Pearson Edexcel

The discriminant: two distinct roots

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- A quadratic equation is an equation in the form $ax^2 + bx + c = 0$ where $a \neq 0$.
- For the quadratic function $f(x) = a(x + p)^2 + q$, the graph of $y = f(x)$ has a turning point at $(-p, q)$
- For the quadratic equation $ax^2 + bx + c = 0$, the expression $b^2 - 4ac$ is called the discriminant. The value of the discriminant shows how many roots f(x) has:
  - If $b^2 - 4ac > 0$ then the quadratic function has two distinct real roots.
  - If $b^2 - 4ac = 0$ then the quadratic function has one repeated real root.
  - If $b^2 - 4ac < 0$ then the quadratic function has no real roots.

Practice questions

1 The equation $kx^2 + 4x + (5 - k) = 0$, where k is a constant, has 2 different real solutions for x.

(a) Show that k satisfies

$$k^2 - 5k + 4 > 0.$$

(b) Hence find the set of possible value of k.

2 The equation $x^2 + (k - 3)x + (3 - 2k) = 0$, where k is a constant, has two distinct real roots.

(a) Show that k satisfies

$$k^2 + 2k - 3 > 0$$

(b) Find the set of possible values of k.

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Answers

1 (a) $b^2 - 4ac > 0 \Rightarrow 16 - 4k(5 - k) > 0$ or equiv., e.g. $16 > 4k(5 - k)$

So $k^2 - 5k + 4 > 0$ (Allow any order of terms,

e.g. $4 - 5k + k^2 > 0$)

(b) $k < 1$ or $k > 4$

2 (a) $b^2 - 4ac = (k-3)^2 - 4(3-2k)$

$k^2 - 6k + 9 - 4(3-2k) > 0$ or $(k-3)^2 - 12 + 8k > 0$ or better

$\underline{k^2 + 2k - 3 > 0}$ *

(b) $(k+3)(k-1)[=0]$

Critical values are $k = 1$ or -3

(choosing "outside" region)

$\underline{k > 1 \text{ or } k < -3}$