 10^{-8} \text{ m}^2$$

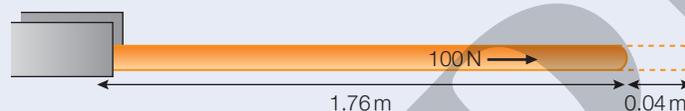
$$\sigma = \frac{F}{A} = \frac{100}{3.8 \times 10^{-8}}$$

$$\therefore \sigma = 2.63 \times 10^9 \text{ Pa}$$

$$E = \frac{\sigma}{\epsilon} = \frac{2.63 \times 10^9}{0.023}$$

$$E = 1.16 \times 10^{11} \text{ Pa}$$

(Note that the stress has been calculated from the original data and the unrounded value has been used in this calculation to give the value shown here.)



▲ **fig C** Copper has a Young modulus, $E = 1.16 \times 10^{11} \text{ Pa}$.

CHECKPOINT

- What is the strain of an aluminium wire if it extends from 97 cm to 1.04 m?
- What is the tensile stress in a vertical steel wire that has a diameter of 0.40 mm and a 1.0 kg mass hanging from it?
- Use these data to find the Young modulus of human hair: $d = 0.1 \text{ mm}$; $x = 12 \text{ cm}$; $F = 0.60 \text{ N}$; $\Delta x = 1.8 \text{ mm}$.
- (a) What is the compressive stress in each upper leg bone of a 7 tonne elephant if the bone is a vertical cylinder with a diameter 25 cm and the elephant is standing normally?
(b) If the Young modulus of elephant bone is 19 GPa, and the bones are originally 95 cm long, how much would they reduce in length if the elephant stood up on its back legs? What assumption do you need to make?

SUBJECT VOCABULARY

stress a proportionate measure of the force on a sample:

$$\text{stress (pascals Pa, or } \text{N m}^{-2}\text{)} = \frac{\text{force (N)}}{\text{cross-sectional area (m}^2\text{)}}$$

$$\sigma = \frac{F}{A}$$

strain a proportionate measure of the extension (or compression) of a sample:

$$\text{strain (no units)} = \frac{\text{extension (m)}}{\text{original length (m)}}$$

$$\epsilon = \frac{\Delta x}{x}$$

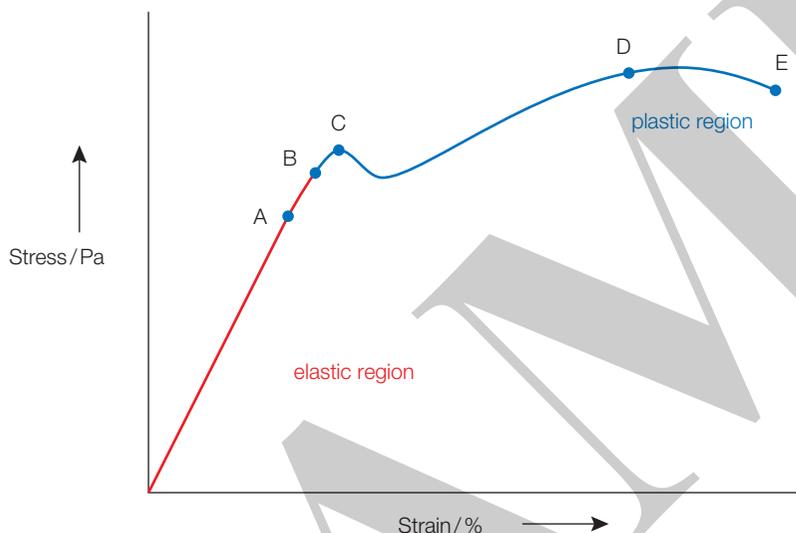
Young modulus the stiffness constant for a material, equal to the stress divided by its corresponding strain

LEARNING OBJECTIVES

- Interpret stress–strain graphs.
- Understand and apply the terms limit of proportionality, elastic limit, yield point, breaking stress, elastic deformation and plastic deformation in relation to stress–strain graphs.

STRESS–STRAIN ANALYSIS

From the definition of the Young modulus, the stress should be proportional to the strain if a material is undergoing elastic deformation. Therefore we should get a straight-line graph if we plot stress against strain. When this is done, we do find a straight-line relationship for small stresses. Once the limit of proportionality is passed, the internal structure of the material starts to behave differently. This means that the graph starts to curve. Depending on the material under test, the graph will go through various phases as the molecular structure of each material determines its response to increasing stress. Eventually, the stress will become too great, and the material will fracture. At this point, the line on the graph must end, as no further data can be obtained about the material. It cannot withstand higher stresses, so the graph cannot be plotted any further.



▲ **fig A** The stress–strain graph for a metal gives detailed information about how the material behaves under different levels of stress. The gradient of the straight-line portion of the graph will be equal to the value of the Young modulus.

LEARNING TIP

It can be confusing to see the sections of a stress–strain graph where the line goes down. In reality, these graphs are generated from results of materials testing in which the strain is continuously increased on a machine that can measure the stress within a material. This stress can go down if there is a change in the arrangement of the molecular structure of the material.

Fig A is a generalised stress–strain graph for a metal, such as copper. Most metals will follow the shape shown here, and there are various areas of interest on the graph.

In the straight-line portion from the origin to **point A**, the metal extends elastically, and will return to its original size and shape when the force is removed. The gradient of the straight-line portion of the graph is equal to the Young modulus for the metal.

Point A is the limit of proportionality. Slightly beyond this point, the metal may still behave elastically, but it cannot be relied upon to increase strain in proportion to the stress.

Point B is the elastic limit. Beyond this point, the material is permanently deformed and will not return to its original size and shape, even when the stress is completely released.

Point C is the **yield point**, beyond which the material undergoes a sudden increase in extension as its atomic substructure is significantly re-organised. The metal ‘gives’ just beyond its yield point as the metal’s atoms slip past each other to new positions where the stress is reduced.

LEARNING TIP

It is quite common for the limit of proportionality, the elastic limit and the yield point to be in the same place on the graph. They have been separated here to explain the different ideas behind each.

Point D represents the highest possible stress within this material. It is called the Ultimate Tensile Stress, or UTS, σ_U .

Point E is the fracture stress, or breaking stress. It is the value that the stress will be in the material when the sample breaks.

