31 Geometry and measures: Vectors

Learning objectives

- Apply addition, subtraction and multiplication by a scalar of vectors
- Diagrammatic and column representation of vectors
- Use vectors to construct geometrical arguments and proof
- Transform shapes using rotation, reflection, enlargement and translation

Specification links

- G1, G6, G7, G8, G24, G25,

Starter activity

- **Length of line segments; 5 minutes; page 198**
  Full instructions are given on the activity sheet.

Main activities

- **Introducing vectors; 25 minutes; page 199**
  Show the student standard vector notation ($\overrightarrow{AB}$ or **a**) and challenge them to write the column vector that describes the movement between different pairs of points. Establish which vectors are equivalent ($\overrightarrow{BA}$ and $\overrightarrow{DC}$, $\overrightarrow{CE}$ and $\overrightarrow{BF}$, $\overrightarrow{HG}$ and $\overrightarrow{IC}$), and extend this to vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$, $\overrightarrow{EC}$ and $\overrightarrow{FB}$ and $\overrightarrow{GH}$ and $\overrightarrow{CI}$ therefore being equivalent. Explain that parallel vectors are multiples of one another, and that you can show they are parallel by joining the points with straight lines.
- **Vectors and proof; 15 minutes; page 200**
  Spend some time discussing what it means for vectors to be parallel and co-linear. Encourage the student to sketch diagrams to support their understanding of these problems.

Plenary activity

- **Vector facts; 5 minutes**
  Give the student five minutes to write as many facts as they can about vectors and how to solve vector problems.

Homework activity

- **Transformations; 20 minutes; page 201**
  Full instructions are given on the activity sheet.

Support idea

- **Introducing vectors** Support the student by drawing the horizontal and then vertical ‘journey’ between the points.

Extension idea

- **Vectors and proof** Ask the student to give a vector parallel to $\begin{pmatrix} a \\ b \end{pmatrix}$ and to define what it means for three points to be co-linear.

Progress and observations

STARTER ACTIVITY: LENGTH OF LINE SEGMENTS — TIMING: 5 MINS

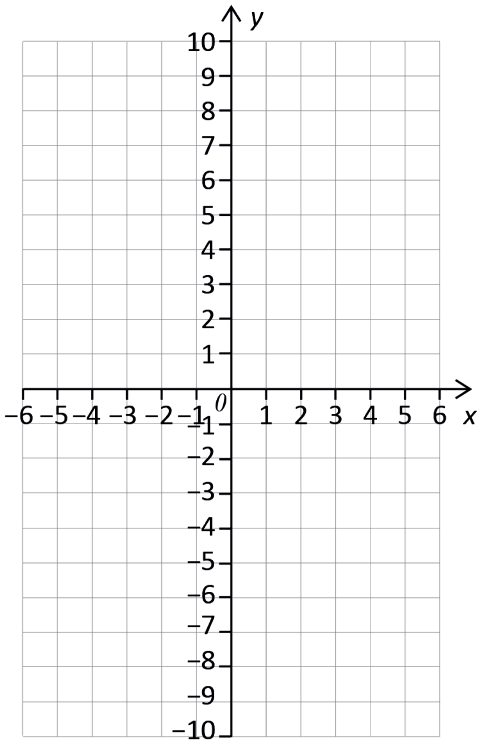
LEARNING OBJECTIVES

- Calculate the length of a line segment

EQUIPMENT

- ruler

1. On the coordinate axis, mark the points A (–2, 5), B (3, –4) and C (–3, –7).



2. Work out the exact length of each of these line segments.

a) AB ....

b) BC ....

c) AC ....

