11 - 19

Sample

1 Data collection

9 Constant acceleration

Edexcel AS and A level Mathematics

NEW FOR 2017

Statistics and Mechanics

Year 1/AS

Pearson

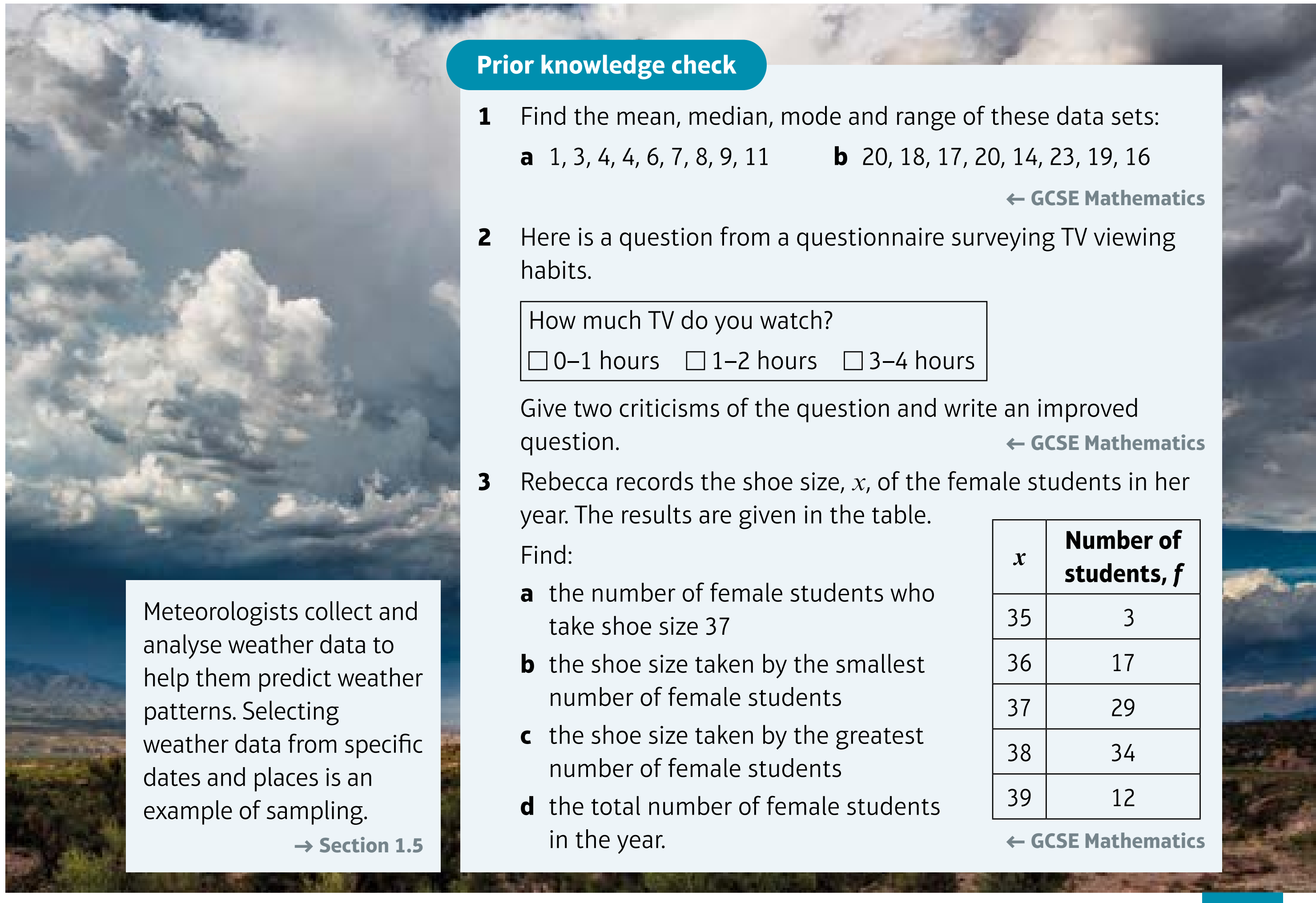
Data collection

1

Objectives

After completing this chapter you should be able to:

- Understand 'population', 'sample' and 'census', and comment on the advantages and disadvantages of each → pages 2–3
- Understand the advantages and disadvantages of simple random sampling, systematic sampling, stratified sampling, quota sampling and opportunity sampling → pages 4–9
- Define qualitative, quantitative, discrete and continuous data, and understand grouped data → pages 9–10
- Understand the large data set and how to collect data from it, identify types of data and calculate simple statistics → pages 11–16



Meteorologists collect and analyse weather data to help them predict weather patterns. Selecting weather data from specific dates and places is an example of sampling.

→ Section 1.5

Prior knowledge check

1 Find the mean, median, mode and range of these data sets:

a 1, 3, 4, 4, 6, 7, 8, 9, 11 **b** 20, 18, 17, 20, 14, 23, 19, 16

← GCSE Mathematics

2 Here is a question from a questionnaire surveying TV viewing habits.

How much TV do you watch?
☐ 0–1 hours ☐ 1–2 hours ☐ 3–4 hours

Give two criticisms of the question and write an improved question. ← GCSE Mathematics

3 Rebecca records the shoe size, x, of the female students in her year. The results are given in the table.

Find:

a the number of female students who take shoe size 37

b the shoe size taken by the smallest number of female students

c the shoe size taken by the greatest number of female students

d the total number of female students in the year.

x	Number of students, f
35	3
36	17
37	29
38	34
39	12

← GCSE Mathematics

1.1 Populations and samples

■ **In statistics, a population is the whole set of items that are of interest.**

For example, the population could be the items manufactured by a factory or all the people in a town.

Information can be obtained from a population. This is known as raw data.

■ **A census observes or measures every member of a population.**

■ **A sample is a selection of observations taken from a subset of the population which is used to find out information about the population as a whole.**

There are a number of advantages and disadvantages of both a census and a sample.

	Advantages	Disadvantages
Census	• It should give a completely accurate result	• Time consuming and expensive • Cannot be used when the testing process destroys the item • Hard to process large quantity of data
Sample	• Less time consuming and expensive than a census • Fewer people have to respond • Less data to process than in a census	• The data may not be as accurate • The sample may not be large enough to give information about small sub-groups of the population

The size of the sample can affect the validity of any conclusions drawn.

- The size of the sample depends on the required accuracy and available resources.
- Generally, the larger the sample, the more accurate it is, but you will need greater resources.
- If the population is very varied, you need a larger sample than if the population were uniform.
- Different samples can lead to different conclusions due to the natural variation in a population.

■ **Individual units of a population are known as sampling units.**

■ **Often sampling units of a population are individually named or numbered to form a list called a sampling frame.**

Example 1

A supermarket wants to test a delivery of avocados for ripeness by cutting them in half.

a Suggest a reason why the supermarket should not test all the avocados in the delivery.

The supermarket tests a sample of 5 avocados and finds that 4 of them are ripe.
They estimate that 80% of the avocados in the delivery are ripe.

b Suggest one way that the supermarket could improve their estimate.

a Testing all the avocados would mean that there were none left to sell.

When testing a product destroys it, a 'census' is not appropriate.

b They could take a larger sample, for example 10 avocados. This would give a better estimate of the overall proportion of ripe avocados.

In general, larger samples produce more accurate predictions about a population.

Exercise 1A

1 A school uses a census to investigate the dietary requirements of its students.

a Explain what is meant by a census.

b Give one advantage and one disadvantage to the school of using a census.

2 A factory makes safety harnesses for climbers and has an order to supply 3000 harnesses. The buyer wishes to know that the load at which the harness breaks exceeds a certain figure.

a Suggest a reason why a census would not be used for this purpose.

The factory tests four harnesses and the load for breaking is recorded:

320 kg 260 kg 240 kg 180 kg

b The factory claims that the harnesses are safe for loads up to 250 kg. Use the sample data to comment on this claim.

c Suggest one way in which the company can improve their prediction.

3 A city council wants to know what people think about its recycling centre.
The council decides to carry out a sample survey to learn the opinion of residents.

a Write down one reason why the council should not take a census.

b Suggest a suitable sampling frame.

c Identify the sampling units.

4 A manufacturer of microswitches is testing the reliability of its switches. It uses a special machine to switch them on and off until they break.

a Give one reason why the manufacturer should use a sample rather than a census.

The company tests a sample of 10 switches, and obtains the following results:

23 150 25 071 19 480 22 921 7455

b The company claims that its switches can be operated an average of 20 000 times without breaking. Use the sample data above to comment on this claim.

c Suggest one way the company could improve its prediction.

5 A manager of a garage wants to know what their mechanics think about a new pension scheme designed for them. The manager decides to ask all the mechanics in the garage.

a Describe the population the manager will use.

b Write down the main advantage in asking all of their mechanics.

1.2 Sampling

In random sampling, every member of the population has an equal chance of being selected. The sample should therefore be **representative** of the population. Random sampling also helps to remove **bias** from a sample.

There are three methods of random sampling:

- Simple random sampling
- Systematic sampling
- Stratified sampling

- **A simple random sample of size n is one where every sample of size n has an equal chance of being selected.**

To carry out a simple random sample, you need a sampling frame, usually a list of people or things. Each person or thing is allocated a unique number and a selection of these numbers is chosen at random.

There are two methods of choosing the numbers: generating random numbers (using a calculator, computer or random number table) and **lottery** sampling.

In lottery sampling, the members of the sampling frame could be written on tickets and placed into a 'hat'. The required number of tickets would then be drawn out.

Example 2

The 100 members of a yacht club are listed alphabetically in the club's membership book.

The committee wants to select a sample of 12 members to fill in a questionnaire.

a Explain how the committee could use a calculator or random number generator to take a simple random sample of the members.

b Explain how the committee could use a lottery sample to take a simple random sample of the members.

a Allocate a number from 1 to 100 to each member of the yacht club. Use your calculator or a random number generator to generate 12 random numbers between 1 and 100.
Go back to the original population and select the people corresponding to these numbers.

If your calculator generates a number that has already been selected, ignore that number and generate an extra random number.

b Write all the names of the members on (identical) cards and place them into a hat. Draw out 12 names to make up the sample of members.

- **In systematic sampling, the required elements are chosen at regular intervals from an ordered list.**

For example, if a sample of size 20 was required from a population of 100, you would take every fifth person since 100 ÷ 20 = 5.

The first person to be chosen should be chosen at random. So, for example, if the first person chosen is number 2 in the list, the remaining sample would be persons 7, 12, 17 etc.

- **In stratified sampling, the population is divided into mutually exclusive strata (males and females, for example) and a random sample is taken from each.**

The proportion of each strata sampled should be the same. A simple formula can be used to calculate the number of people we should sample from each stratum:

$$\text{The number sampled in a stratum} = \frac{\text{number in stratum}}{\text{number in population}} \times \text{overall sample size}$$

Example 3

A factory manager wants to find out what his workers think about the factory canteen facilities.

The manager decides to give a questionnaire to a sample of 80 workers. It is thought that different age groups will have different opinions.

There are 75 workers between ages 18 and 32.

There are 140 workers between 33 and 47.

There are 85 workers between 48 and 62.

a Write down the name of the method of sampling the manager should use.

b Explain how he could use this method to select a sample of workers' opinions.

a Stratified sampling.

b There are: 75 + 140 + 85 = 300 workers altogether.

Find the total number of workers.

18–32: $\frac{75}{300} \times 80 = 20$ workers.

33–47: $\frac{140}{300} \times 80 = 37\frac{1}{3} \approx 37$ workers.

48–62: $\frac{85}{300} \times 80 = 22\frac{2}{3} \approx 23$ workers.

For each age group find the number of workers needed for the sample.

Where the required number of workers is not a whole number, round to the nearest whole number.

Number the workers in each age group. Use a random number table (or generator) to produce the required quantity of random numbers. Give the questionnaire to the workers corresponding to these numbers.

Each method of random sampling has advantages and disadvantages.

Simple random sampling	
Advantages	**Disadvantages**
• Free of bias • Easy and cheap to implement for small populations and small samples • Each sampling unit has a known and equal chance of selection	• Not suitable when the population size or the sample size is large • A sampling frame is needed

Systematic sampling	
Advantages	**Disadvantages**
• Simple and quick to use • Suitable for large samples and large populations	• A sampling frame is needed • It can introduce bias if the sampling frame is not random

Stratified sampling	
Advantages	**Disadvantages**
• Sample accurately reflects the population structure • Guarantees proportional representation of groups within a population	• Population must be clearly classified into distinct strata • Selection within each stratum suffers from the same disadvantages as simple random sampling

Exercise 1B

1 a The head teacher of an infant school wishes to take a stratified sample of 20% of the pupils at the school. The school has the following numbers of pupils.

Year 1	Year 2	Year 3
40	60	80

Work out how many pupils in each age group will be in the sample.

b Describe one benefit to the head teacher of using a stratified sample.

Problem-solving

When describing advantages or disadvantages of a particular sampling method, always refer to the context of the question.

2 A survey is carried out on 100 members of the adult population of a city suburb. The population of the suburb is 2000. An alphabetical list of the inhabitants of the suburb is available.

a Explain one limitation of using a systematic sample in this situation.

b Describe a sampling method that would be free of bias for this survey.

3 A gym wants to take a sample of its members. Each member has a 5-digit membership number, and the gym selects every member with a membership number ending 000.

a Is this a systematic sample? Give a reason for your answer.

b Suggest one way of improving the reliability of this sample.

