Putting Research into Practice

An overview of the scientific research base of *Prentice Hall Mathematics*, written by the program authors
# Prentice Hall Mathematics: Putting Research Into Practice

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Introduction

Research and Prentice Hall Mathematics

Pearson Prentice Hall is proud of the fact that for over half a century we have used a variety of types of research as a base on which to build our mathematics programs. Over the years, we have worked collaboratively with our authors to make continual program improvements based on empirical, scientific research.

Today’s global economy has increased the need for high levels of literacy and numeracy as key ingredients for attaining a decent standard of living. This, in turn, has further increased the need for decisions about the teaching and learning of mathematics to be based on the sound evidence of research. The purpose of this document is to illustrate how the extensive research base that guided the development of Prentice Hall Mathematics was put into practice.

Three Phases of Research

Prentice Hall Mathematics is based on research that describes how students learn mathematics well and provides classroom-based evidence of program efficacy. The three phases of research described below were integrated into the development of Prentice Hall Mathematics. The goal of establishing such rigorous research methods is to ensure that the program enables all students to learn the mathematics skills and concepts they need for academic success and for everyday life.

Prentice Hall’s research is cyclical and ongoing, and provides evidence of a program’s overall effectiveness based principally on students’ test scores. Previous mathematics programs by Prentice Hall, Scott Foresman, and Addison-Wesley provided a strong basis for success. What we learned about the effectiveness of our previous programs informed the instructional design of our new program.

Pearson Prentice Hall is analyzing students’ pretest and posttest scores with national districts and school-level data of users of the program. This process allows for continual monitoring of performance and learning from results in an ongoing basis.

At Pearson Prentice Hall, our programs benefit from a long history of excellent mathematics programs published by Prentice Hall, Scott Foresman, Addison-Wesley, Allyn & Bacon, and Ginn & Company.

The Pearson Prentice Hall Mathematics Program Efficacy Studies, located at PHSchool.com/MathResearch, examine student achievement data and present case studies across a wide range of demographics. Standardized tests such as SAT-9 and TerraNova, as well as state tests, provide a diverse group of measures.
We also conduct independent, third-party research using quasi-experimental and experimental research designs. A national effect size study comparing the efficacy of Prentice Hall users versus matched non-users was recently completed. A longitudinal, experimental study will begin in Spring 2004. Specific program features such as the Instant Check System™ will be studied as well as the fidelity of implementation and program efficacy. Prentice Hall Mathematics users will be compared to matched non-users and tested using national standardized examinations. This information will help monitor student success, identify how well our program works, and inform the need for revision.

Exploratory Needs Assessment
Along with periodic surveys concerning curriculum issues and challenges, we conducted specific product development research, which included discussions with teachers and advisory panels, focus groups, and quantitative surveys. We explored the specific needs of teachers, students, and other educators regarding each book we developed in Prentice Hall Mathematics.

In conjunction with Prentice Hall authors, secondary research was done to explore educational research about learning. This research was incorporated into our instructional strategy and pedagogy to make a more effective mathematics program.

Formative: Prototype Development and Field Testing
During this phase of research, we worked to develop prototype materials for each course in Prentice Hall Mathematics. Then we tested the materials, including doing field testing with students and teachers and conducting qualitative and quantitative evaluations of different kinds. We received solid feedback about our lesson structure in our early prototype testing. Results were channeled back into the program development for improvement.
Summative: Validation Research

Finally, we conducted and continue to conduct longer-term research based on scientific, experimental designs under actual classroom conditions. This research identifies what works and what can be improved in revisions. We also continue to monitor the program in the market. We talk to our users about what works, and then we begin the cycle over again. This phase involves longitudinal, control-group research designs.

It is important to note that Pearson Prentice Hall uses this research to subsequently inform the development of the next program. Hence, our three phases of scientific research form a cycle that is truly ongoing.

Prentice Hall Program Efficacy Studies

The following Learner Verification Studies were conducted on previous Prentice Hall mathematics programs. The results of these studies support the effectiveness of the programs that led to Prentice Hall Mathematics © 2004:

- **Prentice Hall Middle Grades Math: Tools for Success, Course 1 © 1999 (Grade 6).**
- **Scott Foresman-Addison Wesley Middle School Math, Course 3 © 1999 (Grade 8). Full-year study, 2000–2001 school year.**
- **Prentice Hall Algebra 1 © 2004 (Grade 9). Half-year study, Spring 2002.**
- **Prentice Hall Algebra 1 © 2004 (Grade 9). Full-year study, 2002–2003 school year.**
- **Prentice Hall Mathematics, Course 2 © 2004 (Grade 7). Full-year study, 2002–2003 school year.**

To obtain a copy of these reports, visit PHSchool.com/MathResearch.
The No Child Left Behind Act of 2001 (NCLB)

One of the most prominent features of the No Child Left Behind legislation is the call for scientifically based research to determine the extent to which a program or approach to learning is effective in the classroom. NCLB defines scientifically based research as “research that involves the application of rigorous, systematic, and objective procedures to obtain reliable and valid knowledge relevant to educational activities and programs.”

- It involves rigorous data analyses that are adequate to test the stated hypotheses and justify the general conclusions drawn.
- It relies on measurement or observational methods that provide reliable and valid data.
- It is evaluated using experimental or quasi-experimental designs in which programs or activities are assigned to different conditions, with appropriate controls to evaluate the effects of varying the condition of interest.
- It ensures that experimental studies are presented in sufficient detail and clarity to allow for replication or, at a minimum, to offer the opportunity of building systematically on their findings.
- It has been accepted by a peer-reviewed journal or approved by a panel of independent experts through a comparably objective and scientific review.1

In short, NCLB has an impact on textbook publishers because it has a vast impact on schools and on student learning. As a result, going forward, authors and publishers must do an even better job of researching best practices of what works in the classroom—and then of collecting and interpreting scientific evidence that what they have put “into print” indeed works.

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Adding It Up: Helping Children Learn Mathematics

An 18-month project that reviewed and synthesized relevant research on mathematics teaching and learning resulted in the report *Adding It Up: Helping Children Learn Mathematics*. The core of this book is a discussion of the five strands of mathematical proficiency. These interwoven and interdependent strands essentially define what it means to learn mathematics. In the “popular” version of this book, *Helping Children Learn Mathematics*, these strands are defined as follows:

### Five Strands of Mathematical Proficiency

<table>
<thead>
<tr>
<th>Strand</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1. Understanding</strong></td>
<td>Comprehending mathematical concepts, operations, and relations—knowing what mathematical symbols, diagrams, and procedures mean</td>
</tr>
<tr>
<td><strong>2. Computing</strong></td>
<td>Carrying out mathematical procedures, such as adding, subtracting, multiplying, and dividing numbers flexibly, accurately, efficiently, and appropriately</td>
</tr>
<tr>
<td><strong>3. Applying</strong></td>
<td>Being able to formulate problems mathematically and to devise strategies for solving them, using concepts and procedures appropriately</td>
</tr>
<tr>
<td><strong>4. Reasoning</strong></td>
<td>Using logic to explain and justify a solution to a problem or to extend from something not yet known</td>
</tr>
<tr>
<td><strong>5. Engaging</strong></td>
<td>Seeing mathematics as sensible, useful, and doable—if you work at it—and being willing to do the work</td>
</tr>
</tbody>
</table>

A major conclusion of this book is that U.S. students need more skill and more understanding along with the ability to apply concepts to solve problems, to reason logically, and to see math as sensible, useful, and doable. This conclusion influenced our judgment that what will best challenge and support all students in developing mathematical proficiency is a mathematics program focusing on important ideas that build both skills and understandings for all children.

Putting Research Into Practice

In developing Prentice Hall Mathematics, we identified four critical program areas requiring a strong research base. Because of their importance, we created this document to illustrate the connections between the research base and these program features. The areas are the following:

1. Instructional Design
2. Problem Solving
3. Meeting Individual Needs
4. Approaches to Important Content

There is little doubt that a program’s instructional design, how problem solving is infused, and how individual needs are met must all be research-based if a program is to support all students’ making sense of the mathematics they are taught. In this publication, the Prentice Hall authors not only identify for you the critical content that leads to greater success in higher level mathematics but also the successful approaches to content used throughout Prentice Hall Mathematics. In addition, we share with you the research that supports those approaches.

The pages that follow contain research summaries and citations that support the instructional practices used in Prentice Hall Mathematics and illustrate how we put the large body of available research into practice. We at Pearson Prentice Hall are committed to collecting and sharing the scientific evidence of our programs’ effectiveness. Since the late 1920s, when Prentice Hall published its first mathematics book, our goal at Pearson Prentice Hall has been to help children and teachers be successful.

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“The overriding premise of this book is that all students can and should achieve mathematical proficiency.”
—Helping Children Learn Mathematics, page 10

Since the English and Spanish editions of Prentice Hall Mathematics are parallel, the research base that was used applies to both editions.

