

Indices – problem solving

A LEVEL LINKS

Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the n th root of a
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$
- $a^{-m} = \frac{1}{a^m}$
- The square root of a number produces two solutions, e.g. $\sqrt{16} = \pm 4$.

Example 1 Write $\frac{1}{3x}$ as a single power of x

$\frac{1}{3x} = \frac{1}{3}x^{-1}$	<p>Use the rule $\frac{1}{a^m} = a^{-m}$, note that the fraction $\frac{1}{3}$ remains unchanged</p>
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Example 2 Write $\frac{4}{\sqrt{x}}$ as a single power of x

$\begin{aligned} \frac{4}{\sqrt{x}} &= \frac{4}{x^{\frac{1}{2}}} \\ &= 4x^{-\frac{1}{2}} \end{aligned}$	<ol style="list-style-type: none"> 1 Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$ 2 Use the rule $\frac{1}{a^m} = a^{-m}$
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Practice questions

1 Write the following in the form ax^n .

a	$5\sqrt{x}$	b	$\frac{2}{x^3}$	c	$\frac{1}{3x^4}$
d	$\frac{2}{\sqrt{x}}$	e	$\frac{4}{\sqrt[3]{x}}$	f	3

2 $p = 4^a$ and $q = 4^b$

Write in terms of p and q

a	4^{a+b}	b	4^{b-a}
c	4^{2a}	d	4^{b-2}

3 **a** $2^3 \times 2^n = 2^9$

Work out the value of n .

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b $2x^3 = 128$

Work out the value of x .

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Answers

1 a	$5x^{\frac{1}{2}}$	b	$2x^{-3}$	c	$\frac{1}{3}x^{-4}$
d	$2x^{-\frac{1}{2}}$	e	$4x^{-\frac{1}{3}}$	f	$3x^0$

2 a	pq	b	$\frac{a}{p}$	c	p^2	d	$\frac{q}{16}$
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3 a	6	b	4
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