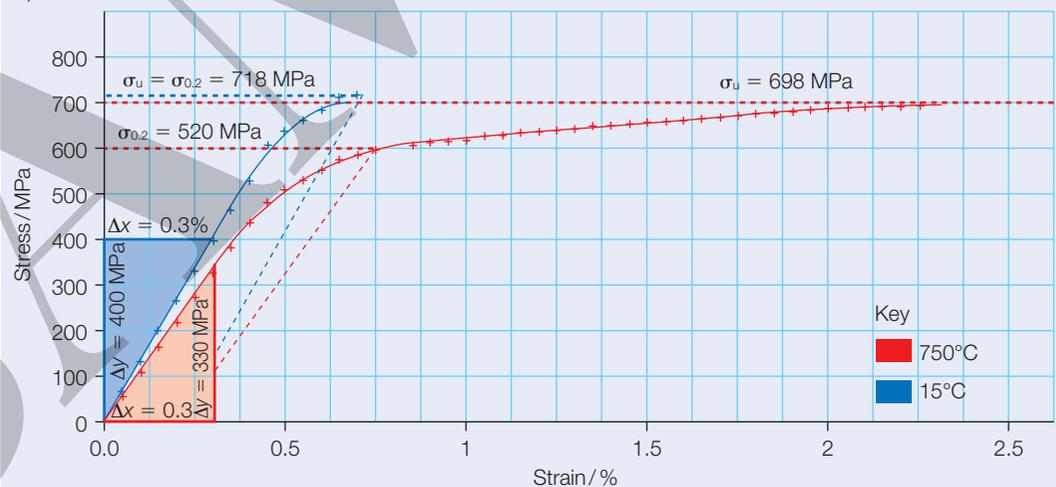
WORKED EXAMPLE

The European Space Agency (ESA) have tested a material known as ‘IMPRESS intermetallic alloy’. **Fig B** illustrates a stress–strain graph for this material at two different temperatures, to see how it would stand up under different conditions.

Considering first the Young modulus, you can see that the ESA have drawn measurements on the graph to make the calculation of the gradient of the straight-line portions of the graphs for each temperature plot. For example, at 15 °C:

$$\begin{aligned} \text{gradient} = E &= \frac{\Delta y}{\Delta x} = \frac{\sigma}{\varepsilon} = \frac{400 \text{ MPa}}{0.3\%} \\ \therefore E &= \frac{400 \times 10^6}{0.003} \\ E &= 1.33 \times 10^{11} \text{ Pa} \end{aligned}$$

The values labelled as $\sigma_{0.2}$ are the stress values at each temperature, which will result in a permanent deformation strain of 0.2%. These are in the plastic region, and the permanent deformations are indicated by the dashed lines connected back to 0.2% on the x-axis.

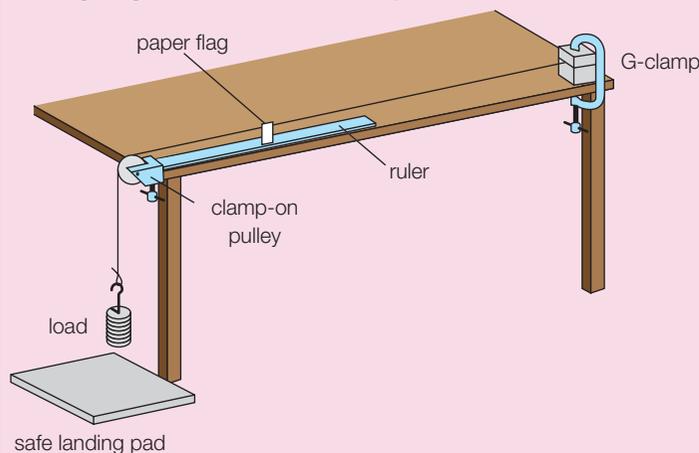


▲ **fig B** Stress–strain graphs, at two temperatures, for the IMPRESS intermetallic alloy, for use in the European space programme.

PRACTICAL SKILLS

CP3

Investigating stress–strain relationships for metals



▲ **fig C** An experiment to find the stress–strain relationships when stretching a metal wire. With detailed analysis of the stress–strain graph of the metal, you can find the Young modulus.

You can perform a simple experiment to measure the stiffness constant – the Young modulus – for a metal by stretching a thin wire. The original length and diameter of the wire must be measured first. Then, using increasing forces as the independent variable, you will need to take measurements of the extension corresponding to each force. There will be some readings during the wire's elastic deformation region which will increase uniformly. Beyond the elastic limit, it will increase with greater extensions for each increase in the load force until, eventually, the fracture stress will be reached and the wire will snap. Safety goggles must be worn during this experiment, as wires that snap under tension are a hazard to the eyes.

The graph in **fig B** has been plotted with the strain on the x -axis, so that when the gradient of the line is calculated this will give you the value for the stiffness constant, or Young modulus:

$$\text{gradient, } m = \frac{\Delta y}{\Delta x} = \frac{\sigma}{\epsilon} = E$$

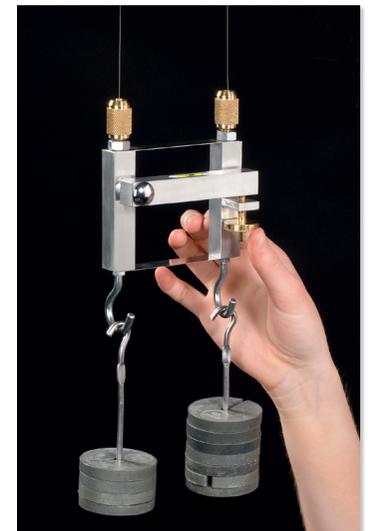
Normal practice is to plot experimental results with the independent variable on the x -axis. Although your experiment will have the stress as the independent variable, it is usual to draw the curve of these results with stress on the y -axis. Plot this graph and find the Young modulus of the metal you used, by finding the gradient of the straight-line portion of the graph.



Safety Note: Use eye protection to prevent injury if the wire snaps. Use a 'drop box' on the landing pad to keep hands and feet out of the 'drop zone'.

EXAM HINT

You need to be able to describe all the steps in an experimental method for finding the Young modulus, including safety points, and how to use the measurements to calculate the Young modulus.



▲ **fig D** Searle's apparatus.

CHECKPOINT

SKILLS PROBLEM SOLVING, CREATIVITY

1. Estimate the limit of proportionality for the IMPRESS alloy at 750 °C, as shown in **fig B**.
2. Searle's apparatus, as shown in **fig D** is used in experiments to find the Young modulus for a metal wire. With this apparatus, the test wire hangs vertically, parallel to an identical control wire. Weights are added to load the test wire only, and its extension, measured as an excess over the length of the control wire rather than from its original length, is calculated. Explain how this setup will allow the experimenter to avoid possible error caused by variations in the room temperature.

