11 - 19 P

Sample

Chapter 4 Collisions in one dimension

NEW FOR 2017

Edexcel AS and A level Further Mathematics

Further Mechanics 1

FM1

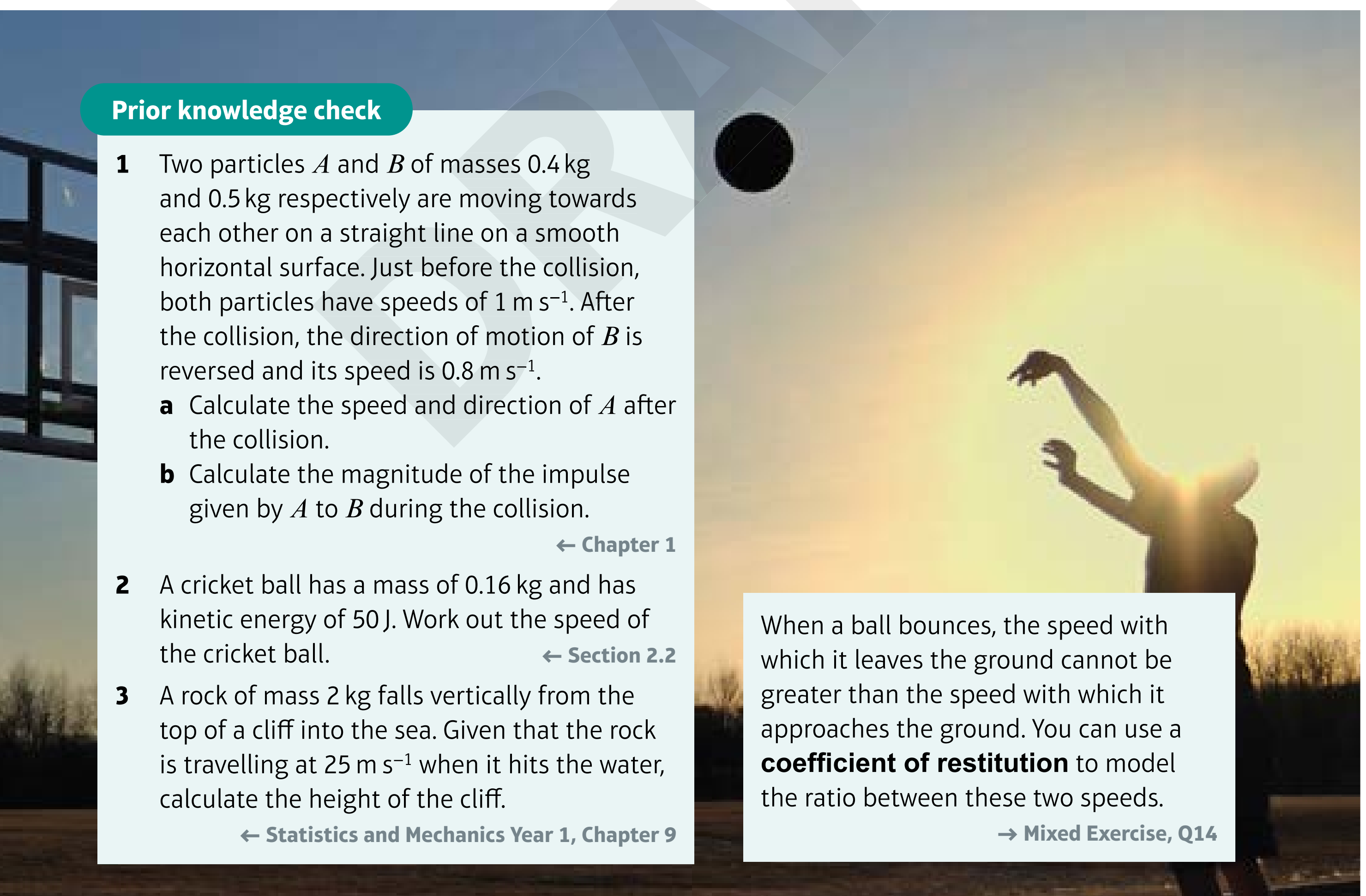
Pearson

4 Collisions in one dimension

Objectives

After completing this chapter you should be able to:

