

Solving quadratic equations

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- A quadratic equation is an equation in the form $ax^2 + bx + c = 0$ where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose products is ac .
- When the product of two numbers is 0, then at least one of the numbers must be 0.
- If a quadratic can be solved it will have two solutions (these may be equal).

Example 2 Solve $2x^2 - 7x - 12 = 0$. Give your solutions in surd form.

$$2x^2 - 7x - 12 = 0$$

$$2\left(x^2 - \frac{7}{2}x\right) - 12 = 0$$

$$2\left[\left(x - \frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2\right] - 12 = 0$$

$$2\left(x - \frac{7}{4}\right)^2 - \frac{49}{8} - 12 = 0$$

$$2\left(x - \frac{7}{4}\right)^2 - \frac{145}{8} = 0$$

$$2\left(x - \frac{7}{4}\right)^2 = \frac{145}{8}$$

$$\left(x - \frac{7}{4}\right)^2 = \frac{145}{4}$$

$$x - \frac{7}{4} = \pm \frac{\sqrt{145}}{2}$$

$$x = \frac{7}{4} \pm \frac{\sqrt{145}}{2}$$

(You can use the Quadratic formula or complete the square)

1 Before completing the square write $ax^2 + bx + c$ in the form

$$a\left(x^2 + \frac{b}{a}x\right) + c$$

2 Now complete the square by writing

$$x^2 + \frac{b}{a}x$$
 in the form

$$\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2$$

3 Expand the square brackets.

4 Simplify.

5 Rearrange the equation to work out x . First, add $\frac{145}{8}$ to both sides.

6 Divide both sides by 2.

7 Square root both sides. Remember that the square root of a value gives two answers.

8 Add $\frac{7}{4}$ to both sides.

So $x = \frac{7}{4} - \frac{\sqrt{145}}{2}$ or $x = \frac{7}{4} + \frac{\sqrt{145}}{2}$	9 Write down both the solutions.
---	---

Practice questions

1 Solve

a $6x^2 + 4x = 0$

b $2x^2 - 7x - 4 = 0$

c $3x^2 - 13x - 10 = 0$

d $3x(x - 1) = 2(x + 1)$

2 Solve $3x^2 + 6x - 2 = 0$

Answers

1 a $x = 0$ or $x = -\frac{2}{3}$

b $x = -\frac{1}{2}$ or $x = 4$

c $x = -\frac{2}{3}$ or $x = 5$

d $x = -\frac{1}{3}$ or $x = 2$

2
$$\frac{-6 \pm \sqrt{6^2 - 4 \times 3 \times -2}}{2 \times 3}$$

$$(x + 1)^2 - 1 - \frac{2}{3} = 0$$

$$x = 0.29 \text{ and } x = -2.29$$