SUBJECT VOCABULARY

yield point a strain value beyond which a material undergoes a sudden and large plastic deformation

2B THINKING BIGGER

GET ROPED IN

SKILLS

CRITICAL THINKING, PROBLEM SOLVING, ANALYSIS, REASONING/ARGUMENTATION, INTERPRETATION, ADAPTIVE LEARNING, INITIATIVE, PRODUCTIVITY, COMMUNICATION

If a manufacturer of climbing ropes is to choose a material for a new rope, what properties would they want the material to have? Apart from low cost, they will need to consider whether it will safely hold climbers, especially when they fall and the rope has to save them.

In this activity, we will look at some important aspects of the properties of some climbing ropes.

VARIOUS SOURCES

PRODUCT	COLOUR	DIAMETER	LENGTHS (m)	WEIGHT (per m)	NO OF FALLS (UIAA)	IMPACT FORCE
PITCH	NAVY BLUE	8.5 mm	50, 60	49 g	13	6.5 kN
PITCH	ORANGE	8.5 mm	50, 60	49 g	13	6.5 kN
ZONE	SUNSET RED	9.8 mm	50, 60, 70, 80	63 g	6	8.7 kN
ORBIT	COBALT BLUE	9.6 mm	50, 60, 70, 80	61 g	8	8.7 kN
COULOIR	RED	8.0 mm	50, 60	43 g	7	6.6 kN
COULOIR	BLUE	8.0 mm	50, 60	43 g	7	6.6 kN



From the website of rope manufacturer DMM International, <http://dmmclimbing.com/products/ropes-&-cord/>

SINGLE ROPE TESTING METHODS FOR UIAA 101

Above: Static elongation finds the standard strain for a rope, giving an indication of its stiffness.

Right: Fall tests indicate the maximum impact force felt by a climber during a fall.

UIAA

From the Safety Standards pages of UIAA, the International Climbing and Mountaineering Federation

ROPE ANCHORING

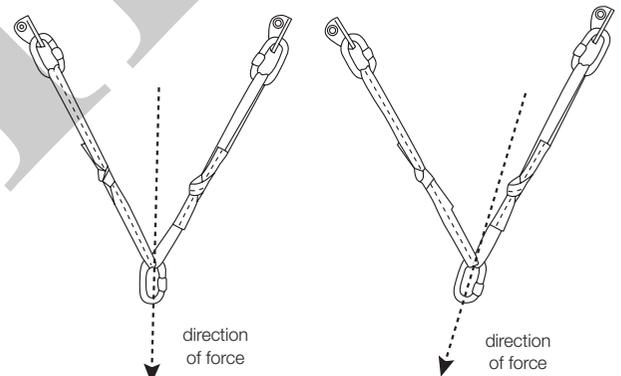


Fig 7-19 a, line representing direction of force **bisects** angle, thus load on the two anchors is equal; **b**, direction of force is to one side, thus load on right-hand anchor is greater than on left-hand anchor.

Angle (in degrees)	Force on each anchor
0	50%
60	60%
90	70%
120	100%
150	190%
170	580%

From the 6th edition of *Mountaineering: The Freedom of the Hills*, a climbing book

SCIENCE COMMUNICATION

- The extracts opposite are from three different sources relating to the use of ropes for rock climbing.
 - Compare and contrast the style of presentation from each source.
 - Discuss the safety considerations that the authors (and their insurers) will have had to think about before publishing the information in each case.

PHYSICS IN DETAIL

Now we will look at the physics in detail. Some of these questions link to topics earlier in this book, so you may need to combine concepts from different areas of physics to work out the answers.

- Calculate the weight of a 50 m length of DMM's *Couloir* rope.
 - Calculate the density of DMM's *Orbit* rope.
- Explain the physics behind the caption in **fig 7-19** from the *Mountaineering* book.
 - The table of angles from the *Mountaineering* book refers to the total angle between the slings (short support ropes) in the symmetrical situation shown in **fig 7-19a**. Explain why the force on each **anchor** would increase as suggested, and calculate how accurately the force percentages have been reported.
- Looking at the fall tests, explain why lower impact force ropes will stretch more.
 - Considering part (a), explain why a larger percentage dynamic elongation, compared with the static elongation, minimises injuries from the rope stopping a fall.
- A recent update of the UIAA rope testing standards includes a method for calculating the energy absorbed by a rope before failure. This is given by the equation:

$$E_{\text{rupt}} = \int_{t_{\text{tens}}}^{t_{\text{rupt}}} F_{(S)} S dS$$

This equation is an integration which means it calculates the sum of all the $F \times S$ values over the range of tensions up to rupture. How could such calculations produce results for ropes as presented in the suggested picture below?



THINKING BIGGER TIP

Include an explanation of what physics term should be used instead of the word 'elongation'. Note that elongation is quoted as a percentage.

ACTIVITY

Write a review comparing the DMM ropes listed in terms of physics properties. Include at least:

- calculations of density, and a commentary about which ropes would float in water.
- calculations of stress in each rope under static elongation, when it must support an 80 kg mass, and assuming the maximum 8% elongation.
- comparison of the Young modulus values for each rope, and compare these with their impact forces transferred in the UIAA fall tests. Is there a relationship?

SUBJECT VOCABULARY

bisects to divide something into two, usually equal, parts

anchor an object, usually on a rope or chain, designed to give support

2B EXAM PRACTICE

1 The pascal is the unit for the Young modulus. Which unit is equivalent?

- A mm/m
- B %
- C N m^{-1}
- D N m^{-2}

[1]

(Total for Question 1 = 1 mark)

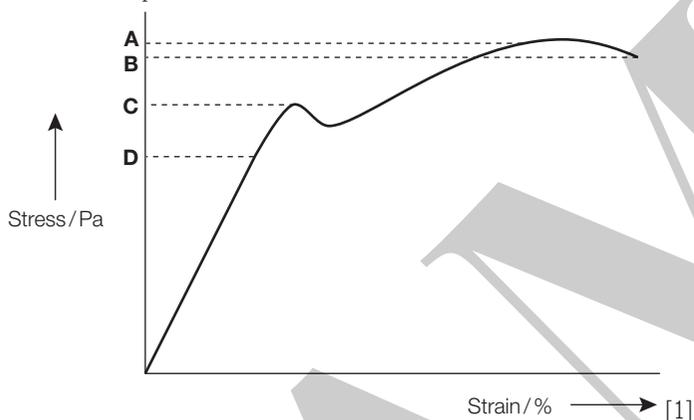
2 A 200 g mass hung on a spring stretches it from 8.2 cm to 9.0 cm. What is the spring constant for this spring?

- A 2.5 N m^{-1}
- B 25 N m^{-1}
- C 250 N m^{-1}
- D $250\,000 \text{ N m}^{-1}$

[1]

(Total for Question 2 = 1 mark)

3 The diagram shows the stress–strain curve for a metal. Which label corresponds to the Ultimate Tensile Stress for this metal?



