

Simplifying algebraic fractions

A LEVEL LINKS

Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Example 1 Factorise $x^2 + 3x - 10$

$b = 3, ac = -10$ <p>So $x^2 + 3x - 10 = x^2 + 5x - 2x - 10$</p> $= x(x + 5) - 2(x + 5)$ $= (x + 5)(x - 2)$	<ol style="list-style-type: none"> 1 Work out the two factors of $ac = -10$ which add to give $b = 3$ (5 and -2) 2 Rewrite the b term ($3x$) using these two factors 3 Factorise the first two terms and the last two terms 4 $(x + 5)$ is a factor of both terms
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Example 2 Factorise $6x^2 - 11x - 10$

$b = -11, ac = -60$ <p>So</p> $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$ $= 3x(2x - 5) + 2(2x - 5)$ $= (2x - 5)(3x + 2)$	<ol style="list-style-type: none"> 1 Work out the two factors of $ac = -60$ which add to give $b = -11$ (-15 and 4) 2 Rewrite the b term ($-11x$) using these two factors 3 Factorise the first two terms and the last two terms 4 $(2x - 5)$ is a factor of both terms
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Example 3 Factorise $4x^2 - 25y^2$

$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$	<p>This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$</p>
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Example 4 Simplify $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$

$\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$ <p>For the numerator: $b = -4, ac = -21$</p> <p>So $x^2 - 4x - 21 = x^2 - 7x + 3x - 21$ $= x(x - 7) + 3(x - 7)$ $= (x - 7)(x + 3)$</p> <p>For the denominator: $b = 9, ac = 18$</p> <p>So $2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$ $= 2x(x + 3) + 3(x + 3)$ $= (x + 3)(2x + 3)$</p> <p>So $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9} = \frac{(x - 7)(x + 3)}{(x + 3)(2x + 3)}$ $= \frac{x - 7}{2x + 3}$</p>	<ol style="list-style-type: none"> 1 Factorise the numerator and the denominator 2 Work out the two factors of $ac = -21$ which add to give $b = -4$ (-7 and 3) 3 Rewrite the b term ($-4x$) using these two factors 4 Factorise the first two terms and the last two terms 5 $(x - 7)$ is a factor of both terms 6 Work out the two factors of $ac = 18$ which add to give $b = 9$ (6 and 3) 7 Rewrite the b term ($9x$) using these two factors 8 Factorise the first two terms and the last two terms 9 $(x + 3)$ is a factor of both terms 10 $(x + 3)$ is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1
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Practice questions

1 Simplify the algebraic fractions.

a $\frac{2x^2 + 4x}{x^2 - x}$

c $\frac{x^2 - 2x - 8}{x^2 - 4x}$

e $\frac{x^2 - x - 12}{x^2 - 4x}$

b $\frac{x^2 + 3x}{x^2 + 2x - 3}$

d $\frac{x^2 - 5x}{x^2 - 25}$

f $\frac{2x^2 + 14x}{2x^2 + 4x - 70}$

2 Simplify

a $\frac{9x^2 - 16}{3x^2 + 17x - 28}$

c $\frac{4 - 25x^2}{10x^2 - 11x - 6}$

b $\frac{2x^2 - 7x - 15}{3x^2 - 17x + 10}$

d $\frac{6x^2 - x - 1}{2x^2 + 7x - 4}$

Extend

3 Simplify $\sqrt{x^2 + 10x + 25}$

4 Simplify $\frac{(x+2)^2 + 3(x+2)^2}{x^2 - 4}$

Answers

1 a $\frac{2(x+2)}{x-1}$

c $\frac{x+2}{x}$

e $\frac{x+3}{x}$

b $\frac{x}{x-1}$

d $\frac{x}{x+5}$

f $\frac{x}{x-5}$

2 a $\frac{3x+4}{x+7}$

c $\frac{2-5x}{2x-3}$

b $\frac{2x+3}{3x-2}$

d $\frac{3x+1}{x+4}$

3 $(x+5)$

4 $\frac{4(x+2)}{x-2}$