- Solve problems involving the direct impact of two particles by using the principle of conservation of momentum and Newton's law of restitution → pages 2–9
- Apply Newton's law of restitution to problems involving the direct collision of a particle with a smooth plane surface → pages 9–12
- Solve problems involving successive direct impacts → pages 12–19
- Find the change in energy due to an impact or the application of an impulse → pages 19–23



Prior knowledge check

1. Two particles A and B of masses 0.4 kg and 0.5 kg respectively are moving towards each other on a straight line on a smooth horizontal surface. Just before the collision, both particles have speeds of 1 m s^{-1}. After the collision, the direction of motion of B is reversed and its speed is 0.8 m s^{-1}.
   a Calculate the speed and direction of A after the collision.
   b Calculate the magnitude of the impulse given by A to B during the collision.
   ← Chapter 1
2. A cricket ball has a mass of 0.16 kg and has kinetic energy of 50 J. Work out the speed of the cricket ball. ← Section 2.2
3. A rock of mass 2 kg falls vertically from the top of a cliff into the sea. Given that the rock is travelling at 25 m s^{-1} when it hits the water, calculate the height of the cliff.
   ← Statistics and Mechanics Year 1, Chapter 9

When a ball bounces, the speed with which it leaves the ground cannot be greater than the speed with which it approaches the ground. You can use a **coefficient of restitution** to model the ratio between these two speeds.
→ Mixed Exercise, Q14

4.1 Direct impact and Newton's law of restitution

You can solve problems involving the direct impact of two particles by using the principle of conservation of momentum and **Newton's law of restitution**.

A direct impact is a collision between particles, of the same size, which are moving along the same straight line. When two particles collides their speeds after the collision depend upon the materials from which they are made.

Newton's law of restitution (sometimes called Newton's experimental law) defines how the speeds of the particles after the collision depend on the nature of the particles as well as their speeds before the collision. This law only holds when the collision takes place in free space or on a smooth surface.

- **Newton's law of restitution states that:**

$$\frac{\textbf{speed of separation of particles}}{\textbf{speed of approach of particles}} = e$$

The constant e is the coefficient of restitution between the particles and $0 \leqslant e \leqslant 1$.

The value of the coefficient of restitution e depends on the materials from which the particles are made. Particles for which $e = 1$ are called **perfectly elastic** particles. Particles for which $e = 0$ are called **perfectly inelastic** particles. Inelastic particles coalesce on impact.

Watch out Table tennis balls and some marbles are very elastic with values for e of approximately 0.95. Tennis balls, golf balls and cricket balls are less elastic with values for e ranging between 0.4 and 0.9. Balls of plasticine are perfectly inelastic and e is zero.

	Perfectly elastic particles (e = 1)	Perfectly inelastic particles (e = 0)
Before collision:	→ v m s^{-1} ← ◯ ◯	→ v m s^{-1} ← ◯ ◯
After collision:	← v m s^{-1} → ◯ ◯	0 m s^{-1} ◯◯

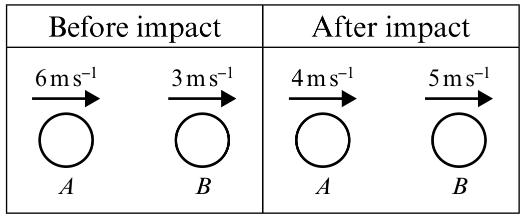
Example 1

In each part of this question the two diagrams show the speeds and directions of motion of two particles A and B just before and just after a collision. The particles move on a smooth horizontal plane. Find the coefficient of restitution e in each case.

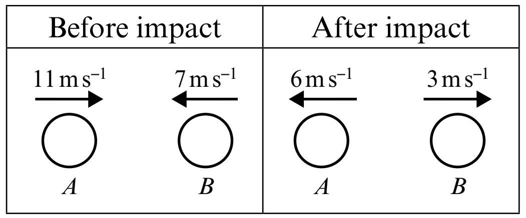
a

Before impact		After impact	
8 m s^{-1} →	At rest	At rest	2 m s^{-1} →
◯ A	◯ B	◯ A	◯ B

b



c



a The speed of approach is $8 - 0 = 8\text{ m s}^{-1}$

The speed of separation is $2 - 0 = 2\text{ m s}^{-1}$

$e = \frac{2}{8} = \frac{1}{4}$

b The speed of approach is $6 - 3 = 3\text{ m s}^{-1}$

The speed of separation is $5 - 4 = 1\text{ m s}^{-1}$

$e = \frac{1}{3}$

c The speed of approach is $11 + 7 = 18\text{ m s}^{-1}$

The speed of separation is $6 + 3 = 9\text{ m s}^{-1}$

$e = \frac{9}{18} = \frac{1}{2}$

Find the difference in the velocities before impact, called the speed of approach.

