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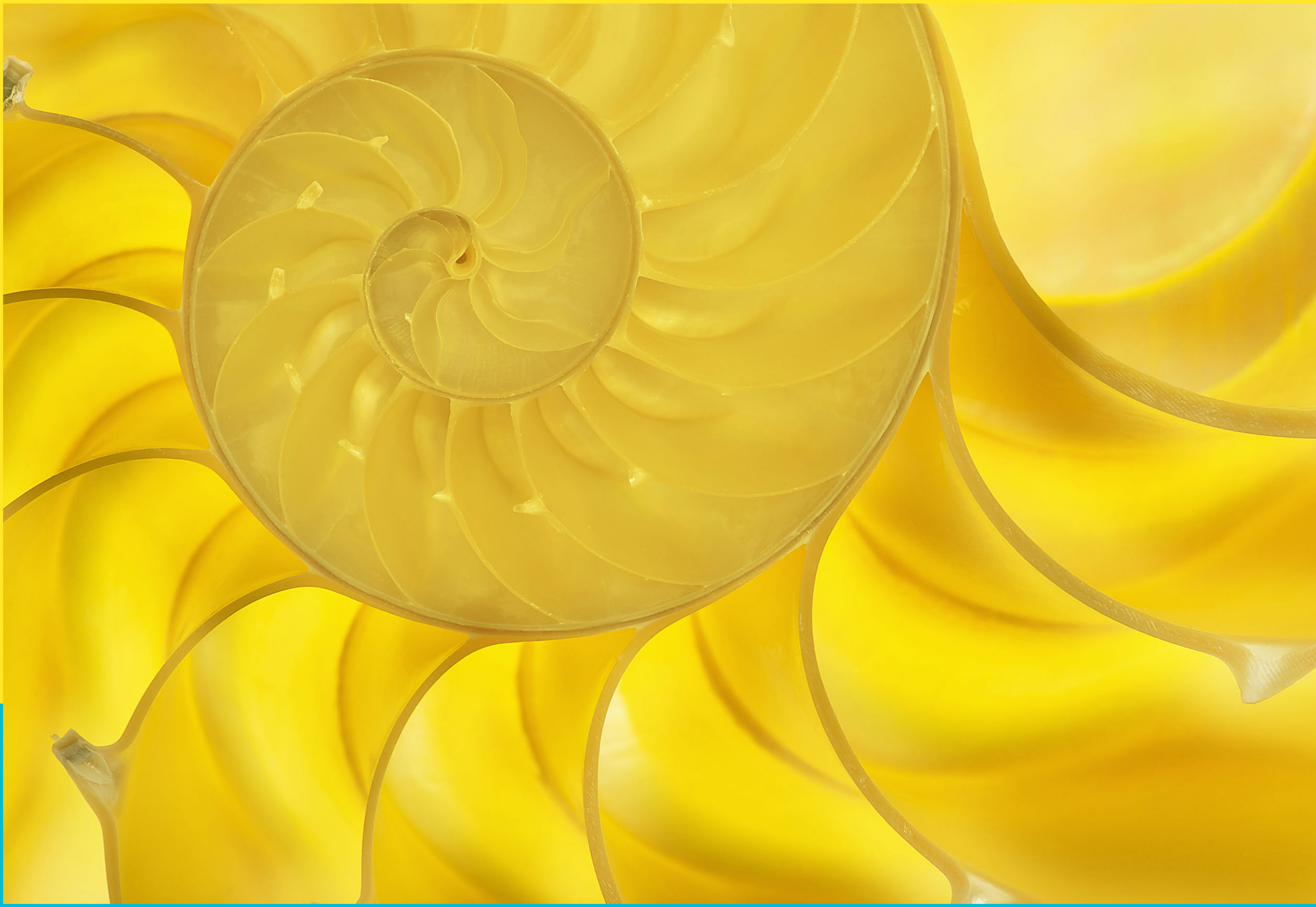

EDEXCEL INTERNATIONAL GCSE (9–1)

MATHEMATICS A

Student Book 1

1

David Turner, Ian Potts

eBook included

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MATHEMATICS A

Student Book 1

David Turner
Ian Potts

UNIT 2

2 is the first and only even prime number. $\sqrt{2}$ cannot be written as an exact fraction; this defines it as an irrational number. If n is a whole number (integer) bigger than 0, the value of $n^2 + n$ is always divisible by 2. Fermat's Last Theorem states that there are no integers x, y, z which have a solution to $x^n + y^n = z^n$ when n is bigger than 2.

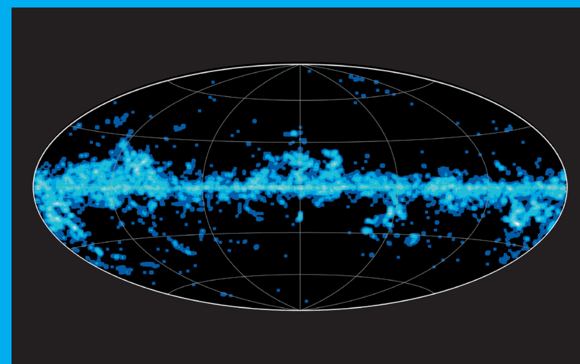
NUMBER 2

The smallest measurable thing in the Universe is the *Planck length* which if written in full is 0.000 000 000 000 000 000 000 000 000 000 000 016 2 metres.

The size of the observable universe is approximately a giant sphere of diameter 880 000 000 000 000 000 000 000 000 metres.

These numbers can both be written more conveniently in a simpler format called standard form. The first length is 1.62×10^{-35} m and the second measurement is 8.8×10^{26} m.

Max Planck (1858–1947)



LEARNING OBJECTIVES

- Write a number in standard form
- Calculate with numbers in standard form
- Work out a percentage increase and decrease
- Solve real-life problems involving percentages

BASIC PRINCIPLES

- $10^2 \times 10^3 = 10^5 \Rightarrow 10^m \times 10^n = 10^{m+n}$
- $10^2 \div 10^3 = \dfrac{1}{10^1} = 10^{-1} \Rightarrow 10^m \div 10^n = 10^{m-n}$
- $(10^3)^2 = 10^6 \Rightarrow (10^m)^n = 10^{mn}$

STANDARD FORM

Standard form is used to express large and small numbers more efficiently.

KEY POINT

- **Standard form** is always written as $a \times 10^b$, where a is between 1 and 10, but is never equal to 10 and b is an **integer** which can be positive or negative.

STANDARD FORM WITH POSITIVE INDICES

EXAMPLE 1

SKILL: REASONING

Write 8 250 000 in standard form.

$8\,250\,000 = 8.25 \times 1\,000\,000 = 8.25 \times 10^6$

EXAMPLE 2

SKILL: REASONING

Write 3.75×10^5 as an ordinary number.

$3.75 \times 10^5 = 3.75 \times 100\,000 = 375\,000$

ACTIVITY 1

SKILL: ADAPTIVE LEARNING

In the human brain, there are about 100 000 000 000 neurons, and over the human lifespan 1 000 000 000 000 000 neural connections are made.

Write these numbers in standard form.

Calculate the approximate number of neural connections made per second in an average human lifespan of 75 years.

EXERCISE 1

Write each of these in standard form.

1 ▶ 456

2 ▶ 67.8

3 ▶ 123.45

4 ▶ 67 million

5 ▶ 568

6 ▶ 38.4

7 ▶ 706.05

8 ▶ 123 million

Write each of these as an ordinary number.

9 ▶ 4×10^3

10 ▶ 5.6×10^4

11 ▶ 4.09×10^6

12 ▶ 6.789×10^5

13 ▶ 5.6×10^2

14 ▶ 6.5×10^4

15 ▶ 7.97×10^6

16 ▶ 9.876×10^5

17 ▶ The approximate area of all the land on Earth is 10^8 square miles. The area of the British Isles is 10^5 square miles. How many times larger is the Earth's area?

18 ▶ The area of the surface of the largest known star is about 10^{15} square miles. The area of the surface of the Earth is about 10^{11} square miles. How many times larger is the star's area?

