Pearson Edexcel

Points of intersection of graphs

A LEVEL LINKS
Scheme of work: 1e. Graphs – cubic, quartic and reciprocal

Key points

- To sketch the graph of a function, find the points where the graph intersects the axes.
- To find where the curve intersects the y-axis substitute $x = 0$ into the function.
- To find where the curve intersects the x-axis substitute $y = 0$ into the function.
- Where appropriate, mark and label the asymptotes on the graph.
- Asymptotes are lines (usually horizontal or vertical) which the curve gets closer to but never touches or crosses. Asymptotes usually occur with reciprocal functions. For example, the asymptotes for the graph of $y = \frac{a}{x}$ are the two axes (the lines $y = 0$ and $x = 0$).
- At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
- A double root is when two of the solutions are equal. For example $(x – 3)^2(x + 2)$ has a double root at $x = 3$.
- When there is a double root, this is one of the turning points of a cubic function.

Practice question

1 (*a*) On separate axes sketch the graphs of

(i) $y = -3x + c$, where c is a positive constant,

(ii) $y = \frac{1}{x} + 5$

On each sketch show the coordinates of any point at which the graph crosses the y-axis and the equation of any horizontal asymptote.

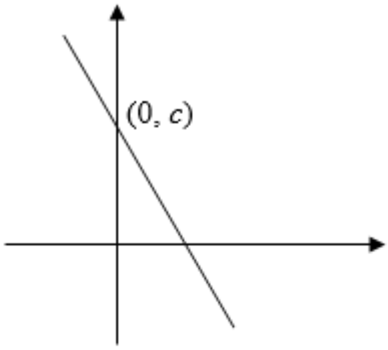
Given that $y = -3x + c$, where c is a positive constant, meets the curve $y = \frac{1}{x} + 5$ at two distinct points,

(*b*) show that $(5 - c)^2 > 12$

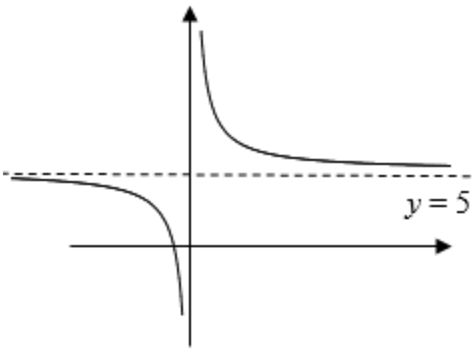
(*c*) Hence find the range of possible values for c.

Answers

1 (a)(i)



(ii)



(b)

$$b^2 - 4ac = (5-c)^2 - 4 \times 1 \times 3$$

$$(5-c)^2 > 12\,*$$

(c) $(5-c)^2 = 12 \Rightarrow (c =) 5 \pm \sqrt{12}$ **or**

$(5-c)^2 = 12 \Rightarrow c^2 - 10c + 13 = 0$

or $\Rightarrow (c =) \dfrac{{}^{-}10 \pm \sqrt{(-10)^2 - 4 \times 13}}{2}$

$$0 < c < 5 - \sqrt{12},\ c > 5 + \sqrt{12}$$