

# Quadratic inequalities

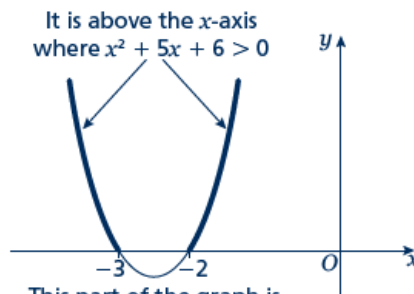
## A LEVEL LINKS

Scheme of work: 1d. Inequalities – linear and quadratic (including graphical solutions)

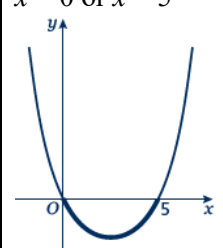
## Key points

- First replace the inequality sign by = and solve the quadratic equation.
- Sketch the graph of the quadratic function.
- Use the graph to find the values which satisfy the quadratic inequality.

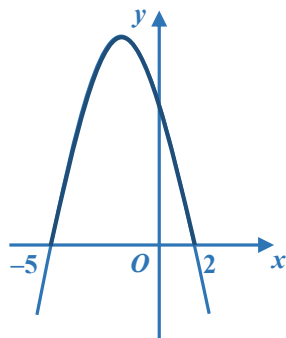
**Example 1** Find the set of values of  $x$  which satisfy  $x^2 + 5x + 6 > 0$

$x^2 + 5x + 6 = 0$ $(x + 3)(x + 2) = 0$ $x = -3 \text{ or } x = -2$  $x < -3 \text{ or } x > -2$	<ol style="list-style-type: none"> <li>1 Solve the quadratic equation by factorising.</li> <li>2 Sketch the graph of <math>y = (x + 3)(x + 2)</math></li> <li>3 Identify on the graph where <math>x^2 + 5x + 6 &gt; 0</math>, i.e. where <math>y &gt; 0</math></li> <li>4 Write down the values which satisfy the inequality <math>x^2 + 5x + 6 &gt; 0</math></li> </ol>
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**Example 2** Find the set of values of  $x$  which satisfy  $x^2 - 5x \leq 0$

$x^2 - 5x = 0$ $x(x - 5) = 0$ $x = 0 \text{ or } x = 5$  $0 \leq x \leq 5$	<ol style="list-style-type: none"> <li>1 Solve the quadratic equation by factorising.</li> <li>2 Sketch the graph of <math>y = x(x - 5)</math></li> <li>3 Identify on the graph where <math>x^2 - 5x \leq 0</math>, i.e. where <math>y \leq 0</math></li> <li>4 Write down the values which satisfy the inequality <math>x^2 - 5x \leq 0</math></li> </ol>
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**Example 3** Find the set of values of  $x$  which satisfy  $-x^2 - 3x + 10 \geq 0$

<p> <math>-x^2 - 3x + 10 = 0</math>  <math>(-x + 2)(x + 5) = 0</math>  <math>x = 2</math> or <math>x = -5</math> </p>  <p> <math>-5 \leq x \leq 2</math> </p>	<ol style="list-style-type: none"> <li>1 Solve the quadratic equation by factorising.</li> <li>2 Sketch the graph of <math>y = (-x + 2)(x + 5) = 0</math></li> <li>3 Identify on the graph where <math>-x^2 - 3x + 10 \geq 0</math>, i.e. where <math>y \geq 0</math></li> </ol> <ol style="list-style-type: none"> <li>3 Write down the values which satisfy the inequality <math>-x^2 - 3x + 10 \geq 0</math></li> </ol>
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## Practice questions

- 1 Find the set of values of  $x$  for which  $(x + 7)(x - 4) \leq 0$
- 2 Find the set of values of  $x$  for which  $x^2 - 4x - 12 \geq 0$
- 3 Find the set of values of  $x$  for which  $2x^2 - 7x + 3 < 0$
- 4 Find the set of values of  $x$  for which  $4x^2 + 4x - 3 > 0$
- 5 Find the set of values of  $x$  for which  $12 + x - x^2 \geq 0$

Find the set of values which satisfy the following inequalities.

- 6  $x^2 + x \leq 6$
- 7  $x(2x - 9) < -10$
- 8  $6x^2 \geq 15 + x$
- 9 (a)  $4(x - 2) \blacktriangleright 2x + 1$   
 (b)  $(2x - 3)(x + 5) > 0$   
 (c) **both**  $4(x - 2) \blacktriangleright 2x + 1$  **and**  $(2x - 3)(x + 5) > 0$

## Answers

1  $-7 \leq x \leq 4$

2  $x \leq -2$  or  $x \geq 6$

3  $\frac{1}{2} < x < 3$

4  $x < -\frac{3}{2}$  or  $x > \frac{1}{2}$

5  $-3 \leq x \leq 4$

6  $-3 \leq x \leq 2$

7  $2 < x < 2\frac{1}{2}$

8  $x \leq -\frac{3}{2}$  or  $x \geq \frac{5}{3}$

9 (a)  $x \blacktriangleleft 4.5$

(b)  $x < -5, x > 1.5$

(c)  $x < -5, 1.5 < x \blacktriangleright 4.5$