

Equation of a straight line

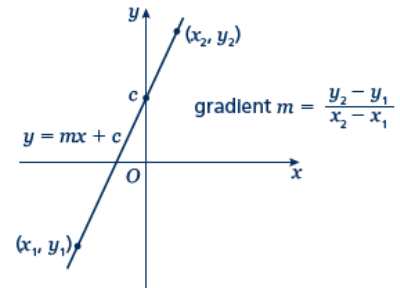
A LEVEL LINKS

Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- A straight line has the equation $y = mx + c$, where m is the gradient and c is the y -intercept (where $x = 0$).
- The equation of a straight line can be written in the form $ax + by + c = 0$, where a , b and c are integers.
- When given the coordinates (x_1, y_1) and (x_2, y_2) of two points on a line the gradient is calculated using the

$$\text{formula } m = \frac{y_2 - y_1}{x_2 - x_1}$$



Example 1 A straight line has gradient $-\frac{1}{2}$ and y -intercept 3.

Write the equation of the line in the form $ax + by + c = 0$.

$m = -\frac{1}{2} \text{ and } c = 3$ $\text{So } y = -\frac{1}{2}x + 3$ $\frac{1}{2}x + y - 3 = 0$ $x + 2y - 6 = 0$	<ol style="list-style-type: none"> 1 A straight line has equation $y = mx + c$. Substitute the gradient and y-intercept given in the question into this equation. 2 Rearrange the equation so all the terms are on one side and 0 is on the other side. 3 Multiply both sides by 2 to eliminate the denominator.
--	---

Example 2 Find the gradient and the y -intercept of the line with the equation $3y - 2x + 4 = 0$.

$3y - 2x + 4 = 0$ $3y = 2x - 4$ $y = \frac{2}{3}x - \frac{4}{3}$ $\text{Gradient} = m = \frac{2}{3}$ $y\text{-intercept} = c = -\frac{4}{3}$	<ol style="list-style-type: none"> 1 Make y the subject of the equation. 2 Divide all the terms by three to get the equation in the form $y = \dots$ 3 In the form $y = mx + c$, the gradient is m and the y-intercept is c.
--	---

Example 3 Find the equation of the line which passes through the point (5, 13) and has gradient 3.

$m = 3$ $y = 3x + c$ $13 = 3 \times 5 + c$ $13 = 15 + c$ $c = -2$ $y = 3x - 2$	<ol style="list-style-type: none"> 1 Substitute the gradient given in the question into the equation of a straight line $y = mx + c$. 2 Substitute the coordinates $x = 5$ and $y = 13$ into the equation. 3 Simplify and solve the equation. 4 Substitute $c = -2$ into the equation $y = 3x + c$
--	---

Example 4 Find the equation of the line passing through the points with coordinates (2, 4) and (8, 7).

$x_1 = 2, x_2 = 8, y_1 = 4 \text{ and } y_2 = 7$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 4}{8 - 2} = \frac{3}{6} = \frac{1}{2}$ $y = \frac{1}{2}x + c$ $4 = \frac{1}{2} \times 2 + c$ $c = 3$ $y = \frac{1}{2}x + 3$	<ol style="list-style-type: none"> 1 Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line. 2 Substitute the gradient into the equation of a straight line $y = mx + c$. 3 Substitute the coordinates of either point into the equation. 4 Simplify and solve the equation. 5 Substitute $c = 3$ into the equation $y = \frac{1}{2}x + c$
--	---

Practice questions

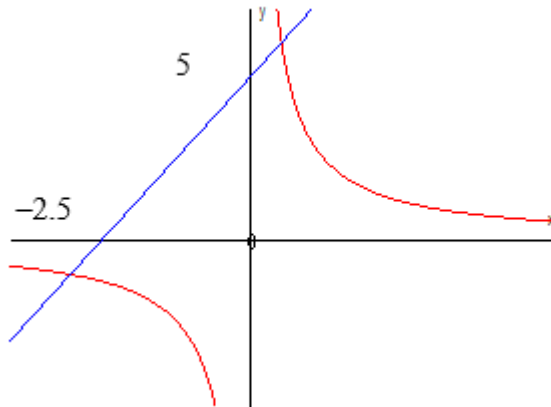
1 The curve C has equation $y = \frac{3}{x}$ and the line l has equation $y = 2x + 5$.

(a) Sketch the graphs of C and l , indicating clearly the coordinates of any intersections with the axes.

(b) Find the coordinates of the points of intersection of C and l .

Answers

1 (a)



(b) $2x + 5 = \frac{3}{x}$

$$2x^2 + 5x - 3 = 0 \quad \text{or} \quad 2x^2 + 5x = 3$$

$$(2x - 1)(x + 3) = 0$$

$$x = -3 \quad \text{or} \quad \frac{1}{2}$$

$$y = \frac{3}{-3} \quad \text{or} \quad 2 \times (-3) + 5 \quad \text{or} \quad y = \frac{3}{\frac{1}{2}} \quad \text{or} \quad 2 \times \left(\frac{1}{2}\right) + 5$$

Points are $(-3, -1)$ and $(\frac{1}{2}, 6)$