Calculate these, and write each answer in standard form.

19 ▶ $(2 \times 10^4) \times (4.2 \times 10^5)$

20 ▶ $(6.02 \times 10^5) \div (4.3 \times 10^3)$

21 ▶ $(4.5 \times 10^{12}) \div (9 \times 10^{10})$

22 ▶ $(2.5 \times 10^4) + (2.5 \times 10^5)$

EXERCISE 1*

Write each of these in standard form.

1 ▶ 45 089

2 ▶ 87 050

3 ▶ 29.83 million

4 ▶ 0.076 54 billion

Q4 HINT
1 billion = 10^9

Calculate these, and write each answer in standard form.

5 ▶ 10×10^2

6 ▶ $(10^3)^2$

7 ▶ $\dfrac{10^9}{10^4}$

8 ▶ 10 million $\div 10^6$

9 ▶ $10^{12} \times 10^9$

10 ▶ $(10^2)^4$

11 ▶ $10^7 \div 10^7$

12 ▶ $\dfrac{10^{12}}{1\ \text{million}}$

Calculate these, and write each answer in standard form.

13 ▶ $(5.6 \times 10^5) + (5.6 \times 10^6)$

14 ▶ $(4.5 \times 10^4) \times (6 \times 10^3)$

15 ▶ $(3.6 \times 10^4) \div (9 \times 10^2)$

16 ▶ $(7.87 \times 10^4) - (7.87 \times 10^3)$

Calculate these, and write each answer in standard form.

17 ▶ $(4.5 \times 10^5)^3$

18 ▶ $(3 \times 10^8)^5$

19 ▶ $10^{12} \div (4 \times 10^7)$

20 ▶ $(3.45 \times 10^8) + 10^6$

21 ▶ $10^9 - (3.47 \times 10^7)$

22 ▶ $10^{16} \div (2.5 \times 10^{12})$

You will need the information in this table to answer Questions 23, 24 and 25.

CELESTIAL BODY (OBJECT IN SPACE)	APPROXIMATE DISTANCE FROM EARTH (MILES)
Sun	10^8
Saturn	10^9
Andromeda Galaxy (nearest major galaxy)	10^{19}
Quasar OQ172 (one of the remotest objects known)	10^{22}

Copy and complete these sentences.

23 ▶ The Andromeda Galaxy is … times further away from the Earth than Saturn.

24 ▶ The quasar OQ172 is … times further away from the Earth than the Andromeda Galaxy.

25 ▶ To make a scale model showing the distances of the four bodies from the Earth, a student marks the Sun 1 cm from the Earth. How far along the line should the other three celestial bodies (objects in space) be placed?

STANDARD FORM WITH NEGATIVE INDICES

ACTIVITY 2

SKILL: ADAPTIVE LEARNING

Copy and complete the table.

DECIMAL FORM	FRACTION FORM OR MULTIPLES OF 10	STANDARD FORM
0.1	$\frac{1}{10} = \frac{1}{10^1}$	1×10^{-1}
	$\frac{1}{100} = \frac{1}{10^2}$	
0.001		
0.0001		
		1×10^{-5}

KEY POINT

- $10^{-n} = \frac{1}{10^n}$

EXAMPLE 3

SKILL: REASONING

Write these **powers** of 10 as decimal numbers: **a** 10^{-2} **b** 10^{-6}

a $10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$

b $10^{-6} = \frac{1}{10^6} = \frac{1}{1000000} = 0.000001$

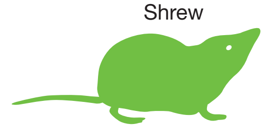
ACTIVITY 3

SKILL: ADAPTIVE LEARNING

Write down the **mass** of each of the first three objects in grams

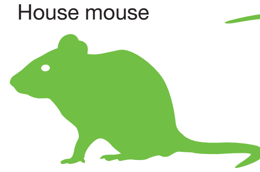
- in ordinary numbers
- in standard form.

Shrew



10^{23} kg

House mouse

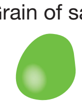


10^{22} kg

Copy and complete these statements.

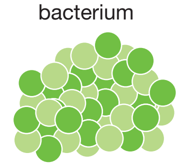
- A house mouse is … times heavier than a pigmy shrew.
- A shrew is … times heavier than a grain of sand.
- A grain of sand is 100 000 times lighter than a …

Grain of sand



10^{27} kg

Staphylococcus bacterium



10^{215} kg

- A shrew is 10 000 times heavier than a …
- A … is 100 million times heavier than a …
- A house mouse is … 10 000 billion times heavier than a …

EXAMPLE 4

SKILL: REASONING

Write 0.987 in standard form.

Write the number between 1 and 10 first.

$$0.987 = 9.87 \times \frac{1}{10} = 9.87 \times 10^{-1}$$

To display this on your calculator, press [9] [.] [8] [7] [×] [×10^x] [(−)] [1]

EXAMPLE 5

SKILL: REASONING

Write 3.75×10^{-3} as an ordinary number.

Write the number between 1 and 10 first.

$$3.75 \times 10^{-3} = 3.75 \times \frac{1}{10^3} = 0.00375$$

EXERCISE 2

Write each number in standard form.

1 ▶ 0.1
2 ▶ 0.01
3 ▶ 0.001
4 ▶ 0.0001
5 ▶ $\frac{1}{1000}$
6 ▶ $\frac{1}{100}$
7 ▶ 10
8 ▶ 1

Write each number as an ordinary number.

9 ▶ 10^{-3}
10 ▶ 10^{-5}
11 ▶ 1.2×10^{-3}
12 ▶ 8.7×10^{-1}
13 ▶ 10^{-6}
14 ▶ 10^{-4}
15 ▶ 4.67×10^{-2}
16 ▶ 3.4×10^{-4}

Write each number in standard form.

17 ▶ 0.543
18 ▶ 0.0708
19 ▶ 0.007
20 ▶ 0.0009
21 ▶ 0.67
22 ▶ 0.000 707
23 ▶ 100
24 ▶ 1000

Write each as an ordinary number.

25 ▶ $10^{-2}\times10^{4}$ **27** ▶ $10^{2}\div10^{-2}$ **29** ▶ $(3.2\times10^{-2})\times(4\times10^{3})$

26 ▶ $10^{3}\times10^{-1}$ **28** ▶ $10^{3}\div10^{-3}$ **30** ▶ $(2.4\times10^{-2})\div(8\times10^{-1})$

EXERCISE 2*

8

Write each as an ordinary number.

1 ▶ $10^{3}\times10^{-2}$ **3** ▶ $10^{-2}+10^{-3}$ **5** ▶ $10^{-4}\times10^{2}$ **7** ▶ $10^{-3}+10^{-4}$

2 ▶ $10^{-1}\times10^{-2}$ **4** ▶ $10^{-1}-10^{-3}$ **6** ▶ $10^{-3}\times10^{-1}$ **8** ▶ $10^{-3}-10^{-1}$

Write each number in standard form.

9 ▶ $10\div10^{-2}$ **11** ▶ $10^{-1}\div10^{-2}$ **13** ▶ $10^{3}\div10^{-1}$ **15** ▶ $10^{-2}\div10^{-4}$

10 ▶ $10^{2}\div10^{-2}$ **12** ▶ $10^{-4}\div10^{-3}$ **14** ▶ $10^{-1}\div10^{3}$ **16** ▶ $10^{-5}\div10^{-2}$

9

Write each number in standard form.

17 ▶ $(4\times10^{2})^{-2}$ **21** ▶ $(5\times10^{2})^{-2}$

18 ▶ $(4\times10^{-2})^{2}$ **22** ▶ $(5\times10^{-2})^{2}$

19 ▶ $(6.9\times10^{3})\div10^{-4}$ **23** ▶ $(4.8\times10^{2})\div10^{-3}$

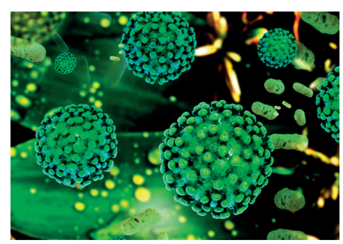
20 ▶ $10^{-3}\div(2\times10^{-2})$ **24** ▶ $10^{-2}\div(5\times10^{-3})$

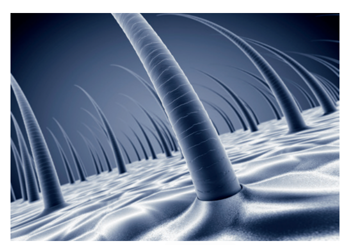
You will need this information to answer Questions 25 and 26.

