

# Points of intersection of graphs

## A LEVEL LINKS

Scheme of work: 1e. Graphs – cubic, quartic and reciprocal

## Key points

- To sketch the graph of a function, find the points where the graph intersects the axes.
- To find where the curve intersects the  $y$ -axis substitute  $x = 0$  into the function.
- To find where the curve intersects the  $x$ -axis substitute  $y = 0$  into the function.
- Where appropriate, mark and label the asymptotes on the graph.
- Asymptotes are lines (usually horizontal or vertical) which the curve gets closer to but never touches or crosses. Asymptotes usually occur with reciprocal functions. For example, the asymptotes for the graph of  $y = \frac{a}{x}$  are the two axes (the lines  $y = 0$  and  $x = 0$ ).
- At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
- A double root is when two of the solutions are equal. For example  $(x - 3)^2(x + 2)$  has a double root at  $x = 3$ .
- When there is a double root, this is one of the turning points of a cubic function.

## Practice question

1. (a) On the axes below, sketch the graph of

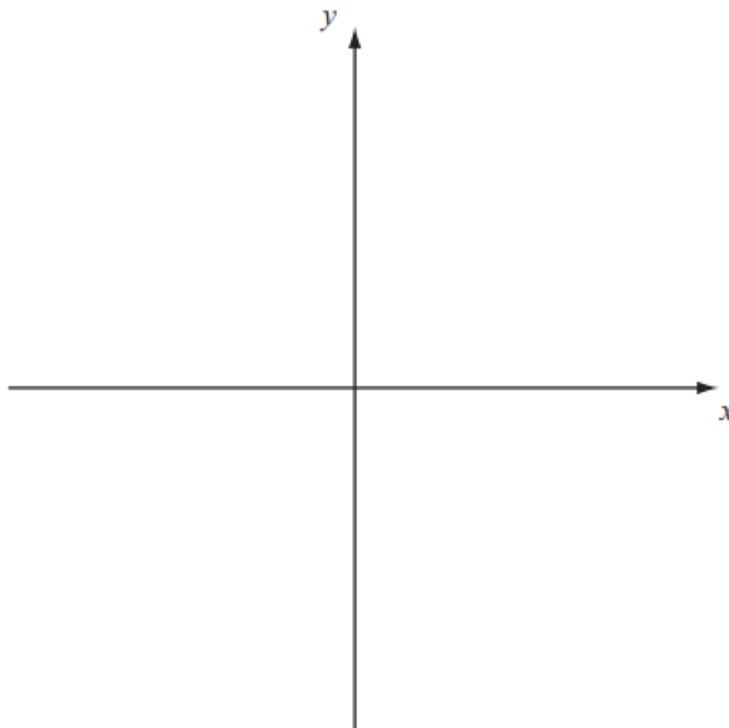
(i)  $y = x(x + 2)(3 - x)$

(ii)  $y = -\frac{2}{x}$

Show clearly the coordinates of all points where the curves cross the coordinate axis.

(b) Using your sketch state, giving a reason, the number of real solutions to the equation

$$x(x + 2)(3 - x) + \frac{2}{x} = 0$$



2. The point  $P(1, a)$  lies on the curve with equation  $y = (x + 1)^2(2 - x)$ .

(a) Find the value of  $a$ .

(b) On the axes below sketch the curves with the following equations:

