Non-linear simultaneous equations on graphs

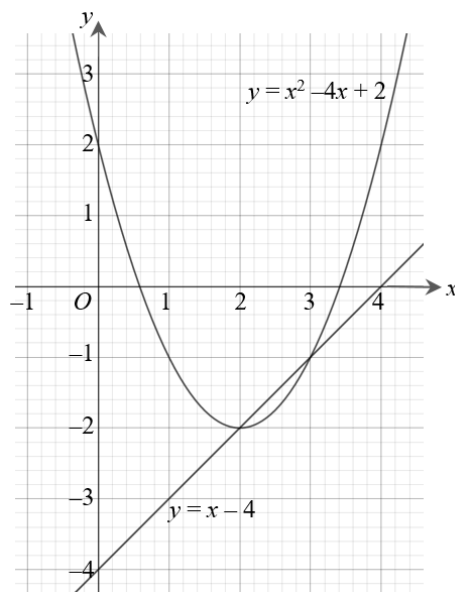| A LEVEL LINKS |
| --- |
| **Scheme of work:** 1c. Equations – quadratic/linear simultaneous |

Key points

- You can solve any pair of simultaneous equations by drawing the graph of both equations and finding the point/points of intersection.

Example 1 Solve the simultaneous equations $y = x - 4$ and $y = x^2 - 4x + 2$ graphically.

x	0	1	2	3	4
y	2	−1	−2	−1	2

1 Construct a table of values and calculate the points for the quadratic equation.

$y = x^2 - 4x + 2$

$y = x - 4$

2 Plot the graph.

3 Plot the linear graph on the same grid using the gradient and y-intercept.
$y = x - 4$ has gradient 1 and y-intercept –4.

The line and curve intersect at
$x = 3$, $y = -1$ and $x = 2$, $y = -2$

Check:
First equation $y = x - 4$:
$-1 = 3 - 4$ YES
$-2 = 2 - 4$ YES
Second equation $y = x^2 - 4x + 2$:
$-1 = 3^2 - 4 \times 3 + 2$ YES
$-2 = 2^2 - 4 \times 2 + 2$ YES

4 The solutions of the simultaneous equations are the points of intersection.

5 Check your solutions by substituting the values into both equations.

Practice questions

1. (a)   Show that the x-coordinates of the points of intersection of

$$y = x(4 - x) \text{ and } y = x^2(7 - x)$$

are given by the solutions to the equations $x(x^2 - 8x + 4) = 0$

(b)   The point A lies on both of the curves and the x and y coordinates of A are both positive. Find the exact coordinates of A, leaving your answer in the form $(p + q\sqrt{3}, r + s\sqrt{3})$, where p, q, r, and s are integers.

Answers

1   (a) $x(4-x) = x^2(7-x)$ $(0 =)x[7x - x^2 - (4 - x)]$

$(0 =)x[7x - x^2 - (4 - x)]$ (o.e.)

$0 = x(x^2 - 8x + 4)$ *

(b) $(4 - 2\sqrt{3}, -12 + 8\sqrt{3})$