[1]

(Total for Question 3 = 1 mark)

4 Which is the correct definition for the 'elastic limit' for a metal wire?

- A The maximum extension that the wire can have, which is still proportional to the load applied.
- B The highest value that the stress can ever reach within this wire.
- C The value that the stress will be in the wire when it breaks.
- D The maximum extension or compression that the wire can undergo and still return to its original dimensions when the force is removed.

[1]

(Total for Question 4 = 1 mark)

5 When a spring is extended by Δx , elastic strain energy, E_{el} , is stored in the spring. If the extension changes, then the elastic strain energy also changes.

Which row in the table correctly gives the new elastic strain energy for the change in extension given?

	New extension	New elastic strain energy
A	$2\Delta x$	$2E_{el}$
B	$\frac{\Delta x}{2}$	$2E_{el}$
C	$2\Delta x$	$4E_{el}$
D	$\frac{\Delta x}{2}$	$4E_{el}$

[1]

(Total for Question 5 = 1 mark)

6 You are asked to find the Young modulus for a metal using a sample of wire.

- (a) Describe the apparatus you would use, the measurements you would take and explain how you would use them to determine the Young modulus for the metal. [8]
- (b) State **one** safety precaution you would take. [1]
- (c) Explain **one** experimental precaution you would take to ensure you obtain accurate results. [2]

(Total for Question 6 = 11 marks)

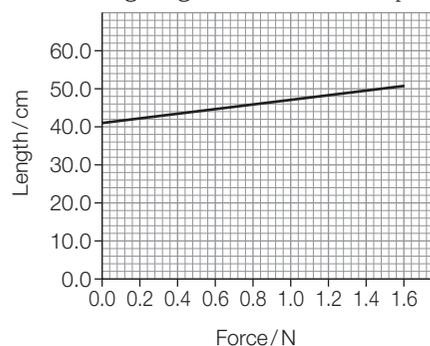
7 The photograph shows a tin bought from a joke shop. When the lid is removed, a long spring, covered in fabric to resemble a snake, flies out of the tin.



The spring on its own is shown here.



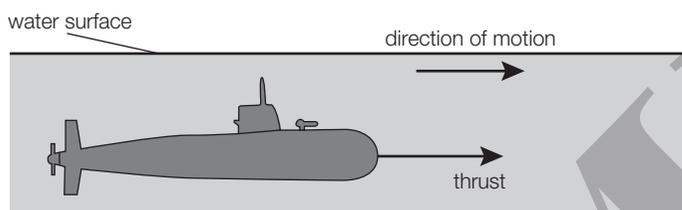
The graph shows length against force for the spring.



- (a) Explain whether the spring obeys Hooke's law. [2]
- (b) Show that the spring constant k of the spring is about 20 N m^{-1} . [3]
- (c) The original length of the spring is 41.0 cm and the length of the tin is 9.0 cm .
- (i) Calculate the force that must be applied to the spring to get it into the tin. [2]
- (ii) Calculate the energy stored in the spring when it is compressed to fit into the tin. [2]
- (d) In fact, the bottom of the tin contains a device that makes a squeak when the spring is released, making the internal length of the tin less than 9.0 cm . Explain the effect this has on the speed at which the spring leaves the tin. [3]

(Total for Question 7 = 12 marks)

- 8 The diagram shows a submarine and one of the forces acting on it. The submarine moves at a constant depth and speed in the direction shown.



- (a) State two equations that show the relationship between the forces acting on the submarine. [2]
- (b) The submarine has a volume of 7100 m^3 . Show that the weight of the submarine is about $7 \times 10^7 \text{ N}$. Density of seawater = 1030 kg m^{-3} [2]
- (c) The submarine can control its depth by changing its weight. This is done by adjusting the amount of water held in ballast tanks. As the submarine dives to greater depths the increased pressure of the surrounding water produces a compressive strain.
- (i) Explain what is meant by compressive strain. [1]
- (ii) This decreases the volume of the submarine. Explain the action that should be taken to maintain a constant depth as the volume of the submarine is decreased. [2]
- (iii) The submarine is made from steel. Suggest why a material, such as fibreglass, which has a much smaller Young modulus than steel would be unsuitable at greater depths. [2]

(Total for Question 8 = 9 marks)

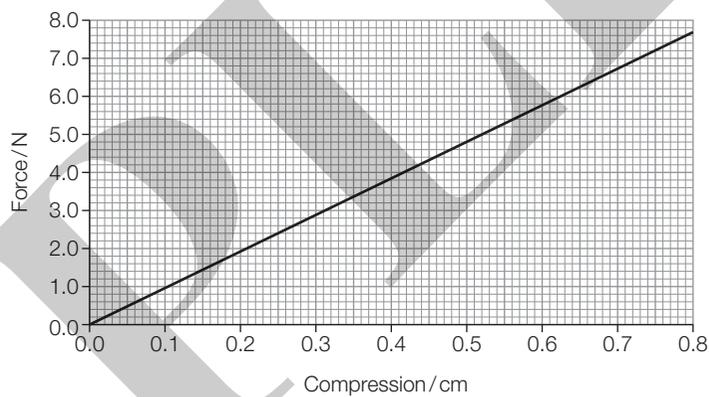
- 9 The photograph shows a device for swatting flies.



The device consists of a handle, a spring and a disc as shown in the photograph below.



When the button is pushed, the compressed spring is released, launching the disc at the fly.



- (a) Show that the force constant for the spring is about 1000 N m^{-1} . [2]
- (b) The spring is 6.3 cm long. When it is compressed in the device, the length of the spring is reduced to 1.6 cm . Assuming that the spring obeys Hooke's law throughout the compression, show that the energy stored in the spring before releasing the disc is about 1 J . [2]
- (c) The disc and spring have a combined mass of 9.4 g .
- (i) Show that the maximum speed at which they can be launched is about 15 m s^{-1} . [2]
- (ii) State an assumption that you have made. [1]
- (d) The disc is launched horizontally at a fly on the wall 3.0 m away.
- (i) Calculate the velocity of the disc as it hits the wall. Ignore the effects of air resistance. [4]
- (ii) The fly is 20 cm below the horizontal level at which the disc is launched. Show that the disc is close enough to hit the fly if it does not move. The disc has a radius of 3 cm . [3]
- (e) Suggest an advantage of the disc used over a solid disc. [1]

(Total for Question 9 = 15 marks)

MATHS SKILLS

In order to be able to develop your skills, knowledge and understanding in Physics, you will need to have developed your mathematical skills in a number of key areas. This section gives more explanation and examples of some key mathematical concepts you need to understand. Further examples relevant to your International AS/A level Physics studies are given throughout the book. In particular, the Working as a Physicist section explores important ideas about units and estimation.

