

Sketching quartics

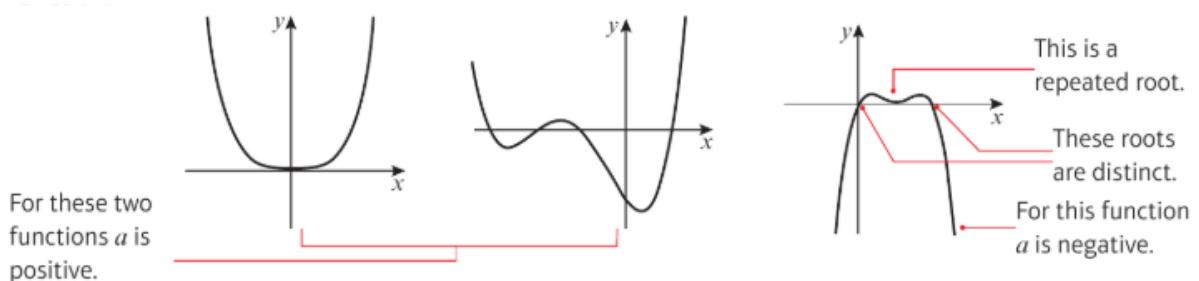
A LEVEL LINKS

Scheme of work: 1e. Graphs – cubic, quartic and reciprocal

Key points

- To quartic function has the form $f(x) = ax^4 + bx^3 + cx^2 + dx + e$, where a, b, c, d and e are real numbers and a is non-zero.

The graph of a quartic function can take several different forms, depending on the exact nature of the function.



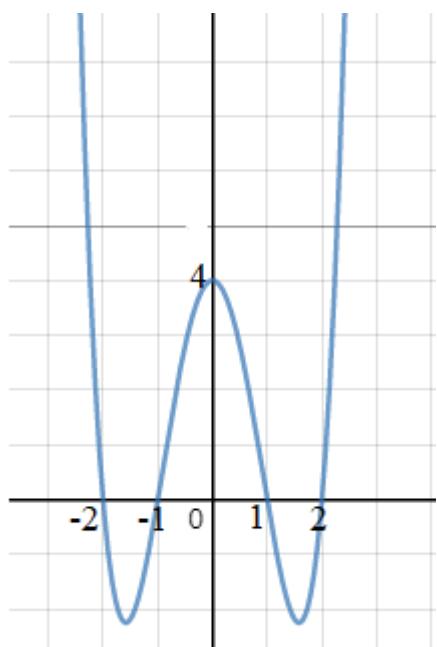
You can sketch the graph of a quartic function by finding the roots of the function.

- To sketch the graph of a function, find the points where the graph intersects the axes.
- To find where the curve intersects the y -axis substitute $x = 0$ into the function.
- To find where the curve intersects the x -axis substitute $y = 0$ into the function.
- At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
- A double root is when two of the solutions are equal. For example $(x - 3)^2(x + 2)^2$ has a double repeated root at $x = 3$ and $x = -2$.
- When there is a double root, this is one of the turning points of a cubic function.



