

# Simplifying algebraic fractions

## A LEVEL LINKS

**Scheme of work:** 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

**Example 1** Factorise  $x^2 + 3x - 10$

$$b = 3, ac = -10$$

$$\begin{aligned} \text{So } x^2 + 3x - 10 &= x^2 + 5x - 2x - 10 \\ &= x(x + 5) - 2(x + 5) \\ &= (x + 5)(x - 2) \end{aligned}$$

- 1 Work out the two factors of  $ac = -10$  which add to give  $b = 3$  (5 and -2)
- 2 Rewrite the  $b$  term ( $3x$ ) using these two factors
- 3 Factorise the first two terms and the last two terms
- 4 ( $x + 5$ ) is a factor of both terms

**Example 2** Factorise  $6x^2 - 11x - 10$

$$b = -11, ac = -60$$

$$\begin{aligned} \text{So } 6x^2 - 11x - 10 &= 6x^2 - 15x + 4x - 10 \\ &= 3x(2x - 5) + 2(2x - 5) \\ &= (2x - 5)(3x + 2) \end{aligned}$$

- 1 Work out the two factors of  $ac = -60$  which add to give  $b = -11$  (-15 and 4)
- 2 Rewrite the  $b$  term ( $-11x$ ) using these two factors
- 3 Factorise the first two terms and the last two terms
- 4 ( $2x - 5$ ) is a factor of both terms

**Example 3** Factorise  $4x^2 - 25y^2$

$$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$$

This is the difference of two squares as the two terms can be written as  $(2x)^2$  and  $(5y)^2$



**Example 4** Simplify  $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$

$$\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$$

For the numerator:

$$b = -4, ac = -21$$

So

$$x^2 - 4x - 21 = x^2 - 7x + 3x - 21$$

$$= x(x - 7) + 3(x - 7)$$

$$= (x - 7)(x + 3)$$

For the denominator:

$$b = 9, ac = 18$$

So

$$2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$$

$$= 2x(x + 3) + 3(x + 3)$$

$$= (x + 3)(2x + 3)$$

So

$$\frac{x^2 - 4x - 21}{2x^2 + 9x + 9} = \frac{(x - 7)(x + 3)}{(x + 3)(2x + 3)}$$

$$= \frac{x - 7}{2x + 3}$$

**1** Factorise the numerator and the denominator

**2** Work out the two factors of  $ac = -21$  which add to give  $b = -4$  (-7 and 3)

**3** Rewrite the  $b$  term ( $-4x$ ) using these two factors

**4** Factorise the first two terms and the last two terms

**5**  $(x - 7)$  is a factor of both terms

**6** Work out the two factors of  $ac = 18$  which add to give  $b = 9$  (6 and 3)

**7** Rewrite the  $b$  term ( $9x$ ) using these two factors

**8** Factorise the first two terms and the last two terms

**9**  $(x + 3)$  is a factor of both terms

**10**  $(x + 3)$  is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1

## Practice questions

1 Simplify the algebraic fractions.

a 
$$\frac{2x^2 + 4x}{x^2 - x}$$

c 
$$\frac{x^2 - 2x - 8}{x^2 - 4x}$$

e 
$$\frac{x^2 - x - 12}{x^2 - 4x}$$

b 
$$\frac{x^2 + 3x}{x^2 + 2x - 3}$$

d 
$$\frac{x^2 - 5x}{x^2 - 25}$$

f 
$$\frac{2x^2 + 14x}{2x^2 + 4x - 70}$$

2 Simplify

a 
$$\frac{9x^2 - 16}{3x^2 + 17x - 28}$$

c 
$$\frac{4 - 25x^2}{10x^2 - 11x - 6}$$

b 
$$\frac{2x^2 - 7x - 15}{3x^2 - 17x + 10}$$

d 
$$\frac{6x^2 - x - 1}{2x^2 + 7x - 4}$$

## Extend

3 Simplify  $\sqrt{x^2 + 10x + 25}$

4 Simplify 
$$\frac{(x+2)^2 + 3(x+2)^2}{x^2 - 4}$$

## Answers

$$1 - a \frac{2(x+2)}{x-1} \quad b \quad \frac{x}{x-1}$$

$$\mathbf{c} \quad \frac{x+2}{x} \qquad \qquad \mathbf{d} \quad \frac{x}{x+5}$$

$$\mathbf{e} \quad \frac{x+3}{x} \qquad \qquad \mathbf{f} \quad \frac{x}{x-5}$$

$$2 \quad \mathbf{a} \quad \frac{3x+4}{x+7} \quad \mathbf{b} \quad \frac{2x+3}{3x-2}$$

$$\mathbf{c} \quad \frac{2-5x}{2x-3} \qquad \mathbf{d} \quad \frac{3x+1}{x+4}$$

$$3(x + 5)$$

$$4 \quad \frac{4(x+2)}{x-2}$$