## PERESON EDEMCEL

 TNTERNETONEL CCSE (9-1) REVISE Mathematics 2RYTSTON CTTD
## Higher



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A small bit of small print
Edexcel publishes Sample Assessment Material and the Specification on its website. This is the official content and this book should be used in conjunction with it. The questions in Now try this have been written to help you practise every topic in the book. Remember: the real exam questions may not look like this.

## Factors and primes

The factors of a number are any numbers that divide into it exactly. A prime number has exactly two factors. The prime numbers are 2, 3, 5, 7, 11, 13, 17, 19 and so on.

## Prime ractors

If a number is a factor of another number and it is a prime number then it is called a prime factor. You use a factor tree to find prime factors.

Remember to circle the

$$
\begin{array}{rlr}
84 & =2 \times 2 \times 3 \times 7 . & \text { Remember to put in the multiplication signs. } \\
& =2^{2} \times 3 \times 7 . & \text { This is called a product of prime factors. }
\end{array}
$$

prime factors as you go along. The order doesn't matter.

The highest common factor (HCF) of two numbers is the highest number that is a factor of both numbers.

The lowest common multiple (LCM) of two numbers is the lowest number that is a multiple of both numbers.

## Worked example

(a) Write 108 as the product of its prime factors. Give your answer in index form.
(3 marks)

$108=2 \times 2 \times 3 \times 3 \times 3=2^{2} \times 3^{3}$
(b) Work out the highest common factor (HCF) of 108 and 24.
(2 marks)
$108=$ (2) $\times$ (2) $\times(3) \times 3 \times 3$
$24=(2) \times(2) \times 2$
HCF is $2 \times 2 \times 3=12$
(c) Work out the lowest common multiple (LCM) of 108 and 24.
(2 marks)
LCM $=12 \times 3 \times 3 \times 2=216$

Draw a factor tree. Continue until every branch ends with a prime number. This question asks you to write your answer in index form. This means you need to use powers to say how many times each prime number occurs in the product.

## Check it!

$2^{2} \times 3^{3}=4 \times 27=108$

To find the HCF, circle all the prime numbers which are common to both products of prime factors. 2 appears twice in both products so you have to circle it twice. Multiply the circled numbers together to find the HCF.

To find the LCM multiply the HCF by each unshared prime from part (b).

## Now try this

1 (a) Express 980 as a product of its prime factors.
(3 marks)
(b) Find the highest common factor (HCF) of 980 and 56.
(2 marks)
$2 X=2 \times 3^{5} \times 7^{2} \quad Y=3^{2} \times 5 \times 7$
(a) Find the highest common factor (HCF) of $X$ and $Y$.
(2 marks)
(b) Find the lowest common multiple (LCM) of $X$ and $Y$.
(2 marks)

## Algebraic expressions

You need to be able to work with algebraic expressions confidently. For a reminder about using the index laws with numbers have a look at pages 2 and 3.

You can use the index laws to simplify algebraic expressions.
$a^{m} \times a^{n}=a^{m+n}$
$x^{4} \times x^{3}=x^{4+3}=x^{7}$
$\frac{a^{m}}{a^{n}}=a^{m-n}$
$m^{8} \div m^{2}=m^{8-2}=m^{6}$
$\left(a^{m}\right)^{n}=a^{m n}$
$\left(n^{2}\right)^{4}=n^{2 \times 4}=n^{8}$


You can square or cube a whole expression.

$$
\begin{aligned}
&\left(4 x^{3} y\right)^{2}=(4)^{2} \times\left(x^{3}\right)^{2} \times(y)^{2} \\
&=16 x^{6} y^{2} \\
& 16=(4)^{2} \\
&\left(x^{3}\right)^{2}=x^{3} \times 2=x^{6} \begin{array}{l}
\text { You need to } \\
\text { square everything } \\
\text { inside the } \\
\text { brackets. }
\end{array}
\end{aligned}
$$

Remember that if a letter appears on its own then it has the power 1.

(3)Algebraic expressions may also contain negative and fractional indices.

$$
\begin{aligned}
& a^{-m}=\frac{1}{a^{m}} \\
& \left(c^{2}\right)^{-3}=c^{2 \times-3}=c^{-6}=\frac{1}{c^{6}} \\
& a^{\frac{1}{n}}=\sqrt[n]{a}
\end{aligned}
$$

$$
\begin{aligned}
\left(8 p^{3}\right)^{\frac{1}{3}} & =(8)^{\frac{1}{3}} \times\left(p^{3}\right)^{\frac{1}{3}} \\
& =\sqrt[3]{8} \times p^{3 \times \frac{1}{3}} \\
& =2 p
\end{aligned}
$$

## One at a time

When you are multiplying expressions:

1. Multiply any number parts first.
2. Add the powers of each letter to work out the new power.

$$
\begin{gathered}
6 p^{2} q \times 3 p^{3} q^{2}=18 p^{5} q^{3} \\
6 \times 3=18 \\
p^{2} \times p^{3}=p^{2+3}=p^{5}
\end{gathered} \prod_{q \times q^{2}=q^{1+2}=q^{3}}
$$

## When you are dividing expressions:

1. Divide any number parts first.
2. Subtract the powers of each letter to work out the new power.

$$
\begin{aligned}
& 12 \div 3=4 \\
& \frac{12 a^{5} b^{3}}{3 a^{2} b^{2}}=4 a^{3} b
\end{aligned} b^{3} \div b^{2}=b^{3-2}=b
$$