4 A head of sixth form wants to get the opinion of year 12 and year 13 students about the facilities available in the common room. The table shows the numbers of students in each year.

	Year 12	Year 13
Male	70	50
Female	85	75

a Suggest a suitable sampling method that might be used to take a sample of 40 students.

b How many students from each gender in each of the two years should the head of sixth form ask?

5 A factory manager wants to get information about the ways their workers travel to work. There are 480 workers in the factory, and each has a clocking-in number. The numbers go from 1 to 480. Explain how the manager could take a systematic sample of size 30 from these workers.

6 The director of a sports club wants to take a sample of members. The members each have a unique membership number. There are 121 members who play cricket, 145 members who play hockey and 104 members who play squash. No members play more than one sport.

a Explain how the director could take a simple random sample of 30 members and state one disadvantage of this sampling method.

The director decides to take a stratified sample of 30 members.

b State one advantage of this method of sampling.

c Work out the number of members who play each sport that the director should select for the sample.

1.3 Non-random sampling

There are two types of non-random sampling that you need to know:

- Quota sampling
- Opportunity sampling

■ **In quota sampling, an interviewer or researcher selects a sample that reflects the characteristics of the whole population.**

The population is divided into groups according to a given characteristic. The size of each group determines the proportion of the sample that should have that characteristic.

As an interviewer, you would meet people, assess their group and then, after interview, allocate them into the appropriate quota.

This continues until all quotas have been filled. If a person refuses to be interviewed or the quota into which they fit is full, then you simply ignore them and move on to the next person.

■ **Opportunity sampling consists of taking the sample from people who are available at the time the study is carried out and who fit the criteria you are looking for.**

Notation Opportunity sampling is sometimes called **convenience sampling**.

This could be the first 20 people you meet outside a supermarket on a Monday morning who are carrying shopping bags, for example.

There are advantages and disadvantages of each type of sampling.

Quota sampling	
Advantages	**Disadvantages**
• Allows a small sample to still be representative of the population • No sampling frame required • Quick, easy and inexpensive • Allows for easy comparison between different groups within a population	• Non-random sampling can introduce bias • Population must be divided into groups, which can be costly or inaccurate • Increasing scope of study increases number of groups, which adds time and expense • Non-responses are not recorded as such

Opportunity sampling	
Advantages	**Disadvantages**
• Easy to carry out • Inexpensive	• Unlikely to provide a representative sample • Highly dependent on individual researcher

Exercise 1C

1 Interviewers in a shopping centre collect information on the spending habits from a total of 40 shoppers.

a Explain how they could collect the information using:
i quota sampling **ii** opportunity sampling

b Which method is likely to lead to a more representative sample?

2 Describe the similarities and differences between quota sampling and stratified random sampling.

3 An interviewer asks the first 50 people he sees outside a fish and chip shop on a Friday evening about their eating habits.

a What type of sampling method did he use?

b Explain why the sampling method may not be representative.

c Suggest two improvements he could make to his data collection technique.

4 A researcher is collecting data on the radio-listening habits of people in a local town. She asks the first 5 people she sees on Monday morning entering a supermarket. The number of hours per week each person listens is given below:

4 7 6 8 2

a Use the sample data to work out a prediction for the average number of hours listened per week for the town as a whole.

b Describe the sampling method used and comment on the reliability of the data.

c Suggest two improvements to the method used.

5 In a research study on the masses of wild deer in a particular habitat, scientists catch the first 5 male deer they find and the first 5 female deer they find.

a What type of sampling method are they using?

b Give one advantage of this method.

The masses of the sampled deer are listed below.

Male (kg)	75	80	90	85	82
Female (kg)	67	72	75	68	65

c Use the sample data to compare the masses of male and female wild deer.

d Suggest two improvements the scientists could make to the sampling method.

6 The heights, in metres, of 20 ostriches are listed below:

1.8, 1.9, 2.3, 1.7, 2.1, 2.0, 2.5, 2.7, 2.5, 2.6, 2.3, 2.2, 2.4, 2.3, 2.2, 2.5, 1.9, 2.0, 2.2, 2.5

a Take an opportunity sample of size five from the data.

b Starting from the second data value, take a systematic sample of size five from the data.

c Calculate the mean height for each sample.

d State, with reasons, which sampling method is likely to be more reliable.

Hint An example of an opportunity sample from this data would be to select the first five heights from the list.

1.4 Types of data

■ **Variables or data associated with numerical observations are called quantitative variables or quantitative data.**

For example, you can give a number to shoe size so shoe size is a quantitative variable.

■ **Variables or data associated with non-numerical observations are called qualitative variables or qualitative data.**

For example, you can't give a number to hair colour (blonde, red, brunette). Hair colour is a qualitative variable.

■ **A variable that can take any value in a given range is a continuous variable.**

For example, time can take any value, e.g. 2 seconds, 2.1 seconds, 2.01 seconds etc.

■ **A variable that can take only specific values in a given range is a discrete variable.**

For example, the number of girls in a family is a discrete variable as you can't have 2.65 girls in a family.

Large amounts of data can be displayed in a frequency table or as grouped data.

■ **When data is presented in a grouped frequency table, the specific data values are not shown. The groups are more commonly known as classes.**
- **Class boundaries tell you the maximum and minimum values that belong in each class.**
- **The midpoint is the average of the class boundaries.**
- **The class width is the difference between the upper and lower class boundaries.**

Example 4

The lengths, x mm, to the nearest mm, of the forewings of a random sample of male adult butterflies are measured and shown in the table.

Length of forewing (mm)	Number of butterflies, f
30–31	2
32–33	25
34–36	30
37–39	13

a State whether length is
  i quantitative or qualitative
  ii discrete or continuous.

b Write down the class boundaries, midpoint and class width for the class 34–36.

a i Quantitative
ii Continuous

b Class boundaries 33.5 mm, 36.5 mm

Midpoint = $\frac{1}{2}(33.5 + 36.5) = 35$ mm

Class width = $36.5 - 33.5 = 3$ mm

Watch out Be careful when finding class boundaries for continuous data. The data values have been rounded to the nearest mm, so the upper class boundary for the 30–31 mm class is 31.5 mm.

Exercise 1D

1 State whether each of the following variables is qualitative or quantitative.

a Height of a tree
b Colour of car
c Time waiting in a queue
d Shoe size
e Names of pupils in a class

2 State whether each of the following quantitative variables is continuous or discrete.

a Shoe size
b Length of leaf
c Number of people on a bus
d Weight of sugar
e Time required to run 100 m
f Lifetime in hours of torch batteries

3 Explain why:

a ‘Type of tree’ is a qualitative variable
b ‘The number of pupils in a class’ is a discrete quantitative variable
c ‘The weight of a collie dog’ is a continuous quantitative variable.

4 The distribution of the masses of two-month-old lambs is shown in the grouped frequency table.

Mass, m (kg)	Frequency
$1.2 \leqslant m < 1.3$	8
$1.3 \leqslant m < 1.4$	28
$1.4 \leqslant m < 1.5$	32
$1.5 \leqslant m < 1.6$	22

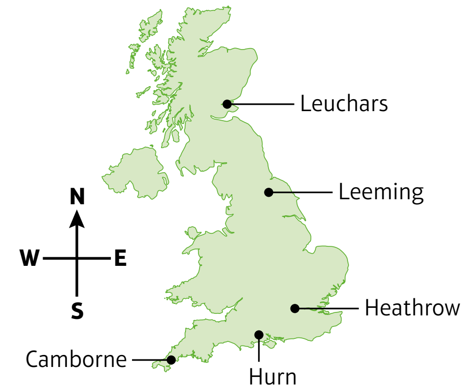
Hint The class boundaries are given using inequalities, so the values given in the table are the actual class boundaries.

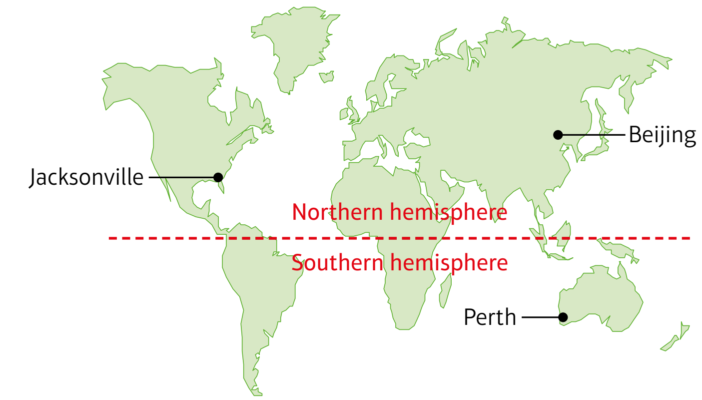
a Write down the class boundaries for the third group.
b Work out the midpoint of the second group.
c Work out the class width of the first group.

1.5 The large data set

You will need to answer questions based on real data in your exam. Some of these questions will be based on weather data from the **large data set** provided by Edexcel.

The data set consists of weather data samples provided by the Met Office for five UK weather stations and three overseas weather stations over two set periods of time: May to October 1987 and May to October 2015. The weather stations are labelled on the maps below.





The large data set contains data for a number of different variables at each weather station:

- **Daily mean temperature** in °C – this is the average of the hourly temperature readings during a 24-hour period.
- **Daily total rainfall** including solid precipitation such as snow and hail, which is melted before being included in any measurements – amounts less than 0.05 mm are recorded as ‘tr’ or ‘trace’
- **Daily total sunshine** recorded to the nearest tenth of an hour
- **Daily mean wind direction and windspeed** in knots, averaged over 24 hours from midnight to midnight. Mean wind directions are given as bearings and as cardinal (compass) directions. The data for mean windspeed is also categorised according to the **Beaufort scale**

Beaufort scale	Descriptive term	Average speed at 10 metres above ground
0	Calm	Less than 1 knot
1–3	Light	1 to 10 knots
4	Moderate	11 to 16 knots
5	Fresh	17 to 21 knots

Notation A **knot** (kn) is a ‘nautical mile per hour’. 1 kn = 1.15 mph.

- **Daily maximum gust** in knots – this is the highest instantaneous windspeed recorded. The direction from which the maximum gust was blowing is also recorded
- **Daily maximum relative humidity**, given as a percentage of air saturation with water vapour. Relative humidities above 95% give rise to misty and foggy conditions

Watch out For the overseas locations, the only data recorded are:
- Daily mean temperature
- Daily total rainfall
- Daily mean pressure
- Daily mean windspeed

- **Daily mean cloud cover** measured in 'oktas' or eighths of the sky covered by cloud
- **Daily mean visibility** measured in decametres (Dm). This is the greatest horizontal distance at which an object can be seen in daylight
- **Daily mean pressure** measured in hectopascals (hPa)

Any missing data values are indicated in the large data set as n/a or 'not available'.

Data from Hurn for the first days of June 1987 is shown to the right.

You are expected to be able to take a sample from the large data set, identify different types of data and calculate statistics from the data.

- **If you need to do calculations on the large data set in your exam, the relevant extract from the data set will be provided.**

HURN			© Crown Copyright Met Office 1987			
Date	Daily mean temperature (°C)	Daily total rainfall (mm)	Daily total sunshine (hrs)	Daily mean windspeed (kn)	Daily mean windspeed (Beaufort conversion)	Daily maximum gust (kn)
01/6/1987	15.1	0.6	4.5	7	Light	19
02/6/1987	12.5	4.7	0	7	Light	22
03/6/1987	13.8	tr	5.6	11	Moderate	25
04/6/1987	15.5	5.3	7.8	7	Light	17
05/6/1987	13.1	19.0	0.5	10	Light	33
06/6/1987	13.8	0	8.9	19	Fresh	46
07/6/1987	13.2	tr	3.8	11	Moderate	27
08/6/1987	12.9	1	1.7	9	Light	19
09/6/1987	11.2	tr	5.4	6	Light	19
10/6/1987	9.2	1.3	9.7	4	Light	n/a
11/6/1987	12.6	0	12.5	6	Light	18
12/6/1987	10.4	0	11.9	5	Light	n/a
13/6/1987	9.6	0	8.6	5	Light	15
14/6/1987	10.2	0	13.1	5	Light	18
15/6/1987	9.2	3.7	7.1	4	Light	25
16/6/1987	10.4	5.6	8.3	6	Light	25
17/6/1987	12.8	0.1	5.3	10	Light	27
18/6/1987	13.0	7.4	3.2	9	Light	24
19/6/1987	14.0	tr	0.4	12	Moderate	33
20/6/1987	12.6	0	7.7	6	Light	17

Example 5

Look at the extract from the large data set given above.

a Describe the type of data represented by daily total rainfall.

Alison is investigating daily maximum gust. She wants to select a sample of size 5 from the first 20 days in Hurn in June 1987. She uses the first two digits of the date as a sampling frame and generates five random numbers between 1 and 20.

b State the type of sample selected by Alison.

c Explain why Alison's process might not generate a sample of size 5.

a Continuous quantitative data.
b Simple random sample
c Some of the data values are not available (n/a).

Watch out Although you won't need to recall specific data values from the large data set in your exam, you will need to know the limitations of the data set and the approximate range of values for each variable.

Example 6

Using the extract from the large data set on the previous page, calculate:

a the mean daily mean temperature for the first five days of June in Hurn in 1987

b the median daily total rainfall for the week of 14th June to 20th June inclusive.

The median daily total rainfall for the same week in Perth was 19.0 mm. Karl states that more southerly countries experience higher rainfall during June.

c State with a reason whether your answer to part **b** supports this statement.

a $15.1 + 12.5 + 13.8 + 15.5 + 13.1 = 70.0$
$70.0 \div 5 = 14.0\,°C$ (1 d.p.)