ARITHMETIC AND NUMERICAL COMPUTATION

USING STANDARD FORM

Dealing with very large or small numbers can be difficult. To make them easier to handle, you can write them in the format $a \times 10^b$. This is called standard form.

To change a number from decimal form to standard form:

- Count the number of positions you need to move the decimal point by until it is directly to the right of the first number which is not zero.
- This number is the index number that tells you how many multiples of 10 you need. If the original number was a decimal, your index number must be negative.

Here are some examples:

DECIMAL NOTATION	STANDARD FORM NOTATION
0.000 000 012	1.2×10^{-8}
15	1.5×10^1
1000	1.0×10^3
3 700 000	3.7×10^6

WORKED EXAMPLE

Calculate the momentum of a bullet of mass 5.2 grams fired at a speed of 825 m s^{-1} in kg m s^{-1} .

The momentum of this bullet would be:

$$\begin{aligned}p &= m \times v \\p &= 5.2 \times 10^{-3} \times 825 \\p &= 4290 \times 10^{-3} \\p &= 4.29 \text{ kg m s}^{-1}\end{aligned}$$

ALGEBRA

CHANGING THE SUBJECT OF AN EQUATION

It can be very helpful to rearrange an equation to express the variable that you are interested in in terms of the variables it is related to. Always remember that any operation that you apply to one side of the equation must also be applied to the other side.

WORKED EXAMPLE

Consider the following equation which relates v = velocity, u = initial velocity, a = acceleration and s = displacement:

$$v^2 = u^2 + 2as$$

If we wished to rearrange this equation to make u the subject, we would first subtract $2as$ from each side to obtain:

$$v^2 - 2as = u^2$$

Now to obtain the formula in terms of u , we have to 'undo' the square term by doing the opposite of squaring for each side. In essence, we have to take the square root of each side:

$$\sqrt{u^2} = \sqrt{v^2 - 2as}$$

$$u = \sqrt{v^2 - 2as}$$

HANDLING DATA

USING SIGNIFICANT FIGURES

Often when you do a calculation, your answer will have many more figures than you need. Using an appropriate number of significant figures will help you to interpret results in a meaningful way.

Remember the 'rules' for significant figures:

- The first significant figure is the first figure which is not zero.
- Digits 1–9 are always significant.
- Zeros which come after the first significant figure are significant unless the number has already been rounded.

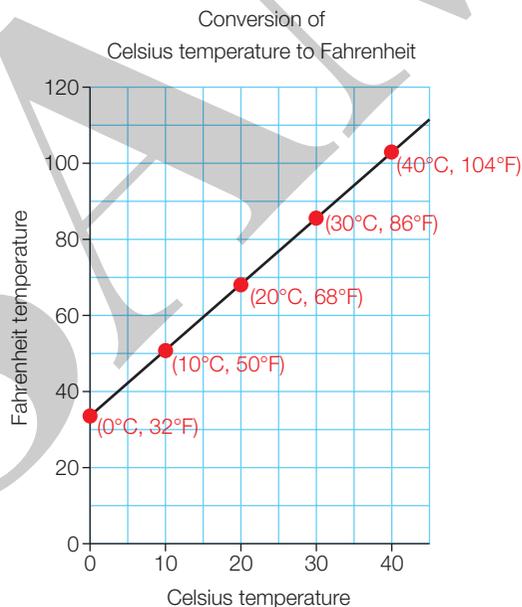
Here are some examples:

EXACT NUMBER	TO ONE S.F.	TO TWO S.F.	TO THREE S.F.
45 678	50 000	46 000	45 700
45 000	50 000	45 000	45 000
0.002 755	0.003	0.002 8	0.002 76

GRAPHS

UNDERSTAND THAT $y = mx + c$ REPRESENTS A LINEAR RELATIONSHIP

Two variables are in a linear relationship if they increase at a constant rate in relation to one another. If you plotted a graph with one variable on the x -axis and the other variable on the y -axis, you would get a straight line. Any linear relationship can be represented by the equation $y = mx + c$ where the gradient of the line is m and the value at which the line crosses the y -axis is c . An example of a linear relationship is the relationship between degrees Celsius and degrees Fahrenheit, which can be represented by the equation $F = \frac{9}{5}C + 32$ where C is temperature in degrees Celsius and F is temperature in degrees Fahrenheit.



DRAW AND USE THE SLOPE OF A TANGENT TO A CURVE AS A MEASURE OF A RATE OF CHANGE

Sir Isaac Newton was fascinated by rates of change. He drew tangents to curves at various points to find the rates of change of graphs as part of his journey towards discovering the calculus – an amazing branch of mathematics. He argued that the gradient of a curve at a given point is exactly equal to the gradient of the tangent of a curve at that point.

Technique:

- 1 Use a ruler to draw a tangent to the curve.
- 2 Calculate the gradient of the tangent using the technique given for a linear relationship. This is equal to the gradient of the curve at the point of the tangent.
- 3 State the unit for your answer.

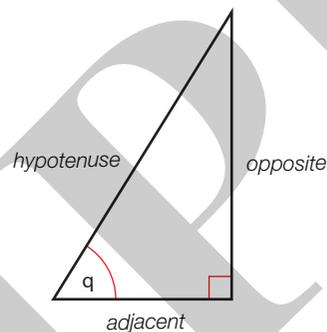
GEOMETRY AND TRIGONOMETRY**USING SIN, COS AND TAN WITH RIGHT-ANGLED TRIANGLES**

For a right-angled triangle, Pythagoras' Theorem can be used to find the length of a side given the length of two other sides. For problems that involve angles as well as side lengths, we can use the ratios sine, cosine and tangent. These ratios are called trigonometric ratios.

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$



To help you to remember these rules, you can learn a mnemonic for the letters 'SOH CAH TOA'. For example:

Some **O**ld **H**orses

Can **A**lways **H**ear

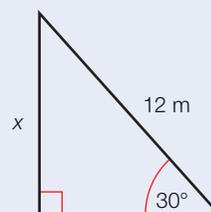
Their **O**wners **A**pproaching

FINDING THE LENGTH OF AN UNKNOWN SIDE

Follow this four-step procedure:

WORKED EXAMPLE

Calculate the length of side x .

**1 Summary of data**

First, write down all of the data involved:

Angle = 30°

Hypotenuse = 12 m

Opposite = x

2 Selection

Next, select the appropriate trigonometric ratio using 'SOH CAH TOA'. It can be helpful to place a small tick above any variable involved.

