

Midpoints and vectors

A LEVEL LINKS

Scheme of work: 5a. Definitions, magnitude/direction, addition and scalar multiplication

Practice questions

1

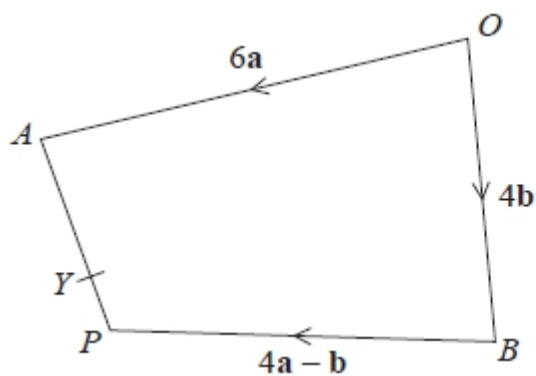


Diagram NOT
accurately drawn

$OBPA$ is a quadrilateral.

$$\vec{OA} = 6\mathbf{a}$$

$$\vec{OB} = 4\mathbf{b}$$

$$\vec{BP} = 4\mathbf{a} - \mathbf{b}$$

Y is the point on AP such that $AY : YP = 2 : 1$

Show that \vec{OY} is parallel to the vector $7\mathbf{a} + 3\mathbf{b}$

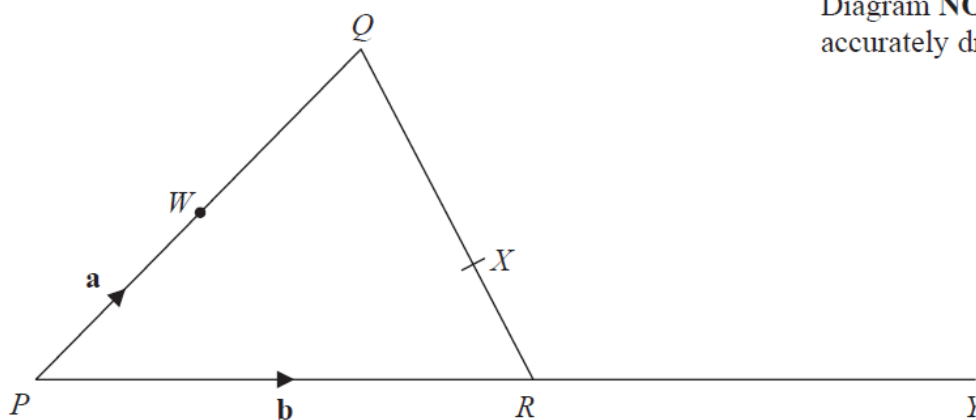


Diagram **NOT**
accurately drawn

PQR is a triangle.
The midpoint of PQ is W .
 X is the point on QR such that $QX : XR = 2 : 1$
 PRY is a straight line.

$$\vec{PW} = \mathbf{a} \quad \vec{PR} = \mathbf{b}$$

(a) Find, in terms of \mathbf{a} and \mathbf{b} ,

(i) \vec{QR}

.....

(ii) \vec{QX}

.....

(iii) \vec{WX}

.....

R is the midpoint of the straight line PRY .

(b) Use a vector method to show that WXY is a straight line.

3

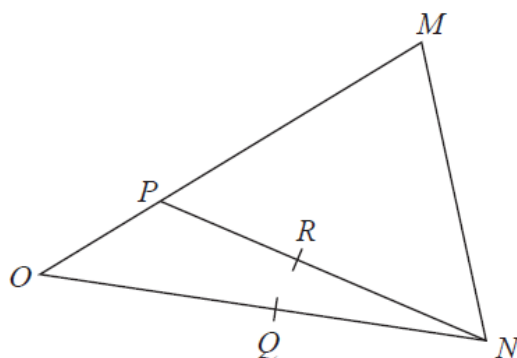


Diagram **NOT**
accurately drawn

OMN is a triangle.