The mean is the sum of the data values divided by the number of data values. The data values are given to 1 d.p. so give your answer to the same degree of accuracy.

b The values are: 0, 3.7, 5.6, 0.1, 7.4, tr, 0
In ascending order: 0, 0, tr, 0.1, 3.7, 5.6, 7.4
The median is the middle value so 0.1 mm.

Trace amounts are slightly larger than 0. If you need to do a numerical calculation involving a trace amount you can treat it as 0.

c Perth is in Australia, which is south of the UK, and the median rainfall was higher (19.0 mm > 0.1 mm). However, this is a very small sample from a single location in each country so does not provide enough evidence to support Karl's statement.

Online Use your calculator to find the mean and median of discrete data.

Problem-solving Don't just look at the numerical values. You also need to consider whether the sample is large enough, and whether there are other geographical factors which could affect rainfall in these two locations.

Exercise 1E

1 From the eight weather stations featured in the large data set, write down:
  a the station which is furthest north
  b the station which is furthest south
  c an inland station
  d a coastal station
  e an overseas station.

2 Explain, with reasons, whether daily maximum relative humidity is a discrete or continuous variable.

Questions 3 and 4 in this exercise use the following extracts from the large data set.

LEEMING **© Crown Copyright Met Office 2015**

Date	Daily mean temperature (°C)	Daily total rainfall (mm)	Daily total sunshine (hrs)	Daily mean windspeed (kn)
01/06/2015	8.9	10	5.1	15
02/06/2015	10.7	tr	8.9	17
03/06/2015	12.0	0	10.0	8
04/06/2015	11.7	0	12.8	7
05/06/2015	15.0	0	8.9	9
06/06/2015	11.6	tr	5.4	17
07/06/2015	12.6	0	13.9	10
08/06/2015	9.4	0	9.7	7
09/06/2015	9.7	0	12.1	5
10/06/2015	11.0	0	14.6	4

HEATHROW **© Crown Copyright Met Office 2015**

Date	Daily mean temperature (°C)	Daily total rainfall (mm)	Daily total sunshine (hrs)	Daily mean windspeed (kn)
01/06/2015	12.1	0.6	4.1	15
02/06/2015	15.4	tr	1.6	18
03/06/2015	15.8	0	9.1	9
04/06/2015	16.1	0.8	14.4	6
05/06/2015	19.6	tr	5.3	9
06/06/2015	14.5	0	12.3	12
07/06/2015	14.0	0	13.1	5
08/06/2015	14.0	tr	6.4	7
09/06/2015	11.4	0	2.5	10
10/06/2015	14.3	0	7.2	10

(P) **3** **a** Work out the mean of the daily total sunshine for the first 10 days of June 2015 in:
i Leeming
ii Heathrow.

b Work out the range of the daily total sunshine for the first 10 days of June 2015 in:
i Leeming
ii Heathrow.

c Supraj says that the further north you are, the fewer the number of hours of sunshine. State, with reasons, whether your answers to parts **a** and **b** support this conclusion.

Hint State in your answer whether Leeming is north or south of Heathrow.

(P) **4** Calculate the mean daily total rainfall in Heathrow for the first 10 days of June 2015. Explain clearly how you dealt with the data for 2/6/2015, 5/6/2015 and 8/6/2015.

(P) **5** Dominic is interested in seeing how the average monthly temperature changed over the summer months of 2015 in Jacksonville. He decides to take a sample of two days every month and average the temperatures before comparing them.

a Give one reason why taking two days a month might be:
i a good sample size
ii a poor sample size.

b He chooses the first day of each month and the last day of each month. Give a reason why this method of choosing days might not be representative.

c Suggest a better way that he can choose his sample of days.

(P) **6** The table shows the mean daily temperatures at each of the eight weather stations for August 2015:

	Camborne	Heathrow	Hurn	Leeming	Leuchars	Beijing	Jacksonville	Perth
Mean daily mean temp (°C)	15.4	18.1	16.2	15.6	14.7	26.6	26.4	13.6

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a Give a geographical reason why the temperature in August might be lower in Perth than in Jacksonville.

b Comment on whether this data supports the conclusion that coastal locations experience lower average temperatures than inland locations.

(P) **7** Brian calculates the mean cloud coverage in Leeming in September 1987. He obtains the answer 9.3 oktas. Explain how you know that Brian's answer is incorrect.

(E/P) **8** The large data set provides data for 184 consecutive days in 1987. Marie is investigating daily mean windspeeds in Camborne in 1987.

a Describe how Marie could take a systematic sample of 30 days from the data for Camborne in 1987. **(3 marks)**

b Explain why Marie's sample would not necessarily give her 30 data points for her investigation. **(1 mark)**

Large data set

You will need access to the large data set and spreadsheet software to answer these questions.

1 a Find the mean daily mean pressure in Beijing in October 1987.

b Find the median daily rainfall in Jacksonville in July 2015.

c i Draw a grouped frequency table for the daily mean temperature in Heathrow in July and August 2015. Use intervals $10 \leqslant t < 15$, etc.

ii Draw a histogram to display this data.

iii Draw a frequency polygon for this data.

Hint You can use the **CountIf** command in a spreadsheet to work out the frequency for each class.

2 a i Take a simple random sample of size 10 from the data for daily mean windspeed in Leeming in 1987.

ii Work out the mean of the daily windspeeds using your sample.

b i Take a sample of the last 10 values from the data for daily mean windspeed in Leuchars in 1987.

ii Work out the mean of the daily mean windspeeds using your sample.

c State, with reasons, which of your samples is likely to be more representative.

d Suggest two improvements to the sampling methods suggested in part **a**.

e Use an appropriate sampling method and sample size to estimate the mean windspeeds in Leeming and Leuchars in 1987. State with a reason whether your calculations support the statement 'Coastal locations are likely to have higher average windspeeds than inland locations'.

Mixed exercise 1

1 The table shows the daily mean temperature recorded on the first 15 days in May 1987 at Heathrow.

Day of month	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Daily mean temp (°C)	14.6	8.8	7.2	7.3	10.1	11.9	12.2	12.1	15.2	11.1	10.6	12.7	8.9	10.0	9.5

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a Use an opportunity sample of the first 5 dates in the table to estimate the mean daily mean temperature at Heathrow for the first 15 days of May 1987.

b Describe how you could use the random number function on your calculator to select a simple random sample of 5 dates from this data.

Hint Make sure you describe your sampling frame.

c Use a simple random sample of 5 dates to estimate the mean daily mean temperature at Heathrow for the first 15 days of May 1987.

d Use all 15 dates to calculate the mean daily mean temperature at Heathrow for the first 15 days of May 1987. Comment on the reliability of your two samples.

2 a Give one advantage and one disadvantage of using:

i a census **ii** a sample survey.

b It is decided to take a sample of 100 from a population consisting of 500 elements. Explain how you would obtain a simple random sample from this population.

3 a Explain briefly what is meant by:

i a population **ii** a sampling frame.

b A market research organisation wants to take a sample of:

i owners of diesel motor cars in the UK

ii persons living in Oxford who suffered injuries to the back during July 1996.

Suggest a suitable sampling frame in each case.

4 Write down one advantage and one disadvantage of using:

a stratified sampling **b** simple random sampling.

5 The managing director of a factory wants to know what the workers think about the factory canteen facilities. 100 people work in the offices and 200 work on the shop floor.

The factory manager decides to ask the people who work in the offices.

a Suggest a reason why this is likely to produce a biased sample.

b Explain briefly how the factory manager could select a sample of 30 workers using:

i systematic sampling **ii** stratified sampling **iii** quota sampling.

6 There are 64 girls and 56 boys in a school.

Explain briefly how you could take a random sample of 15 pupils using:

a simple random sampling **b** stratified sampling.

7 As part of her statistics project, Deepa decided to estimate the amount of time A-level students at her school spent on private study each week. She took a random sample of students from those studying arts subjects, science subjects and a mixture of arts and science subjects. Each student kept a record of the time they spent on private study during the third week of term.

a Write down the name of the sampling method used by Deepa.

b Give a reason for using this method and give one advantage this method has over simple random sampling.

8 A conservationist is collecting data on African springboks. She catches the first five springboks she finds and records their masses.

a State the sampling method used.

b Give one advantage of this type of sampling method.

The data is given below:

70 kg 76 kg 82 kg 74 kg 78 kg.

c State, with a reason, whether this data is discrete or continuous.

d Calculate the mean mass.

A second conservationist collects data by selecting one springbok in each of five locations. The data collected is given below:

79 kg 86 kg 90 kg 68 kg 75 kg.

e Calculate the mean mass for this sample.

f State, with a reason, which mean mass is likely to be a more reliable estimate of the mean mass of African springboks.

g Give one improvement the second conservationist could make to the sampling method.

E **9** Data on the daily total rainfall in Beijing during 2015 is gathered from the large data set. The daily total rainfall (in mm) on the first of each month is listed below:

May 1st	9.0
June 1st	0.0
July 1st	1.0
August 1st	32.0
September 1st	4.1
October 1st	3.0

a State, with a reason, whether or not this sample is random. **(1 mark)**

b Suggest two alternative sampling methods and give one advantage and one disadvantage of each in this context. **(2 marks)**

c State, with a reason, whether the data is discrete or continuous. **(1 mark)**

d Calculate the mean of the six data values given above. **(1 mark)**

e Comment on the reliability of this value as an estimate for the mean daily total rainfall in Beijing during 2015. **(1 mark)**

Large data set

You will need access to the large data set and spreadsheet software to answer these questions.

a Take a systematic sample of size 18 for the daily maximum relative humidity in Camborne during 1987.

b Give one advantage of using a systematic sample in this context.

c Use your sample to find an estimate for the mean daily maximum relative humidity in Camborne during 1987.

d Comment on the reliability of this estimate. Suggest one way in which the reliability can be improved.

Summary of key points

1 • In statistics, a **population** is the whole set of items that are of interest.
• A **census** observes or measures every member of a population.

2 • A sample is a selection of observations taken from a subset of the population which is used to find out information about the population as a whole.
• Individual units of a population are known as **sampling units**.
• Often sampling units of a population are individually named or numbered to form a list called a **sampling frame**.

3 • A **simple random sample** of size n is one where every sample of size n has an equal chance of being selected.
• In **systematic sampling**, the required elements are chosen at regular intervals from an ordered list.
• In **stratified sampling**, the population is divided into mutually exclusive strata (males and females, for example) and a random sample is taken from each.
• In **quota sampling**, an interviewer or researcher selects a sample that reflects the characteristics of the whole population.
• **Opportunity sampling** consists of taking the sample from people who are available at the time the study is carried out and who fit the criteria you are looking for.

4 • Variables or data associated with numerical observations are called **quantitative variables** or **quantitative data**.
• Variables or data associated with non-numerical observations are called **qualitative variables** or **qualitative data**.

5 • A variable that can take any value in a given range is a **continuous variable**.
• A variable that can take only specific values in a given range is a **discrete variable**.

6 • When data is presented in a grouped frequency table, the specific data values are not shown. The groups are more commonly known as **classes**.
• Class boundaries tell you the maximum and minimum values that belong in each class.
• The midpoint is the average of the class boundaries.
• The class width is the difference between the upper and lower class boundaries.

7 If you need to do calculations on the large data set in your exam, the relevant extract from the data set will be provided.

9 Constant acceleration

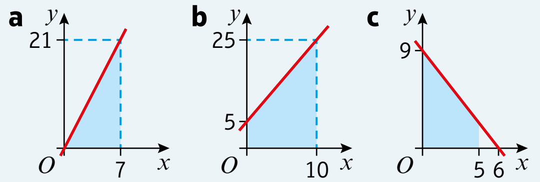
Objectives

After completing this chapter you should be able to:

- Understand and interpret displacement–time graphs → pages 131–133
- Understand and interpret velocity–time graphs → pages 133–136
- Derive the constant acceleration formulae and use them to solve problems → pages 137–146
- Use the constant acceleration formulae to solve problems involving vertical motion under gravity → pages 146–152

Prior knowledge check

1 For each graph find:
i the gradient
ii the shaded area under the graph.



2 A car travels for 45 minutes at an average speed of 35 mph. Find the distance travelled.

3 **a** Solve the simultaneous equations:

$$3x - 2y = 9$$
$$x + 4y + 4 = 0$$

b Solve $2x^2 + 3x - 7 = 0$. Give your answers to 3 s.f.

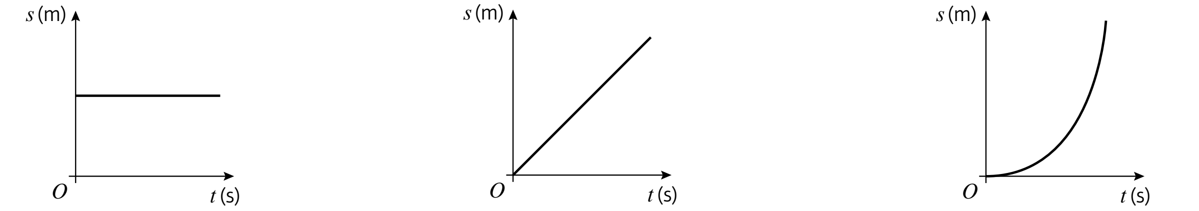
A body falling freely under **gravity** can be modelled as having **constant acceleration**. You can use this to estimate the time it will take a cliff diver to reach the water.

