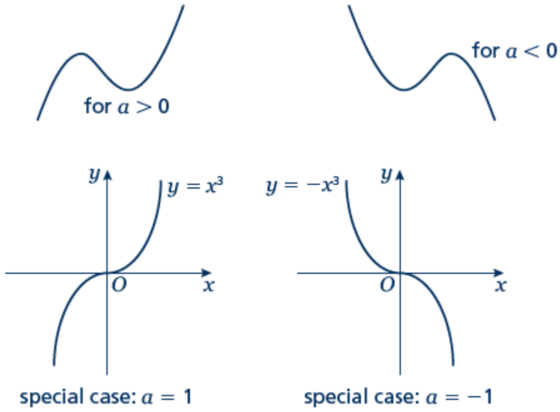
Transformation of cubic functions

A LEVEL LINKS
Scheme of work: 1e. Graphs – cubic, quartic and reciprocal

Key points

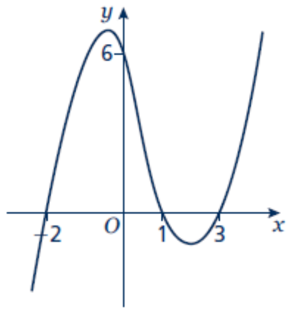
- The graph of a cubic function, which can be written in the form $y = ax^3 + bx^2 + cx + d$, where $a \neq 0$, has one of the shapes shown here.



- To sketch the graph of a function, find the points where the graph intersects the axes.
- To find where the curve intersects the y-axis substitute $x = 0$ into the function.
- To find where the curve intersects the x-axis substitute $y = 0$ into the function.
- At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
- A double root is when two of the solutions are equal. For example $(x - 3)^2(x + 2)$ has a double root at $x = 3$.
- When there is a double root, this is one of the turning points of a cubic function.

Example 1 Sketch the graph of $y = (x - 3)(x - 1)(x + 2)$

<table>
<tr><td colspan="2">To sketch a cubic curve find intersects with both axes and use the key points above for the correct shape.</td></tr>
<tr><td>When $x = 0$, $y = (0 - 3)(0 - 1)(0 + 2)$
$= (-3) \times (-1) \times 2 = 6$
The graph intersects the y-axis at (0, 6)

When $y = 0$, $(x - 3)(x - 1)(x + 2) = 0$
So $x = 3$, $x = 1$ or $x = -2$
The graph intersects the x-axis at
(−2, 0), (1, 0) and (3, 0)
</td><td>1 Find where the graph intersects the axes by substituting $x = 0$ and $y = 0$. Make sure you get the coordinates the right way around, (x, y).
2 Solve the equation by solving $x - 3 = 0$, $x - 1 = 0$ and $x + 2 = 0$

3 Sketch the graph.
$a = 1 > 0$ so the graph has the shape:
</td></tr>
</table>

Practice questions

1 Sketch the following graphs

(a) $y = 2x^3$

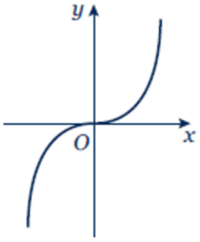
(b) $y = x(x – 2)(x + 2)$

(c) $y = (x + 1)(x + 4)(x – 3)$

(d) $y = (x + 1)(x – 2)(1 – x)$

Answers

1



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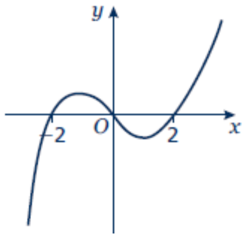

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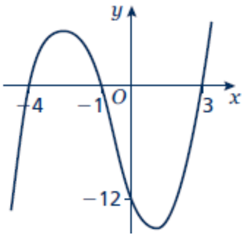

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