Cough virus 9.144×10^{-6} mm **diameter**

Human hair 5×10^{-2} mm diameter

Pin 6×10^{-1} mm diameter





25 ▶ How many viruses, to the nearest thousand, can be placed in a straight line across the width of a human hair?

26 ▶ How many viruses, to the nearest thousand, can be placed in a straight line across the width of a pin?

27 ▶ The **radius** of the nucleus of a hydrogen atom is 1×10^{-12} mm. How many hydrogen atoms would fit in a straight line across a human hair of diameter 0.06 mm?

28 ▶ The average mass of a grain of sand is 10^{-4} g. How many grains of sand are there in 2 kg?

29 ▶ Find a sensible method to work out $(3.4 \times 10^{23}) + (3.4 \times 10^{22})$ without a calculator.

10 **30 ▶** A molecule of water is a very small thing, so small that its volume is 10^{-27} m³.

a How many molecules are there in 1 m³ of water?
If you wrote your answer in full, how many zero digits would there be?

b If you assume that a water molecule is in the form of a **cube**, show that its side length is 10^{-9} m.

c If a number of water molecules were placed touching each other in a straight line, how many would there be in a line 1 cm long?

d The volume of a cup is 200 cm³.
How many molecules of water would the cup hold?

e If all the molecules in the cup were placed end to end in a straight line, how long would the line be?

f Take the **circumference** of the Earth to be 40 000 km.
How many times would the line of molecules go around the Earth?

PERCENTAGES

Percentages are numbers without a dimension that help us make fast judgements. Values are scaled to be out of 100. Percentages appear frequently in everyday life. They can be used to compare quantities and work out a percentage change such as profit or loss.

x AS A PERCENTAGE OF y

EXAMPLE 6

SKILL: REASONING

Calculate $5 as a percentage of $80.

Express the **ratio** as a fraction and multiply by 100.

\$5 as a percentage of \$80 = $\frac{5}{80} \times 100 = 6.25\%$

KEY POINT

- To calculate x as a percentage of y: $\frac{x}{y} \times 100$

x PERCENT OF y

EXAMPLE 7

SKILL: REASONING

Calculate 5% of 80 kg.

1% of 80 kg = $\frac{80}{100}$ so 5% = $5 \times \frac{80}{100} = 80 \times \frac{5}{100} = 80 \times 0.05 = 4$ kg

KEY POINT

- To calculate x percent of y: 1% of $y = \frac{y}{100}$ so x% of $y = x \times \frac{y}{100} = y \times \left(\frac{x}{100}\right)$

  The $\left(\frac{x}{100}\right)$ part of the last expression is the **multiplying factor**.

  5% of a quantity can be found by using a multiplying factor of 0.05.

  95% of a quantity can be found by using a multiplying factor of 0.95 and so on.

PERCENTAGE CHANGE

EXAMPLE 8

SKILL: REASONING

Olive measures Salma's height as 95 cm. Some time later she measures her height as 1.14 m.

Work out the percentage increase in Salma's height.

$$\text{Percentage change} = \frac{\text{value of change}}{\text{original value}} \times 100 = \frac{114 - 95}{95} \times 100 = +20\%$$

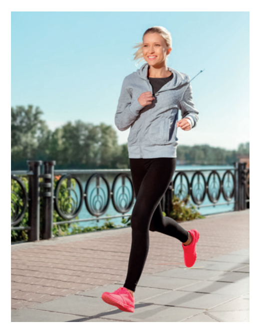
Salma's height has changed by +20%.

To compare units it is necessary to be consistent. In the above example, centimetres were the units used.

EXAMPLE 9

SKILL: REASONING

Kerry improves her 400 m running time from 72 s to 63 s.



What was Kerry's percentage improvement?

$$\text{Percentage change} = \frac{\text{value of change}}{\text{original value}} \times 100 = \frac{72 - 63}{72} \times 100 = -12.5\%$$

Kerry's time has changed by -12.5%.

KEY POINT

- Percentage change = $\frac{\text{value of change}}{\text{original value}} \times 100$

EXERCISE 3

8

1 ▶ Find €12 as a percentage of €60.

2 ▶ Find 15 km as a percentage of 120 km.

3 ▶ Find $180 as a percentage of $3600.

4 ▶ Find 2500 kg as a percentage of 62 500 kg.

5 ▶ Find 5% of 110 km/h.

6 ▶ Find 15% of 80°C.

7 ▶ Find 30% of 420 m^2.

8 ▶ Find 70% of 24 hrs.

7

9 ▶ Pavel's pocket money increases from €12 per week to €15 per week. Work out the percentage increase in his pocket money.

10 ▶ India's swimming time decreases from 32 s to 24 s. Work out the percentage decrease in her time.

EXERCISE 3*

6

1 ▶ Find 175p as a percentage of £35.

2 ▶ Find 2.5 km as a percentage of 15 000 m.

3 ▶ Find $25 000 as a percentage of $1 million.

4 ▶ Find 375 g as a percentage of 15 kg.

5 ▶ Find 15% of the area of a square of side 12 cm.

6 ▶ Find 85% of the volume of a cube of side 12 cm.

7

7 ▶ Find 2.5% of 10% of 1×10^6 m^3.

8 ▶ Find 90% of 36% of 2.5×10^3 db (decibels).

9 ▶ What is the percentage error in using $\frac{22}{7}$ as an approximation to π ?

10 ▶ Find the percentage change in the 100 m sprint World Records for the

a Men's record since 1891

b Women's record since 1922.

MEN'S 100 m SPRINT WORLD RECORD		
Year	Time	Holder
1891	10.80 s	Cary, USA
2009	9.58 s	Bolt, Jamaica

WOMEN'S 100 m SPRINT WORLD RECORD		
Year	Time	Holder
1922	13.60 s	Mejzlikova, Czechoslovakia
1988	10.49 s	Griffith-Joyner, USA

PERCENTAGE INCREASE AND DECREASE

To increase a value by $R\%$ it is necessary to have the original value plus $R\%$.

Therefore, we multiply it by a **factor** of $(1 + \frac{R}{100})$.

EXAMPLE 10

SKILL: REASONING

In 2015, the Kingda Ka Roller Coaster at Six Flags (USA) had the largest vertical drop of 139 m. If the designers want to increase this height by 12%, what will the new height be?

New height = original height × $(1 + \frac{12}{100})$ = 139 × 1.12 = 155.68 m

To decrease a value by $R\%$ it is necessary to have the original value minus $R\%$.

Therefore, we multiply it by a factor of $(1 - \frac{R}{100})$.

EXAMPLE 11

SKILL: REASONING

In 2015, the world record for the 100 m swimming butterfly in the female Paralympian S12 class was held by Joanna Mendak (Poland) with a time of 65.1 secs.

If this world record is reduced by 5%, what will the new time be?

New time = original time × $(1 - \frac{5}{100})$ = 65.1 × 0.95 = 61.845 s = 61.85 s (2 d.p.)

Note: this is the same calculation as finding 95% of the original time, so reducing a quantity by 25% is the same as finding 75% of the value and so on.

KEY POINTS

- To increase a quantity by $R\%$, multiply it by $1 + \frac{R}{100}$
- To decrease a quantity by $R\%$, multiply it by $1 - \frac{R}{100}$

PERCENTAGE CHANGE	MULTIPLYING FACTOR
+25%	1.25
+75%	1.75
–25%	0.75
–75%	0.25

PERCENTAGE INCREASE AND DECREASE

If a quantity gains value over time it has **appreciated** or gone through an inflation. It can happen for a number of reasons, often a ***greater demand*** or a ***smaller supply*** can push prices up. Houses, rare antiques and rare minerals are typical examples.

