# Non-linear inequalities

**A LEVEL LINKS**
**Scheme of work:** 1d. Inequalities – where the unknown is the denominator of a fraction, (including graphical solutions)

## Key points

- Multiply both sides of the inequality by the denominator ***squared***.
- Re-arrange to obtain an inequality where one side is zero and the other side is a quadratic expression.
- Replace the inequality sign by = and solve the quadratic equation.
- Sketch the graph of the quadratic function.
- Use the graph to find the values which satisfy the quadratic inequality.

**Example 1** Find the set of values which satisfy the inequality $\frac{1}{x-2} \geqslant 1 \ , \ x \neq 2$

$\frac{1}{x-2} \geqslant 1$

$x-2 \geqslant (x-2)^2$

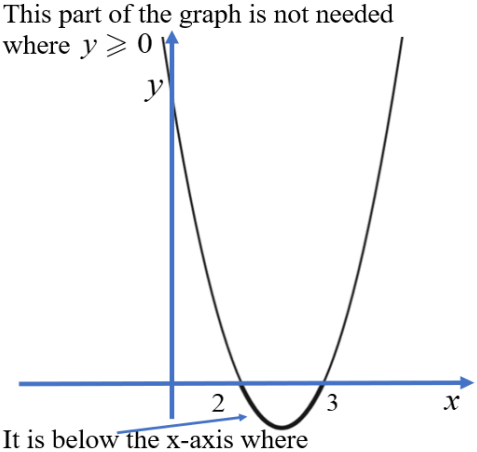
$0 \geqslant x^2 - 5x + 6$

$x^2 - 5x + 6 = 0$

$(x-2)(x-3) = 0$

$x = 2 \ \text{ or } \ x = 3$

This part of the graph is not needed where $y \geqslant 0$




It is below the x-axis where $0 \geqslant x^2 - 5x + 6$ where $y \leqslant 0$

$2 < x \leqslant 3$

**1** Multiply both sides of the inequality by $(x-2)^2$

**2** Re-arrange for a quadratic inequality.

**3** Replace $\geqslant$ with $=$

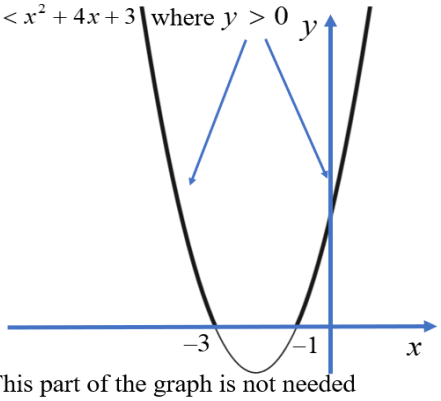
**4** Solve the equation.

**5** Sketch the graph.

**6** Identify on the graph where $0 \geqslant x^2 - 5x + 6$ where $y \leqslant 0$

**7** Write down the solutions which satisfy the inequality $\frac{1}{x-2} \geqslant 1 \ , \ x \neq 2$

**Example 2** Find the set of values which satisfy the inequality $\frac{x-1}{x+1}<2$ , $x \neq -1$

| | |
|---|---|
| $\frac{x-1}{x+1} < 2$<br>$x^2 - 1 < 2(x+1)^2$<br>$0 < x^2 + 4x + 3$<br>$x^2 + 4x + 3 = 0$<br>$(x+1)(x+3) = 0$<br>$x = -1$ or $x = -3$<br>It is above the x-axis where $0 < x^2 + 4x + 3$ where $y > 0$<br>  <br><br>This part of the graph is not needed where $y < 0$<br>$x < -3$ or $x > -1$ | **1** Multiply both sides of the inequality by $(x+1)^2$<br>**2** Re-arrange for a quadratic inequality.<br>**3** Replace $\geq$ with $=$<br>**4** Solve the equation.<br>**5** Sketch the graph.<br>**6** Identify on the graph where $0 < x^2 + 4x + 3$ where $y > 0$<br>**7** Write down the solutions which satisfy the inequality $\frac{x-1}{x+1}<2$ , $x \neq -1$ |

## Practice questions

Find the set of values which satisfy the following inequalities.

**1** $\frac{3}{x} > 1$ , $x \neq 0$

**2** $\frac{1}{x-1} > -1$ , $x \neq 1$

**3** $\frac{2-x}{x+2} \leqslant 0$ , $x \neq -2$

Pearson
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# Answers

**1** $0 < x \leqslant 3$

**2** $x < 0$ or $x > 1$

**3** $x < -2$ and $x \geqslant 2$