Find the difference in the velocities after impact, called the speed of separation.

Find e using

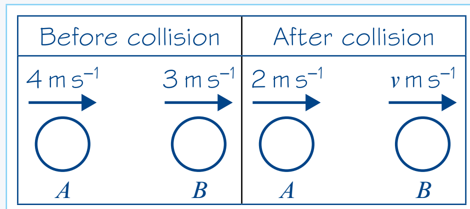
$$\frac{\text{speed of separation of particles}}{\text{speed of approach of particles}} = e$$

Watch out The particles are moving in **opposite directions** so the speed of approach/separation will be the sum of the speeds of each particle.

Example 2

Two particles A and B are travelling in the same direction on a smooth surface with speeds 4 m s^{-1} and 3 m s^{-1} respectively. They collide directly, and immediately after the collision continue to travel in the same direction with speeds 2 m s^{-1} and $v\text{ m s}^{-1}$ respectively.

Given that the coefficient of restitution between A and B is $\frac{1}{3}$, find v.



$$\frac{\text{speed of separation of particles}}{\text{speed of approach of particles}} = e$$

$\frac{v - 2}{4 - 3} = \frac{1}{3}$

$v - 2 = \frac{1}{3}$

So $v = 2\frac{1}{3}$

Substitute the speed of approach, $4 - 3$, and the speed of separation, $v - 2$, then make v the subject of the formula.

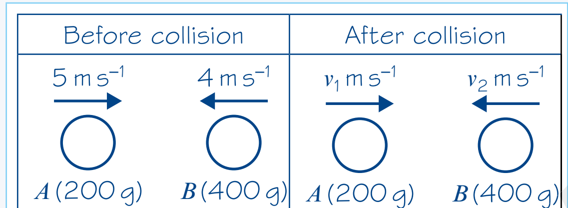
You can use the principle of conservation of linear momentum together with Newton's law of restitution to solve problems involving two unknown velocities.

Links For particles of masses m_1 and m_2 colliding in a straight line, with initial velocities u_1 and u_2 respectively, and final velocities v_1 and v_2 respectively:
$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ ← Section 1.2

Example 3

Two particles A and B of masses 200 g and 400 g respectively are travelling in opposite directions towards each other on a smooth surface with speeds 5 m s^{-1} and 4 m s^{-1} respectively. They collide directly, and immediately after the collision have velocities $v_1\text{ m s}^{-1}$ and $v_2\text{ m s}^{-1}$ respectively, measured in the direction of motion of A before the collision.

Given that the coefficient of restitution between A and B is $\frac{1}{2}$, find v_1 and v_2.



Using conservation of linear momentum for the system (→)

$0.2 \times 5 + 0.4 \times (-4) = 0.2v_1 + 0.4v_2$

$1 - 1.6 = 0.2v_1 + 0.4v_2$

$-0.6 = 0.2v_1 + 0.4v_2$

$-3 = v_1 + 2v_2 \quad (1)$

$$\frac{\text{speed of separation of particles}}{\text{speed of approach of particles}} = e$$

$\frac{v_2 - v_1}{5 + 4} = \frac{1}{2}$

$v_2 - v_1 = 4\frac{1}{2} \quad (2)$

Eliminating v_1 between equations (1) and (2) gives

$v_2 = \frac{1}{2}$

Substituting this value into equation (1) gives

$v_1 = -4$

Problem-solving The final velocities are measured in the direction of motion of A before the collision, so choose this as the positive direction. Initially, B is moving in the opposite direction, so u_2 will be negative.

Calculate the speed of approach and the speed of separation and substitute into Newton's law of restitution.

Solve the simultaneous equations (1) and (2) to find the values of v_1 and v_2.

Example 4

Two balls P and Q have masses $3m$ and $4m$ respectively. They are moving in opposite directions towards each other along the same straight line on a smooth level floor. Immediately before they collide, P has speed $3u$ and Q has speed $2u$. The coefficient of restitution between P and Q is e.

