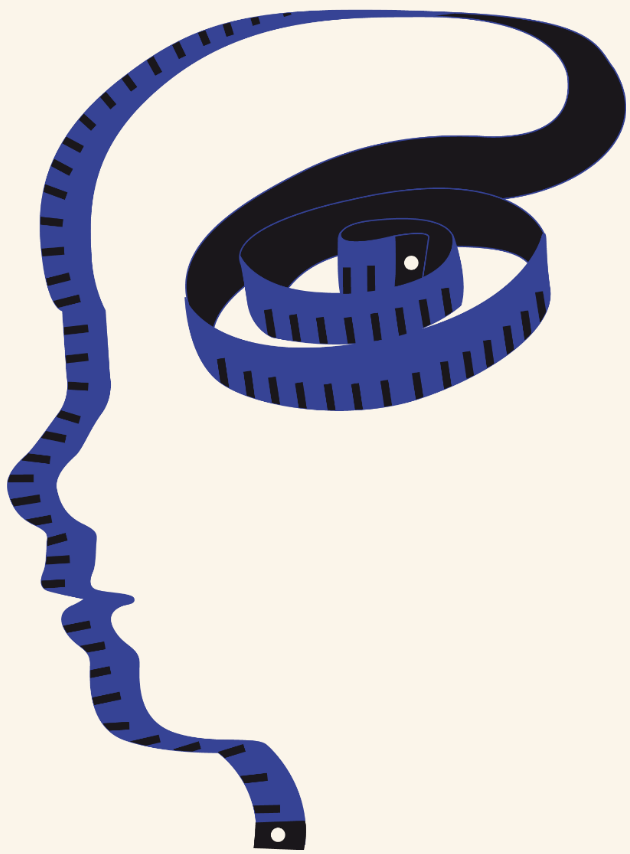
Abacus
Efficacy Research

Pillar 3: Images and Models

June, 2014



Prepared by: Dr Naomi Norman, Director, Techademic Ltd

Pillar 3: Images and models

Summary

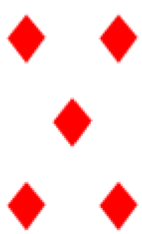
This is a brief examination of some of the most cited education research literature on 'Images and Models'.

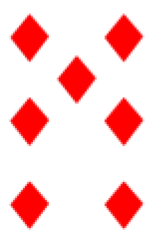
It suggests that pupils who make use of visual images and models do better in mathematics. In particular, images to represent numbers, such as dots set out in arrays, or frames, support pupils in their mathematical thinking and fluency. Furthermore, models, such as number lines, employed in different ways, can support pupils in developing mental strategies, as well as in moving from an understanding of problems in context, to the necessary abstract thinking required to experience success in mathematics.

3.1 The research:

- 3.1.1 According to Owens (2003), visual imagery in learning mathematics involves an ability to create a picture in the mind.
- 3.1.2 Hershkowitz et al. (1989, p.75) state that the purpose of visual imagery is for 'thinking about and developing previously unknown ideas and advancing understandings'.
- 3.1.3 Thomas and Harkness (2012, p.232) claim that the 'use of mental imagery signifies a conceptual milestone in a child's development of numeracy'.
- 3.1.4 Sharma (2013) speaks of the significance of pupils developing visual images to move beyond counting. In particular, the usefulness of images of 'visual clusters' of numbers, such as:

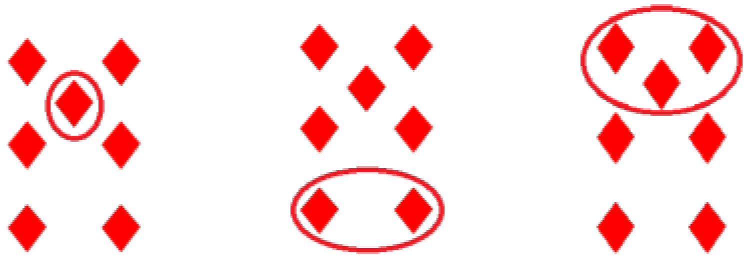
5: or 7:





Sharma (2013, p.12) writes that, 'when the child sees the visual cluster of 7 objects (as on a playing card), the child learns the sight facts:

7 = 1 + 6 = 2 + 5 = 3 + 4 without counting.

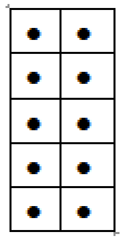


These sight facts, with formal strategies of addition and subtraction based on decomposition/recomposition provide children a strong base to extend their addition and subtraction facts mastery beyond 10... ' Sharma (2013, p.17) goes on to state that 'without the idealized image of these numbers and the decomposition/recomposition process, children have difficulty in developing fluency in number relationships'.