MAIN ACTIVITY: INTRODUCING VECTORS — TIMING: 25 MINS

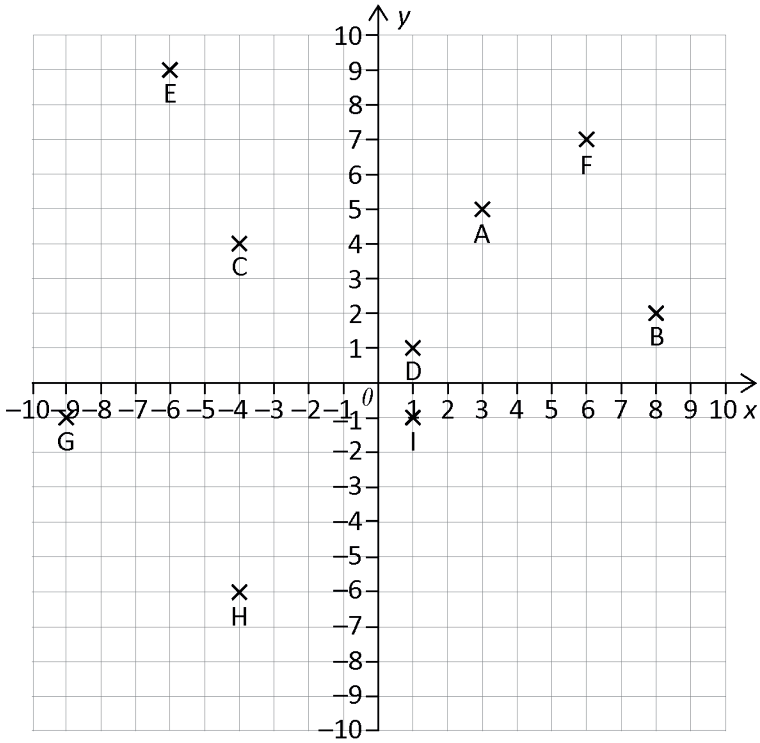
LEARNING OBJECTIVES

- Apply addition, subtraction and multiplication by a scalar of vectors
- Diagrammatic and column representation of vectors

EQUIPMENT

- ruler

1. Points A to I have been plotted on the coordinate grid opposite.



a) $\overrightarrow{BJ} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$. Mark point J on the diagram.

b) Write $\overrightarrow{JB}$ as a column vector.

....

c) Which vector on the diagram is parallel to vector $\overrightarrow{BJ}$? Explain how you could work this out given just the column vector for each possible pair of points.

....

....

....

d) Write three vectors in column vector form that would be parallel to $\overrightarrow{AF}$.

....

e) If you knew vector $\overrightarrow{XY}$, explain how you could easily write down vector $\overrightarrow{YX}$.

....

2. Calculate the magnitude of vector $\overrightarrow{JB}$.

....

3. Work out the resultant of $\overrightarrow{AB} + \overrightarrow{BC}$. Explain why this is equal to $\overrightarrow{AC}$.

....

MATHS — HIGHER

AQA

MAIN ACTIVITY: VECTORS AND PROOF — TIMING: 15 MINS

LEARNING OBJECTIVES

- Use vectors to construct geometrical arguments and proof

EQUIPMENT

none

1. Using the vectors below, write each of the following vector calculations as a single column vector.

$$\mathbf{a} = \begin{pmatrix} -2 \\ 5 \end{pmatrix} \qquad \mathbf{b} = \begin{pmatrix} 3 \\ 8 \end{pmatrix} \qquad \mathbf{c} = \begin{pmatrix} 4 \\ -10 \end{pmatrix} \qquad \mathbf{d} = \begin{pmatrix} 0 \\ -7 \end{pmatrix}$$

a) $2\mathbf{b}$ ....

b) $\mathbf{b} + \mathbf{c}$ ....

c) $5\mathbf{a} - \mathbf{d}$ ....

d) $-\mathbf{a}$ ....

e) $-3\mathbf{a} + 2\mathbf{d}$ ....

2. Explain how you know that d is parallel to the *y*-axis.

....

3. Prove that c is parallel to a.

....

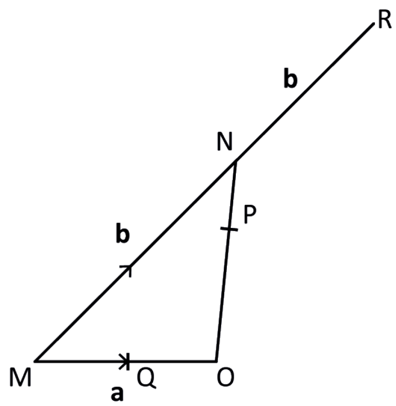
4. Look at this vector diagram.