P is the point on OM such that $OP = \frac{1}{4} OM$

Q is the midpoint of ON

R is the midpoint of PN

$$\vec{OP} = \mathbf{p} \quad \vec{OQ} = \mathbf{q}$$

(a) Find, in terms of \mathbf{p} and \mathbf{q} ,

(i) \vec{MN}

.....

(ii) \vec{PR}

.....

(b) Use a vector method to prove that QR is parallel to OP .

Answers

1 $\frac{2}{3}(7\mathbf{a} + 3\mathbf{b})$

M1 for correct vector for \overrightarrow{OY} , or \overrightarrow{AP}

eg $(\overrightarrow{OY}) = \overrightarrow{OA} + \overrightarrow{AY}$, or $\overrightarrow{OY} = \overrightarrow{OA} + \frac{2}{3}\overrightarrow{AP}$ may include terms in \mathbf{a} and \mathbf{b} , eg. $6\mathbf{a} + \overrightarrow{AY}$, $4\mathbf{b} + 4\mathbf{a} - \mathbf{b} + \overrightarrow{PY}$ or $\overrightarrow{AP} = \overrightarrow{AO} + \overrightarrow{OB} + \overrightarrow{BP}$ or $-6\mathbf{a} + 4\mathbf{b} + 4\mathbf{a} - \mathbf{b}$ or $-2\mathbf{a} + 3\mathbf{b}$

M1 for $(\overrightarrow{AY}) = \frac{2}{3}\overrightarrow{AP}$ or $\frac{1}{3}(-6\mathbf{a} + 4\mathbf{b} + 4\mathbf{a} - \mathbf{b})$ or $\frac{1}{3}(-2\mathbf{a} + 3\mathbf{b})$

or $(\overrightarrow{PY}) = \frac{1}{3}\overrightarrow{PA}$ or $\frac{1}{3}(-4\mathbf{a} + \mathbf{b} - 4\mathbf{b} + 6\mathbf{a})$ or $\frac{1}{3}(2\mathbf{a} - 3\mathbf{b})$

M1 for correct expression for \overrightarrow{OY} in terms of \mathbf{a} and \mathbf{b} , eg $\frac{14}{3}\mathbf{a} + 2\mathbf{b}$

C1 for $\frac{2}{3}(7\mathbf{a} + 3\mathbf{b})$ and " \overrightarrow{OY} is parallel to the vector $7\mathbf{a} + 3\mathbf{b}$ "
oe

2 (a)(i) $\mathbf{b} - 2\mathbf{a}$

(ii) $\frac{2}{3}\mathbf{b} - \frac{4}{3}\mathbf{a}$

(iii) $\frac{2}{3}\mathbf{b} - \frac{1}{3}\mathbf{a}$

(b) $\overrightarrow{WY} = -\mathbf{a} + 2\mathbf{b}$ oe or $\overrightarrow{XY} = \frac{2}{3}(-\mathbf{a} + 2\mathbf{b})$ oe

Conclusion using correct vectors eg. $\overrightarrow{WY} = 2\mathbf{b} - \mathbf{a}$ $\overrightarrow{XY} = \frac{2}{3}(-\mathbf{a} + 2\mathbf{b})$

$$\overrightarrow{XY} = \frac{2}{3}\overrightarrow{WY}$$

3 (a)(i) $2\mathbf{q} - 4\mathbf{p}$

(ii) $\mathbf{q} - \frac{1}{2}\mathbf{p}$ oe

(b) Must be shown (one method shown only)

$$(\overrightarrow{QR} =) -\mathbf{q} + \mathbf{p} + \mathbf{q} - \frac{1}{2}\mathbf{p} \text{ or } \frac{1}{2}\mathbf{p} \text{ oe}$$

$$\begin{aligned}(\overrightarrow{QR} =) \frac{1}{2}\mathbf{p} \text{ and } \overrightarrow{QR} = 0.5\overrightarrow{OP} \text{ or} \\ (\overrightarrow{QR} =) \frac{1}{2}\mathbf{p} \text{ and } \overrightarrow{OP} = 2\overrightarrow{QR}\end{aligned}$$