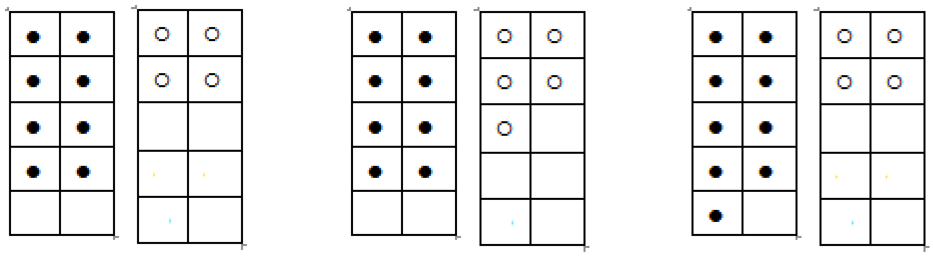
- 3.1.5 Mulligan, Prescott and Michelmore (2003, p.24) state that 'a student may recognise patterns of two or three dots in a rectangular array of six dots and hence see it as a structure rather than as six individual items. It has been shown that 7 to 11 year-olds whose early mathematical representations show pattern and structure have a better chance of achieving well in mathematics later on.

- 3.1.6 Yackel (2001, p.28) writes of visual imagery to encourage fluency with number. In her work with primary children, she flashes images of dots that represent quantities two or three times 'for a few seconds' each. She writes that 'the purpose of the flashing approach is to encourage students to develop mental images of the quantities shown'.

- 3.1.7 One series of tasks devised by Yackel (2001) make use of a double ten frame:



Yackel (2001) maintains that the double ten frame not only helps children to develop mental images, but also to 'organize their thinking and reasoning'. She gives this example of a series of tasks:



'A child might notice that in Task 2 there is one more dot in the second frame than in the Task 1.

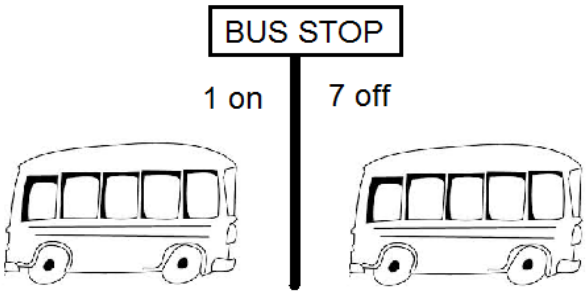
So the answer has to be one more than it was in Task 1. Similarly, some child might relate Task 3 to

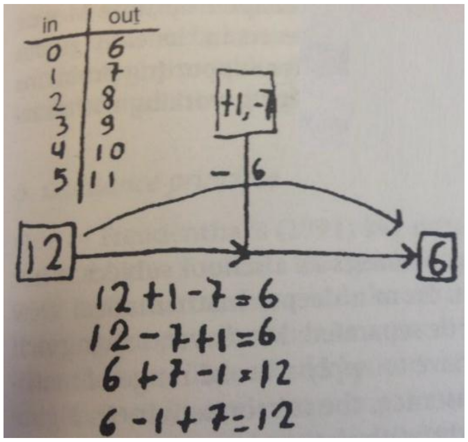
Task 2 by noticing that if one dot from the first frame were moved to the second frame, the task would be the same as in Task 2. So the result is the same. Another child might relate Task 3 to Task 1 by noticing that there is one more dot in the first frame than in the first frame of Task 1, so the answer has to be one more. Our experience is that the visual imagery of the dots on the ten frame provides support for students' use of thinking strategies in a way that posing the same tasks in purely symbolic notation, that is, as 8 + 4, 8 + 5 and 9 + 4 would not' (Yackel, 2001,p.28).

- 3.1.8 According to Anghileri (2001) the empty number line is a useful model for visual imagery. She writes that 'removing all calibration from the number line has enabled children to use it flexibly for 'jumps' of any size, in either direction, providing imagery to encourage and support mental strategies' (2001, p.10). These mental strategies may involve counting forwards or backwards (in ones, twos, multiples of 5s, 10s, 100s...) or in adding or subtracting.

- 3.1.9 van den Heuvel-Panhuizen (2001, p.52) states that 'Models serve as an important device for bridging the gap between informal, context-related mathematics and more formal mathematics'. She writes that, 'First, the students develop strategies closely connected to the context. Later on, certain aspects of the context situation can become more general which means that the context can assume, more or less, the character of a model, and as such can give support for solving other, but related problems.'

Van den Heuvel-Panhuizen (2001, p.52-53) gives an example of changes in numbers of passengers at a bus stop. She demonstrates that ‘Later on the bus context has become a ‘model for’ understanding all kinds of number sentences. Then the students can go far beyond the real bus context. They can even use the model for backwards reasoning’.





- 3.1.10 van den Heuvel-Panhuizen (2001) specifically advocates ‘longitudinal models’, where ‘there is a strong focus on the connection between what is learned earlier and what will be learned later’ (p.53). She uses the number line as a ‘powerful example’, describing how it is introduced as ‘a beaded necklace... for all kinds of counting activities. [Then in later years] ‘this chain of beads subsequently becomes... an empty number line for supporting addition and subtractions..., [then] a double number line for supporting problems on ratio..., [then] a fraction/percentage bar for supporting working with fractions and percentages’ (p.53 – 54).