By modelling the balls as smooth spheres and the floor as a smooth horizontal plane,

a show that the speed of Q after the collision is $\frac{u}{7}(15e + 1)$.

b Given that the direction of motion of P is unchanged, find the range of possible values of e.

c Given that the magnitude of the impulse of P on Q is $\frac{80mu}{9}$, find the value of e.

a

Before impact		After impact	
$3u$ →	← $2u$	v_1 →	v_2 →
P ($3m$)	Q ($4m$)	P ($3m$)	Q ($4m$)

Choose a positive direction and draw a diagram showing masses and velocities before and after the impact. Use v_1 and v_2 for the unknown velocities after impact.

Using conservation of linear momentum for the system (→):

$9mu - 8mu = 3mv_1 + 4mv_2$

$\Rightarrow 3v_1 + 4v_2 = u \quad (1)$

Use $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$, noting that u_2 is negative as Q is moving in the opposite direction.

Newton's law of restitution gives

$\frac{v_2 - v_1}{3u + 2u} = e$

$\Rightarrow v_2 - v_1 = 5eu \quad (2)$

Calculate the speed of approach and the speed of separation then substitute into Newton's law of restitution.

Eliminating v_1 between equations (1) and (2) gives

$7v_2 = 15ue + u$

$v_2 = \frac{u}{7}(15e + 1)$

Solve the simultaneous equations (1) and (2) to find the value of v_2.

b Substituting this value into equation (2) gives

$v_1 = \frac{u}{7}(15e + 1) - 5eu$

$v_1 = \frac{u}{7}(1 - 20e)$

Now find the value of v_1 by substituting the value of v_2 from part **a** into equation (2).

As $v_1 > 0$,

$\frac{u}{7}(1 - 20e) > 0$

So $e < \frac{1}{20}$

Problem-solving

As the direction of motion of P is unchanged by the impact, v_1 must be positive.

c Impulse of P on Q = change in momentum of Q

$= 4mv_2 - 4m(-2u)$

$= \frac{4mu}{7}(15e + 1) + 8mu$

$= \frac{60mu}{7}(1 + e)$

$m_2v_2 - m_2u_2$

However the impulse is given as $\frac{80mu}{9}$

So $\frac{60mu}{7}(1 + e) = \frac{80mu}{9}$

$(1 + e) = \frac{28}{27}$

$e = \frac{1}{27}$

Exercise 4A

1 In each part of this question the two diagrams show the speeds and directions of motion of two particles A and B just before and just after a collision. The particles move on a smooth horizontal plane. Find the coefficient of restitution e in each case.

	Before collision		After collision	
a	$6\,\mathrm{m\,s^{-1}}$ → A	At rest B	At rest A	$4\,\mathrm{m\,s^{-1}}$ → B
b	$4\,\mathrm{m\,s^{-1}}$ → A	$2\,\mathrm{m\,s^{-1}}$ → B	$2\,\mathrm{m\,s^{-1}}$ → A	$3\,\mathrm{m\,s^{-1}}$ → B
c	$9\,\mathrm{m\,s^{-1}}$ → A	← $6\,\mathrm{m\,s^{-1}}$ B	← $3\,\mathrm{m\,s^{-1}}$ A	$2\,\mathrm{m\,s^{-1}}$ → B

2 In each part of this question, the two diagrams show the speeds and directions of motion of two particles A and B just before a collision, and their velocities relative to the initial directon of A just after a collision. The particles move on a smooth horizontal plane. The masses of A and B and the coefficients of restitution e are also given. Find the values of v_1 and v_2 in each case.

