

# Quadratic inequalities

**A LEVEL LINKS**

**Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

## Key points

- Completing the square lets you write a quadratic equation in the form  $p(x + q)^2 + r = 0$ .

## Examples

**Example 1** Find the set of values of  $x$  for which  $x^2 + 4x + 4 > 0$

$$x^2 + 4x + 4 > 0$$

$$(x + 2)(x + 2) > 0$$

$$x > -2$$

**Example 2** Find the set of values of  $x$  for which  $2x^2 - x - 1 < 0$

$$2x^2 + 6x + 4 < 0$$

$$(2x + 1)(x - 1) < 0$$

$$-\frac{1}{2} < x < 1$$

## Practice questions

- 1** Find the set values of  $x$  for which
  - (a)  $4x - 5 > 15 - x$
  - (b)  $x(x - 4) > 12$
  
- 2** The equation  $x^2 + (k - 3)x + (3 - 2k) = 0$ , where  $k$  is a constant, has two distinct real roots.
  - (a) Show that  $k$  satisfies
$$k^2 + 2k - 3 > 0.$$
  - (b) Find the set of possible values of  $k$ .
  
- 3** The equation  $kx^2 + 4x + (5 - k) = 0$ , where  $k$  is a constant, has 2 different real solutions for  $x$ .
  - (a) Show that  $k$  satisfies
$$k^2 - 5k + 4 > 0.$$
  - (b) Hence find the set of possible values of  $k$ .
  
- 4** Find the set of values of  $x$  for which
  - (a)  $2(3x + 4) > 1 - x$  (2)
  - (b)  $3x^2 + 8x - 3 < 0$  (4)

## Answers

1 (a)  $x > 4$

(b)  $x < -2, x > 6$

2 (a)  $b^2 - 4ac = (k-3)^2 - 4(3-2k)$   
 $k^2 - 6k + 9 - 4(3-2k) > 0 \quad \text{or} \quad (k-3)^2 - 12 + 8k > 0 \quad \text{or better}$   
 $k^2 + 2k - 3 > 0$  \*

(b)  $k > 1, k < -3$

3 (a)  $b^2 - 4ac > 0 \Rightarrow 16 - 4k(5-k) > 0 \quad \text{or equiv., e.g. } 16 > 4k(5-k)$   
So  $k^2 - 5k + 4 > 0$  (Allow any order of terms, e.g.  $4 - 5k + k^2 > 0$ )

(b)  $k < 1, k > 4$

4 (a)  $x > -1$

(b)  $-3 < x < \frac{1}{3}$