→ Exercise 9E Q1

9.1 Displacement–time graphs

You can represent the motion of an object on a displacement–time graph. Displacement is always plotted on the vertical axis and time on the horizontal axis.

In these graphs s represents the displacement of an object from a given point in metres and t represents the time taken in seconds.



There is no change in the displacement over time and the object is stationary.

The displacement increases at a constant rate over time and the object is moving with constant velocity.

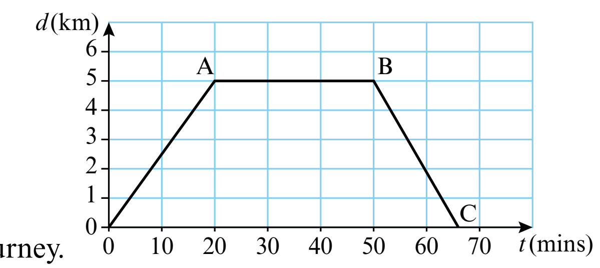
The displacement is increasing at a greater rate as time increases. The velocity is increasing and the object is accelerating.

- **Velocity is the rate of change of displacement.**
  - **On a displacement–time graph the gradient represents the velocity.**
  - **If the displacement–time graph is a straight line, then the velocity is constant.**
- **Average velocity** = $\dfrac{\textbf{displacement from starting point}}{\textbf{time taken}}$
- **Average speed** = $\dfrac{\textbf{total distance travelled}}{\textbf{time taken}}$

Example 1

A cyclist rides in a straight line for 20 minutes. She waits for half an hour, then returns in a straight line to her starting point in 15 minutes. This is a displacement–time graph for her journey.

a Work out the average velocity for each stage of the journey in km h^{-1}.
b Write down the average velocity for the whole journey.
c Work out the average speed for the whole journey.



a Journey from O to A: time = 20 mins; displacement = 5 km

Average velocity = $\frac{5}{20}$ = 0.25 km min^{-1}

$0.25 \times 60 = 15$ km h^{-1}

Journey from A to B: no change in displacement so average velocity = 0

Journey from B to C: time = 15 mins; displacement = −5 km

Average velocity = $\frac{-5}{15} = -\frac{1}{3}$ km min^{-1}

$-\frac{1}{3} \times 60 = -20$ km h^{-1}

To convert from km min^{-1} to km h^{-1} multiply by 60.

A horizontal line on the graph indicates the cyclist is stationary.

The cyclist starts with a displacement of 5 km and finishes with a displacement of 0 km, so the change in displacement is −5 km, and velocity will be negative.

b The displacement for the whole journey is 0 so average velocity is 0.

c Total time = 65 mins

Total distance travelled is 5 + 5 = 10 km

Average speed = $\frac{10}{65} = \frac{2}{13}$ km min^{-1}

$\frac{2}{13} \times 60 = 9.2$ km h^{-1} (2 s.f.)

At C the cyclist has returned to the starting point.

The cyclist has travelled 5 km away from the starting point and then 5 km back to the starting point.

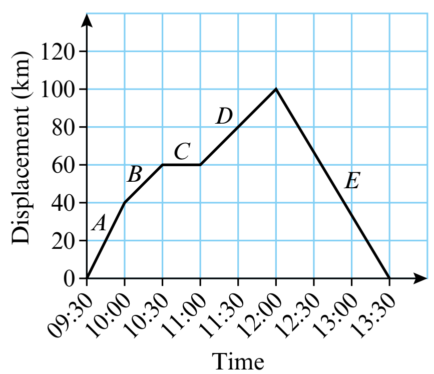
Exercise 9A

1 This is a displacement–time graph for a car travelling along a straight road. The journey is divided into 5 stages labelled A to E.

a Work out the average velocity for each stage of the journey.

b State the average velocity for the whole journey.

c Work out the average speed for the whole journey.



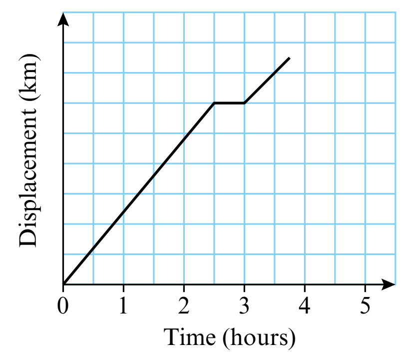
(P) 2 Khalid drives from his home to a hotel. He drives for $2\frac{1}{2}$ hours at an average velocity of 60 km h^{-1}. He then stops for lunch before continuing to his hotel. The diagram shows a displacement–time graph for Khalid's journey.

a Work out the displacement of the hotel from Khalid's home.

b Work out Khalid's average velocity for his whole journey.

Problem-solving

You need to work out the scale on the vertical axis.



(P) 3 Sarah left home at 10:00 and cycled north in a straight line. The diagram shows a displacement–time graph for her journey.

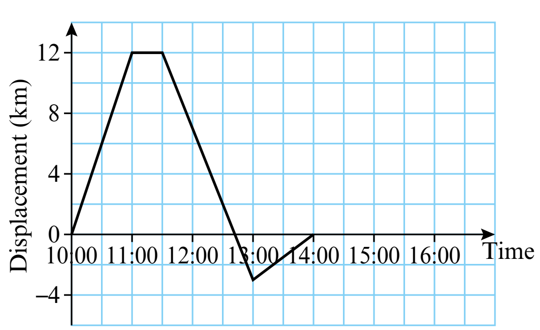
a Work out Sarah's velocity between 10:00 and 11:00.

On her return journey, Sarah continued past her home before returning.

b Estimate the time that Sarah passed her home.

c Work out Sarah's velocity for each of the last two stages of her journey.

d Calculate Sarah's average speed for her entire journey.



(P) 4 A ball is thrown vertically up in the air and falls to the ground. This is a displacement–time graph for the motion of the ball.

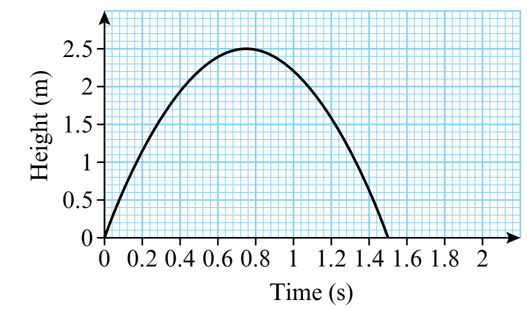
a Find the maximum height of the ball and the time at which it reaches that height.

b Write down the velocity of the ball when it reaches its highest point.

c Describe the motion of the ball:

i from the time it is thrown to the time it reaches its highest point

ii after reaching its highest point.

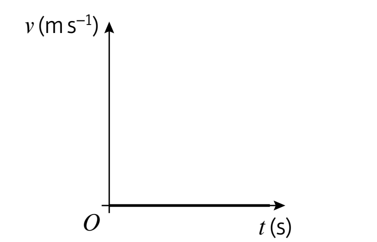


Hint To describe the motion you should state the direction of travel of the ball and whether it is accelerating or decelerating.

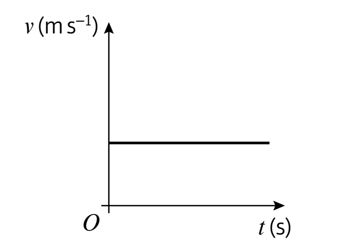
9.2 Velocity–time graphs

You can represent the motion of an object on a velocity–time graph. Velocity is always plotted on the vertical axis and time on the horizontal axis.

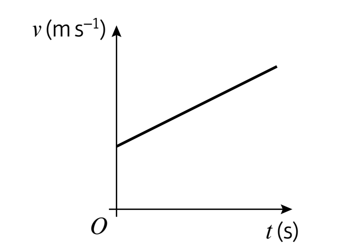
In these graphs v represents the velocity of an object in metres per second and t represents the time taken in seconds.



The velocity is zero and the object is stationary.



The velocity is unchanging and the object is moving with constant velocity.

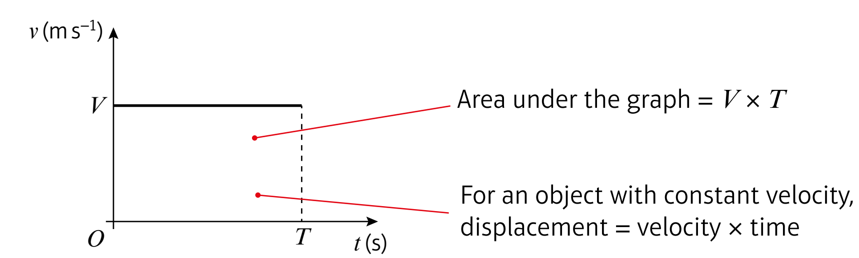


The velocity is increasing at a constant rate and the object is moving with constant acceleration.

- **Acceleration is the rate of change of velocity.**
  - **In a velocity–time graph the gradient represents the acceleration.**
  - **If the velocity–time graph is a straight line, then the acceleration is constant.**

Notation Negative acceleration is sometimes described as deceleration or retardation.

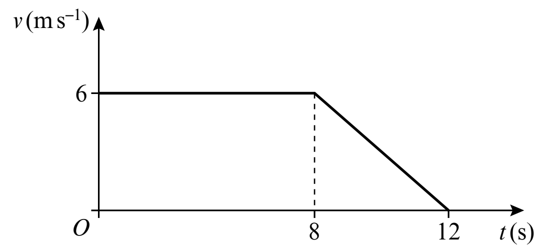
This velocity–time graph represents the motion of an object travelling in a straight line at constant velocity V m s^{-1} for time T seconds.



- **The area between a velocity–time graph and the horizontal axis represents the distance travelled.**
  - **For motion in a straight line with positive velocity, the area under the velocity–time graph up to a point t represents the displacement at time t.**

Example 2

The figure shows a velocity–time graph illustrating the motion of a cyclist moving along a straight road for a period of 12 seconds. For the first 8 seconds, she moves at a constant speed of 6 m s^{-1}. She then decelerates at a constant rate, stopping after a further 4 seconds.



a Find the displacement from the starting point of the cyclist after this 12 second period.

b Work out the rate at which the cyclist decelerates.

a The displacement s after 12 s is given by the area under the graph.

$s = \frac{1}{2}(a + b)h$

$= \frac{1}{2}(8 + 12) \times 6$

$= 10 \times 6 = 60$

The displacement of the cyclist after 12 s is 60 m.

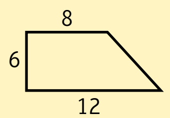
b The acceleration is the gradient of the slope.

$a = \frac{-6}{4} = -1.5$

The deceleration is 1.5 m s^{-2}.

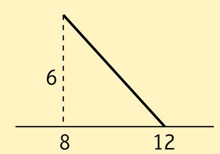
Model the cyclist as a particle moving in a straight line.

The displacement is represented by the area of the trapezium with these sides.



You can use the formula for the area of a trapezium to calculate this area.

The gradient is given by $\frac{\text{difference in the } v\text{-coordinates}}{\text{difference in the } t\text{-coordinates}}$



Example 3

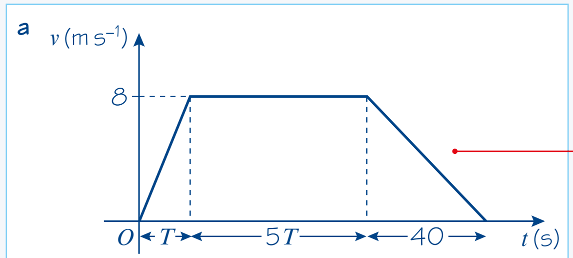
A particle moves along a straight line. The particle accelerates uniformly from rest to a velocity of 8 m s^{-1} in T seconds. The particle then travels at a constant velocity of 8 m s^{-1} for $5T$ seconds. The particle then decelerates uniformly to rest in a further 40 s.

a Sketch a velocity–time graph to illustrate the motion of the particle.

Given that the total displacement of the particle is 600 m:

b find the value of T.

a



If the particle accelerates from rest and decelerates to rest this means the initial and final velocities are zero.

Online Explore how the area of the trapezium changes as the value of T changes using technology.

b The area of the trapezium is:

$s = \frac{1}{2}(a + b)h$

$= \frac{1}{2}(5T + 6T + 40) \times 8$

$= 4(11T + 40)$

The displacement is 600 m.

$4(11T + 40) = 600$

$44T + 160 = 600$

$T = \frac{600 - 160}{44} = 10$

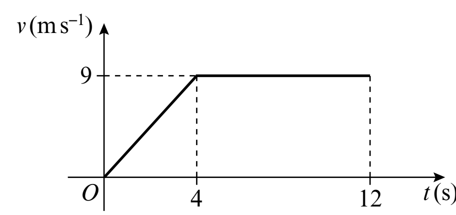
The length of the shorter of the two parallel sides is $5T$. The length of the longer side is $T + 5T + 40 = 6T + 40$.

Problem-solving

The displacement is equal to the area of the trapezium. Write an equation and solve it to find T.

Exercise 9B

1 The diagram shows the velocity–time graph of the motion of an athlete running along a straight track. For the first 4 s, he accelerates uniformly from rest to a velocity of 9 m s^{-1}.
This velocity is then maintained for a further 8 s. Find:



a the rate at which the athlete accelerates

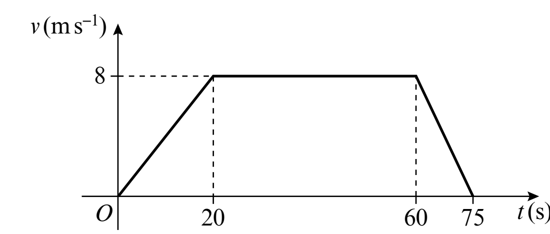
b the displacement from the starting point of the athlete after 12 s.