“...To teach mathematics well, they [teachers] must themselves be proficient in mathematics, at a much deeper level than their students. . . . and they must have a repertoire of teaching practices that can promote [mathematical] proficiency.”
—Helping Children Learn Mathematics, page 31
Instructional design refers to the structure of a lesson as presented in the student textbook. The design of a lesson significantly influences the content that students are to learn, how the content is taught, and, most importantly, how well the content is learned.

In building the instructional design of Prentice Hall Mathematics, the authors gave special emphasis to the following areas, each of which is discussed in detail below.

• Consistent lesson structure
• Development of both skills and understandings
• Effective learning activities and questioning strategies
• Embedded, ongoing assessment
Consistent Lesson Structure

Jitendra et al. (1999), in examining the instructional design of seven mathematics programs, found that all learners, but particularly special-needs learners, are aided when lessons are built around a consistent structure. Leinhardt (1988) found that effective lessons cannot be “homogeneous blobs of teacher talk or student seat work.” In contrast, she found that effective lessons consist of short segments, each with specific goals.

The lessons in Prentice Hall Mathematics are built on a design of “structured instruction” consisting of a carefully crafted sequence of lesson parts.

1. Check Prerequisite Skills
2. Learn—Examples and Investigations
3. Check Understanding
4. Practice

Research Says: Communicating Lesson Goals

Jitendra et al. (1999) found that each lesson should make it clear to both teacher and students both the skills and the concepts that are to be learned.

Putting Research Into Practice

Prentice Hall Mathematics communicates both procedural and conceptual objectives to students and teachers in the What You’ll Learn . . . And Why and the Lesson Preview for each lesson. Each lesson objective is then reiterated at the point at which it is taught within the lesson. The Math Background section of the Teacher’s Edition goes into further depth for each lesson by providing the underlying Math Understandings associated with each lesson.

5. Jitendra et al., op. cit., p. 76.
Research Says: Prerequisite Skills
Gagne and Driscoll (1988) state that the learning of skills typically requires the explicit prior development of simpler component skills—prerequisite skills.6

Putting Research Into Practice
In Prentice Hall Mathematics, lessons begin with a Check Skills You’ll Need section that assesses prior knowledge necessary for the new lesson.

Research Says: Ample Examples
Trafton (1983) states that each lesson should present a sufficient number and range of examples prior to independent practice.7

Putting Research Into Practice
In Prentice Hall Mathematics, Examples are broken into clear and meaningful steps and address the key variations of the newly introduced skills, including real-world applications of the skills. Each Example is followed by Check Understanding exercises for immediate feedback to the student and teacher.

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Research Says: Ample Practice

Jitendra et al. (1999) found that each lesson should provide an adequate number of practice exercises on the new skill.\(^8\)

Putting Research Into Practice

In *Prentice Hall Mathematics*, exercise sets within lessons provide ample practice so that key mathematical variations can be addressed and learning deficiencies identified. The exercise sets are organized into five parts to provide address different needs:

1. **Practice by Example**—Exercises that refer students back to specific examples within the lesson for help
2. **Apply Your Skills**—Skill exercises and multi-step application problems that may incorporate skills and concepts from previous lessons
3. **Challenge**—Exercises to stretch students’ thinking
4. **Test Prep**—Exercises that address the concepts from the day’s lesson in the formats commonly found on high-stakes tests—multiple choice, short response, extended response, gridded response, reading comprehension, quantitative comparison.
5. **Mixed Review**—Maintains skills developed in earlier lessons

\(^8\) Jitendra, et al., op. cit., pp. 69–79.
Research Says: Continual Practice and Review

A number of studies have shown that continual practice and review promotes student achievement at all grade levels. The authors of *Adding It Up (2001)* conclude that “distributed practice is based on the principle that mastery is achieved gradually and once achieved is maintained through regular practice.”

Putting Research Into Practice

*Prentice Hall Mathematics* employs a planned program of review, distributed over time, covering important ideas and skills to enhance retention. **Mixed Review** is part of each practice set in every lesson, and the reviewed lesson is clearly identified for the student beside each problem. Additionally, many exercises in the Apply Your Skills and Challenge sections of the exercises incorporate skills and concepts from prior lessons.

Additional Examples

The instructional design of *Prentice Hall Mathematics* is based on a **consistent lesson structure**. For additional examples, see the following Student Edition pages:

<table>
<thead>
<tr>
<th>Course</th>
<th>Pages</th>
<th>Course</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Course 1</td>
<td>148–152</td>
<td>Algebra 1</td>
<td>268–273</td>
</tr>
<tr>
<td>Course 2</td>
<td>187–190</td>
<td>Geometry</td>
<td>294–297</td>
</tr>
<tr>
<td>Course 3</td>
<td>331–336</td>
<td>Algebra 2</td>
<td>187–194</td>
</tr>
<tr>
<td>Pre-Algebra</td>
<td>288–291</td>
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</tbody>
</table>

Skills and Understandings

In the past, many textbooks focused only on procedural knowledge or skills. Even when these skills were developed conceptually, the requisite understandings were often not made clear to either the students or the teacher. However, research indicates a clear connection between conceptual understanding and skill development. Kloosterman and Gainey (1993) state: “Students who can connect mathematical procedures with underlying concepts are likely to apply those procedures correctly rather than in an arbitrary, inappropriate fashion.”

Hiebert et al. (1997) found that the development of conceptual knowledge with procedural skills provides students with a sense of confidence in what they are doing because the skills are developed meaningfully. The development of both skill acquisition and conceptual understandings is a cornerstone of Prentice Hall Mathematics.

Sufficient practice of important skills and ideas has long been accepted as an important component of effective instruction. Based on various research studies, Sutton and Krueger (2002) found that practice is essential for learning mathematics. They also found that it is important that students understand the skills being practiced so that they do not inadvertently practice incorrect skills. Practice helps to inform students that mathematics is well structured and that there is power in the mastery of important skills. Prentice Hall Mathematics builds appropriate, leveled practice into every lesson—focusing on both the skills and concepts developed in that lesson. This design feature encourages teachers to reach all learners by allowing teachers to customize instruction to match students’ needs.

Research Says: Checking Understanding

Good et al. (1983) found that a design of direct instruction that balances a strong component of conceptual development with appropriate practice to consolidate main ideas has a positive effect on student achievement.

Putting Research Into Practice

The Examples within the lessons of Prentice Hall Mathematics provide step-by-step development of concepts and skills. Each step within the Examples is consistently justified or explained so that students understand the “why” behind the mathematics.

Following every Example in the Student Edition, the authors provide Check Understanding exercises that allow the teacher to evaluate whether students have the understandings necessary before moving on to the next idea.


Instructional Design: Skills and Understandings

Research Says: Approaches to Problem Solving
Reeves and Reeves (2003) state: “students (should) have conceptual understanding of topics as a base on which to build and have both creative approaches and traditional routines at their fingertips so that they have choices about the way they do mathematics.”

Putting Research Into Practice
In Prentice Hall Mathematics, the connection between skills and conceptual understanding is enhanced by showing students that there is often more than one way to think about how to apply skills to solve a problem. A featured section More than One Way presents two students’ solutions to the same problem and encourages discussion of comparisons of the methods.

The Problem-Solving Strategy lessons in every chapter of the texts develop students’ abilities to both compare strategies for solving a problem and combine strategies for solving a problem.

Research Says: Professional Development
In reporting on research in teacher effectiveness, Kilpatrick, Swafford, and Findell (2001) state: “Teachers’ understanding of their students’ work and the progress they are making relies on the teachers’ own understanding of the mathematics and their ability to use that understanding to make sense of what the students are doing.”

Putting Research Into Practice
Prentice Hall Mathematics is designed to help teachers teach both skills and understandings. At the lesson level, Math Background notes are provided and include Math Understandings that address the key ideas that teachers must have in order to effectively teach the lesson.

The Prentice Hall Mathematics Chapter Facilitators’ Guides provide tools for face-to-face, interactive sessions on content and pedagogy of each chapter. The support text Math Across the Grades provides background information on how key strands are developed across grade levels. LessonLab Courses take advantage of the power of the Internet to deliver in-depth and collaborative “lesson studies,” like those featured in The International Mathematics and Science Study (TIMSS).

Additional Examples
In Prentice Hall Mathematics, both skills and understandings are infused throughout the program. For additional examples, see the following Student Edition pages:

<table>
<thead>
<tr>
<th>Course</th>
<th>Pages</th>
<th>Course</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Course 1</td>
<td>190–191, 226</td>
<td>Algebra 1</td>
<td>106–107, 247–248</td>
</tr>
<tr>
<td>Course 2</td>
<td>246–248, 575</td>
<td>Geometry</td>
<td>181, 435</td>
</tr>
<tr>
<td>Pre-Algebra</td>
<td>129, 367–368</td>
<td></td>
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</tr>
</tbody>
</table>

Learning Activities and Questioning Strategies

Students are not blank slates and do not learn by simply sitting passively in the classroom listening to the teacher. Wood and Turner-Vorbeck (2001) argue strongly that teacher pedagogical and mathematical knowledge is not sufficient for effective instruction. It is critical that students be involved in the lessons, listen to each other, and engage in a variety of interactions with the material being studied.16

Lampert and Cobb (2003) found evidence that talking about mathematics can give rise to opportunities for students to learn by reflecting on and objectifying prior activity. A further consequence of such activity is that students often develop an ability to organize the results of prior mathematical reasoning by searching for patterns.17

The authors of Prentice Hall Mathematics have incorporated numerous opportunities for students to interact with the subject matter and to reflect on the ideas that they are learning. Investigations are included both prior to and within lessons to engage students in working together to explore rich mathematical contexts. Students are continually encouraged to justify their thinking and express their ideas in writing. Additionally, every Example in Prentice Hall Mathematics is followed by Check Understanding questions that help the teacher gauge student understanding at critical points of instruction.