If a quantity loses value over time it has **depreciated** or gone through a deflation. It can happen for a number of reasons, often a ***smaller demand*** or a ***greater supply*** can push prices down. Cars, oil and some toys are typical examples.

EXERCISE 4

6

1 ▶ Copy and complete the following table.

ORIGINAL VALUE	PERCENTAGE INCREASE	MULTIPLYING FACTOR	NEW VALUE
20	5		
180	95		
360		1.30	
2500		1.70	

2 ▶ Copy and complete the following table.

ORIGINAL VALUE	PERCENTAGE DECREASE	MULTIPLYING FACTOR	NEW VALUE
20	5		
180	95		
360		0.70	
2500		0.30	

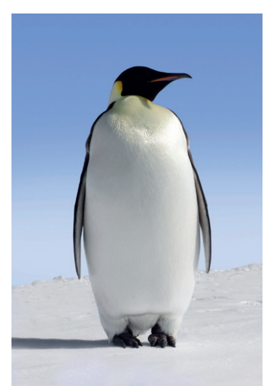
3 ▶ Increase $1500 by

a 1% **b** 99% **c** 10% **d** 90%

4 ▶ Decrease 500 kg by

a 1% **b** 99% **c** 10% **d** 90%

5 ▶ An Emperor Penguin weighs 40 kg and gains 70% of its weight before losing its feathers so that it can survive the extreme temperatures of Antarctica. Find the penguin's weight just before it loses its feathers.



6 ▶ A bottlenose dolphin weighs 650 kg while carrying its baby calf. After it gives birth to the calf its weight is reduced by 4%. Find the dolphin's weight just after giving birth.

7 ▶ Madewa pays $12 000 into an investment and it appreciates by 12% after one year. Find the value of Madewa's investment after a year.

8 ▶ Iris buys a new car for $45 000 and it depreciates by 12% after one year. Find the value of Iris' car after a year.

9 ▶ A rare sculpture is worth €120 000 and appreciates by 8% p.a. Find the value of the sculpture after one year.

10 ▶ A rare stamp is worth €2500 and depreciates by 8% p.a. Find the value of the stamp after one year.

EXERCISE 4*

6

1 ▶ Copy and complete the following table.

ORIGINAL VALUE	PERCENTAGE INCREASE	MULTIPLYING FACTOR	NEW VALUE
60 secs			75 secs
50 kg			80 kg
		1.25	125 km/h
	20		1500 m

2 ▶ Copy and complete the following table.

ORIGINAL VALUE	PERCENTAGE DECREASE	MULTIPLYING FACTOR	NEW VALUE
75 secs			60 secs
80 kg			50 kg
120 km/h		0.60	
1500 m	20		

3 ▶ A $24 box of luxury chocolates is sold in Canada where the inflation **rate** is 2% p.a. Find the new price of these chocolates in Canada after a year.

4 ▶ The cost of oil is $45 per barrel (a standard unit) and the price goes through a deflation rate of 12% p.a. Find the new price of a barrel after one year.

7

5 ▶ A Persian rug is worth £5750. It goes through an increase of 5% followed by a second increase of 12%. Find the price of the rug after the second increase.

6 ▶ A super-size hi-definition TV costs £7500. It goes through a decrease of 10% followed by a second decrease of 12% in the sales. Find the price of the TV after the second decrease.

7 ▶ The temperature in Doha, Qatar on 1 June is 40°C. Over the next two days this temperature increases by 10% followed by a decrease of 10%. Find the temperature in Doha on 3 June.

8 ▶ A loud clap of thunder is measured at a noise level of 120 decibels (the unit for measuring sound). The next two thunderclaps register a decrease of 20% followed by a 25% increase in noise level. How loud, in decibels, is the third thunderclap?

9 ▶ A circular drop of oil has a radius of 10 cm. If this radius increases by 5% then by 10% and finally by 15%, find the new area of the circle. (Area of circle $A = \pi r^2$)

10 ▶ A circular drop of oil has a diameter of 10 cm. If this diameter decreases by 5% then by 10% and finally by 15%, find the new circumference of the circle. (Circumference of circle = $2\pi r$)

EXERCISE 5

6 → 8

REVISION

1 ▶ Write 275 000 in standard form.

2 ▶ Write 0.0275 in standard form.

3 ▶ Write 3.5×10^3 as an ordinary number.

4 ▶ Write 3.5×10^{-3} as an ordinary number.

5 ▶ Find 18% of $360 000.

6 ▶ Write 240 m as a percentage of 12 000 m.

7 ▶ Luke's salary changes from €75 000 p.a. to €100 000 p.a. Find the percentage increase in Luke's salary.

8 ▶ Mari's watch gains 3 minutes every hour. Find the percentage error in Mari's watch at the end of one hour.

9 ▶ Increase $350 by 17.5%.

10 ▶ Decrease $350 by 17.5%.

EXERCISE 5*

6 → 9

REVISION

1 ▶ Write $(4.5 \times 10^3) \times (5 \times 10^3)$ as an ordinary number.

2 ▶ Write 0.1 + 0.02 + 0.003 in standard form.

3 ▶ Write $5.3 \times 10^4 + 5.3 \times 10^3$ as an ordinary number.

4 ▶ Write $\dfrac{2.5 \times 10^3 \times 6 \times 10^2}{3 \times 10^{-6}}$ in standard form.

5 ▶ Find 15% of the **perimeter** of a square of area 1024 m^2.

6 ▶ Write a time of 1 second as a percentage of 1 day. Express your answer in standard form to 3 s.f.

7 ▶ When Fredrick buys a cup of coffee he is given change of €1.65 when he should have received €1.50. Find the percentage error.

8 ▶ Find the percentage error in x when it is estimated to be y and $y > x$.

9 ▶ Erika's toy ski chalet is valued at €450. Its value increases by 10% then decreases by 10% the year after. What is the value of Erika's toy after these two changes?

10 ▶ Akintade makes the following purchases and sales:

a He buys a jewel for \$180, then sells it for \$216. Find his percentage profit.

b He buys a toy car for \$150, then sells it for \$120. Find his percentage loss.

EXAM PRACTICE: NUMBER 2

1 Write the following numbers in standard form.

a 4500
b 3 million
c 0.0075
d a quarter **[4]**

2 Write the following as ordinary numbers.

a 1.2×10^3
b 5.8×10^6
c 4.5×10^{-1}
d 9.3×10^{-3} **[4]**

3 Write the following in standard form to 3 s.f.

a $(2.5 \times 10^2) \times (1.7 \times 10^5)$
b $\frac{7.3\times10^6}{2.1\times10^3}$
c $(7.3 \times 10^5) + (7.3 \times 10^4)$ **[6]**

4 The human body contains about 60% water. How many kg of water are contained in a 75 kg man? **[2]**

5 Between 2010 and 2015 the human population of India grew from 1.21×10^9 to 1.29×10^9. The world population in 2015 was 7.39 billion.

Find the percentage

a of the world population that lived in India in 2015
b change in the Indian population from 2010 to 2015. **[4]**

6 A square has its side length increased by 10%. Find the percentage increase in the area of the square. **[2]**

7 The Womens' World Record Marathon time has improved by 34.82% from Dale Grieg's (UK) time of 3 hrs 27 mins 45 s in 1964 to Paula Radcliffe's (UK) time in 2003. Find Paula Radcliffe's World Record time. **[3]**

[Total 25 marks]

CHAPTER SUMMARY: NUMBER 2

STANDARD FORM

Standard form is used to express large and small numbers more efficiently.

A number in standard form looks like this:

2.5×10^6

This part (2.5) is written as a number between 1 and 10. This part (10^6) is written as a power of 10.

For negative powers of 10: $10^{-n} = \frac{1}{10^n}$

It is always written as $a \times 10^b$, where $1 \le a < 10$ and b is an integer which can be positive or negative.