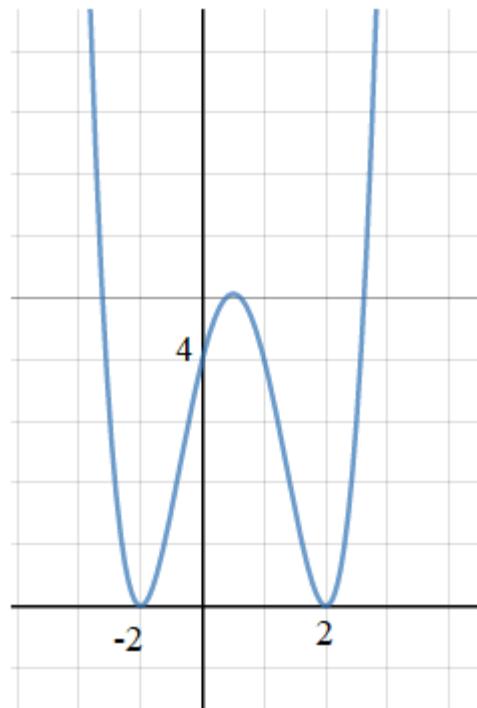
Example 1

Sketch the graph of $y = (x - 1)(x + 3)(x + 1)(x - 2)$



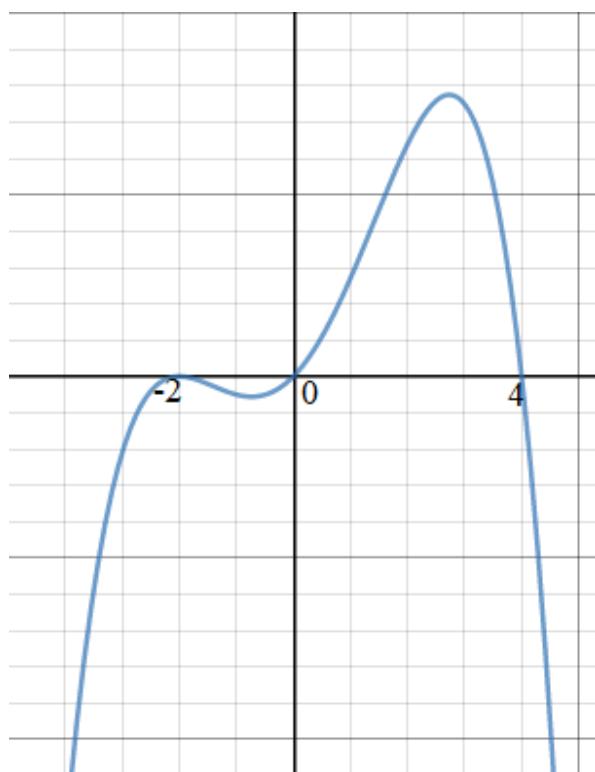
Example 2

Sketch the graph of $y = (x - 2)^2(x + 1)^2$



Example 3

Sketch the graph of $y = x(x + 2)^2(4 - x)$



Practice questions

1 Sketch the following graphs

(a) $y = (x - 1)(x + 2)(x + 3)(x + 4)$

(b) $y = -(x - 1)^2 (x + 3)^2$

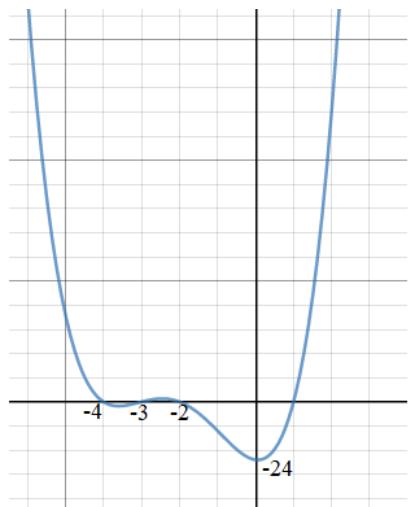
(c) $y = x (x - 5)^3$

(d) $y = (x + 2)^3 (4 - x)$

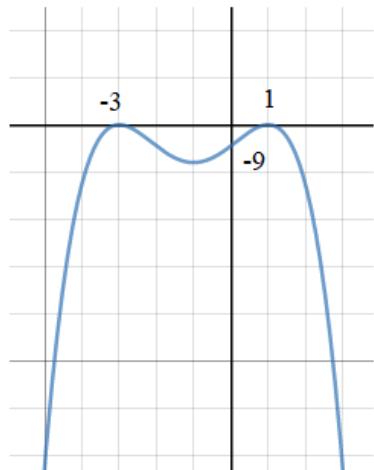
(e) $y = (x + 5)^4$

Answers

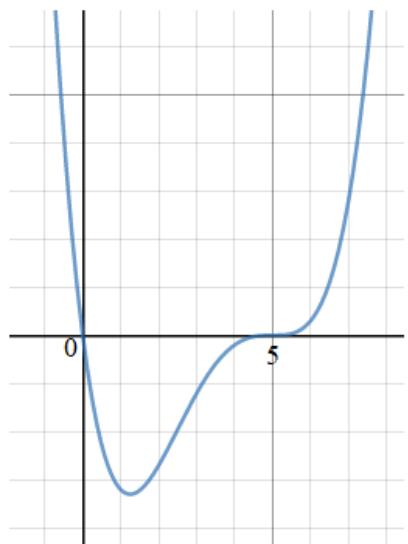
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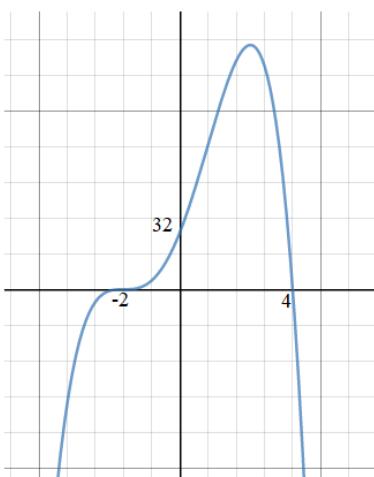
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