

18 ALGEBRA: TRIGONOMETRIC GRAPHS AND TRANSFORMATIONS

LEARNING OBJECTIVES

- Recognise, sketch and interpret graphs of trigonometric functions
- Sketch translations and reflections of a given function

SPECIFICATION LINKS

- A12, A13

STARTER ACTIVITY

- **Shapes of graphs; 5 minutes; page 120**
Full instructions are given on the activity sheet.

MAIN ACTIVITIES

- **Trigonometric functions; 15 mins; page 121**
Explain to the student that the worksheet shows all three trigonometric functions. Students must identify each graph. Draw the student's attention to the values of $\tan x$ which cannot be calculated (where they tend to infinity). Ask the student to use the graphs to give values of $\sin x$, $\cos x$ and $\tan x$. Similarly, ask the student to give approximate values of x given values of $\sin x$, $\cos x$ and $\tan x$. Draw the student's attention to the fact that $-1 \leq \sin x \leq 1$ and $-1 \leq \cos x \leq 1$.
- **Transformations of functions; 25 minutes; page 122**
To support question 1, ask the student to sketch the graphs of $y = x^2 + 2$. Model each transformation by choosing values for 'a' and sketching the transformation with the student. Discuss how each transformation changes the graph. Move on to question 2, discussing the effects that each transformation will have on particular points.

PLENARY ACTIVITY

- **How has it been transformed?; 5 minutes**
Sketch the graph of $y = x^2$, then sketch a transformation of $y = x^2$. Challenge the student to write down the equation of the transformed graph. Repeat for different transformations. Draw attention to the fact that $y = x^2$ and $y = (-x)^2$ are equivalent since the graph of $y = x^2$ is symmetrical about the y-axis.

HOMEWORK ACTIVITY

- **Vlogging; 60 minutes; page 123**
Full instructions are given on the activity sheet.

SUPPORT IDEA

- **Transformations of functions** Provide support for question 1 by considering the effect numerically, e.g. draw up a table of values for $y = x^2 + 2$, then compare this to the values for $y = 2(x^2 + 2)$. Which values are altered? How are they altered?

EXTENSION IDEA

- **Transformations of functions** Ask the student to sketch transformations of the trigonometric functions, e.g. $y = \sin x + 1$ or $y = -\cos x$, marking on points of intersection with the axes and maximum and minimum points.

PROGRESS AND OBSERVATIONS

STARTER ACTIVITY: SHAPES OF GRAPHS

TIMING: 5 MINS

LEARNING OBJECTIVES

- Recognise and interpret linear, quadratic, cubic and reciprocal graphs

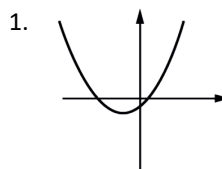
EQUIPMENT

none

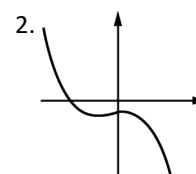
- Draw lines to match the equation of each graph to its diagram. Explain to your tutor how you know which goes with which.



