Pearson Edexcel

Solving linear and non-linear simultaneous equations by substitution

A LEVEL LINKS
Scheme of work: Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- Two equations are simultaneous when they are both true at the same time.
- Solving simultaneous equations in two unknowns involves finding the value of each unknown which works for both equations.
- Find an expression for one of the unknowns from one of the equations.
- It's usually easier to start with the linear equation.
- Substitute this expression in the other equation to obtain an equation with only one unknown.
- Solve this equation to obtain values for one of the unknowns.
- Substitute these values into the first equation to find values for the second unknown.
- The subsitution method is the method most commonly used for A level. This is because it is the method used to solve linear and nno-linear simultaneous equations.

Example 1 Solve the simultaneous equations $y = 2x - 1$ and $y = x^2 - 4$

$2x - 1 = x^2 - 4$	**1** Substitute $2x - 1$ for y in the linear equation
$x^2 - 2x - 3 = 0$	**2** Rearrange to obtain a quadratic equation whose RHS is zero
$(x+1)(x-3) = 0$	**3** Factorize
$x = -1$ and $x = 3$	**4** Find two values for x
$x = -1,\ y = -3$ $x = 3,\ y = 5$	**5** Substitute each of these values in turn into the other equation to find two values for y
Check: Equation 1: $-3 = 2(-1) - 1$ YES $5 = 2(3) - 1$ YES Equation 2: $-1 = (-1)^2 - 4$ YES $5 = (3)^2 - 4$ YES	**6** Substitute both values for x and y into both equations to check your answers.

Example 2

Solve simultaneously, $y = x + 1$ and $y = 1 + \frac{4}{x}$

$x + 1 = 1 + \frac{4}{x}$	**1** Substitute $x + 1$ for y in the linear equation
$x^2 + x = x + 4$	**2** Multiply both sides by x
$x^2 - 4 = 0$	**3** Rearrange to obtain a quadratic equation whose RHS is zero
$(x - 2)(x + 2) = 0$	**3** Factorize
$x = 2$ and $x = -2$	**4** Find two values for x
$x = 2,\ y = 3$ $x = -2, y = -1$	**5** Substitute each of these values in turn into the other equation to find two values for y
Check: Equation 1: $3 = 2 + 1$ YES $-1 = -2 + 1$ YES Equation 2: $3 = 1 + (4 \div 2)$ YES $-1 = 1 + (4 \div (-2))$ YES	**6** Substitute both values for x and y into both equations to check your answers.

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Practice questions

Solve these simultaneous equations.

1 $xy = 9$ and $y = x$

2 $x^2 + y^2 = 50$ and $y = x$

3 $xy - 3 = 16$ and $x - 19y = 0$

4 $x - 2y = 3$ and $(x-4)^2 + (y-3)^2 = 25$

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Answers

1 $x = -3,\ y = -3$ and $x = 3,\ y = 3$

2 $x = -5,\ y = -5$ and $x = 5,\ y = 5$

3 $x = -19,\ y = -1$ and $x = 19,\ y = 1$

4 $x = 1,\ y = -1$ and $x = 9,\ y = 3$