2 A car is moving along a straight road. When $t = 0$ s, the car passes a point A with velocity 10 m s^{-1} and this velocity is maintained until $t = 30$ s. The driver then applies the brakes and the car decelerates uniformly, coming to rest at the point B when $t = 42$ s.

a Sketch a velocity–time graph to illustrate the motion of the car.

b Find the distance from A to B.

E **3** The diagram shows the velocity–time graph of the motion of a cyclist riding along a straight road. She accelerates uniformly from rest to 8 m s^{-1} in 20 s. She then travels at a constant velocity of 8 m s^{-1} for 40 s. She then decelerates uniformly to rest in 15 s. Find:



a the acceleration of the cyclist in the first 20 s of motion **(2 marks)**

b the deceleration of the cyclist in the last 15 s of motion **(2 marks)**

c the displacement from the starting point of the cyclist after 75 s. **(2 marks)**

E **4** A motorcyclist starts from rest at a point S on a straight race track. He moves with constant acceleration for 15 s, reaching a velocity of 30 m s^{-1}. He then travels at a constant velocity of 30 m s^{-1} for T seconds. Finally he decelerates at a constant rate coming to rest at a point F, 25 s after he begins to decelerate.

a Sketch a velocity–time graph to illustrate the motion. **(3 marks)**

Given that the distance between S and F is 2.4 km:

b calculate the time the motorcyclist takes to travel from S to F. **(3 marks)**

E **5** A train starts from a station X and moves with constant acceleration of $0.6\,\mathrm{m\,s^{-2}}$ for 20 s. The velocity it has reached after 20 s is then maintained for T seconds. The train then decelerates from this velocity to rest in a further 40 s, stopping at a station Y.

a Sketch a velocity–time graph to illustrate the motion of the train. **(3 marks)**

Given that the distance between the stations is 4.2 km, find:

b the value of T **(3 marks)**

c the distance travelled by the train while it is moving with constant velocity. **(2 marks)**

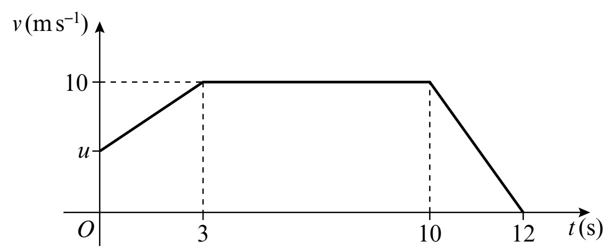
E **6** A particle moves along a straight line. The particle accelerates from rest to a velocity of $10\,\mathrm{m\,s^{-1}}$ in 15 s. The particle then moves at a constant velocity of $10\,\mathrm{m\,s^{-1}}$ for a period of time. The particle then decelerates uniformly to rest. The period of time for which the particle is travelling at a constant velocity is 4 times the period of time for which it is decelerating.

a Sketch a velocity–time graph to illustrate the motion of the particle. **(3 marks)**

Given that the displacement from the starting point of the particle after it comes to rest is 480 m

b find the total time for which the particle is moving. **(3 marks)**

E **7** A particle moves 100 m in a straight line. The diagram is a sketch of a velocity–time graph of the motion of the particle. The particle starts with velocity $u\,\mathrm{m\,s^{-1}}$ and accelerates to a velocity of $10\,\mathrm{m\,s^{-1}}$ in 3 s. The velocity of $10\,\mathrm{m\,s^{-1}}$ is maintained for 7 s and then the particle decelerates to rest in a further 2 s. Find:



a the value of u **(3 marks)**

b the acceleration of the particle in the first 3 s of motion. **(3 marks)**

E **8** A motorcyclist M leaves a road junction at time $t = 0$ s. She accelerates from rest at a rate of $3\,\mathrm{m\,s^{-2}}$ for 8 s and then maintains the velocity she has reached. A car C leaves the same road junction as M at time $t = 0$ s. The car accelerates from rest to $30\,\mathrm{m\,s^{-1}}$ in 20 s and then maintains the velocity of $30\,\mathrm{m\,s^{-1}}$. C passes M as they both pass a pedestrian.

a On the same diagram, sketch velocity–time graphs to illustrate the motion of M and C. **(3 marks)**

b Find the distance of the pedestrian from the road junction. **(3 marks)**

Challenge

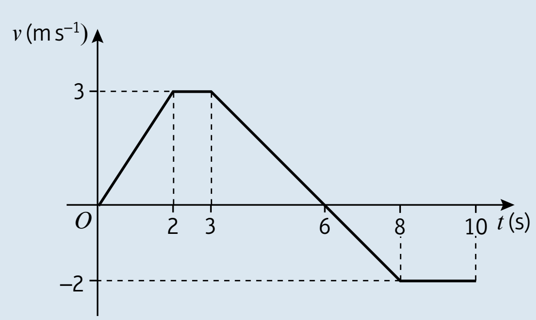
The graph shows the velocity of an object travelling in a straight line during a 10-second time interval.

a After how long did the object change direction?

b Work out the total distance travelled by the object.

c Work out the displacement from the starting point of the object after:

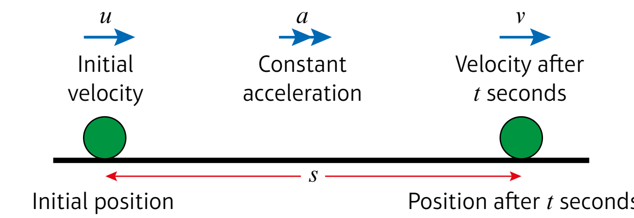
i 6 seconds **ii** 10 seconds.



9.3 Constant acceleration formulae 1

A standard set of letters is used for the motion of an object moving in a straight line with constant acceleration.

- s is the displacement.
- u is the initial velocity.
- v is the final velocity.
- a is the acceleration.
- t is the time.

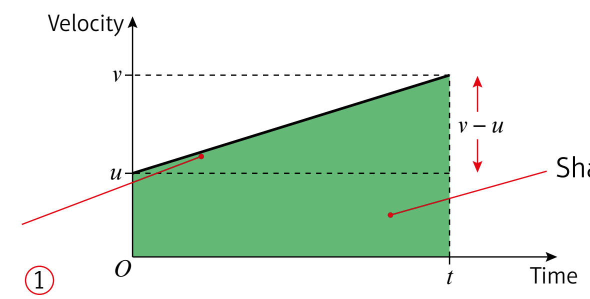


You can use these letters to label a velocity–time graph representing the motion of a particle moving in a straight line accelerating from velocity u at time 0 to velocity v at time t.

Gradient of line $= \dfrac{v-u}{t}$

$a = \dfrac{v-u}{t}$

Rearranging $v = u + at$ ①



Shaded area $= \left(\dfrac{u+v}{2}\right)t$

$s = \left(\dfrac{u+v}{2}\right)t$ ②

- $\boldsymbol{v = u + at}$ ①
- $\boldsymbol{s = \left(\dfrac{u+v}{2}\right)t}$ ②

Hint Formula ① does not involve s and formula ② does not involve a.

You need to know how to derive these formulae from the velocity–time graph.

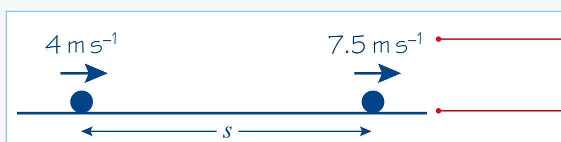
Links These formulae can also be derived using calculus. → Chapter 11

Example 4

A cyclist is travelling along a straight road. She accelerates at a constant rate from a velocity of $4\,\mathrm{m\,s^{-1}}$ to a velocity of $7.5\,\mathrm{m\,s^{-1}}$ in 40 seconds. Find:

a the distance she travels in these 40 seconds

b her acceleration in these 40 seconds.



$u = 4,\ v = 7.5,\ t = 40,\ s = ?,\ a = ?$

Start by drawing a diagram.

Model the cyclist as a particle.

Write down the values you know and the values you need to find.

a $s = \left(\frac{u + v}{2}\right)t$

$= \left(\frac{4 + 7.5}{2}\right) \times 40$

$= 230$

The distance the cyclist travels is 230 m.

b $v = u + at$

$7.5 = 4 + 40a$

$a = \frac{7.5 - 4}{40} = 0.0875$

The acceleration of the cyclist is $0.0875\ \mathrm{m\,s^{-2}}$.

You need a and you know v, u and t so you can use $v = u + at$.

Substitute the values you know into the formula. You can solve this equation to find a.

You could rearrange the formula before you substitute the values:

$a = \frac{v - u}{t}$

In real-life situations values for the acceleration are often quite small.
Large accelerations feel unpleasant and may be dangerous.

Example 5

A particle moves in a straight line from a point A to a point B with constant deceleration $1.5\ \mathrm{m\,s^{-2}}$. The velocity of the particle at A is $8\ \mathrm{m\,s^{-1}}$ and the velocity of the particle at B is $2\ \mathrm{m\,s^{-1}}$. Find:

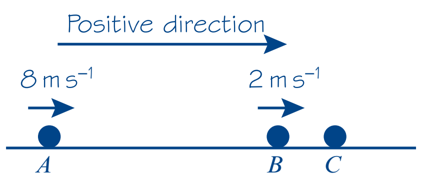
a the time taken for the particle to move from A to B

b the distance from A to B.

After reaching B the particle continues to move along the straight line with constant deceleration $1.5\ \mathrm{m\,s^{-2}}$. The particle is at the point C 6 seconds after passing through the point A. Find:

c the velocity of the particle at C

d the distance from A to C.



$u = 8,\ v = 2,\ a = -1.5,\ t = ?,\ s = ?$

a $v = u + at$

$2 = 8 - 1.5t$

$1.5t = 8 - 2$

$t = \frac{8 - 2}{1.5} = 4$

The time taken to move from A to B is 4 s.

b $s = \left(\frac{u + v}{2}\right)t$

$= \left(\frac{8 + 2}{2}\right) \times 4 = 20$

The distance from A to B is 20 m.

Problem-solving

It's always a good idea to draw a sketch showing the positions of the particle. Mark the positive direction on your sketch, and remember that when the particle is **decelerating**, your value of a will be **negative**.

You can use your answer from part **a** as the value of t.

c $u = 8,\ a = -1.5,\ t = 6,\ v = ?$

$v = u + at$

$= 8 + (-1.5) \times 6$

$= 8 - 9 = -1$

The velocity of the particle is $1\ \mathrm{m\,s^{-1}}$ in the direction $\overrightarrow{BA}$.

d $s = \left(\frac{u + v}{2}\right)t$

$= \left(\frac{8 + (-1)}{2}\right) \times 6$

The distance from A to C is 21 m.

The velocity at C is negative. This means that the particle is moving from right to left.

Remember that to specify a velocity it is necessary to give speed and direction.

Make sure you use the correct sign when substituting a negative value into a formula.

Convert all your measurements into base SI units before substituting values into the formulae.

Example 6

A car moves from traffic lights along a straight road with constant acceleration. The car starts from rest at the traffic lights and 30 seconds later the car passes a speed-trap where it is registered as travelling at $45\ \mathrm{km\,h^{-1}}$. Find:

a the acceleration of the car **b** the distance between the traffic lights and the speed-trap.

$45\ \mathrm{km\,h^{-1}} = 45 \times \frac{1000}{3600}\ \mathrm{m\,s^{-1}} = 12.5\ \mathrm{m\,s^{-1}}$

$u = 0,\ v = 12.5,\ t = 30,\ a = ?,\ s = ?$

a $v = u + at$

$12.5 = 0 + 30a$

$a = \frac{12.5}{30} = \frac{5}{12}$

The acceleration of the car is $\frac{5}{12}\ \mathrm{m\,s^{-2}}$

b $s = \left(\frac{u + v}{2}\right)t$

$= \left(\frac{0 + 12.5}{2}\right) \times 30 = 187.5$

The distance between the traffic lights and the speed-trap is 187.5 m.

Convert into SI units, using:
1 km = 1000 m
1 hour = 60 × 60 s = 3600 s

Model the car as a particle and draw a diagram.

The car starts from rest, so the initial velocity is zero.

This is an exact answer. If you want to give an answer using decimals, you should round to three significant figures.

Exercise 9C

1 A particle is moving in a straight line with constant acceleration $3\,\text{m s}^{-2}$. At time $t = 0$, the velocity of the particle is $2\,\text{m s}^{-1}$. Find the velocity of the particle at time $t = 6$ s.

2 A car is approaching traffic lights. The car is travelling with velocity $10\,\text{m s}^{-1}$. The driver applies the brakes to the car and the car comes to rest with constant deceleration in 16 s. Modelling the car as a particle, find the deceleration of the car.

3 A car accelerates uniformly while travelling on a straight road. The car passes two signposts 360 m apart. The car takes 15 s to travel from one signpost to the other. When passing the second signpost, it has velocity $28\,\text{m s}^{-1}$. Find the velocity of the car at the first signpost.

4 A cyclist is moving along a straight road from A to B with constant acceleration $0.5\,\text{m s}^{-2}$. Her velocity at A is $3\,\text{m s}^{-1}$ and it takes her 12 seconds to cycle from A to B. Find:

a her velocity at B

b the distance from A to B.

5 A particle is moving along a straight line with constant acceleration from a point A to a point B, where $AB = 24$ m. The particle takes 6 s to move from A to B and the velocity of the particle at B is $5\,\text{m s}^{-1}$. Find:

a the velocity of the particle at A

b the acceleration of the particle.

