

Points of intersection of graphs

A LEVEL LINKS

Scheme of work: 1e. Graphs – cubic, quartic and reciprocal

Key points

- To sketch the graph of a function, find the points where the graph intersects the axes.
- To find where the curve intersects the y -axis substitute $x = 0$ into the function.
- To find where the curve intersects the x -axis substitute $y = 0$ into the function.
- Where appropriate, mark and label the asymptotes on the graph.
- Asymptotes are lines (usually horizontal or vertical) which the curve gets closer to but never touches or crosses. Asymptotes usually occur with reciprocal functions. For example, the asymptotes for the graph of $y = \frac{a}{x}$ are the two axes (the lines $y = 0$ and $x = 0$).
- At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
- A double root is when two of the solutions are equal. For example $(x - 3)^2(x + 2)$ has a double root at $x = 3$.
- When there is a double root, this is one of the turning points of a cubic function.

Practice question

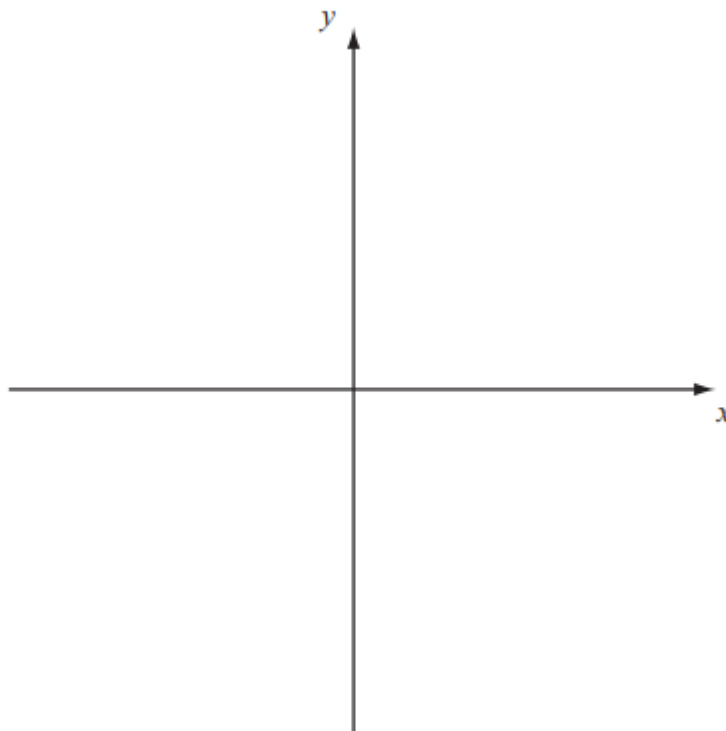
1. (a) On the same axes sketch the graphs of the curves with equations

(i) $y = x^2(x - 2)$,

(ii) $y = x(6 - x)$,

and indicate on your sketches the coordinates of all the points where the curves cross the x-axis.

(b) Use algebra to find the coordinates of the points where the graphs intersect.



2. The curve C has equation $y = \frac{3}{x}$ and the line l has equation $y = 2x + 5$.

(a) Sketch the graphs of C and l , indicating clearly the coordinates of any intersections with the axes.

(b) Find the coordinates of the points of intersection of C and l .

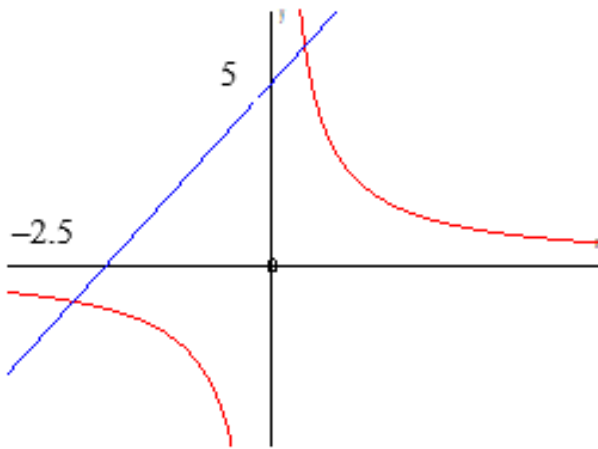
Answers

1 (a)



(b) (3, 9) (-2, -16)

2 (a)



(b) $2x + 5 = \frac{3}{x}$

$$2x^2 + 5x - 3 = 0$$

$$(2x - 1)(x + 3) = 0$$

$$x = -3 \text{ or } \frac{1}{2}$$

$$y = \frac{3}{-3} \text{ or } 2 \times (-3) + 5 \quad \text{or} \quad y = \frac{3}{\frac{1}{2}} \text{ or } 2 \times \left(\frac{1}{2}\right) + 5$$

Points are $(-3, -1)$ and $(\frac{1}{2}, 6)$