$1000 = 1 \times 10^3$, $0.001 = 1 \times 10^{-3}$ are two numbers written in standard form.

$10^m \times 10^n = 10^{m+n}$
$10^m \div 10^n = 10^{m-n}$
$(10^m)^n = 10^{mn}$

PERCENTAGES

To calculate x as a percentage of y: $\frac{x}{y} \times 100$

To calculate x percent of y:

1% of $y = \frac{y}{100}$ so $x\%$ of $y = x \times \frac{y}{100} \quad = y \times \left(\frac{x}{100}\right)$

The $\left(\frac{x}{100}\right)$ part of the last expression is the multiplying factor.

5% of a quantity can be found by using a multiplying factor of 0.05.

95% of a quantity can be found by using a multiplying factor of 0.95 and so on.

$1\% = \frac{1}{100} = 0.01$ $\qquad 10\% = \frac{10}{100} = \frac{1}{10} = 0.1$

$50\% = \frac{50}{100} = \frac{1}{2} = 0.5$ $\qquad 75\% = \frac{75}{100} = \frac{3}{4} = 0.75$

PERCENTAGE CHANGE

$$\text{Percentage change} = \frac{\text{value of change}}{\text{original value}} \times 100$$

Per annum (p.a.) is frequently used and means per year.

PERCENTAGE INCREASE AND DECREASE

To increase a quantity by $R\%$, multiply it by $1 + \frac{R}{100}$

To decrease a quantity by $R\%$, multiply it by $1 - \frac{R}{100}$

PERCENTAGE CHANGE	MULTIPLYING FACTOR
+5%	1.05
+95%	1.95
–5%	0.95
–95%	0.05

ALGEBRA 2

You might think that 9999 is the largest number that can be written using just four digits, however, we can write much larger numbers using index notation. A 15-year-old person has been alive for about 5×10^8 seconds, the universe is about 10^{17} seconds old and the number of atoms in the observable universe has been estimated at 10^{80}. It is amazing that four digits can represent such an incredibly large number!

LEARNING OBJECTIVES

- Multiply and divide algebraic fractions
- Add and subtract algebraic fractions
- Solve equations with roots and powers
- Use the rules of indices (to simplify algebraic expressions)
- Solve inequalities and show the solution on a number line

BASIC PRINCIPLES

- Simplifying number fractions: $\frac{9}{12} = \frac{3}{4}$, $\frac{2}{3} \div \frac{1}{3} = \frac{2}{3} \times \frac{3}{1} = 2$, $\frac{2}{3} + \frac{1}{4} = \frac{8+3}{12} = \frac{11}{12}$
- Solving equations means doing the same to both sides to get the unknown on one side by itself.
- $10^4 = 10 \times 10 \times 10 \times 10$
- $x < y$ means 'x is less than y' or 'y is greater than x'.
- $x \geq y$ means 'x is greater than or equal to y' or 'y is less than or equal to x'.

SIMPLIFYING ALGEBRAIC FRACTIONS

Algebraic fractions are simplified in the same way as number fractions.

MULTIPLICATION AND DIVISION

EXAMPLE 1

Simplify $\frac{4x}{6x}$

$$\frac{^{2}\cancel{4}x}{_{3}\cancel{6}x} = \frac{2\cancel{x}^{1}}{3\cancel{x}_{1}} = \frac{2}{3}$$

EXAMPLE 2

Simplify $\frac{3x^2}{6x}$

$$\frac{3x^2}{6x} = \frac{^{1}\cancel{3} \times x \times {}^{1}\cancel{x}}{_{2}\cancel{6} \times {}_{1}\cancel{x}} = \frac{x}{2}$$

EXAMPLE 3

Simplify $(27xy^2) \div (60x)$

$$(27xy^2) \div (60x) = \frac{27xy^2}{60x} = \frac{^{9}\cancel{27} \times {}^{1}\cancel{x} \times y \times y}{_{20}\cancel{60} \times {}_{1}\cancel{x}} = \frac{9y^2}{20}$$

EXERCISE 1

7 → 8

Simplify these.

1 ▶ $\frac{4x}{x}$

2 ▶ $\frac{6y}{2}$

3 ▶ $(6x)\div(3x)$

4 ▶ $\frac{12a}{4b}$

5 ▶ $\frac{3ab}{6a}$

6 ▶ $(9a)\div(3b)$

7 ▶ $\frac{12c^2}{3c}$

8 ▶ $\frac{4a^2}{8a}$

9 ▶ $\frac{12x}{3x^2}$

10 ▶ $\frac{8ab^2}{4ab}$

11 ▶ $\frac{3a}{15ab^2}$

12 ▶ $(3a^2b^2)\div(12ab^2)$

EXERCISE 1*

7 → 8

Simplify these.

1 ▶ $\frac{5y}{10y}$

2 ▶ $\frac{12a}{6ab}$

3 ▶ $(3xy)\div(12y)$

4 ▶ $\frac{3a^2}{6a}$

5 ▶ $\frac{10b}{5b^2}$

6 ▶ $(18a)\div(3ab^2)$

7 ▶ $\frac{3a^2b^2}{6ab^3}$

8 ▶ $\frac{15abc}{5a^2b^2c^2}$

9 ▶ $(3a^2)\div(12ab^2)$

10 ▶ $\frac{abc^3}{(abc)^3}$

11 ▶ $\frac{150a^3b^2}{400a^2b^3}$

12 ▶ $\frac{45x^3y^4z^5}{150x^5y^4z^3}$

EXAMPLE 4

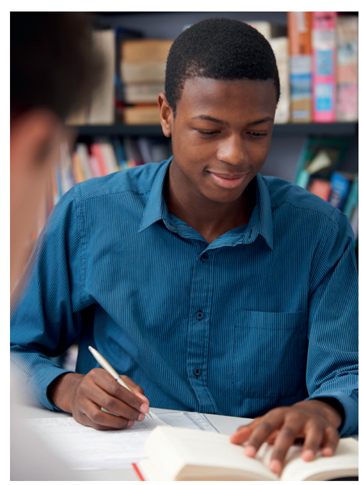
Simplify $\frac{3x^2}{y}\times\frac{y^3}{x}$

$$\frac{3x^2}{y}\times\frac{y^3}{x}=\frac{3\times x\times {}^1\cancel{x}}{{}_1\cancel{y}}\times\frac{{}^1\cancel{y}\times y\times y}{{}_1\cancel{x}}=3xy^2$$

EXAMPLE 5

Simplify $\frac{2x^2}{y}\div\frac{2x}{5y^3}$

$$\frac{2x^2}{y}\div\frac{2x}{5y^3}=\frac{{}^1\cancel{2}\times x\times {}^1\cancel{x}}{{}_1\cancel{y}}\times\frac{5\times {}^1\cancel{y}\times y\times y}{{}_1\cancel{2}\times {}_1\cancel{x}}=5xy^2$$



KEY POINT

- To divide by a fraction, turn the fraction upside down and multiply.

$$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc}$$

EXERCISE 2

Simplify these.

1 ▶ $\frac{3x}{4} \times \frac{5x}{3}$

2 ▶ $\frac{x^2y}{z} \times \frac{xz^2}{y^2}$

3 ▶ $\frac{x^2}{y} \times \frac{z}{x^2} \times \frac{y}{z}$

4 ▶ $\frac{4c \times 7c^2}{7 \times 5c}$

5 ▶ $\frac{3x}{4} \div \frac{x}{8}$

6 ▶ $4 \div \frac{8}{ab}$

7 ▶ $\frac{2b}{3} \div 4$

8 ▶ $\frac{2x}{3} \div \frac{2x}{3}$

9 ▶ $\frac{2x}{y^2} \div \frac{x}{y}$

10 ▶ $\frac{5ab}{c^2} \div \frac{10a}{c}$

EXERCISE 2*

Simplify these.