Research Says: Hands-On Activities

In a meta-analysis of 60 research studies, Sowell (1989) found that for students of all ages, mathematics achievement is increased and students’ attitudes toward mathematics are improved with the long-term use of manipulative materials.18

In a study of over 7,000 students, Wenglinsky (2000) found that students whose teachers conduct hands-on learning activities outperform their peers by more than 70% of a grade level in math on the National Assessment of Educational Progress (NAEP).19

Putting Research Into Practice

In Prentice Hall Mathematics, concepts are often introduced via Investigations in the Student Edition or Hands-On Activities booklet that involve manipulative materials. Electronic versions of manipulatives are also available within the Interactive Textbook online and on CD-ROM. These electronic versions can be manipulated in ways that physical models cannot, allowing students to experience multiple representations of concepts. Hands-on activities are also provided in the Alternative Assessment activities included with each lesson in the Teacher’s Edition.

For information on the use of multiple representations, see pages 48–56 in this document.

from Geometry Student Edition, page 303

Research Says: Good Questions

Research has found that most middle grades teachers ask many questions during the course of their lessons, but most of the questions are at a low cognitive level. Good questioning includes challenging students to think about the main ideas in the lesson and go beyond simple recall or parroting of what has just been said. Questioning also increases student-to-student interaction so that students’ ideas, expressed in their terminology, are shared with the class. (Hiebert et al., 1997).

Putting Research Into Practice

In Prentice Hall Mathematics, questioning is a driving force to both engage students and offer ongoing assessment data for the teacher. Teachers use student responses to the Check Understanding questions that follow each Example to gauge whether students understand the concepts or whether follow-up is needed. The Teacher’s Edition contains additional questions for teachers to pose in the Teaching Notes, Alternative Assessment, and Closure sections.

Research Says: Communicating Mathematically

“If students are to engage in mathematical argumentation and produce mathematical evidence, they will need to talk or write in ways that expose their reasoning to one another and to their teacher.” (Lampert and Cobb, 2003).

Putting Research Into Practice

Writing in Math is an exercise in every lesson. Selected chapters also have a full-page lesson devoted to Writing Strategies that encourage clear expression of students’ ideas. The writing strategies include Writing to Explain, Writing to Compare, Writing to Justify, and Writing to Persuade.

Additional Examples

In Prentice Hall Mathematics, learning activities and questioning strategies are embedded throughout the program. For additional examples, see the following Student Edition pages:

<table>
<thead>
<tr>
<th>Course</th>
<th>Pages</th>
<th>Course</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Course 1</td>
<td>69, 71, 92-93, 558, 621</td>
<td>Algebra 1</td>
<td>38-39, 95, 198-199, 201, 261</td>
</tr>
<tr>
<td>Course 2</td>
<td>35, 37, 241, 253, 267</td>
<td>Geometry</td>
<td>127, 145, 184, 295, 303</td>
</tr>
<tr>
<td>Course 3</td>
<td>227, 271, 453, 488</td>
<td>Algebra 2</td>
<td>100-101, 120, 144-145, 300, 424</td>
</tr>
<tr>
<td>Pre-Algebra</td>
<td>139, 305, 317-319, 597</td>
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</table>


Embedded Assessment

Research evidence suggests that in middle grades and high school mathematics, it is particularly important that assessment practices are ongoing and integrated into the fabric of the daily classroom routine. In his review of the research and literature on the importance of integrating mathematics assessment with instruction, Webb (1992) argued that assessment should be used “to make informed decisions throughout instruction based on current information available about what a student knows and about what a student is striving to know.”

Prentice Hall Mathematics is designed so that student readiness and understanding are continually evaluated. The program provides a seamless process of assessment, diagnosis, and intervention—making it possible for teachers to use real-time data to meet the needs of individual students. The assessment and intervention features build on the evidence that learning is enhanced when teachers have access to the knowledge that learners bring to the lesson, use this knowledge as part of instruction, monitor students’ changing conceptions as the lesson proceeds, and provide appropriate intervening instruction (De Corte et al., 1996).

Research Says: Ongoing Assessment

Based on their research synthesis, Bredekamp and Copple (1997) concluded that assessment should be ongoing, strategic, and purposeful. The methods of assessment should be appropriate to the age and experiences of students, and the results should be used to benefit them—in adapting curriculum and teaching to meet their developmental and learning needs.

According to the findings of Bransford et al. (2000), “Formative assessments—ongoing assessments designed to make students’ thinking visible to both teachers and students—are essential. They permit the teacher to grasp the students’ preconceptions, understand where the students are in the ‘developmental corridor’ from informal to formal thinking, and design instruction accordingly.”

Putting Research Into Practice

To provide teachers with a means to assess student readiness and understanding at each step of the way, Prentice Hall Mathematics texts feature the Instant Check System™. This system includes the following assessment components to assess readiness and understanding before, during, and after each lesson:

- Diagnosing Readiness before each chapter assesses students’ knowledge of key prerequisite vocabulary, skills, and concepts.
- Check Skills You’ll Need evaluates knowledge of key prerequisite skills prior to each lesson. A quick review of concepts may be all that students need to experience success with the day’s lesson.
- Check Understanding exercises allow teachers to assess students’ understanding following each Example in the lesson. These informal assessments allow the teacher to gauge whether students are ready to move on or whether intervention is needed.
- Checkpoint Quizzes provide students with opportunities for ongoing assessment, twice in every chapter.

Research Says: Student Progress Reports

Stecker and Fuchs (2000) found that mathematics students at Grades 2–8 for whom teachers tailored instructional adjustments based on individual performance data performed significantly better on a global achievement test than did their matched partners whose instructional adjustments were not based on their own assessment data.27

In a meta-analysis of eighteen studies, Baker et al. (2001) found that providing precise information on student progress and special areas of students’ strengths and weaknesses in mathematics can enhance achievement for low-performing students. This is particularly true if the information is precise as to which topics or concepts require additional practice or reteaching. They found that “students do benefit if they are given specific software lessons or curricula” to immediately address relevant topics.28

Putting Research Into Practice

The Math Online Intervention System that supports Prentice Hall Mathematics provides timely reports to teachers that track mastery by skill, or by state or national standard. Teachers receive top-down data on which students have “Mastered,” “Not Mastered,” or “Not Attempted” each skill or standard. As such, Math Online Intervention provides on-going monitoring of adequate yearly progress (AYP)—enabling students to improve performance in class and to achieve test success.

Additional Examples

In Prentice Hall Mathematics, embedded assessment is a feature of every lesson. For additional examples, see the following Student Edition pages:

<table>
<thead>
<tr>
<th>Course</th>
<th>Pages</th>
<th>Course</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Course 1</td>
<td>320, 326–327, 334</td>
<td>Algebra 1</td>
<td>72, 115, 304–307</td>
</tr>
<tr>
<td>Course 2</td>
<td>468, 480–481, 488</td>
<td>Geometry</td>
<td>178, 186-188, 201</td>
</tr>
<tr>
<td>Course 3</td>
<td>116, 136–139, 141</td>
<td>Algebra 2</td>
<td>114, 135-137, 147</td>
</tr>
<tr>
<td>Pre-Algebra</td>
<td>346, 357–359</td>
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</tbody>
</table>


Problem solving has a long history in the mathematics curriculum. Stanic and Kilpatrick (1989) did a historical analysis of problem solving in mathematics starting with early Greek and Chinese writings and continuing to the mathematics curricula found in schools today. They identified three themes that have characterized the role of problem solving in mathematics: problem solving as skill, problem solving as context, and problem solving as art.29 Schroeder and Lester (1989) studied approaches to “teaching problem solving” found in school mathematics curricula since the early 1980s. They identified three approaches: teaching about problem solving, teaching for problem solving, and teaching via problem solving.30

The Stanic and Kilpatrick themes, together with the Lester and Schroeder work, provide ways to organize and report the research on problem solving that underlies the Prentice Hall Mathematics instructional materials.

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Teaching About Problem Solving

In this approach, problem solving is viewed as a skill in which component parts of the process can be taught and learned separately. After students learn the component parts, the parts can be combined for students to solve real problems. In teaching about problem solving, Prentice Hall Mathematics addresses problem-solving skills and problem-solving strategies.