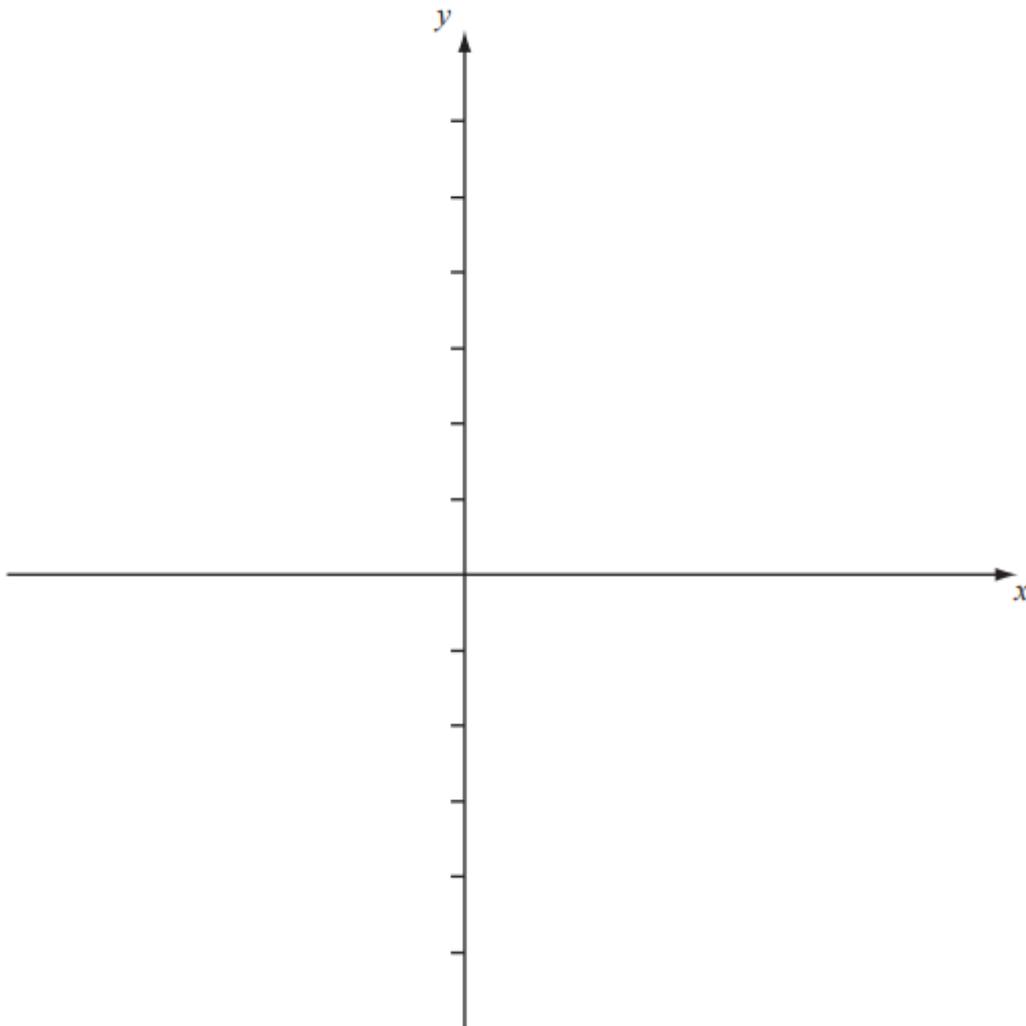
(i)  $y = (x + 1)^2(2 - x)$ ,

(ii)  $y = \frac{2}{x}$ .

On your diagram show clearly the coordinates of any points at which the curves meet the axes.

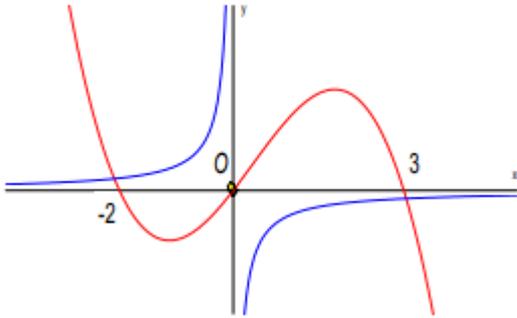
With reference to your diagram in part (b) state the number of real solutions to the equation

$$(x + 1)^2(2 - x) = \frac{2}{x}.$$



## Answers

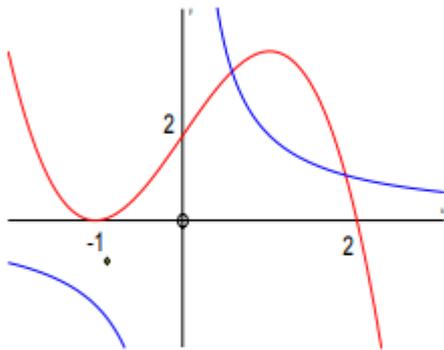
1 (a)



(b) 2 solutions

2 (a) (1, 4)

(b)



(c) 2 intersections therefore 2 roots