1 ▶ $\frac{4a}{3} \times \frac{5a}{2} \times \frac{3a}{5}$

2 ▶ $\frac{3x^2y}{z^3} \times \frac{z^2}{xy}$

3 ▶ $\frac{45}{50} \times \frac{p^2}{q} \times \frac{q^3}{p}$

4 ▶ $\frac{3x}{y} \div \frac{6x^2}{y}$

5 ▶ $\frac{15x^2y}{z} \div \frac{3xz}{y^2}$

6 ▶ $\frac{2x}{y} \times \frac{3y}{4x} \times \frac{2y}{3}$

7 ▶ $\left(\frac{x}{2y}\right)^3 \times \frac{2x}{3} \div \frac{2}{9y^2}$

8 ▶ $\frac{\sqrt{a^3b^2}}{6a^3} \times \frac{3a^5b}{(a^3b^2)^2} \div \frac{ab}{\sqrt{a^3b^2}}$

ADDITION AND SUBTRACTION

EXAMPLE 6

Simplify $\frac{a}{4} + \frac{b}{5}$

$$\frac{a}{4} + \frac{b}{5} = \frac{5a + 4b}{20}$$

EXAMPLE 7

Simplify $\frac{3x}{5} - \frac{x}{3}$

$$\frac{3x}{5} - \frac{x}{3} = \frac{9x - 5x}{15} = \frac{4x}{15}$$

EXAMPLE 8

Simplify $\frac{2}{3b} + \frac{1}{2b}$

$$\frac{2}{3b} + \frac{1}{2b} = \frac{4 + 3}{6b} = \frac{7}{6b}$$

EXAMPLE 9

Simplify $\frac{3 + x}{7} - \frac{x - 2}{3}$

$$\frac{3 + x}{7} - \frac{x - 2}{3} = \frac{3(3 + x) - 7(x - 2)}{21} = \frac{9 + 3x - 7x + 14}{21} = \frac{23 - 4x}{21}$$

Remember to use brackets here. Note **sign** change.

EXERCISE 3

Simplify these.

1 ▶ $\frac{x}{3}+\frac{x}{4}$

2 ▶ $\frac{a}{3}-\frac{a}{4}$

3 ▶ $\frac{a}{3}+\frac{b}{4}$

4 ▶ $\frac{2x}{3}-\frac{x}{4}$

5 ▶ $\frac{2a}{7}+\frac{3a}{14}$

6 ▶ $\frac{a}{4}+\frac{b}{3}$

7 ▶ $\frac{2a}{3}-\frac{a}{2}$

8 ▶ $\frac{a}{4}+\frac{2b}{3}$

EXERCISE 3*

Simplify these.

1 ▶ $\frac{x}{6}+\frac{2x}{9}$

2 ▶ $\frac{2a}{3}-\frac{3a}{7}$

3 ▶ $\frac{2x}{5}+\frac{4y}{7}$

4 ▶ $\frac{3a}{4}+\frac{a}{3}-\frac{5a}{6}$

5 ▶ $\frac{3}{2b}+\frac{4}{3b}$

6 ▶ $\frac{2}{d}+\frac{3}{d^2}$

7 ▶ $\frac{2-x}{5}+\frac{3-x}{10}$

8 ▶ $\frac{y+3}{5}-\frac{y+4}{6}$

9 ▶ $\frac{x-3}{3}+\frac{x+5}{4}-\frac{2x-1}{6}$

10 ▶ $\frac{a}{a-1}-\frac{a-1}{a}$

SOLVING EQUATIONS WITH ROOTS AND POWERS

EXAMPLE 10

Solve $3x^2 + 4 = 52$.

$3x^2 + 4 = 52$ (Subtract 4 from both sides)

$3x^2 = 48$ (Divide both sides by 3)

$x^2 = 16$ (Square root both sides)

$x = \pm 4$

Check: $3 \times 16 + 4 = 52$

Note: –4 is also an answer because $(-4) \times (-4) = 16$.

EXAMPLE 11

Solve $5\sqrt{x} = 50$.

$5\sqrt{x} = 50$ (Divide both sides by 5)

$\sqrt{x} = 10$ (Square both sides)

$x = 100$

Check: $5 \times \sqrt{100} = 50$

EXAMPLE 12

Solve $\frac{\sqrt{x+5}}{3} = 1$.

$\frac{\sqrt{x+5}}{3} = 1$ (Multiply both sides by 3)

$\sqrt{x+5} = 3$ (Square both sides)

$x + 5 = 9$ (Subtract 5 from both sides)

$x = 4$

Check: $\frac{\sqrt{4+5}}{3} = 1$

KEY POINT

- To solve equations, do the same operations to both sides.

EXERCISE 4

Solve these equations.

1 ▶ $4x^2 = 36$

2 ▶ $\frac{x^2}{3} = 12$

3 ▶ $x^2 + 5 = 21$

4 ▶ $\frac{x^2}{2} + 5 = 37$

5 ▶ $2x^2 + 5 = 23$

6 ▶ $5x^2 - 7 = -2$

7 ▶ $\frac{x+12}{5} = 5$

8 ▶ $\frac{x^2+4}{5} = 4$

9 ▶ $\sqrt{x} + 27 = 31$

10 ▶ $4\sqrt{x} + 4 = 40$

EXERCISE 4*

Solve these equations.

1 ▶ $4x^2 + 26 = 126$

2 ▶ $\frac{x^2}{7} - 3 = 4$

3 ▶ $\frac{x^2 - 11}{7} = 10$

4 ▶ $1 = \frac{\sqrt{x+4}}{2}$

5 ▶ $\sqrt{\frac{x-3}{4}} + 5 = 6$

6 ▶ $\frac{40 - 2x^2}{2} = 4$

7 ▶ $22 = 32 - \frac{2x^2}{5}$

8 ▶ $(3+x)^2 = 169$

9 ▶ $\sqrt{\frac{3x^2+5}{2}} + 4 = 8$

10 ▶ $\sqrt{3 + \frac{\left(4+\sqrt{x+3}\right)^2}{6}} = 3$

POSITIVE INTEGER INDICES

$10 \times 10 \times 10 \times 10$ is written in a shorter form as 10^4. In the same way, $a \times a \times a \times a$ is written as a^4.

To help you to understand how the rules of indices work, look carefully at these examples.

KEY POINTS

OPERATION	EXAMPLE	RULES
Multiplying	$a^4 \times a^2 = (a \times a \times a \times a) \times (a \times a) = a^6 = a^{4+2}$	Add the indices ($a^m \times a^n = a^{m+n}$)
Dividing	$a^4 \div a^2 = \dfrac{a \times a \times a \times a}{a \times a} = a^2 = a^{4-2}$	Subtract the indices ($a^m \div a^n = a^{m-n}$)
Raising to a **power**	$(a^4)^2 = (a \times a \times a \times a) \times (a \times a \times a \times a) = a^8 = a^{4\times 2}$	Multiply the indices $(a^m)^n = a^{mn}$

EXAMPLE 13

Use the rules of indices to simplify $6^3 \times 6^4$. Then use your calculator to check the answer.

$6^3 \times 6^4 = 6^7 = 279\,936$ (Add the indices)

6 $x^{\blacksquare}$ 7 =

EXAMPLE 14

Simplify $9^5 \div 9^2$.

$9^5 \div 9^2 = 9^3 = 729$ (Subtract the indices)

9 $x^{\blacksquare}$ 3 =

EXAMPLE 15

Simplify $(4^2)^5 = 4^{10}$.

$(4^2)^5 = 4^{10} = 1\,048\,576$ (Multiply the indices)

4 $x^{\blacksquare}$ 1 0 =

Some answers become very large after only a few multiplications.

EXERCISE 5

Use the rules of **indices** to simplify these. Then use your calculator to calculate the answer.

6

1 ▶ $2^4 \times 2^6$

2 ▶ $4^3 \times 4^4$

3 ▶ $2^{10} \div 2^4$

4 ▶ $\dfrac{7^{13}}{7^{10}}$

5 ▶ $(2^3)^4$

6 ▶ $(6^2)^4$

Use the rules of indices to simplify these.

