



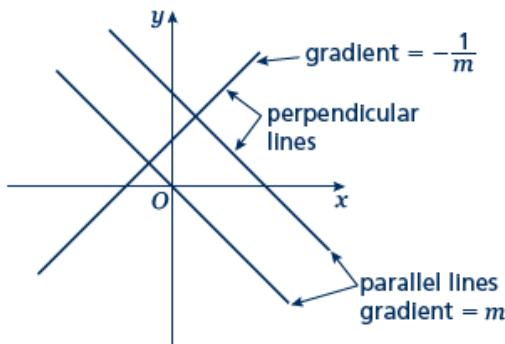
Perpendicular and parallel lines

A LEVEL LINKS

Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- When lines are parallel they have the same gradient.
- A line perpendicular to the line with equation $y = mx + c$ has gradient $-\frac{1}{m}$.



Examples

Example 1 Find the equation of the line parallel to $y = 2x + 4$ which passes through the point $(4, 9)$.

$$\begin{aligned}y &= 2x + 4 \\m &= 2 \\y &= 2x + c \\9 &= 2 \times 4 + c \\9 &= 8 + c \\c &= 1 \\y &= 2x + 1\end{aligned}$$

- As the lines are parallel they have the same gradient.
- Substitute $m = 2$ into the equation of a straight line $y = mx + c$.
- Substitute the coordinates into the equation $y = 2x + c$
- Simplify and solve the equation.
- Substitute $c = 1$ into the equation $y = 2x + c$

Example 2 Find the equation of the line perpendicular to $y = 2x - 3$ which passes through the point $(-2, 5)$.

$$\begin{aligned}y &= 2x - 3 \\m &= 2 \\-\frac{1}{m} &= -\frac{1}{2} \\y &= -\frac{1}{2}x + c \\5 &= -\frac{1}{2} \times (-2) + c \\5 &= 1 + c \\c &= 4 \\y &= -\frac{1}{2}x + 4\end{aligned}$$

- As the lines are perpendicular, the gradient of the perpendicular line is $-\frac{1}{m}$.
- Substitute $m = -\frac{1}{2}$ into $y = mx + c$.
- Substitute the coordinates $(-2, 5)$ into the equation $y = -\frac{1}{2}x + c$
- Simplify and solve the equation.
- Substitute $c = 4$ into $y = -\frac{1}{2}x + c$.

Example 3 A line passes through the points $(0, 5)$ and $(9, -1)$.

Find the equation of the line which is perpendicular to the line and passes through its midpoint.

$$x_1 = 0, x_2 = 9, y_1 = 5 \text{ and } y_2 = -1$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 5}{9 - 0}$$

$$= \frac{-6}{9} = -\frac{2}{3}$$

$$-\frac{1}{m} = \frac{3}{2}$$

$$y = \frac{3}{2}x + c$$

$$\text{Midpoint} = \left(\frac{0+9}{2}, \frac{5+(-1)}{2} \right) = \left(\frac{9}{2}, 2 \right)$$

$$2 = \frac{3}{2} \times \frac{9}{2} + c$$

$$c = -\frac{19}{4}$$

$$y = \frac{3}{2}x - \frac{19}{4}$$

1 Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line.

2 As the lines are perpendicular, the gradient of the perpendicular line is $-\frac{1}{m}$.

3 Substitute the gradient into the equation $y = mx + c$.

4 Work out the coordinates of the midpoint of the line.

5 Substitute the coordinates of the midpoint into the equation.

6 Simplify and solve the equation.

7 Substitute $c = -\frac{19}{4}$ into the equation

$$y = \frac{3}{2}x + c$$
.

Practice questions

1 Work out whether these pairs of lines are parallel, perpendicular or neither.

a $y = 2x + 3$
 $y = 2x - 7$

b $y = 3x$
 $2x + y - 3 = 0$

c $y = 4x - 3$
 $4y + x = 2$

d $3x - y + 5 = 0$
 $x + 3y = 1$

e $2x + 5y - 1 = 0$
 $y = 2x + 7$

f $2x - y = 6$
 $6x - 3y + 3 = 0$

2 Find the equation of the line parallel to each of the given lines and which passes through each of the given points.

a $y = 3x + 1$ (3, 2)
c $2x + 4y + 3 = 0$ (6, -3)

b $y = 3 - 2x$ (1, 3)
d $2y - 3x + 2 = 0$ (8, 20)

Answers

1	a	Parallel	b	Neither	c	Perpendicular
	d	Perpendicular	e	Neither	f	Parallel
2	a	$y = 3x - 7$	b	$y = -2x + 5$		
	c	$y = -\frac{1}{2}x$	d	$y = \frac{3}{2}x + 8$		