

Abacus Efficacy Research

Pillar 2: Number Facts

May, 2014



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Summary

This is a brief examination of some of the most cited education research literature on 'Number facts'.

It suggests that developing automatic recall of number facts (such as number bonds and multiplication tables), along with the strategies to derive them, is a necessary precursor to experiencing success in mathematics in later Primary years and into Secondary. For this reason, 'Number facts' is important as one of the 'Four Pillars' of Abacus.

2.1 The research:

- 2.1.1 According to Ofsted's study of Good practice in primary mathematics: evidence from 20 successful schools (2011, p.6):
'Good recall of number facts, such as multiplication tables and number bonds, are considered by the schools to be essential precursors for learning traditional vertical algorithms (methods) for addition, subtraction, multiplication and division.'
- 2.1.2 Sun and Zhang (2001, p.28) state that 'The ability to automatically recall facts strengthens mathematical ability, mental mathematics, and higher order mathematical learning. Without this automation students have difficulty performing advanced operations.'
- 2.1.3 Westwood (2009, p.14) agrees with Sun and Zhang, stating that 'Knowing basic number facts with a high degree of automaticity allows children to deal swiftly and effectively with the solution of problems. They are able to focus full attention and mental effort beyond the lower-order simple steps in a calculation to the higher-order processes involved in problem-solving.'
- 2.1.4 Sun and Zhang (2001, p.28) confirm that 'A mastery of lower-order skills [i.e. that involve number facts] instils confidence in students'.
- 2.1.5 Vukovic and Siegel (2010, p.34) state that '...deficient number fact knowledge is a defining feature of mathematical difficulty'.
- 2.1.6 Tait-McCutcheon, Drake and Sherley (2011, p.323) criticise 'Many resources... [for] tasks in which students need to utilise basic facts, but fail to remind students of the strategies they should be practising to gain mastery. Rather, they seem to treat basic fact fluency and mastery as the recall of instant answers in isolation.'
- 2.1.7 According to Thornton (1990, p.241) there are three phases to learning number facts:

Phase 1: Understand the concept;

Phase 2: Learn strategies or procedures to derive answers to unknown facts;

Phase 3: Practice so facts are memorised to the point of automatic recall.

- 2.1.8 Tait-McCutcheon, Drake and Sherley (2011, p.329) develop Thornton's model, stating that 'The process of basic fact learning can be described as moving through a set of points.

Point A: Fact selection

Point B: Strategy teaching

Point C: Targeted practice

Point D: Knowledge: Recall and derivation.

In this model, new facts are first introduced and constructed through the use of efficient mental strategies that build on what is already known [For example, if a child knows $5 + 5$ is 10, then $5 + 6$ can be introduced as being one more.]... Once proficiency has been achieved the facts are practiced to encourage accurate and speedy recall. Recall and derivation occur at point D when students are able to quickly and efficiently recall the answer to a problem as a fact, and can start to use the established fact to derive further facts.'

- 2.1.9 Cooke & Reichard (1996) find that, when practicing, the interspersion of known and unknown facts increases the speed at which facts are committed to, maintained in, and retrieved from longterm memory. It also assists in the remediation of errors from previous practice sessions.
- 2.1.10 In their work on number fact retrieval, Baroody, Bajwa and Eiland (2009, p.77) elaborate on the particular purposes of targeted practice, stating that 'practice is useful if its primary purpose is to provide students an opportunity to discover patterns or relations or to ensure reasoning strategies become automatic (once a child has [already] discovered patterns and relations)'
- 2.1.11 In their work to determine why some pupils struggle with number facts, Jordon, Hanich and Kaplan (2003, p.118) find that 'Children with poor fact mastery... do not show relative difficulties in reading and language but do show weaknesses in non-verbal reasoning'. For example, they may struggle with pattern recognition.
- 2.1.12 According to Silbert, Carnine and Stein (1990), the order that facts are introduced can assist students to become proficient in learning and recalling basic multiplication facts. They suggest that facts that can be learned easily should be presented first during practice (e.g., 0, 1, 10, 2, 5, 9), then they should be followed by the more demanding multiplication sequences (e.g., 4, 7, 3, 8, and 6).
- 2.1.13 Thornton (1990, p.241) find that 'Learning subtraction number facts typically is more difficult for children than mastering addition facts'
- 2.1.14 According to Thornton (1990, p.245-246) it is important to 'allow more time for children to consolidate newly-learned addition facts before trying to apply them to obtain solutions for subtraction.' Then, 'focus systematically on the subtraction-

addition relationship through manipulative-based and other 'add to check' activities during subtraction number fact work'.

- 2.1.15 According to Baroody (2006, p.29) in the teaching of number facts, 'instruction should: concentrate on "fact families," not individual facts [and]...encourage children to build on what they already know. For example, mastering subtraction combinations is easier if children understand that such combinations are related to complementary and previously learned addition combinations (e.g., $5 - 3$ can be thought of as $3 + ? = 5$).'
- 2.1.16 According to Carpenter and Moser (1983), relating number facts gives an organizational structure to the facts, and provides children with a logical basis for storing addition and subtraction combinations in long-term memory.
- 2.1.17 Baroody, Bajwa and Eiland (2009, p.70) observe that 'Research indicates that children typically progress through three phases in learning a basic combination or a family of related combinations:

Phase 1: Counting strategies (using object or verbal counting to determine answers)

Phase 2: Reasoning strategies (using known facts and relations to deduce the answer of an unknown combination)

Phase 3: Mastery [efficiently producing answers from a memory network]...

The first two phases are characterized by conscious or deliberate and, thus, relatively slow cognitive processes. Phase 3 is characterized by non-conscious or automatic and, thus, relatively fast cognitive processes.
- 2.1.18 According to Steinberg (1985), the use of related facts is accompanied by a decrease in more basic counting strategies.
- 2.1.19 Hasselbring, Goin and Bransford (1988) find that, when practicing, students should be given a specific time to respond to a question or a constant time delay, typically starting at five seconds and gradually reducing to one and a half seconds. The reduction of the response time forces the student to abandon inefficient counting strategies and attempt to retrieve the answer from memory.
- 2.1.20 Baroody, Bajwa and Eiland (2009, p.71) state that 'Recognising commutativity in addition and multiplication is an important part of learning number facts, as unknown combinations can be treated as known combinations (e.g. if 5×8 is unknown, then it can be treated as 8×5 , which may be known), and 'reduce by half the amount of practice needed to master such combinations. This may help to account for why children more easily memorize addition and multiplication combinations than subtraction and division combinations [which are not commutative]'

2.2 References:

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