A. Problem-Solving Skills

Most common in teaching about problem solving is the teaching of a process (or set of heuristics), along with the skills associated with such a process. Polya (1957) articulated a problem-solving process as one involving this series of phases: Understanding the Problem, Devising a Plan, Carrying Out the Plan, and Looking Back.31

The phases of the problem-solving process used in Prentice Hall Mathematics are based on those described by Polya. They are Read and Understand, Plan and Solve, and Look Back and Check. The program addresses a wide variety of problem-solving skills. A list of those skills is shown below.

On a broader level, the program affects other skills related to problem-solving in math, such as Reading in Math and Writing in Math. Of particular focus are skills associated with reading and writing related to problem solving. Throughout the program, special lessons on topics such as too much or too little information develop reading comprehension skills. Special lessons on writing in math develop students’ abilities to write appropriate answers and solutions to problems. The kinds of writing prompts used in Prentice Hall Mathematics are modeled after those found on major kinds of state and national assessments and focus on writing to explain, compare, and persuade.

B. Problem-Solving Strategies

Research evidence shows that there should be direct instruction on the teaching of problem-solving strategies, such as draw a picture, make a table, look for a pattern, write an equation, and so on. Suydam (1980) found that problem-solving performance is enhanced by teaching students to use a variety of strategies. Hembree and Marsh (1993) found that problem-solving practice without direct instruction on strategies did not produce improvement. They found that developing skill with diagrams gave the most pronounced effects on problem-solving performance (at all grades), followed by training in the translation of words into symbols.

Explicit instruction on problem-solving strategies occurs in Prentice Hall Mathematics. A list of the strategies that are taught and applied in the program is shown at the right. Opportunities to choose, compare, or combine strategies are also included throughout the program.

Charles and Lester (1984) found that students can learn how and when to use problem-solving strategies to successfully solve problems when provided with explicit instruction on the strategies. In particular, experimental classes scored significantly higher than the control classes on measures of ability to understand problems, plan solution strategies, and get correct results.

Problem-solving strategies are developed in Prentice Hall Mathematics. Some lessons develop specific strategies like write an equation. Others illustrate how multiple strategies can be used together to solve problems. Still others illustrate how the same problem can be solved using different strategies.

C. A Key Strategy: Writing an Equation

One of the most important goals for algebra is to develop in students the ability to represent mathematical relationships using expressions and equations. “Proficiency with representational activities involves conceptual understanding of the mathematical concepts, operations, and relations expressed in the verbal information, and it involves strategic competence to formulate and represent that information with algebraic equations and expressions.”

---

**Research Says: Writing an Equation**

“Representational activities of algebra interact with well-established natural language based habits.”

Also, students’ underlying understanding of mathematical concepts can affect their ability to translate real-world relationships into algebraic expressions and equations.

**Putting Research Into Practice**

To develop students’ abilities to translate words to algebraic expressions and equations, *Prentice Hall Mathematics* has a plan for developing the strategy “write an equation.” As students move through Courses 1–3, more of the strategy lessons focus on writing an equation.

**Additional Examples**

*Prentice Hall Mathematics* promotes teaching about problem solving throughout the program. For additional examples, see the following Student Edition pages:

<table>
<thead>
<tr>
<th>Course</th>
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<tbody>
<tr>
<td>Course 1</td>
<td>Pages 206–207</td>
<td>Algebra 1</td>
<td>Pages 103–107, 330, 714–741</td>
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<tr>
<td>Course 2</td>
<td>Pages 160–162</td>
<td>Geometry</td>
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</tr>
<tr>
<td>Course 3</td>
<td>Pages 217–218</td>
<td>Algebra 2</td>
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</tr>
<tr>
<td>Pre-Algebra</td>
<td>Pages 430–431</td>
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Teaching for Problem Solving

Teaching for problem solving has the longest tradition in the school mathematics curriculum. “Applications” and “context” are words that are often used to describe this approach. Essentially, in this approach mathematics is taught for its use in the real world. Students are given real-world problems that can be solved using a newly developed skill. The challenge for students is to translate the real-world situation into a representation, for example, an equation that can be solved to answer the question in the problem.

In Prentice Hall Mathematics, rich problem-solving experiences related to the real world that are realistic to students are embedded in daily and special lessons. Students are encouraged to use the familiar aspects of real-world situations to model and solve problems. Students are not likely to ask, “When will I ever use this mathematics?”

“Studies in almost every domain of mathematics have demonstrated that problem solving provides an important context in which students can learn about number and other mathematical topics.”

—Adding It Up: Helping Children Learn Mathematics, page 420

Research Says: Real-World Applications

“We know from years of unsuccessful experience that for most students, decontextualized learning does not last. Students retain what they learn from their own efforts to address challenging problems that arise from situations that resonate with their own interests.”

Putting Research Into Practice

The problems in Prentice Hall Mathematics provide a wide variety of real-world contexts as well as many contexts related to the student’s everyday life. Many are interdisciplinary in nature.

Research Says: Linking Subject Matter to Contexts
Stanic and Kilpatrick (1989) found that students who have trouble solving mathematical problems in school can solve comparable problems in out-of-school situations that are more meaningful to them. As such, teachers should make an effort to link subject matter to contexts that are realistic to students.39

Putting Research Into Practice
Prentice Hall Mathematics provides not just real-world problems but also realistic problems. Every chapter has an applications lesson using data and contexts from books published by Dorling Kindersley (DK). These lessons provide real reasons to use mathematics.

Additional Examples
Prentice Hall Mathematics promotes teaching for problem solving throughout the program. For additional examples, see the following Student Edition pages:

<table>
<thead>
<tr>
<th>Course</th>
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<tr>
<td>Course 2</td>
<td>Pages 126-127, 291, 342-343, 586</td>
<td>Geometry</td>
<td>Pages 110-111, 144, 238-239, 327</td>
</tr>
<tr>
<td>Pre-Algebra</td>
<td>Pages 122-123, 249, 284-285, 409</td>
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</table>

Teaching Through Problem Solving

In this approach, “problems are valued not only as a purpose for learning mathematics but also as a primary means of doing so.” Instruction starts by giving students a problem task to solve. The task may or may not be a real-world problem, but it must be a task for which a way to solve the problem is not immediately obvious to the student. However, students must be able to solve the problem using previously learned methods. The problem is solved by the student (or by groups of students), and the solutions are shared with the class. The student’s thinking and work on the problem help him or her connect prior mathematical ideas to the new ones experienced in solving the problem. The teacher needs to make explicit connections to new language, and as needed, to new concepts and procedures.

Van de Walle (2004) found that the current thinking of a wide segment of mathematics education researchers is that “most, if not all, important mathematics concepts and procedures can best be taught through problem solving.” Research shows that this approach to developing mathematical concepts and skills promotes understanding of the underlying mathematical ideas.

In Prentice Hall Mathematics, problem solving is not just found in special lessons. Rather, the language, skills, and strategies of problem solving are embedded throughout core concept and skill lessons.

“We believe that if we want students to understand mathematics, it is more helpful to think of understanding as something that results from solving problems, rather than something we can teach directly.”
—James Hiebert, et al. in Making Sense: Teaching and Learning Mathematics with Understanding, page 25

“In a study of over 7,000 students, Wenglinsky found that students whose teachers conduct hands-on learning activities outperform their peers by more than 70% of a grade level in math on the National Assessment of Educational Progress (NAEP)”

**Research Says: Embedding Problem Solving Throughout Core Concepts and Skill Development**

Grouws and Cebulla (2000) found that students are able to learn new skills and concepts while they are solving challenging problems. They found that it is not necessary for teachers to focus first on skill development and then move on to problem solving. Both can be done together. In fact, students who develop conceptual understanding through problem solving early perform best on procedural knowledge later.42

Hiebert (2003) argues that conceptual understanding is the goal of teaching mathematics through problem solving. In analyzing the work of several researchers, he concluded that mathematical understanding "is supported best through a delicate balance among engaging students in solving challenging problems, examining increasingly better solution methods, and providing information for students at just the right times."43

**Putting Research Into Practice**

In *Prentice Hall Mathematics*, problem-based *Investigations* in the Student Edition are often used to introduce lessons to engage students in developing key mathematics ideas.

**Additional Examples**

*Prentice Hall Mathematics* promotes *teaching through problem solving* throughout the program. For additional examples, see the following Student Edition pages:

<table>
<thead>
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<tr>
<td>Course 1</td>
<td>Pages 283–284</td>
<td>Algebra 1</td>
<td>Pages 282–286</td>
</tr>
<tr>
<td>Course 3</td>
<td>Pages 258–260</td>
<td>Algebra 2</td>
<td>Pages 130–131, 248–251</td>
</tr>
<tr>
<td>Pre-Algebra</td>
<td>Pages 243–245</td>
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Meeting Individual Needs

by Sadie Bragg, Ed.D

Equity and quality in mathematics education often imply providing every student with both an equal and a quality learning experience in mathematics. Hiebert et al. (1997) define equity as follows: “Every learner – bilingual students, handicapped students, students of all ethnic groups, students who live in poverty, girls and boys – can learn mathematics with understanding. In order to do this, each student must have access to learning with understanding.”