7 ▶ $a^3 \times a^2$

8 ▶ $c^6 \div c^2$

9 ▶ $(e^2)^3$

10 ▶ $a^2 \times a^3 \times a^4$

11 ▶ $\dfrac{c^8}{c^3}$

12 ▶ $2 \times 6 \times a^4 \times a^2$

13 ▶ $2a^3 \times 3a^2$

14 ▶ $2(e^4)^2$

EXERCISE 5*

Use the rules of indices to simplify these. Then use your calculator to calculate the answer. Give your answers **correct** to 3 **significant figures** and in **standard form**.

1 ▶ $6^6 \times 6^6$

2 ▶ $7^{12} \div 7^6$

3 ▶ $(8^3)^4$

4 ▶ $4(4^4)^4$

Use the rules of indices to simplify these.

5 ▶ $a^5 \times a^3 \times a^4$

6 ▶ $(12c^9) \div (4c^3)$

7 ▶ $5(e^2)^4$

8 ▶ $(2g^4)^3$

9 ▶ $3(2j^3)^4$

10 ▶ $3m(2m^2)^3$

11 ▶ $3a^2(3a^2)^2$

12 ▶ $\dfrac{2a^8 + 2a^8}{2a^8}$

13 ▶ $\dfrac{12b^8}{6b^4} + 6b^4$

14 ▶ $\dfrac{b^4 + b^4 + b^4 + b^4 + b^4 + b^4}{b^4}$

INEQUALITIES

NUMBER LINES

EXAMPLE 16

These are examples of how to show **inequalities** on a number line.

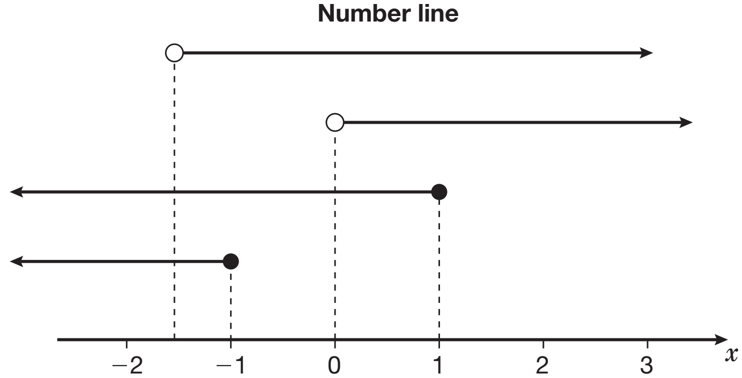
Inequality | **Number line** | **Integer solutions**

$x > -1.5$ — $\{-1, 0, 1, 2, \ldots\}$

$x > 0$ — $\{1, 2, 3, 4, \ldots\}$

$x \leqslant 1$ — $\{1, 0, -1, -2, \ldots\}$

$x \leqslant -1$ — $\{-1, -2, -3, -4, \ldots\}$



SOLVING LINEAR INEQUALITIES

Inequalities are solved in the same way as algebraic equations, EXCEPT that when multiplying or dividing by a negative number, the inequality sign is reversed.

EXAMPLE 17

Solve the inequality $4 < x \leq 10$. Show the result on a number line.

$4 < x \leq 10$ (Split the inequality into two parts)

$4 < x$ and $x \leq 10$

$x > 4$ and $x \leq 10$

Note: x cannot be equal to 4.

EXAMPLE 18

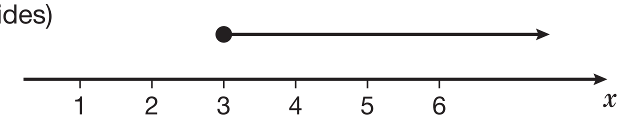
Solve the inequality $4 \geq 13 - 3x$. Show the result on a number line.

$4 \geq 13 - 3x$ (Add $3x$ to both sides)

$3x + 4 \geq 13$ (Subtract 4 from both sides)

$3x \geq 9$ (Divide both sides by 3)

$x \geq 3$



EXAMPLE 19

Solve the inequality $5 - 3x < 1$. List the four smallest **integers** in the solution set.

$5 - 3x < 1$ (Subtract 5 from both sides)

$-3x < -4$ (Divide both sides by −3, so reverse the inequality sign)

$x > \frac{-4}{-3}$

$x > 1\frac{1}{3}$

So the four smallest integers are 2, 3, 4 and 5.

EXAMPLE 20

Solve the inequality $x \leq 5x + 1 < 4x + 5$. Show the inequality on a number line.

$x \leq 5x + 1 < 4x + 5$ (Split the inequality into two parts)

a $x \leq 5x + 1$ (Subtract $5x$ from both sides)

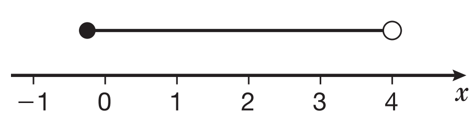
$-4x \leq 1$ (Divide both sides by −4, so reverse the inequality sign)

$x \geq -\frac{1}{4}$

b $5x + 1 < 4x + 5$ (Subtract $4x$ from both sides)

$x + 1 < 5$ (Subtract 1 from both sides)

$x < 4$



KEY POINTS

- $x > 4$ means that x cannot be equal to 4.
- $x \geq 4$ means that x can be equal to 4 or greater than 4.
- When finding the solution set of an inequality:
  Collect up the algebraic term on one side.
  When multiplying or dividing both sides by a negative number, reverse the inequality sign.

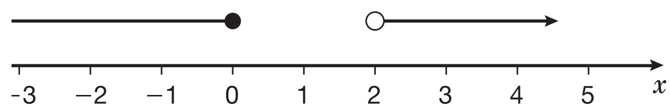
EXERCISE 6

5 → 8

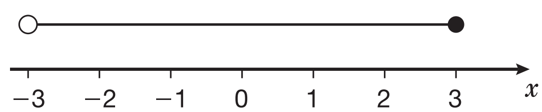
Insert the correct symbol, <, > or =.

1 ▶ $-3 \square 3$ **2 ▶** $30\% \square \frac{1}{3}$ **3 ▶** $-3 \square -4$ **4 ▶** $0.3 \square \frac{1}{3}$

5 ▶ Write down the inequalities represented by this number line.



6 ▶ Write down the single inequality represented by this number line.



Solve the inequality, and show the result on a number line.

7 ▶ $x - 3 > 2$

8 ▶ $x - 3 \leq 1$

9 ▶ $4 < 7 - x$

10 ▶ $10 \geq 13 - 2x$

11 ▶ $4x \geq 3x + 9$

12 ▶ $6x + 3 < 2x + 19$

13 ▶ $2(x + 3) < x + 6$

14 ▶ $5(x - 1) > 2(x + 2)$

Solve these inequalities.

15 ▶ $3 > x + 5$

16 ▶ $-2x \leq 10$

17 ▶ $3 > 2x + 5$

18 ▶ $x - 4 \geq 3x$

19 ▶ $2(x - 1) \leq 5x$

20 ▶ $2(x - 3) \leq 5(x + 3)$

Solve these inequalities. List the integers in each solution set.

21 ▶ $4 < x \leq 6$

22 ▶ $2 < x \leq 4.5$

23 ▶ $-1 < x \leq 1.5$

24 ▶ $2 \leq 2x < x + 5$

25 ▶ $4 < 2x + 1 \leq 7$

EXERCISE 6*

1 ▶ Write down the inequalities represented by this number line.

Explain why your two answers cannot be combined into a single inequality.

Solve the inequality and show the result on a number line.

2 ▶ $3x \le x + 5$

3 ▶ $5x + 3 < 2x + 19$

4 ▶ $3(x + 3) < x + 12$

5 ▶ $2(x - 1) > 7(x + 2)$

6 ▶ $\frac{x}{2} - 3 \ge 3x - 8$

7 ▶ $-7 < 3x - 2 \le 11$

8 ▶ $x < 2x + 1 \le 7$

9 ▶ Find the largest **prime number** y that **satisfies** $4y \le 103$.

10 ▶ List the integers that satisfy both the inequalities.

$-3 \le x < 4$ and $x > 0$

11 ▶ Solve the inequality, then list the four largest integers in the solution set.

$$\frac{x+1}{4} \ge \frac{x-1}{3}$$

EXERCISE 7

REVISION

Simplify these.