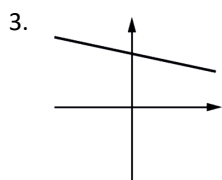
A: $x = 5$



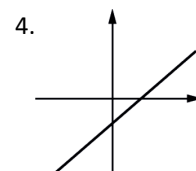
B: $y = 2x - 3$



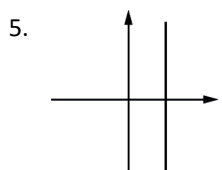
C: $y = -\frac{1}{2}x + 5$



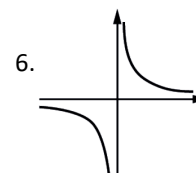
D: $y = x^2 + 2x - 1$



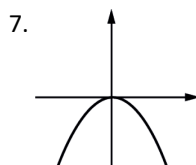
E: $y = -x^2$



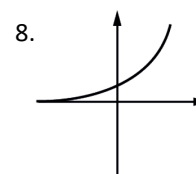
F: $y = -2x^3 - 3x^2 + 2x - 12$



G: $y = 3^x$



H: $y = \frac{1}{x}$



MAIN ACTIVITY: TRIGONOMETRIC FUNCTIONS

TIMING: 15 MINS

LEARNING OBJECTIVES

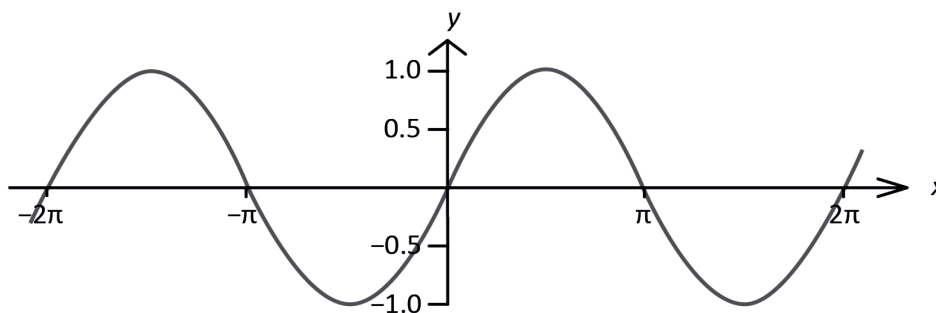
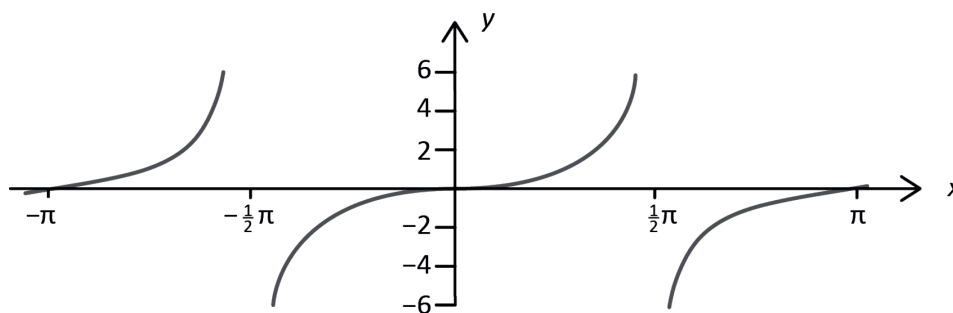
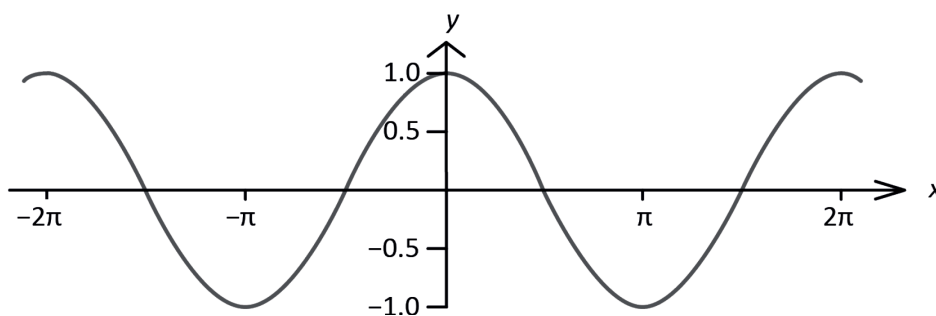
- Recognise, sketch and interpret graphs of the trigonometric functions $y = \sin x$, $y = \cos x$ and $y = \tan x$

EQUIPMENT

none



- These graphs show the three different trigonometric functions. Identify each graph and give the values your tutor asks for.



MAIN ACTIVITY: TRANSFORMATIONS OF FUNCTIONS

TIMING: 25 MINS

LEARNING OBJECTIVES

- Recognise, sketch and interpret translations and reflections of a given function

EQUIPMENT

none

1. Explain how each of these transformations changes the graph of $y = f(x)$.

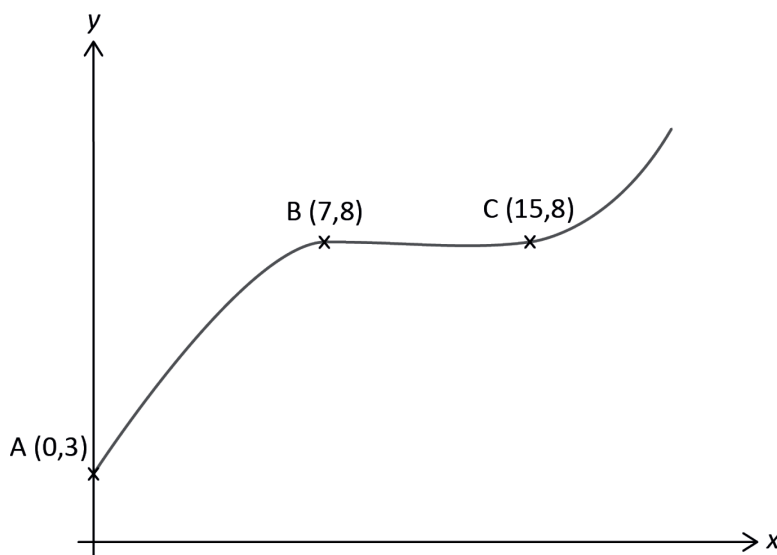
- $y = f(x) + a$
- $y = f(x + a)$
- $y = f(x) - a$
- $y = f(x - a)$
- $y = f(-x)$
- $y = -f(x)$
- $y = af(x)$
- $y = f(ax)$