In this case we see:

✓✓ SOH ✓ CAH ✓✓ TOA

3 Substitution

Write out the trigonometric ratio that you have chosen. You may need to rearrange it to express the value you are interested in.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{opposite} = \sin A \times \text{hypotenuse}$$

$$\text{opposite} = \sin 30 \times 12 \text{ m}$$

4 Calculate and check

Calculate your answer using the required function on your scientific calculator. Then perform three checks: check that your answer is sensible, check whether it needs to be rounded and check whether you have used the correct units.

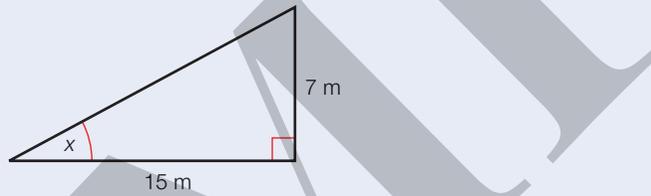
$$x = 0.5 \times 12 \text{ m} = 6 \text{ m}$$

FINDING AN UNKNOWN ANGLE

Follow the same four-step procedure. The only difference is that you will need to use the inverse function on your calculator.

WORKED EXAMPLE

Calculate the angle x .



1 Summary of data

Opposite = 7 m
 Adjacent = 15 m
 Angle = x

2 Selection

✓ SOH ✓✓ CAH ✓✓ TOA

The correct ratio to use is tangent.

3 Substitution

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\theta = \tan^{-1} \left(\frac{\text{opposite}}{\text{adjacent}} \right)$$

$$\theta = \tan^{-1} \left(\frac{7}{15} \right)$$

4 Calculate and check

$$x = 25.01689$$

$$x = 25.0^\circ \text{ (1 decimal place)}$$

APPLYING YOUR SKILLS

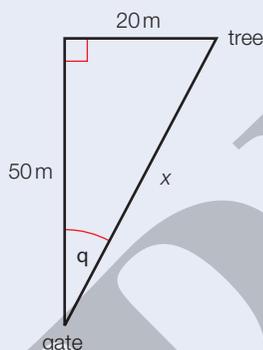
You will often find that you need to use more than one maths technique to answer a question. In this section, we will look at four example questions and consider which maths skills are required and how to apply them.

WORKED EXAMPLE

A tree stands in a field. The tree is 50 m north of the gate to the field and is 20 m east of the gate.

- Calculate the distance of the tree from the gate.
- Calculate the angle of the direction to the tree from north.

You recognise a right angle triangle. You need to use Pythagoras.



- $$x^2 = 50^2 + 20^2$$

$$x^2 = 2500 + 400 = 2900$$

$$x = \sqrt{2900}$$

$$x = 54 \text{ m (2s.f.)}$$
- $$\tan \theta = \frac{20}{50}$$

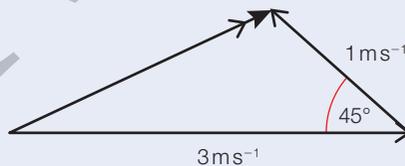
$$\text{so } \theta = 22^\circ$$

WORKED EXAMPLE

A boat sails due east (090°) at 3 m s^{-1} . The tide comes from the south-east (135°) at 1 m s^{-1} .

Draw a vector diagram to show the movement of the boat over the ground and calculate the speed.

You need to draw a diagram that represents the information in the question. If you draw a scale diagram, choose a simple scale like $1 \text{ cm} : 10 \text{ m}$.



Now we must split both of the vectors up into two components, one going north and one going east.

North The boat speed vector 3 m s^{-1} has no component going north. The tide speed vector of 1 m s^{-1} has a component $1 \text{ m s}^{-1} \times \sin 45 = \mathbf{0.707 \text{ m s}^{-1} \text{ going north}}$.

East The boat speed vector 3 m s^{-1} is going due east. The tide speed vector of 1 m s^{-1} has a component $1 \text{ m s}^{-1} \times \cos 45 = -0.707 \text{ m s}^{-1}$ going east; the minus sign means that the component is to the west and so we must subtract it since it reduces the boat speed to the east. So the **boat speed to the east** is $(3 - 0.707) = 2.293$ or $\mathbf{2.29 \text{ m s}^{-1}}$ using 3 significant figures.

Now we must combine the north and east components to find the net speed. You recognise a right angle triangle. You need to use Pythagoras.

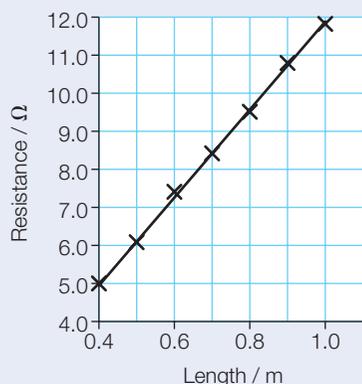
$$(\text{speed})^2 = 0.707^2 + 2.29^2 = 0.500 + 5.24 = 5.74$$

$$\text{so speed} = \sqrt{5.74} = 2.40 \text{ m s}^{-1}.$$

WORKED EXAMPLE

A student measures the resistivity of some metal in the form of a wire. He starts by looking up the definition of resistivity which is $\rho = \frac{R \times A}{l}$ where R is the resistance of a length l of a wire of cross-sectional area A .

He measures the resistance of different lengths of the wire and plots a graph of his readings.



- (a) Explain whether the readings should give a line of best fit that passes through the origin.
- (b) He measures the diameter of the wire as 0.234 mm. Determine the cross-sectional area A .
- (c) Take measurements from the graph to show that the gradient m of the line of best fit is about $m = 11.7 \Omega m^{-1}$ and hence calculate a value for the resistivity of the metal of the wire.

- (a) You need to recall the equation of a straight line $y = mx + c$ where the data is plotted on the x and y axes and m is the gradient and c is the intercept on the y -axis.

You must then compare this with the equation for resistivity where he has measured R and l .

Since $R = \rho \times \frac{l}{A}$ we can see that if R is plotted on the y -axis and l on the x -axis then the gradient of the line will be $\frac{\rho}{A}$ which is a constant so the line will be straight. There are no other terms in the resistivity equation so the value for c will be zero and the line should pass through the origin.

Note that the origin is not marked on this graph, it starts at (0.4, 4.0)

- (b) You need to remember the formula for the area of a circle $A = \pi \times r^2$

Note you can only measure the diameter of a piece of wire so you must divide by 2 to get the radius r .

$$\text{So } A = \left(0.234 \times \frac{10^{-2}}{2}\right)^2 = 4.30 \times 10^{-8} \text{ m}^2$$

- (c) Take measurements from the graph to show that the gradient m of the line of best fit is about $m = 11.7 \Omega m^{-1}$ and hence calculate a value for the resistivity of the metal of the wire.

