

Chapter 4 – Graphs and transformations

? Example 1 – Transformation of cubic functions

Sketch the curves with the following equations and show the points where they cross the coordinate axes.

a. $y = (x - 2)(1 - x)(1 + x)$ **b.** $y = x(x + 1)(x + 2)$

? Example 2 – Transformation of cubic functions

Sketch the following curves.

a. $y = (x - 1)^2(x + 1)$ **b.** $y = x^3 - 2x^2 - 3x$ **c.** $y = (x - 2)^3$

? Example 3 – Transformation of cubic functions

Sketch the curve with equation $y = (x - 1)(x^2 + x + 2)$.

? Example 4 – Sketching quartics

Sketch the following curves:

a. $y = (x + 1)(x + 2)(x - 1)(x - 2)$ **b.** $y = x(x + 2)^2(3 - x)$
c. $y = (x - 1)^2(x - 3)^2$

? Example 5 – Sketching reciprocal graphs

Sketch on the same diagram:

a. $y = \frac{4}{x}$ and $y = \frac{12}{x}$ **b.** $y = -\frac{1}{x}$ and $y = -\frac{3}{x}$ **c.** $y = \frac{4}{x^2}$ and $y = \frac{10}{x^2}$

? Example 6 – Points of intersection of graphs

- a.** On the same diagram sketch the curves with equations $y = x(x - 3)$ and $y = x^2(1 - x)$.
b. Find the coordinates of the points of intersection.

? Example 7 – Points of intersection of graphs

- a.** On the same diagram sketch the curves with equations $y = x^2(3x - a)$ and $y = \frac{b}{x}$, where a and b are positive constants.
b. State, giving a reason, the number of real solutions to the equation $x^2(3x - a) - \frac{b}{x} = 0$

? Example 8 – Points of intersection of graphs

- a.** Sketch the curves $y = \frac{4}{x^2}$ and $y = x^2(x - 3)$ on the same axes.
b. Using your sketch, state, with a reason, the number of real solutions to the equation $x^4(x - 3) - 4 = 0$.

? Example 9 – Transformation of cubic functions

Sketch the graphs of:

a. $y = x^2$ **b.** $y = (x - 2)^2$ **c.** $y = x^2 + 2$

? Example 10 – Translating graphs

$f(x) = x^3$

$g(x) = x(x - 2)$

Sketch the following graphs, indicating any points where the curves cross the axes:

- a.** $y = f(x + 1)$
b. $y = g(x + 1)$

? Example 11 – Translating reciprocals

Given that $h(x) = \frac{1}{x}$, sketch the curve with equation $y = h(x) + 1$ and state the equations of any asymptotes and intersections with the axes.

? Example 12 – Stretching quadratics

Given that $f(x) = 9 - x^2$, sketch the curves with equations:

a. $y = f(2x)$ **b.** $y = 2f(x)$

? Example 13 – Stretching cubics

- a.** Sketch the curve with equation $y = x(x - 2)(x + 1)$.
b. On the same axes, sketch the curves $y = 2x(2x - 2)(2x + 1)$ and $y = -x(x - 2)(x + 1)$.

? Example 14 – Reflecting quadratics

On the same axes sketch the graphs of $y = f(x)$, $y = f(-x)$ and $y = -f(x)$ where $f(x) = x(x + 2)$.

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? Example 15 – Inequalities and simultaneous equations

The following diagram shows a sketch of the curve $f(x)$ which passes through the origin.

The points $A(1, 4)$ and $B(3, 1)$ also lie on the curve.

Sketch the following:

a. $y = f(x + 1)$ **b.** $y = f(x - 1)$ **c.** $y = f(x) - 4$

d. $2y = f(x)$ **e.** $y - 1 = f(x)$

In each case you should show the positions of the images of the points O , A and B .