Pelavin and Kane (1990) found from their research that when low-income and minority students do take courses in algebra and geometry and aspire to attend college, they not only go to college at about the same rate as the majority group, but they also succeed at about the same rate.

A central goal of Prentice Hall Mathematics is to provide the resources teachers need to reach students of various ability levels and learning styles so that they can help all their students be successful and understand mathematics.

The Equity Principle

“Excellence in mathematics education requires high expectations and strong support for all students.”

—NCTM Principles and Standards for School Mathematics, p. 12


Meeting Individual Needs: Instructional Design to Meet Individual Needs

Instructional Design to Meet Individual Needs

“Instructional design” refers to the structure of a lesson as presented in the student textbook. The design of a lesson significantly influences the content that students are to learn, how the content is taught, and most importantly, how well the content is learned.46 The National Center for Education Statistics reported that in the 2000 National Assessment of Educational Progress (NAEP) test, the majority of students at all three grade levels (fourth, eighth, and twelfth) indicated they did mathematics textbook problems in school every day. Eighth and twelfth graders who reported doing textbook problems in school had higher average scores than students who reported doing textbook problems less frequently.47 Therefore, it seems that the textbook is a major factor in determining the school curriculum.

To meet the needs of all students, Prentice Hall Mathematics gave special attention to building the instructional design of the following four areas:

• Reading and writing in math
• Differentiating instruction
• Assessment of learning
• Assessment for learning

These components of the Prentice Hall Mathematics curriculum provide numerous opportunities to promote access and attainment for the diverse needs of all students.

The Curriculum Principle

“A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics and well articulated across the grades.”

—NCTM Principles and Standards for School Mathematics, p. 14

Communication in Math Through Reading and Writing

Reading and writing in mathematics build communication skills. “Because the discourse of the mathematics class reflects messages about what it means to know mathematics, what makes something true or reasonable, and what doing mathematics entails, it is central to both what students learn about mathematics as well as how they learn it. Therefore, the discourse of the mathematics class should be founded on mathematical ways of knowing and ways of communication.” (NCTM, 1991)48 “If students are to engage in mathematical argumentation and produce mathematical evidence, they will need to talk or write in ways that expose their reasoning to one another and to their teacher. These activities are about communication and the use of language.” (Lampert and Cobb, 2003)49

Prentice Hall Mathematics has a consistent emphasis on critical thinking and reasoning through reading and writing in mathematics, and provides a variety of activities to help students to read and write mathematically.

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Research Says: Reading Math
Siegel et. al (1997) found that reading and a variety of related mathematical activities can be orchestrated to help students understand some technical mathematical content (such as distance, the congruence of triangles, or specific properties of geometric figures) as well as their significance and potential applications. Helping students make sense of mathematical concepts or procedures, seeing connections between mathematics and real life, developing broader views of mathematics, developing strategies for sharing information, and valuing students’ own ideas and those of others are among the purposes they illustrated for reading in mathematics.50

Putting Research Into Practice
In Prentice Hall Mathematics, the Reading Math lessons help students read more effectively so that they can write, speak, and think mathematically. Some of these include Reading an Example, Reading Diagrams and Words, Reading Comprehension, Reading to Analyze Errors, and Reading for Problem Solving. The Reading Math tips within a lesson help students to read and understand the language of mathematics. In addition to the Reading Math tips and features, Prentice Hall Mathematics provides Understanding Vocabulary exercises in the Chapter Review that are designed to help students understand and correctly use the vocabulary presented in the chapter.

To emphasize reading and math literacy, there are unique Reading and Math Literacy blackline masters that supplement the coverage of the text. The Interactive Textbook also provides reading support with audio.

Research Says: Writing in Mathematics

Masingila and Prus-Wisniowska (1996) found that writing in the mathematics classroom can be a means through which mathematical understanding is developed and assessed. Mathematical understanding can be developed through students' critical thinking put into words. They assert that writing can help students make their tacit knowledge and thoughts more explicit so that they can look at, and reflect on, their knowledge and thoughts.51

Putting Research Into Practice

Every lesson in Prentice Hall Mathematics incorporates Writing exercises that help students learn to explain, describe, or conjecture in a mathematical situation. Emphasis is placed on writing as it relates to Critical Thinking, Reasoning, Developing Proof, Error Analysis, and Open-Ended questions. When students write about mathematical processes, it can be a valuable way for them to make sense of these processes for themselves.

Each text contains lessons on Writing Extended Responses and Writing Short Responses to help students understand how answers to rubric-scored questions are evaluated. Other writing lessons on Writing in Math address Writing to Explain, Writing to Justify Steps, Writing to Compare, and Writing to Persuade.

Differentiating Instruction

All students are not the same, and do not all learn in the same manner. “If all students were the same, all day, every day, a teacher’s job would be simple – and boring. Researchers would develop one comprehensive theory of learning and prescribe the most effective model of instruction. Teachers would simply follow the recipe to produce high levels of success for all their students. The reality, however, is that students are different, one from the other. Teachers have many teaching strategies that they can implement, but the challenge is to find the combination of strategies that will enable all students to reach their full potential.” (Stiff et al., 1993)52

Wilson (1993) noted the following “Research suggests that carefully planned activities and efforts to involve students lead to understanding. Since individual learners have different ways of making sense out of ideas it is not sufficient for a learner to listen to and practice the teacher’s ideas. Each student must be actively involved in the learning process.”53

Ladson-Billings (1995) concluded that all students can be successful in mathematics when the instruction assumes that all students are capable of mastering the subject matter.54

All students, regardless of their special needs, deserve to have equal access to meaningful mathematics and science learning. While all students differ, requiring teachers to adapt their teaching methods, students with special needs present a particular challenge. Most can learn the important concepts of mathematics and science, but they learn differently. For those special learners, teachers strive to provide optional avenues to high achievement, but not a short-cut to lower standards.55

Central to the Prentice Hall Mathematics program is its goal to provide the resources to allow teachers to differentiate instruction to reach all students, to include activities that can be adapted to provide the necessary accommodations, and to help all students learn mathematics and become mathematically proficient. Such resources focus on the following areas:

- Ability Levels
- Inclusion
- Learning Styles
- Languages

Research Says: Tasks for Students of Differing Ability Levels

Stipek (1996) summarized the effects of difficulty level of task on student performance:
“Tasks perceived by the student as difficult (in relation to his or her skill level) engender lower expectations for success, perceptions of control, and perceptions of self-efficacy than easy tasks. Giving students easy assignments, however, is not an effective strategy for maintaining positive achievement-related cognitions. Although easy tasks produce high expectations for success, students usually assume that success on easy tasks does not require high ability. As a result, easily achieved success does not contribute to positive judgments of competence, nor does it produce feelings of pride and satisfaction.”

Putting Research Into Practice

The Prentice Hall Mathematics program addresses all ability levels with exercises organized by level of difficulty. These leveled exercises allow teachers to easily craft just the right assignments for individual students.

- The A exercises—Practice by Example—provide all students with the same opportunity to review skills and concepts. These exercises are keyed to the examples in the lesson.
- The B exercises—Apply Your Skills—ensure understanding of skills and provide practice for problem solving by connecting skills and concepts to real-world situations.
- The C exercises—Challenge Problems—provide students with opportunities to learn the more difficult mathematical concepts and skills and perform tasks that build on prior understanding.

The Prentice Hall Mathematics program also addresses the needs of advanced learners. The Challenge exercises and the Extension lessons give these students opportunities to solve problems that extend and stretch their thinking. The Teacher’s Edition includes Teaching Notes that address Advanced Learners. There are also Enrichment pages in the Grab & Go Files and Connections to Pre-Calculus Masters that give students opportunities to go beyond the mathematical skills and concepts in the lesson.

Research Says: Inclusion

Merritt (2003) indicated that "Underlying inclusion is the premise that students should be educated with their peers in the least-restrictive environment for as much of the day as possible. Because inclusion is individualized, it will look different for every student."

Putting Research Into Practice

The Prentice Hall Mathematics program provides numerous opportunities to individualize instruction so that all students can learn. Specifically, in the Teacher’s Edition there are Teaching Notes in each lesson to address the needs of those students who require certain accommodations to be successful and to understand the mathematics in the lesson. For example, there is support for English Language Learners, strategies for Reading in Math and Writing in Math, and Inclusion.

Each lesson includes Teaching Notes that provide suggestions for showing alternative teaching methods, tips for asking students questions about the lesson, and other useful strategies that teachers can implement so that all students can reach their full potential. Teachers are also provided with Guided Problem Solving Masters, Hands On Activities masters, a Multilingual Handbook, and Reading and Math Literacy Masters.