Gradient = rise over run, so take measurements from the line of best fit – not the plots

$$\text{Gradient} = \frac{11.9 - 4.9}{1.0 - 0.4} = \frac{7}{0.6} = 11.7 = \frac{\rho}{A}$$

$$\text{So } \rho = 11.7 \Omega m^{-1} \times 4.30 \times 10^{-8} \text{ m}^2 = 5.03 \times 10^{-8} \Omega m$$

WORKED EXAMPLE

A ball is dropped from a height of 1.50 m above the ground.

- (a) Show that it hits the ground at about 5.4 m s⁻¹.
- (b) The ball is now thrown horizontally at a height of 1.5 m with a horizontal velocity of 20 m s⁻¹. Calculate the angle at which it hits the ground.

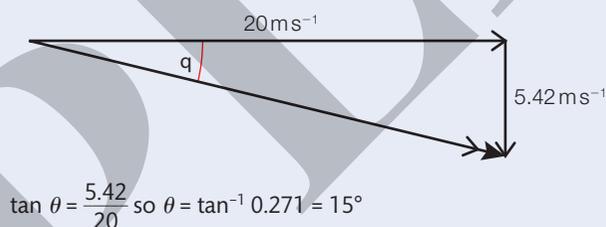
- (a) You need a kinematics equation. You know $u = 0 \text{ m s}^{-1}$, $g = 9.81 \text{ m s}^{-2}$ and $s = 1.50 \text{ m}$. You need to find v .

The equation you need is $v^2 = u^2 + 2as$

$$\text{So } v^2 = 0 + 2 \times 9.81 \times 1.50 = 29.4 \text{ so } v = \sqrt{29.4} = 5.42 \text{ m s}^{-1}$$

- (b) Remember for projectiles that the horizontal velocity remains constant. While the ball is falling it is travelling horizontally at a constant velocity of 20 m s⁻¹

It hits the ground horizontally at 20 m s⁻¹ and vertically at 5.42 m s⁻¹.



PREPARING FOR YOUR EXAMS

IAS AND IAL OVERVIEW

The Pearson Edexcel International Advanced Subsidiary in Physics and the Pearson Edexcel International Advanced Level in Physics are modular qualifications. The International Advanced Subsidiary can be claimed on completion of the International Advanced Subsidiary (IAS) units. The International Advanced Level (IAL) can be claimed on completion of all the units (IAS and IA2 units).

- International AS students will sit three exam papers. The IAS qualification can either be standalone or contribute 50% of the marks for the International Advanced Level.
- International A level students will sit six exam papers, the three IAS papers and three IAL papers.

The tables below give details of the exam papers for each qualification.

IAS Papers	Unit 1: Mechanics and Materials	Unit 2: Waves and Electricity*	Unit 3: Practical Skills in Physics 1
Topics covered	Topics 1–2	Topics 3–4	Topics 1–4
% of the IAS qualification	40%	40%	20%
Length of exam	1 hour 30 minutes	1 hour 30 minutes	1 hour 20 minutes
Marks available	80 marks	80 marks	50 marks
Question types	multiple-choice short open open-response calculation extended writing	multiple-choice short open open-response calculation extended writing	short open open-response calculation extended writing
Mathematics	For Unit 1 and Unit 2, a minimum of 32 marks will be awarded for mathematics at Level 2 or above. For Unit 3, a minimum of 20 marks will be awarded for mathematics at Level 2 or above.		

* This paper will contain some synoptic questions which require knowledge and understanding from Unit 1.

IAL Papers	Unit 4: Further Mechanics, Fields and Particles**	Unit 5: Thermodynamics, Radiation, Oscillations and Cosmology†	Unit 6: Practical Skills in Physics 2
Topics covered	Topics 5–7	Topics 8–11	Topics 5–11
% of the IAL qualification	20%	20%	10%
Length of exam	1 hour 45 minutes	1 hour 45 minutes	1 hours 20 minutes
Marks available	90 marks	90 marks	50 marks
Question types	multiple-choice short open open-response calculation extended writing	multiple-choice short open open-response calculation extended writing	short open open-response calculation extended writing synoptic
Mathematics	For Unit 4 and Unit 5, a minimum of 36 marks will be awarded for mathematics at Level 2 or above. For Unit 6, a minimum of 20 marks will be awarded for mathematics at Level 2 or above.		

** This paper will contain some synoptic questions which require knowledge and understanding from Units 1 and 2.

† This paper will contain some synoptic questions which require knowledge and understanding from Units 1, 2 and 4.

EXAM STRATEGY

ARRIVE EQUIPPED

Make sure you have all of the correct equipment needed for your exam. As a minimum you should take:

- pen (black ink or ball-point pen)
- pencil (HB)
- rule (ideally 30 cm)
- rubber (make sure it's clean and doesn't smudge the pencil marks or rip the paper)
- calculator (scientific).

MAKE SURE YOUR ANSWERS ARE LEGIBLE

Your handwriting does not have to be perfect but the examiner must be able to read it! When you're in a hurry it's easy to write key words that are difficult to understand.

PLAN YOUR TIME

Note how many marks are available on the paper and how many minutes you have to complete it. This will give you an idea of how long to spend on each question. Be sure to leave some time at the end of the exam for checking answers. A rough guide of a minute a mark is a good start, but short answers and multiple choice questions may be quicker. Longer answers might require more time.

UNDERSTAND THE QUESTION

Always read the question carefully and spend a few moments working out what you are being asked to do. The command word used will give you an indication of what is required in your answer.

Be scientific and accurate, even when writing longer answers. Use the technical terms you've been taught.

Always show your working for any calculations. Marks may be available for individual steps, not just for the final answer. Also, even if you make a calculation error, you may be awarded marks for applying the correct technique.

PLAN YOUR ANSWER

In questions marked with an *, marks will be awarded for your ability to structure your answer logically showing how the points that you make are related or follow on from each other where appropriate. Read the question fully and carefully (at least twice!) before beginning your answer.

MAKE THE MOST OF GRAPHS AND DIAGRAMS

Diagrams and sketch graphs can earn marks – often more easily and quickly than written explanations – but they will only earn marks if they are carefully drawn.

- If you are asked to read a graph, pay attention to the labels and numbers on the x and y axes. Remember that each axis is a number line.
- If asked to draw or sketch a graph, always ensure you use a sensible scale and label both axes with quantities and units. If plotting a graph, use a pencil and draw small crosses or dots for the points.
- Diagrams must always be neat, clear and fully labelled.

CHECK YOUR ANSWERS

For open-response and extended writing questions, check the number of marks that are available. If three marks are available, have you made three distinct points?