2. This graph shows $y = f(x)$.

- Sketch these graphs on the axes.

- $y = f(x + 5)$
- $y = 3f(x)$
- $y = f(-x)$
- $y = f(x) - 7$

- Mark the coordinates of the new points of A, B and C.



HOMEWORK ACTIVITY: VLOGGING

TIMING: 60 MINS

LEARNING OBJECTIVES

- Recognise, sketch and interpret graphs of trigonometric functions
- Sketch translations and reflections of a given function

EQUIPMENT

- video recording equipment such as a smartphone (if available)



1. **Create a vlog (video blog) about transformations of graphs. You could simply write the script if you'd prefer. Sketch some graphs to illustrate your vlog or script. If you want to challenge yourself, try using the trigonometric functions as examples!**

Make sure you include all of these transformations:

- $y = f(x) + a$: translation through $\begin{pmatrix} 0 \\ a \end{pmatrix}$
- $y = f(x + a) - a$: translation through $\begin{pmatrix} -a \\ 0 \end{pmatrix}$
- $y = f(x) - a$: translation through $\begin{pmatrix} 0 \\ -a \end{pmatrix}$
- $y = f(x - a) + a$: translation through $\begin{pmatrix} a \\ 0 \end{pmatrix}$
- $y = f(-x)$: reflected in the y -axis
- $y = -f(x)$: reflected in the x -axis
- $y = af(x)$: stretched parallel to the y -axis with scale factor a
- $y = f(ax)$: stretched parallel to the x -axis with scale factor $\frac{1}{a}$

18 ANSWERS

STARTER ACTIVITY: SHAPES OF GRAPHS

1. 1: D 2: F 3: C 4: B 5: A 6: H 7: E 8: G

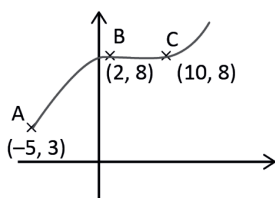
MAIN ACTIVITY: TRIGONOMETRIC FUNCTIONS

1. The first graph is $\cos x$, the second graph is $\tan x$, the third graph is $\sin x$. Other answers will depend on the questions asked.

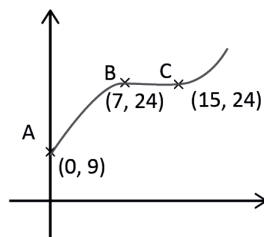
MAIN ACTIVITY: TRANSFORMATIONS OF FUNCTIONS

1. a) $y = f(x) + a$ is a translation through $\begin{pmatrix} 0 \\ a \end{pmatrix}$ b) $y = f(x + a)$ is a translation through $\begin{pmatrix} -a \\ 0 \end{pmatrix}$
 c) $y = f(x) - a$ is a translation through $\begin{pmatrix} 0 \\ -a \end{pmatrix}$ d) $y = f(x - a)$ is a translation through $\begin{pmatrix} a \\ 0 \end{pmatrix}$
 e) $y = f(-x)$ is a reflection in the y -axis f) $y = -f(x)$ is a reflection in the x -axis
 g) $y = af(x)$ is a stretch parallel to the y -axis with scale factor a h) $y = f(ax)$ is a stretch parallel to the x -axis with scale factor $\frac{1}{a}$

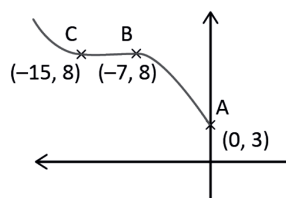
2. i)



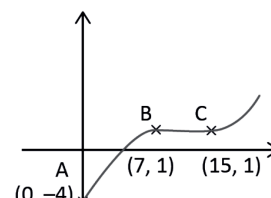
ii)



iii)



iv)



HOMEWORK ACTIVITY: VLOGGING

1. Check the student's vlog and graph sketches.