Research Says: Learning Styles

Keefe (1979) defines learning style as the characteristic behaviors of learners that serve as relatively stable indicators of how they perceive, interact with, and respond to the learning environment.58

Kahn (2003) stated “It is essential that all students have the opportunity to participate in inquiry-based lessons in mathematics and science. Such lessons are highly relevant to students’ lives, provide visual and tactile cues, offers opportunities for peer communication, and encourage skills that are necessary for future learning and employment.”59

Putting Research Into Practice

The Prentice Hall Mathematics program provides teachers with alternative strategies for reaching tactile learners, auditory learners, and visual learners. Tactile learners, who often prefer the use of manipulatives, sometimes use both visual and tactile learning styles.

Each Prentice Hall text features Investigations that allow students to explore concepts that will be developed further in the lesson. The Investigations provide a teaching strategy for tactile learners. In this feature, students may use manipulatives, graph paper, technology, and other resources to investigate the context of the lesson.

In addition, the Prentice Hall Mathematics program includes blackline masters for Hands-on Activities and Technology Activities. Each type of blackline master addresses the needs of students with diverse learning styles.

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The *Prentice Hall Mathematics* program is designed such that all students, regardless of their learning styles or special needs, have opportunities to be engaged in meaningful mathematics learning.

**Research Says: Languages**

Olivares (1996) concluded that because mathematical language proficiency affects the ability to communicate “about” mathematics, Limited English Proficiency (LEP) students should be exposed to a learning environment that focuses on mathematical communication in the second language. LEP students can always rely on the computational skills they developed in their first language, but until they achieve mathematical language proficiency in English, they will continue being mathematically limited in the new language.

**Putting Research Into Practice**

To meet the needs of LEP learners, the *Prentice Hall Mathematics* program has **Teaching Notes for English Language Learners** throughout the lessons. These notes stress ways in which teachers can modify the lessons to accommodate LEP learners. Some examples of these modifications include having students interpret the meaning of words used in the context of the lesson and find other meanings of the words; having them read and interpret graphs, charts, and pictures; and having them create their own word problems or express real-world problems in mathematical terms.

There are specific resources provided for students whose first language is Spanish. These resources include a Student Edition—Spanish Version, Spanish Practice Workbook, Spanish Reading and Math Literacy Masters, Spanish Assessment Resources, and a Multilingual Handbook. The *Prentice Hall Mathematics* program also includes an **English/Spanish Illustrated Glossary** in the Student Edition.

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Achieving Assessment Success for All Students

Assessment is integral to mathematics instruction. It should inform as well as guide teachers as they make instructional decisions.

Research shows that assessment needs to measure and describe a student’s growth and achievement in all domains of mathematics and at all levels of thinking. Because of this, there should be questions at all levels of thinking, of varying degrees of difficulty, and in all content domains.61

Student assessment should occur often and with a variety of different measures. Prentice Hall Mathematics provides an ongoing assessment strand that addresses assessment for learning and assessment of learning. The formative assessment features – before and during instruction – offer a variety of modalities that speak to different kinds of learners. The summative assessment features – after instruction – further prepare students for success in today’s tests.

A. Assessment for Learning

Kilpatrick and Swafford (2002) reported that all assessments need to support the development of mathematical proficiency. They need to measure the five strands of proficiency – understanding, computing, applying, reasoning, and engaging – and their integration. By doing so, assessments will provide opportunities for students to become proficient rather than taking time away from this goal.62

Prentice Hall Mathematics provides an ongoing assessment strand called the Instant Check System™. This begins within the lesson strand instruction and continues throughout the program components.

The Assessment Principle

“Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.”

—NCTM Principles and Standards for School Mathematics, p. 22

Meeting Individual Needs: Achieving Assessment Success for All Students

Research Says: Assessment of Prerequisite Skills
Gagne and Driscole (1988) found that the learning of skills typically requires the explicit prior development of simpler component skills (prerequisite skills).63

Putting Research Into Practice
At the beginning of every chapter or each lesson, students complete the Diagnosing Readiness and Check Skills You’ll Need exercises to see what prerequisite skills they may need to review before they begin the chapter or the lesson. The Teacher’s Edition prescribes specific Examples and Exercises that students can do for intervention.

Research Says: Assessment of Fundamental Ideas
The National Research Council (2002) indicated that understanding refers to a student’s grasp of fundamental mathematical ideas. Students with understanding know more than isolated facts and procedures.64

Putting Research Into Practice
Every lesson in the Prentice Hall Mathematics program includes numerous worked-out Examples, each followed by Check Understanding questions that students can do on their own. As skills and concepts are introduced, these questions focus students on the mathematics being presented and allow them to assess their understanding.

B. Assessment of Learning

The No Child Left Behind legislation (NCLB) clearly challenges school districts across the nation to raise expectations for all students. NCLB indicates that all students, regardless of ability level or learning disability, must participate in standardized testing to gauge what has been learned. So all students must become comfortable with not only the mathematics but also the formats of the standardized tests that are used to assess learning.

Meeting this challenge will necessitate the use of high-quality mathematical instruction as well as an integrated approach to assessment with a variety of assessment tools. Providing student feedback and obtaining student input is a valuable, essential part of the assessment process.

The Prentice Hall Mathematics program provides teachers with a range of tests and other assessment activities and instruments that enhance students' learning and allow them to build and show their mathematics proficiency.

Research Says: Preparing for Standardized Tests

Wilson and Kenny (2003) indicated that “teachers must understand the kinds of large-scale assessments that their students take and must think about the extent to which those tests can affect classroom instruction and assessment activities.”

Putting Research Into Practice

The Prentice Hall Mathematics program builds readiness and confidence for testing success while enhancing student learning. The program’s unique Assessment System prepares every student for high-stakes tests.

The Standardized Test Prep exercises, found in every lesson, give students daily practice with the types of test question formats that they will encounter on state and national tests. Each chapter also features Test Prep pages as well as Test-Taking Strategies that teach students strategies to be successful and give them practice in the skills they need to pass standardized tests.

Approaches to Important Content

By Dan Kennedy, Ph.D.; Randall I. Charles, Ph.D.; & Art Johnson, Ed.D

In this section of Putting Research into Practice, the Prentice Hall Mathematics authors discuss the approaches to important content incorporated in the series that lead students to success in mathematics, and the research base associated with them. First, Dan Kennedy discusses the mathematics that students will need to prepare for the real world, collegiate mathematics, and high-stakes assessments. Then, Randy Charles discusses the role of representations in the development of mathematical understanding. And finally, Art Johnson discusses the development of proof in geometry.

The Mathematics That Students Will Need

by Dan Kennedy, Ph.D

Surely the oldest defense of mathematics in the curriculum (and the reason that it is studied throughout one's formal education) is that all students will “need mathematics in the future.” Not only is this truer than ever in the modern world, but it is truer in more ways than ever before. It is not enough that students must be prepared to confront a world that is increasingly data-driven and reliant on technology. They must also be prepared for high-stakes testing and (for many) for the challenges of higher level mathematics classes such as "Mathematics is the key to opportunity. No longer just the language of science, mathematics now contributes in direct and fundamental ways to business, finance, health, and defense. For students, it opens doors to careers. For citizens, it enables informed decisions. For nations, it provides knowledge to compete in a technological economy. To participate in the world of the future, America must tap the power of mathematics.”

calculus. Since teachers often see these three goals as incompatible, especially for all students, it is more important than ever that they have well-designed instructional materials with a realistic, coherent vision of what their students will, in fact, need in the future. That is precisely the vision that experienced classroom teachers and knowledgeable specialists in mathematics education have brought to the Prentice Hall Mathematics series.

Mathematics Preparation for the Modern World

In a 2001 paper The Case for Quantitative Literacy, a research team from the National Council on Education and the Disciplines (NCED) observes that “despite years of study and life experience in an environment immersed in data, many educated adults remain functionally innumerate. Most U.S. students leave high school with quantitative skills far below what they need to live well in today’s society; businesses lament the lack of technical and quantitative skills of prospective employees, and virtually every college finds that many students need remedial mathematics.” The problem is not a lack of mathematics in the schools, but rather the inability of graduates to use that mathematics in context later on. In that respect, students have not been getting the mathematics they really need.

Prentice Hall Mathematics bases its program on sound research that has shown how students learn mathematics most effectively. The series fosters understanding for all students using structured instruction based on research and best practices.

Structured instruction is a consistent instructional approach found throughout the Prentice Hall series. The elements of structured instruction found within lessons include the following:

• Timely diagnosis and intervention
• Opportunities for active learning
• Making important mathematical ideas explicit
• Use of visual representations
• Emphasizing both skills and understandings

“The standards specify that instruction should be developed from problem situations. As long as the situations are familiar, conceptions are created from objects, events, and relationships in which operations and strategies are well understood. In this way, students develop a framework of support that can be drawn upon in the future, when rules may well have been forgotten but the structure of the situation remains embedded in the memory as a foundation for reconstruction.”

Structured instruction gives students a framework in which they can build their mathematics skills incrementally while continually solving problems in real-world contexts they can understand.

