

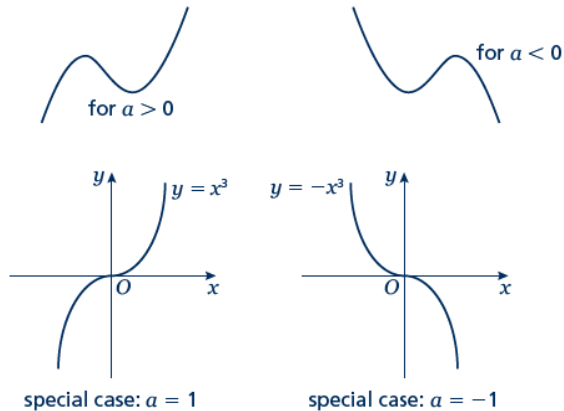
Transformation of cubic functions

A LEVEL LINKS

Scheme of work: 1e. Graphs – cubic, quartic and reciprocal

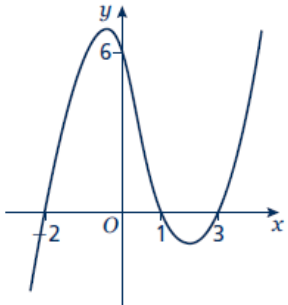

Key points

- The graph of a cubic function, which can be written in the form $y = ax^3 + bx^2 + cx + d$, where $a \neq 0$, has one of the shapes shown here.

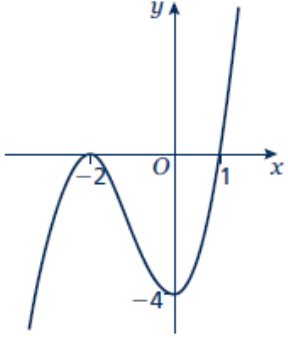



- To sketch the graph of a function, find the points where the graph intersects the axes.
- To find where the curve intersects the y -axis substitute $x = 0$ into the function.
- To find where the curve intersects the x -axis substitute $y = 0$ into the function.
- At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
- A double root is when two of the solutions are equal. For example $(x - 3)^2(x + 2)$ has a double root at $x = 3$.
- When there is a double root, this is one of the turning points of a cubic function.

Example 1 Sketch the graph of $y = (x - 3)(x - 1)(x + 2)$

To sketch a cubic curve find intersects with both axes and use the key points above for the correct shape.	
<p>When $x = 0$, $y = (0 - 3)(0 - 1)(0 + 2)$ $= (-3) \times (-1) \times 2 = 6$ The graph intersects the y-axis at $(0, 6)$</p> <p>When $y = 0$, $(x - 3)(x - 1)(x + 2) = 0$ So $x = 3$, $x = 1$ or $x = -2$ The graph intersects the x-axis at $(-2, 0)$, $(1, 0)$ and $(3, 0)$</p> 	<ol style="list-style-type: none"> Find where the graph intersects the axes by substituting $x = 0$ and $y = 0$. Make sure you get the coordinates the right way around, (x, y). Solve the equation by solving $x - 3 = 0$, $x - 1 = 0$ and $x + 2 = 0$ Sketch the graph. $a = 1 > 0$ so the graph has the shape: 

Example 2 Sketch the graph of $y = (x + 2)^2(x - 1)$

To sketch a cubic curve find intersects with both axes and use the key points above for the correct shape.	
<p>When $x = 0$, $y = (0 + 2)^2(0 - 1)$ $= 2^2 \times (-1) = -4$ The graph intersects the y-axis at $(0, -4)$</p> <p>When $y = 0$, $(x + 2)^2(x - 1) = 0$ So $x = -2$ or $x = 1$</p> <p>$(-2, 0)$ is a turning point as $x = -2$ is a double root. The graph crosses the x-axis at $(1, 0)$</p> 	<ol style="list-style-type: none"> Find where the graph intersects the axes by substituting $x = 0$ and $y = 0$. Solve the equation by solving $x + 2 = 0$ and $x - 1 = 0$ $a = 1 > 0$ so the graph has the shape: 

Practice questions

1 Sketch the following graphs

a $y = (x - 3)^2(x + 1)$

b $y = (x - 1)^2(x - 2)$

2 The curve C has equation

$$y = (x + 3)(x - 1)^2.$$

Sketch C , showing clearly the coordinates of the points where the curve meets the coordinate axes.

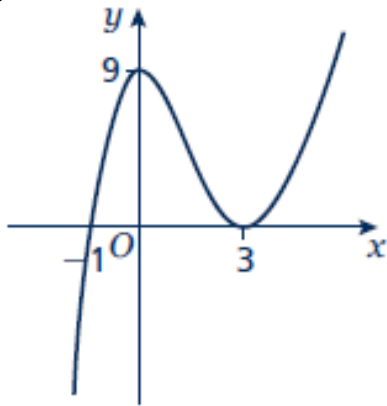
3 The curve C has equation

$$y = (x + 1)^2(2 - x).$$

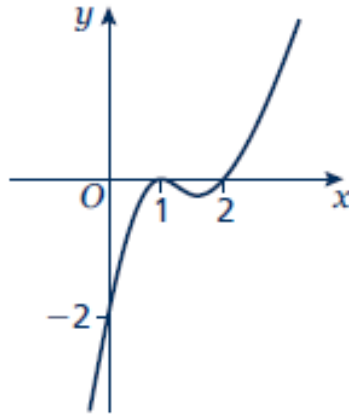
Sketch C , showing clearly the coordinates of the points where the curve meets the coordinate axes.

Answers

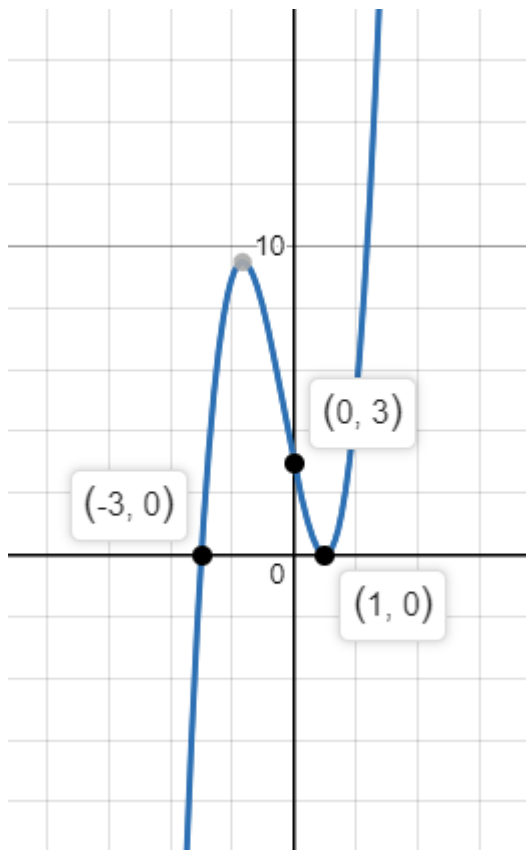
1 a



b



2



3