	Before collision		After collision	
a $e = \frac{1}{2}$	$6\,\mathrm{m\,s^{-1}}$ → A (0.25 kg)	At rest B (0.5 kg)	$v_1\,\mathrm{m\,s^{-1}}$ → A (0.25 kg)	$v_2\,\mathrm{m\,s^{-1}}$ → B (0.5 kg)
b $e = 0.25$	$4\,\mathrm{m\,s^{-1}}$ → A (2 kg)	$2\,\mathrm{m\,s^{-1}}$ → B (3 kg)	$v_1\,\mathrm{m\,s^{-1}}$ → A (2 kg)	$v_2\,\mathrm{m\,s^{-1}}$ → B (3 kg)
c $e = \frac{1}{7}$	$8\,\mathrm{m\,s^{-1}}$ → A (3 kg)	← $6\,\mathrm{m\,s^{-1}}$ B (1 kg)	$v_1\,\mathrm{m\,s^{-1}}$ → A (3 kg)	$v_2\,\mathrm{m\,s^{-1}}$ → B (1 kg)
d $e = \frac{2}{3}$	$6\,\mathrm{m\,s^{-1}}$ → A (400 g)	← $6\,\mathrm{m\,s^{-1}}$ B (400 g)	$v_1\,\mathrm{m\,s^{-1}}$ → A (400 g)	$v_2\,\mathrm{m\,s^{-1}}$ → B (400 g)
e $e = \frac{1}{5}$	$3\,\mathrm{m\,s^{-1}}$ → A (5 kg)	← $12\,\mathrm{m\,s^{-1}}$ B (4 kg)	$v_1\,\mathrm{m\,s^{-1}}$ → A (5 kg)	$v_2\,\mathrm{m\,s^{-1}}$ → B (4 kg)

3 A small smooth sphere A of mass 1 kg is travelling along a straight line on a smooth horizontal plane with speed $4\,\text{m s}^{-1}$ when it collides with a second smooth sphere B of the same radius, with mass 2 kg and travelling in the same direction as A with speed $2.5\,\text{m s}^{-1}$. After the collision, A continues in the same direction with speed $2\,\text{m s}^{-1}$.

a Find the speed of B after the collision.

b Find the coefficient of restitution for the spheres.

4 Two spheres A and B are of equal radius and have masses 2 kg and 6 kg respectively. A and B move towards each other along the same straight line on a smooth horizontal surface with velocities $4\,\text{m s}^{-1}$ and $6\,\text{m s}^{-1}$ respectively. If the coefficient of restitution is $\frac{1}{5}$, find the velocities of the spheres after the collision and the magnitude of the impulse given to each sphere.

(P) 5 Two particles P and Q of masses $2m$ and $3m$ respectively are moving in opposite directions towards each other. Each particle is travelling with speed u. Given that Q is brought to rest by the collision, find the speed of P after the collision, and the coefficient of restitution between the particles.

6 Two particles A and B are travelling along the same straight line in the same direction on a smooth horizontal surface with speeds $3u$ and u respectively. Particle A catches up and collides with particle B. If the mass of B is twice that of A and the coefficient of restitution is e, find, in terms of e and u, expressions for the speeds of A and B after the collision.

7 Two identical particles of mass m are projected towards each other along the same straight line on a smooth horizontal surface with speeds of $2u$ and $3u$. After the collision, the directions of motion of both particles are reversed. Show that this implies that the coefficient of restitution e satisfies the inequality $e > \frac{1}{5}$

(P) 8 Two particles A and B of masses m and km respectively are placed on a smooth horizontal plane. Particle A is made to move on the plane with speed u so as to collide directly with B, which is at rest. After the collision B moves with speed $\frac{3}{10}u$.

a Find, in terms of u and the constant k, the speed of A after the collision.

b By using Newton's law of restitution, show that $\frac{7}{3} < k < \frac{17}{3}$

Problem-solving

In part **b** use the limits of e to set up an inequality.

(P) 9 Two particles A and B of masses m and $3m$ respectively are placed on a smooth horizontal plane. Particle A is made to move on the plane with speed $2u$ so as to collide directly with B, which is moving in the same direction with speed u. After the collision B moves with speed ku, where k is a constant.

a Find, in terms of u and the constant k, the speed of A after the collision.

b By using Newton's law of restitution show that $\frac{5}{4} < k < \frac{3}{2}$.