$\overrightarrow{MO} = \mathbf{a}$ $\qquad \overrightarrow{MN} = \mathbf{b}$

Q is the mid-point of line MO. OP is two thirds of the length of ON.

Prove that QPR is a straight line.

Hint: First work out $\overrightarrow{QO}$ and $\overrightarrow{ON}$, then work out $\overrightarrow{OP}$ and $\overrightarrow{PN}$. Work out and simplify $\overrightarrow{QP}$ and $\overrightarrow{PR}$, then show that $\overrightarrow{QP}$ is a multiple of $\overrightarrow{PR}$.



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HOMEWORK ACTIVITY: TRANSFORMATIONS   TIMING: 20 MINS

LEARNING OBJECTIVES

- Describe translations as 2-D vectors
- Describe the changes and invariance achieved by combinations of rotations, reflections and enlargements

EQUIPMENT

- ruler

1. Shape A has been drawn on the coordinate grid below.

a) Translate shape A through $\begin{pmatrix} -5 \\ 2 \end{pmatrix}$.

Label the shape B.

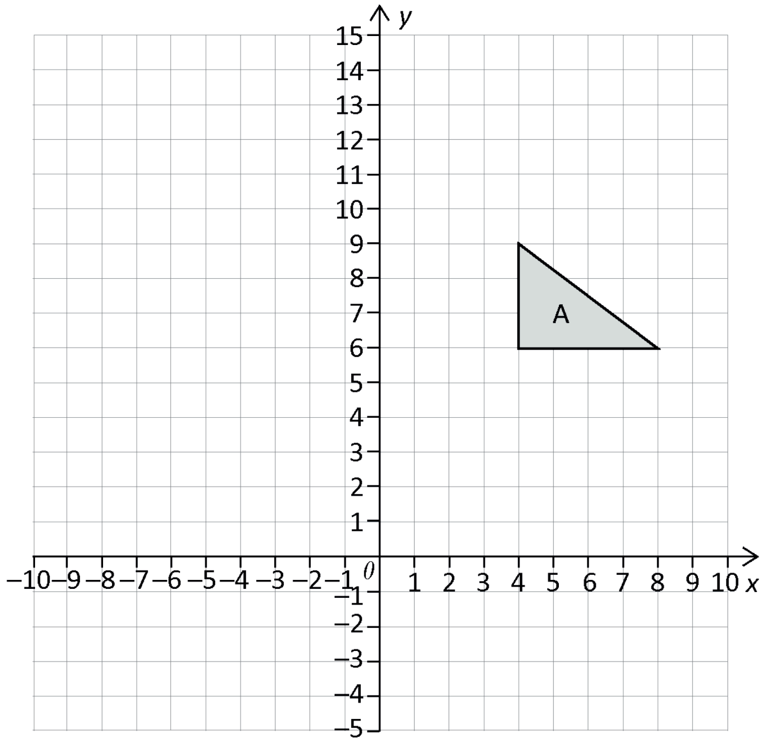
b) Reflect B in the line $x = -2$. Label the shape C.

c) Rotate C through 90° about the point (0, 8). Label the shape D.

d) Reflect D in the line $y = 8$. Label the shape E.

e) Rotate E 270° about the point (–2, 0). Label the shape F.

f) Describe the single transformation that will move shape F to shape A.



....

g) What can you say about all the triangles on the diagram?

....

h) Write down three vectors parallel to the vector that moved A to B. ....

2. Enlarge shape A:

a) about (10, 10) with scale factor 2. Label the shape G.

b) about the point (2, 3) with scale factor $-\frac{1}{2}$. Label the shape H.

31 ANSWERS

STARTER ACTIVITY: LENGTH OF LINE SEGMENTS

1. Check plotted points.
2. a) $\sqrt{97}$ b) $3\sqrt{5}$ c) $\sqrt{145}$

MAIN ACTIVITY: INTRODUCING VECTORS

1. a) Point J should be at (0, 7). b) $\begin{pmatrix}1\\2\end{pmatrix}$ c) $\overrightarrow{AD}$ since it is a multiple of $\begin{pmatrix}1\\2\end{pmatrix}$.

d) Any vectors of the form $\begin{pmatrix}3\mathbf{a}\\2\mathbf{a}\end{pmatrix}$. e) Change the signs of both parts of the vector.

