

Quadratic inequalities

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Completing the square lets you write a quadratic equation in the form $p(x + q)^2 + r = 0$.

Examples

Example 1 Find the set of values of x for which $x^2 + 4x + 4 > 0$

$x^2 + 4x + 4 > 0$ $(x + 2)(x + 2) > 0$ $x > -2$	
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Example 2 Find the set of values of x for which $2x^2 - x - 1 < 0$

$2x^2 + 6x + 4 < 0$ $(2x + 1)(x - 1) < 0$ $-\frac{1}{2} < x < 1$	
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Practice questions

1 Find the set values of x for which

(a) $4x - 5 > 15 - x$

(b) $x(x - 4) > 12$

2 The equation $x^2 + (k - 3)x + (3 - 2k) = 0$, where k is a constant, has two distinct real roots.

(a) Show that k satisfies

$$k^2 + 2k - 3 > 0.$$

(b) Find the set of possible values of k .

3 The equation $kx^2 + 4x + (5 - k) = 0$, where k is a constant, has 2 different real solutions for x .

(a) Show that k satisfies

$$k^2 - 5k + 4 > 0.$$

(b) Hence find the set of possible values of k .

4 Find the set of values of x for which

(a) $2(3x + 4) > 1 - x$

(2)

(b) $3x^2 + 8x - 3 < 0$

(4)

Answers

- 1 (a) $x > 4$
- (b) $x < -2, x > 6$
- 2 (a) $b^2 - 4ac = (k-3)^2 - 4(3-2k)$
 $k^2 - 6k + 9 - 4(3-2k) > 0$ or $(k-3)^2 - 12 + 8k > 0$ or better
 $k^2 + 2k - 3 > 0$ *
- (b) $k > 1, k < -3$
- 3 (a) $b^2 - 4ac > 0 \Rightarrow 16 - 4k(5-k) > 0$ or equiv., e.g. $16 > 4k(5-k)$
So $k^2 - 5k + 4 > 0$ (Allow any order of terms, e.g. $4 - 5k + k^2 > 0$)
- (b) $k < 1, k > 4$
- 4 (a) $x > -1$
- (b) $-3 < x < \frac{1}{3}$