Abacus
Efficacy Research

Pillar 1: Place Value

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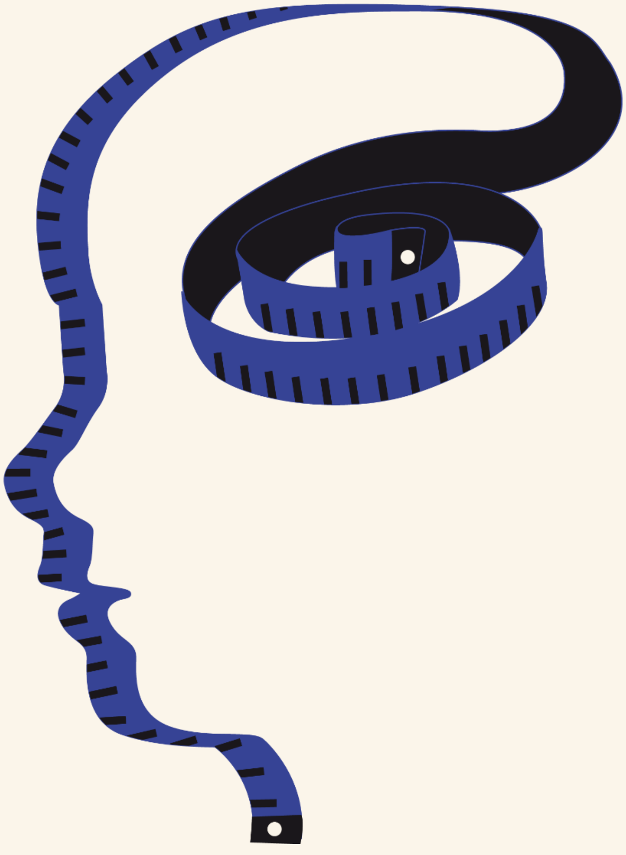

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Pillar 1: Place Value

The research:

- 1.1.1 According to Ofsted's study of Good practice in primary mathematics: evidence from 20 successful schools (2011):

  'Understanding of place value... [is an] essential precursor for learning traditional vertical algorithms (methods) for addition, subtraction, multiplication and division' (p.6).

  'A good understanding of place value is considered to be of paramount importance. This is supported by a wide range of practical equipment including base-10 apparatus, 100 squares, bead strings, place-value cards and number lines' (p.10).

  'In the schools visited, the teachers frequently emphasised place value. [For example] that multiplication by 48... is the sum of multiplying by 40 (four 10s, and hence the 0) and by 8 (units), rather than simply by 4 and by 8. Such attention to mathematical precision is an important element of these schools' success' (p.28).

- 1.1.2 According to Schwartz & Kenney (2008), there are three 'big ideas' associated with place value:

  Basic digits – these are just a few symbols, reused in different positions;

  Position – this gives the size of a quantity;

  Base – in base 10, units are counted up to 9, then groups are counted (in 10s), then groups of groups are counted (in 10s of 10s, i.e. 100s) and so on.

  Pupils need to understand all three ideas. In particular, position and base, because they provide the foundation to later understanding of computation.

- 1.1.3 According to Ross (1989, p.47), the number system is characterised by the following four properties:
  i. Positional property – the quantities represented by the individual digits are determined by the position they hold in the whole numeral.
  ii. Base-ten property – the values of the positions increase in powers from right to left.
  iii. Multiplicative property – the value of an individual digit is found by multiplying the face value of the digit by the value assigned to its position.
  iv. Additive property – the quantity represented by the whole numeral is the sum of the values represented by the individual digits.

  To understand place value the student must coordinate and synthesize... [this] subordinate knowledge... [For example,] a student who understands place value knows not only that the numeral 52 can be used to represent "how many" for a collection of fifty-two objects but also that the digit on the right represents two of them, the digit on the left represents fifty of them (five sets of ten), and that 52 is the sum of the quantities represented by the individual digits'.

