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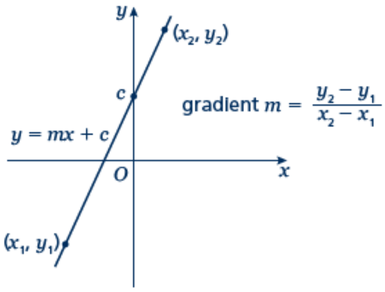
Distances and areas

A LEVEL LINKS
Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- A straight line has the equation $y = mx + c$, where m is the gradient and c is the y-intercept (where $x = 0$).
- The equation of a straight line can be written in the form $ax + by + c = 0$, where a, b and c are integers.
- When given the coordinates (x_1, y_1) and (x_2, y_2) of two points on a line the gradient is calculated using the formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$



Example 1 A straight line has gradient $-\dfrac{1}{2}$ and y-intercept 3.

Write the equation of the line in the form $ax + by + c = 0$.

$m = -\dfrac{1}{2}$ and $c = 3$ So $y = -\dfrac{1}{2}x + 3$ $\dfrac{1}{2}x + y - 3 = 0$ $x + 2y - 6 = 0$	**1** A straight line has equation $y = mx + c$. Substitute the gradient and y-intercept given in the question into this equation. **2** Rearrange the equation so all the terms are on one side and 0 is on the other side. **3** Multiply both sides by 2 to eliminate the denominator.

Example 2 Find the gradient and the y-intercept of the line with the equation $3y - 2x + 4 = 0$.

$3y - 2x + 4 = 0$ $3y = 2x - 4$ $y = \dfrac{2}{3}x - \dfrac{4}{3}$ Gradient $= m = \dfrac{2}{3}$ y-intercept $= c = -\dfrac{4}{3}$	**1** Make y the subject of the equation. **2** Divide all the terms by three to get the equation in the form $y = \ldots$ **3** In the form $y = mx + c$, the gradient is m and the y-intercept is c.

Example 3 Find the equation of the line which passes through the point (5, 13) and has gradient 3.

$m = 3$ $y = 3x + c$	**1** Substitute the gradient given in the question into the equation of a straight line $y = mx + c$.
$13 = 3 \times 5 + c$	**2** Substitute the coordinates $x = 5$ and $y = 13$ into the equation.
$13 = 15 + c$ $c = -2$	**3** Simplify and solve the equation.
$y = 3x - 2$	**4** Substitute $c = -2$ into the equation $y = 3x + c$

Example 4 Find the equation of the line passing through the points with coordinates (2, 4) and (8, 7).

$x_1 = 2,\ x_2 = 8,\ y_1 = 4$ and $y_2 = 7$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-4}{8-2} = \frac{3}{6} = \frac{1}{2}$	**1** Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line.
$y = \frac{1}{2}x + c$	**2** Substitute the gradient into the equation of a straight line $y = mx + c$.
$4 = \frac{1}{2} \times 2 + c$	**3** Substitute the coordinates of either point into the equation.
$c = 3$	**4** Simplify and solve the equation.
$y = \frac{1}{2}x + 3$	**5** Substitute $c = 3$ into the equation $y = \frac{1}{2}x + c$

Practice questions

1 The line l_1 passes through the point (9, –4) and has gradient $\frac{1}{3}$.

(*a*) Find an equation for l_1 in the form $ax + by + c = 0$, where a, b and c are integers.

The line l_2 passes through the origin O and has gradient –2. The lines l_1 and l_2 intersect at the point P.

(*b*) Calculate the coordinates of P.

Given that l_1 crosses the y-axis at the point C,

(*c*) calculate the exact area of $\triangle OCP$.

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2 (*a*) Find an equation of the line joining $A(7, 4)$ and $B(2, 0)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(*b*) Find the length of AB, leaving your answer in surd form.

The point C has coordinates $(2, t)$, where $t > 0$, and $AC = AB$.

(*c*) Find the value of t.

(*d*) Find the area of triangle ABC.

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Answers

1 (a) $y-(-4)=\frac{1}{3}(x-9)$ or $\frac{y-(-4)}{x-9}=\frac{1}{3}$

$3y-x+21=0$

(b) Equation of l_2 is: $y=-2x$ (o.e.)

Solving l_1 and l_2 : $-6x-x+21=0$

p is point where $x_p=3$, $y_p=-6$

(c) (l_1 is $y=\frac{1}{3}x-7$) C is (0, -7) or OC = 7

Area of $\Delta OCP=\frac{1}{2}OC\times x_p$, $=\frac{1}{2}\times 7\times 3=10.5$ or $\frac{21}{2}$

2 (a) $m_{AB}=\frac{4-0}{7-2}$ $\left(=\frac{4}{5}\right)$

Equation of AB is: $y-0=\frac{4}{5}(x-2)$ or $y-4=\frac{4}{5}(x-7)$

$4x-5y-8=0$

(b) $(AB=)\sqrt{(7-2)^2+(4-0)^2}$

$=\sqrt{41}$

(c) Using isos triangle with $AB=AC$ then $t=2\times y_A=2\times 4=8$

(d) Area of triangle $=\frac{1}{2}t\times(7-2)$

$=$ 20