6 A particle moves in a straight line from a point A to a point B with constant deceleration $1.2\,\text{m s}^{-2}$. The particle takes 6 s to move from A to B. The speed of the particle at B is $2\,\text{m s}^{-1}$ and the direction of motion of the particle has not changed. Find:

a the speed of the particle at A

b the distance from A to B.

P 7 A train, travelling on a straight track, is slowing down with constant deceleration $0.6\,\text{m s}^{-2}$. The train passes one signal with speed $72\,\text{km h}^{-1}$ and a second signal 25 s later. Find:

a the velocity, in km h^{-1}, of the train as it passes the second signal

b the distance between the signals.

Hint Convert the speeds into m s^{-1} before substituting.

8 A particle moves in a straight line from a point A to a point B with a constant deceleration of $4\,\text{m s}^{-2}$. At A the particle has velocity $32\,\text{m s}^{-1}$ and the particle comes to rest at B. Find:

a the time taken for the particle to travel from A to B

b the distance between A and B.

E 9 A skier travelling in a straight line up a hill experiences a constant deceleration. At the bottom of the hill, the skier has a velocity of $16\,\text{m s}^{-1}$ and, after moving up the hill for 40 s, he comes to rest. Find:

a the deceleration of the skier **(2 marks)**

b the distance from the bottom of the hill to the point where the skier comes to rest. **(4 marks)**

E 10 A particle is moving in a straight line with constant acceleration. The points A, B and C lie on this line. The particle moves from A through B to C. The velocity of the particle at A is $2\,\text{m s}^{-1}$ and the velocity of the particle at B is $7\,\text{m s}^{-1}$. The particle takes 20 s to move from A to B.

a Find the acceleration of the particle. **(2 marks)**

The velocity of the particle is C is $11\,\text{m s}^{-1}$. Find:

b the time taken for the particle to move from B to C **(2 marks)**

c the distance between A and C. **(3 marks)**

E 11 A particle moves in a straight line from A to B with constant acceleration $1.5\,\text{m s}^{-2}$. It then moves along the same straight line from B to C with a different acceleration. The velocity of the particle at A is $1\,\text{m s}^{-1}$ and the velocity of the particle at C is $43\,\text{m s}^{-1}$. The particle takes 12 s to move from A to B and 10 s to move from B to C. Find:

a the velocity of the particle at B **(2 marks)**

b the acceleration of the particle as it moves from B to C **(2 marks)**

c the distance from A to C. **(3 marks)**

E/P 12 A cyclist travels with constant acceleration $x\,\text{m s}^{-2}$, in a straight line, from rest to $5\,\text{m s}^{-1}$ in 20 s. She then decelerates from $5\,\text{m s}^{-1}$ to rest with constant deceleration $\frac{1}{2}x\,\text{m s}^{-2}$. Find:

a the value of x **(2 marks)**

b the total distance she travelled. **(4 marks)**

Problem-solving You could sketch a velocity–time graph of the cyclist's motion and use the area under the graph to find the total distance travelled.

E/P 13 A particle is moving with constant acceleration in a straight line. It passes through three points, A, B and C, with velocities $20\,\text{m s}^{-1}$, $30\,\text{m s}^{-1}$ and $45\,\text{m s}^{-1}$ respectively. The time taken to move from A to B is t_1 seconds and the time taken to move from B to C is t_2 seconds.

a Show that $\frac{t_1}{t_2} = \frac{2}{3}$. **(3 marks)**

Given also that the total time taken for the particle to move from A to C is 50 s:

b find the distance between A and B. **(5 marks)**

Challenge

A particle moves in a straight line from A to B with constant acceleration. The particle moves from A with velocity $3\,\text{m s}^{-1}$. It reaches point B with velocity $5\,\text{m s}^{-1}$ t seconds later.

One second after the first particle leaves point A, a second particle also starts to move in a straight line from A to B with constant acceleration. Its velocity at point A is $4\,\text{m s}^{-1}$ and it reaches point B with velocity $8\,\text{m s}^{-1}$ at the same time as the first particle.

Find:

a the value of t

b the distance between A and B.

Problem-solving The time taken for the second particle to travel from A to B is $(t - 1)$ seconds.

9.4 Constant acceleration formulae 2

You can use the formulae $v = u + at$ and $s = \left(\frac{u+v}{2}\right)t$ to work out three more formulae.

You can eliminate t from the formulae for constant acceleration.

$t = \frac{v-u}{a}$ — Rearrange the formula $v = u + at$ to make t the subject.

$s = \left(\frac{u+v}{2}\right)\left(\frac{v-u}{a}\right)$ — Substitute this expression for t into $s = \left(\frac{u+v}{2}\right)t$.

$2as = v^2 - u^2$

- $\mathbf{v^2 = u^2 + 2as}$ — Multiply out the brackets and rearrange.

You can also eliminate v from the formulae for constant acceleration.

$s = \left(\frac{u+u+at}{2}\right)t$ — Substitute $v = u + at$ into $s = \left(\frac{u+v}{2}\right)t$.

$s = \left(\frac{2u}{2} + \frac{at}{2}\right)t$

$s = \left(u + \frac{1}{2}at\right)t$ — Multiply out the brackets and rearrange.

- $\mathbf{s = ut + \frac{1}{2}at^2}$

Finally, you can eliminate u by substituting into this formula:

$s = (v - at)t + \frac{1}{2}at^2$ — Substitute $u = v - at$ into $s = ut + \frac{1}{2}at^2$.

- $\mathbf{s = vt - \frac{1}{2}at^2}$

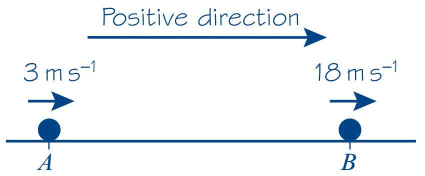
■ **You need to be able to use and to derive the five formulae for solving problems about particles moving in a straight line with constant acceleration.**

- $\mathbf{v = u + at}$
- $\mathbf{s = \left(\frac{u+v}{2}\right)t}$
- $\mathbf{v^2 = u^2 + 2as}$
- $\mathbf{s = ut + \frac{1}{2}at^2}$
- $\mathbf{s = vt - \frac{1}{2}at^2}$

Watch out These five formulae are sometimes referred to as the **kinematics formulae** or ***suvat* formulae**. They are given in the formulae booklet.

Example 7

A particle is moving along a straight line from A to B with constant acceleration $5\,\text{m s}^{-2}$. The velocity of the particle at A is $3\,\text{m s}^{-1}$ in the direction $\overrightarrow{AB}$. The velocity of the particle at B is $18\,\text{m s}^{-1}$ in the same direction. Find the distance from A to B.



$a = 5,\ u = 3,\ v = 18,\ s = ?$ — Write down the values you know and the values you need to find. This will help you choose the correct formula.

$v^2 = u^2 + 2as$ — t is not involved so choose the formula that does not have t in it.

$18^2 = 3^2 + 2 \times 5 \times s$

$324 = 9 + 10s$

$s = \frac{324 - 9}{10} = 31.5$ — Substitute in the values you are given and solve the equation for s. This gives the distance you were asked to find.

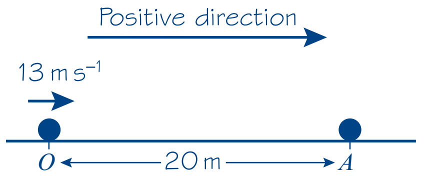
$AB = 31.5\,\text{m}$

Example 8

A particle is moving in a straight horizontal line with constant deceleration $4\,\text{m s}^{-2}$. At time $t = 0$ the particle passes through a point O with speed $13\,\text{m s}^{-1}$ travelling towards a point A, where $OA = 20\,\text{m}$. Find:

a the times when the particle passes through A

b the value of t when the particle returns to O.



a $a = -4,\ u = 13,\ s = 20,\ t = ?$ — The particle is decelerating so the value of a is negative. You are told the values of a, u and s and asked to find t. You are given no information about v and are not asked to find it so you choose the formula without v.

$s = ut + \frac{1}{2}at^2$

$20 = 13t - \frac{1}{2} \times 4t^2$

$= 13t - 2t^2$

$2t^2 - 13t + 20 = 0$

$(2t - 5)(t - 4) = 0$

$t = \frac{5}{2}$, or $t = 4$ — There are two answers. Both are correct. The particle moves from O to A, goes beyond A and then turns round and returns to A.

The particle moves through A twice, $2\frac{1}{2}$ seconds and 4 seconds after moving through O.

Problem-solving When you use $s = ut + \frac{1}{2}at^2$ with an unknown value of t you obtain a quadratic equation in t. You can solve this equation by factorising, or using the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

b The particle returns to O when $s = 0$.

$s = 0,\ u = 13,\ a = -4,\ t = ?$

$s = ut + \frac{1}{2}at^2$

$0 = 13t - 2t^2$

$= t(13 - 2t)$

$t = 0$, or $t = \frac{13}{2}$

The particle returns to O 6.5 seconds after it first passed through O.

When the particle returns to O, its displacement (distance) from O is zero.

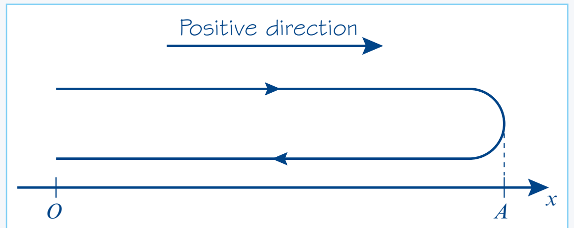
The first solution ($t = 0$) represents the starting position of the particle. The other solution ($t = \frac{13}{2}$) tells you when the particle returns to O.

Example 9

A particle P is moving on the x-axis with constant deceleration 2.5 m s^{-2}. At time $t = 0$, the particle P passes through the origin O, moving in the positive direction of x with speed 15 m s^{-1}. Find:

a the time between the instant when P first passes through O and the instant when it returns to O

b the total distance travelled by P during this time.



a $a = -2.5,\ u = 15,\ s = 0,\ t = ?$

$s = ut + \frac{1}{2}at^2$

$0 = 15t - \frac{1}{2} \times 2.5 \times t^2$

$0 = 60t - 5t^2$

$= 5t(12 - t)$

$t = 0,\ t = 12$

The particle P returns to O after 12 s.

b $a = -2.5,\ u = 15,\ v = 0,\ s = ?$

$v^2 = u^2 + 2as$

$0^2 = 15^2 - 2 \times 2.5 \times s$

$5s = 15^2 = 225$

$s = \frac{225}{5} = 45$

The distance $OA = 45$ m.

The total distance travelled by P is $2 \times 45\text{ m} = 90\text{ m}$.

Problem-solving

Before you start, draw a sketch so you can see what is happening. The particle moves through O with a positive velocity. As it is decelerating it slows down and will eventually have zero velocity at a point A, which you don't yet know. As the particle is still decelerating, its velocity becomes negative, so the particle changes direction and returns to O.

When the particle returns to O, its displacement (distance) from O is zero.

Multiply by 4 to get whole-number coefficients.

At the furthest point from O, labelled A in the diagram, the particle changes direction. At that point, for an instant, the particle has zero velocity.

In the 12 s the particle has been moving it has travelled to A and back. The total distance travelled is twice the distance OA.

Exercise 9D

1 A particle is moving in a straight line with constant acceleration 2.5 m s^{-2}. It passes a point A with velocity 3 m s^{-1} and later passes through a point B, where $AB = 8$ m. Find the velocity of the particle as it passes through B.

2 A car is accelerating at a constant rate along a straight horizontal road. Travelling at 8 m s^{-1}, it passes a pillar box and 6 s later it passes a sign. The distance between the pillar box and the sign is 60 m. Find the acceleration of the car.

3 A cyclist travelling at 12 m s^{-1} applies her brakes and comes to rest after travelling 36 m in a straight line. Assuming that the brakes cause the cyclist to decelerate uniformly, find the deceleration.

4 A train is moving along a straight horizontal track with constant acceleration. The train passes a signal with a velocity of 54 km h^{-1} and a second signal with a velocity of 72 km h^{-1}. The distance between the two signals is 500 m. Find, in m s^{-2}, the acceleration of the train.

5 A particle moves along a straight line, with constant acceleration, from a point A to a point B where $AB = 48$ m. At A the particle has velocity 4 m s^{-1} and at B it has velocity 16 m s^{-1}. Find:

a the acceleration of the particle

b the time the particle takes to move from A to B.

6 A particle moves along a straight line with constant acceleration 3 m s^{-2}. The particle moves 38 m in 4 s. Find:

a the initial velocity of the particle

b the final velocity of the particle.

7 The driver of a car is travelling at 18 m s^{-1} along a straight road when she sees an obstruction ahead. She applies the brakes and the brakes cause the car to slow down to rest with a constant deceleration of 3 m s^{-2}. Find:

a the distance travelled as the car decelerates

b the time it takes for the car to decelerate from 18 m s^{-1} to rest.

8 A stone is sliding across a frozen lake in a straight line. The initial speed of the stone is 12 m s^{-1}. The friction between the stone and the ice causes the stone to slow down at a constant rate of 0.8 m s^{-2}. Find:

a the distance moved by the stone before coming to rest

b the speed of the stone at the instant when it has travelled half of this distance.

