

Abacus Efficacy Research

Pillar 4: Doubling and Halving

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Prepared by: Dr Naomi Norman, Director, Techademic Ltd

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Summary

This is a brief examination of some of the most cited education research literature on 'Doubling and Halving'. It suggests that learning frequently occurring doubles and halves by heart, along with being taught the strategy of doubling and halving, assists pupils in their times tables fluency. It further suggests that extending the strategy so pupils can recognise connections (for example between doubles and near doubles, or between a number divided by 8 and the same number divided by 16) encourages flexibility at mental calculation, as well as the development of the necessary reasoning skills to experience success in harder mathematics later on.

4.1 The research:

- 4.1.1 According to Jansen and Pollmann (2001, p.200) western societies have an innate 'favouritism for' doubling and halving. They give examples of the 'denominations of coins and notes... (1 euro, 2 euros, 5 euro, 10 euro, 20 euro etc)... the years when we celebrate jubilees (5, 10, 25, 50, etc)... [and the] partition [of] our time periods into two halves and four quarters...' (p.200–201). They state that, 'There is a 'natural' propensity for halving and doubling quantities. This propensity – be it innate or 'just easily learned' – seems to be independent from the counting system that is being used' (p.201).
- 4.1.2 In their examination of good practice in primary mathematics, Ofsted (2011) describe Year 2 children's use of doubles in the context of money: 'Year 2 pupils in one school worked in pairs to calculate the cost of buying six lemons at 5p each. They then went on to finding the cost of 12 and then 24 lemons...Most of the pupils (eventually in some cases) realised that doubling each answer gave the next answer' (p.22). They also describe Year 6 pupils who, having worked out $432 \div 8 = 54$, could 'spot the connection' for working out $432 \div 16$, 'with only initial hesitation over whether the answer would be doubled or halved' (p.25). Ofsted (2011) state, 'A key message of this report is that pupils whose understanding of number, its structures and relationships, is developed alongside their proficiency with arithmetic have the grounding so necessary for future learning, particularly of algebra' (p.26).
- 4.1.3 Wigley (1996, p.1) writes of the use of doubling as a means for helping students with 'facts not yet memorised [but that] can be generated from others'. For example, 'tables x2, x4 and x8 can be generated by repeated doubling. Since x10 is easy, x5 can be generated by x10 and then halving... x6 (double x3)'. He states that, 'Anyone who can use these methods understands the number operations and is more truly fluent than someone who can only chant tables. Appropriately structured, such an approach builds knowledge and confidence' (p.1).
- 4.1.4 According to Anghileri (2006, p.49), 'teachers play a vital role... through signals they send about the knowledge and ways of thinking and knowing that are valued'. She gives an example of three students' solutions for solving $6 + 7$: one by counting ("6, 7, 8, 9, 10, 11, 12, 13"); another by using number bonds to 10 ("7 and 3 makes 10 and 3 more makes 13"); and a final solution by using near doubles ("6 add 6 and one more is 13"). Anghileri suggests that teachers should emphasize the strategy of doubling.

- 4.1.5 Threlfall (2002, p.37) expresses concern about teaching a range of mental strategies and then expecting pupils to choose between them. He states, '54 – 28 is done by children in a number of ways. Sometimes it involves near doubles – "Half 54 is 27...", sometimes breaking the second number into parts "I took off 20 then..." and sometimes by rounding "First I took off 30..." Which of these is the most appropriate?' Threlfall (2002, p.44) proposes that 'Teaching towards flexible mental calculation must include extensive development of factual knowledge about numbers', as looking at the numbers will help pupils choose the best strategy. For example, as well as the halving and doubling strategy, pupils must to be taught common and useful double and half number facts (such as doubles up to 20, half of 100 is 50; half of 50 is 25 and so on.)
- 4.1.6 Davis (2009, p.27) reports the 'importance of learning doubles by heart, in order that they can be used within a number of strategies'. She gives examples of teaching doubles up to double 5 in Foundation and up to double 20 in Year 1.
- 4.1.7 Hartnett (2007, p.349) lists a range of doubling and halving strategies:
 - Use a double or near double to add or subtract
 - Double to multiply by 2
 - Double, double to multiply by 4
 - Double, double, double to multiply by 8
 - Half to divide by 2
 - Half, half to divide by 4
 - Half, half, half to divide by 8
 - Double and halve
- 4.1.8 Fielker (1986, p.34) describes asking a class of 9-year olds to use their calculators to double 7, stating that 'They arrived at it [14] by adding another 7'. Fielker (1986, p.34) comments, 'We can see that children of 9 or 10 have a decided feeling that doubling is more to do with addition than with multiplication. Even if we are not sure about this when children are working in their heads, then doing it on the calculator forces them into making an explicit choice about what key to press.'
- 4.1.9 Steinberg (1985) taught pupils to use doubles (e.g. 4 + 4, 5 + 5), doubles ± 1 (e.g. 6 + 7, 7 + 8), doubles ± 2 (e.g. 6 + 8, 6 + 4), reverse doubles (e.g. 12 – 6), reverse doubles ± 1 (e.g. 15 – 7, 13 – 7) and reverse doubles ± 2 (e.g. 16 – 7, 12 – 7). However, she specifically states that, 'to use the doubles + 1 [or + any number] strategy, the associative principle is needed (e.g., $6 + 7 = 6 + (6 + 1) = (6 + 6) + 1$) [and pupils must also] understand that when an addend is increased, the sum is increased by the same amount (p.351–352).
- 4.1.10 Steinberg (1985) advised giving pupils sufficient practice to develop understanding and move beyond 'rote procedural learning' where they remember rules like "double the first addend and add 1". (p.349). She described the usefulness of manipulatives, such as cubes: 'the children modeled the problems with Unifix cubes of two different colors. To solve $6 + 7 = ?$, for example, 6 cubes of one color were placed next to 7 cubes of another color. Two ways of relating the sum of $6 + 7$ to the nearest doubles or "equal" rows of cubes were discussed and shown (1 more than $6 + 6$ or 1 less than $7 + 7$). (p.340).

- 4.1.11 Seven of the thirteen children involved in Steinberg's research "transferred their knowledge of the doubles + 1 strategy and used the doubles - 1 strategy before it was taught in class. Similarly, after learning the doubles + 1 strategy (e.g., $6 + 7 = (6 + 6) + 1$), seven children applied the idea of doubles in other problems and used doubles + 3 or doubles + 4 strategies. For example, four children solved the problem $5 + 9 = ?$ by breaking it up as $(5 + 5) + 4$." (Steinberg, 1985, p.349)
- 4.1.12 According to Steinberg (1985, p.351), children had more difficulty with the use of doubles to solve subtraction than addition problems.
- 4.1.13 According to Fielker's (1986) research, pupils have more difficulty with halving than with doubling. He suggests that this stems 'from an inability to transform multiplication into division' (p.35).
- 4.1.14 Anghileri (2001, p.96) wrote of research where students chose to use doubling and halving.
- However, she noted that 'In some cases, pupils were successful at reaching the correct total but had difficulty extracting the correct answer from the written record they had made... [because of] poorly structured recording that involved partial sums written erratically across the work space.'

4.2 References:

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