

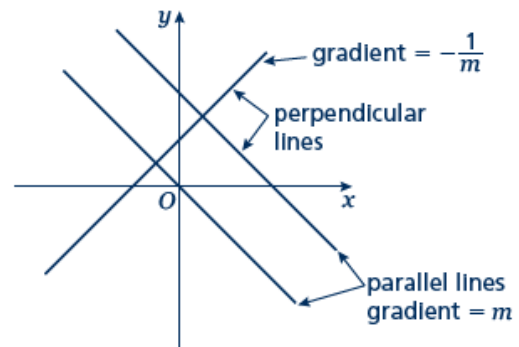
# Perpendicular and parallel lines

## A LEVEL LINKS

Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

## Key points

- When lines are parallel they have the same gradient.
- A line perpendicular to the line with equation  $y = mx + c$  has gradient  $-\frac{1}{m}$ .



## Examples

**Example 1** Find the equation of the line parallel to  $y = 2x + 4$  which passes through the point  $(4, 9)$ .

$y = 2x + 4$ $m = 2$ $y = 2x + c$ $9 = 2 \times 4 + c$ $9 = 8 + c$ $c = 1$ $y = 2x + 1$	<ol style="list-style-type: none"> <li>1 As the lines are parallel they have the same gradient.</li> <li>2 Substitute <math>m = 2</math> into the equation of a straight line <math>y = mx + c</math>.</li> <li>3 Substitute the coordinates into the equation <math>y = 2x + c</math></li> <li>4 Simplify and solve the equation.</li> <li>5 Substitute <math>c = 1</math> into the equation <math>y = 2x + c</math></li> </ol>
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**Example 2** Find the equation of the line perpendicular to  $y = 2x - 3$  which passes through the point  $(-2, 5)$ .

$y = 2x - 3$ $m = 2$ $-\frac{1}{m} = -\frac{1}{2}$ $y = -\frac{1}{2}x + c$ $5 = -\frac{1}{2} \times (-2) + c$ $5 = 1 + c$ $c = 4$ $y = -\frac{1}{2}x + 4$	<ol style="list-style-type: none"> <li>1 As the lines are perpendicular, the gradient of the perpendicular line is <math>-\frac{1}{m}</math>.</li> <li>2 Substitute <math>m = -\frac{1}{2}</math> into <math>y = mx + c</math>.</li> <li>3 Substitute the coordinates <math>(-2, 5)</math> into the equation <math>y = -\frac{1}{2}x + c</math></li> <li>4 Simplify and solve the equation.</li> <li>5 Substitute <math>c = 4</math> into <math>y = -\frac{1}{2}x + c</math>.</li> </ol>
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**Example 3** A line passes through the points (0, 5) and (9, -1).  
Find the equation of the line which is perpendicular to the line and passes through its midpoint.

$x_1 = 0, x_2 = 9, y_1 = 5 \text{ and } y_2 = -1$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 5}{9 - 0}$ $= \frac{-6}{9} = -\frac{2}{3}$ $-\frac{1}{m} = \frac{3}{2}$ $y = \frac{3}{2}x + c$ $\text{Midpoint} = \left( \frac{0+9}{2}, \frac{5+(-1)}{2} \right) = \left( \frac{9}{2}, 2 \right)$ $2 = \frac{3}{2} \times \frac{9}{2} + c$ $c = -\frac{19}{4}$ $y = \frac{3}{2}x - \frac{19}{4}$	<ol style="list-style-type: none"> <li>1 Substitute the coordinates into the equation <math>m = \frac{y_2 - y_1}{x_2 - x_1}</math> to work out the gradient of the line.</li> <li>2 As the lines are perpendicular, the gradient of the perpendicular line is <math>-\frac{1}{m}</math>.</li> <li>3 Substitute the gradient into the equation <math>y = mx + c</math>.</li> <li>4 Work out the coordinates of the midpoint of the line.</li> <li>5 Substitute the coordinates of the midpoint into the equation.</li> <li>6 Simplify and solve the equation.</li> <li>7 Substitute <math>c = -\frac{19}{4}</math> into the equation <math>y = \frac{3}{2}x + c</math>.</li> </ol>
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## Practice questions

1 Work out whether these pairs of lines are parallel, perpendicular or neither.

**a**  $y = 2x + 3$   
 $y = 2x - 7$

**b**  $y = 3x$   
 $2x + y - 3 = 0$

**c**  $y = 4x - 3$   
 $4y + x = 2$

**d**  $3x - y + 5 = 0$   
 $x + 3y = 1$

**e**  $2x + 5y - 1 = 0$   
 $y = 2x + 7$

**f**  $2x - y = 6$   
 $6x - 3y + 3 = 0$

2 Find the equation of the line parallel to each of the given lines and which passes through each of the given points.

**a**  $y = 3x + 1$  (3, 2)

**b**  $y = 3 - 2x$  (1, 3)

**c**  $2x + 4y + 3 = 0$  (6, -3)

**d**  $2y - 3x + 2 = 0$  (8, 20)

## Answers

- 1**   **a**   Parallel                      **b**   Neither                      **c**   Perpendicular  
      **d**   Perpendicular                **e**   Neither                      **f**   Parallel
- 2**   **a**    $y = 3x - 7$                       **b**    $y = -2x + 5$   
      **c**    $y = -\frac{1}{2}x$                         **d**    $y = \frac{3}{2}x + 8$