(E) 10 A particle P of mass m is moving with speed $4u$ on a smooth horizontal plane. The particle collides directly with a particle Q of mass $3m$ moving with speed $2u$ in the same direction as P. The coefficient of restitution between P and Q is e.

a Show that the speed of Q after the collision is $\frac{u}{2}(5 + e)$. **(6 marks)**

b Find the speed of P after the collision, giving your answer in terms of e. **(4 marks)**

c Show that the direction of motion of P is unchanged by the collision. **(2 marks)**

d Given that the magnitude of the impulse of P on Q is $2mu$, find the value of e. **(4 marks)**

Challenge

Two particles P and Q of masses $3m$ kg and m kg respectively move towards each other in a straight line in opposite directions on a smooth horizontal surface. P has initial speed $2\,\text{m s}^{-1}$ and Q has initial speed $u\,\text{m s}^{-1}$. After P and Q collide, both particles move in the same direction that P was originally moving. Given that, after the impact, Q moves with twice the speed of P and that the coefficient of restitution between P and Q is $\frac{1}{4}$, show that $u = \frac{14}{9}\,\text{m s}^{-1}$.

4.2 Direct collision with a smooth plane

You can also apply Newton's law of restitution to problems involving the direct collision of a particle with a smooth plane surface perpendicular to the direction of motion of the particle.

In the figure a particle is shown moving horizontally with speed u before impact with a vertical plane surface. After impact the particle moves in the opposite direction with speed v.

The speed of the particle after the impact depends on the speed of the particle before the impact and the coefficient of restitution e between the particle and the plane.

- **For the direct collision of a particle with a smooth plane Newton's law of restitution can be written as:**

$$\frac{\textbf{speed of rebound}}{\textbf{speed of approach}} = e$$

Example 5

A particle collides normally with a fixed vertical plane. The diagram shows the speeds of the particle before and after the collision. Find the value of the coefficient of restitution e.

Notation If a particle collides **normally** then its direction of motion immediately before the instant of impact is perpendicular to the plane.

Using Newton's law of restitution

$$e = \frac{\text{speed of rebound}}{\text{speed of approach}}$$

$$= \frac{2}{8} = \frac{1}{4}$$

This is the same as $e = \frac{2-0}{8-0}$

The coefficient of restitution is $\frac{1}{4}$

Example 6

A small sphere collides normally with a fixed vertical wall. Before the impact the sphere is moving with a speed of $4\,\text{m s}^{-1}$ on a smooth horizontal floor. The coefficient of restitution between the sphere and the wall is 0.2. Find the speed of the sphere after the collision.

Using Newton's law of restitution

$$e = \frac{\text{speed of rebound}}{\text{speed of approach}}$$

$$0.2 = \frac{v}{4}$$

Let the speed of the sphere after the collision be $v\,\text{m s}^{-1}$.

So $v = 0.8$

The speed of the sphere after the collision is $0.8\,\text{m s}^{-1}$.

Example 7

A particle falls 22.5 cm from rest onto a smooth horizontal plane. It then rebounds to a height of 10 cm. Find the coefficient of restitution between the particle and the plane. Give your answer to 2 significant figures.

As particle falls:

Use $v^2 = u^2 + 2as$

The particle is falling under gravity so use the appropriate constant acceleration formula to find its speed when it hits the plane.

with $u = 0$, $s = 0.225$ and $a = g$

$v^2 = 0.45g$

$v = 2.1$

$v\,\text{m s}^{-1}$ is the speed of the particle when it hits the plane.

After impact:

Use $v^2 = u^2 + 2as$

with $v = 0$, $s = 0.1$ and $a = -g$

After it rebounds it initially moves upwards under gravity. As the upward direction is taken as positive here, the acceleration is negative.

$u^2 = 0.2g$

$u = 1.4$

$u\,\text{m s}^{-1}$ is the rebound speed of the particle.

Using Newton's law of restitution

$$e = \frac{\text{speed of rebound}}{\text{speed of approach}}$$

$$= \frac{1.4}{2.1} = \frac{2}{3}$$

The coefficient of restitution is 0.67 (2 s.f.).

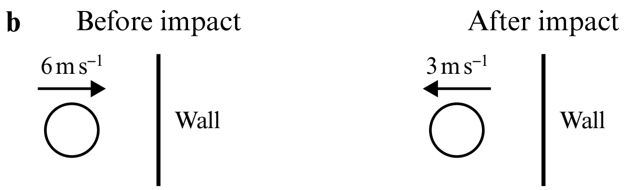
Exercise 4B

Whenever a numerical value of g is required, take $g = 9.8\,\text{m s}^{-2}$.