Multiple approaches are emphasized throughout the program, teaching students through constant experience how to recognize the mathematics that affects their lives. As the NCED study concludes: “To enable students to become numerate, teachers must encourage them to see and use mathematics in everything they do. Numeracy is driven by issues that are important to people in their lives and work, not by future needs of the few who make professional use of mathematics or statistics.”

Mathematics Preparation for Collegiate Mathematics

Notwithstanding the need for all students to be quantitatively literate in society, there is also a basic toolbox of additional mathematical background that they must accumulate simply to understand higher-level courses in science and mathematics. In attempting to build this toolbox of skills most efficiently, textbooks in the past have deliberately presented algebra and geometry devoid of context in order to emphasize patterns and algorithms, thereby contributing to the innumeracy problem mentioned above. Now that context has been restored, teachers need assurance that the manipulative skills are still being taught.

Prentice Hall Mathematics has incorporated the best of research-based pedagogy into its textbooks from middle school through calculus, giving students and teachers a smooth path to higher learning that sacrifices none of the necessary concepts. By teaming authors from various levels together to produce this series, Prentice Hall has produced a coherent, consistent, developmentally sound path to higher-level mathematics—an approach that knows where it is going and can assume where it has been. The advantages are many:

- **Consistent Pedagogy.** Students learn not only the “what” and the “how” but also the “why.” Research has shown that this leads to long-term retention of skills. A National Research Council publication, *How People Learn: Brain, Mind, Experience, and School*, notes that “Learning with understanding is more likely to promote transfer than simply memorizing information from a text or a lecture…when the transfer of learning is measured, the advantages of learning with understanding are likely to be revealed.”

> “Textbooks and other instructional materials in the United States need to support the learning of all five strands of mathematical proficiency (understanding, computing, applying, reasoning, engaging). They should develop the core content of mathematics in a focused way and with continuity across grade levels.”
• **Spiraling Without Redundancy.** The courses can enjoy the benefits of intentional spiraling without turning students off with needless redundancy. For example, in the past, Algebra 2 chapters have looked like more rigorous versions of Algebra 1 chapters, often repeating the same exercises. In *Prentice Hall Mathematics*, chapters of Algebra 1 spiral into lessons of Algebra 2.

• **Consistent Conceptual Development.** Concepts that *should* be developed slowly, like variables and coordinate geometry, can be developed consistently over several years.

• **Monitoring Progress Within Instruction.** Prentice Hall provides a unique *Instant Check System™* that is built into each text in the series to diagnose weaknesses and assess mastery before, during, and after each lesson’s instruction. The *Instant Check System™* teaches students to be independent learners, reading the textbooks for their own understanding. There are *Diagnosing Readiness* exercises to recall previous knowledge at the beginning of each chapter, *Check Skills You’ll Need* exercises at the beginning of each lesson to review requisite skills, and *Check Understanding* exercises after each example to provide immediate feedback on what has been learned. Once students catch on, they can enjoy the benefits of a similar design throughout the Prentice Hall series, straight through calculus.

• **Consistent Technology Integration.** The incorporation of technology is treated consistently throughout the series, an advantage that can truly be appreciated by teachers who have struggled with textbooks that either treat technology differently at different levels or else re-introduce it at every level as if students have never seen it before.

• **Seamless Transition for Each Course.** Graduating to the next textbook does not have to mean such unnecessary distractions as new notation, new kinds of teacher support materials, unfamiliar design, or different approaches to pedagogy. Students and teachers can truly pick up where the previous course left off.
Adapting to Trends in College Mathematics

About the time that the National Council of Teachers of Mathematics began fomenting change in the K–12 curriculum, college mathematicians began changing the nature of the college courses into which the K–12 curriculum fed. In 1993, the National Science Foundation sponsored a conference, *Preparing for A New Calculus*, during which the participants drafted a set of “fundamental principles” that they felt should characterize courses intended to prepare students for twenty-first century college mathematics.

These fundamental principles (many of which are reflected in the NCTM *Curriculum and Evaluation Standards for School Mathematics*) are hallmarks of the *Prentice Hall Mathematics* series at every level. The table below lists Prentice Hall’s response to the conference’s recommendations for what prerequisite courses should include to prepare students for calculus.

<table>
<thead>
<tr>
<th>Recommendations for Calculus Preparation from the Preparing for a New Calculus Conference</th>
<th>Prentice Hall’s Response in the <em>Prentice Hall Mathematics</em> series</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cover fewer topics, but each topic should be covered more thoroughly and with more emphasis on fundamental concepts.</td>
<td>Trimmed, moved, or modified material to give more attention to the important topics and to keep the books at a teachable size.</td>
</tr>
<tr>
<td>Place less emphasis on complex manipulative skills.</td>
<td>Continually used mathematics in context in examples and exercises, while providing students with ample exercises for practicing (and later reinforcing) manipulative skills.</td>
</tr>
<tr>
<td>Teach students to think and reason mathematically, not just to perform routine operations. This higher level of expectation should be reflected in exercises, project examinations, and written assignments.</td>
<td>Provided leveled exercises with built-in help—including critical thinking, error analysis, developing proof, and open-ended questions—that teach students to think and reason mathematically.</td>
</tr>
<tr>
<td>Approach each topic symbolically, graphically, numerically, and in verbal and written form with the aim of helping students construct appropriate mental representations of mathematical concepts.</td>
<td>Stressed multiple representations (teaching students to think graphically, numerically, algebraically, and verbally) at every level of the series, culminating in the precalculus and calculus textbooks, which incorporate them into their titles. This “Rule of Four,” a defining theme of modern college courses, motivates much of the pedagogy in <em>Prentice Hall Mathematics</em>.</td>
</tr>
<tr>
<td>Emphasize modeling the real world and develop problem-solving skills.</td>
<td>Stressed modeling and real-world problem-solving in every chapter of every book. The Dorling-Kindersley Real-World Snapshots also take that reality to a new level.</td>
</tr>
<tr>
<td>Make use of all appropriate calculator and computer technologies for graphing, numerical computations, and symbolic manipulation. The full power of technology should be introduced in the service of learning the mathematics.</td>
<td>Incorporated technology almost matter-of-factly, always in service of learning the mathematics. Ancillary materials are available with extra graphing calculator explorations, while computer-based applications range from textbook examples to the Interactive Textbook online and on CD-ROM.</td>
</tr>
<tr>
<td>Promote experimentation and conjecturing.</td>
<td>Consistently encouraged experimentation and conjecturing, from <em>Investigations</em> in the earlier books to <em>Explorations</em> throughout the pre-calculus and calculus texts.</td>
</tr>
</tbody>
</table>

*Source: The Mathematical Association of America/Notes Number 36: Preparing for a New Calculus (Conference Proceedings)*
### Recommendations for Calculus Preparation from the Preparing for a New Calculus Conference

- Provide a solid foundation in mathematics that prepares students to read and learn mathematical material at a comparable level on their own, and to apply what has been learned in new situations.

### Prentice Hall’s Response in the Prentice Hall Mathematics series

- Encouraged students to read the textbook and take charge of their own learning through the Instant Check System™, building skills that will serve them well in all future courses.

- Simultaneously serve mathematics and the physical sciences, the biological sciences, the social sciences, and other fields. The mathematics included should be presented as an elegant, unified, and powerful subject that describes processes in all of these areas.

- Featured a rich array of applications from all branches of the academic tree in every textbook in the series. Not only does this prepare students for the new college courses, as noted above, but it also contributes to their quantitative literacy.

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It should be noted that the goals for preparing students for today’s college courses are not that different from the goals of preparing students for the modern workplace. As the Mathematical Sciences Education Board of the National Research Council wrote in 1989 in *Everybody Counts, a Report to the Nation on the Future of Mathematics Education*, “As colleges, universities, and continuing education attract significant fractions of the population, schools must now prepare all students for some type of postsecondary study. The level of literacy formerly associated with the few who entered college must now be a goal for all.”

### Preparing Students for High-Stakes Assessments

Many teachers wish they could worry about preparing students for life in a mathematics-based world and survival in higher-level courses and let the high-stakes assessments take care of themselves, but that is often an unrealistic strategy. Preparation for such assessments has become another example of “mathematics students will need.” In fact, a national survey of 1,091 public school teachers conducted for *Education Week* in 2000 found that 67 percent of teachers felt that their teaching had become excessively focused on state tests, while 65 percent of the math teachers surveyed were concentrating on tested information to the detriment of other important areas of learning.

Schools want to prepare their students for the future, yet they realize that there are gates to that future that are guarded by standardized tests. Teachers need some assurance that students who learn “the mathematics they need” in the courses they are teaching will have whatever else they need to succeed in the high-stakes assessments.
The *Prentice Hall Mathematics* series addresses this situation by making test preparation a strategically embedded component of its ongoing assessment at every level. Not only do the exercises encourage creative thinking in addition to manipulation, but they also come in various standardized test formats to acclimate students to the types of questions they could possibly see. Appropriate problem-solving strategies for various kinds of test items are discussed in the context of the mathematics being tested. In the Prentice Hall series the test preparation is in the service of the mathematics rather than vice-versa; teachers and students need never feel that they have interrupted the course to prepare for the test.

