

## 29 GEOMETRY AND MEASURES: TRIGONOMETRIC VALUES

### LEARNING OBJECTIVES

- Know the exact values of  $\sin \theta$  and  $\cos \theta$  for  $\theta = 0^\circ, 30^\circ, 45^\circ, 60^\circ$  and  $90^\circ$
- Know the exact value of  $\tan \theta$  for  $\theta = 0^\circ, 30^\circ, 45^\circ$  and  $60^\circ$
- Recognise, sketch and interpret graphs of the trigonometric functions

### SPECIFICATION LINKS

- G21, G22, G23, A12

### STARTER ACTIVITY

- **Ordering numbers; 5 minutes; page 186**

Remind the student that an exact value can be given by leaving the number in surd form. Encourage the student to estimate the value of each surd (knowing that  $\sqrt{1}=1$  and  $\sqrt{4}=2$  tells you that  $\sqrt{2}$  and  $\sqrt{3}$  must be between 1 and 2).

### MAIN ACTIVITIES

- **Special triangles and trigonometric graphs; 20 minutes; page 187**

Look at the special triangles and work through how to work out the values of  $\sin$ ,  $\cos$  and  $\tan$  of  $60^\circ$ ,  $30^\circ$  and  $45^\circ$  using these triangles. Explain to the student that they must learn these values. Look at the graphs of  $\sin x$ ,  $\cos x$  and  $\tan x$  and discuss their properties. Read the values of  $\sin$  and  $\cos$  for  $0^\circ$  and  $90^\circ$  from the graphs and discuss why  $\tan 90^\circ$  cannot be found. Use the graphs to give approximate values of the  $\sin/\cos/\tan$  of various angles. Discuss the symmetry of the graphs and how this can be used to work out other trigonometric values; for example,  $\sin(-45^\circ)$ .

- **Working with non-right-angled triangles; 20 minutes; page 188**

The student will need to learn the three formulae on this activity sheet. Encourage them to write the formulae on revision cards or photograph them using their phones so they can refer back to them while revising.

Work through the activity, discussing which formula to use in each circumstance. Ensure that the student always labels the sides and angles ( $a$ ,  $b$ ,  $c$  and  $A$ ,  $B$ ,  $C$  respectively) before attempting to use the formulae.

### PLENARY ACTIVITY

- **What's my angle?; 5 minutes**

Sketch two right-angled triangles, one with sides of length 1 cm, 1 cm and  $\sqrt{2}$  cm, and one with sides of length 2 cm, 1 cm and  $\sqrt{3}$  cm. Invite the student to write the angles of the triangles in the correct places.

### HOMEWORK ACTIVITY

- **Trigonometry revision; 60 minutes; page 189**

Full instructions are given on the activity sheet.

### SUPPORT IDEA

- **Working with non-right-angled triangles** Model how to use the sine and cosine rules. Emphasise the importance of correctly labelling the sides. You may also wish to give the student the rearranged cosine rule formula:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

### EXTENSION IDEA

- **Working with non-right-angled triangles** Extend to 3-D shapes, asking the student to use the sine and/or cosine rule to calculate the missing lengths/angles within 3-D shapes.

### PROGRESS AND OBSERVATIONS

## STARTER ACTIVITY: ORDERING NUMBERS

**TIMING: 5 MINS**

### LEARNING OBJECTIVES

- Order numbers including fractions, decimals and surds

### EQUIPMENT

none

**1. Write these numbers in ascending order.**

$$\frac{1}{2}$$

$$\sqrt{2}$$

$$1$$

$$5^2$$

.....

.....

.....

.....

**2. Write these numbers in descending order.**

$$\frac{3}{4}$$

$$\sqrt{3}$$

$$-1.7$$

$$4.9$$

.....

.....

.....

.....



## MAIN ACTIVITY: SPECIAL TRIANGLES AND TRIGONOMETRIC GRAPHS

**TIMING: 20 MINS**

### LEARNING OBJECTIVES

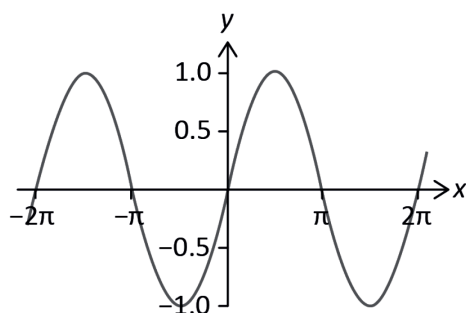
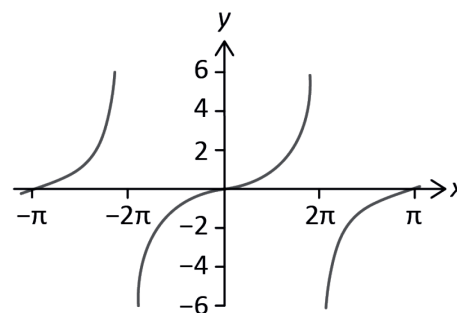
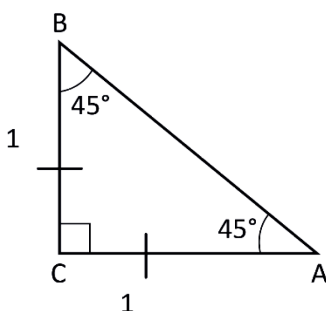
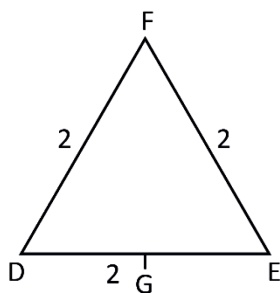
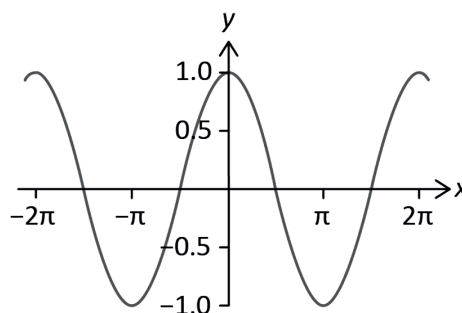
- Know the exact values of  $\sin \theta$  and  $\cos \theta$  for  $\theta = 0^\circ, 30^\circ, 45^\circ, 60^\circ$  and  $90^\circ$
- Know the exact value of  $\tan \theta$  for  $\theta = 0^\circ, 30^\circ, 45^\circ$  and  $60^\circ$
- Recognise, sketch and interpret graphs of the trigonometric functions