9 A particle is moving along a straight line OA with constant acceleration 2.5 m s^{-2}. At time $t = 0$, the particle passes through O with speed 8 m s^{-1} and is moving in the direction OA. The distance OA is 40 m. Find:

a the time taken for the particle to move from O to A

b the speed of the particle at A. Give your answers to one decimal place.

10 A particle travels with uniform deceleration 2 m s^{-2} in a horizontal line. The points A and B lie on the line and $AB = 32$ m. At time $t = 0$, the particle passes through A with velocity 12 m s^{-1} in the direction $\overrightarrow{AB}$. Find:

a the values of t when the particle is at B

b the velocity of the particle for each of these values of t.

E/P **11** A particle is moving along the x-axis with constant deceleration 5 m s^{-2}. At time $t = 0$, the particle passes through the origin O with velocity 12 m s^{-1} in the positive direction. At time t seconds the particle passes through the point A with x-coordinate 8. Find:

a the values of t **(3 marks)**

b the velocity of the particle as it passes through the point with x-coordinate -8. **(3 marks)**

Problem-solving

The particle will pass through A twice. Use $s = ut + \frac{1}{2}at^2$ to set up and solve a quadratic equation.

E **12** A particle P is moving on the x-axis with constant deceleration 4 m s^{-2}. At time $t = 0$, P passes through the origin O with velocity 14 m s^{-1} in the positive direction. The point A lies on the axis and $OA = 22.5\text{ m}$. Find:

a the difference between the times when P passes through A **(4 marks)**

b the total distance travelled by P during the interval between these times. **(3 marks)**

E/P **13** A car is travelling along a straight horizontal road with constant acceleration. The car passes over three consecutive points A, B and C where $AB = 100\text{ m}$ and $BC = 300\text{ m}$. The speed of the car at B is 14 m s^{-1} and the speed of the car at C is 20 m s^{-1}. Find:

a the acceleration of the car **(3 marks)**

b the time take for the car to travel from A to C. **(3 marks)**

E/P **14** Two particles P and Q are moving along the same straight horizontal line with constant accelerations 2 m s^{-2} and 3.6 m s^{-2} respectively. At time $t = 0$, P passes through a point A with speed 4 m s^{-1}. One second later Q passes through A with speed 3 m s^{-1}, moving in the same direction as P.

a Write down expressions for the displacements of P and Q from A, in terms of t, where t seconds is the time after P has passed through A. **(2 marks)**

b Find the value of t where the particles meet. **(3 marks)**

c Find the distance of A from the point where the particles meet. **(3 marks)**

Problem-solving

When P and Q meet, their displacements from A are equal.

E/P **15** In an orienteering competition, a competitor moves in a straight line past three checkpoints, P, Q and R, where $PQ = 2.4\text{ km}$ and $QR = 11.5\text{ km}$. The competitor is modelled as a particle moving with constant acceleration. She takes 1 hour to travel from P to Q and 1.5 hours to travel from Q to R. Find:

a the acceleration of the competitor

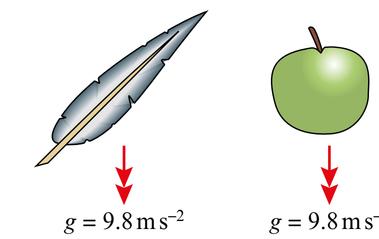
b her speed at the instant she passes P. **(7 marks)**

9.5 Vertical motion under gravity

You can use the formulae for constant acceleration to model an object moving vertically under gravity.

- **The force of gravity causes all objects to accelerate towards the earth. If you ignore the effects of air resistance, this acceleration is constant. It does not depend on the mass of the object.**

As the force of gravity does not depend on mass, this means that in a vacuum an apple and a feather would both accelerate downwards at the same rate.



On earth, the acceleration due to gravity is represented by the letter g and is approximately 9.8 m s^{-2}.

The actual value of the acceleration can vary by very small amounts in different places due to the changing radius of the earth and height above sea level.

- **An object moving vertically under gravity can be modelled as a particle with a constant downward acceleration of $g = 9.8\text{ m s}^{-2}$.**

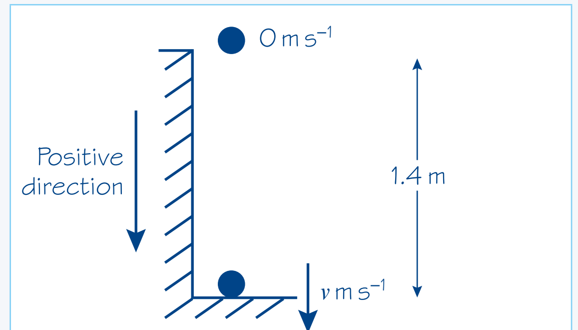
Watch out In mechanics questions you will always use $g = 9.8\text{ m s}^{-2}$ unless a question specifies otherwise. However, if a different value of g is specified (e.g. $g = 10\text{ m s}^{-2}$ or $g = 9.81\text{ m s}^{-2}$) the degree of accuracy in your answer should be chosen to be consistent with this value.

When solving problems about vertical motion you can choose the positive direction to be either upwards or downwards. Acceleration due to gravity is always downwards, so if the positive direction is upwards then $g = -9.8\text{ m s}^{-2}$.

Notation The total time that an object is in motion from the time it is projected (thrown) upwards to the time it hits the ground is called the **time of flight**. The initial speed is sometimes called the **speed of projection**.

Example 10

A book falls off the top shelf of a bookcase. The shelf is 1.4 m above a wooden floor. Find:
a the time the book takes to reach the floor, **b** the speed with which the book strikes the floor.



Model the book as a particle moving in a straight line with a constant acceleration of magnitude 9.8 m s^{-2}.

As the book is moving downwards throughout its motion, it is sensible to take the downwards direction as positive.

a $s = 1.4$

$a = +9.8$ — You have taken the downwards direction as positive and gravity acts downwards. Here the acceleration is positive.

$u = 0$ — Assume the book has an initial speed of zero.

$t = ?$

$s = ut + \frac{1}{2}at^2$ — Choose the formula without v.

$1.4 = 0 + \frac{1}{2} \times 9.8 \times t^2$

$t^2 = \frac{1.4}{4.9} = 0.2857\ldots$ — Solve the equation for t^2 and use your calculator to find the positive square root.

$t = \sqrt{0.2857\ldots} = 0.5345\ldots$

The time taken for the book to reach the floor is 0.53 s, to two significant figures. — Give the answer to two significant figures to be consistent with the degree of accuracy used for the value of g.

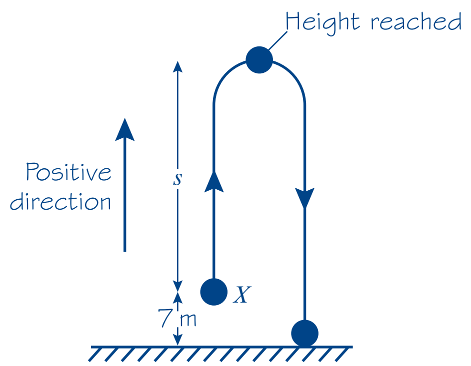
b $s = 1.4$
$a = 9.8$
$u = 0$
$v = ?$
$v^2 = u^2 + 2as$
$= 0^2 + 2 \times 9.8 \times 1.4 = 27.44$
$v = \sqrt{27.44} = 5.238... \approx 5.2$
The book hits the floor with speed $5.2\,\text{m}\,\text{s}^{-1}$, to two significant figures.

Choose the formula without t.

Use unrounded values in your calculations, but give your final answer correct to two significant figures.

Example 11

A ball is projected vertically upwards, from a point X which is 7 m above the ground, with speed $21\,\text{m}\,\text{s}^{-1}$. Find: **a** the greatest height above the ground reached by the ball, **b** the time of flight of the ball.



Problem-solving

In this sketch the upward and downwards motion have been sketched side by side. In reality they would be on top of one another, but drawing them separately makes it easier to see what is going on. Remember that X is 7 m above the ground, so mark this height on your sketch.

a $u = 21$
$v = 0$
$a = -9.8$
$s = ?$
$v^2 = u^2 + 2as$
$0^2 = 21^2 + 2 \times (-9.8) \times s = 441 - 19.6s$
$s = \frac{441}{19.6} = 22.5$
$(22.5 + 7)\,\text{m} = 29.5\,\text{m}$
Greatest height is 30 m (2 s.f.)

At its highest point, the ball is turning round. For an instant, it is neither going up nor down, so its speed is zero.

22.5 m is the distance the ball has moved above X but X is 7 m above the ground. You must add on another 7 m to get the greatest height above the ground reached by the ball.

b $s = -7$
$u = 21$
$a = -9.8$
$t = ?$
$s = ut + \frac{1}{2}at^2$
$-7 = 21t - 4.9t^2$
$4.9t^2 - 21t - 7 = 0$

The time of flight is the total time that the ball is in motion from the time that it is projected to the time that it stops moving. Here the ball will stop when it hits the ground. The point where the ball hits the ground is 7 m below the point from which it was projected so $s = -7$.

Online Use your calculator to check solutions to quadratic equations quickly.

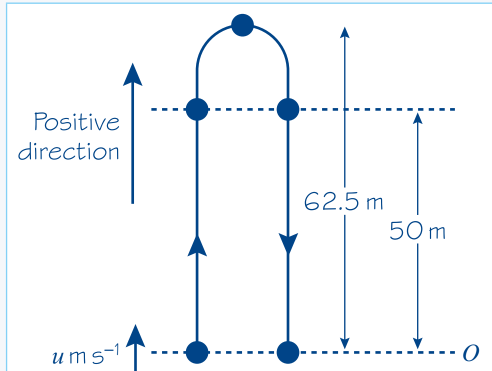
$t = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$
$= \frac{-(-21) \pm \sqrt{((-21)^2 - 4 \times 4.9 \times (-7))}}{2 \times 4.9}$
$= \frac{21 \pm \sqrt{578.2}}{9.8} \approx \frac{21 \pm 24.046}{9.8}$
$t \approx 4.5965,$
or $t \approx -0.3108$
Time of flight is 4.6 s (2 s.f.)

Rearrange the equation and use the quadratic formula.

Take the positive answer and round to two significant figures.

Example 12

A particle is projected vertically upwards from a point O with speed $u\,\text{m}\,\text{s}^{-1}$. The greatest height reached by the particle is 62.5 m above O. Find: **a** the value of u, **b** the total time for which the particle is 50 m or more above O.



The particle will pass through the point 50 m above O twice: once on the way up and once on the way down.

a $v = 0$
$s = 62.5$
$a = -9.8$
$u = ?$
$v^2 = u^2 + 2as$
$0^2 = u^2 + 2 \times (-9.8) \times 62.5$
$u^2 = 1225$
$u = \sqrt{1225} = 35\,\text{m}\,\text{s}^{-1}$

There is no t, so you choose the formula without t.

In this part, you obtain an exact answer, so there is no need for approximation.

b $s = 50$
$u = 35$
$a = -9.8$
$t = ?$
$s = ut + \frac{1}{2}at^2$
$50 = 35t - 4.9t^2$
$4.9t^2 - 35t + 50 = 0$
$t = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$
$= \frac{35 \pm \sqrt{(35^2 - 4 \times 4.9 \times 50)}}{9.8}$

Two values of t need to be found: one on the way up and one on the way down.

Write the equation in the form $ax^2 + bx + c = 0$ and use the quadratic formula.

$= \frac{35 \pm \sqrt{245}}{9.8} \approx \frac{35 \pm 15.6525}{9.8}$

$t \approx 5.1686\ldots$, or $t \approx 1.9742\ldots$

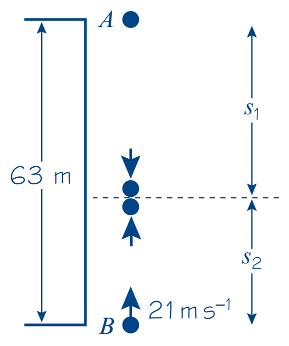
$(5.1686\ldots) - (1.9742\ldots) \approx 3.194$

Particle is 50 m or more above O for 3.2 s (2 s.f.)

Between these two times the particle is always more than 50 m above O. You find the total time for which the particle is 50 m or more above O by finding the difference of these two values.

Example 13

A ball A falls vertically from rest from the top of a tower 63 m high. At the same time as A begins to fall, another ball B is projected vertically upwards from the bottom of the tower with speed 21 m s^{-1}. The balls collide. Find the distance of the point where the balls collide from the bottom of the tower.



Problem-solving

You must take special care with problems where objects are moving in different directions. Here A is moving downwards and you will take the acceleration due to gravity as positive. However, B is moving upwards so for B the acceleration due to gravity is negative.

For A, the motion is downwards

$u = 0$

$a = 9.8$

$s = ut + \frac{1}{2}at^2$

$s_1 = 4.9t^2$

You cannot find s_1 at this stage. You have to express it in terms of t.

For B, the motion is upwards

$u = 21$

$a = -9.8$

As B is moving upwards, the acceleration due to gravity is negative.

$s = ut + \frac{1}{2}at^2$

$s_2 = 21t - 4.9t^2$

You now have expressions for s_1 and s_2 in terms of t.

The height of the tower is 63 m.

$s_1 + s_2 = 63$

$4.9t^2 + (21t - 4.9t^2) = 63$

$21t = 63$

$t = 3$

Adding together the two distances gives the height of the tower. You can write this as an equation in t.

$s_2 = 21t - 4.9t^2$

$= 21 \times 3 - 4.9 \times 3^2 = 18.9$

You have found t but you were asked for the distance from the bottom of the tower. Substitute your value for t into your equation for s_2.