1 ▶ $\frac{3y}{y}$

2 ▶ $\frac{4x}{4}$

3 ▶ $\frac{9x^2}{3x}$

4 ▶ $\frac{2a}{3} \times \frac{6}{a}$

5 ▶ $\frac{6b}{4} \div \frac{3b}{2a}$

6 ▶ $\frac{10x^2}{3} \times \frac{9}{5x}$

7 ▶ $\frac{y}{4} + \frac{y}{5}$

8 ▶ $\frac{x}{3} - \frac{x}{5}$

9 ▶ $\frac{2a}{5} + \frac{b}{10}$

Solve these.

10 ▶ $\frac{x^2}{2} + 2 = 10$

11 ▶ $\frac{x^2+2}{2} = 19$

12 ▶ $\sqrt{\frac{4+x}{6}} = 2$

Use the rules of indices to simplify these.

13 ▶ $a^4 \times a^6$

14 ▶ $b^7 \div b^5$

15 ▶ $(c^4)^3$

Rewrite each expression and insert the correct symbol <, > or = in the box.

16 ▶ $-2 \; \square \; -3$

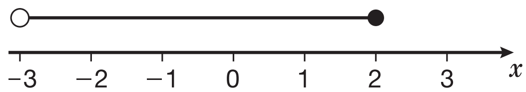
17 ▶ $\frac{1}{8} \; \square \; \frac{1}{7}$

18 ▶ $0.0009 \; \square \; 0.01$

19 ▶ $0.1 \; \square \; 10\%$

20 ▶ Write down the single inequality represented by this number line.

What is the smallest integer that x can be?



Solve the inequality and show each result on a number line.

21 ▶ $x - 4 > 1$

22 ▶ $5x \leq 3x + 9$

23 ▶ $5(x - 2) \geq 4(x - 2)$

24 ▶ Solve the inequality $x + 5 \leq 6x$.

25 ▶ List the integers in the solution set $3 \leq x < 5$.

EXERCISE 7*

REVISION

Simplify these.

1 ▶ $\frac{20a}{5b}$

2 ▶ $\frac{35x^2}{7xy}$

3 ▶ $\frac{12ab^2}{48a^2b}$

4 ▶ $\frac{2a}{b} \times \frac{b^2}{4a}$

5 ▶ $\frac{30}{xy^2} \div \frac{6x^2}{x^2y}$

6 ▶ $\frac{(3a)^2}{7b} \div \frac{a^3}{14b^2}$

7 ▶ $\frac{3a}{2} + \frac{a}{10}$

8 ▶ $\frac{2}{3b} + \frac{3}{4b} - \frac{5}{6b}$

9 ▶ $\frac{x+1}{7} - \frac{x-3}{21}$

Solve these.

10 ▶ $3x^2 + 5 = 32$

11 ▶ $2 = \frac{\sqrt{2x} + 2}{2}$

12 ▶ $\sqrt{100 - 4x^2} = 6$

Use the rules of indices to simplify each expression.

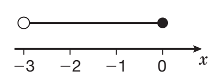
13 ▶ $a^5 \times a^6 \div a^7$

14 ▶ $(2b^3)^2$

15 ▶ $3c(3c^2)^3$

16 ▶ Write down the single inequality represented by the number line.

What is the smallest integer that satisfies the inequality?



Solve the inequality and show each result on a number line.

17 ▶ $7x + 3 < 2x - 19$

18 ▶ $2(x - 1) < 5(x + 2)$

19 ▶ $\frac{x-2}{5} \geq \frac{x-3}{3}$

20 ▶ Find the largest prime number y which satisfies $3y - 11 \leq 103$.

21 ▶ List the integers which satisfy both these inequalities simultaneously.

$-3.5 < x < 3$ and $4x + 1 \leq x + 2$

EXAM PRACTICE: ALGEBRA 2

In questions 1–6, simplify as much as possible.

1 $\frac{12xy^2}{3x}$ **[1]**

2 $(5xy^2) \div (15x^2y)$ **[1]**

3 $\frac{a}{b^3} \times \frac{ab}{c} \times \frac{b^2c}{a^2}$ **[1]**

4 $\frac{3x^2}{y^2} \div \frac{x^2}{y}$ **[1]**

5 $\frac{x}{4} - \frac{x}{6}$ **[2]**

6 $\frac{x}{9} + \frac{2x}{3}$ **[2]**

In questions 7–9, solve for x.

7 $2x^2 + 13 = 63$ **[3]**

8 $\frac{x^2 - 11}{7} = 10$ **[3]**

9 $\frac{\sqrt{x+4}}{4} = 1$ **[2]**

10 Simplify

a $3(q^3)^2$

b $p^5 \div p^3$

c $x^8 \times x^{12}$ **[3]**

11 Solve the inequality $10 \leq 7 - x$ and show the result on a number line. **[3]**

12 List the integer solutions of $3 \leq 3x < x + 6$. **[3]**

[Total 25 marks]

CHAPTER SUMMARY: ALGEBRA 2

SIMPLIFYING ALGEBRAIC FRACTIONS

$$\frac{5a}{\cancel{4b}_2} \times \frac{\cancel{2b}}{3} = \frac{5a}{6} \qquad \frac{5a}{12} + \frac{2b}{3} = \frac{5a+8b}{12} \qquad \frac{5a}{12} - \frac{2b}{3} = \frac{5a-8b}{12}$$

To divide by a fraction, turn the fraction upside down and multiply.

$$\frac{5a}{12} \div \frac{2b}{3} = \frac{5a}{\cancel{12}_4} \times \frac{\cancel{3}}{2b} = \frac{5a}{8b}$$

POSITIVE INTEGER INDICES

When multiplying, add the indices.

$a^m \times a^n = a^{m+n}$

When dividing, subtract the indices.

$a^m \div a^n = a^{m-n}$

When raising to a power, multiply the indices.

$(a^m)^n = a^{mn}$

SOLVING EQUATIONS WITH ROOTS AND POWERS

The way to solve equations is to isolate the unknown letter by systematically doing the same operation to both sides.

Always check your answer.

Solve $3x^2 - 4 = 71$

$3x^2 - 4 = 71$	(Add 4 to both sides)
$3x^2 = 75$	(Divide both sides by 3)
$x^2 = 25$	(Square root both sides)
$x = \pm 5$	(Note there are two answers)

Check: $3 \times (\pm 5)^2 - 4 = 71$

Solve $\frac{\sqrt{y+3}}{4} - 2 = 1$

$\frac{\sqrt{y+3}}{4} - 2 = 1$	(Add 2 to both sides)
$\frac{\sqrt{y+3}}{4} = 3$	(Multiply both sides by 4)
$\sqrt{y+3} = 12$	(Square both sides)
$y + 3 = 144$	(Subtract 3 from both sides)
$y = 141$	

Check: $\frac{\sqrt{141+3}}{4} - 2 = 1$

INEQUALITIES

Inequalities are solved in the same way as algebraic equations, EXCEPT that when multiplying or dividing by a negative number the inequality sign is reversed.

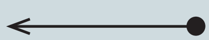
$2(x - 3) \le 5(x - 3)$	(Expand brackets)
$2x - 6 \le 5x - 15$	(Add 15 to both sides)
$2x + 9 \le 5x$	(Subtract $2x$ from both sides)
$9 \le 3x$	(Divide both sides by 3)

$3 \le x$ or $x \ge 3$

$x > 3$ means that x cannot be equal to 3.

$x \ge 3$ means that x can be equal to 3 or greater than 3.

A solid circle means

$\ge$  or $\le$ 

An open circle means

$>$  or $<$ 

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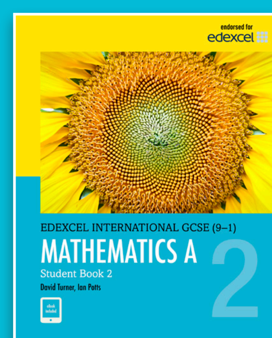

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