For calculations, read through each stage of your working. Substituting your final answer into the original question can be a simple way of checking that the final answer is correct. Another simple strategy is to consider whether the answer seems sensible. Pay particular attention to using the correct units.

SAMPLE EXAM ANSWERS

QUESTION TYPE: MULTIPLE CHOICE

The unit of electric current is the ampere. One ampere is equivalent to:

- A 0.1 C s^{-1} B 1 C s^{-1}
C 0.1 s C^{-1} D 0.1 C s

[1]

In all types of Physics exam questions, using the correct unit in your answer is vital. Make sure you are totally familiar with all the units you need, as well as the order of magnitude prefixes, like 'mega-'. ←

Question analysis

- Multiple choice questions look easy until you try to answer them. Very often they require some working out and thinking. A good approach is to work out the answer as if it was an open response question, and then find your answer among the choices.
- In multiple choice questions you are given the correct answer along with three incorrect answers (called distractors). You need to select the correct answer and put a cross in the box of the letter next to it.
- If you change your mind, put a line through the box () and then mark your new answer with a cross () .

Multiple choice questions always have one mark and the answer is given! For this reason students often make the mistake of thinking that they are the easiest questions on the paper. Unfortunately, this is not the case. These questions often require several answers to be worked out and an error in one of them will lead to the wrong answer being selected. The three incorrect answers supplied (distractors) will feature the answers that students arrive at if they make typical or common errors. The trick is to answer the question before you look at any of the answers.

Average student answer

D 0.1 C s

← One ampere (1A) is the movement of one coulomb (1C) of charge per second (1s). Therefore the correct answer is B.

COMMENTARY

This is an incorrect answer because:

- The student did not know the definition of an ampere and chose an answer with incorrect units. ←

← If you have any time left at the end of the paper go back and check your answer to each part of a multiple choice question so that a slip like this does not cost you a mark.

QUESTION TYPE: SHORT OPEN

Write the equation for calculating the total resistance of three resistors in parallel.

[1]

Question analysis

- Generally one piece of information is required for each mark given in the question. There is one mark available for this question and so one piece of information is required.
- Clarity and brevity are the keys to success on short open questions. For one mark, it is not always necessary to write complete sentences.

← The command word write is used when you are being asked to write down an equation. Since only one mark is available, your equation needs to be completely correct to gain the mark.

Average student answer

$$\frac{1}{R^1} + \frac{1}{R_2} + \frac{1}{R_3}$$

← While this answer is the correct idea, it would not get the mark as the student has not written an equation as the question asked.

COMMENTARY

This is an incorrect answer because:

- The student did not give a complete equation. This is easy to see because there is no equals sign!

QUESTION TYPE: OPEN RESPONSE

Describe a circuit that could be used to control the heating in a greenhouse to come on when the ambient temperature drops below a certain value.

[3]

The command word in this question is *describe*. This means that you need to give an account of something. You do not need to include a justification or reason.

Question analysis

- With any question worth three or more marks, think about your answer and the points that you need to make before you write anything down. Keep your answer concise, and the information you write down relevant to the question. You will not gain marks for writing down physics that is not relevant to the question (even if correct) but it will cost you time.
- Remember that you can use bullet points or diagrams in your answer.

Average student answer

A potential divider circuit with a thermistor in it could do this job. This controls the heating so it was on when too cold.

In many answers, a clear (labelled) diagram can easily gain marks. Here it is essential.

COMMENTARY

This is a weak answer because:

- The student has suggested including a thermistor but has not described what it would do.
- The student's answer restates information from the question. This will never score marks so is a poor use of time and space.

QUESTION TYPE: EXTENDED WRITING

The maximum speed limit on motorways has not been raised since the 1960s. However, developments in engineering mean that modern cars have much higher top speeds than cars in the past and can stop safely in shorter distances.

Discuss the risks and benefits of raising the maximum speed limits on motorways.

[4]

It is reasonable to assume that there will be equal numbers of marks available for each side of the argument so you should balance the viewpoints you give accordingly. However, you should also remember that marks will also be available for giving an overall conclusion so you should be careful not to omit that.

Question analysis

- All extended writing answers need to discuss the physics behind the scenario. To gain full marks, your answer must be complete and coherent.
- It is vital to plan out your answer before you write it down. There is always space given on an exam paper to do this so just jot down the points that you want to make before you answer the question in the space provided. This will help to ensure that your answer is coherent and logical and that you don't end up contradicting yourself. However, once you have written your answer go back and cross these notes out so that it is clear they do not form part of the answer.

Average student answer

It would be more dangerous because with more momentum, people wouldn't be able to stop in time. Weaving in and out of the lanes would be more common as people try to get past each other at higher speeds, which would also be more dangerous. Shouldn't be done.

There is no need to make judgements in physics questions. Explaining the risks and benefits is all that is required. Including your own opinion is likely to cause you to forget to include points from the other side of the argument, just like this student has done.

COMMENTARY

This is a weak answer because:

- The phrase 'more dangerous' does not explain a specific event that will be dangerous and more likely at higher speeds.
- The student needs to explain the physics behind why these dangerous events become more likely at higher speeds.
- This student's mention of momentum does not follow up with why momentum increase means cars take longer to stop, and so it would not score a mark.
- There must be at least one benefit explained in order to score full marks.

QUESTION TYPE: CALCULATION

An electric motor takes 45.0 s to lift a mass of 800 kg through a vertical height of 14.0 m. The potential difference across the motor is 230 V and the current is 13.0 A.

Calculate the efficiency of the motor.

[3]

The command word here is calculate. This means that you need to obtain a numerical answer to the question, showing relevant working. If the answer has a unit, this must be included.

Question analysis

- The important thing with calculations is that you must show your working clearly and fully. The correct answer on the line will gain all the available marks, however, an incorrect answer can gain all but one of the available marks if your working is shown and is correct.
- Show the calculation that you are performing at each stage and not just the result. When you have finished, look at your result and see if it is sensible.

Average student answer

$$\text{GPE} = 800 \text{ kg} \times 9.81 \text{ m s}^{-2} \times 14 \text{ m} = 109\,900 \text{ J}$$

$$P = 230 \text{ V} \times 13.0 \text{ A} = 2990 \text{ W}$$

$$E = P \times t = 2990 \text{ W} \times 45 \text{ s} = 134\,600 \text{ J}$$

$$\text{efficiency} = 109\,900 \text{ J} / 134\,600 \text{ J} = 0.816$$

Answers using any standard unit will be acceptable (providing correctly converted). Here, giving the final answer as 81.6% would also score full marks.

COMMENTARY

This is a strong answer because:

- The answer has been laid out very clearly so that even if an error had been made then a mark could have been awarded for part of the calculation and errors carried forward lose no further marks.