The balls collide 19 m from the bottom of the tower, to two significant figures.

Exercise 9E

1 A cliff diver jumps from a point 28 m above the surface of the water. Modelling the diver as a particle moving freely under gravity with initial velocity 0, find:

a the time taken for the diver to hit the water

b the speed of the diver when he hits the water.

2 A particle is projected vertically upwards with speed 20 m s^{-1} from a point on the ground. Find the time of flight of the particle.

3 A ball is thrown vertically downward from the top of a tower with speed 18 m s^{-1}. It reaches the ground in 1.6 s. Find the height of the tower.

4 A pebble is catapulted vertically upwards with speed 24 m s^{-1}. Find:

a the greatest height above the point of projection reached by the pebble

b the time taken to reach this height.

5 A ball is projected upwards from a point which is 4 m above the ground with speed 18 m s^{-1}. Find:

a the speed of the ball when it is 15 m above its point of projection

b the speed with which the ball hits the ground.

6 A particle P is projected vertically downwards from a point 80 m above the ground with speed 4 m s^{-1}. Find:

a the speed with which P hits the ground

b the time P takes to reach the ground.

7 A particle P is projected vertically upwards from a point X. Five seconds later P is moving downwards with speed 10 m s^{-1}. Find:

a the speed of projection of P

b the greatest height above X attained by P during its motion.

8 A ball is thrown vertically upwards with speed 21 m s^{-1}. It hits the ground 4.5 s later. Find the height above the ground from which the ball was thrown.

9 A stone is thrown vertically upward from a point which is 3 m above the ground, with speed 16 m s^{-1}. Find:

a the time of flight of the stone

b the total distance travelled by the stone.

P 10 A particle is projected vertically upwards with speed 24.5 m s^{-1}. Find the total time for which it is 21 m or more above its point of projection.

E/P 11 A particle is projected vertically upwards from a point O with speed u m s^{-1}. Two seconds later it is still moving upwards and its speed is $\frac{1}{3}u$ m s^{-1}. Find:

a the value of u **(3 marks)**

b the time from the instant that the particle leaves O to the instant that it returns to O. **(4 marks)**

Problem-solving

Use $v = u + at$ and substitute $v = \frac{1}{3}u$.

E/P **12** A ball A is thrown vertically downwards with speed 5 m s^{-1} from the top of a tower block 46 m above the ground. At the same time as A is thrown downwards, another ball B is thrown vertically upwards from the ground with speed 18 m s^{-1}. The balls collide. Find the distance of the point where A and B collide from the point where A was thrown. **(5 marks)**

E/P **13** A ball is released from rest at a point which is 10 m above a wooden floor. Each time the ball strikes the floor, it rebounds with three-quarters of the speed with which it strikes the floor. Find the greatest height above the floor reached by the ball

Problem-solving

Consider each bounce as a separate motion.

a the first time it rebounds from the floor **(3 marks)**

b the second time it rebounds from the floor. **(4 marks)**

Challenge

1 A particle P is projected vertically upwards from a point O with speed 12 m s^{-1}. One second after P has been projected from O, another particle Q is projected vertically upwards from O with speed 20 m s^{-1}. Find: **a** the time between the instant that P is projected from O and the instant when P and Q collide, **b** the distance of the point where P and Q collide from O.

2 A stone is dropped from the top of a building and two seconds later another stone is thrown vertically downwards at a speed of 25 m s^{-1}. Both stones reach the ground at the same time. Find the height of the building.

Mixed exercise 9

1 A car accelerates in a straight line at a constant rate, starting from rest at a point A and reaching a velocity of 45 km h^{-1} in 20 s. This velocity is then maintained and the car passes a point B 3 minutes after leaving A.

a Sketch a velocity–time graph to illustrate the motion of the car.

b Find the displacement of the car from its starting point after 3 minutes.

2 A particle is moving on an axis Ox. From time $t = 0$ s to time $t = 32$ s, the particle is travelling with constant velocity 15 m s^{-1}. The particle then decelerates from 15 m s^{-1} to rest in T seconds.

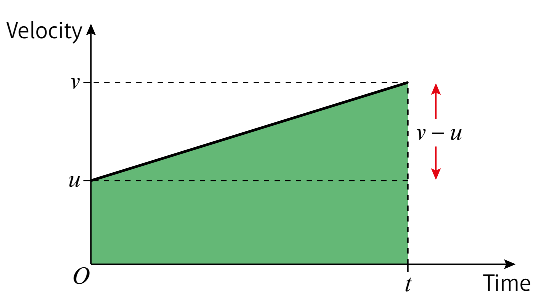
a Sketch a velocity–time graph to illustrate the motion of the particle.

The total distance travelled by the particle is 570 m.

b Find the value of T.

c Sketch a displacement–time graph illustrating the motion of the particle.

P **3** The velocity–time graph represents the motion of a particle moving in a straight line accelerating from velocity u at time 0 to velocity v at time t.



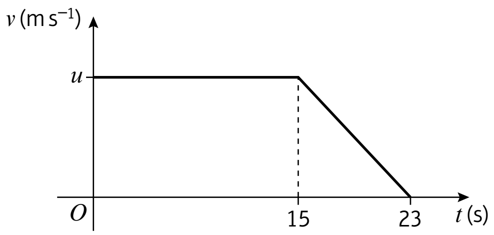
a Use the graph to show that:

i $v = u + at$ **ii** $s = \left(\frac{u+v}{2}\right)t$

b Hence show that:

i $v^2 = u^2 + 2as$ **ii** $s = ut + \frac{1}{2}at^2$ **iii** $s = vt - \frac{1}{2}at^2$

P **4** The diagram is a velocity–time graph representing the motion of a cyclist along a straight road. At time $t = 0$ s, the cyclist is moving with velocity $u\text{ m s}^{-1}$. The velocity is maintained until time $t = 15$ s, when she slows down with constant deceleration, coming to rest when $t = 23$ s. The total distance she travels in 23 s is 152 m. Find the value of u.



5 A car travelling on a straight road slows down with constant deceleration. The car passes a road sign with velocity 40 km h^{-1} and a post box with velocity of 24 km h^{-1}. The distance between the road sign and the post box is 240 m. Find, in m s^{-2}, the deceleration of the car.

6 A particle P is moving along the x-axis with constant deceleration 2.5 m s^{-2}. At time $t = 0$ s, P passes through the origin with velocity 20 m s^{-1} in the direction of x increasing. At time $t = 12$ s, P is at the point A. Find:

a the distance OA **b** the total distance P travels in 12 s.

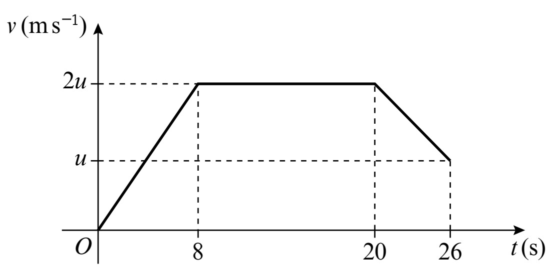
7 A ball is thrown vertically downward from the top of a tower with speed 6 m s^{-1}. The ball strikes the ground with speed 25 m s^{-1}. Find the time the ball takes to move from the top of the tower to the ground.

8 A child drops a ball from a point at the top of a cliff which is 82 m above the sea. The ball is initially at rest. Find:

a the time taken for the ball to reach the sea **b** the speed with which the ball hits the sea.

c State one physical factor which has been ignored in making your calculation.

P **9** A particle moves 451 m in a straight line. The diagram shows a speed–time graph illustrating the motion of the particle. The particle starts at rest and accelerates at a constant rate for 8 s reaching a speed of $2u\text{ m s}^{-1}$. The particle then travels at a constant speed for 12 seconds before decelerating uniformly, reaching a speed of $u\text{ m s}^{-1}$ at time $t = 26$ s. Find:



a the value of u

b the distance moved by the particle while its speed is less than $u\text{ m s}^{-1}$.

E/P **10** A train is travelling with constant acceleration along a straight track. At time $t = 0$ s, the train passes a point O travelling with velocity 18 m s^{-1}. At time $t = 12$ s, the train passes a point P travelling with velocity 24 m s^{-1}. At time $t = 20$ s, the train passes a point Q. Find:

a the speed of the train at Q **(5 marks)**

b the distance from P to Q. **(2 marks)**

E **11** A particle moves along a straight line, from a point X to a point Y, with constant acceleration. The distance from X to Y is 104 m. The particle takes 8 s to move from X to Y and the speed of the particle at Y is 18 m s^{-1}. Find:

a the speed of the particle at X **(3 marks)**

b the acceleration of the particle. **(2 marks)**

The particle continues to move with the same acceleration until it reaches a point Z. At Z the speed of the particle is three times the speed of the particle at X.

c Find the distance XZ. **(4 marks)**

(E) **12** A pebble is projected vertically upwards with speed $21\,\text{m s}^{-1}$ from a point 32 m above the ground. Find:

a the speed with which the pebble strikes the ground **(3 marks)**

b the total time for which the pebble is more than 40 m above the ground. **(4 marks)**

c Sketch a velocity–time graph for the motion of the pebble from the instant it is projected to the instant it hits the ground, showing the values of t at any points where the graph intercepts the horizontal axis. **(4 marks)**

(E) **13** A car is moving along a straight road with uniform acceleration. The car passes a checkpoint A with speed $12\,\text{m s}^{-1}$ and another checkpoint C with speed $32\,\text{m s}^{-1}$. The distance between A and C is 1100 m.

a Find the time taken by the car to move from A to C. **(2 marks)**

b Given that B is the midpoint of AC, find the speed with which the car passes B. **(2 marks)**

(E/P) **14** A particle is projected vertically upwards with a speed of $30\,\text{m s}^{-1}$ from a point A. The point B is h metres above A. The particle moves freely under gravity and is above B for a time 2.4 s. Calculate the value of h. **(5 marks)**

(E/P) **15** Two cars A and B are moving in the same direction along a straight horizontal road. At time $t = 0$, they are side by side, passing a point O on the road. Car A travels at a constant speed of $30\,\text{m s}^{-1}$. Car B passes O with a speed of $20\,\text{m s}^{-1}$, and has constant acceleration of $4\,\text{m s}^{-2}$. Find:

a the speed of B when it has travelled 78 m from O **(2 marks)**

b the distance from O of A when B is 78 m from O **(3 marks)**

c the time when B overtakes A. **(4 marks)**

(E/P) **16** A car is being driven on a straight stretch of motorway at a constant velocity of $34\,\text{m s}^{-1}$, when it passes a velocity restriction sign S warning of road works ahead and requiring speeds to be reduced to $22\,\text{m s}^{-1}$. The driver continues at her velocity for 2 s after passing S. She then reduces her velocity to $22\,\text{m s}^{-1}$ with constant deceleration of $3\,\text{m s}^{-2}$, and continues at the lower velocity.

a Draw a velocity–time graph to illustrate the motion of the car after it passes S. **(2 marks)**

b Find the shortest distance before the road works that S should be placed on the road to ensure that a car driven in this way has had its velocity reduced to $22\,\text{m s}^{-1}$ by the time it reaches the start of the road works. **(4 marks)**

(E/P) **17** A train starts from rest at station A and accelerates uniformly at $3x\,\text{m s}^{-2}$ until it reaches a velocity of $30\,\text{m s}^{-1}$. For the next T seconds the train maintains this constant velocity. The train then decelerates uniformly at $x\,\text{m s}^{-2}$ until it comes to rest at a station B. The distance between the stations is 6 km and the time taken from A to B is 5 minutes.

a Sketch a velocity–time graph to illustrate this journey. **(2 marks)**

b Show that $\dfrac{40}{x} + T = 300$. **(4 marks)**

c Find the value of T and the value of x. **(2 marks)**

d Calculate the distance the train travels at constant velocity. **(2 marks)**

e Calculate the time taken from leaving A until reaching the point halfway between the stations. **(3 marks)**

Challenge

A ball is projected vertically upwards with speed $10\,\text{m s}^{-1}$ from a point X, which is 50 m above the ground. T seconds after the first ball is projected upwards, a second ball is dropped from X. Initially the second ball is at rest. The balls collide 25 m above the ground. Find the value of T.

Summary of key points

1. Velocity is the **rate of change** of displacement.
   On a displacement–time graph the **gradient** represents the velocity.
   If the displacement–time graph is a straight line, then the velocity is constant.
2. Average velocity $= \dfrac{\text{displacement from starting point}}{\text{time taken}}$
3. Average speed $= \dfrac{\text{total distance travelled}}{\text{time taken}}$
4. Acceleration is the **rate of change** of velocity.
   In a velocity–time graph the **gradient** represents the acceleration.
   If the velocity–time graph is a straight line, then the acceleration is constant.
5. The area between a velocity–time graph and the horizontal axis represents the distance travelled.
   For motion in a straight line with positive velocity, the area under the velocity–time graph up to a point t represents the displacement at time t.
6. You need to be able to use and to derive the five formulae for solving problems about particles moving in a straight line with constant acceleration.
   - $v = u + at$
   - $s = \left(\dfrac{u+v}{2}\right)t$
   - $v^2 = u^2 + 2as$
   - $s = ut + \frac{1}{2}at^2$
   - $s = vt - \frac{1}{2}at^2$
7. The force of **gravity** causes all objects to accelerate towards the earth. If you ignore the effects of air resistance, this acceleration is constant. It does not depend on the mass of the object.
8. An object moving vertically in a straight line can be modelled as a particle with a constant downward acceleration of $g = 9.8\,\text{m s}^{-2}$.

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