## Worked example

Simplify fully
(a) $m^{2}+m^{2}+m^{2}+m^{2}$
$4 m^{2}$
(b) $\left(x^{3}\right)^{3}$
(c) $\frac{4 y^{2} \times 3 y^{7}}{6 y}$
$\frac{4 y^{2} \times 3 y^{7}}{6 y}=\frac{12 y^{9}}{6 y}=2 y^{8}$
(a) This is four lots of $\mathrm{m}^{2}$, so you write it as $4 \times \mathrm{m}^{2}$ or $4 \mathrm{~m}^{2}$
(b) Use $\left(a^{m}\right)^{n}=a^{m n}$
(c) Start by simplifying the top part of the fraction. Do the number part first then the powers. Use $a^{m} \times a^{n}=a^{m+n}$ Next divide the expressions. Divide the number part, then divide the indices using $\frac{a^{m}}{a^{n}}=a^{m-n}$

## Now try this

1 Simplify $\left(h^{2}\right)^{6}$
2 Simplify fully
(a) $\left(2 a^{5} b\right)^{4}$
(b) $5 x^{4} y^{2} \times 3 x^{3} y^{7}$
(c) $18 d^{8} g^{10} \div 6 d^{2} g^{5}$
(1 mark)
(2 marks)
(2 marks)
(2 marks)


3 (a) Simplify $\left(16 p^{10}\right)^{\frac{1}{2}}$
(2 marks)
(b) Simplify $\left(64 x^{9} y^{2}\right)^{-\frac{1}{3}}$
 $-\square \begin{aligned} & \text { Worked } \\ & \text { solution } \\ & \text { video }\end{aligned}$


Apply the power outside the brackets to everything inside the brackets.

## Speed

This is the formula triangle for speed.


Average speed $=\frac{\text { total distance travelled }}{\text { total time taken }}$
Time $=\frac{\text { distance }}{\text { average speed }} \quad$ LTE R TM
Distance $=$ average speed $\times$ time

## Units

The most common units of speed are:

- metres per second: $\mathrm{m} / \mathrm{s}$
- kilometres per hour: $\mathrm{km} / \mathrm{h}$
- miles per hour: mph.

To convert between measures of speed you need to convert one unit first then the other. Write the new units at each step of your working. To convert $72 \mathrm{~km} / \mathrm{h}$ into $\mathrm{m} / \mathrm{s}$ :
$72 \mathrm{~km} / \mathrm{h} \rightarrow 72 \times 1000=72000 \mathrm{~m} / \mathrm{h}$
$72000 \mathrm{~m} / \mathrm{h} \rightarrow 72000 \div 3600=20 \mathrm{~m} / \mathrm{s}$ 1 hour $=60 \times 60=3600$ seconds

## Worked example

The speed of light in a vacuum is approximately $1.08 \times 10^{9} \mathrm{~km} / \mathrm{h}$.
Light from the Sun takes approximately 8 minutes and 15 seconds to travel to Earth.
Estimate the distance from the Earth to the Sun.
(3 marks)


8 mins 15 secs $=8.25$ mins $=\frac{8.25}{60}$ $=0.1375$ hours
$D=S \times T$
$=1.08 \times 10^{9} \times 0.1375$
$=1.485 \times 10^{8} \mathrm{~km}$

Be careful with the units. You need to convert 12 minutes and 15 minutes into hours before doing your calculations.

## Minutes and hours

For questions on speed, you need to be able to convert between minutes and hours.
Remember there are 60 minutes in 1 hour.
To convert from minutes to hours you divide by 60 .
24 minutes $=0.4$ hours

$$
\frac{24}{60}=\frac{2}{5}=0.4
$$

To convert from hours to minutes you multiply by 60 .
0.2 hours $=12$ minutes
$3.2 \times 60=192$
3.2 hours $=3$ hours 12 minutes

## Speed checklist

Draw formula triangle.
Make sure units match.
Give units with answer.

If you're answering questions involving speed, distance and time you must always make sure that the units match. Speed is given in km/h here, so convert the time into hours before calculating.

## Now try this

Rosa lives in Durham and works in Newcastle. She takes the train to work every day.
Last Tuesday her train journey to work took
12 minutes, at an average speed of $108 \mathrm{~km} / \mathrm{h}$.
Her journey home from work took 15 minutes.
Calculate Rosa's average speed on her journey home.
(3 marks)
$\square$
$\square$

## Mean，median and mode

You can analyse data by calculating statistics like the mean，median and mode．


Do not round your answer

Median


## Worked example

Kayla has eight numbered cards．


She removes two cards．The mean value of the remaining cards is 4 ． Which two cards could Kayla have removed？Give one possible answer．
（4 marks）
$6 \times 4=24$
$1+2+3+4+5+6+7+8$
$=36$
$36-24=12$
The removed cards add up to 12 so
Kayla could have removed 7 and 5
Check：
$\frac{1+2+3+4+6+8}{6}=4 \checkmark$

You can work out the sum of the 6 remaining cards using this formula：
Sum of values $=$ mean $\times$ number of values Subtract this sum from the sum of all 8 cards．This tells you the sum of the 2 cards Kayla removed．The removed cards were either 5 and 7 or 8 and 4 ．

## Check it！

Work out the mean of the remaining 6 cards．

## Which average works best？

|  | ह， | $\beta^{3}$ |
| :---: | :---: | :---: |
| Mean | Uses all the data | Affected by extreme values |
| Median | Not affected by extreme values | May not be one of the values |
| Mode | Suitable for data that can be described in words | Not always near the middle of the data |

## Now try this

Takeshi scored these marks out of 20 in six maths tests．

$$
\begin{array}{llllll}
11 & 9 & 5 & 13 & 15 & 12
\end{array}
$$

How many marks must he score in the next test so that his new mean mark and his new median mark are the same as each other？

Make sure you check your answer by calculating the new mean and median． Remember that the median is not affected by extreme values．