### EQUIPMENT

none



1. Look at these graphs and triangles and use the given information to complete the table of trigonometric values.


 $y = \sin x$ 

 $y = \cos x$ 
 $y = \tan x$ 

$\theta$	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$
$\sin \theta$					
$\cos \theta$					
$\tan \theta$					

## MAIN ACTIVITY: WORKING WITH NON-RIGHT-ANGLED TRIANGLES TIMING: 20 MINS

### LEARNING OBJECTIVES

- Know and apply the sine rule and cosine rule
- Know and apply the formula  $\text{area} = \frac{1}{2}ab\sin C$  to calculate the area or sides of any triangle

### EQUIPMENT

- ruler
- protractor
- calculator

### 1. Complete each of these formulae.

a) Sine rule:  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

b) Cosine rule:  $a^2 = b^2 + c^2 - 2bc\cos A$

c) Area of a triangle:  $\text{area} = \frac{1}{2}ab\sin C$

### 2. Draw a triangle in the space below. Measure and label two of the sides and an included angle.

- a) Work out the size of all the other sides and angles. Check your answers by measuring them.

- b) Work out the area of the triangle.

### 3. Explain when you would use:

- a) the sine rule

- b) the cosine rule.

### 4. If you knew the area of a triangle and two of the side lengths, how could you work out the angles?

## HOMEWORK ACTIVITY: TRIGONOMETRY REVISION

**TIMING: 60 MINS**

### LEARNING OBJECTIVES

- Know the exact values of  $\sin \theta$  and  $\cos \theta$  for  $\theta = 0^\circ, 30^\circ, 45^\circ, 60^\circ$  and  $90^\circ$
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- Recognise, sketch and interpret graphs of the trigonometric functions

### EQUIPMENT

- card
- large sheet of paper
- scissors



1. Draw a poster or flow chart showing how to use SOHCAHTOA to work out unknown angles and unknown lengths.



2. Cut out these cards and use them to learn the trigonometric values. You could play snap or pairs, display them on the wall, or ask someone to test you.

$\sin 45^\circ$	$\frac{1}{\sqrt{2}}$	$\cos 60^\circ$	$\frac{1}{2}$
$\cos 45^\circ$	$\frac{1}{\sqrt{2}}$	$\tan 60^\circ$	$\sqrt{3}$
$\tan 45^\circ$	1	$\sin 0^\circ$	0
$\sin 30^\circ$	$\frac{1}{2}$	$\cos 0^\circ$	1
$\cos 30^\circ$	$\frac{\sqrt{3}}{2}$	$\tan 0^\circ$	0
$\tan 30^\circ$	$\frac{1}{\sqrt{3}}$	$\sin 90^\circ$	1
$\sin 60^\circ$	$\frac{\sqrt{3}}{2}$	$\cos 90^\circ$	0

## 29 ANSWERS

### STARTER ACTIVITY: ORDERING NUMBERS

1.  $\frac{1}{2}$     1     $\sqrt{2}$      $5^2$
2. 4.9     $\sqrt{3}$      $\frac{3}{4}$     -1.7

### MAIN ACTIVITY: SPECIAL TRIANGLES AND TRIGONOMETRIC GRAPHS

1.

$\theta$	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	

### MAIN ACTIVITY: WORKING WITH NON-RIGHT-ANGLED TRIANGLES

1. a) Sine rule:  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
- b) Cosine rule:  $a^2 = b^2 + c^2 - 2ac$
- c) Area of a triangle:  $\text{area} = \frac{1}{2} ab \sin C$
2. a)–b) Check the student's drawing and calculations.
3. a) When given two sides and an angle opposite one of the sides, or two angles and one side.  
b) When given three sides, or two sides and an included angle.
4. Rearrange the area equation to give  $\sin C = \frac{2 \times \text{area}}{ab}$ , then work out angle C.
- Use the sine or cosine rule to work out the second angle.  
Work out the third angle by subtracting the sum of the first two from  $180^\circ$ .

### HOMEWORK ACTIVITY: TRIGONOMETRY REVISION

1. Check the student's work.