- 3.1.11 Presmeg (1986) identified five different kinds of imagery important to mathematics learning:

    - Concrete pictorial imagery – This involves pictures in the mind that closely relate to the visual appearance of objects. For example, visualising the problem of four apples in a fruit bowl; another five apples are added. How many apples are there altogether?

    - Pattern imagery – This involves visual images that encompass relationships. For example, the model of a number line, where each number is related to its spatial location, so 9 is close to 10, but there is an ‘appropriate’ space between 10 and 100. Alternatively, the model of ten frames which demonstrate how 10 + 3 relates to 9 + 4.

    - Kinesthetic imagery – This involves visual images that also entail muscular activity. For example, using fingers to trace an imagined shape, or to count imaginary steps in a sequence.

    - Dynamic imagery – This involves visualising movement. For example, the reflection, rotation, translation or enlargement of a shape.

- Memory imagery – This involves visualising what was previously seen. For example, what was written on a whiteboard on in an exercise book.

- 3.1.12 Hegarty and Kozhevnikov (1999) and van Garderen (2006) classified the visual images generated by pupils when tackling mathematical word problems. Both researchers found that pattern imagery was positively related to success in mathematical problem solving, while pictorial imagery made little difference to pupils' ability.

- 3.1.13 In work with pupils of varying abilities, van Garderen (2002) found that gifted pupils use visual images significantly more often than pupils with learning difficulties. Furthermore, the imagery used by gifted pupils tended to be pattern imagery. However, when pupils with learning difficulties used visual imagery, it tended to be pictorial imagery.

- 3.1.14 Many researchers relate visual imagery to pupil's spatial ability. For example, Krutetski (1976, p.315) writes that 'the ability to visualise abstract mathematical relationships and the ability for special geometric concepts showed a very high intercorrelation in our experiments'.

- 3.1.15 The spatial visualization ability of seven mathematically gifted pupils (six in grade six and one in grade seven) was studied by Ryu, Chong, and Song (2007). They found that these pupils shared the ability to visualise the rotation of an object, to transform it into a different form, and to manipulate it in their imagination.

3.2 References:

Anghileri, J. (2001). Contrasting approaches that challenge tradition. In J. Anghileri (Ed.), *Principles and Practices in Arithmetic Teaching*. Buckingham: Open University Press.

Hegarty, M., & Kozhevnikov, M. (1999). Types of visual–spatial representations and mathematical problem solving. *Journal of Educational Psychology*, 91, pp. 684–689.

Hershkowitz, R., Ben-Chaim, D., Hoyles, C., Lappan, G., Mitchelmore, M. and Vinner, S. (1989). Psychological aspects of learning geometry. In P. Nesher and J. Kilpatrick (Eds.), *Mathematics and Cognition*. (ICMI Study Series), Cambridge: Cambridge University Press.

Krutetskii, V.A. (1976). *The Psychology of Mathematical Abilities in Schoolchildren*. Chicago: University of Chicago Press.

Mulligan, J. T., Prescott, A., & Mitchelmore, M. C. (2003). Taking a closer look at visual imagery. *Australian Primary Mathematics Classroom*, 8 (4), pp.23–27.

Owens, K. (2003). Investigating and visualising in Space and Geometry. *Australian Primary Mathematics Classroom*, 8 (2), pp.14–19.

Presmeg, N. C. (1986). Visualisation in high school mathematics. *For the Learning of Mathematics*, 6 (3), pp. 42-46.

Ryu, H., Chong, Y., & Song, S. (2007). Mathematically gifted students' spatial visualization ability of solid figures. In Woo, J. H., Lew, H. C., Park, K. S. & Seo, D. Y. (Eds.). *Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4, pp. 137-144. Seoul: PME.

Sharma, M. C. (2013). Keynote speech: *Number Concept, Numbersense, and Numeracy in Common Core State Standards*. NHSAA 2013 Best Practices Conference on Curriculum, Instruction, and Assessment in New Hampshire, pp.1 – 33.

Thomas, J. N. and Harkness, S. S. (2013) Implications for intervention: categorising the quantitative mental imagery of children. *Mathematics Education Research Journal*, 25 (2), pp 231-256.

van den Heuvel-Panhuizen, M. (2001). Realistic mathematics education in the Netherlands. In J. Anghileri (Ed.), *Principles and Practices in Arithmetic Teaching*. Buckingham: Open University Press.

van Garderen, D. (2002). *Visual imaging and mathematical word problem solving by students of varying abilities*. Unpublished doctoral dissertation, University of Miami, Florida.

Yackel, E. (2001). Perspectives on arithmetic from classroom-based research in the United States of America. In J. Anghileri (Ed.), *Principles and Practices in Arithmetic Teaching*. Buckingham: Open University Press.