1 A smooth sphere collides normally with a fixed vertical wall. The two diagrams show the speed and direction of motion of the sphere before and after the collision. In each case, find the value of the coefficient of restitution e.

a

b



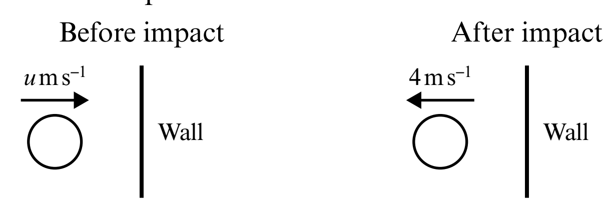
2 A smooth sphere collides normally with a fixed vertical wall. The two diagrams show the speed and direction of motion of the sphere before and after the collision. The value of e is given in each case. Find the speed of the sphere after the collision in each case.

a $e = \frac{1}{2}$

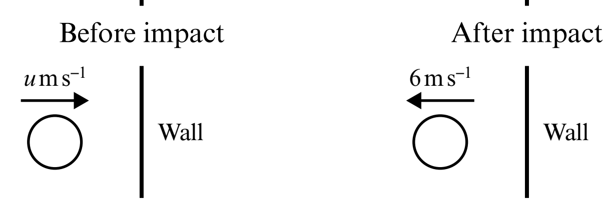
b $e = \frac{1}{4}$

3 A smooth sphere collides normally with a fixed vertical wall. The two diagrams show the speed and direction of motion of the sphere before and after the collision. The value of e is also given. Find the speed of the sphere before the collision in each case.

a $e = \frac{1}{2}$



b $e = \frac{3}{4}$



4 A small smooth sphere of mass 0.3 kg is moving on a smooth horizontal table with a speed of $10\,\text{m s}^{-1}$ when it collides normally with a fixed smooth wall. It rebounds with a speed of $7.5\,\text{m s}^{-1}$. Find the coefficient of restitution between the sphere and the wall.

5 A particle falls 2.5 m from rest onto a smooth horizontal plane. It then rebounds to a height of 1.5 m. Find the coefficient of restitution between the particle and the plane. Give your answer to 2 significant figures.

P **6** A particle falls 3 m from rest onto a smooth horizontal plane. It then rebounds to a height h m. The coefficient of restitution between the particle and the plane is 0.25. Find the value of h.

E/P **7** A small smooth sphere falls from rest onto a smooth horizontal plane. It takes 2 seconds to reach the plane then another 2 seconds to reach the plane a second time. Find the coefficient of restitution between the particle and the plane.

E/P **8** A small smooth sphere falls from rest onto a smooth horizontal plane. It takes 3 seconds to reach the plane. The coefficient of restitution between the particle and the plane is 0.49. Find the time it takes for the sphere to reach the plane a second time.

E/P **9** A particle falls from rest from a height of h m above level ground. After rebounding the ball reaches a maximum height of $\frac{1}{2}h$. Find, in terms of g and h, the height of the ball 1 s after it hits the ground.

Challenge

A particle P falls from rest from a height of h m above level ground. Show that after hitting the ground, the maximum height that the ball reaches is he^2, where e is the coefficient of restitution between the ball and the ground.

4.3 Successive direct impacts

You can solve problems involving successive direct impacts of particles with each other, or with a smooth plane surface. When you are solving such problems, you should draw a clear diagram showing the 'before' and 'after' information for each collision.

Example 8

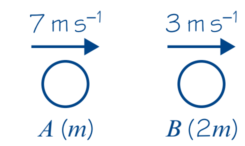
Three spheres A, B and C have masses m, $2m$ and $3m$ respectively. The spheres move along the same straight line on a horizontal plane with A following B, which is following C. Initially the speeds of A, B and C are $7\,\text{m s}^{-1}$, $3\,\text{m s}^{-1}$ and $1\,\text{m s}^{-1}$ respectively, in the direction ABC. Sphere A collides with sphere B and then sphere B collides with sphere C. The coefficient of restitution between A and B is $\frac{1}{2}$ and the coefficient of restitution between B and C is $\frac{1}{4}$.

a Find the velocities of the three spheres after the second collision.

b Explain how you can predict that there will be a further collision between A and B.

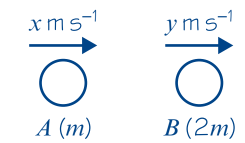
a First collision

Before impact between A and B



Draw a diagram to show the masses, speeds and directions of A and B before the first collision.