**Representations and the Development of Mathematical Understanding**

*by Randall I. Charles, Ph.D.*

“Representations” Are Important in Developing Understanding

One of the most significant changes from the 1989 *Curriculum and Evaluation Standards for School Mathematics* from the National Council of Teachers of Mathematics (NCTM) to the 2000 *Principles and Standards for School Mathematics* (PSSM) was the addition of “representation” to the “Process Standards.” The addition of this standard sends a significant message about teaching and learning mathematics.

“The term *representation* refers both to process and to product.” Representation as a process refers to the act of creating in one’s mind one’s own mental image or images of a mathematical idea; these are sometimes called *internal* or *mental* representations.

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Randall I. Charles, Ph.D.

Author, *Prentice Hall Mathematics, Courses 1–3; Algebra 1; Geometry; Algebra 2*

**Representation Standard**

Instructional programs from prekindergarten through grade 12 should enable all students to—

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.

—Principles and Standards for School Mathematics.

representations. Representation as a product refers to some physical form of that idea such as a picture, a graph, or an equation; these are sometimes called external representations.

For example, consider the mathematical idea of a ratio. The idea of ratio might be associated with mental notions such as these:

- a comparison
- something that can be written in different ways
- all ratios can be written in fractional form but not all ratios are fractions
- related to percents
- matching like terms when using ratios

Obviously, we cannot observe one’s internal or mental representations. Instead, by focusing on external representations and their connections we can infer how a student represents an idea internally and thereby comes to understand that mathematical idea. For the idea of ratio, one might think of external representations such as \(a\) to \(b\), \(a:b\), and \(a/b\). Or one might show a graph of equal ratios as a representation.

Why is the idea of representation so important? Simply stated, the more powerful representations of an idea someone has in his or her mind, and the more these representations are connected to each other, then the better the person understands that mathematical idea. For the example above, the more ways a student can think about the concept of ratio and the more ways that student can show and talk about representations of ratio, the better that student understands ratio. So, with regard to teaching and learning mathematics, Goldin and Shteingold state that using and connecting representations to develop mathematical ideas is the one of the best ways to help students understand mathematics.\(^{67}\)

How Representations Can Be Infused Into the Curriculum

There are many kinds of (external) representations that teachers and students can use in teaching and learning mathematics. The remainder of this section will show and discuss two key categories of representations: diagrams and symbol meanings, and technology tools.

Nickerson (1994)\textsuperscript{68} tells us that the ability to use diagrams in mathematics is critical to learning mathematics. There are at least two reasons why this is so. First, a great deal of mathematics involves understanding and using visual images. Diagrams are one kind of visual image. Also, diagrams provide a vehicle for representing abstract ideas in ways that promote understanding. Students who use and understand diagrams are better able to handle abstract concepts because they have a concrete physical model to work from. Diezmann and English (2001)\textsuperscript{69} present a strong case for developing diagram literacy. They suggest five ideas for ways teachers can develop students’ abilities to use diagrams:

- Explain links between the structure of a problem and a diagram representing that structure;
- Model creating diagrams and explain how the diagram is connected to the problem;
- Have students compare and discuss their diagrams;
- Provide instruction on particular classes of diagrams;
- Have students evaluate diagrams for their appropriateness.

Diesmann and English say that there is a distinction between a drawing and a diagram. They suggest that a good diagram is one that provides a “structural representation” of an idea with unimportant surface features. For example, consider the simple word problem below.

\begin{quote}
Suppose your grandmother sends you five games for your birthday.
The games all have the same weight. The box she mails them in weighs 8 ounces. The total weight is 48 ounces.
\end{quote}

A drawing for this problem might look like this.

\begin{center}
\includegraphics[width=0.3\textwidth]{drawing}
\end{center}

\textit{from Course 2 Student Edition, page 94}


A diagram for this problem might look like this, where $g$ represents the weight of the game.

![Diagram](image)

The diagram shows how the quantities in the problem are related to each other without showing the specific number or kinds of objects in the problem. The diagram is a more powerful representation. The examples that follow are the major types of diagrams used throughout the Prentice Hall series to help students develop a solid understanding of the concepts.

**Mixed Numbers and Improper Fractions**

Symbols and two pictorial models are representations shown and connected to help students understand the idea of mixed numbers and improper fractions.

![Models](image)

A number line can also help you understand improper fractions and mixed numbers.

The number line shows that $1\frac{3}{4} = \frac{7}{4}$.
**Percent and Problem Solving**

Several representations are presented in this example. The real-world story is a verbal representation of a situation. The diagram shows how the quantities in the real-world situation are related to each other, and the equation shows through color coding how the elements of the equation are related to the diagram.

![Example](image1)

*from Course 2 Student Edition, page 312*

**Part-to-Whole Relationships**

The diagram is used to show how the quantities in the real-world problem are related to each other. Then the diagram is used to give meaning to the symbols in the equation.

![Example](image2)

*from Course 1 Student Edition, page 91*
**Number Set Relationships**

A Venn diagram, a pictorial representation, is used to show how number sets are related to each other.

Venn diagrams are also used in Greatest Common Factor (GCF) and Least Common Multiple (LCM) problems.

**Translating to Algebraic Equations**

The Relate-Define-Write representations move students from the real-world problem to the symbolic representation. Also, the annotations with symbols provide verbal representations for the steps in the process.
Technology
The Prentice Hall Interactive Textbooks contain many examples of dynamic interactive representations. These screen captures from the Course 2 Interactive Textbook show multiple representations of ratios.

In this interactive environment, the student can change either visual representation (area or number line) and immediately see the corresponding changes in symbolic representations (fraction, decimal, or percent).

The technology also allows the student to explore part-to-part ratios (such as shaded to non-shaded area) and to toggle between types of ratios to observe the effect on symbolic representation.
Curriculum Design

The use of representations can be promoted in a variety of ways. The examples that follow illustrate three different ways that the Prentice Hall Mathematics authors used to promote representations throughout the series.

Hands-On Investigations

Investigations that engage students in thinking about mathematical ideas through both physical and symbolic representations promote understanding. The example below shows how paper folding can be used to connect to the algorithm for multiplying fractions.

More Than One Way

One of the most important beliefs a teacher can promote with students is the idea that most problems in mathematics can be represented and solved in more than one way. This fact needs to be made explicit through examples.
The notion of problems being solved in more than one way is particularly crucial in problem solving. The example below shows how the same problem is solved in two different ways.

The Development of Proof

By Art Johnson

Learning theories developed by Piaget and Inhelder and by the Van Hieles describe levels of geometric thinking. Research on these theories, as described in *A Research Companion to Principles and Standards for School Mathematics* (NCTM, 2003) indicates that geometry students are often unprepared for writing proofs.

*Prentice Hall Geometry* provides students with multiple opportunities to practice and master the skills necessary for success in writing independent proofs.

Here is how *Prentice Hall Geometry* builds students’ skills—and confidence.

“Unfortunately, more than 70% of students begin high school geometry at Levels 0 or 1 (out of five levels), and only those students who enter at Level 2 (or higher) have a good chance of becoming competent with proof by the end of the course.”

—Shaughnessy & Burger, 1985

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Art Johnson, Ed.D.
Author, Prentice Hall Mathematics, Geometry

from Course 2 Student Edition, pages 252–253
A. Reasoning and Proof

Sometimes developing a proof is a natural way of thinking through a problem.\(^7\)\(^0\) In every lesson, *Prentice Hall Geometry* exercises ask students to explain or justify their reasoning—precisely the skill needed to successfully construct a proof. Research says that teachers should have students justify their thinking at all levels. (Clements, 1992)\(^7\)\(^1\)

The examples that follow illustrate opportunities for students to use and develop reasoning and critical thinking skills throughout the Prentice Hall series.

B. Types of Proof

Research says that students need to attain higher levels of geometric thought before they begin a proof-oriented study of geometry. (Clements, 1992) Prentice Hall Geometry dedicates instruction to reading and writing proofs.

Prentice Hall Geometry also builds students’ confidence by giving them opportunities to complete proofs. In the previous section, you see an example of paragraph proofs. Three types of proof that students use throughout the course are paragraph proofs, flow proofs, and two-column proofs.
C. Developing the Proof Process

Studies that have attempted to involve students in the crucial elements of mathematical reasoning and discourse—conjectures, careful reasoning, and the building of validating arguments that can be scrutinized by others—have shown more positive effects (in proof writing).72 As students’ understanding of the proof process grows, Prentice Hall Geometry requires them to do proofs that they think through themselves. The examples that follow illustrate how students learn how to develop this important skill.

Research for the past 20 years has indicated that most students finish a geometry course without the ability to do proofs. “Only about 30% of students in full-year geometry courses that teach proof reach a 75% mastery level in proof writing.”73 (Senk, 1985) Prentice Hall Geometry has been developed with this research in mind. Our goal in developing our proof-writing strand is to give students the experiences, support, and practice they need so that they can write proofs successfully and independently.