- 1.1.4 According to Ross (1990), there are five stages in children's understanding of place value.

  These are:

  - The child interprets a 2 digit numeral as the whole number it represents, but assigns no meaning to individual digits.
  - Children demonstrate knowledge of the positional property of their individual digits; they know the digit on the right is 'in the ones place' and the digit on the left is 'in the tens place'.
  - The digits are interpreted by their face values in such a way that they sum to the whole. For example, the '2' in '25' is seen to represent two of one kind of object and the "5" is seen to represent five objects of a different kind.
  - The tens digit is interpreted as representing groups of ten, though the understanding is limited and performance is unreliable.
  - Students know that the individual digits in a two digit numeral represent a partitioning of the whole quantity into a tens part and a ones part… Understanding is easily demonstrated and performances are reliable.

- 1.1.5 According to Cotter (2000), the focus on counting by ones can interfere with the development of place value understanding. In the Japanese school system children are discouraged from using only one-by-one counting and are encouraged to also see multi-digit numbers as part-whole concepts from an early stage.

- 1.1.6 According to Sinclair et al (1992), a part-whole understanding is acquired before an appreciation that the tens digit represents sets of ten objects. For example, in a study of 144 year two to year four children undertaken by Thompson & Bramald (2002), '63% of the sample correctly added two two-digit numbers using partitioning: a procedure that demands an appreciation not only of the fact that the whole is the sum of the parts, but that the separate parts can be added and a new whole constructed from these new parts... [Of these], only four... were in the 'excellent understanding of place value' category' (p.11). For example, they might have been able to recognise that the 2 in 23 stands for 'twenty' but not that it stands for 'two tens'.

- 1.1.7 Thompson and Bramald (2002) state that 'what we have up to now called place value should be seen as comprising two separate concepts: quantity value' (i.e. twenty [and] three) and column value (i.e. two tens and three ones)'. They suggest that an understanding of the former develops before the latter, which lends support to the argument for delaying the teaching of tens and units – column value – until later in the curriculum, even until such time as children are to be taught the standard algorithms for the basic operations.

- 1.1.8 The types of questions that 'test' different aspects of place value understanding, include:

  - Can you read this number to me? (Show card with 16 written on it). Please take 16 cubes out of the box.

Can you show me with the cubes what this part (6) of the number means? (Circle the 6).
Can you show me with the cubes what this part (1) of the number means? (Circle the 1).

This aims to test whether children understand that the 1 stands for ten and not just one, and has been used by several researchers (for example, Kamii, 1986; Ross, 1989; Hiebert & Wearne, 1992; Price, 1998 and Thompson & Bramald, 2007).

- What is 25 plus 23? (Show card with 25 + 23 written on it).
  Tell me how you did it.

  Thompson & Smith (1999) classified children's strategies for answering this question, as follows, with the level suggesting increasing 'levels of sophistication' in understanding:
  Level 1 – Counting on
  Level 2 – Manipulating digits
  Level 3 – Partitioning (i.e. 20 + 20 = 40; 5 + 3 = 8; 40 + 8 = 48)
  Level 4 – Mixed method (e.g. 20 + 20 = 40; 40 + 5 = 45; 45 + 3 = 48)
  Level 5 – Sequencing (i.e. 25 + 20 = 45; 45 + 3 = 48)

- Put 3 cubes in the 10s column and four in the ones column and ask, 'What number does this represent?'
  Remove the three cubes from the tens column, and point to the remaining cubes, and say 'These cubes are in the ones column and so we say they represent 'four''.
  'I'm moving the cubes to the next column' (say this as you do it)…
  'Can you tell me how the value of the cubes has changed?'

  This aims to test children's realisation that moving the digits one place to the left is equivalent to making a number ten times bigger. A question like this has been used by researchers Ward (1979), Brown (1981) and Thompson and Bramald (2002).

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