After impact between A and B



Let the velocity of A be $x\,\text{m s}^{-1}$ and let the velocity of B be $y\,\text{m s}^{-1}$ after the impact. Make it clear in your diagram which velocity corresponds to which particle and which direction is positive.

Using conservation of momentum ($\rightarrow$)

$m \times 7 + 2m \times 3 = m \times x + 2m \times y$

$\Rightarrow x + 2y = 13 \qquad (1)$

Use $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

Using Newton's law of restitution

$\dfrac{y - x}{7 - 3} = \dfrac{1}{2}$

$\Rightarrow y - x = 2 \qquad (2)$

Use $\dfrac{\text{speed of separation of particles}}{\text{speed of approach of particles}} = e$

Solving equations (1) and (2) gives

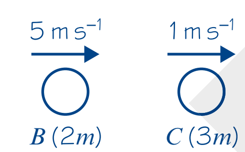
$y = 5$ and $x = 3$

After the first impact, the velocity of A is $3\,\text{m s}^{-1}$ and the velocity of B is $5\,\text{m s}^{-1}$.

Add equations (1) and (2) to eliminate x and to give $3y = 15$, then substitute $y = 5$ into equation (2) to give $x = 3$.

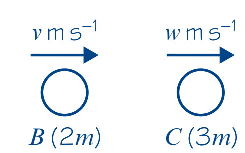
Second collision

Before impact between B and C



Draw a diagram to show the masses, speeds and directions of B and C before the collision.

After impact between B and C



Let the velocities of B and C after the collision be $v\,\text{m s}^{-1}$ and $w\,\text{m s}^{-1}$ respectively.

Using conservation of momentum ($\rightarrow$)

$2m \times 5 + 3m \times 1 = 2m \times v + 3m \times w$

$\Rightarrow 2v + 3w = 13 \qquad (3)$

Using Newton's law of restitution

$\dfrac{w - v}{5 - 1} = \dfrac{1}{4}$

$\Rightarrow w - v = 1 \qquad (4)$

Solving equations (3) and (4) gives

$w = 3$ and $v = 2$

Form two equations and solve them as you did for the first collision.

After the second impact, the velocity of B is 2 m s^{-1} and the velocity of C is 3 m s^{-1}
The velocity of A is 3 m s^{-1}.

A was not involved in the second collision so its final velocity is 3 m s^{-1}.

b As the velocity of A is greater than, and in the same direction as, the velocity of B, there will be a further collision between A and B.

Problem-solving

A plane is assumed to extend infinitely in either direction. In real life, a particle will not continue to move in a straight line and at a constant speed forever.

Example 9

A uniform smooth sphere P of mass $3m$ is moving in a straight line with speed u on a smooth horizontal table. Another uniform smooth sphere Q, of mass m and having the same radius as P, is moving with speed $2u$ in the same straight line as P, but in the opposite direction. The sphere P collides with the sphere Q directly. The velocities of P and Q after the collision are v and w respectively, measured in the direction of motion of P before the collision. The coefficient of restitution between P and Q is e.

a Find expressions for v and w in terms of u and e.

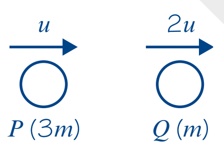
b Show that, if the direction of motion of P is changed by the collision, then $e > \frac{1}{3}$

Following the collision with P, the sphere Q then collides with and rebounds from a vertical wall, which is perpendicular to the direction of motion of Q. The coefficient of restitution between Q and the wall is e'.

c Given that $e = \frac{5}{9}$ and that P and Q collide again in the subsequent motion, show that $e' = \frac{1}{9}$

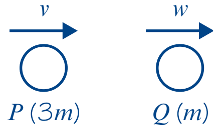
a First collision

Before impact between P and Q



In your diagram ensure that the velocity of Q is in the opposite direction from that of P.

After impact between P and Q



Using conservation of momentum $(\rightarrow)$

$3m \times u - m \times 2u = 3m \times v + m \times w$

$\Rightarrow 3v + w = u \qquad (1)$

Each velocity must be given the correct sign in relation to the positive direction.

Using Newton's law of restitution

$\dfrac{w - v}{u - (-2u)} = e$

$\Rightarrow w - v = 3ue \qquad (2)$

Note that the speed of approach is $u - (-2u)$.

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