2. $\sqrt{5}$
3. $\begin{pmatrix}-7\\-1\end{pmatrix}$ It is equivalent as it describes the same journey.

MAIN ACTIVITY: VECTORS AND PROOF

1. a) $\begin{pmatrix}6\\16\end{pmatrix}$ b) $\begin{pmatrix}7\\-2\end{pmatrix}$ c) $\begin{pmatrix}-10\\32\end{pmatrix}$ d) $\begin{pmatrix}2\\-5\end{pmatrix}$ e) $\begin{pmatrix}6\\-29\end{pmatrix}$
2. The movement parallel to the x-axis is zero.
3. $\mathbf{c} = -2\mathbf{a}$
4. $\overrightarrow{QO} = \frac{1}{2}\mathbf{a}$ and $\overrightarrow{ON} = -\mathbf{a}+\mathbf{b}$ $\overrightarrow{OP} = -\frac{2}{3}\mathbf{a}+\frac{2}{3}\mathbf{b}$ and $\overrightarrow{PN} = -\frac{1}{3}\mathbf{a}+\frac{1}{3}\mathbf{b}$; $\overrightarrow{QP} = -\frac{1}{6}\mathbf{a}+\frac{2}{3}\mathbf{b}$ and $\overrightarrow{PR} = -\frac{1}{3}\mathbf{a}+\frac{4}{3}\mathbf{b}$

$\overrightarrow{QP} = \frac{1}{6}(-\mathbf{a}+4\mathbf{b})$ and $\overrightarrow{PR} = \frac{1}{3}(-\mathbf{a}+4\mathbf{b})$, therefore $\overrightarrow{QP}$ is a multiple of $\overrightarrow{PR}$.

HOMEWORK ACTIVITY: TRANSFORMATIONS

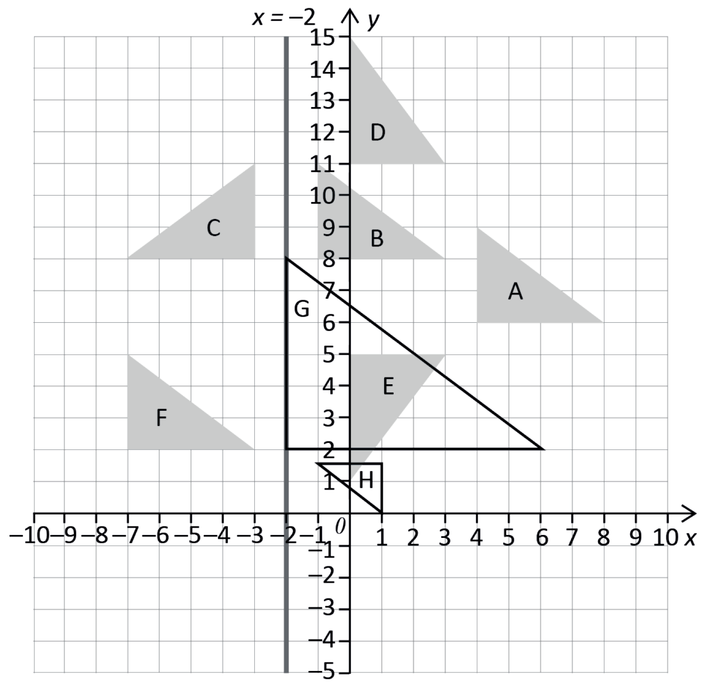
1. a)–e) See diagram.

f) Translate F through vector $\begin{pmatrix}11\\4\end{pmatrix}$

g) They are all congruent.

h) Any vectors of the form $\begin{pmatrix}-5a\\2a\end{pmatrix}$

2. a)–b) See diagram.



GLOSSARY

Vector
The displacement from one position to another; can be pictured geometrically as a line segment; has magnitude and direction

Co-linear
Points lying on the same straight line